Title: PSI 2017/2018 - Cosmology - Lecture 12

Date: Apr 24, 2018 10:15 AM

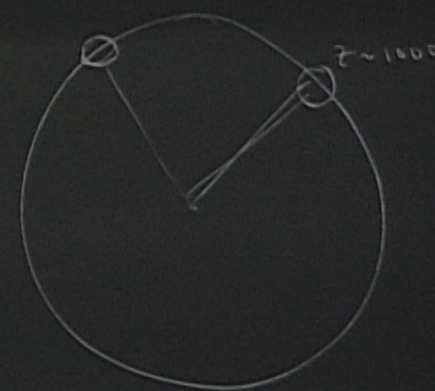
URL: http://pirsa.org/18040024

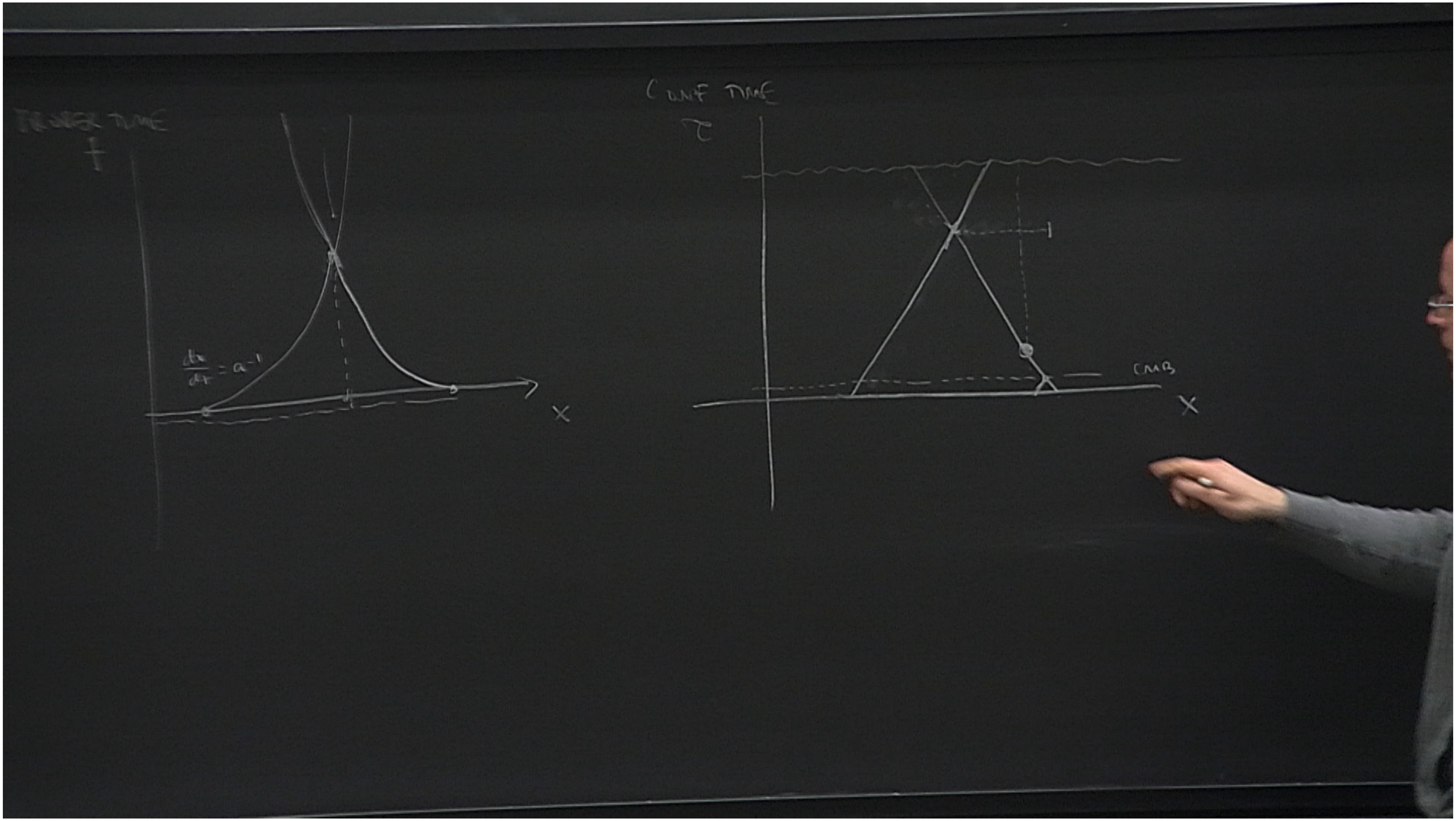
Abstract:

