Title: PSI 2017/2018 - Quantum Integrable Models - Lecture 10

Date: Apr 02, 2018 11:30 AM

URL: http://pirsa.org/18040030

Abstract:

$T \times \bar{T} \quad + \quad J\bar{J}$ perturbation

$T \supseteq \{ \mathcal{O}, \; J\cdot\mathcal{O}$ is non-singular $\}$

remain in the kernel of $\partial_{\bar{w}}$

$$\partial_{\bar{w}} \mathcal{O} + \bar{J} \cdot \left(\cancel{\oint} J \cdot \mathcal{O} \right)$$

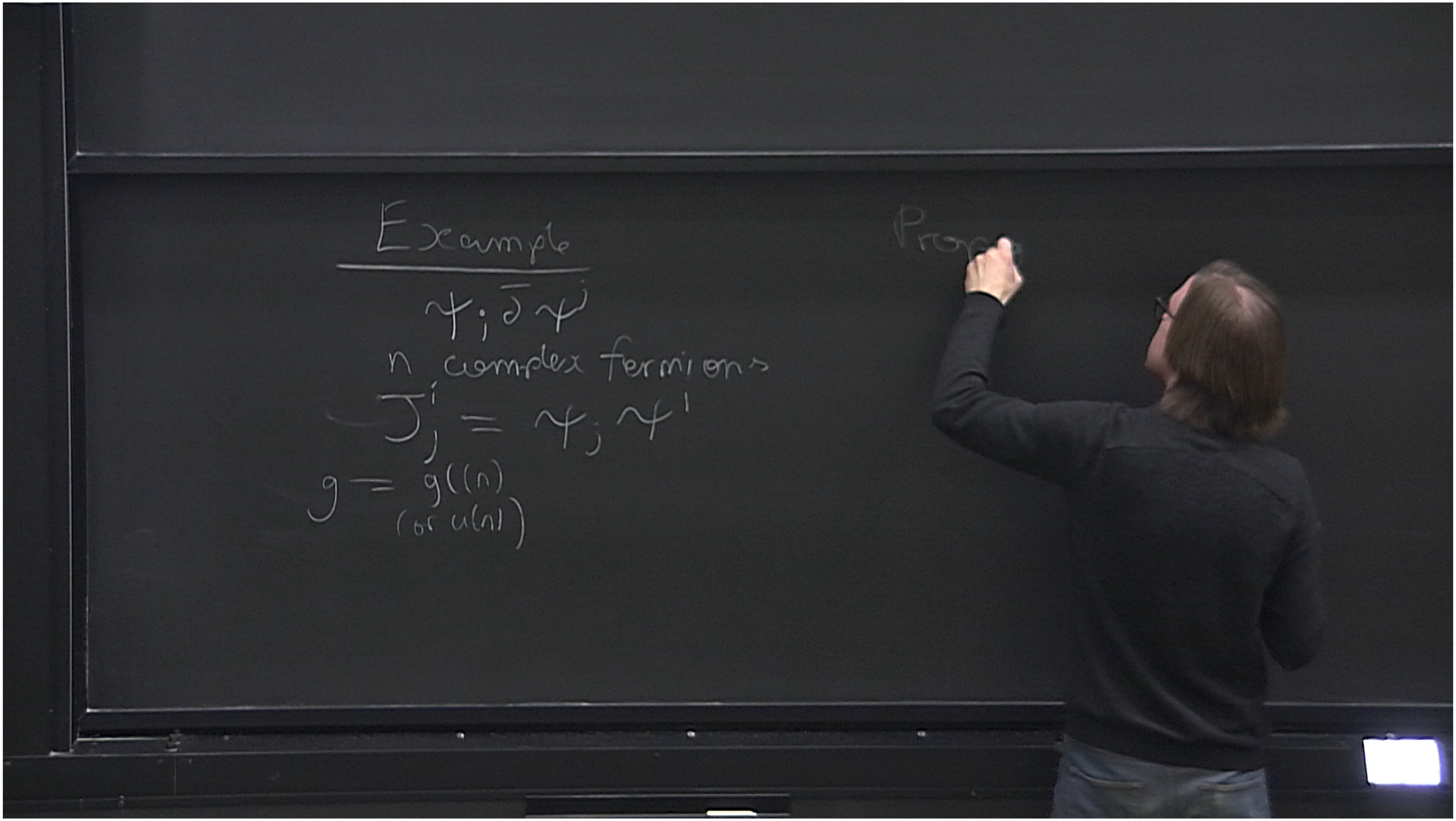
β-function

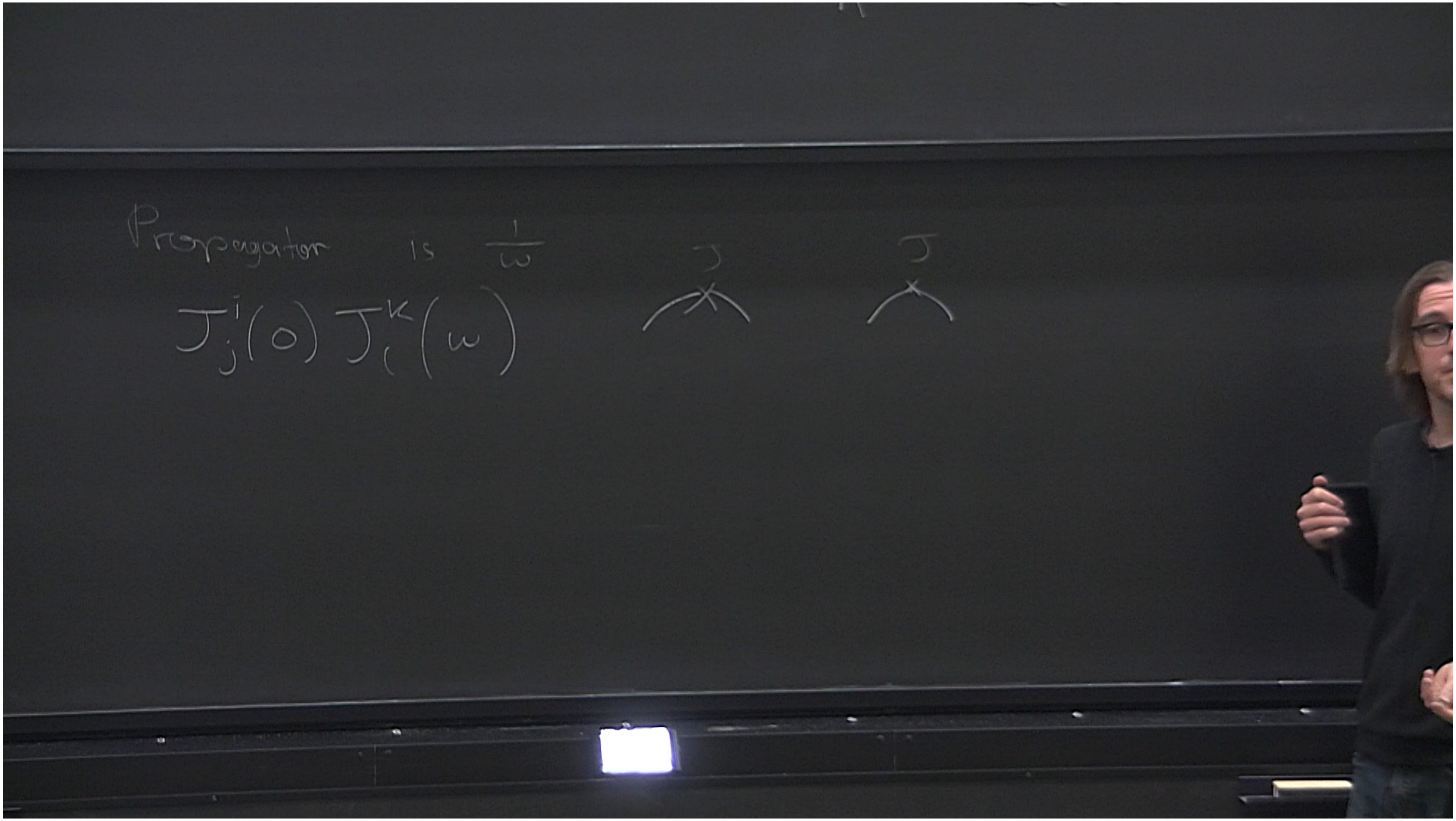
1st thing to understand: OPEs between J_a and J_b in any chiral theory

FORMULA

$$J_a(0) \cdot J_b(w) \sim \frac{1}{w} f_{abc} J_c + \frac{K}{w^2} \delta_{ab}$$

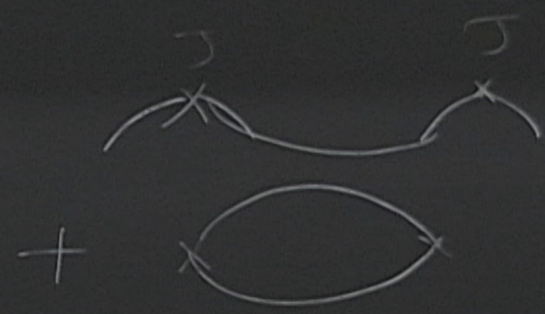
K = Level


Example
ψ_i ∂̄ ψ^j
n complex fermions
J^i_j = ψ_j ψ^i
g = gl(n)
(or u(n))


