

Title: PSI 2017/2018 - Quantum Integrable Models - Lecture 10

Date: Apr 02, 2018 11:30 AM

URL: <http://pirsa.org/18040030>

Abstract:

$$T \times \bar{T} + J\bar{J} \quad \text{perturbation on}$$

$$T \supseteq \left\{ \theta, J \cdot \theta \quad \text{is non-singular?} \right\}$$

↑ remain in the kernel of $\partial_{\bar{u}}$
 $\partial_{\bar{u}} \theta + J \cdot (\cancel{J} \cdot \theta)$

β -function

1st thing to understand: OPEs between J_a and J_b
in any chiral theory

FORMULA

$$J_a(0) \cdot J_b(\omega) \sim \frac{1}{\omega} f_{abc} J_c + \frac{K}{\omega} \delta_{ab}$$

$K = \text{Level}$

Example

$$\psi_j, \bar{\psi}_j$$

n complex fermions

$$J_j^i = \psi_j \psi^i$$

$$g = g(n) \\ (\text{or } u(n))$$

Prop

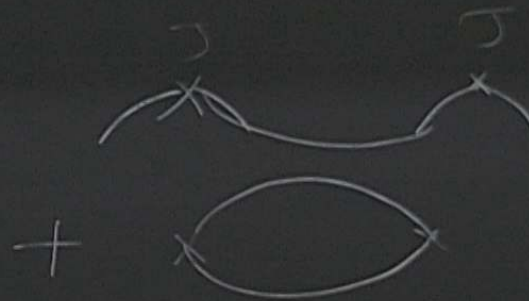
Propagator is $\frac{1}{\omega}$

$$J_j^i(0) J_c^k(\omega)$$



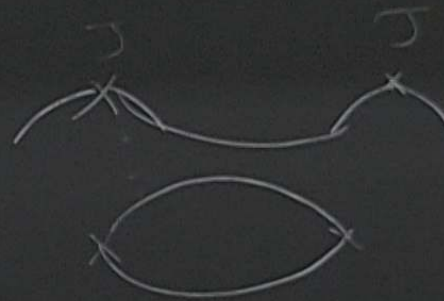
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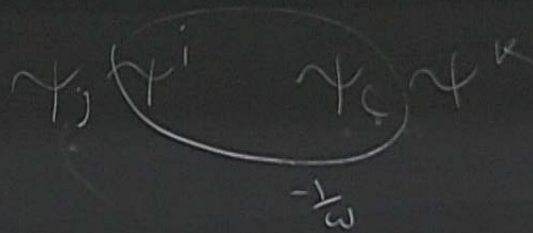
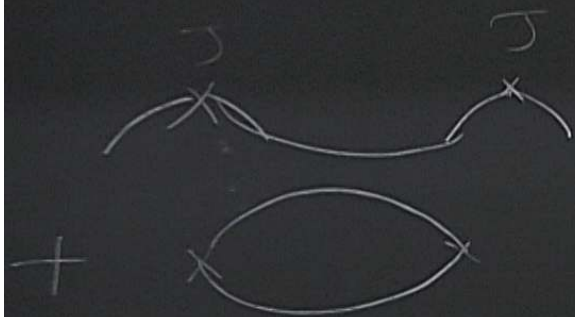


Propagator is $\frac{1}{\omega} \delta_j^i$

$$J_j^i(0) J_i^k(\omega)$$



$$= \frac{1}{\omega} (J_j^i J_i^k \delta_j^k - J_j^i J_i^k \delta_l^i) + \frac{1}{\omega^2} \delta_l^i \delta_j^k$$



$$J_j J_k \delta_l^i$$



$$J^i J^k \delta_{ik}$$

$$\psi_j \psi^i \psi_c \psi^k$$

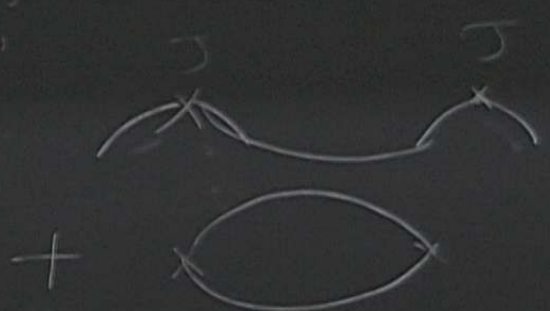
$$-\frac{1}{\omega}$$

If every operator is built from J
 then (if theory is unitary) the model
 is equal to "chiral WZW at level
 K "

