Title: PSI 2017/2018 - Quantum Integrable Models - Lecture 11

Date: Apr 03, 2018 11:30 AM

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Abstract:

Massive Integrable Theories

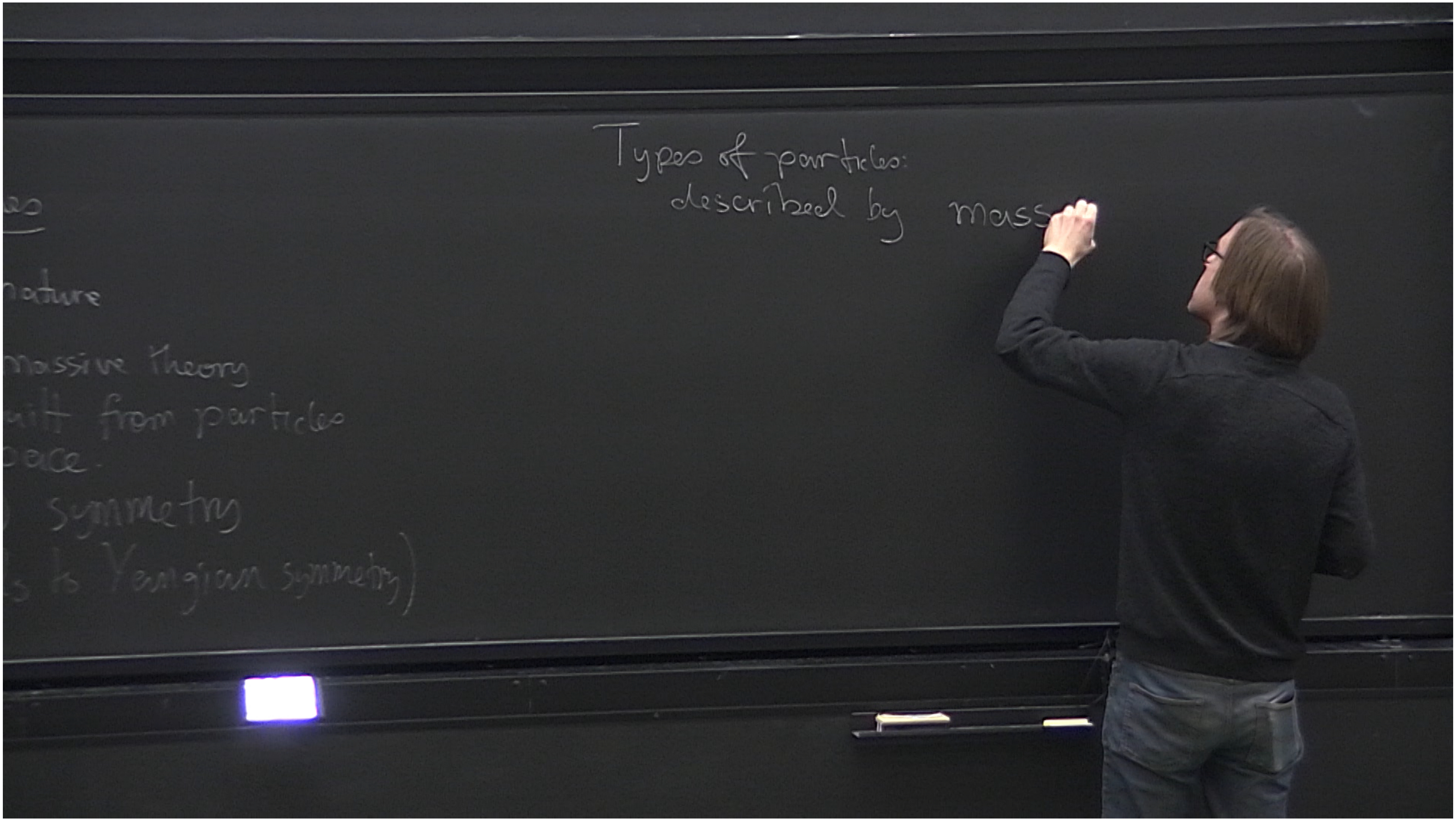
2d, Lorentzian signature

Assume we have a massive theory

- Hilbert space is built from particles localized in space.