Propagator is $\frac{1}{\omega}$
$J^i_j(0)\, J^k_l(\omega)$
J
J

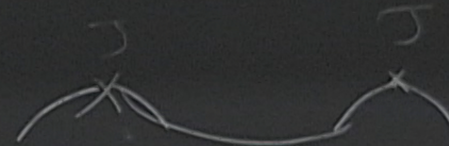
Propagator is $\frac{1}{\omega}$

$J^i_j(0)\, J^k_i(\omega)$



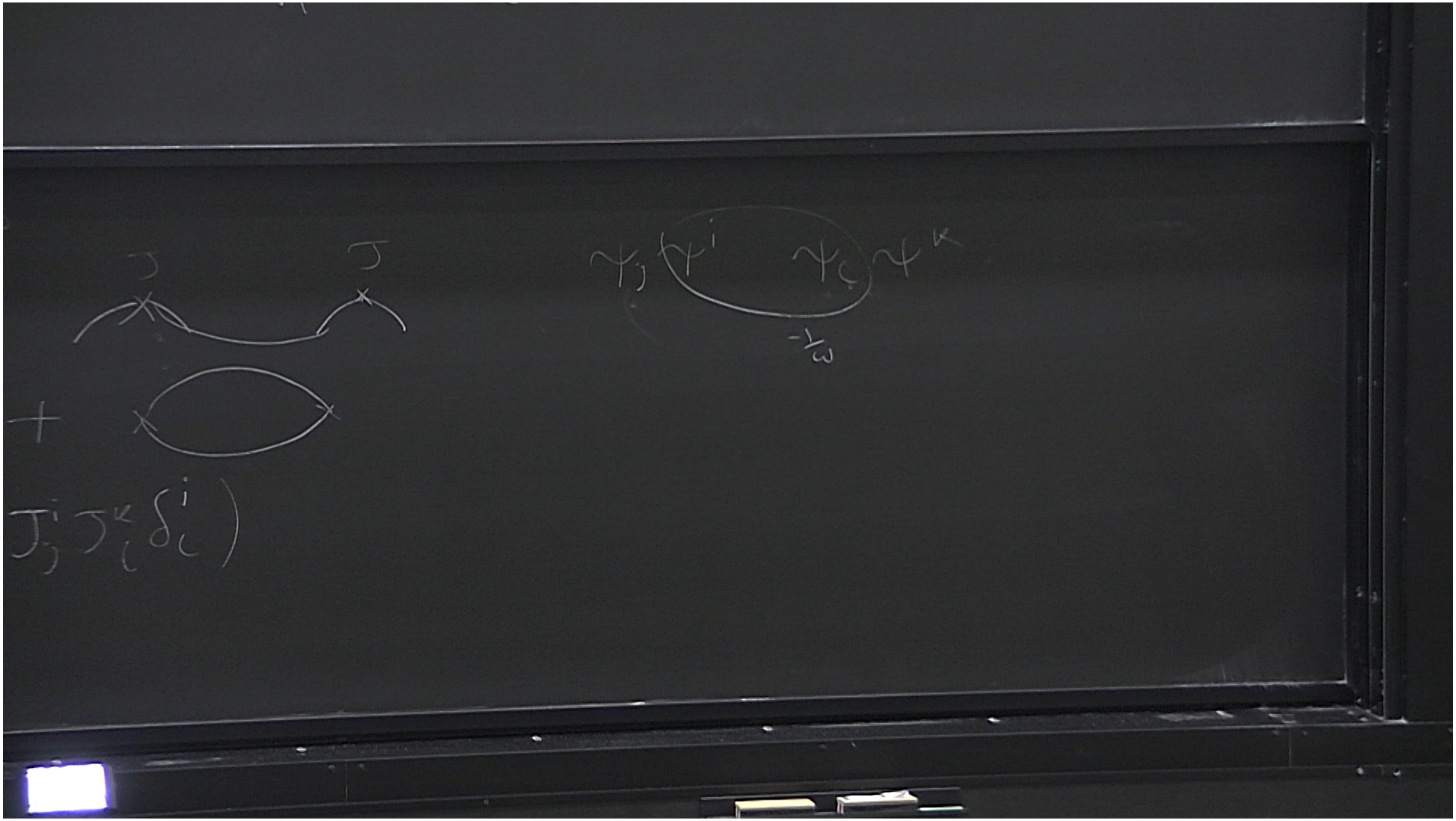
Propagator is $\frac{1}{\omega}\delta^j_i$

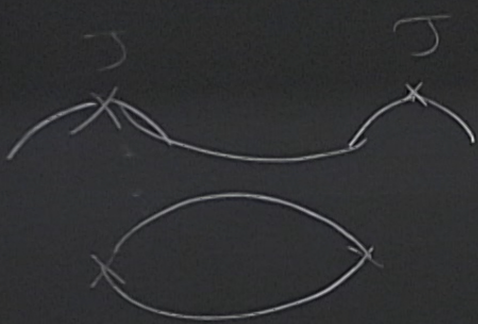
$J^i_j(0)\, J^k_l(\omega)$



$+$

$$= \frac{1}{\omega}\left(J^i_j J^k_l \delta^k_j - J^i_j J^k_l \delta^i_l\right) + \frac{1}{\omega^2}\delta^i_l \delta^k_j$$





$+$

$J_j^i J^k_l \delta^i_l)$

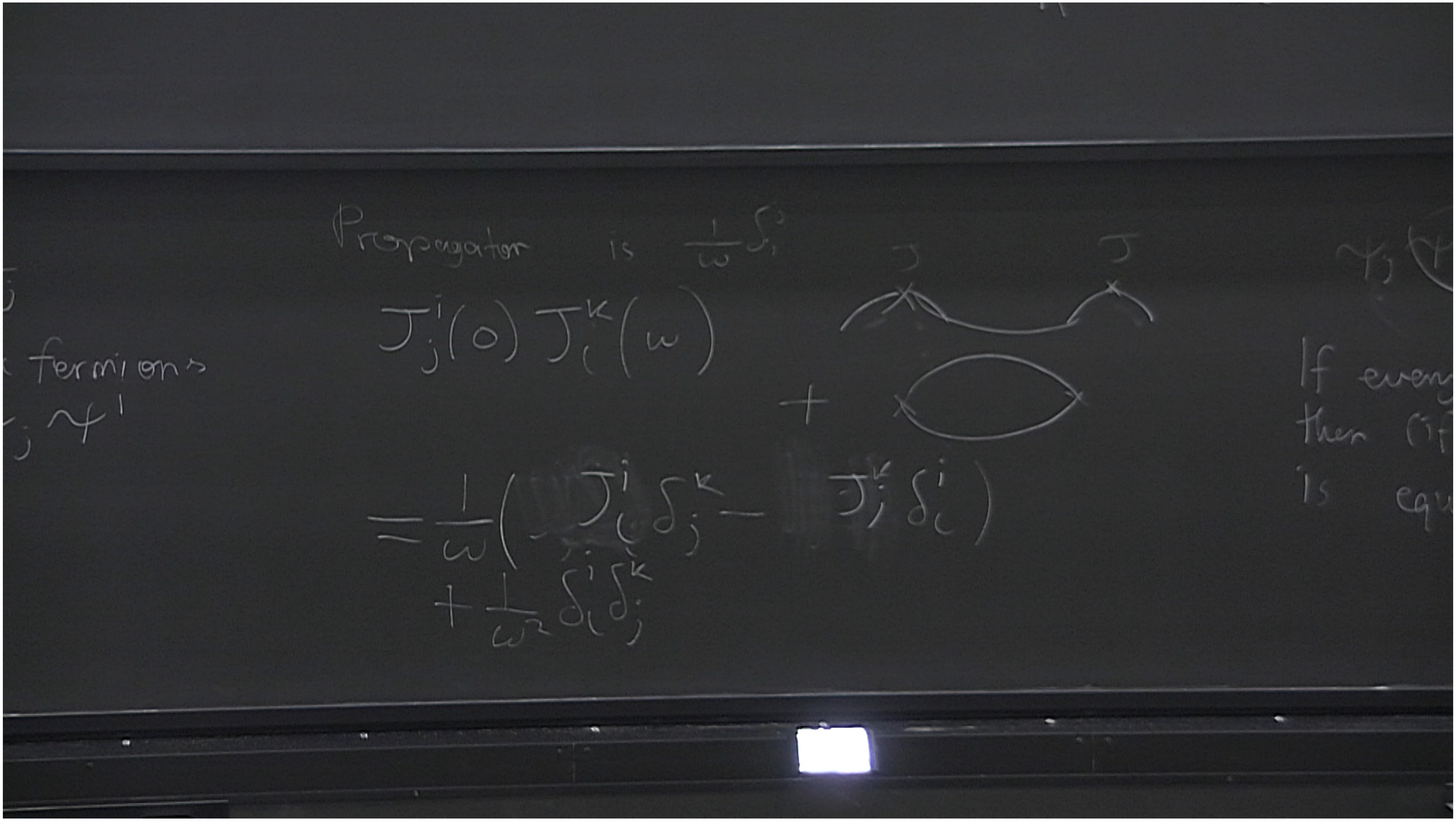
$\psi_j \psi^i \quad \psi_l \psi^k$

$-\frac{1}{\omega}$

If every operator is built from J
then (if theory is unitary) the model
is equal to "chiral WZW at level
k"