$U(n)_I = n$ complex chiral fermions $\psi_i, \bar{\psi}_i$
 $SO(n)_I = n$ real chiral fermions ψ_i, ψ_i

Propagator is $\frac{1}{\omega} \delta_j^i$

$$T_j^i(0) T_c^k(\omega)$$



$$= \frac{1}{\omega} \left(\sum_c T_c^i \delta_j^k - T_j^k \delta_c^i \right) + \frac{1}{\omega^2} \delta_c^i \delta_j^k$$

fermions $\psi_j \sim \psi^i$

If every then (if) is equ

$U(n)_I = n$ complex chiral fermions $\psi_i, \bar{\psi}_i$

$SO(n)_I = n$ real chiral fermions ψ_i, ψ_i

β -function of chiral + a.c. + $\bar{\psi}\psi$

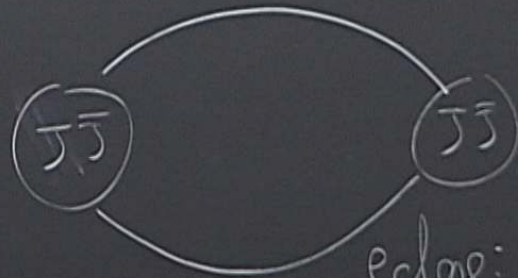
We'll look for 1 loop logarithmic divergences

Introduce a coupling constant $c \bar{\psi}\psi$

Have diagrams like

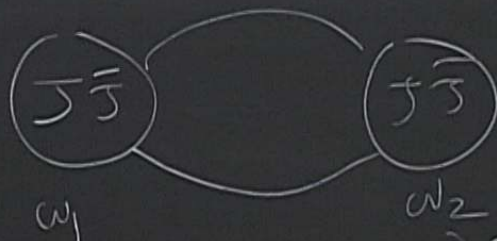
$$\int_{\omega_1, \omega_2} c^2 (\delta \bar{\tau})(\omega_1) (\delta \bar{\tau})(\omega_2)$$


Have diagrams like

$$\int_{w_1, w_2} c^2 (\bar{J} J)(w_1) (\bar{J} J)(w_2)$$


We replace $J(w_1) J(w_2)$ by OPE:
 edge: means

First Diagram

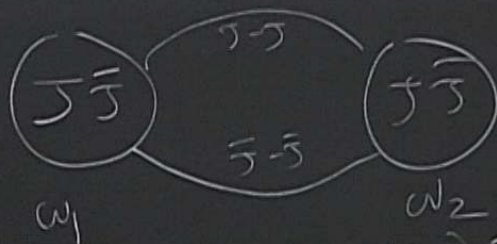


$$\int J(w_1) \bar{J}(w_1) J(w_2) \bar{J}(w_2)$$

$|w_1 - w_2| > \epsilon$



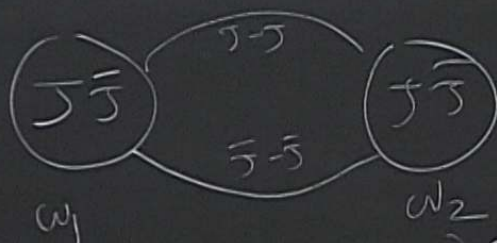
First Diagram



$$\int_{\omega_1} f(z)\bar{f}(z) dz + \int_{\omega_2} f(z)\bar{f}(z) dz$$

$|w_1 - w_2| > \epsilon$

First Diagram



$$\int \bar{J}_a(\omega_1) \bar{J}_a(\omega_1) \bar{J}_b(\omega_2) \bar{J}_b(\omega_2)$$