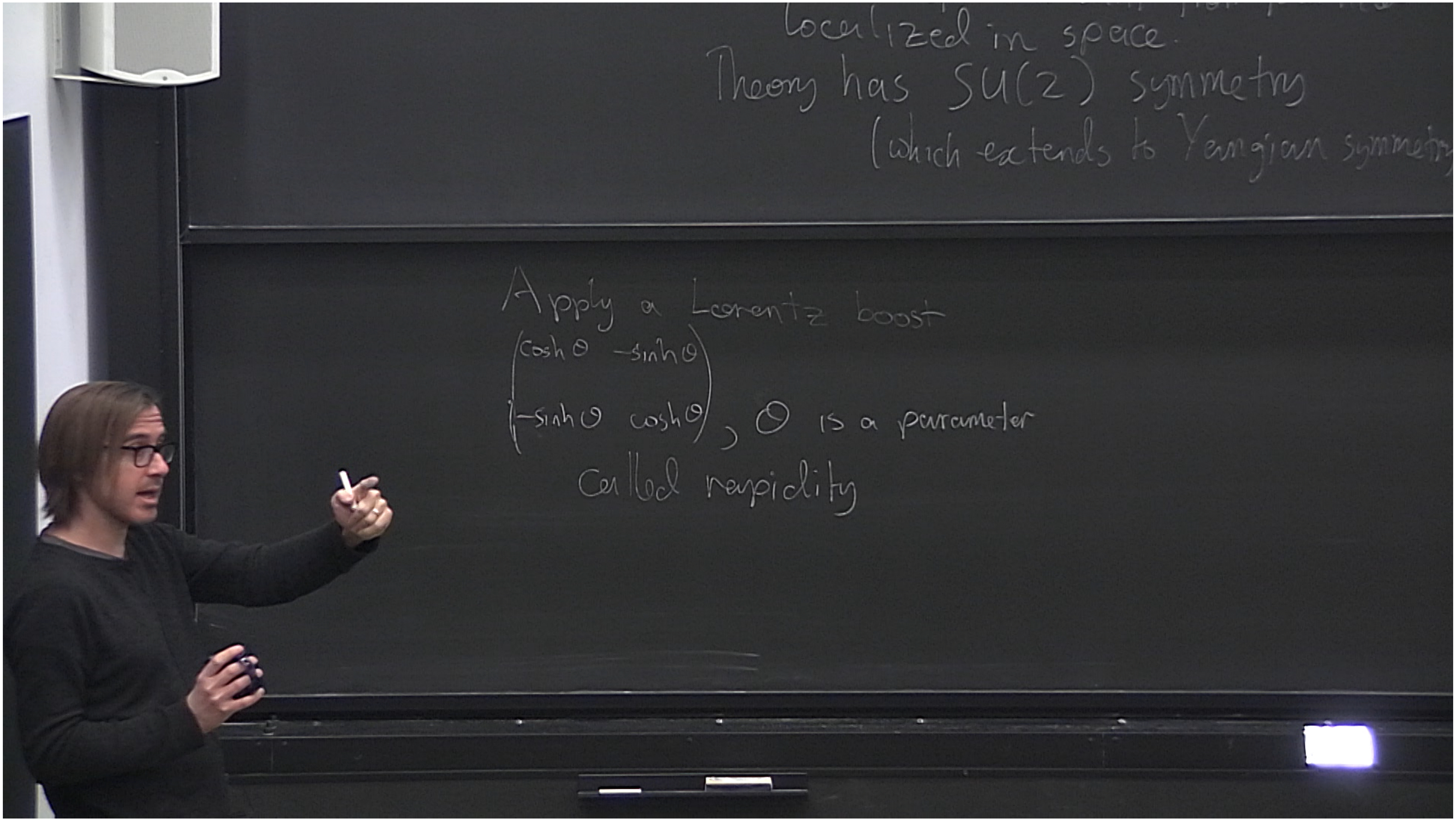
Theory has SU(2) symmetry

(which extends to Yangian symmetry)


Types of particles:
described by mass
nature
massive theory
uilt from particles
pace.
symmetry
s to Yangian symmetry)

Types of particles:
- described by mass m
- And by spin under $SU(2)$ global symmetry.

etry)


localized in space.
Theory has SU(2) symmetry
(which extends to Yangian symmetr
Apply a Lorentz boost
cosh θ -sinh θ
-sinh θ cosh θ
, θ is a parameter
called rapidity

...pace.