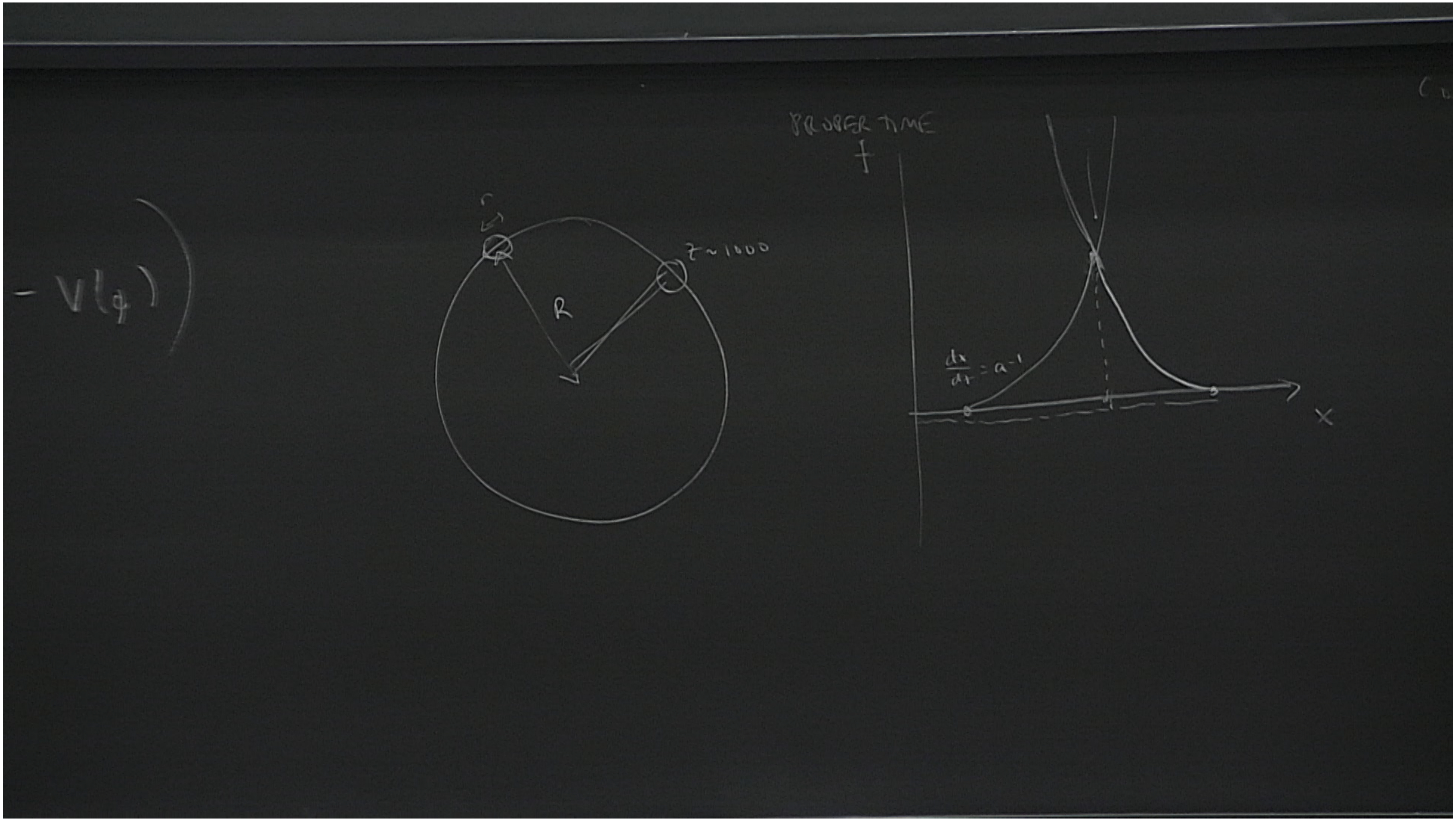
SINGLE-FIELD SLOW-ROLL INFLATION

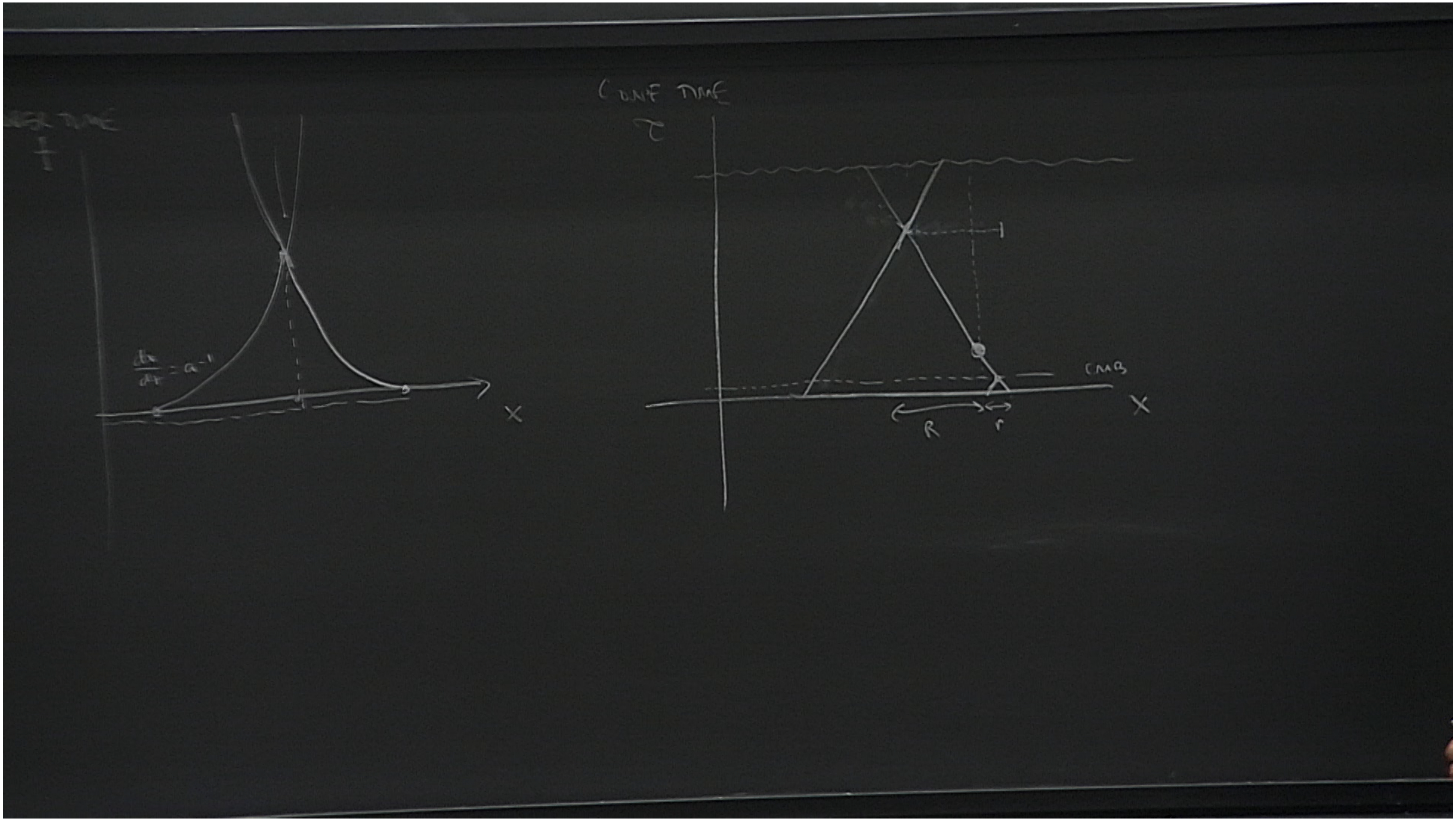
$$S = \int d^4x \sqrt{-g}\left(\frac{M_{pl}^2}{2}R - \frac{1}{2}(\nabla^\mu\phi)(\nabla_\mu\phi) - V(\phi)\right)$$

- HORIZON PROBLEM

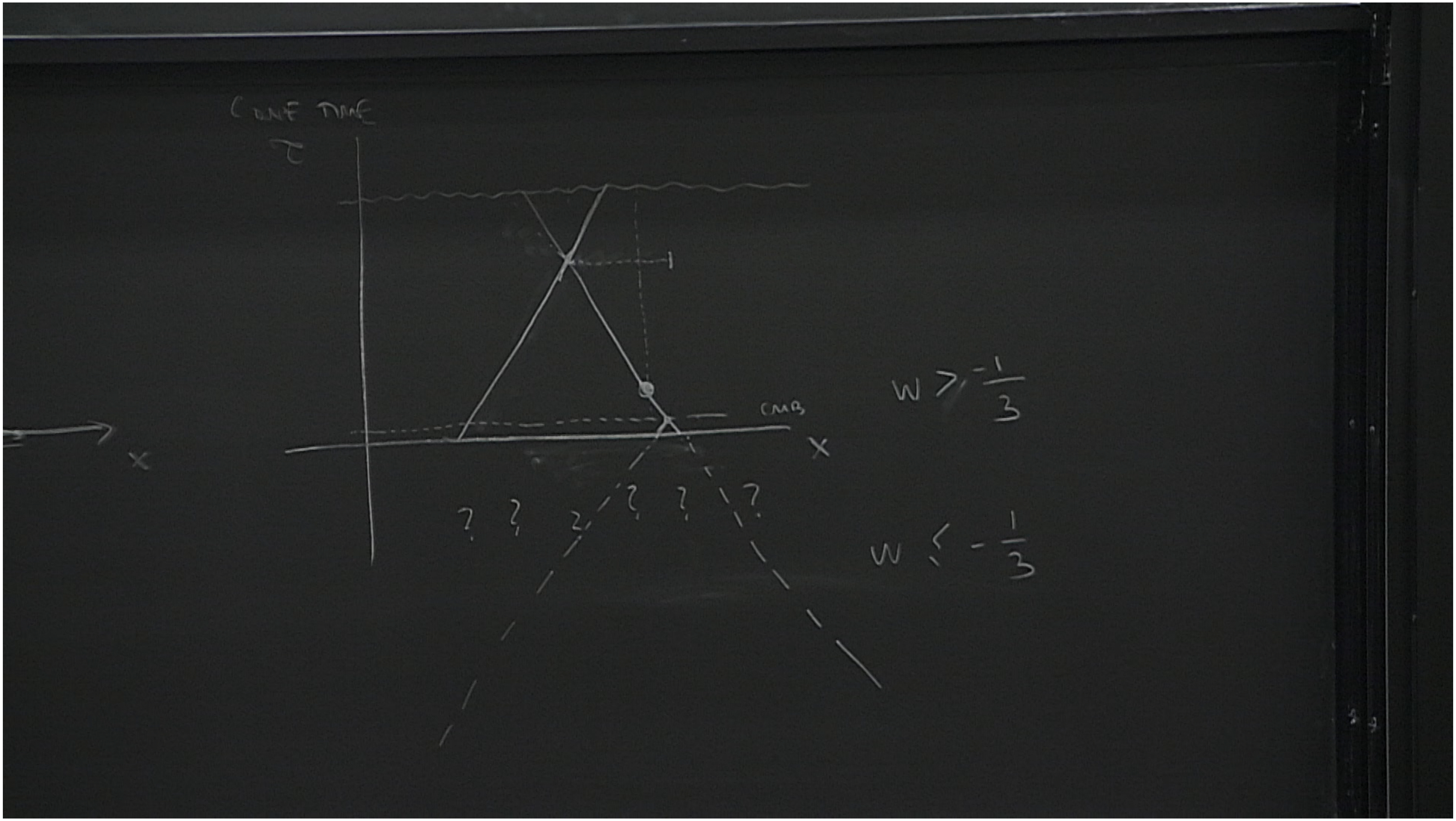



PROPER TIME
t
$\frac{dx}{dt} = a^{-1}$
x
CONF TIME
τ
CMB
x


- V(φ)
R
z ~ 1000
PROPER TIME
t
dx/dt = a⁻¹
x



CONF TIME
τ
$\frac{dx}{dt} = a^{-1}$
X
CMB
R
r
X


CONF TIME
τ
x
X
CMB
? ? ? ? ? ?
w > -1/3
w < -1/3

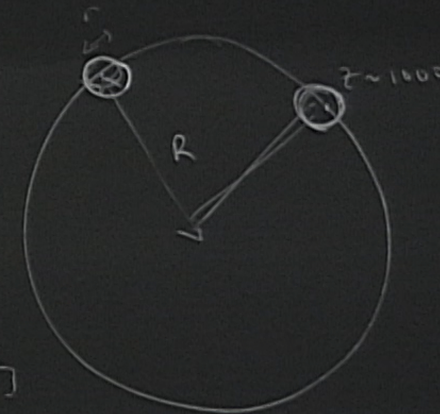
SINGLE-FIELD SLOW-ROLL INFLATION

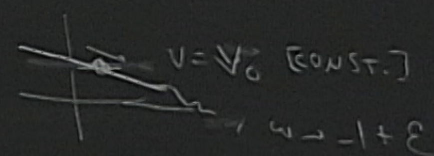
$$S = \int d^4x \sqrt{-g} \left(\frac{M_{pl}^2}{2} R - \frac{1}{2} (\nabla^\mu \phi)(\nabla_\mu \phi) - V(\phi) \right)$$