$| \omega_1 - \omega_2 | > \epsilon$

J-J OPE

$$f_{abc} \frac{\bar{J}_c(\omega_1)}{\omega_1 - \omega_2} + \frac{K_{dab}}{(\omega_1 - \omega_2)^2}$$

$\bar{J}\bar{J}$ OPE

$$f_{abc} \frac{\bar{J}_c(\omega_1)}{\omega_1 - \omega_2} + \frac{K_{dab}}{(\omega_1 - \omega_2)^2}$$

Substitute into integral

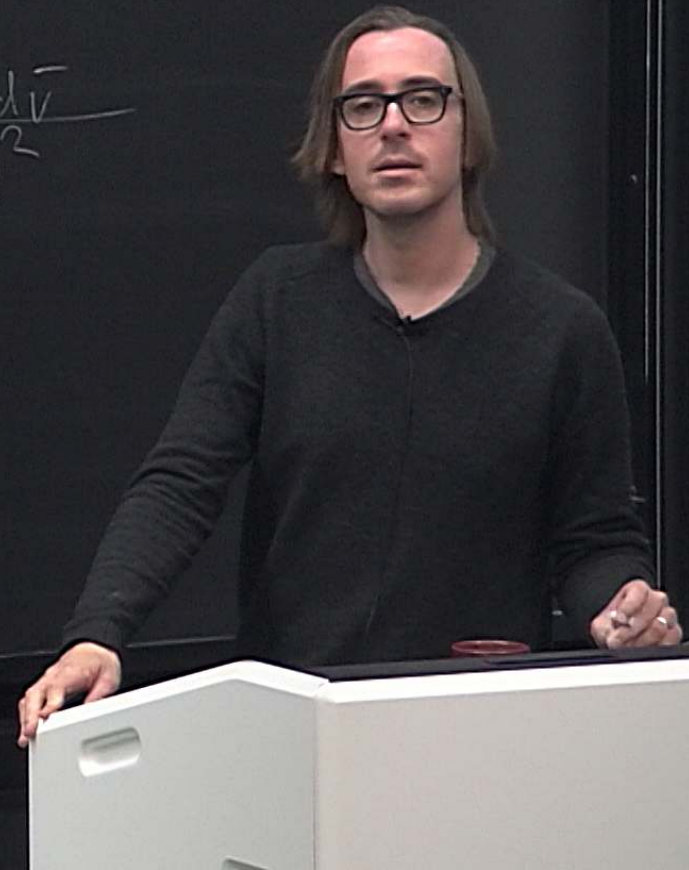
$$v = \omega_1 - \omega_2$$

We find 4 terms $\int_{|v| > \epsilon} \frac{dv d\bar{v}}{|v|^2}$

$$f_{abc} f_{abd} \mathcal{J}_c(\omega_1) \bar{\mathcal{J}}_d(\omega_1)$$

$$+ \frac{K_{cab}}{(\omega_1 - \omega_2)^2}$$

$$+ \frac{K_{cab}}{(\omega_1 - \omega_2)^2}$$



$$\delta_{ab} f_{abc} J_c(\omega_1) \int \frac{d^4 v}{v_1 \sqrt{v^2}}$$

$$\delta_{ab} f_{abc} \mathcal{J}_c(\omega_1) \int \frac{d\nu d\bar{\nu}}{\nu_i \bar{\nu}^2} = 0$$

Only other term:

$$\delta_{ab} \int \frac{d\nu d\bar{\nu}}{|\nu|^4}$$

$$\delta_{ab} f_{abc} \mathcal{J}_c(\omega) \int \frac{d\nu d\bar{\nu}}{\nu_i \sqrt{\nu^2}} = 0$$