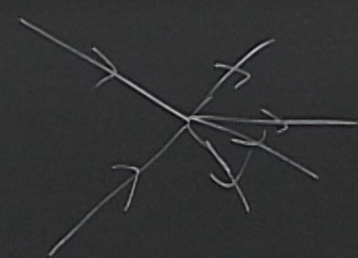
...2) symmetry

...ends to Yangian symmetry)

$p = (p_0, p_1) = (m, 0)$

$\Rightarrow$ momentum is $\begin{pmatrix} m\cosh\theta \\ -m\sinh\theta \end{pmatrix}$

Suppose we have 2 particles of masses m_i, SU(2) spins j_i, rapidities θ_i



They can scatter to produce some number of particles of m_i', θ_i', j_i'

If V_j = spin j rep. of SU(2)

Then the scattering matrix is

$$S : \underbrace{V_{j_1} \otimes V_{j_2}}_{\text{incoming}} \longrightarrow \underbrace{V_{j_1'} \otimes \cdots \otimes V_{j_n'}}_{\text{outgoing}}$$

S must be an intertwiner for $SU(2)$ reps.

[S depends on m_i, rapidities....]

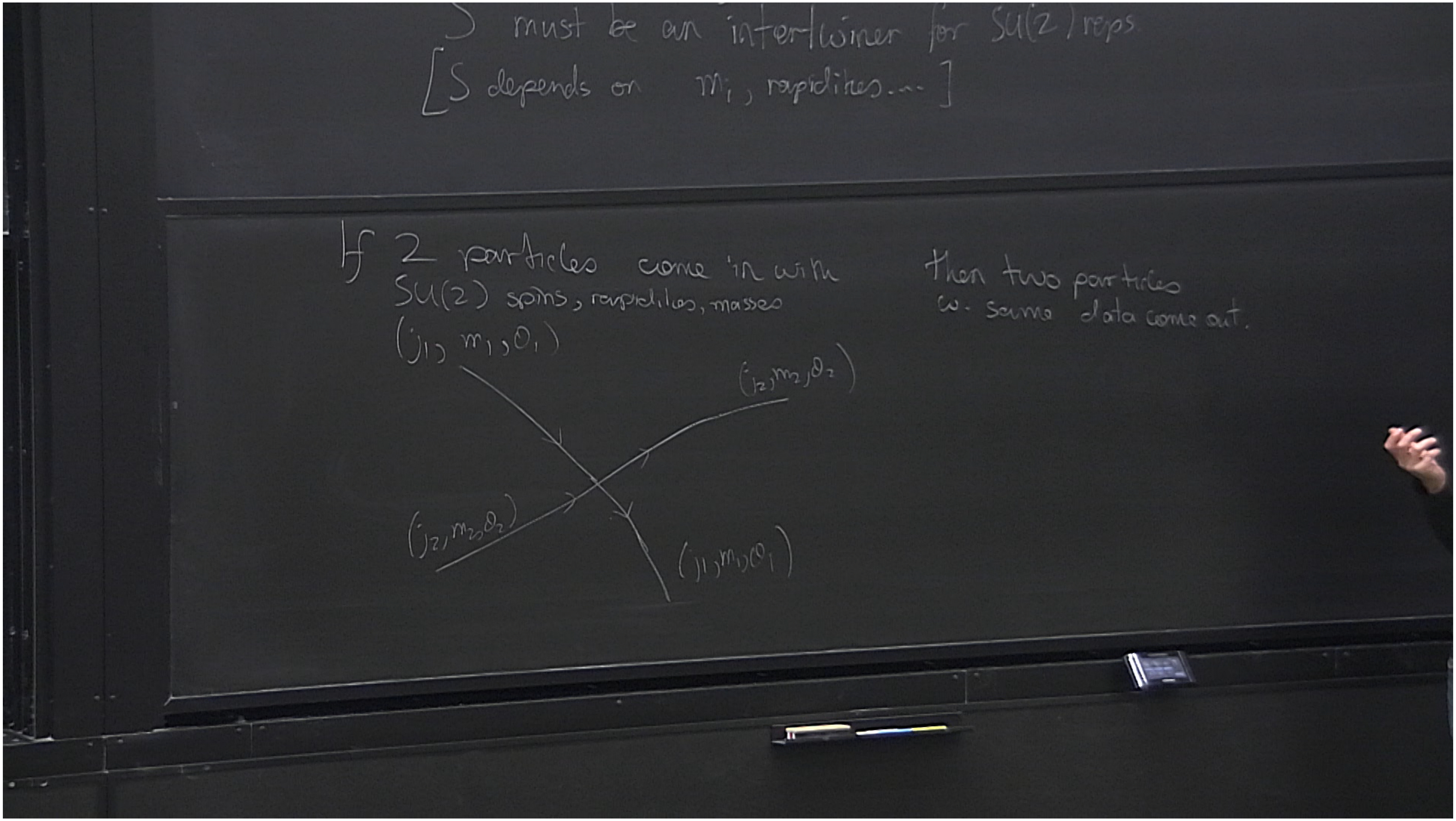
[S depends on m_i, rapidities....]

