Title: PSI 2017/2018 - Quantum Integrable Models - Lecture 12

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Abstract:

GN model

$$\psi_i \partial_{\bar{w}} \psi_i + \bar{\psi}_i \partial_w \bar{\psi}_i + \frac{g}{N} \psi_i \bar{\psi}_i \psi_j \bar{\psi}_j$$

This model has a $\mathbb{Z}/2$ symmetry

$$\psi_i \longrightarrow -\psi_i$$

$$\bar{\psi}_i \longrightarrow \bar{\psi}_i$$

Mass term is $\psi_i \bar{\psi}_i$ – not $\mathbb{Z}/2$ symmetric.

Claim of Gross-Neveu

As $N \to \infty$, the $\mathbb{Z}/2$ symmetry is **spontaneously** broken

The vacuum is not $\mathbb{Z}/2$ symmetric

In this vacuum, the theory is massive.

Mass term is $\psi_i \bar{\psi}_i$ – <u>not</u> $\mathbb{Z}/2$ symmetric.

To understand this, we'll introduce σ a scalar

And rewrite the Lagrangian as

$$\psi_i \partial_{\bar{w}} \psi_i + \bar{\psi}_i \partial_w \bar{\psi}_i + \sigma\left(\sum \psi_i \bar{\psi}_i\right) + \frac{N}{g}\sigma^2$$

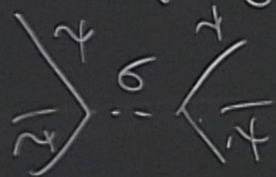
By completing the square

$$\sigma \longrightarrow \sigma - \sum \psi_i \bar{\psi}_i$$

we get the previous Lagrangian $+ \sigma^2$

Alternatively: $\int$ out σ

Propagator is $\frac{g}{N}\,\delta$



gives $\frac{g}{N}\sum \psi_i\bar{\psi}_i\,\psi_j\bar{\psi}_j$

$\mathbb{Z}/2$ symmetry sends $\sigma \longrightarrow -\sigma$.

We'll find in the lowest energy state

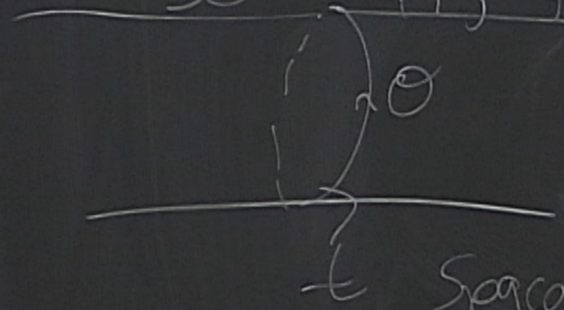
$\langle \sigma \rangle$ is not zero, but is some c

$\sigma = c + \sigma_0$ Perturbing around $\sigma = c$, $\sigma_0 = 0$

The Lagrangian will include the term

$$c \sum \psi_i \bar{\psi}_i$$

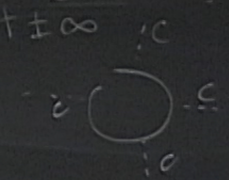
So $\psi_i, \bar{\psi}_i$ have a mass



Space-time = a cylinder

$$\log \int_{\substack{\sigma \to c \\ \text{at } \pm\infty}} e^{i\mathcal{L}(\sigma,\psi,\bar{\psi})} = \log \int_{\substack{\sigma \to 0 \\ \text{at } \pm\infty}} e^{i\mathcal{L}(\sigma + c,\psi,\bar{\psi})}$$

