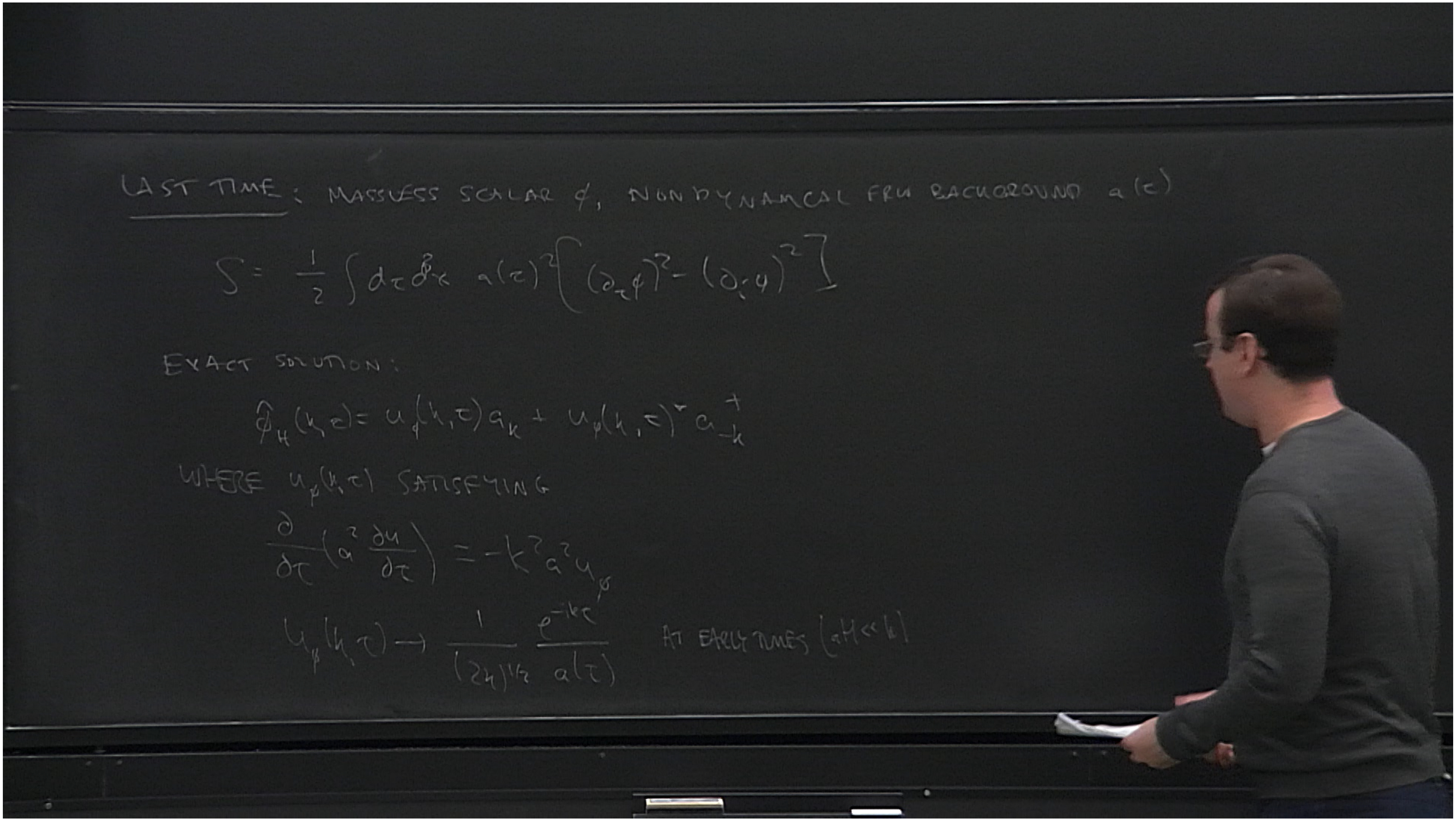
Title: PSI 2017/2018 - Cosmology - Lecture 15

Date: Apr 27, 2018 10:15 AM

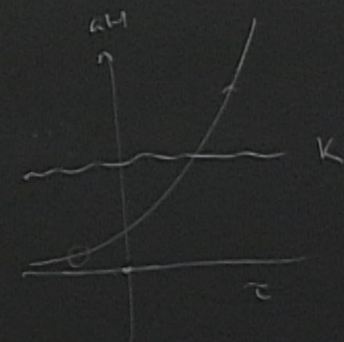
URL: http://pirsa.org/18040027

Abstract:


LAST TIME: MASSLESS SCALAR φ, NONDYNAMICAL FRW BACKGROUND a(τ)
S = 1/2 ∫ dτ d³x a(τ)² [(∂τ φ)² − (∂i φ)²]
EXACT SOLUTION:
φ̂k(k,τ) = uφ(k,τ) ak + uφ(k,τ)* a†−k
WHERE uφ(k,τ) SATISFYING
∂/∂τ (a² ∂u/∂τ) = −k² a² uφ
uφ(k,τ) → 1/(2k)^1/2 e^−ikτ / a(τ)   AT EARLY TIMES (aH << k)

LAST TIME: MASSLESS SCALAR ϕ, NONDYNAMICAL FRW BACKGROUND $a(\tau)$

$$S = \frac{1}{2}\int d\tau\, d^3x\, a(\tau)^2\left[(\partial_\tau\phi)^2 - (\partial_i\phi)^2\right]$$



EXACT SOLUTION:

$$\hat{\phi}_k(k,\tau) = u_\phi(k,\tau)\, a_k + u_\phi(k,\tau)^*\, a^\dagger_{-k}$$

WHERE $u_\phi(k,\tau)$ SATISFYING

$$\frac{\partial}{\partial\tau}\left(a^2 \frac{\partial u}{\partial\tau}\right) = -k^2 a^2 u_\phi$$

$$u_\phi(k,\tau) \rightarrow \frac{1}{(2k)^{1/2}} \frac{e^{-ik\tau}}{a(\tau)}$$ AT EARLY TIMES $(aH \ll k)$

AT

AT LATE TIMES $(aH \gg k)$ $\quad u_\varphi(k,\tau) \to A(k)$ [CONSTANT IN τ]

LATE POWER SPECTRUM: $\quad P(k) = |A(k)|^2$

AT LATE TIMES ($aH \gg k$) $\quad u_\phi(k,\tau) \to A(k)$ [CONSTANT IN τ]

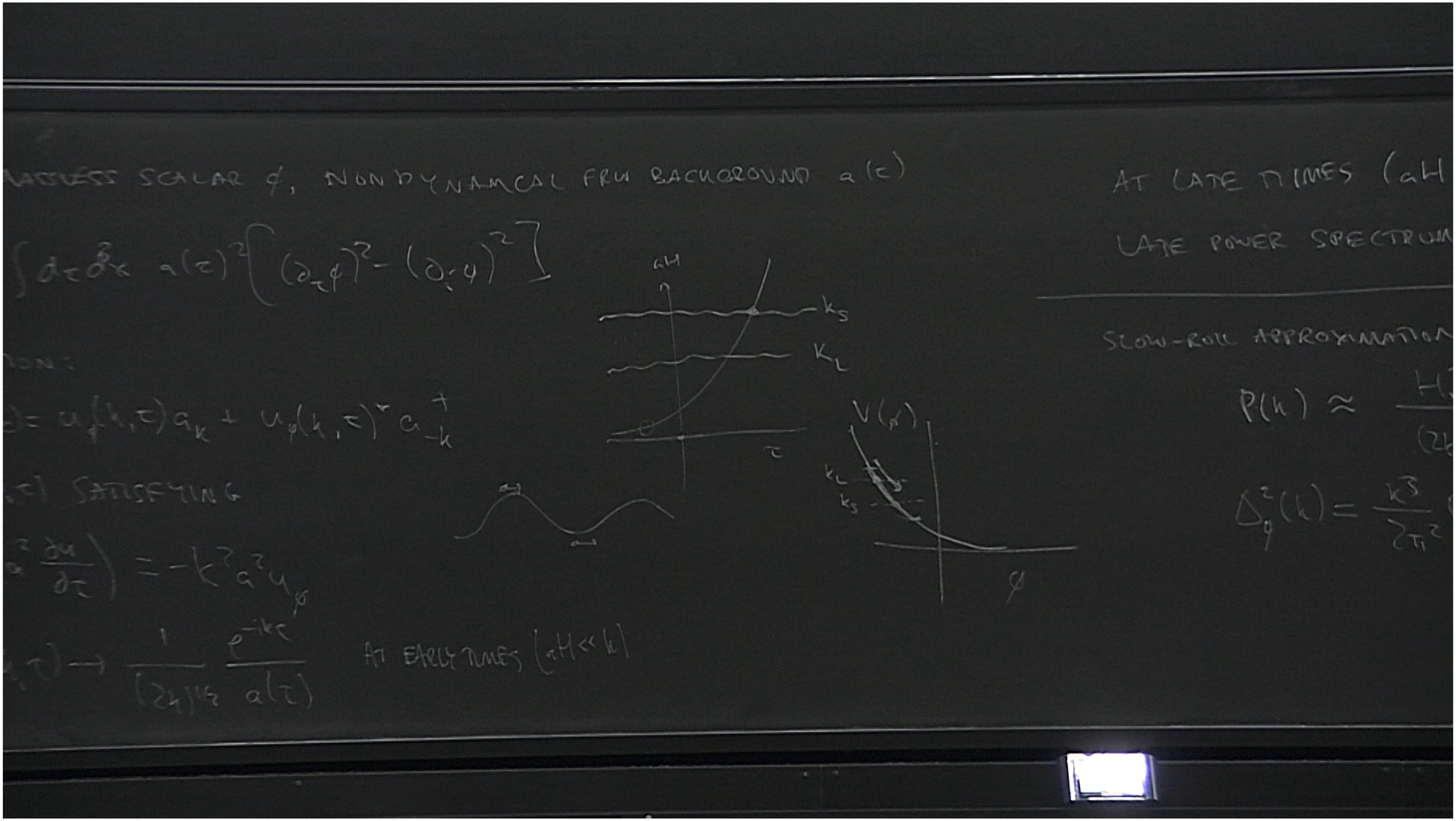
LATE POWER SPECTRUM: $\quad P(k) = |A(k)|^2$