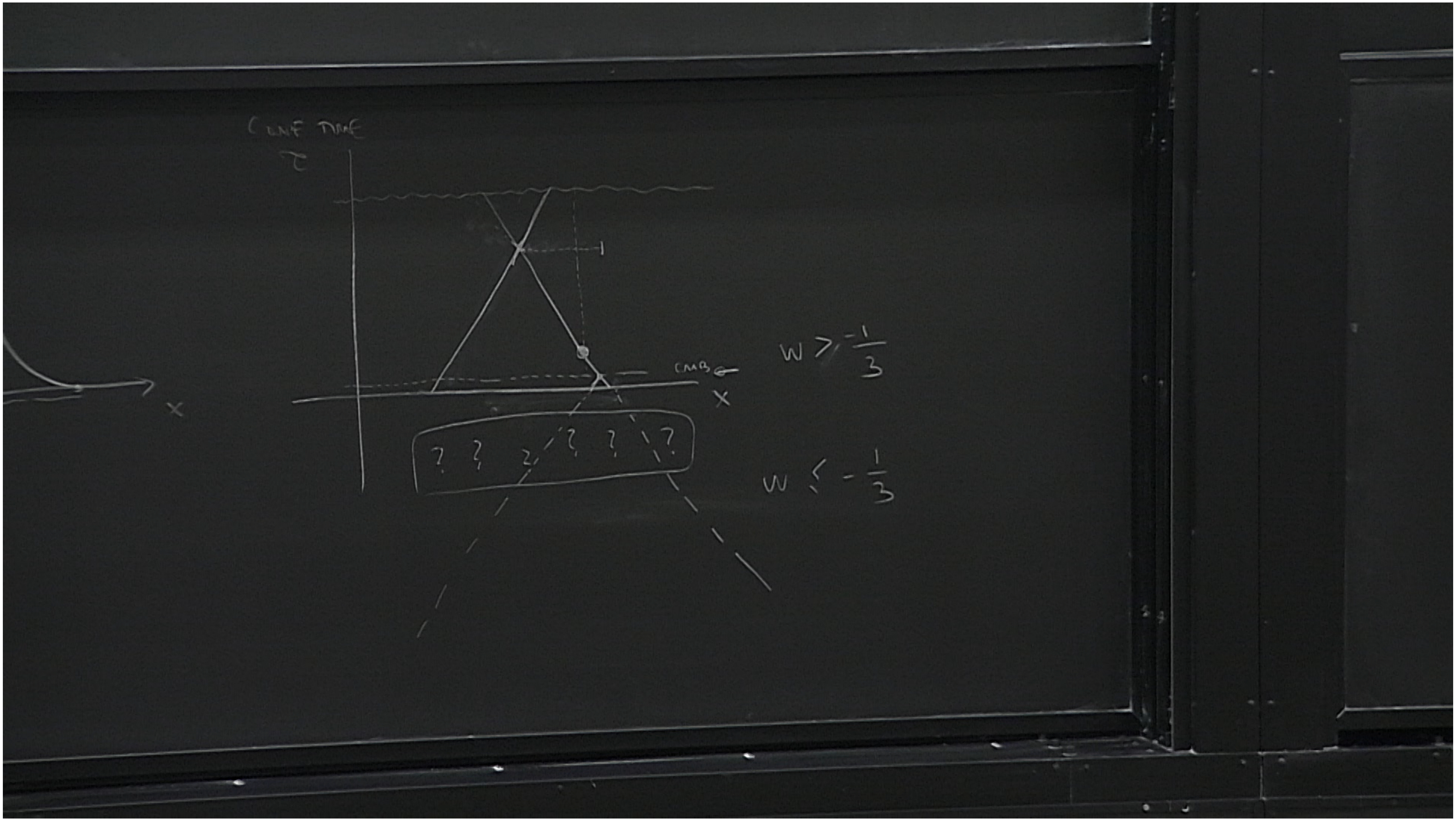
"FLAT"

- HORIZON PROBLEM $\quad w \lesssim -\frac{1}{3}$

$$\int d^4 \sqrt{-g} \left(\Lambda \right)$$





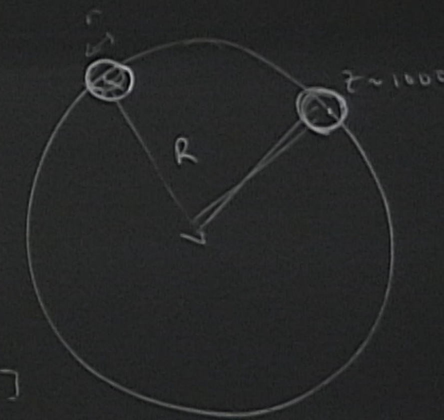

CONF TIME
τ
CMB
x
x
? ? ? ? ? ?
W > -1/3
W ≤ -1/3

$$\int d^4x \sqrt{-g}\,(\Lambda)$$

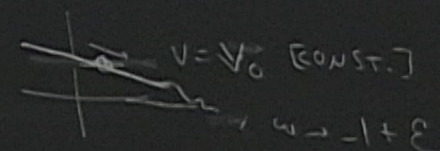
SINGLE-FIELD SLOW-ROLL INFLATION

$$S = \int d^4x \sqrt{-g}\left(\frac{M_{pl}^2}{2}R - \frac{1}{2}(\nabla^\mu\phi)(\nabla_\mu\phi) - V(\phi)\right)$$

"FLAT"



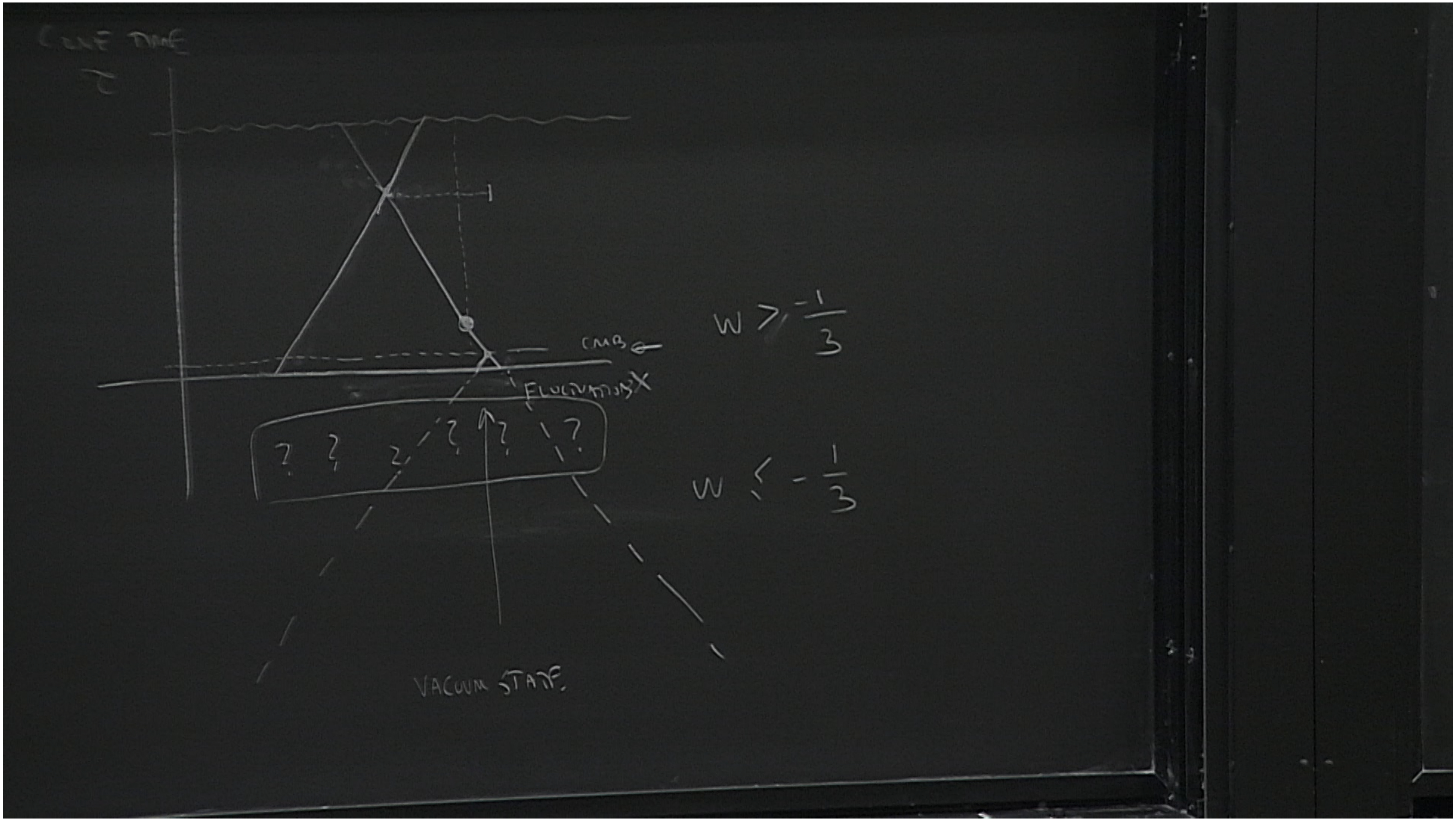
✓ • HORIZON PROBLEM $w \le -\frac{1}{3}$