$U(n)_1 = n$ complex chiral fermions ψ_i, ψ^j

$SO(n)_1 = n$ real chiral fermions ψ_i, ψ_j


Propagator is $\frac{1}{\omega}\delta^j_i$
$J^i_j(0)\, J^k_l(\omega)$
$= \frac{1}{\omega}\left(J^i_l\, \delta^k_j - J^k_j\, \delta^i_l \right)$
$+ \frac{1}{\omega^2}\delta^i_l \delta^k_j$
fermions
If every
then (if
is equ

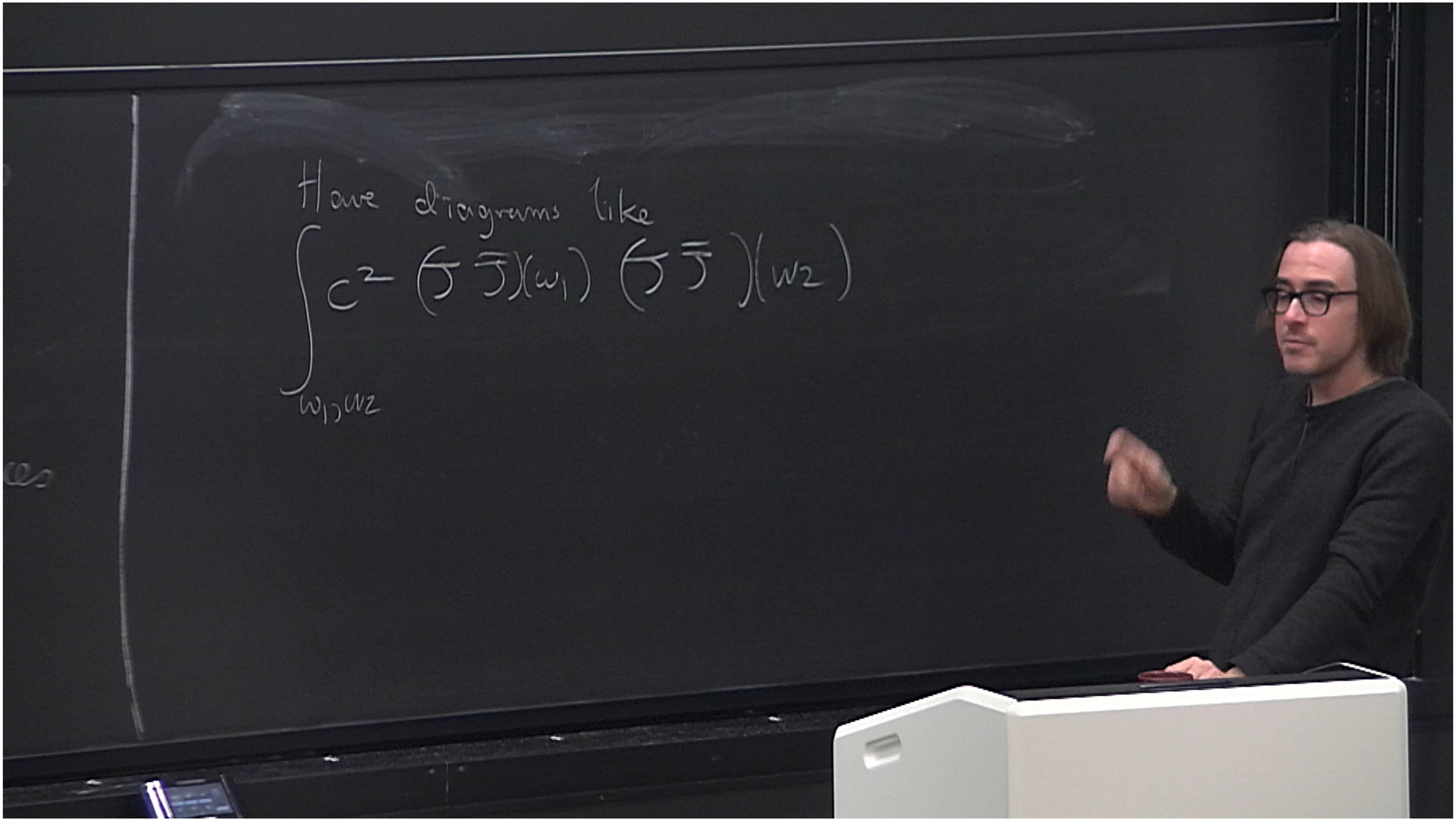
$U(n)_1 = n$ complex chiral fermions ψ_i, ψ^i

$SO(n)_1 = n$ real chiral fermions ψ_i, ψ_i

β-function of chiral + a.c. + $J\bar{J}$

We'll look for 1 loop logarithmic divergences

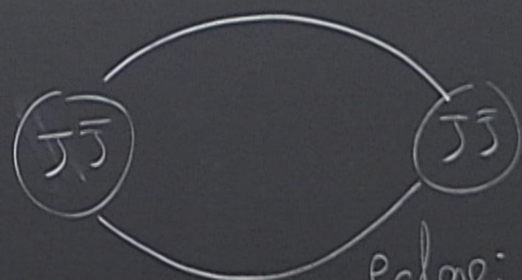
Introduce a coupling constant $c\, J\bar{J}$


Have diagrams like
$\int_{\omega_1, \omega_2} c^2 (J \bar{J})(\omega_1) (J \bar{J})(w_2)$