$$= \sum$$



connected diagrams
with external lines
at which we place $\sigma = c$

Feynman rules

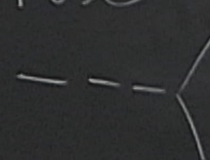
- - - - σ

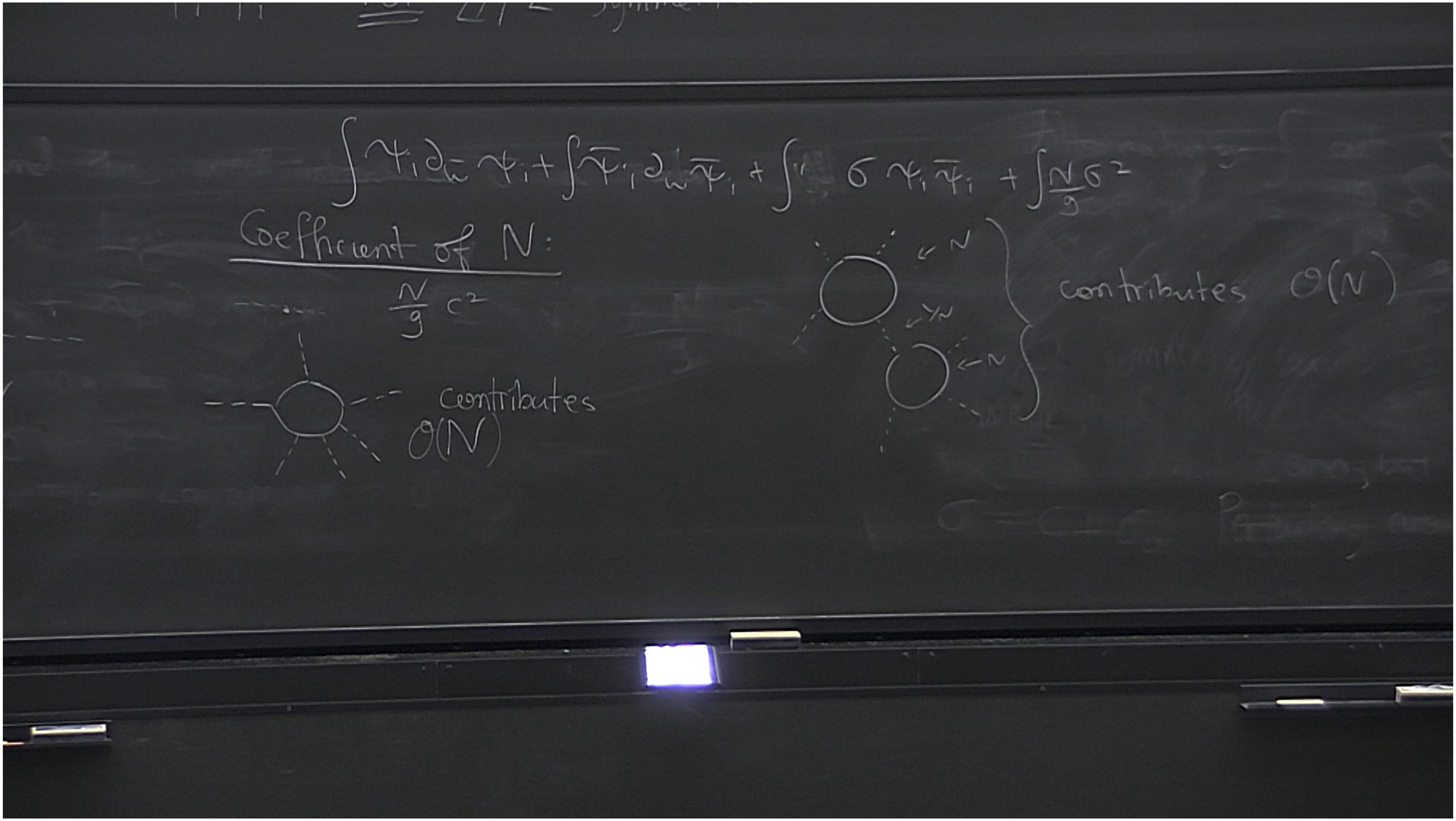
—— $\psi, \bar{\psi}$

σ - propagator - - - - -

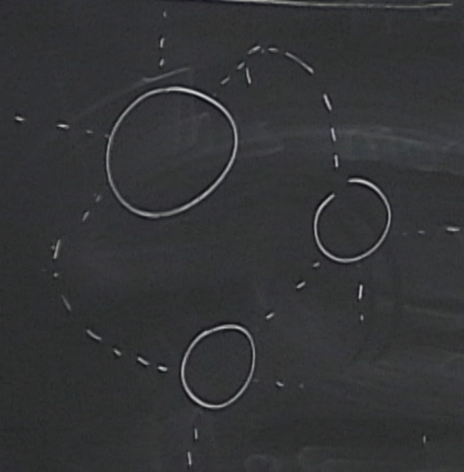
comes with $1/N$

Vertex




$\int \psi_i \partial_{\bar{w}} \psi_i + \int \bar{\psi}_i \partial_w \bar{\psi}_i + \int \sigma \psi_i \bar{\psi}_i + \int \frac{N}{g} \sigma^2$
Coefficient of N:
$\frac{N}{g} c^2$
contributes $O(N)$
$\leftarrow N$
$\leftarrow 1/N$
$\leftarrow N$
contributes $O(N)$

Next diagram



N each circle $\bigcirc$

$\frac{1}{N}$ each σ - propagator

$\mathcal{O}(N^0)$

We find, the most general $\mathcal{O}(N)$ diagram is a tree whose vertices are wheels  and edges are σ-propagators.