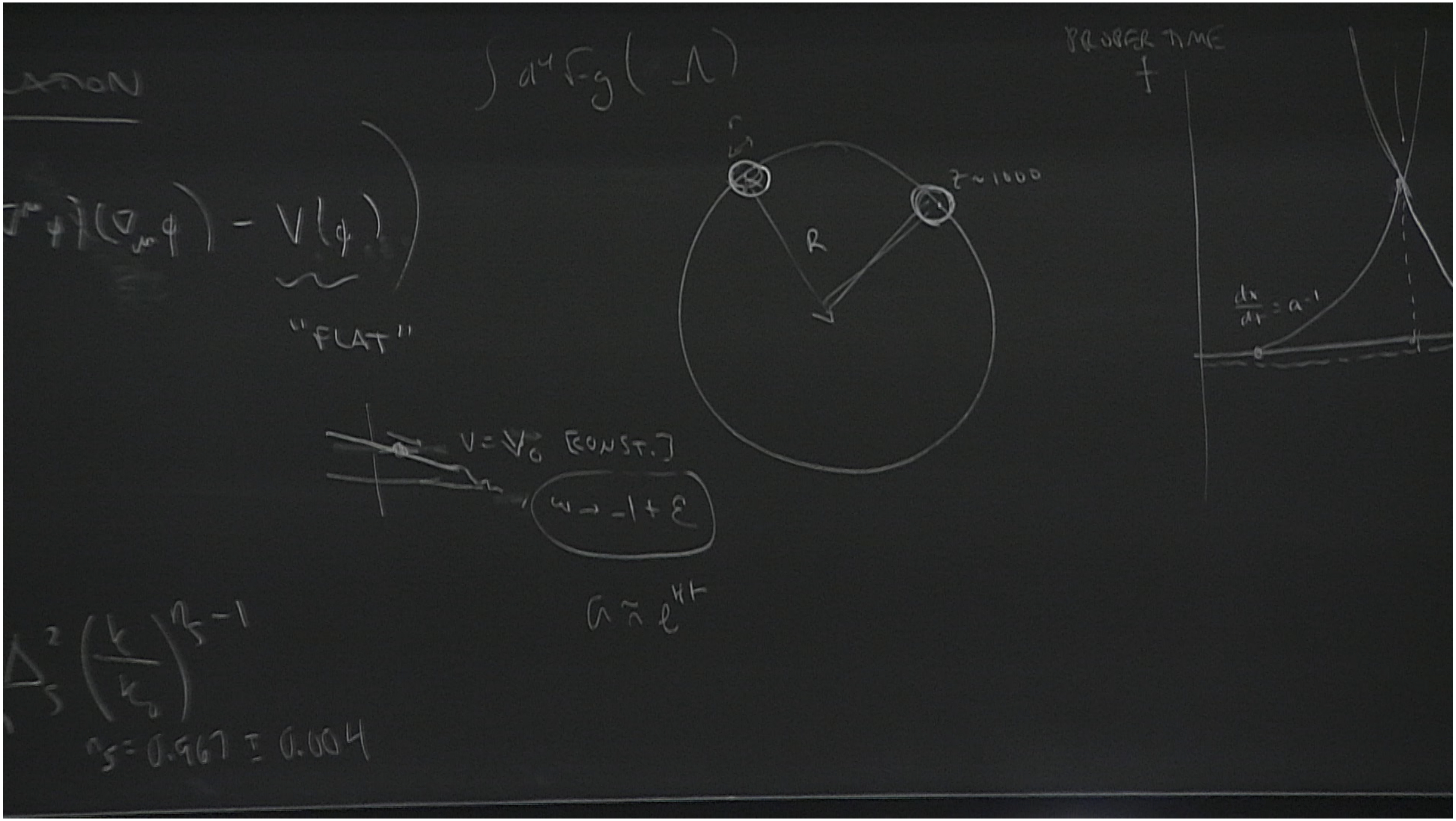


• GENERATES PERTURBATIONS

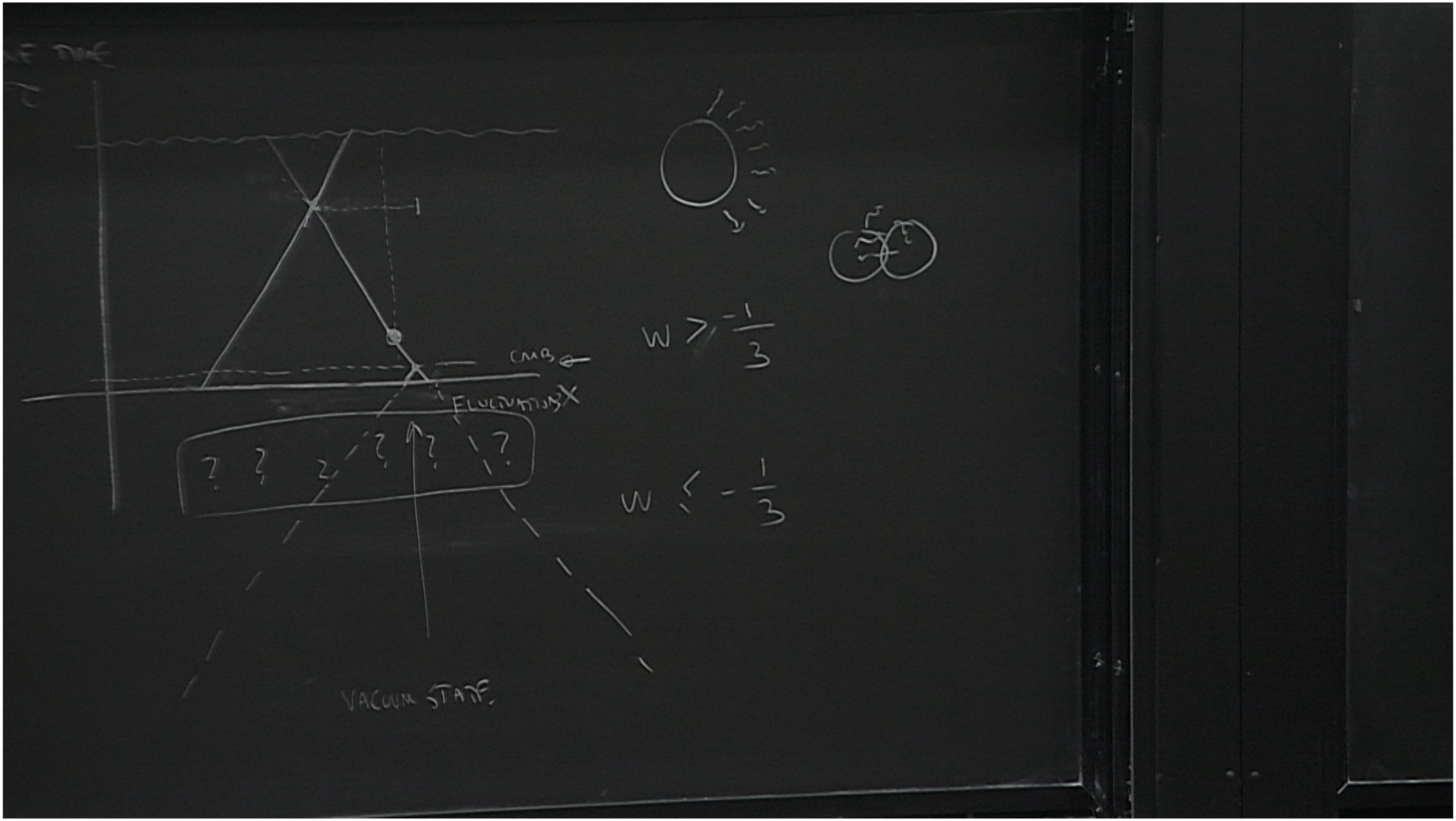
- SCALAR ADIABATIC
- GAUSSIAN TO $\lesssim 0.1\%$
- POWER SPECTRUM $P_s(k) = \Delta_s^2 \left(\frac{k}{k_0}\right)^{n_s - 1}$

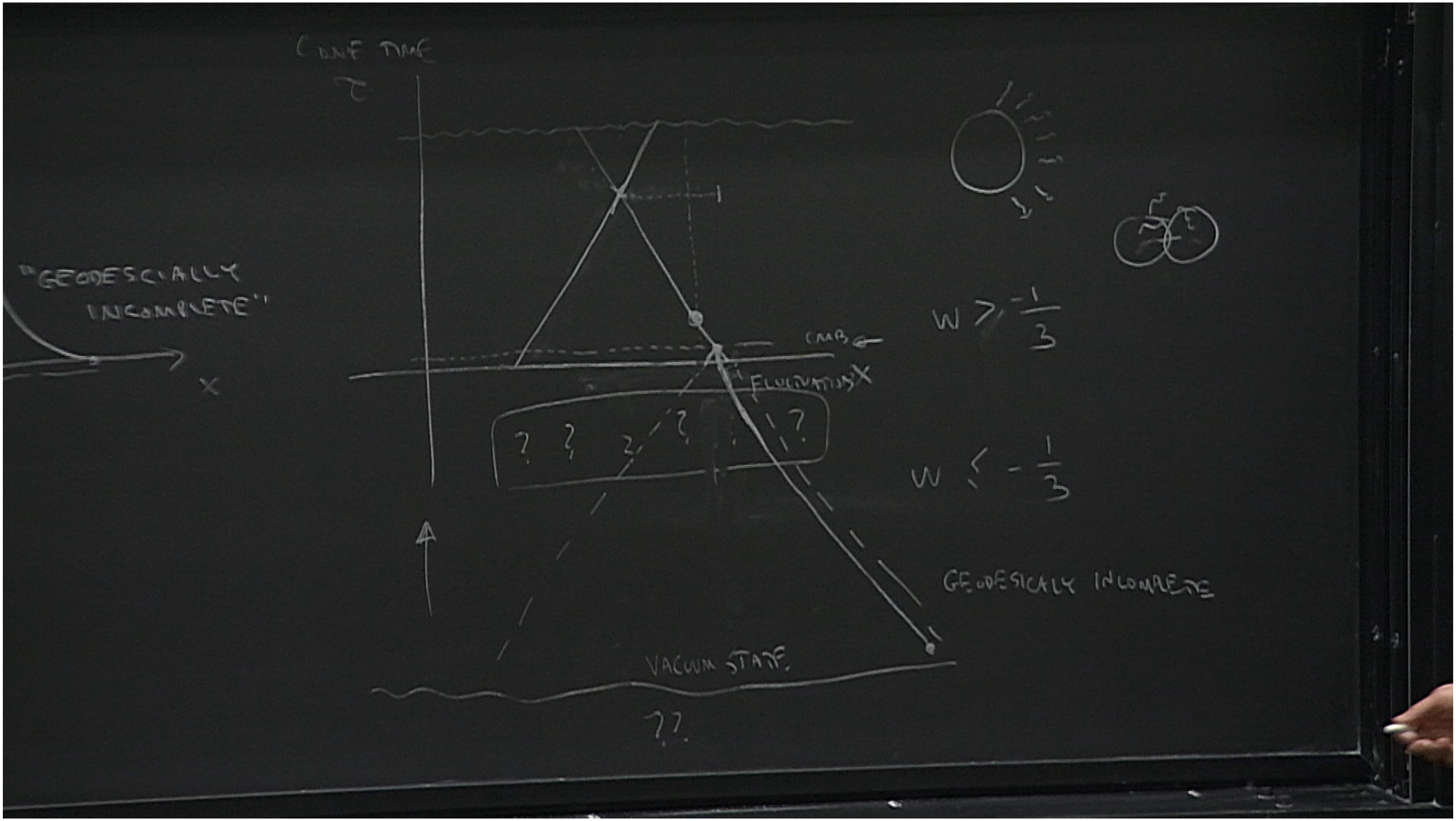
$\Delta_s^2 = (2.11 \pm 0.05)\times 10^{-9}$ $\quad n_s = 0.967 \pm 0.004$