es

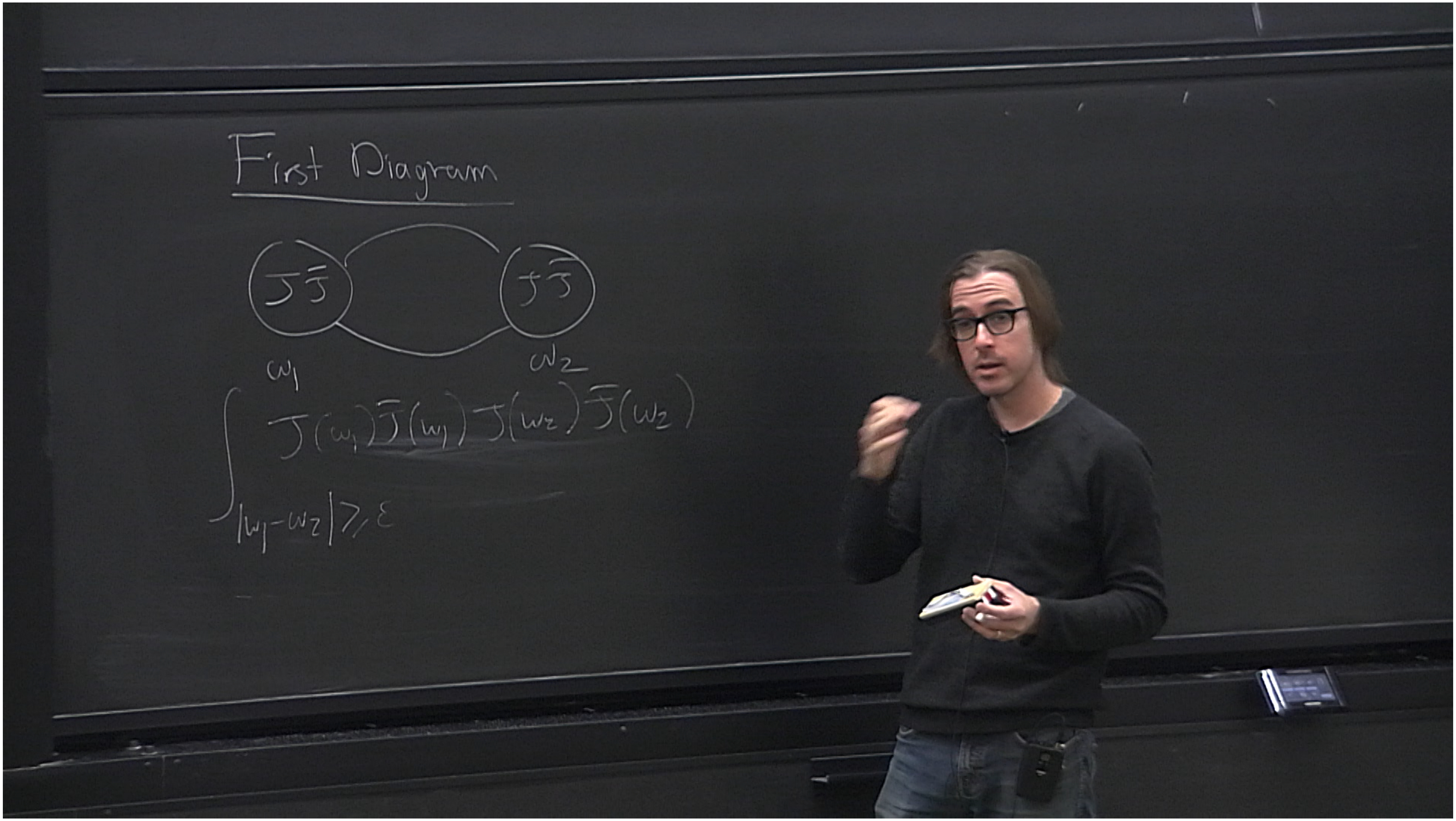
Have diagrams like

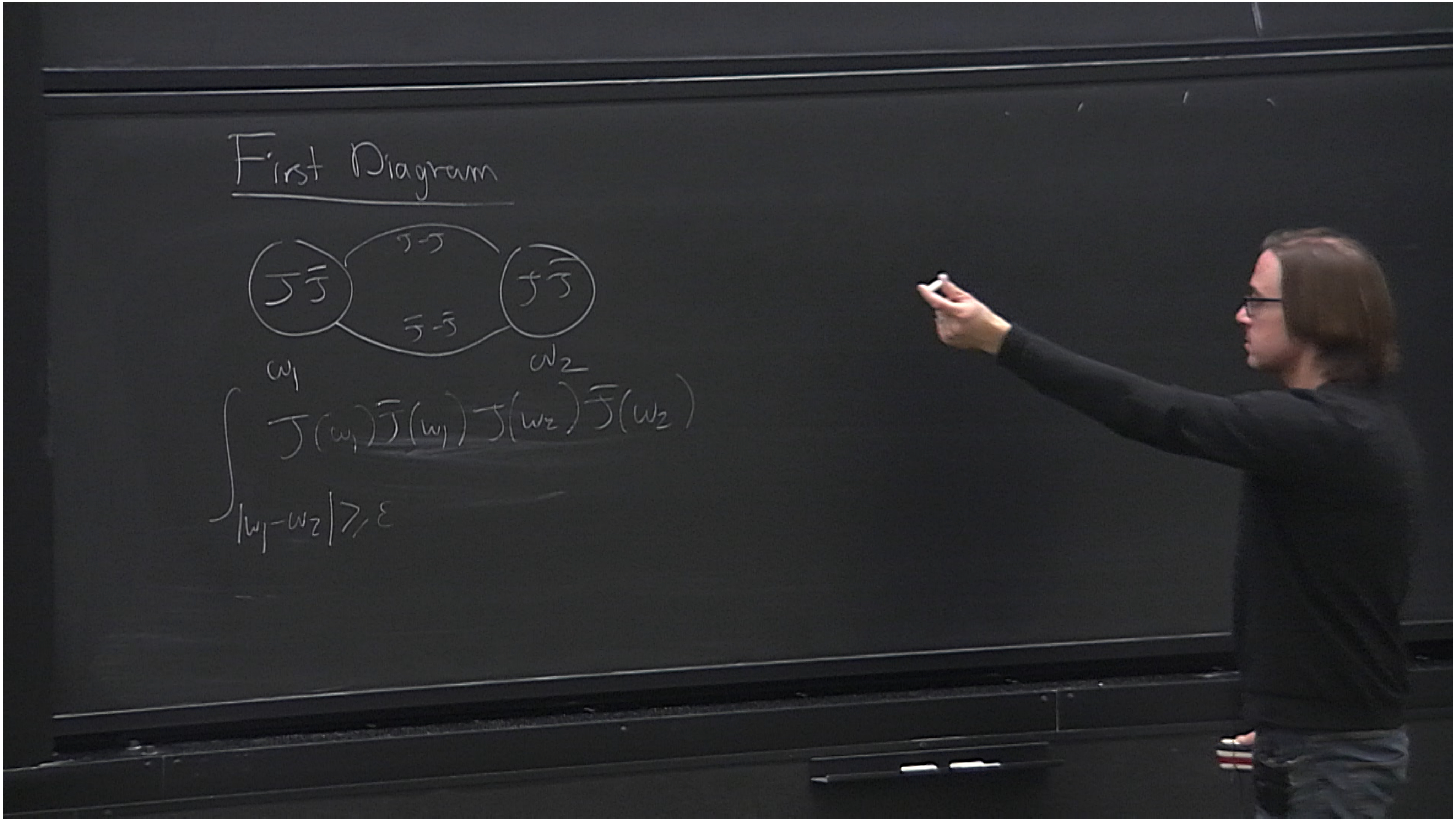
$$\int_{w_1, w_2} c^2 \, (J\bar{J})(w_1) \, (J\bar{J})(w_2)$$



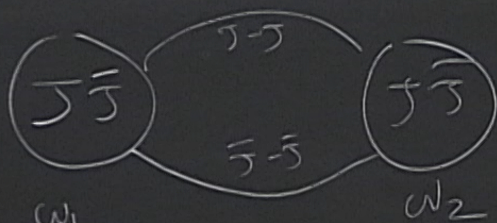
edge: means

we replace $J(w_1)\, J(w_2)$ by OPE.


First Diagram
$J\bar{J}$
$J\bar{J}$
w_1
w_2
$\int_{|w_1-w_2|\geq\varepsilon} J(w_1)\bar{J}(w_1)J(w_2)\bar{J}(w_2)$


First Diagram
$J\bar{J}$
$J-J$
$J\bar{J}$
$\bar{J}-\bar{J}$
w_1
w_2
$\int_{|w_1-w_2|\geq\varepsilon} J(w_1)\bar{J}(w_1)J(w_2)\bar{J}(w_2)$

First Diagram



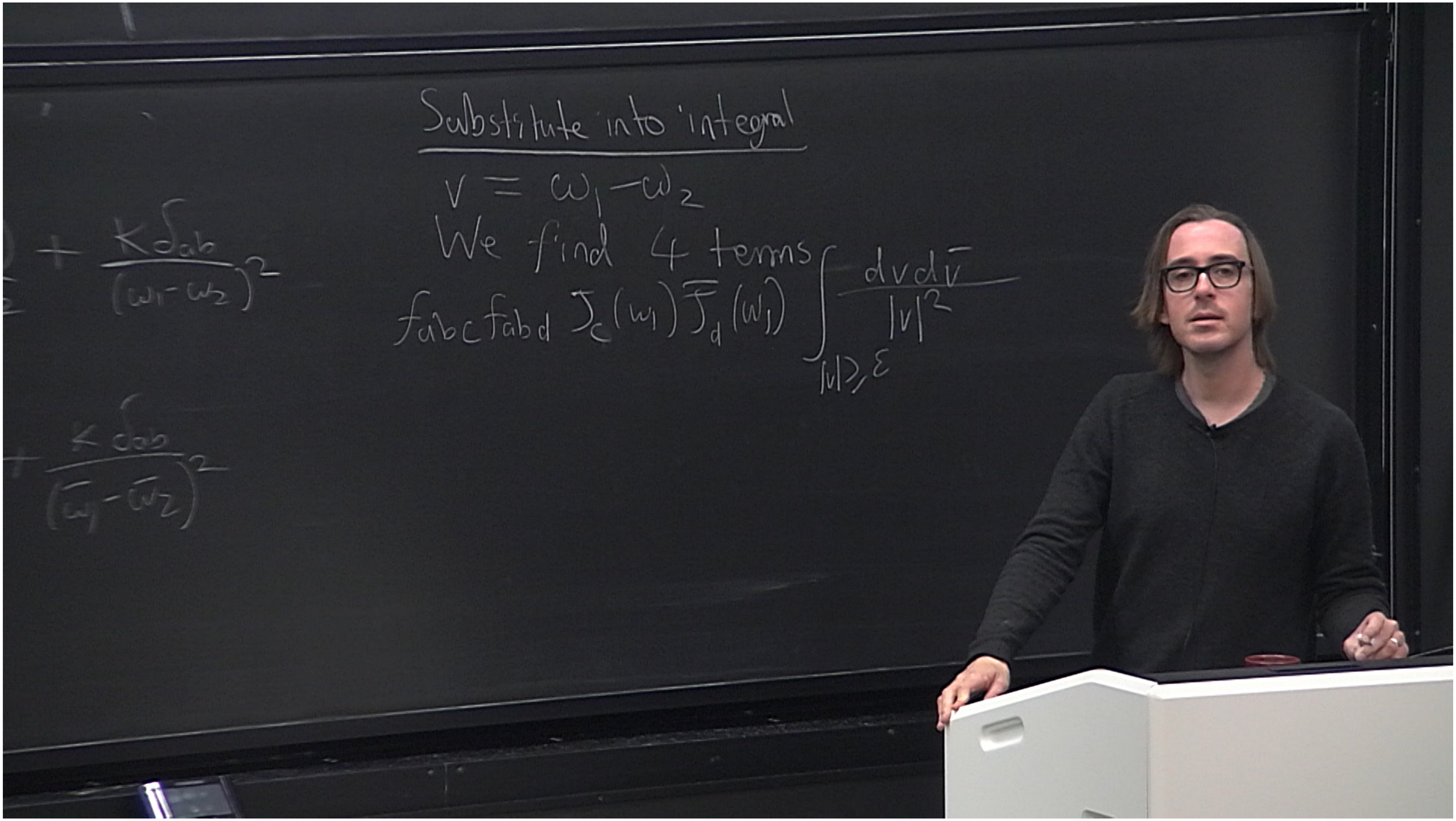
$$\int_{|w_1-w_2|\geq\varepsilon} J_a(w_1)\bar{J}_a(w_1)\,J_b(w_2)\bar{J}_b(w_2)$$