Only other term:

$$\delta_{ab} \int_{|\nu| \geq \epsilon} \frac{d\nu d\bar{\nu}}{|\nu|^4}$$

= 0

By dimensional analysis

$$\int_{|v| \geq \varepsilon} \frac{dv d\bar{v}}{|v|^R} \sim \log \varepsilon$$

$$\int_{|v| \geq \varepsilon} \frac{dv d\bar{v}}{|v|^4} \sim \varepsilon^{-2}$$

= 0

By dimensional analysis
 $\int_{|v| \geq \varepsilon} \frac{dv d\bar{v}}{|v|^R} \sim \log \varepsilon$, (times $f_{abc} f_{abd} \bar{J}_c \bar{J}_d$)

$\int_{|v| \geq \varepsilon} \frac{dv d\bar{v}}{|v|^4} \sim \varepsilon^{-2}$

= 0

By dimensional analysis

$$\int_{|v| \geq \varepsilon} \frac{dv d\bar{v}}{|v|^R} \sim \log \varepsilon$$

$$\int_{|v| \geq \varepsilon} \frac{dv d\bar{v}}{|v|^4} \sim \varepsilon^{-2}$$

(times $f_{abc} f_{abd} J_c J_d$)

FACT

$$\frac{f_{abc} f_{abd}}{f_{abc} f_{abd}} = \delta_{cd} h^v$$

$h^v =$ dual Coxeter num

analysis

$\log \varepsilon$

(times $f_{abc}f_{abd} J_c J_d$)

$$\frac{\text{FACT}}{f_{abc}f_{abd}} = \delta_{cd} h^{\nu}$$

$h^{\nu} = \text{dual Coxeter number (some rational number } \neq 0)$

analysis

$\log \varepsilon$

ε^{-2}

(times $fabcfabd \int_c \int_d$)

FACT

$$fabcfabd = \delta_{cd} h^{\nu}$$

$h^{\nu} = \text{dual Coxeter number (some rational number } \neq 0)$



β -function

$$C_1 \log \varepsilon \bar{J}_a \bar{J}_a + C_2 \varepsilon^{-2}$$

= Counter terms.

$$C_1 \log \varepsilon \bar{J}_a \bar{J}_a + C_2 \varepsilon^{-2}$$

= Counter terms.

Find β -function is again
 $\bar{J}_a \bar{J}_a$

If c = coupling constant, we find

RG flow looks like $c^2 \partial_c +$ higher order terms

β -fn of a σ -model

σ -model on a Kähler manifold has fields

$\beta^i, \bar{\beta}^i, \gamma_j, \bar{\gamma}_j$ Lagrangian

$$\int \beta^i \partial \gamma_j + \int \bar{\beta}^i \partial \bar{\gamma}_j + \int g_{i\bar{j}}(\gamma, \bar{\gamma}) \beta^i \bar{\beta}^j$$

β -fn of a σ -model

σ -model on a Kähler manifold has fields

$\beta^i, \bar{\beta}^i, \gamma_j, \bar{\gamma}_j$ Lagrangian

$$\int \beta^i \partial \gamma_i + \int \bar{\beta}^i \partial \bar{\gamma}_i + \int g_{i\bar{j}}(\gamma, \bar{\gamma}) \beta^i \bar{\beta}^{\bar{j}}$$

$$g_{i\bar{j}} = \delta_{i\bar{j}} + h_{i\bar{j}}(u, \bar{u})$$

We're in normal coordinates: $h_{i\bar{j}}(u, \bar{u})$ is quadratic in u 's

Fact

$$R_{ij}(\bar{u}, \bar{u} = 0) = C \sum \left(\frac{\partial}{\partial u^k} \frac{\partial}{\partial u^k} h_{ij} \right) (\bar{u}, \bar{u} = 0)$$

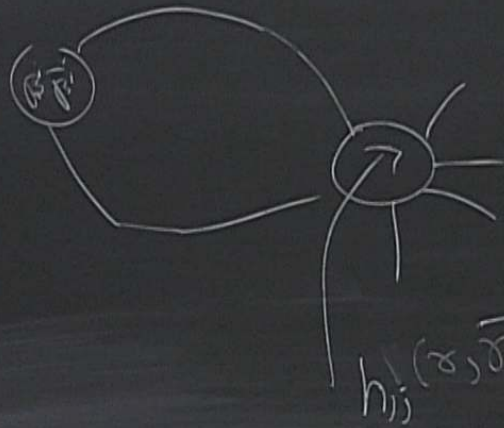
C a constant.