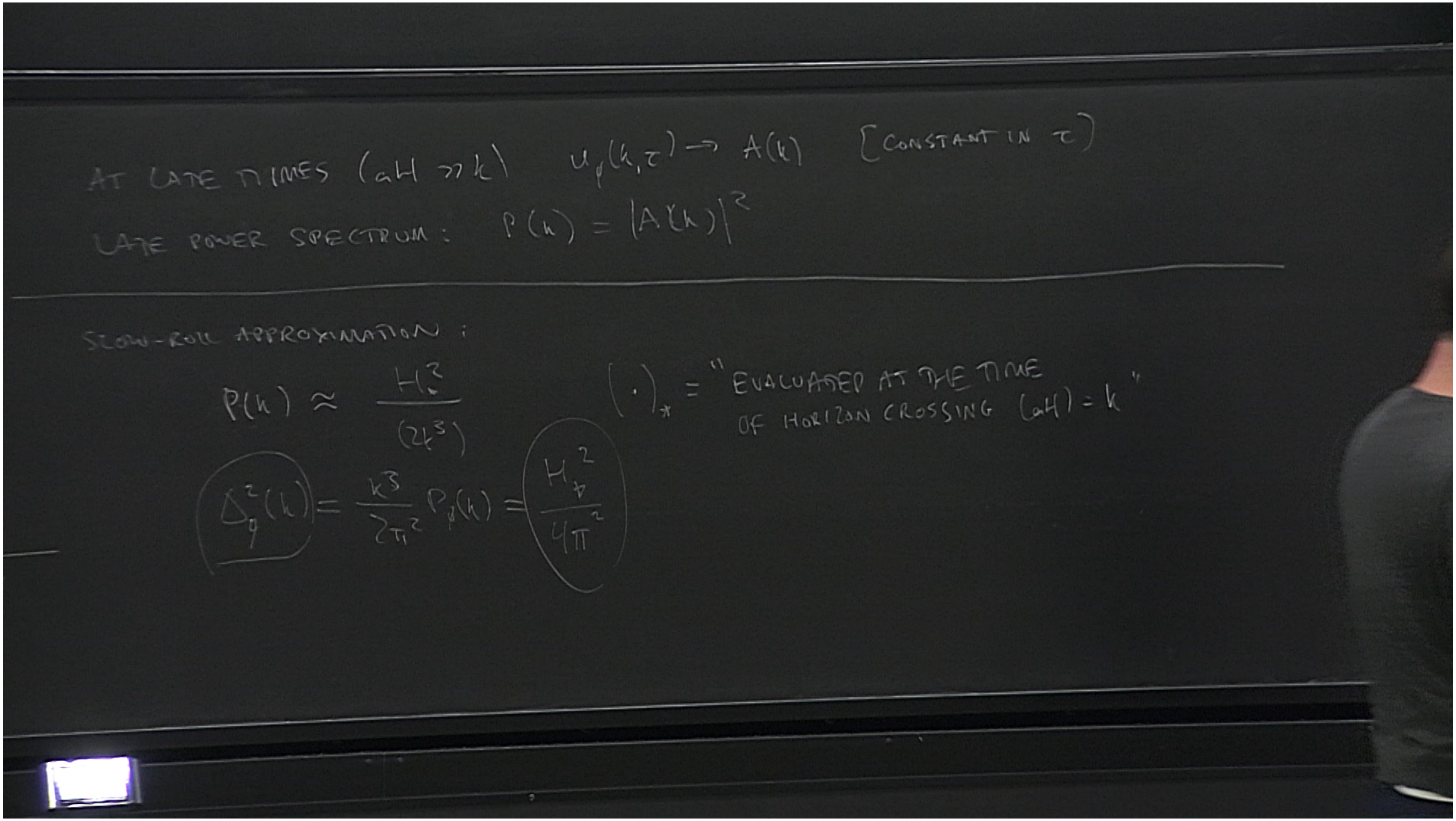
SLOW-ROLL APPROXIMATION:

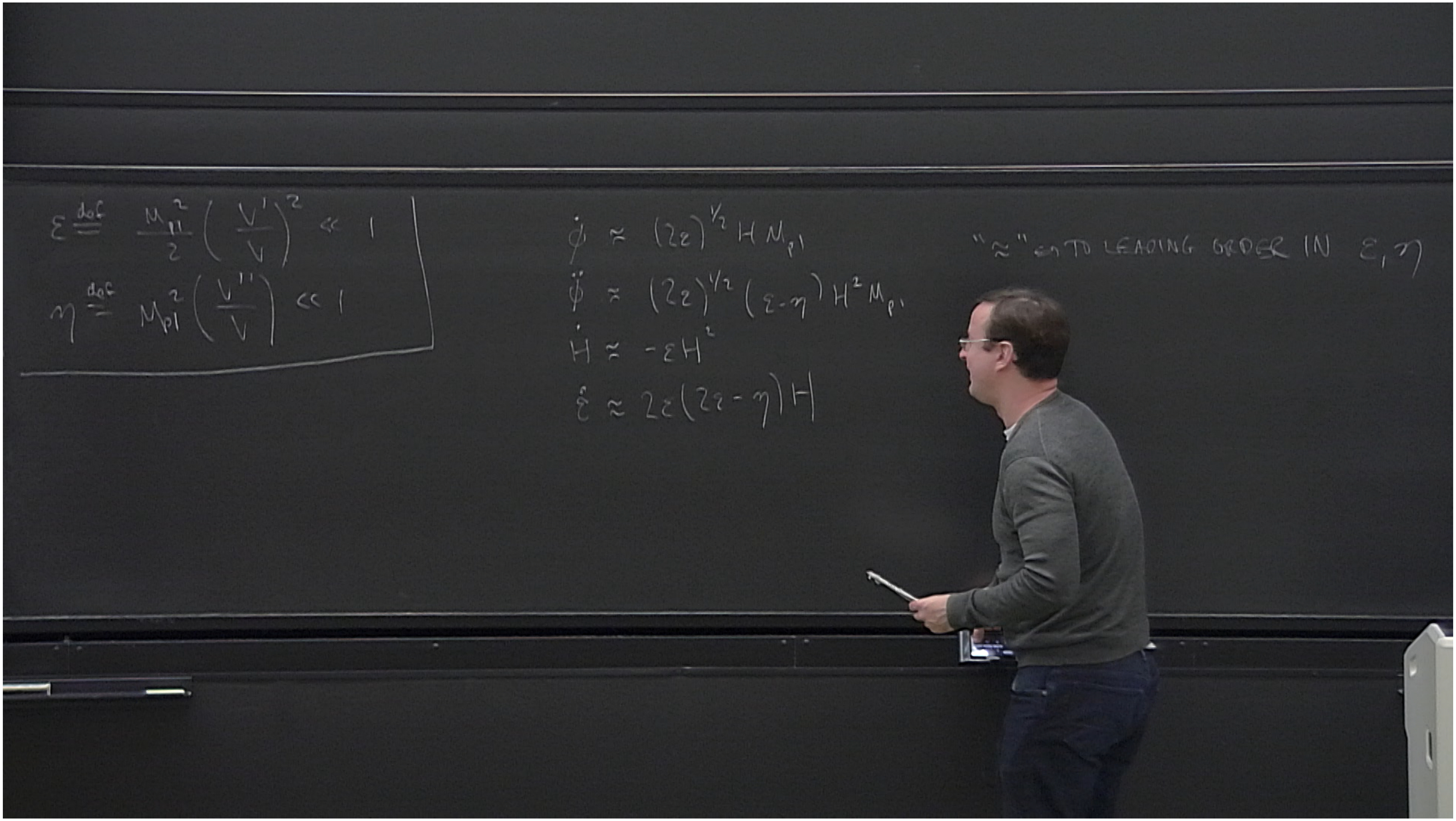
$$P(k) \approx \frac{H_*^2}{(2k^3)}$$

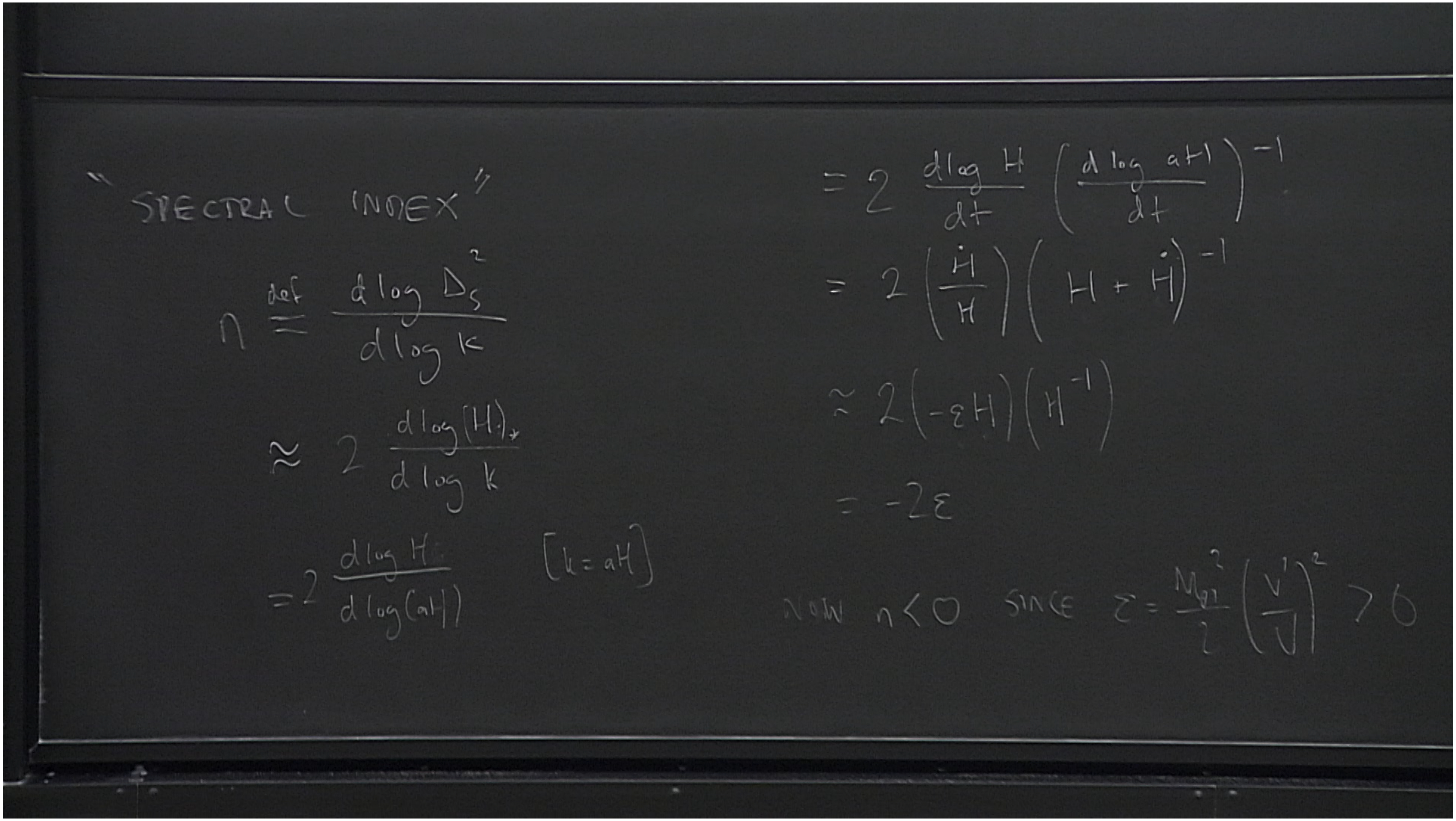
$(\cdot)_* =$ "EVALUATED AT THE TIME OF HORIZON CROSSING $(aH) = k$"