$J-J$ OPE

$$f_{abc}\frac{J_c(w_1)}{w_1-w_2}+\frac{k\delta_{ab}}{(w_1-w_2)^2}$$

$\bar{J}\bar{J}$ OPE

$$f_{abc}\frac{\bar{J}_c(\bar{w}_1)}{\bar{w}_1-\bar{w}_2}+\frac{k\delta_{ab}}{(\bar{w}_1-\bar{w}_2)^2}$$


Substitute into integral
$v = \omega_1 - \omega_2$
We find 4 terms
$f_{abc} f_{abd} J_c(w_1) \bar{J}_d(w_1) \int_{|v| \geq \varepsilon} \frac{dv\, d\bar{v}}{|v|^2}$
$+ \frac{K \delta_{ab}}{(\omega_1 - \omega_2)^2}$
$+ \frac{K \delta_{ab}}{(\bar{\omega}_1 - \bar{\omega}_2)^2}$

$$\delta_{ab} f_{abc} J_c(\omega_1) \int \frac{dv\, d\bar{v}}{v_1 \bar{v}^2}$$

$$\delta_{ab} f_{abc} J_c(w_1) \int \frac{dv\, d\bar{v}}{v_1 \bar{v}^2} = 0$$

$\vdots$

<u>Only other term:</u>

$$\delta_{ab} \int \frac{dv\, d\bar{v}}{|v|^4}$$

$$\delta_{ab} f_{abc} J_c(w_1) \int \frac{dv\, d\bar{v}}{v_1 \bar{v}^2} = 0$$

$\vdots$

<u>Only other term:</u>

$$\delta_{ab} \int_{|v| \geq \varepsilon} \frac{dv\, d\bar{v}}{|v|^4}$$

$= 0$

By dimensional analysis

$$\int_{|v| \geq \varepsilon} \frac{dv\, d\bar{v}}{|v|^2} \sim \log \varepsilon$$

$$\int_{|v| \geq \varepsilon} \frac{dv\, d\bar{v}}{|v|^4} \sim \varepsilon^{-2}$$

$= 0$

By dimensional analysis

$$\int_{|v|\geq \varepsilon} \frac{dv\, d\bar{v}}{|v|^2} \sim \log \varepsilon \; ; \quad (\text{times } f_{a_1bc} f_{a_1bd} J_c \bar{J}_d)$$

$$\int_{|v|\geq \varepsilon} \frac{dv\, d\bar{v}}{|v|^4} \sim \varepsilon^{-2}$$

$= 0$

By dimensional analysis

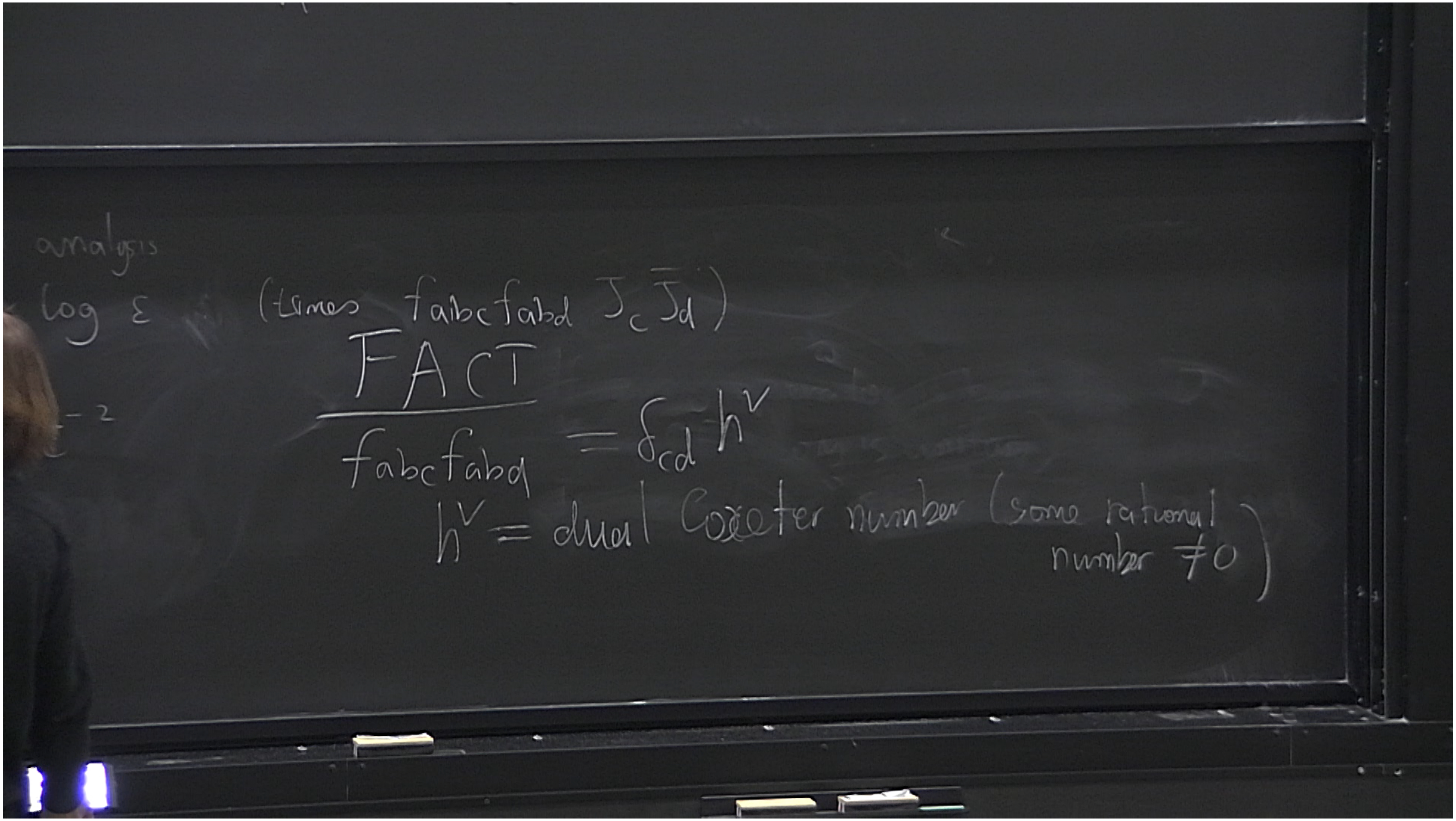
$$\int_{|v|\geq\varepsilon} \frac{dv\,d\bar{v}}{|v|^2} \sim \log \varepsilon \quad (\text{times } f_{abc}f_{abd}\, J_c \bar{J}_d)$$

$$\int_{|v|\geq\varepsilon} \frac{dv\,d\bar{v}}{|v|^4} \sim \varepsilon^{-2}$$

FACT

$$f_{abc}f_{abd} = \delta_{cd}\, h^\vee$$

$h^\vee$ = dual Coxeter num


analysis
log ε
(times $f_{abc} f_{abd} J_c \bar{J}_d$)
$\frac{\text{FACT}}{f_{abc} f_{abd}} = \delta_{cd} h^\vee$
$h^\vee$ = dual Coxeter number (some rational number $\neq 0$)

analysis

$\log \varepsilon$ (times $f_{abc} f_{abd} J_c \bar{J}_d$)