If ρ_a are SU(2) generators then

$$S\,(\rho_a \otimes 1 + 1 \otimes \rho_a) = (\rho_a \otimes \mathbb{1}^{\otimes n-1} + \mathbb{1} \otimes \rho_a \otimes \mathbb{1}^{\otimes n-2} + \ldots)\,S$$

CLAIM

In an integrable theory, there is no particle production:


S must be an intertwiner for SU(2) reps.
[S depends on mi, rapidities....]
If 2 particles come in with
SU(2) spins, rapidities, masses
(j1, m1, θ1)
then two particles
w. same data come out.
(j2, m2, θ2)
(j2, m2, θ2)
(j1, m1, θ1)

Monodromy Matrix

In perturbation theory we had $M^i_j(z) = P\mathrm{Exp}\,\mathcal{L}(z)$

We will assume the non-perturbative theory has a monodromy matrix $M^i_j(z)$ acting on the Hilbert space, with the following properties.

1) If we apply a Lorentz boost by θ,

then $m^i_j(z) \rightsquigarrow m^i_j(z-\theta)$

2) If we have a single particle at rest in a rep. of spin j, then

$$m^a_b(z) = \frac{1}{z}\,\rho^a_{b;j}$$

1) If we apply a Lorentz boost by θ,

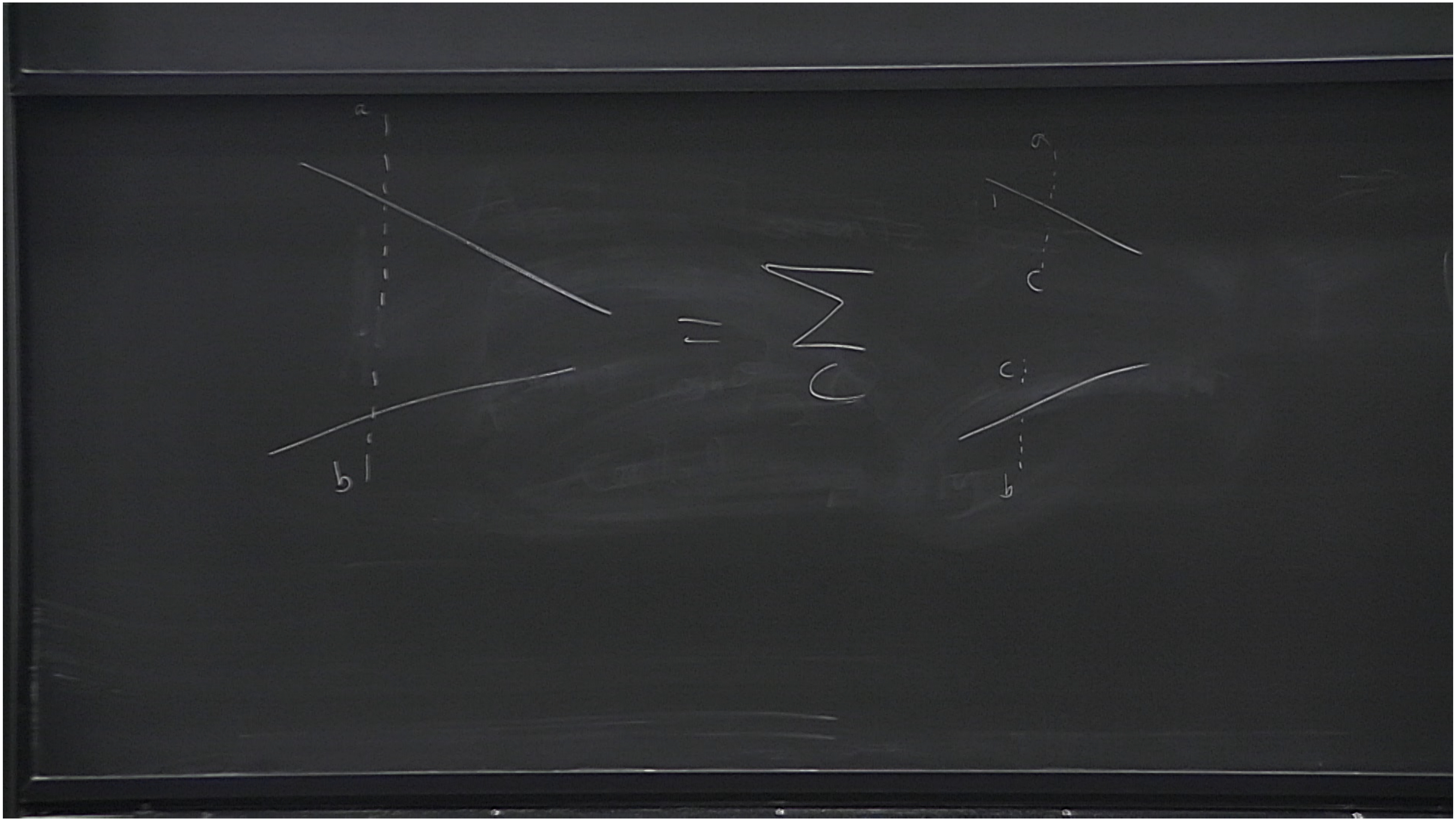
then $m^i_j(z) \rightsquigarrow m^i_j(z-\theta)$