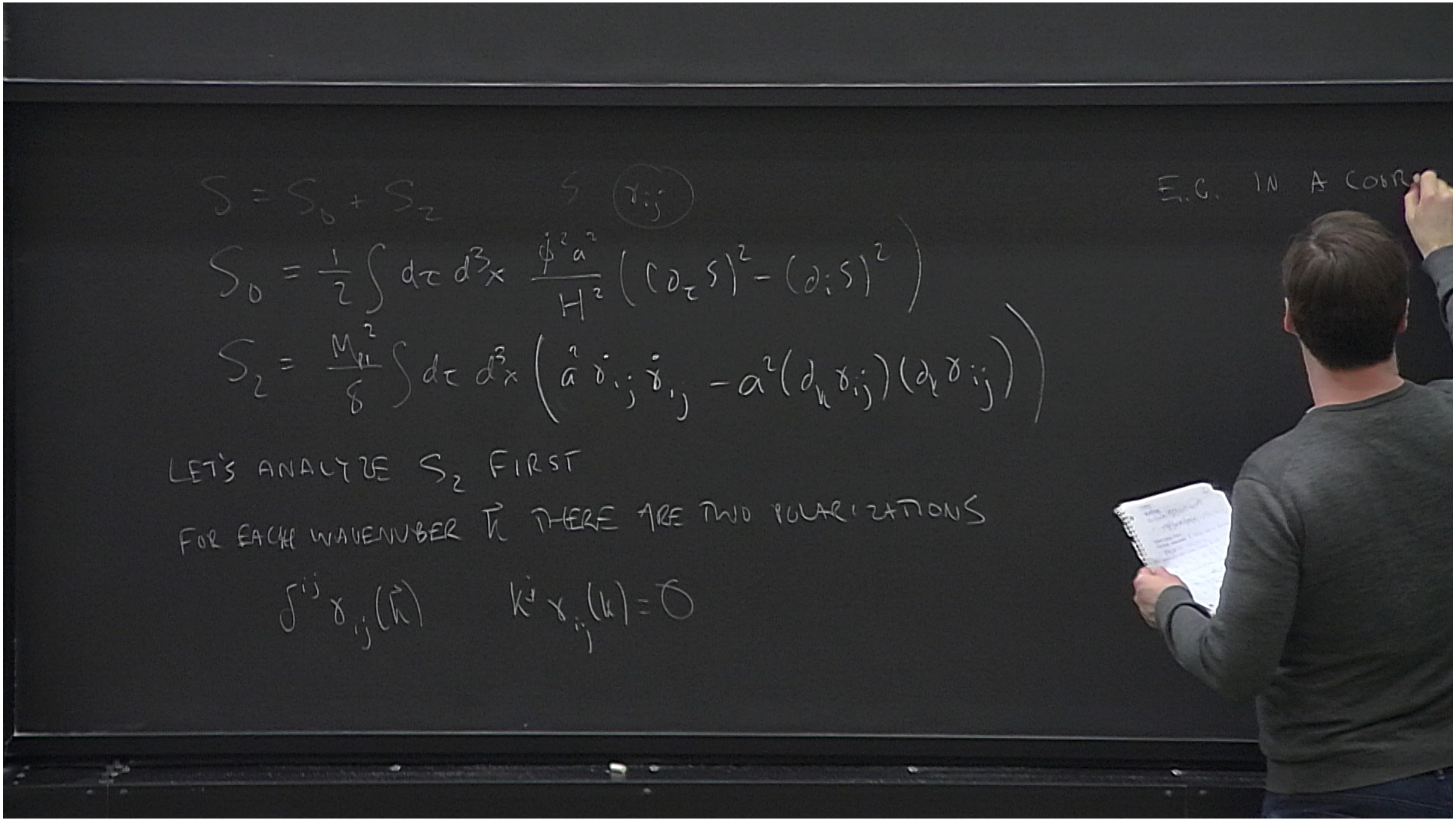
$$\Delta_\phi^2(k) = \frac{k^3}{2\pi^2} P_\phi(k) = \frac{H_*^2}{4\pi^2}$$


MASSLESS SCALAR φ, NONDYNAMICAL FRW BACKGROUND a(τ)
∫dτ d³x a(τ)² [(∂_τφ)² − (∂_iφ)²]
aH
k_S
k_L
τ
= u_φ(k,τ) a_k + u_φ(k,τ)* a†_{-k}
SATISFYING
= −k² a² u_φ
→ 1/(2k)^{1/2} e^{−ikτ}/a(τ)
AT EARLY TIMES (aH << k)
V(φ)
k_L
k_S
φ
AT LATE TIMES (aH
LATE POWER SPECTRUM
SLOW-ROLL APPROXIMATION
P(k) ≈
Δ²_φ(k) = k³/2π²


AT LATE TIMES $(aH \gg k)$ $u_\phi(k,\tau) \to A(k)$ [CONSTANT IN τ]
LATE POWER SPECTRUM: $P(k) = |A(k)|^2$
SLOW-ROLL APPROXIMATION:
$P(k) \approx \frac{H_*^2}{(2k^3)}$
$(\cdot)_* =$ "EVALUATED AT THE TIME OF HORIZON CROSSING $(aH) = k$"
$\Delta_\phi^2(k) = \frac{k^3}{2\pi^2} P_\phi(k) = \frac{H_*^2}{4\pi^2}$


$\varepsilon \stackrel{\text{def}}{=} \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$
$\eta \stackrel{\text{def}}{=} M_{pl}^2\left(\frac{V''}{V}\right) \ll 1$
$\dot\phi \approx (2\varepsilon)^{1/2} H M_{pl}$
$\ddot\phi \approx (2\varepsilon)^{1/2}(\varepsilon-\eta) H^2 M_{pl}$
$\dot H \approx -\varepsilon H^2$
$\dot\varepsilon \approx 2\varepsilon(2\varepsilon-\eta) H$
"$\approx$" ⇒ TO LEADING ORDER IN ε, η


"SPECTRAL INDEX"
$n \overset{def}{=} \frac{d \log \Delta_s^2}{d \log k}$
$\approx 2 \frac{d \log (H)_*}{d \log k}$
$= 2 \frac{d \log H}{d \log (aH)}$ $[k = aH]$
$= 2 \frac{d \log H}{dt} \left(\frac{d \log aH}{dt} \right)^{-1}$
$= 2 \left(\frac{\dot{H}}{H} \right) \left(H + \dot{H} \right)^{-1}$
$\approx 2(-\varepsilon H)(H^{-1})$
$= -2\varepsilon$
NOW $n < 0$ SINCE $\varepsilon = \frac{M_{pl}^2}{2} \left(\frac{V'}{V} \right)^2 > 0$


$S = S_0 + S_2$
ζ γ_{ij}
$S_0 = \frac{1}{2}\int d\tau\, d^3x\, \frac{\dot\phi^2 a^2}{H^2}\left((\partial_\tau \zeta)^2 - (\partial_i \zeta)^2\right)$
$S_2 = \frac{M_{pl}^2}{8}\int d\tau\, d^3x \left(a^2 \dot\gamma_{ij}\dot\gamma_{ij} - a^2(\partial_k \gamma_{ij})(\partial_k \gamma_{ij})\right)$
LET'S ANALYZE S_2 FIRST
FOR EACH WAVENUBER $\vec{k}$ THERE ARE TWO POLARIZATIONS
$\delta^{ij}\gamma_{ij}(\vec{k})$ $k^i\gamma_{ij}(k) = 0$
E.G. IN A COOR

E.G. IN A COORDINATE SYSTEM WHERE $\vec{k} = k\hat{z}$

$$\varepsilon^{(1)}_{ij} = \begin{pmatrix} 1/2 & 0 & \\ 0 & -1/2 & \\ & & 0 \end{pmatrix} \qquad \varepsilon^{(2)}_{ij} = \begin{pmatrix} 0 & 1/2 & \\ 1/2 & 0 & \\ & & 0 \end{pmatrix}$$

SAME NO

$$\gamma_{ij}(k,\tau) = h_1(k,\tau)\,\varepsilon^{(1)}_{ij}(k) + h_2(k,\tau)\,\varepsilon^{(2)}_{ij}(k)$$

$$\gamma^*_{ij}\gamma_{ij} = \frac{1}{2}\left[|h_1|^2 + |h_2|^2\right]$$ ← COSMOLOGISTS NORMALIZATION.

$$S_2 = \frac{M_{pl}^2}{16}\int d\tau \frac{d^3k}{(2\pi)^2}\sum_{s=1}^{2} a(\tau)^2\left[\dot{h}_s(k,\tau)^*\,\dot{h}_s(k,\tau) - k^2\, h_s(k,\tau)^*\, h_s(k,\tau)\right]$$

C. IN A COORDINATE SYSTEM WHERE $\vec{k} = k\hat{z}$

$$\varepsilon^{(1)}_{ij} = \begin{pmatrix} 1/2 & 0 & \\ 0 & -1/2 & \\ & & 0 \end{pmatrix} \qquad \varepsilon^{(2)}_{ij} = \begin{pmatrix} 0 & 1/2 & \\ 1/2 & 0 & \\ & & 0 \end{pmatrix}$$

$$\gamma_{ij}(k,\tau) = h_1(k,\tau)\,\varepsilon^{(1)}_{ij}(k) + h_2(k,\tau)\,\varepsilon^{(2)}_{ij}(k)$$

$$\gamma^*_{ij}\gamma_{ij} = \frac{1}{2}\left[|h_1|^2 + |h_2|^2\right]$$ ← COSMOLOGISTS NORMALIZATION.

$$S_2 = \frac{M_{pl}^2}{16}\int d\tau \frac{d^3k}{(2\pi)^3}\sum_{s=1}^{2} a(\tau)^2\left[\dot{h}_s(k,\tau)^*\dot{h}_s(k,\tau) - k^2 h_s(k,\tau)^* h_s(k,\tau)\right]$$