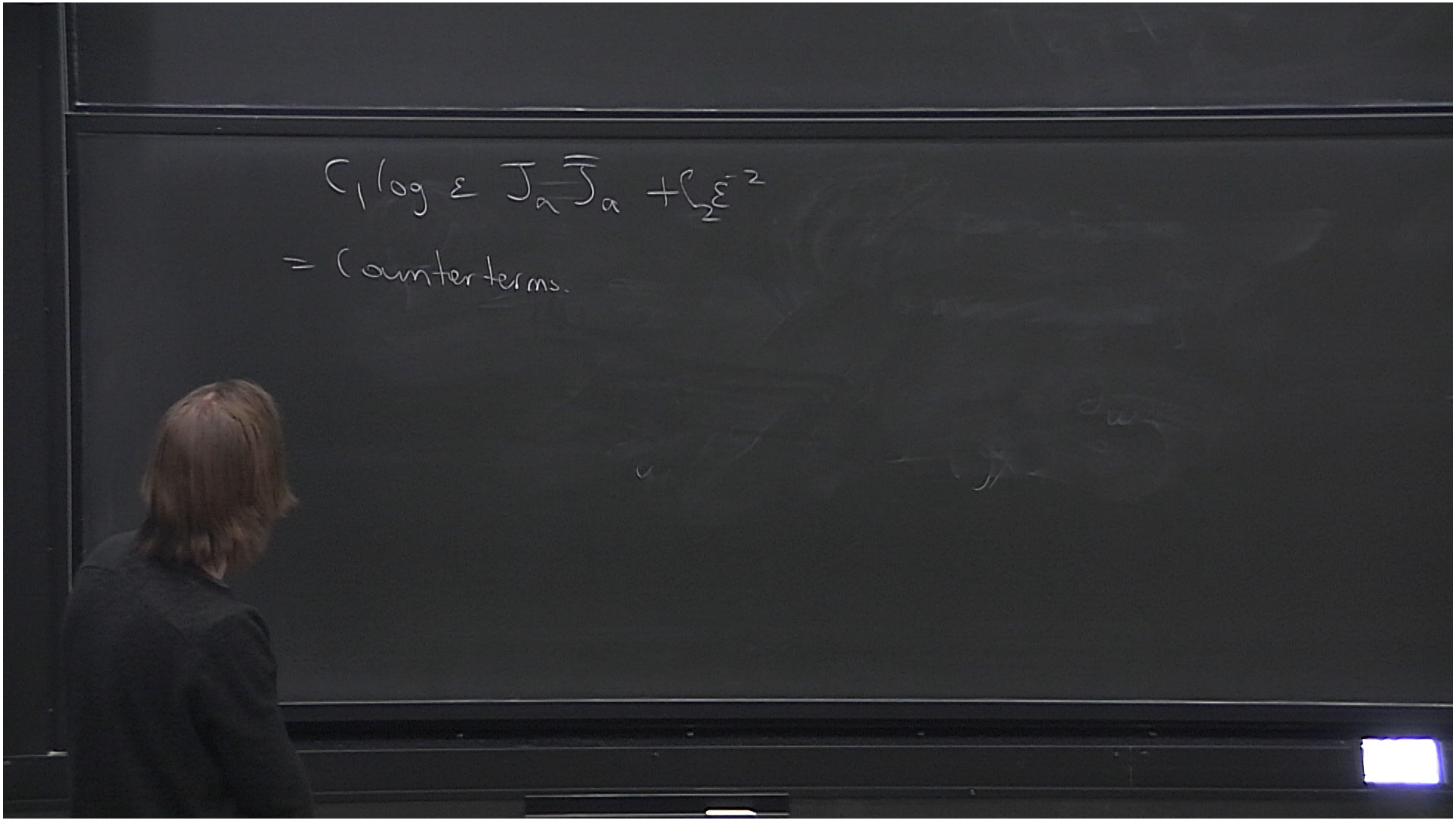
ε^{-2}

FACT

$$f_{abc} f_{abd} = \delta_{cd} h^\vee$$

$h^\vee$ = dual Coxeter number (some rational number $\neq 0$)

β-function


$c_1 \log \varepsilon \, J_a \bar{J}_a + c_2 \varepsilon^{-2}$
= Counterterms.

$c_1 \log \varepsilon \, J_a \bar{J}_a + c_2 \varepsilon^{-2}$

= Counterterms.

Find β-function is again

$J_a \bar{J}_a$

If c = coupling constant, we find

RG flow looks like $c^2 \partial_c$ + higher order terms

β - fn of a σ - model

σ - model on a Kähler manifold has fields

$\beta^i, \gamma_j \quad \bar\beta^i, \bar\gamma_j$ Lagrangian

$$\int \beta^i \bar\partial \gamma_i + \int \bar\beta^i \partial \bar\gamma_i + \int g_{ij}(\gamma, \bar\gamma)\, \beta^i \bar\beta^j$$

β - fn of a σ - model

σ - model on a Kähler manifold has fields $\beta^i, \gamma_i \quad \bar\beta^i, \bar\gamma_i$ Lagrangian

$$\int \beta^i \bar\partial \gamma_i + \int \bar\beta^i \partial \bar\gamma_i + \int g_{i\bar j}(\gamma, \bar\gamma)\, \beta^i \bar\beta^j$$

$$g_{i\bar j} = \delta_{i\bar j} + h_{i\bar j}(u, \bar u)$$

We're in normal coordinates: $h_{i\bar j}(u, \bar u)$ is quadratic in u's

Fact

$$R_{ij}(u,\bar{u}=0) = C\sum\left(\frac{\partial}{\partial u_k}\frac{\partial}{\partial \bar{u}_k} h_{ij}\right)(u,\bar{u}=0)$$

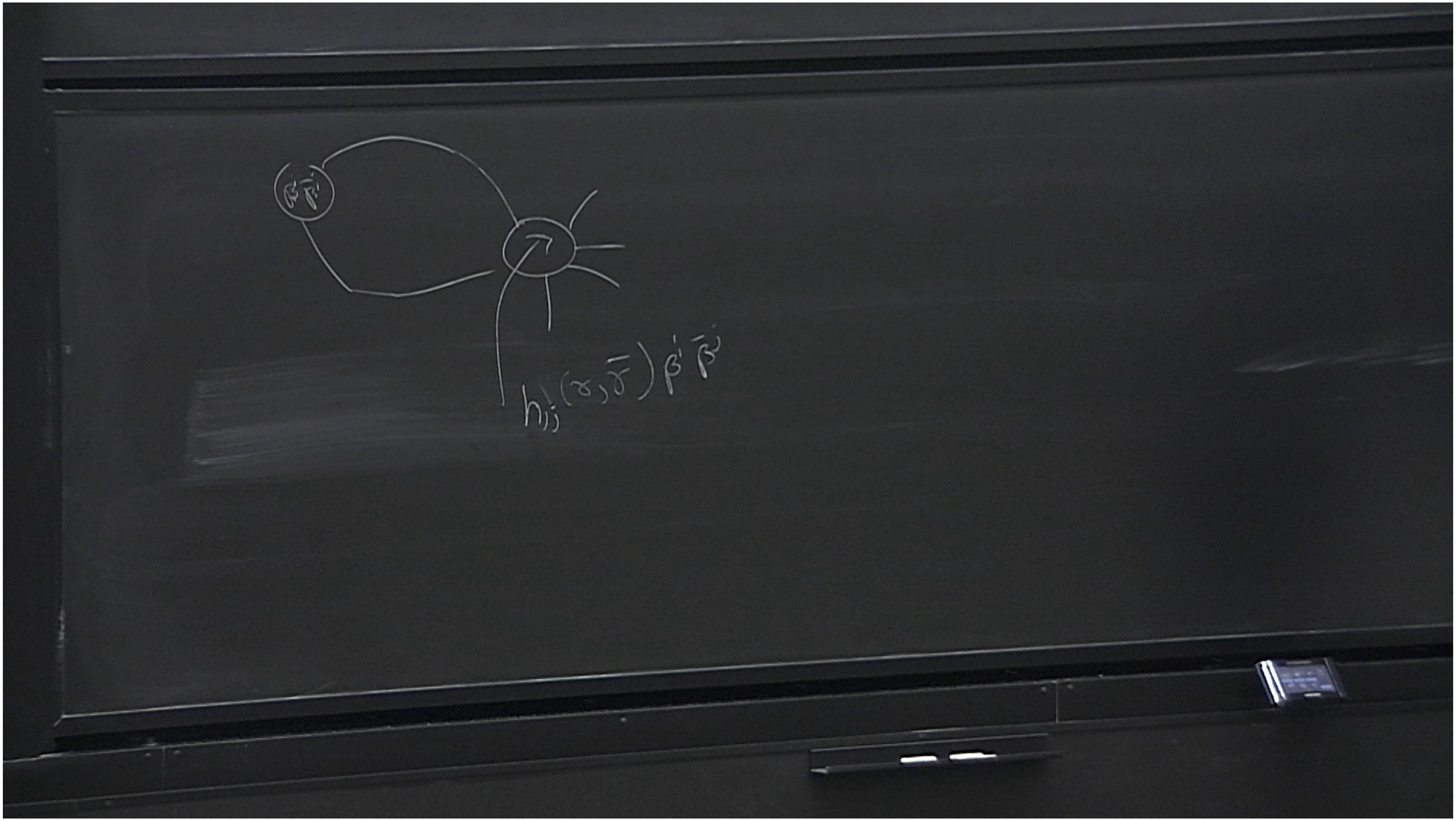
C a constant.