2) If we have a single particle at rest in a rep. of spin j, then

$$m^a_b(z) = \frac{1}{z}\, \rho^a_{b;j}$$ $\leftarrow$ spin j rep. of $su(2)$

3) In a 2 particle rep. particles 1, 2

$$m^a_b(z) = \sum m^a_c(z)_1\, m^c_b(z)_2$$

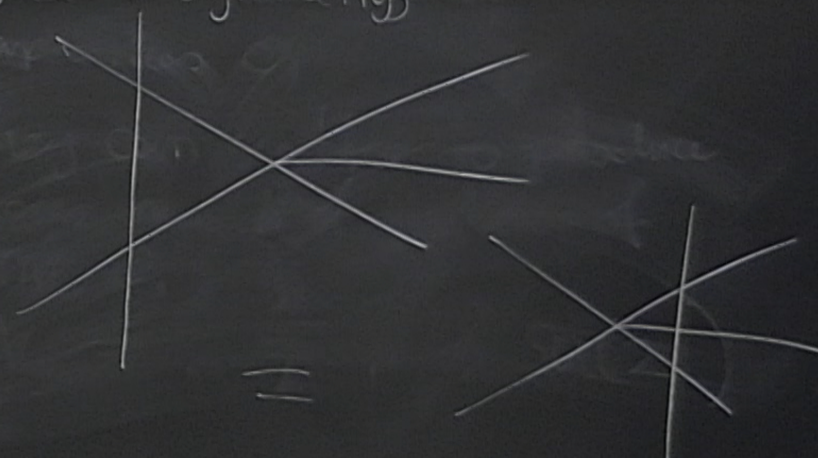

a
b
$= \sum_c$
a
c
c
b

$$m^a_b(z) = \sum m^a_c(z)_1 \, m^c_b(z)_2$$

For the monodromy matrix to be a symmetry,
we must have

$$S m^a_b(z) = m^a_b(z) S$$

for a scattering matrix S



If we have 2 particles of spin $\frac{1}{2}$, rapidities θ_1, θ_2.

Hilbert space is $V_{1/2} \otimes V_{1/2} \simeq \mathbb{C}^2 \otimes \mathbb{C}^2$

Monodromy matrix for the individual particles is

$$\frac{1}{z-\theta_i} \rho^a_b \qquad M^a_b(z) =$$

So, for 2 particles,

$$= \delta^a_b + \frac{1}{z-\theta_1}\rho^a_{b,1} + \frac{1}{z-\theta_2}\rho^a_{b,2} + \frac{1}{(z-\theta_1)(z-\theta_2)}\rho^a_{c,1}\rho^c_{b,2}$$

If we expand $m^a_b(z)$ in powers of $\frac{1}{z}$, we will find

$$\delta^a_b + \frac{1}{z}\underbrace{\left(\rho^a_{b,1} + \rho^a_{b,2}\right)}_{\text{usual action of SU(2) generators}} + \frac{1}{z^2}\underbrace{\left(\theta_1\rho^a_{b,1} + \theta_2\rho^a_{b,2} + \rho^a_{c,1}\rho^c_{b,2}\right)}_{\text{Extra symmetries!}} + O(z^{-3})$$

The statement that

$$m^a_b(z)\, S = S\, m^a_b(z)$$

when expanded in powers of $1/z$, becomes