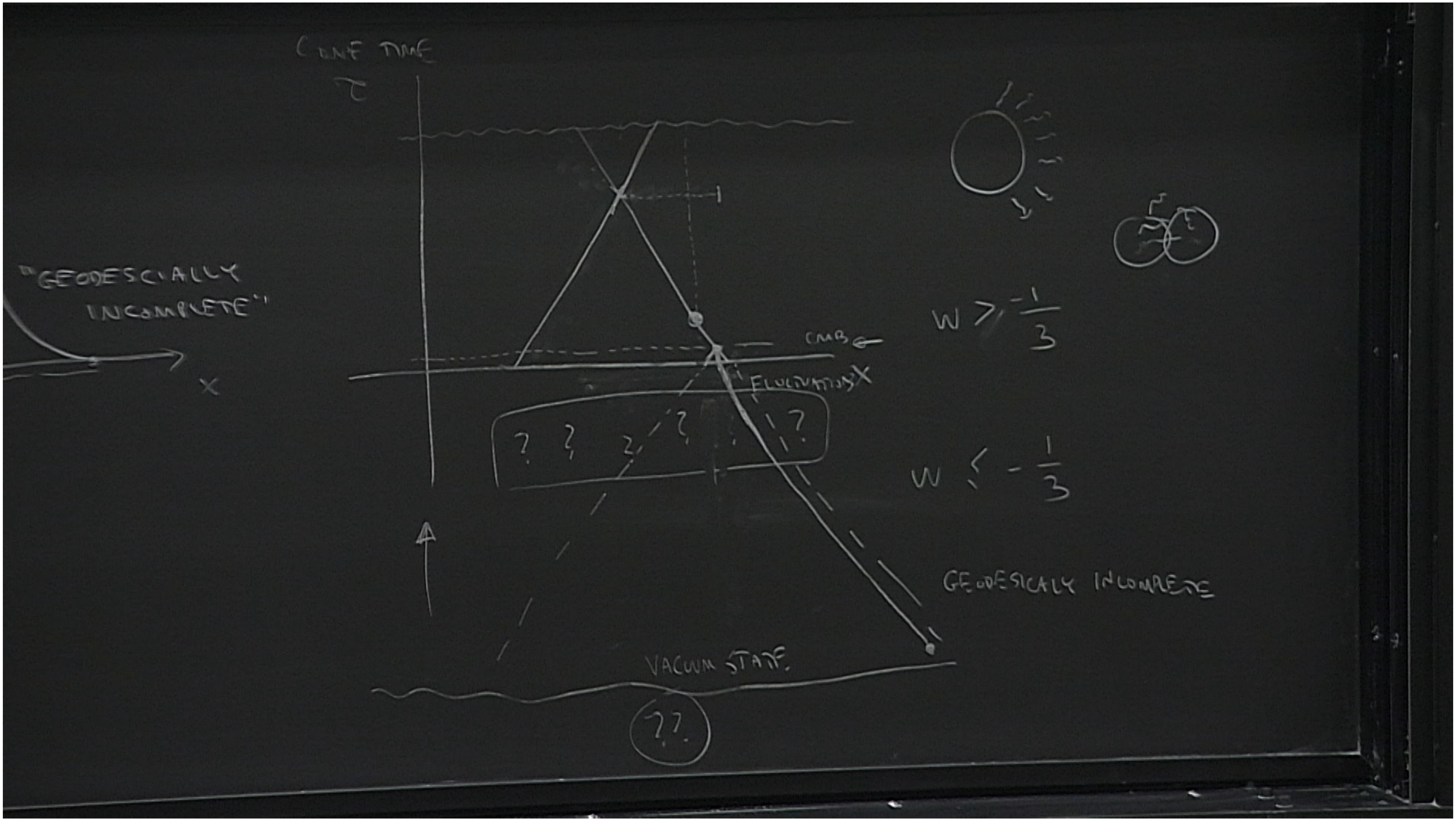

CMB
FLUCTUATIONS
VACUUM STATE
w > -1/3
w < -1/3

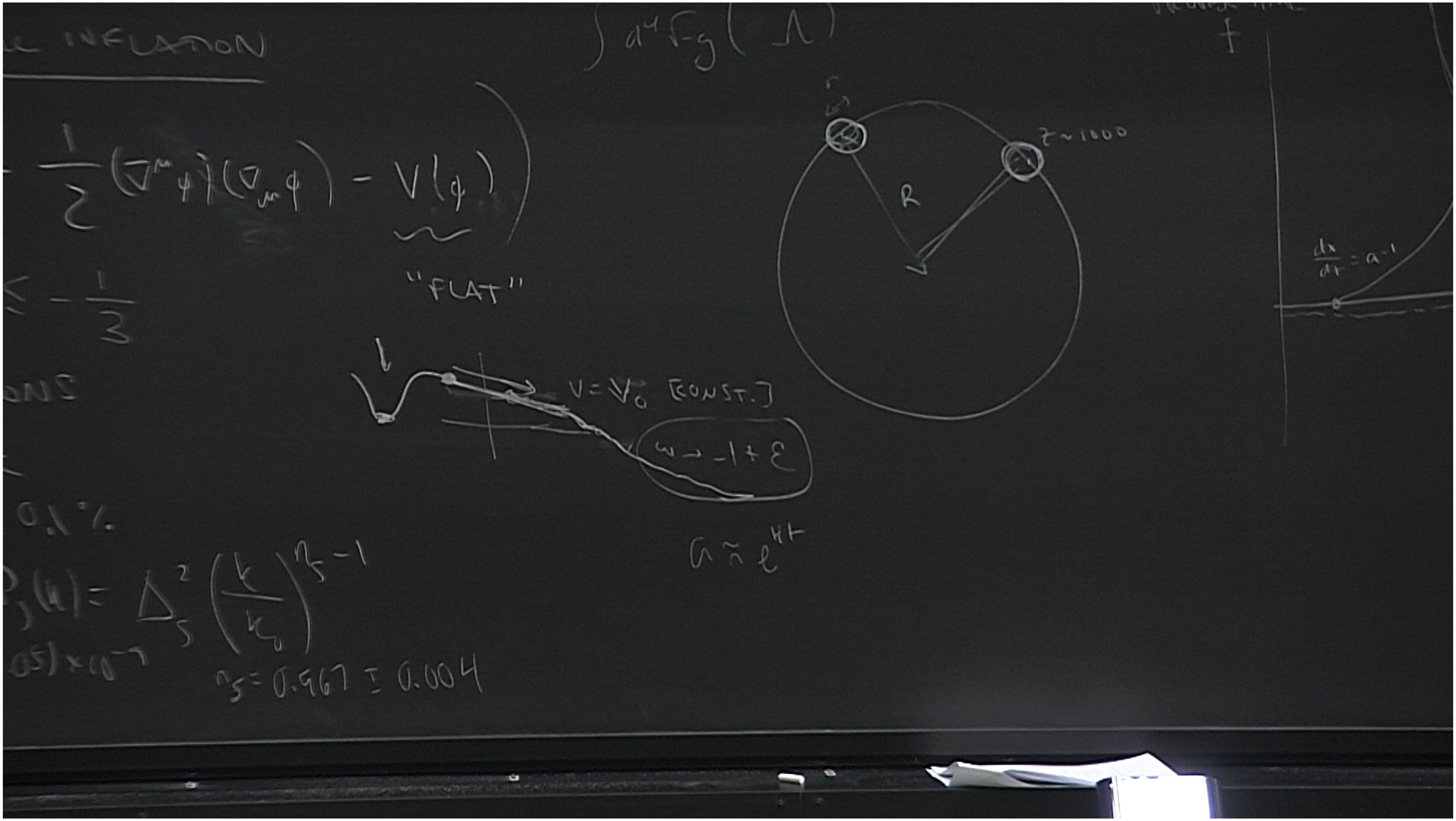


$\int d^4x \sqrt{-g}\,(\Lambda)$
$V(\phi)$
"FLAT"
$V = V_0$ [CONST.]
$w \to -1 + \varepsilon$
$a \approx e^{Ht}$
$\Delta_s^2 \left(\frac{k}{k_0}\right)^{n_s - 1}$
$n_s = 0.967 \pm 0.004$
R
$z \sim 1000$
PROPER TIME
t
$\frac{dx}{dt} = a^{-1}$


CMB
FLUCTUATIONS
VACUUM STATE
w > -1/3
w < -1/3


CONF TIME
τ
"GEODESICALLY INCOMPLETE"
x
CMB
FLUCTUATIONS
? ? ? ? ?
VACUUM STATE
??
w > -1/3
w < -1/3
GEODESICALLY INCOMPLETE


CONF TIME
τ
"GEODESICALLY INCOMPLETE"
X
CMB
FLUCTUATIONS
? ? ? ? ?
W > -1/3
W < -1/3
GEODESICALLY INCOMPLETE
VACUUM STATE
??