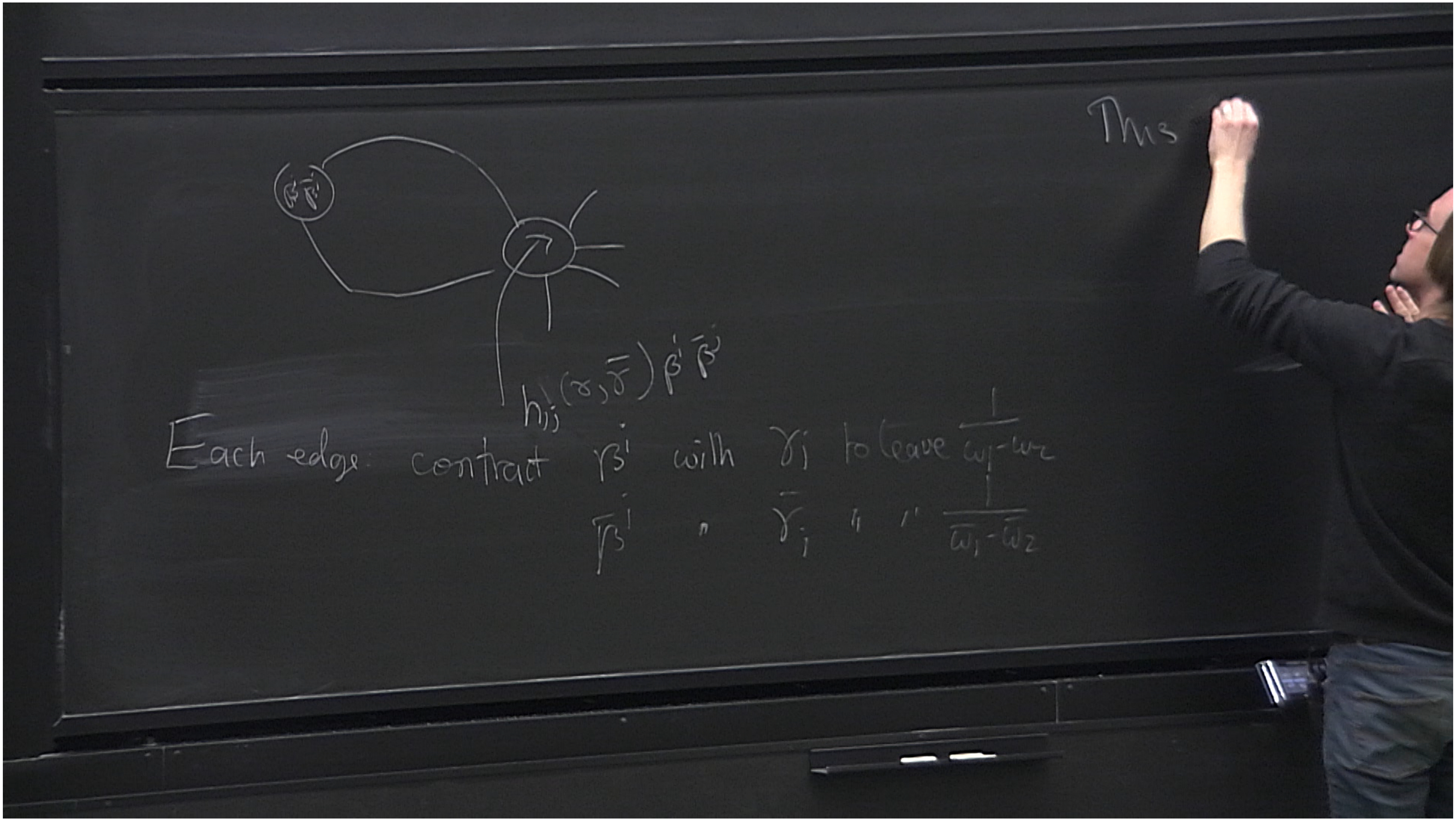
Fact

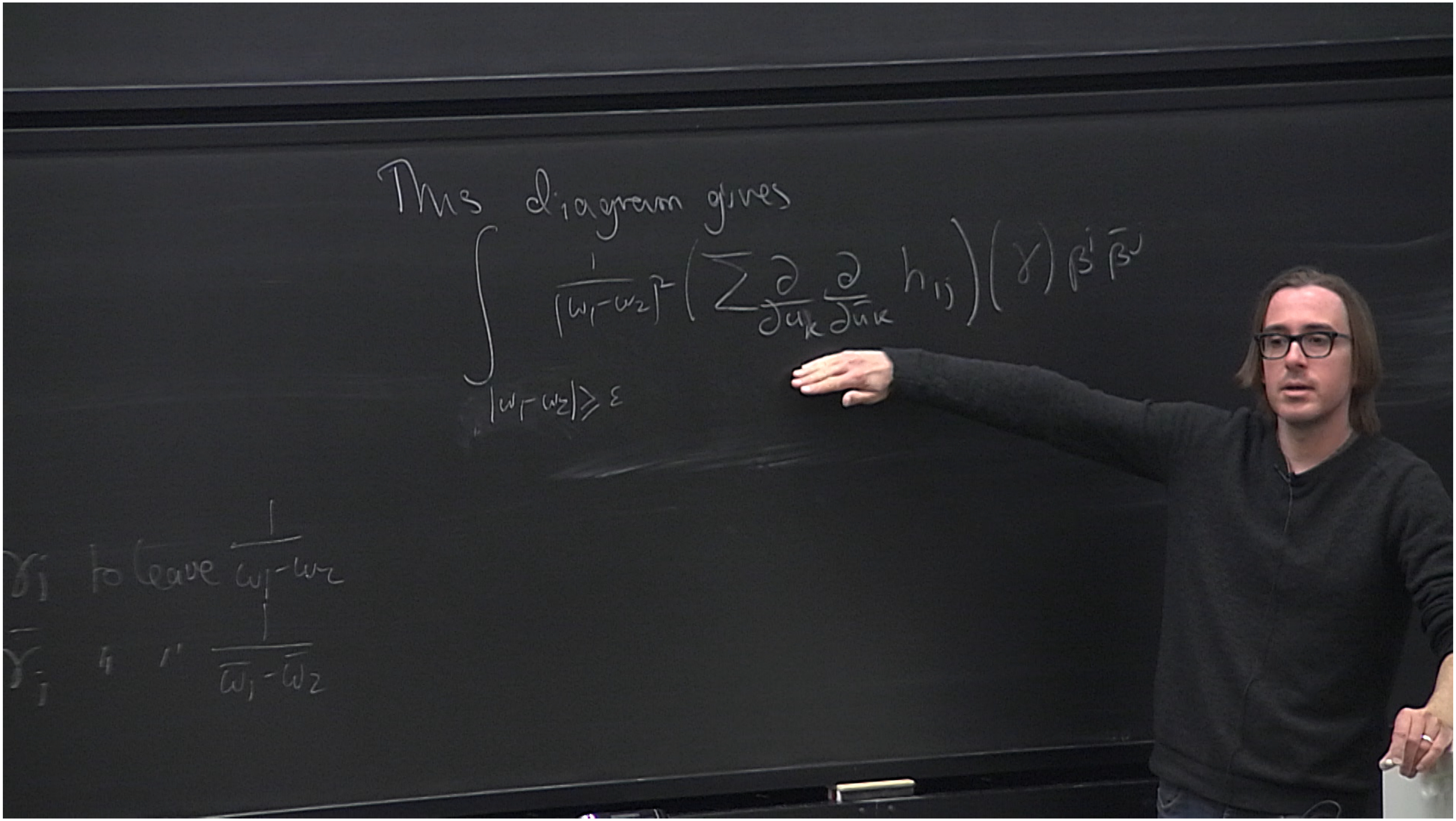
$$R_{ij}(u,\bar{u}=0) = C\sum\left(\frac{\partial}{\partial u_k}\frac{\partial}{\partial \bar{u}_{\bar{k}}} h_{ij}\right)(u,\bar{u}=0)$$