SAME ACTION AS NONDYNAMICAL SCALAR W/ FACTOR $\frac{M_{pl}^2}{8}$

$$u_\phi(k,\tau) \to \frac{1}{(2k)^{1/2}} \frac{e^{-ik\tau}}{a(\tau)}$$ AT EARLY TIMES $[aH \ll k]$

$$\Rightarrow \Delta_h^2(k) = \frac{8}{M_{pl}^2}\left(\frac{H_*^2}{4\pi^2}\right)$$

$$n_t = \frac{d \log \Delta_h^2(k)}{d \log k} = -2\varepsilon < 0$$

GW SPECTRAL INDEX
OR "TENSOR SPECTRAL INDEX"

$$[\phi, p_\phi] = i$$

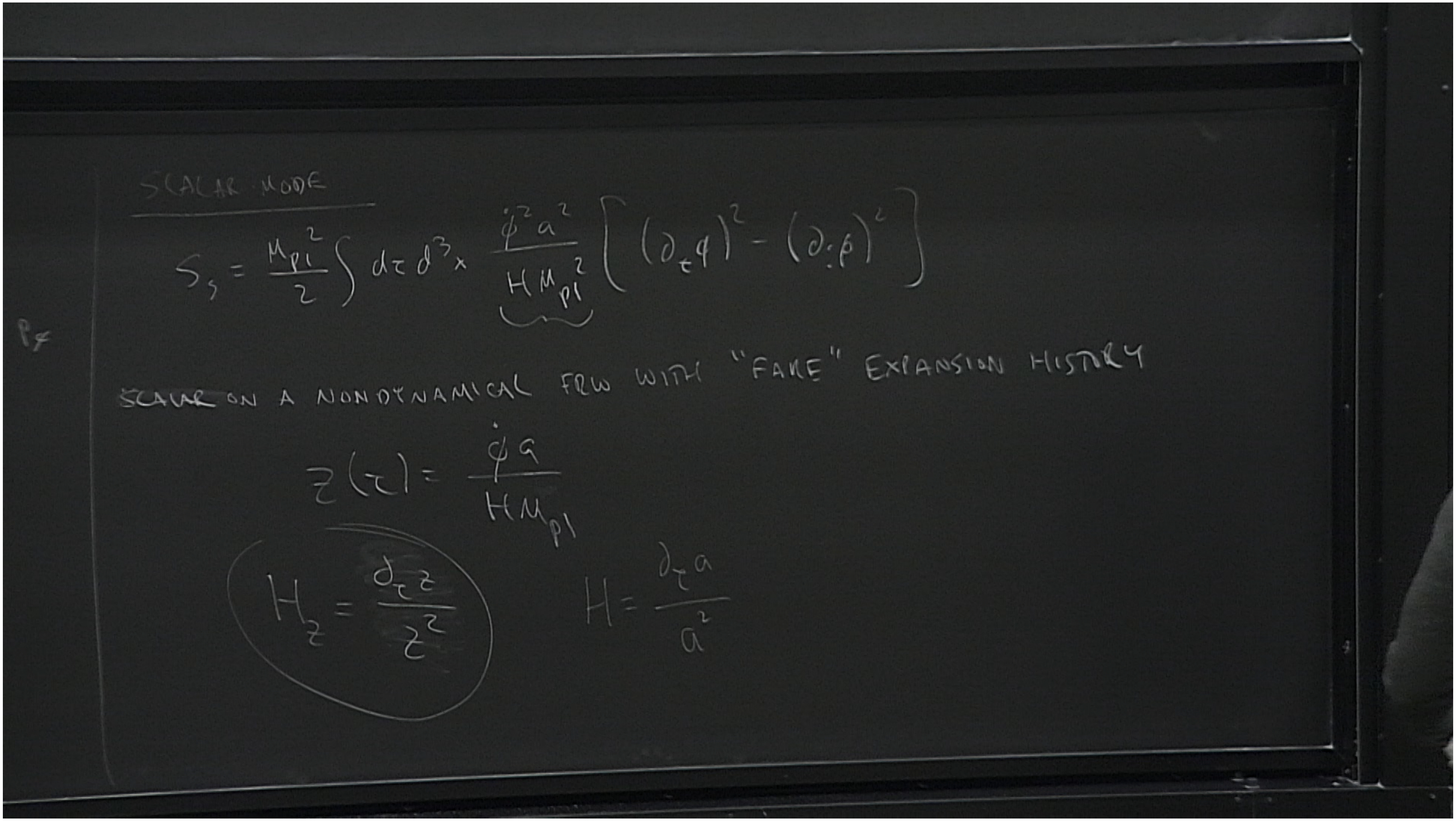
$$\to \frac{M_{pl}^2}{8}[\phi, p_\phi] = i$$

$$u \to \left(\frac{M_{pl}^2}{8}\right)^{-1/2} u$$

$$P(k) = |u|^2 \to \left(\frac{M_{pl}^2}{8}\right)^{-1} P(k)$$

$$p_\phi = \frac{\partial S}{\partial \phi}$$

$$p_\phi \to \frac{M_{pl}^2}{8} p_\phi$$


SCALAR MODE
$S_\zeta = \frac{M_{pl}^2}{2}\int d\tau\, d^3x\, \frac{\dot\phi^2 a^2}{H M_{pl}^2}\left[(\partial_\tau \zeta)^2 - (\partial_i \zeta)^2\right]$
SCALAR ON A NONDYNAMICAL FRW WITH "FAKE" EXPANSION HISTORY
$z(\tau) = \frac{\dot\phi a}{H M_{pl}}$
$H_z = \frac{\partial_\tau z}{z^2}$
$H = \frac{\partial_\tau a}{a^2}$

$$z = \frac{\dot{\phi} a}{H M_{pl}} \simeq (2\varepsilon)^{1/2} a$$

$$H_z = \frac{\partial_\tau z}{z^2}$$

$$= \frac{1}{z^2} a \frac{d}{dt}\left(\frac{\dot{\phi} a}{H M_{pl}}\right)$$

$$= \frac{1}{z^2} a \left(\frac{\ddot{\phi} a}{H} + \frac{\dot{\phi} a H}{H} - \frac{\dot{\phi} a \dot{H}}{H^2}\right)$$

$$\simeq \frac{a}{(2\varepsilon) a^2 M_{pl}} \left[(2\varepsilon)^{1/2} a H M_{pl} \right]$$

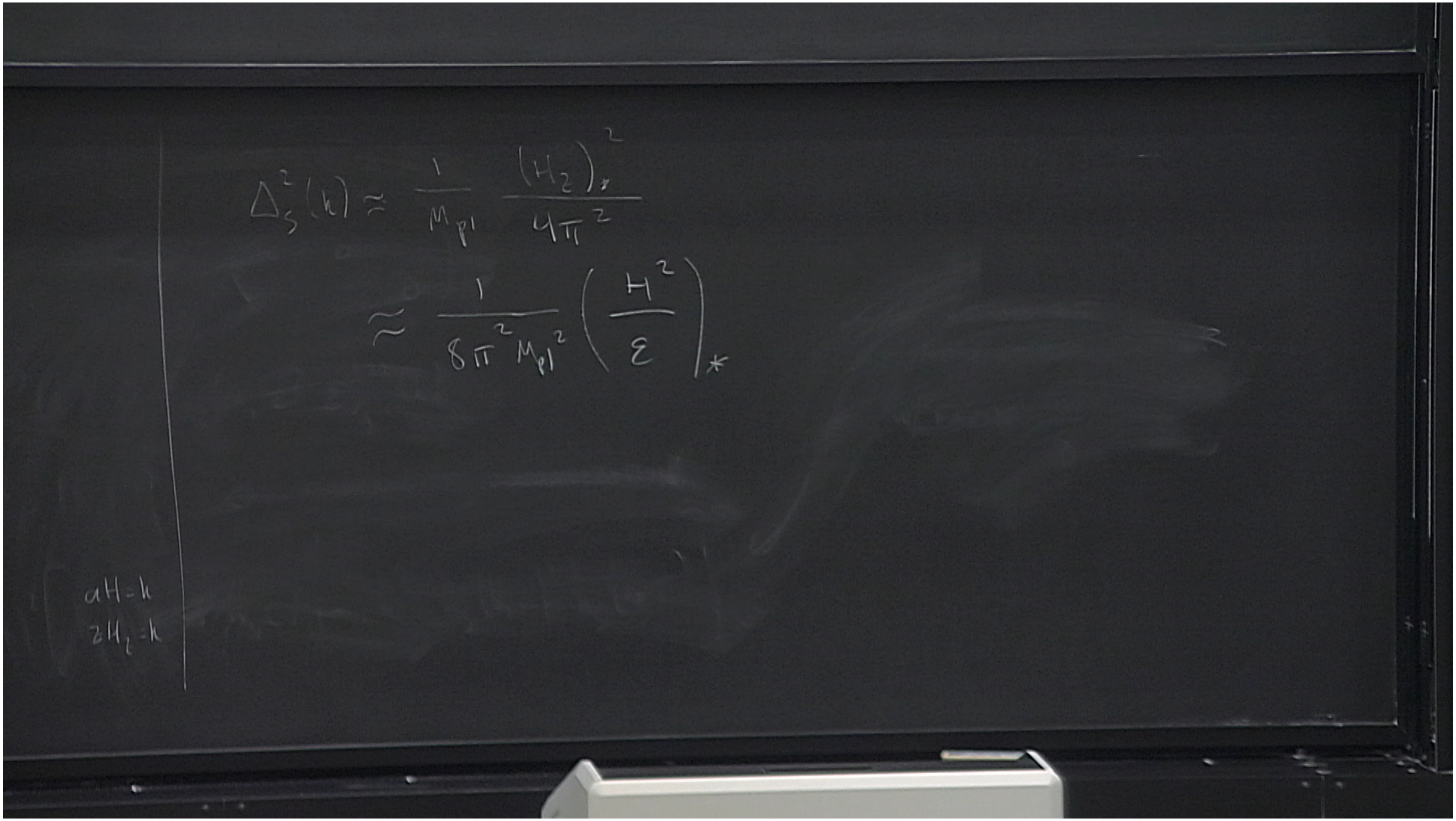
$$\simeq \frac{H}{(2\varepsilon)^{1/2}}$$