INFLATION
"FLAT"
V = V0 [CONST.]
w → -1 + ε
z ~ 1000
R
a ~ e^Ht
ns = 0.967 ± 0.004
0.1%

$$S = \int d^4x \sqrt{g} \left(\frac{M_{pl}^2}{2} R - \frac{1}{2} (\nabla\phi)^2 - V(\phi) \right)$$

$$T_{\mu\nu} = (\nabla_\mu \phi)(\nabla_\nu \phi) - \frac{1}{2} (\nabla_\rho \phi)(\nabla^\rho \phi)\, g_{\mu\nu} - V(\phi)\, g_{\mu\nu}$$

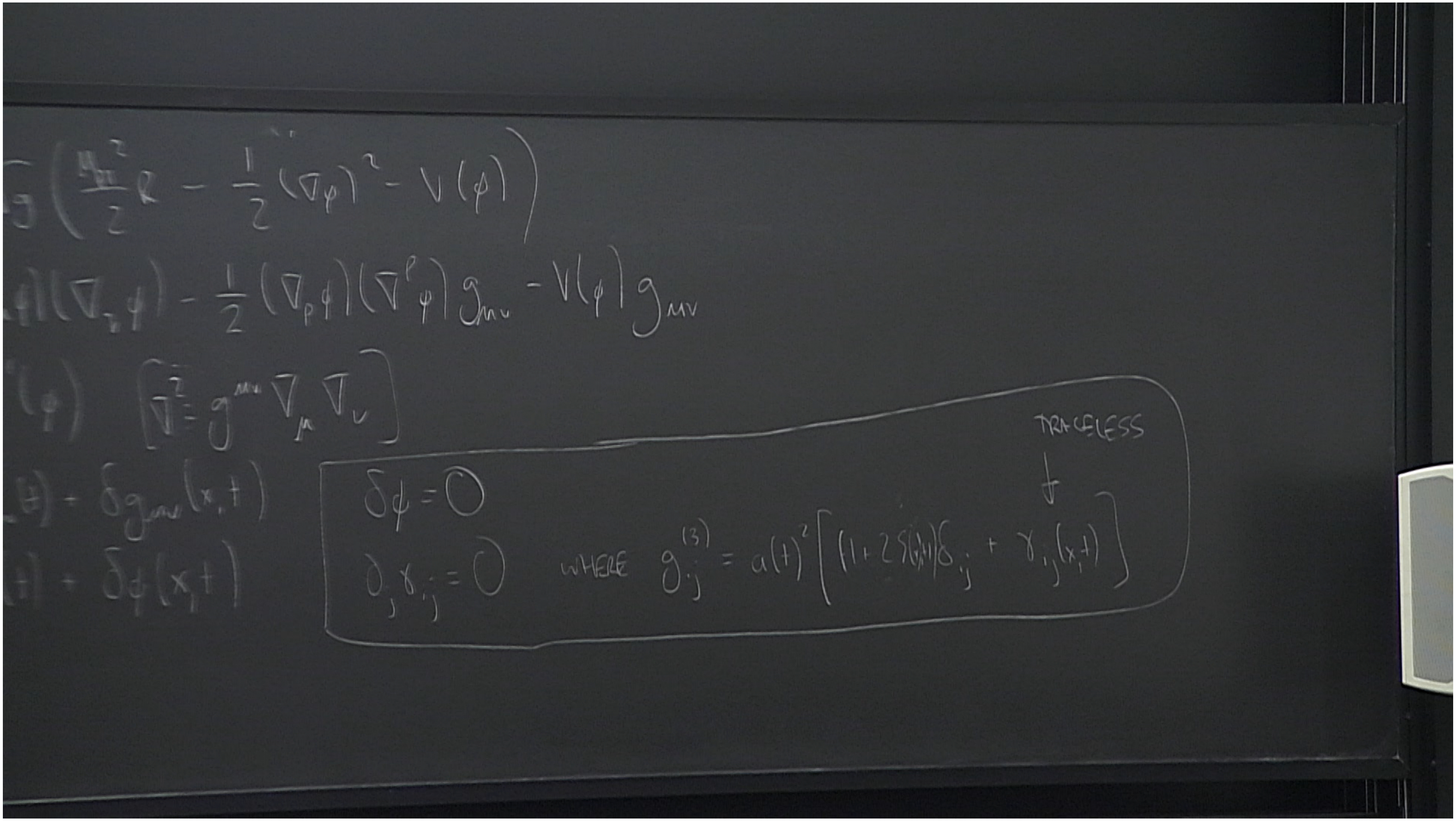
$$\nabla^2 \phi = V'(\phi) \qquad \left[\nabla^2 = g^{\mu\nu} \nabla_\mu \nabla_\nu \right]$$

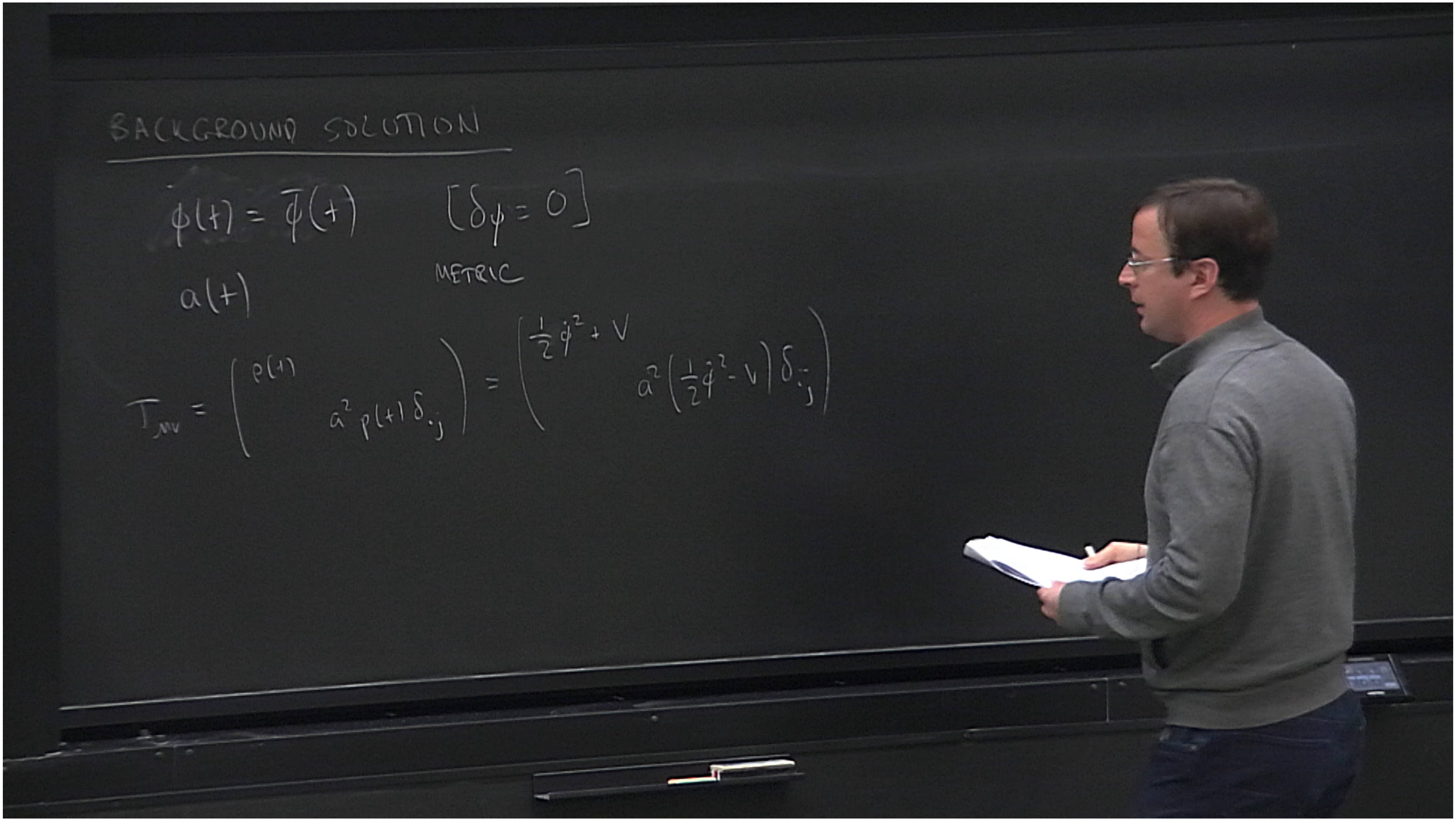
$$g_{\mu\nu}(x,t) = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(x,t)$$