Fact

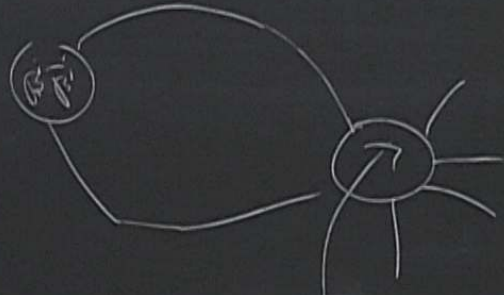
$$R_{ij}(u, \bar{u}=0) = C \sum \left(\frac{\partial}{\partial u_k} \frac{\partial}{\partial \bar{u}_k} h_{ij} \right) (u, \bar{u}=0)$$

C a constant.

When we compute β -fn, to first order in h ,
the only diagram that contributes is



$$h_{ij}(\alpha, \bar{\alpha}) \beta^{\bar{1}} \beta^{\bar{2}}$$



$$h_{ij}(\alpha, \bar{\alpha}) \beta^i \bar{\beta}^j$$

Each edge: contract β^i with γ_i to leave $\frac{1}{\omega_1 - \omega_2}$
 $\bar{\beta}^i$ " $\bar{\gamma}_i$ " " $\frac{1}{\bar{\omega}_1 - \bar{\omega}_2}$

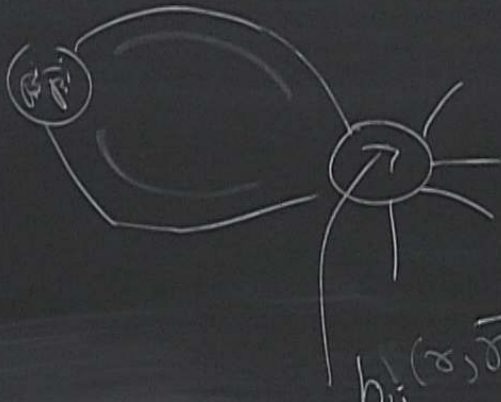
Thus



This diagram gives

$$\int_{|\omega_1 - \omega_2| \geq \epsilon} \frac{1}{|\omega_1 - \omega_2|^2} \left(\sum \frac{\partial}{\partial u_k} \frac{\partial}{\partial \bar{u}_k} h_{ij} \right) (\gamma) \beta^i \bar{\beta}^j$$

γ_1 to leave $\frac{1}{\omega_1 - \omega_2}$
 $\bar{\gamma}_1$ " " $\frac{1}{\bar{\omega}_1 - \bar{\omega}_2}$



$$h_{ij}(\alpha, \bar{\alpha}) \beta^i \bar{\beta}^j$$

Each edge contract γ^i with $\bar{\gamma}_i$ to leave $\frac{1}{\omega_1 - \omega_2}$
 $\bar{\gamma}^i$ " $\bar{\gamma}_i$ " " $\frac{1}{\omega_1 - \omega_2}$

This diagram gives

$$\int_{|\omega_1 - \omega_2| \geq \epsilon} \frac{1}{|\omega_1 - \omega_2|^2} \left(\sum \frac{\partial}{\partial \omega} \right)$$

This diagram gives

$$\int_{|w_1 - w_2| \geq \varepsilon} \frac{1}{|w_1 - w_2|^2} \left(\sum_k \frac{\partial}{\partial x_k} \frac{\partial}{\partial \bar{x}_k} h_{ij} \right) (\gamma) \beta^i \bar{\beta}^j$$

$$= (\log \varepsilon) \text{Ric}_{ij}^{(0)} \beta^i \bar{\beta}^j + \begin{matrix} \text{(non-sing terms)} \\ \text{(terms depending} \\ \text{on } \gamma) \end{matrix}$$