Trees w more than one vertex contribute 0.

E.g.

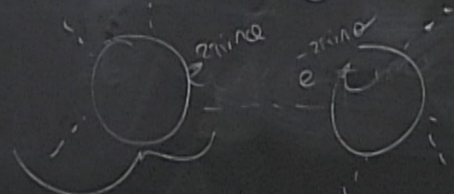
Note integrating over $\sigma = \sigma_0 + c$, $\sigma_0 \longrightarrow 0$ at $\pm\infty$

If we Fourier transform σ_0 in the θ-direction

$$\sigma_0 = \sum_{n \neq 0} \lambda_n e^{2\pi i n \theta}$$

Propagator involves

$$e^{2\pi i n \theta} \otimes e^{-2\pi i n \theta}$$



O_i as not inv. under rotation of θ

Next, we'll compute
the amplitudes

For simplicity, go back to
Euclidean signature and work
on a torus.

$w \sim w+1$

$w \sim w+i$

Also, make our fermions

anti-periodic under $w \to w+1$

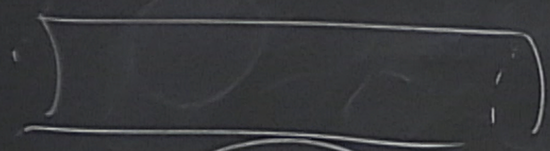
periodic " $w \to w+i$

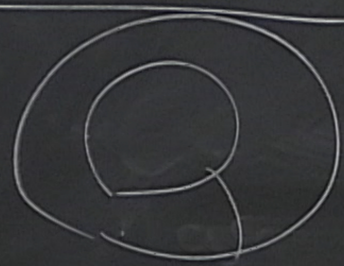
Space of fields.

$$\psi_i = \sum_{n \text{ odd}} c_{n,m} e^{\pi i n \theta_1} e^{2\pi i m \theta_2}$$

$\bar{\psi}_i$

Path integral is only over the non-constant fluctuations of σ.

$\langle c|$  $|c\rangle$

$=$ 

c

We need to calculate $\psi, \overline{\psi}$ propagators.

If $F_{n,m} = e^{\pi i n \theta_1} e^{2\pi i m \theta_2}$

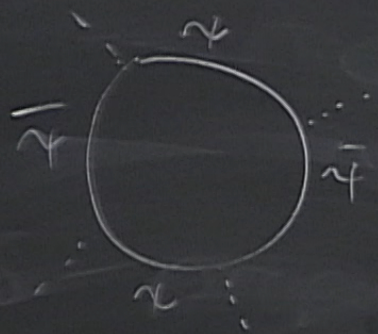
$$\partial_{\bar{w}} F_{n,m} = (n - im) F_{n,m}$$

$$\partial_{w} F_{n,m} = (n + im) F_{n,m}$$

ψ - propagator is $\partial_{\bar{w}}^{-1} : F_{n,m} \longrightarrow \frac{1}{n - im} F_{n,m}$

$\overline{\psi}$ " $\partial_{w}^{-1} : F_{n,m} \longrightarrow \frac{1}{n + im} F_{n,m}$

So, a diagram like



$$\longrightarrow \frac{c^4}{2}\,\mathrm{Tr}\;\partial_{\bar w}^{-1}\partial_w^{-1}\partial_{\bar w}^{-1}\partial_w^{-1}$$

$$= \frac{1}{2}\sum_{\substack{n,m \\ n\ \text{odd}}} c^4 \frac{1}{(n^2+m^2)^2}$$

$2k$ vertices $\Longrightarrow \frac{1}{k}$ symmetry factor.

$\bar{\psi}$ " $\quad \partial_w^{-1} : F_{n,m} \to \frac{1}{n+im} F_{n,m}$

Summing all diagrams, and including the $\frac{N}{g}c^2$ factor we had, we get

$$\frac{N}{g}c^2 + N\sum_{k>0}\sum_{n,m}\frac{1}{k}\frac{c^{2k}}{\left(\frac{n^2}{r^2}+\frac{m^2}{r^2}\right)^k}$$

$$W(c) = \frac{N}{g}c^2 + N\log\left(1 + c^2\sum_{\substack{n,m\\ n\neq 0}}\frac{1}{n^2+m^2}\right)$$

Solve $W'(c) = 0$

Insert $\sigma = \sigma_0 + c$, c solves this eq^n. $\quad \sigma\psi_i\bar{\psi}_i \Rightarrow \underbrace{c\,\psi_i\bar{\psi}_i}_{\text{mass term}}$