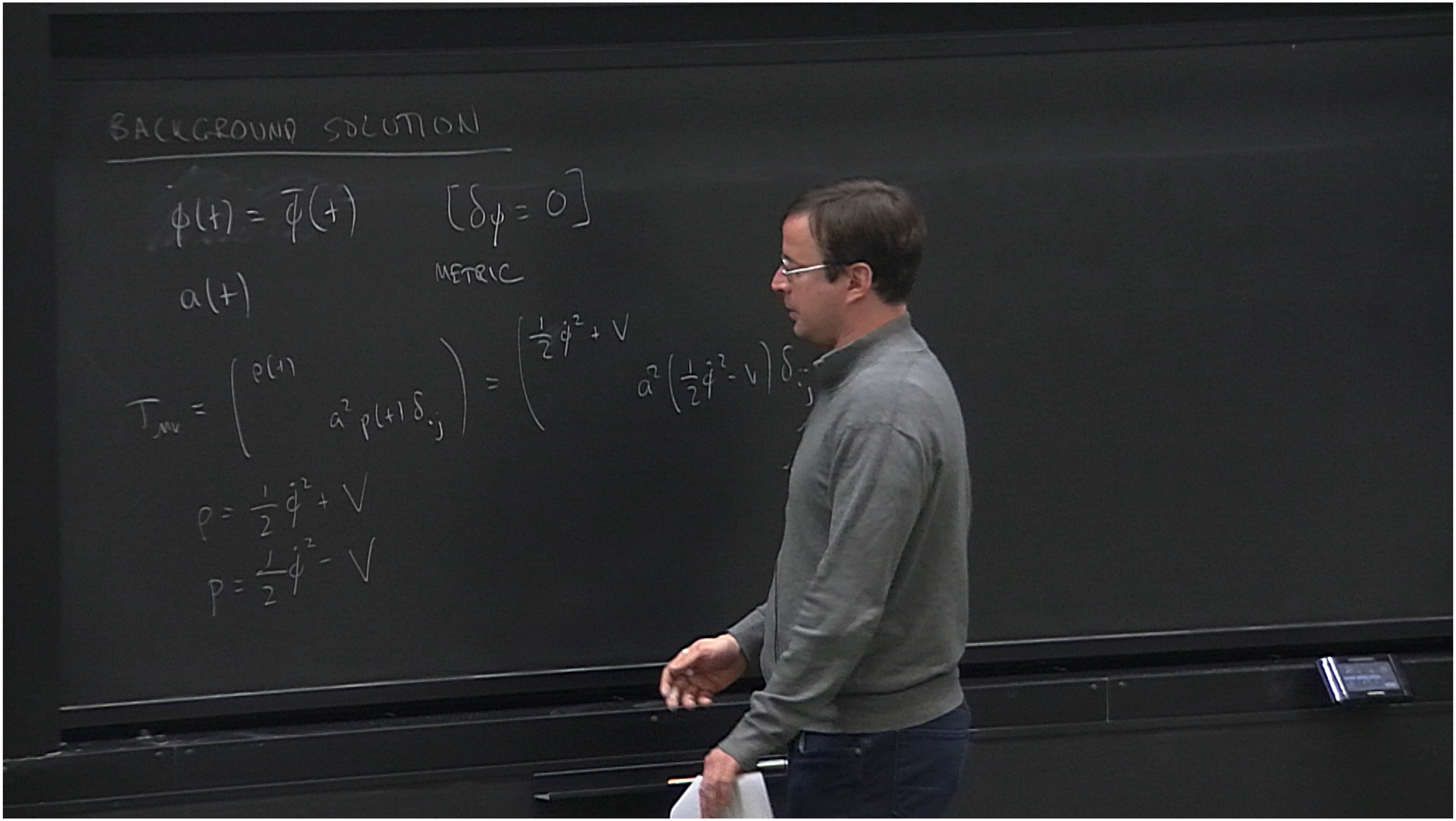
$$\phi(x,t) = \bar{\phi}(t) + \delta\phi(x,t)$$

$$\boxed{\begin{aligned} \delta\phi &= 0 \\ \partial_j \gamma_{ij} &= 0 \end{aligned}}$$

WHERE $g^{(3)}_{ij} = a(t)^2 \left[(1 + 2\zeta(x,t))\delta_{ij} + \gamma_{ij}(x,t) \right.$


$\sqrt{-g}\left(\frac{M_{pl}^2}{2}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right)$
$(\nabla_\mu\phi)(\nabla_\nu\phi) - \frac{1}{2}(\nabla_\rho\phi)(\nabla^\rho\phi)g_{\mu\nu} - V(\phi)g_{\mu\nu}$
$V'(\phi)$ $\left[\nabla^2 = g^{\mu\nu}\nabla_\mu\nabla_\nu\right]$
$(t) + \delta g_{\mu\nu}(x,t)$
$(t) + \delta\phi(x,t)$
TRACELESS
$\delta\phi = 0$
$\partial_j \gamma_{ij} = 0$ WHERE $g^{(3)}_{ij} = a(t)^2\left[(1+2\zeta(x,t))\delta_{ij} + \gamma_{ij}(x,t)\right]$


BACKGROUND SOLUTION
$\phi(t) = \bar{\phi}(t)$
$[\delta\phi = 0]$
$a(t)$
METRIC
$T_{\mu\nu} = \begin{pmatrix} \rho(t) & \\ & a^2 p(t) \delta_{ij} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\dot{\phi}^2 + V & \\ & a^2\left(\frac{1}{2}\dot{\phi}^2 - V\right)\delta_{ij} \end{pmatrix}$


BACKGROUND SOLUTION
$\phi(t) = \bar{\phi}(t)$ $[\delta\phi = 0]$
$a(t)$
METRIC
$T_{\mu\nu} = \begin{pmatrix} \rho(t) & \\ & a^2 p(t)\delta_{ij} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\dot{\phi}^2 + V & \\ & a^2\left(\frac{1}{2}\dot{\phi}^2 - V\right)\delta_{ij} \end{pmatrix}$
$\rho = \frac{1}{2}\dot{\phi}^2 + V$
$p = \frac{1}{2}\dot{\phi}^2 - V$

BACKGROUND SOLUTION

$$\phi(t) = \bar{\phi}(t) \qquad [\delta\phi = 0]$$

$a(t)$ METRIC

$$T_{\mu\nu} = \begin{pmatrix} \rho(t) & \\ & a^2 p(t)\,\delta_{ij} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\dot{\phi}^2 + V & \\ & a^2\left(\frac{1}{2}\dot{\phi}^2 - V\right)\delta_{ij} \end{pmatrix}$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V$$

$$p = \frac{1}{2}\dot{\phi}^2 - V$$

EQ OF MOTION FOR ϕ: $\nabla^2\phi = V'(\phi)$

BACKGROUND METRIC IS FRW: $\nabla^2\phi = -\frac{\partial^2\phi}{\partial t^2} - 3H\frac{\partial\phi}{\partial t} + \cancel{a^{-2}\partial_i^2\phi}$

$$\Rightarrow \boxed{\begin{aligned} &\ddot\phi + 3H\dot\phi + V'(\phi) = 0 \\ &H^2 = \frac{1}{3M_{pl}^2}\left(\frac{1}{2}\dot\phi^2 + V\right) \end{aligned}}$$

EQS OF MOTION FOR BACKGROUND

$$\dot H = -\frac{1}{2M_{pl}^2}(\rho + p) = -\frac{\dot\phi^2}{2M_{pl}^2}$$

(REDUNDANT)

"SLOW ROLL" CONDITIONS ON $V(\phi)$

$$\ddot{\phi} + \underbrace{3H\dot{\phi}}_{\text{"HUBBLE FRICTION"}} + V'(\phi) = 0$$

SLOW ROLL APPROXIMATION TO EQS OF MOTION

$$\Rightarrow \boxed{\begin{array}{c} 3H\dot{\phi} + V'(\phi) = 0 \\ H^2 = \dfrac{V}{3M_{pl}^2} \end{array}}$$