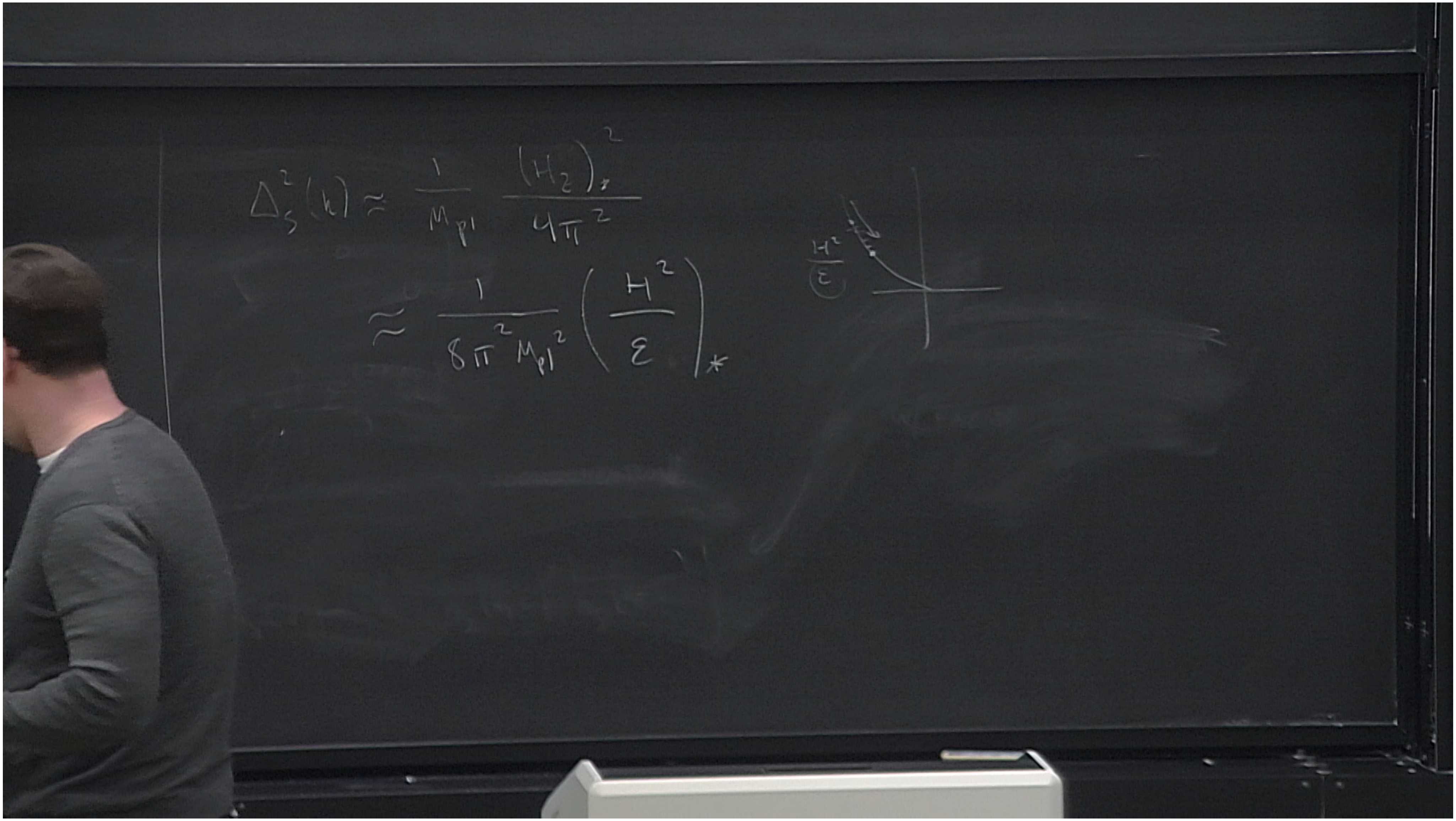
NOTE $aH \simeq z H_z$

ONE NOTION OF HORIZON CROSSING $(\cdot)_*$

$aH = k$

$z H_z = k$


$\Delta_s^2(k) \simeq \frac{1}{M_{pl}} \frac{(H_2)_*^2}{4\pi^2}$
$\approx \frac{1}{8\pi^2 M_{pl}^2} \left(\frac{H^2}{\varepsilon} \right)_*$
$aH = k$
$zH_2 = k$


$\Delta_s^2(k) \approx \frac{1}{M_{pl}} \frac{(H_2)_*^2}{4\pi^2}$
$\approx \frac{1}{8\pi^2 M_{pl}^2} \left(\frac{H^2}{\varepsilon}\right)_*$
$\frac{H^2}{\varepsilon}$

$$z^2 \left(H \quad H \quad H^2 \right)$$

ONE NOTION OF HORIZON CROSSING $(\ )_*$

"SCALAR SPECTRAL INDEX"

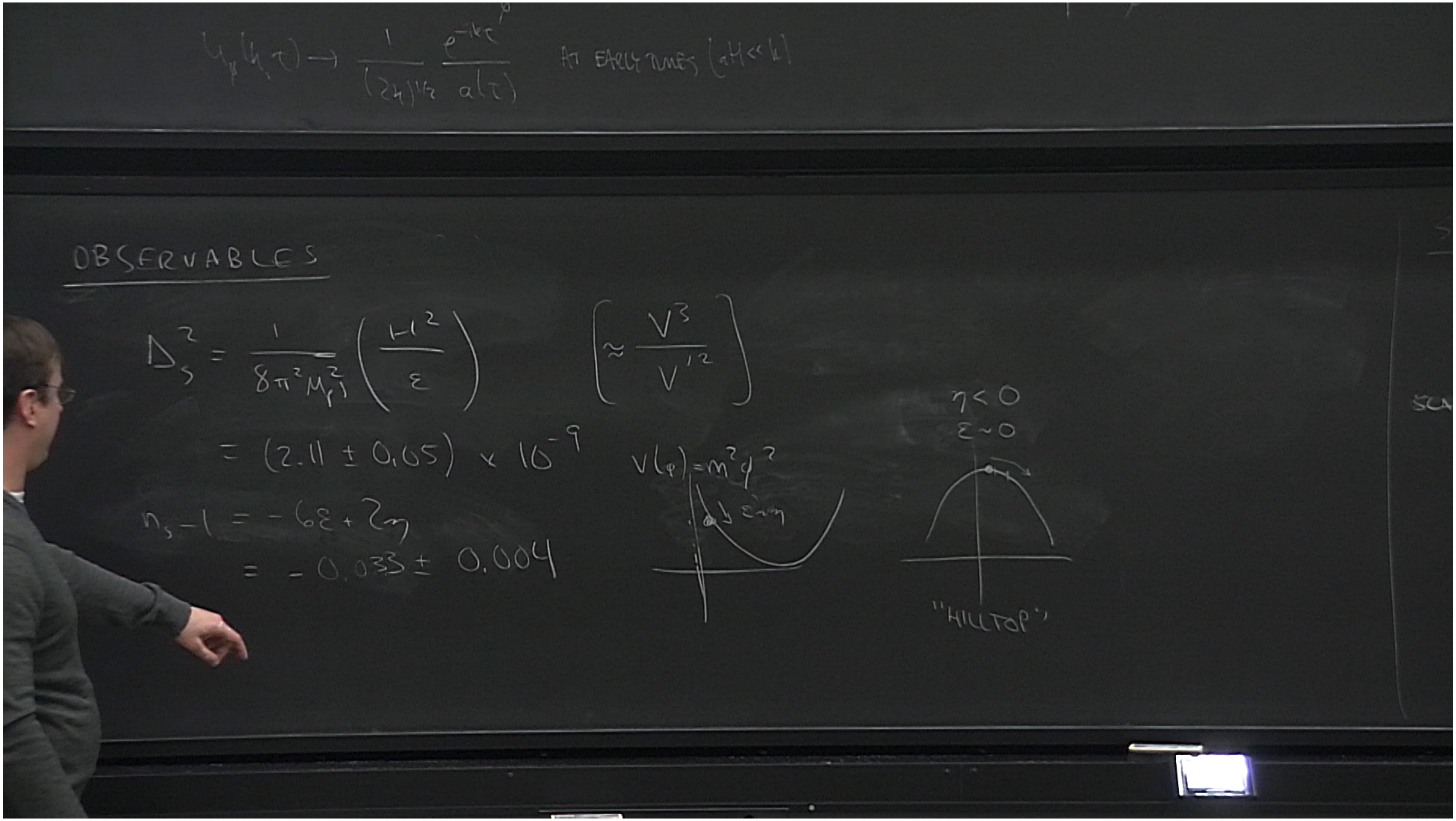
$$n_s - 1 = \frac{d\log\Delta^2_{\mathcal{R}}(k)}{d\log k} \approx \frac{d\log(H^2/\varepsilon)}{d\log(aH)}$$

↑ HISTORICAL!

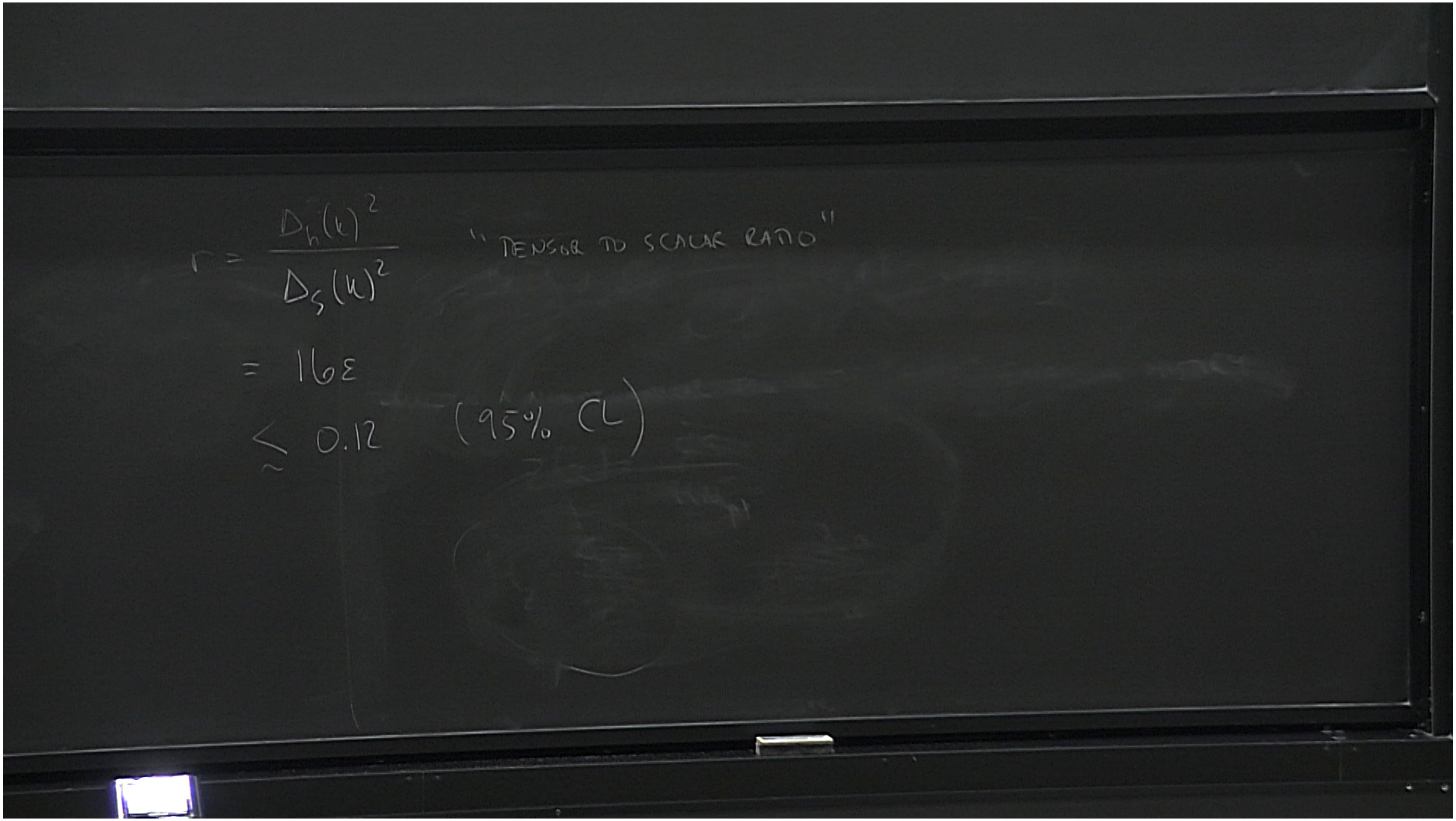
$$= \frac{d\log(H^2/\varepsilon)}{dt}\left(\frac{d\log(aH)}{dt}\right)^{-1}$$