This diagram gives

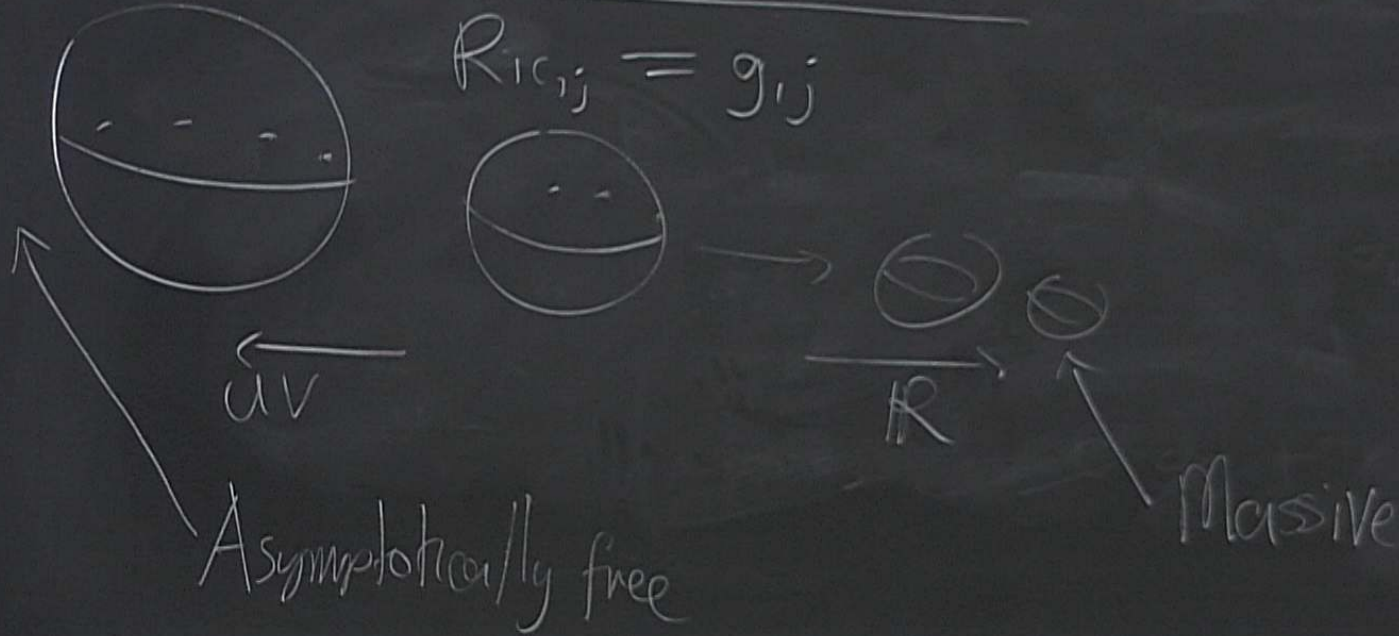
$$\int_{|w_1 - w_2| \geq \varepsilon} \frac{1}{|w_1 - w_2|^2} \left(\sum \frac{\partial}{\partial u_k} \frac{\partial}{\partial \bar{u}_k} h_{ij} \right) (\gamma) \beta^i \bar{\beta}^j$$

$$= (\log \varepsilon) \text{Ricci}_{ij}^{(0)} \beta^i \bar{\beta}^j + \begin{matrix} \text{(non-sing terms)} \\ \text{(terms depending} \\ \text{on } \gamma) \end{matrix}$$

In fact β - $\bar{\beta}$ - $\text{tr} = \text{Ricci}$
Everywhere

5-model on S^3

$$Ric_{ij} = g_{ij}$$



Similarly,

Gross Neveu / Thirring models are
expected to be massive.

BUT

Massive particles are non-perturbative.

Similarly, Gross-Neveu / Thirring models are
expected to be massive.

BUT Massive particles are non-perturbative
Euclidean Signature

Signature of massive theory is that ϕ creates a particle
then $\langle \phi(0), \phi(x) \rangle \sim e^{-m|x|}$