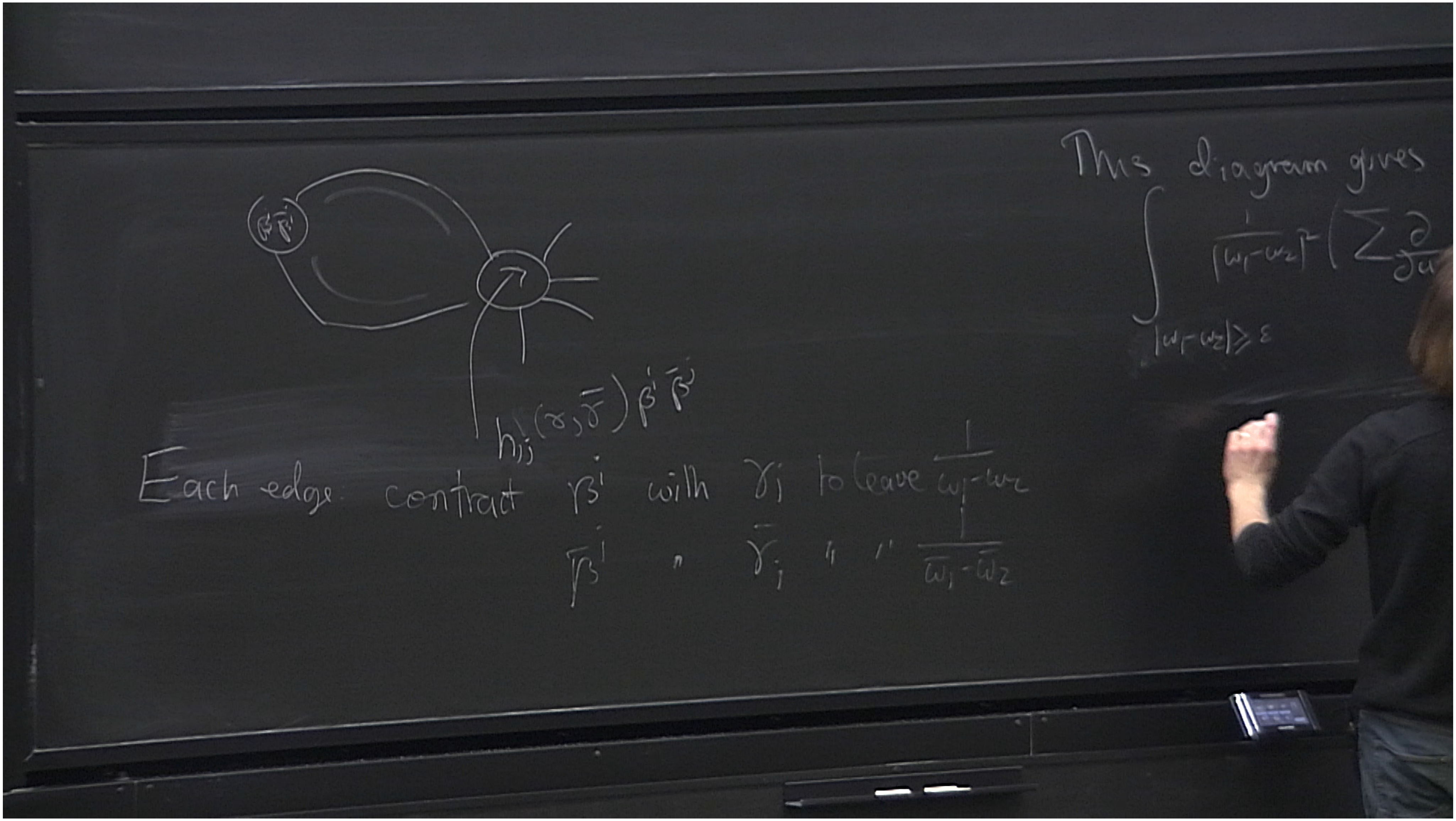
C a constant.

When we compute β-fn, to first order in h, the only diagram that contributes is


$h_{ij}(\gamma,\bar{\gamma})\beta^i\bar{\beta}^j$


This
$\beta^i \bar\beta^i$
$h_{ij}(\gamma,\bar\gamma)\,\beta^i\bar\beta^j$
Each edge: contract β^i with γ_i to leave $\frac{1}{\omega_1-\omega_2}$
$\bar\beta^i$ " $\bar\gamma_i$ " " $\frac{1}{\bar\omega_1-\bar\omega_2}$


This diagram gives
$\int_{|w_1-w_2|\geq\varepsilon} \frac{1}{|w_1-w_2|^2}\left(\sum \frac{\partial}{\partial u_k}\frac{\partial}{\partial \bar{u}_k} h_{ij}\right)(\gamma)\beta^i\bar{\beta}^j$
γ_i to leave $\frac{1}{w_1-w_2}$
$\bar{\gamma}_j$ " " $\frac{1}{\bar{w}_1-\bar{w}_2}$


$h_{ij}(\gamma,\bar\gamma)\beta^i\bar\beta^j$
Each edge: contract β^i with γ_i to leave $\frac{1}{w_1-w_2}$
$\bar\beta^i$ " $\bar\gamma_i$ " " $\frac{1}{\bar w_1-\bar w_2}$
This diagram gives
$\int_{|w_1-w_2|\geq\varepsilon} \frac{1}{|w_1-w_2|^2}\left(\sum \frac{\partial}{\partial u}\right.$

This diagram gives

$$\int_{|w_1-w_2|\geq \varepsilon} \frac{1}{|w_1-w_2|^2}\left(\sum \frac{\partial}{\partial u_k}\frac{\partial}{\partial \bar{u}_k} h_{ij}\right)(\gamma)\,\beta^i\bar{\beta}^j$$

$$= (\log \varepsilon)\, Ric^{(0)}_{ij}\beta^i\bar{\beta}^j + (\text{non-sing terms}) + (\text{terms depending on } \gamma)$$

ase $\frac{1}{w_1-w_2}$

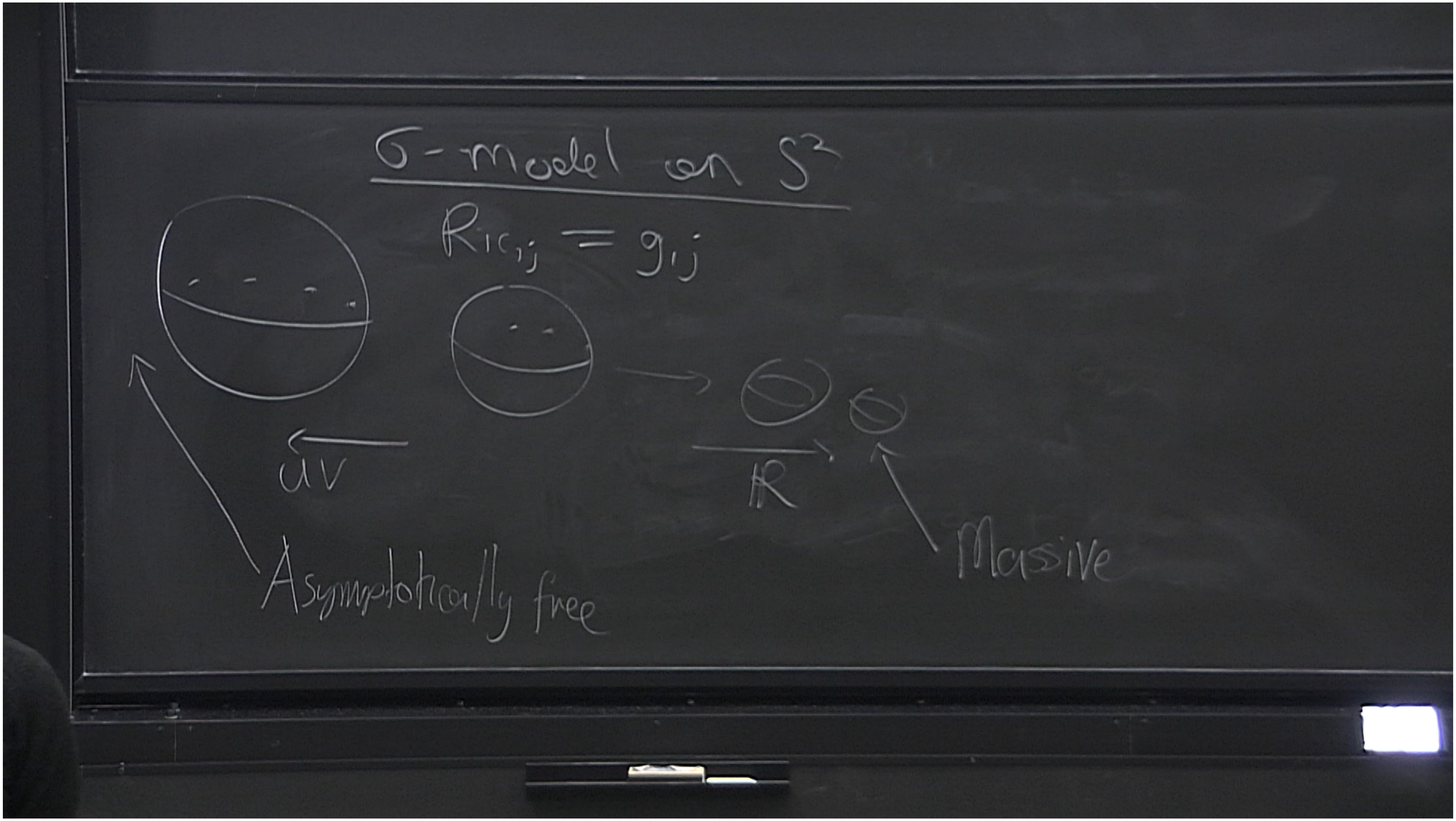
$\frac{1}{\bar{w}_1-\bar{w}_2}$

This diagram gives

$$\int_{|w_1-w_2|\geq \varepsilon} \frac{1}{|w_1-w_2|^2}\left(\sum \frac{\partial}{\partial u_k}\frac{\partial}{\partial \bar{u}_k} h_{ij}\right)(\gamma)\,\beta^i\bar{\beta}^j$$

$$= (\log \varepsilon)\, \mathrm{Ricci}_{ij}(0)\,\beta^i\bar{\beta}^j + (\text{non-sing terms}) + (\text{terms depending on } \gamma)$$

In fact β-fn = Ricci Everywhere


σ-model on S^2
$Ric_{ij} = g_{ij}$
UV
IR
Massive
Asymptotically free

Similarly, Gross-Neveu / Thirring models are expected to be massive.

BUT Massive particles are non-perturbative

Similarly, Gross-Neveu / Thirring models are expected to be massive.

But Massive particles are non-perturbative

Euclidean Signature

Signature of massive theory is that $\mathcal{O}$ creates a particle

then $\langle \mathcal{O}(0), \mathcal{O}(x) \rangle \sim e^{-m^2 |x|^2}$