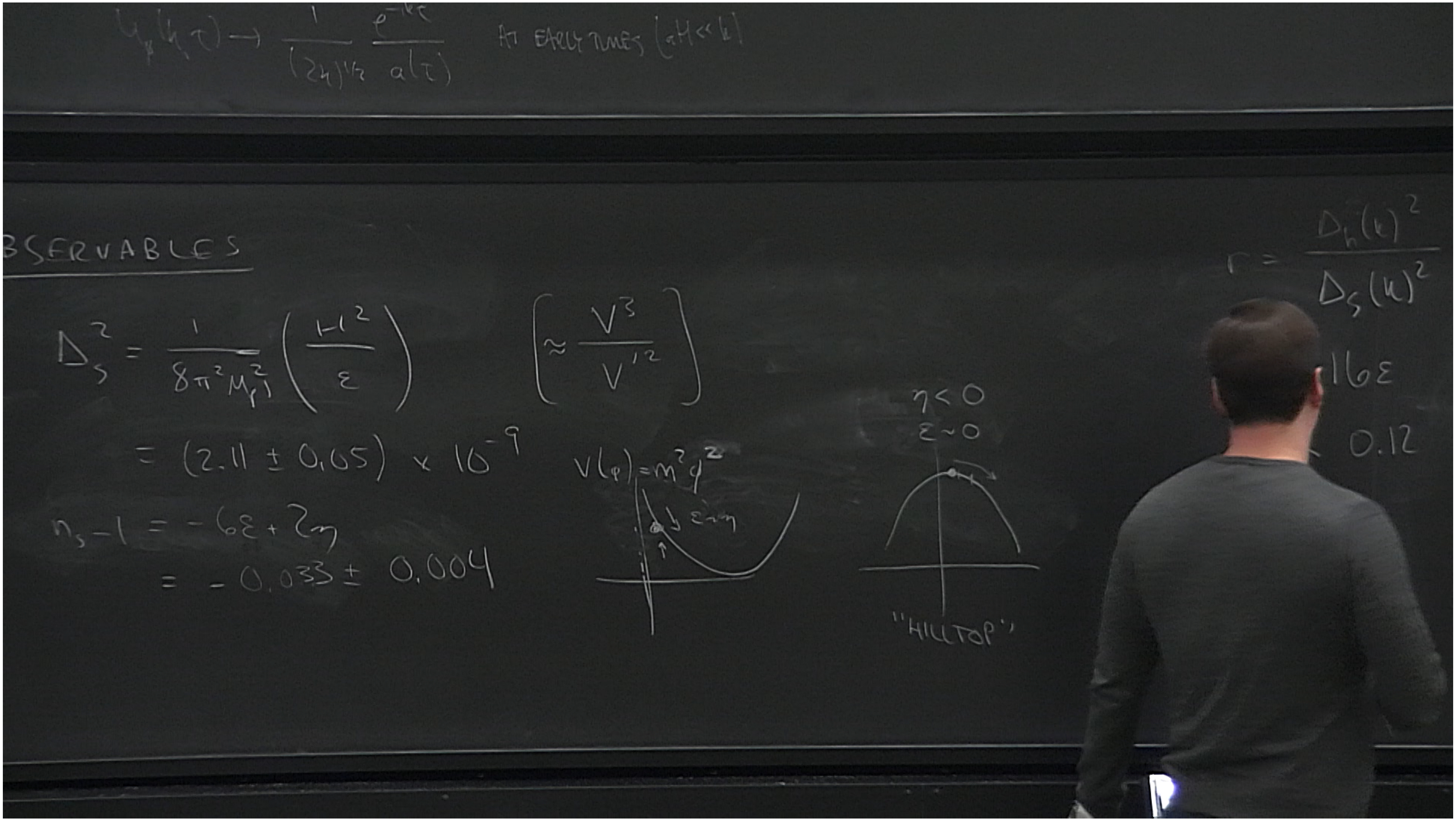
$$= \left[2\frac{\dot H}{H} - \frac{\dot\varepsilon}{\varepsilon}\right]\left[H + \dot H\right]^{-1}$$

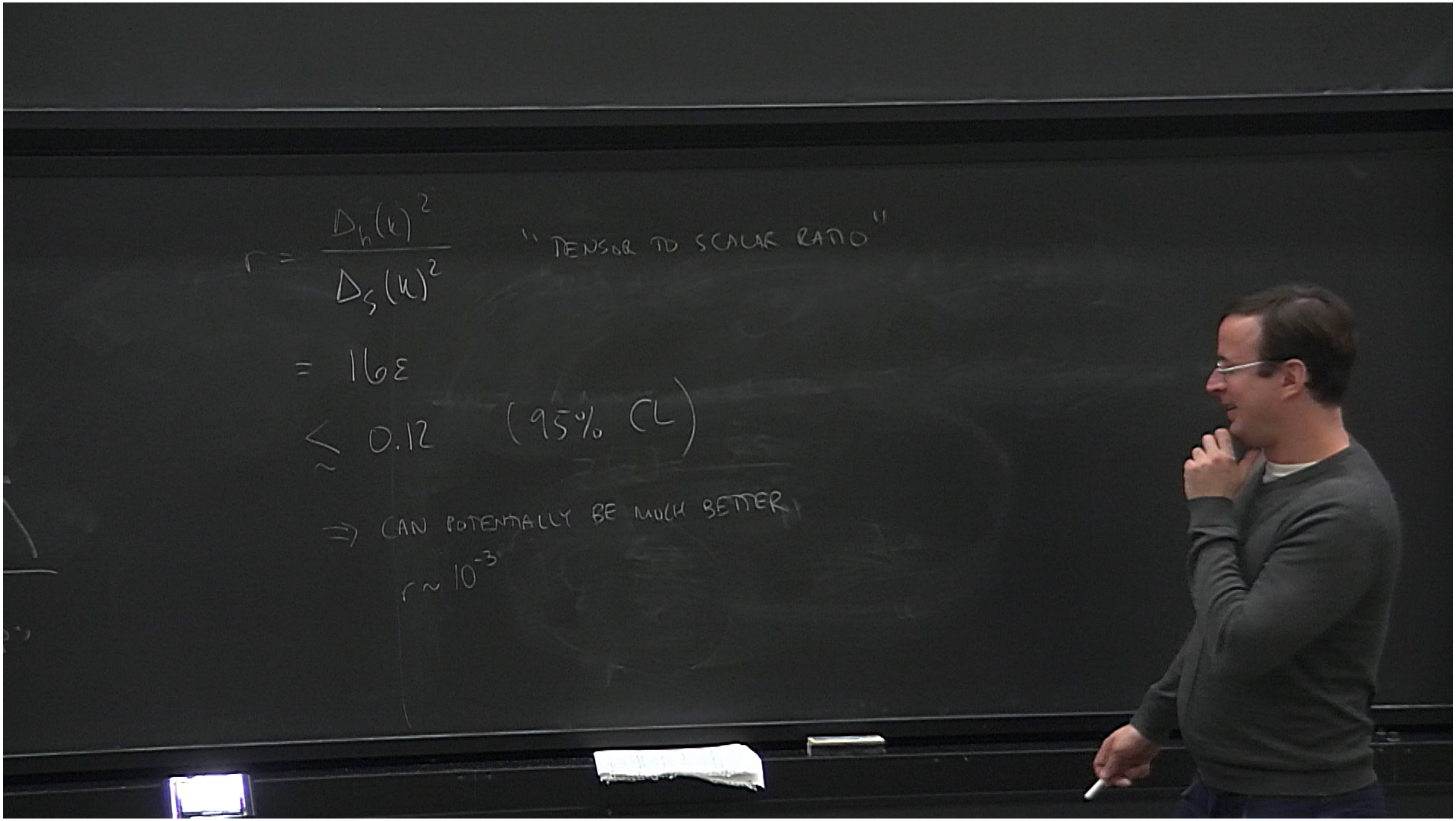
$$\approx \left[-2\varepsilon H - 2(2\varepsilon - \eta)H\right]H^{-1} = \boxed{-6\varepsilon + 2\eta}$$



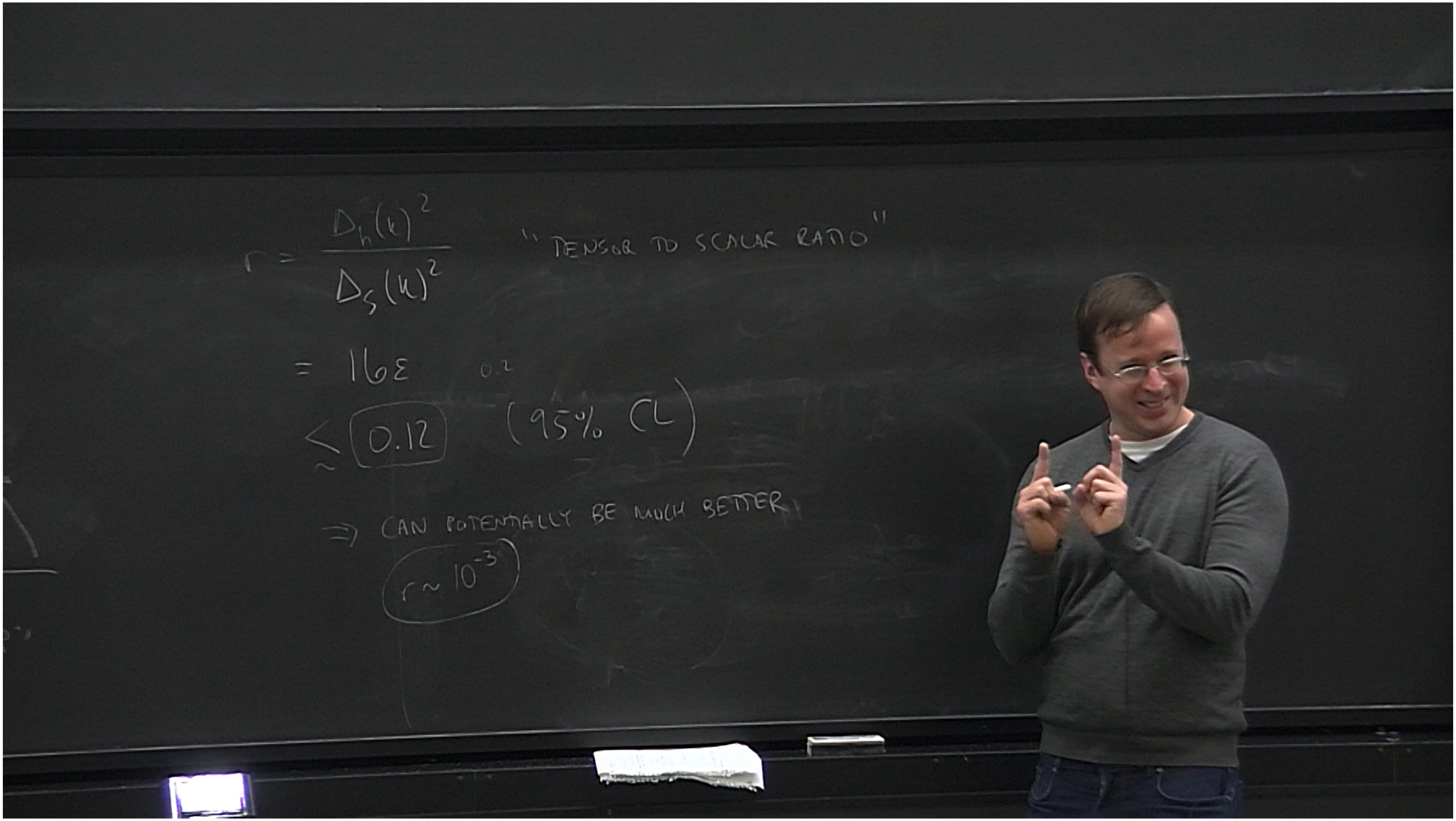
$u_\phi(k,\tau) \to \frac{1}{(2k)^{1/2}} \frac{e^{-ik\tau}}{a(\tau)}$ AT EARLY TIMES (aH << k)
OBSERVABLES
$\Delta_s^2 = \frac{1}{8\pi^2 M_{pl}^2}\left(\frac{H^2}{\varepsilon}\right)$ $\left(\approx \frac{V^3}{V'^2}\right)$
$= (2.11 \pm 0.05) \times 10^{-9}$
$n_s - 1 = -6\varepsilon + 2\eta$
$= -0.033 \pm 0.004$
$V(\phi) = m^2\phi^2$
$\varepsilon \sim \eta$
$\eta < 0$
$\varepsilon \sim 0$
"HILLTOP"