z^0: trivial

z^{-1}: S is an $SU(2)$ intertwiner

z^{-2}: an interesting new constraint on S.

spin j, then

ve theory

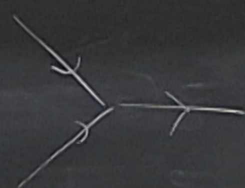
$: \; m^a_b(z) = \frac{1}{z}\rho^a_{b;j} + \delta^a_b$ — ρ: spin j rep. of $su(2)$

3) In a 2 particle rep. particles 1, 2

rties

$$m^a_b(z) = \sum m^a_c(z)_1 \, m^c_b(z)_2$$

Example: 2 particles of spin 1 form a bound state of spin 1



omes

Unique $SU(2)$ symmetric map $S: V_1 \otimes V_1 \to V_1$

S is determined up to scale.

What are rapidities?

If we

$\delta^a_b +$

... in powers of $1/z$) becomes

z^0: trivial)

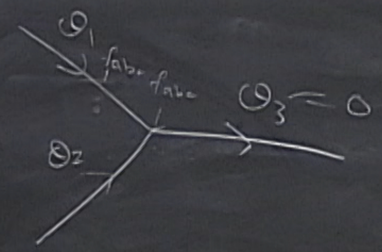
z^{-1}: S is an $SU(2)$ intertwiner

z^{-2}: an interesting new constraint on S

Unique $SU(2)$ symmetric map

S is determined up to scale

What are rapidities?



Coeff. of z^{-2} before scattering is

$$(\theta_1 \rho^a_{b,1} + \theta_2 \rho^a_{b,2} + \rho^a_{c,1}\rho^c_{b,2})$$

After scattering: it's zero.

So, we must have

$$S(\theta_1 \rho^a_{b,1} + \theta_2 \rho^a_{b,2} + \rho^a_{c,1}\rho^c_{b,2}) = 0$$

Using algebra

First two terms

$\Rightarrow$ a multiple of ρ^a_b acting on outgoing states

Third term: some other multiple of ρ^a_b

$\Rightarrow$ A linear equation by which θ_1, θ_2 can be fixed.

YBE For Scattering Matrix

Consider 3-3 scattering