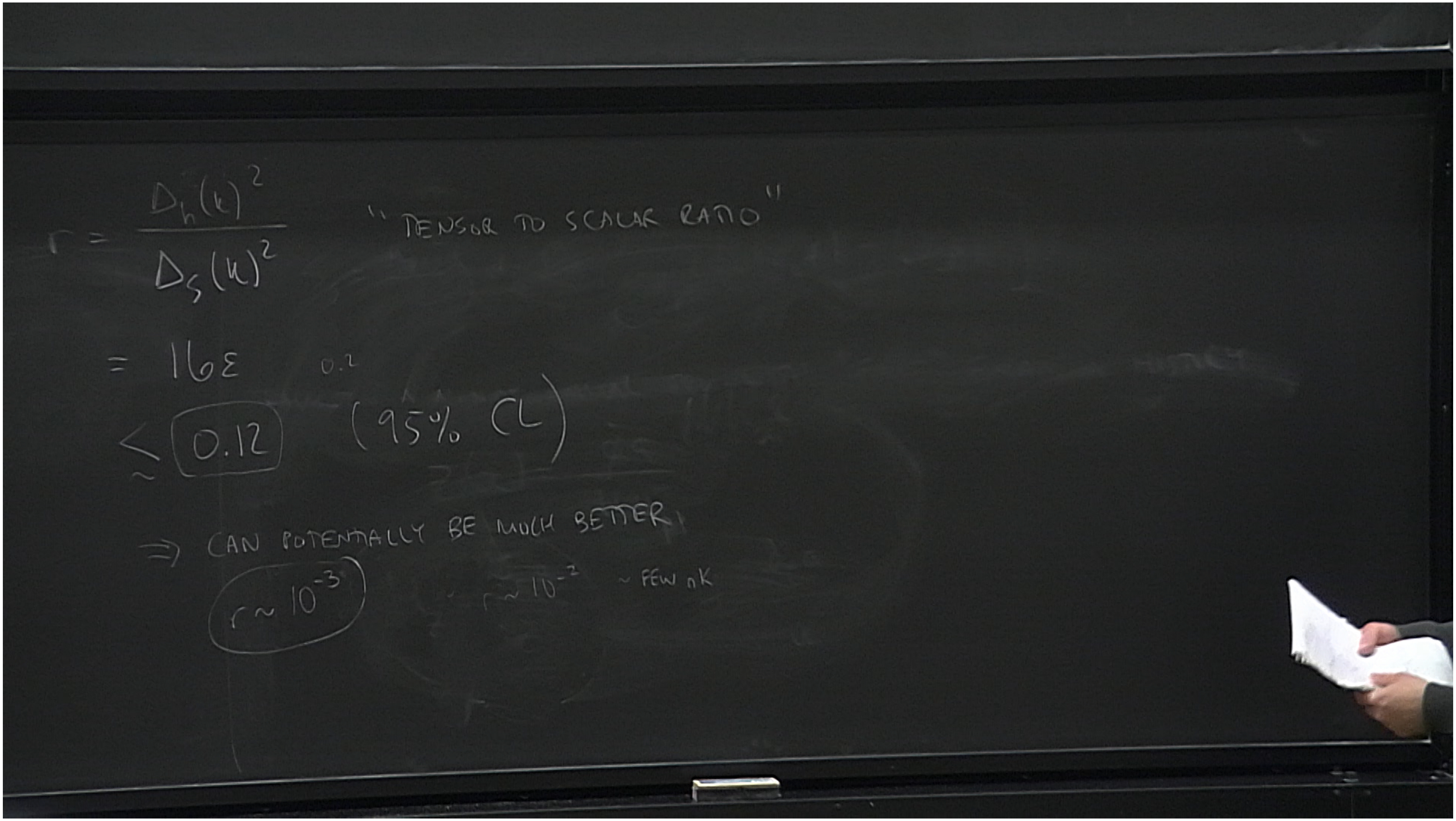

r = Δ_h(k)² / Δ_s(k)²
"TENSOR TO SCALAR RATIO"
= 16ε
≲ 0.12 (95% CL)

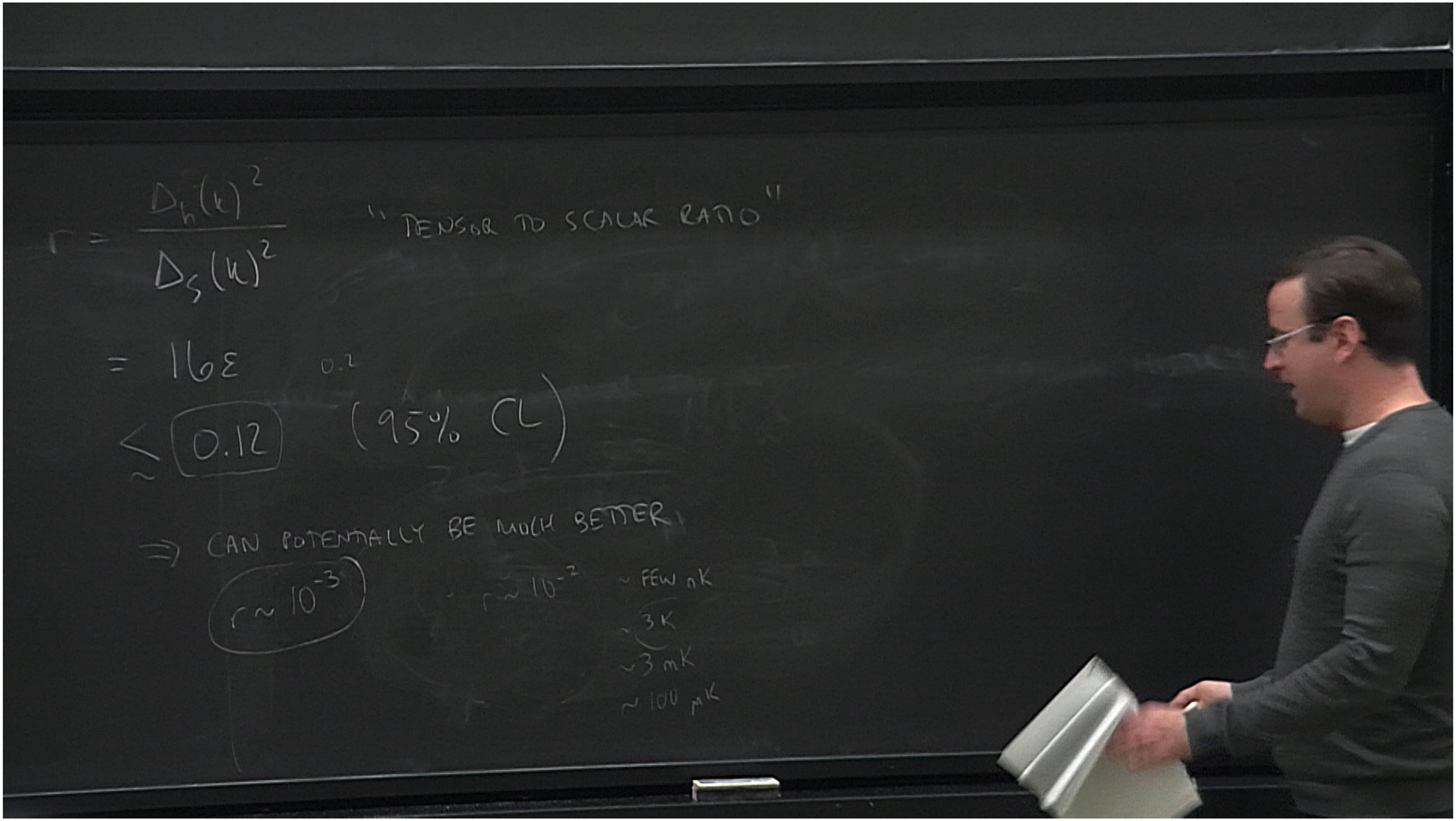

AT EARLY TIMES
BSERVABLES
$\Delta_s^2 = \frac{1}{8\pi^2 M_{pl}^2}\left(\frac{H^2}{\varepsilon}\right)$
$\left(\approx \frac{V^3}{V'^2}\right)$
$= (2.11 \pm 0.05) \times 10^{-9}$
$n_s - 1 = -6\varepsilon + 2\eta$
$= -0.033 \pm 0.004$
$V(\varphi) = m^2\varphi^2$
$\varepsilon \sim \eta$
$\eta < 0$
$\varepsilon \sim 0$
"HILLTOP"
$r = \frac{\Delta_h(k)^2}{\Delta_s(k)^2}$
16ε
0.12



r = $\frac{\Delta_h(k)^2}{\Delta_s(k)^2}$
"TENSOR TO SCALAR RATIO"
= 16ε
≲ 0.12 (95% CL)
⇒ CAN POTENTIALLY BE MUCH BETTER
r ~ 10^{-3}


r = Dh(k)² / Ds(k)²
"TENSOR TO SCALAR RATIO"
= 16ε
0.2
≲ 0.12 (95% CL)
⇒ CAN POTENTIALLY BE MUCH BETTER
r ~ 10⁻³


r = Δh(k)² / Δs(k)²
"TENSOR TO SCALAR RATIO"
= 16ε
0.2
≲ 0.12 (95% CL)
⇒ CAN POTENTIALLY BE MUCH BETTER
r ~ 10⁻³
r ~ 10⁻²
~ FEW nK



r = Δh(k)² / Δs(k)²
"TENSOR TO SCALAR RATIO"
= 16ε
0.2
≲ 0.12 (95% CL)
⇒ CAN POTENTIALLY BE MUCH BETTER
r ~ 10⁻³
r ~ 10⁻²
~ FEW nK
~ 3 K
~ 3 mK
~ 100 μK

• If r IS DETECTED

$$E_{infl} = \rho^{1/4}$$

$$= \left(3 M_{pl}^2 H^2\right)^{1/4}$$

$$= r^{1/4} \left(\frac{3\pi^2 \Delta_s^2}{2}\right)^{1/4} M_{pl}$$

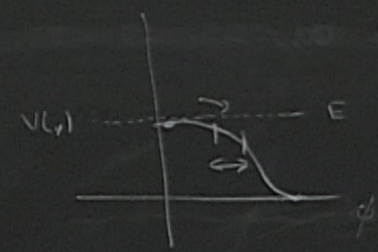
(MEASURED)

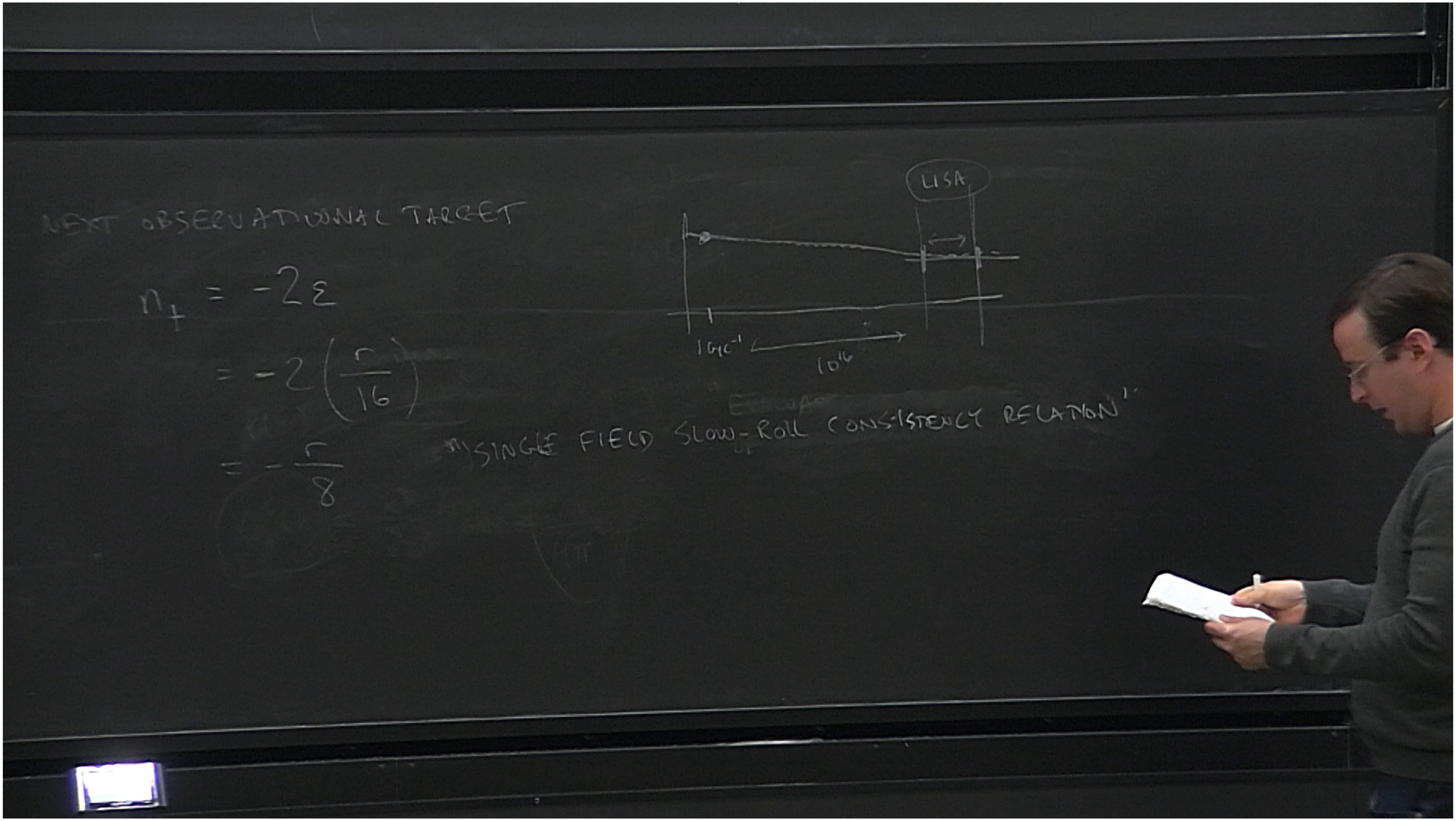
$$= r^{1/4} \left(3.23 \times 10^{16}\ \text{GeV}\right)$$

$\Rightarrow E_{infl} = $ GUT SCALE

$\Rightarrow$ FIRST TWO DERIVATIVES OF $V(\phi)$

(c)

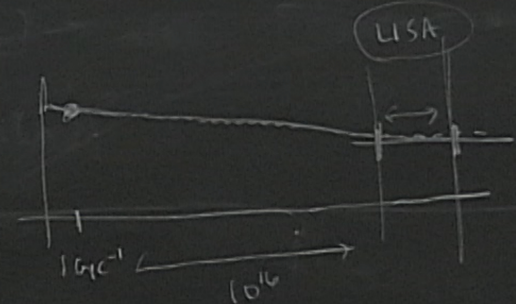



NEXT OBSERVATIONAL TARGET
$n_t = -2\varepsilon$
$= -2\left(\frac{r}{16}\right)$
$= -\frac{r}{8}$
LISA
1 Gpc⁻¹
10¹⁶
"SINGLE FIELD SLOW-ROLL CONSISTENCY RELATION"

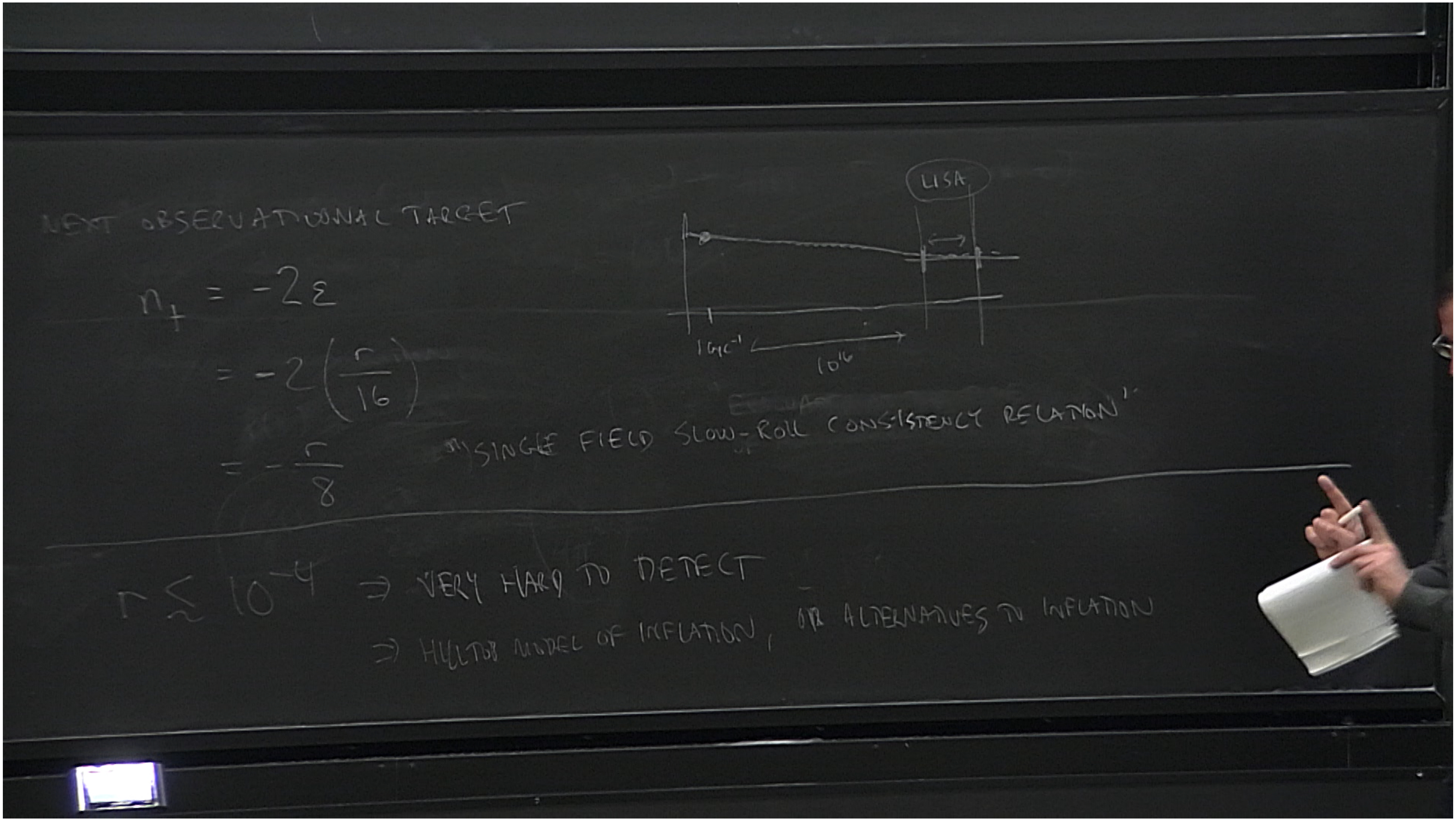
NEXT OBSERVATIONAL TARGET

$$n_t = -2\varepsilon = -2\left(\frac{r}{16}\right) = -\frac{r}{8}$$

"SINGLE FIELD SLOW-ROLL CONSISTENCY RELATION"



$r \lesssim 10^{-4} \Rightarrow$ VERY HARD TO DETECT


NEXT OBSERVATIONAL TARGET
LISA
$n_t = -2\varepsilon$
$= -2\left(\frac{r}{16}\right)$
$= -\frac{r}{8}$
"SINGLE FIELD SLOW-ROLL CONSISTENCY RELATION"
10^{16}
$r \lesssim 10^{-4}$ ⇒ VERY HARD TO DETECT
⇒ HILLTOP MODEL OF INFLATION, OR ALTERNATIVES TO INFLATION