ϕ

ϕ_{SR}

$$[\dot{\phi}]_{SR} = -\frac{V'}{3H}$$

$$\boxed{[\dot{\phi}]_{SR} = -M_{pl}\frac{V'}{(3V)^{1/2}}}$$

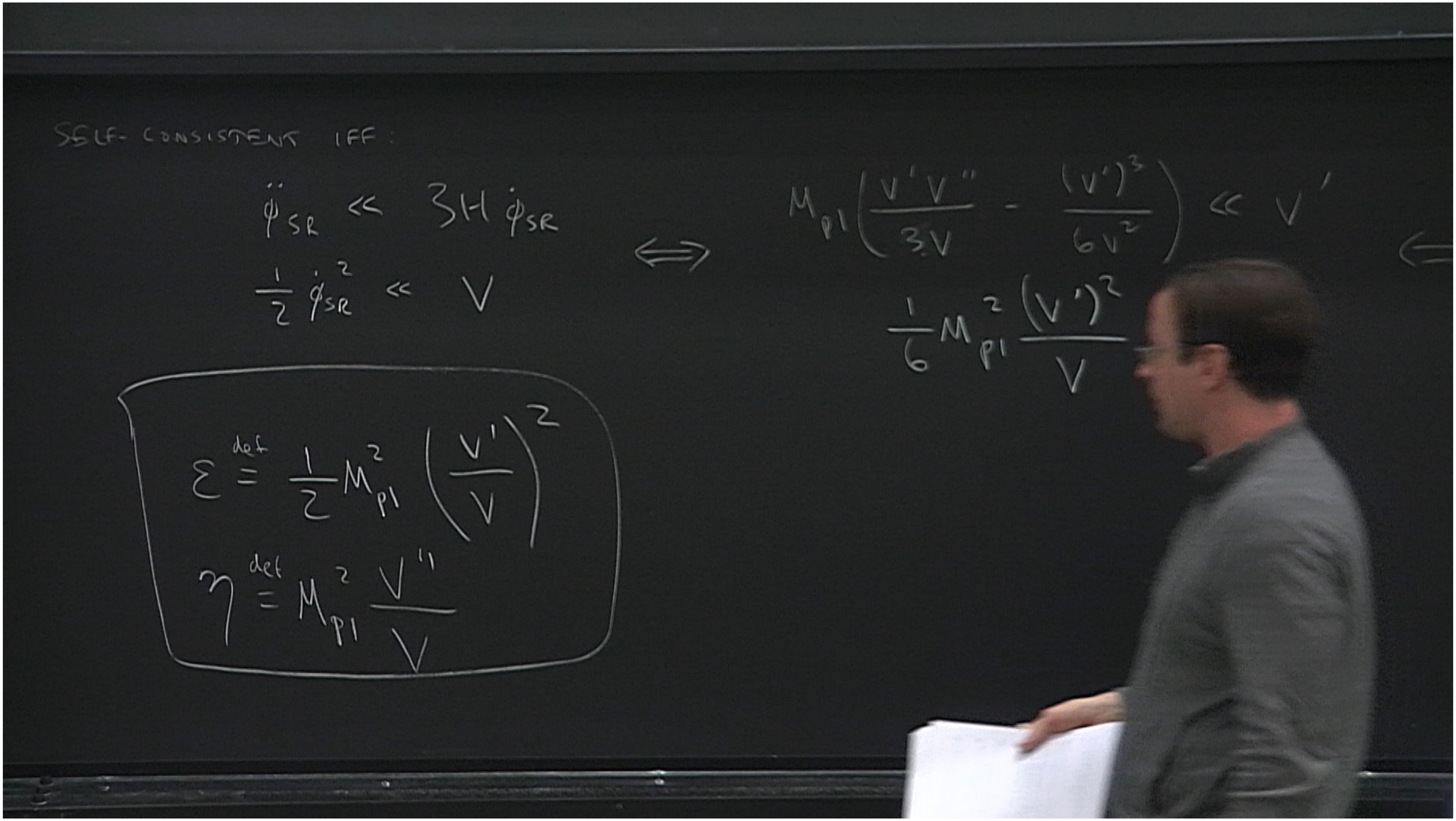
$$[\ddot{\phi}]_{SR} = \frac{d}{dt}[\dot{\phi}]_{SR}$$

$$= -M_{pl}\left(\frac{V''}{(3V)^{1/2}} - \frac{1}{2}\frac{V'}{(3V)^{1/2}V}\right)[\dot{\phi}]_{SR}$$

$$= -M_{pl}\left(\frac{V''}{(3V)^{1/2}} - \frac{1}{2}\frac{V'}{(3V)^{1/2}V}\right)\left(-M_{pl}\frac{V'}{(3V)^{1/2}}\right)$$

$$\boxed{[\ddot{\phi}]_{SR} = M_{pl}^{2}\left(\frac{V'V''}{3V} - \frac{(V')^{3}}{6V^{2}}\right)}$$

$$\boxed{[H]_{SR} = M_{pl}^{-1}\left(\frac{V}{3}\right)^{1/2}}$$


SELF-CONSISTENT IFF:
$\ddot{\phi}_{SR} \ll 3H\dot{\phi}_{SR}$
$\frac{1}{2}\dot{\phi}_{SR}^2 \ll V$
$\Longleftrightarrow$
$M_{pl}\left(\frac{V'V''}{3V} - \frac{(V')^3}{6V^2}\right) \ll V'$
$\frac{1}{6}M_{pl}^2\frac{(V')^2}{V}$
$\varepsilon \stackrel{\text{def}}{=} \frac{1}{2}M_{pl}^2\left(\frac{V'}{V}\right)^2$
$\eta \stackrel{\text{def}}{=} M_{pl}^2\frac{V''}{V}$

$$\frac{V'V''}{3V} - \frac{(V')^3}{6V^2}\Big) \ll V' \quad \Longleftrightarrow \quad \frac{\eta - \varepsilon}{3} \ll 1 \quad \Longleftrightarrow \quad \varepsilon \ll 1$$

$$\frac{1}{6} M_{pl}^2 \frac{(V')^2}{V} \ll V \qquad \frac{\varepsilon}{3} \ll 1 \qquad \eta \ll 1$$

PERTURBATIONS $\left(\delta g_{\mu\nu}(x,t),\ \delta\phi(x,t)\right)$

3 DEGREES OF F

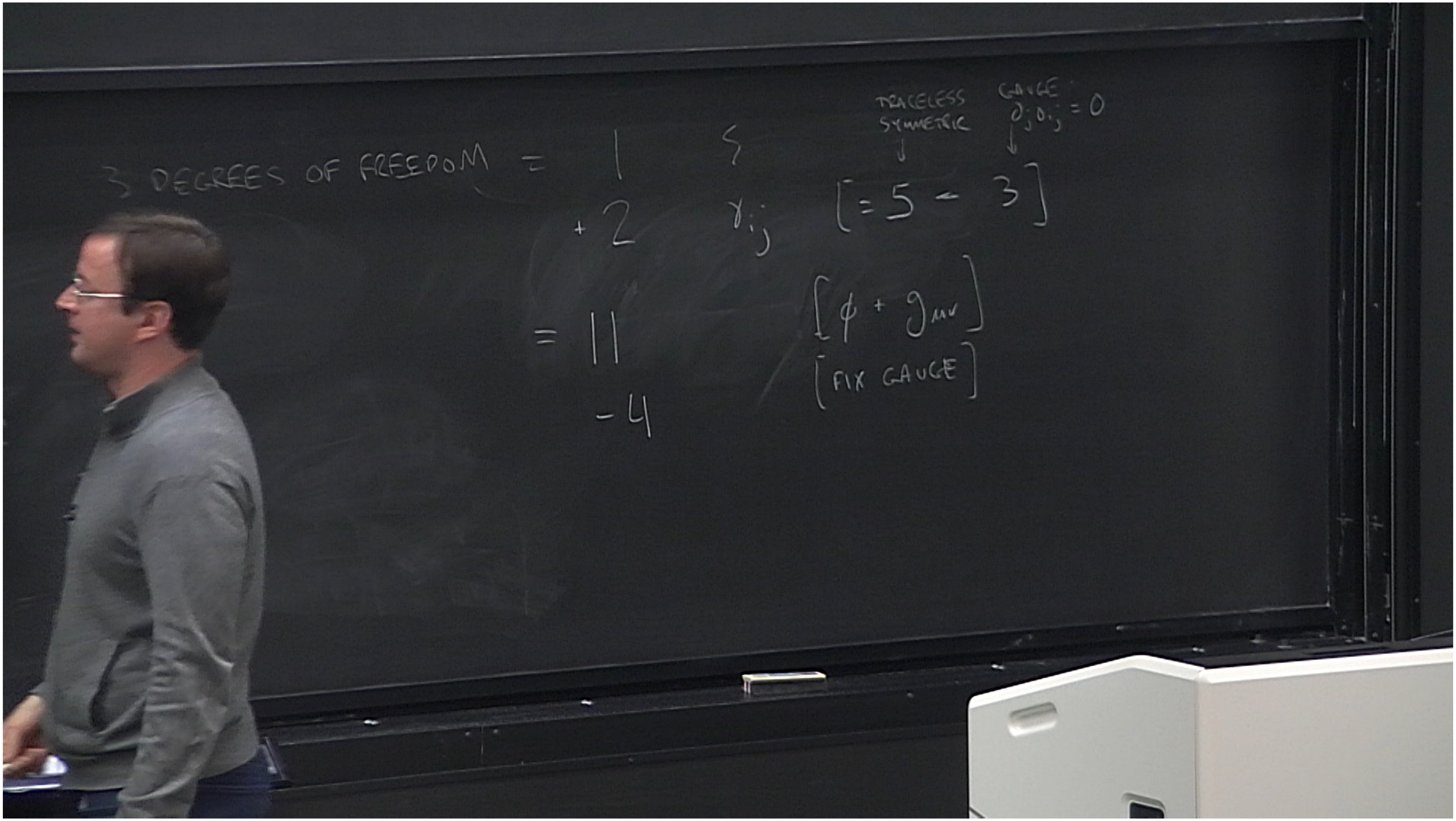
$$S = \frac{M_{pl}^2}{2}\int d^4x\sqrt{-g}\left(\frac{M_{pl}^2}{2} - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right)$$

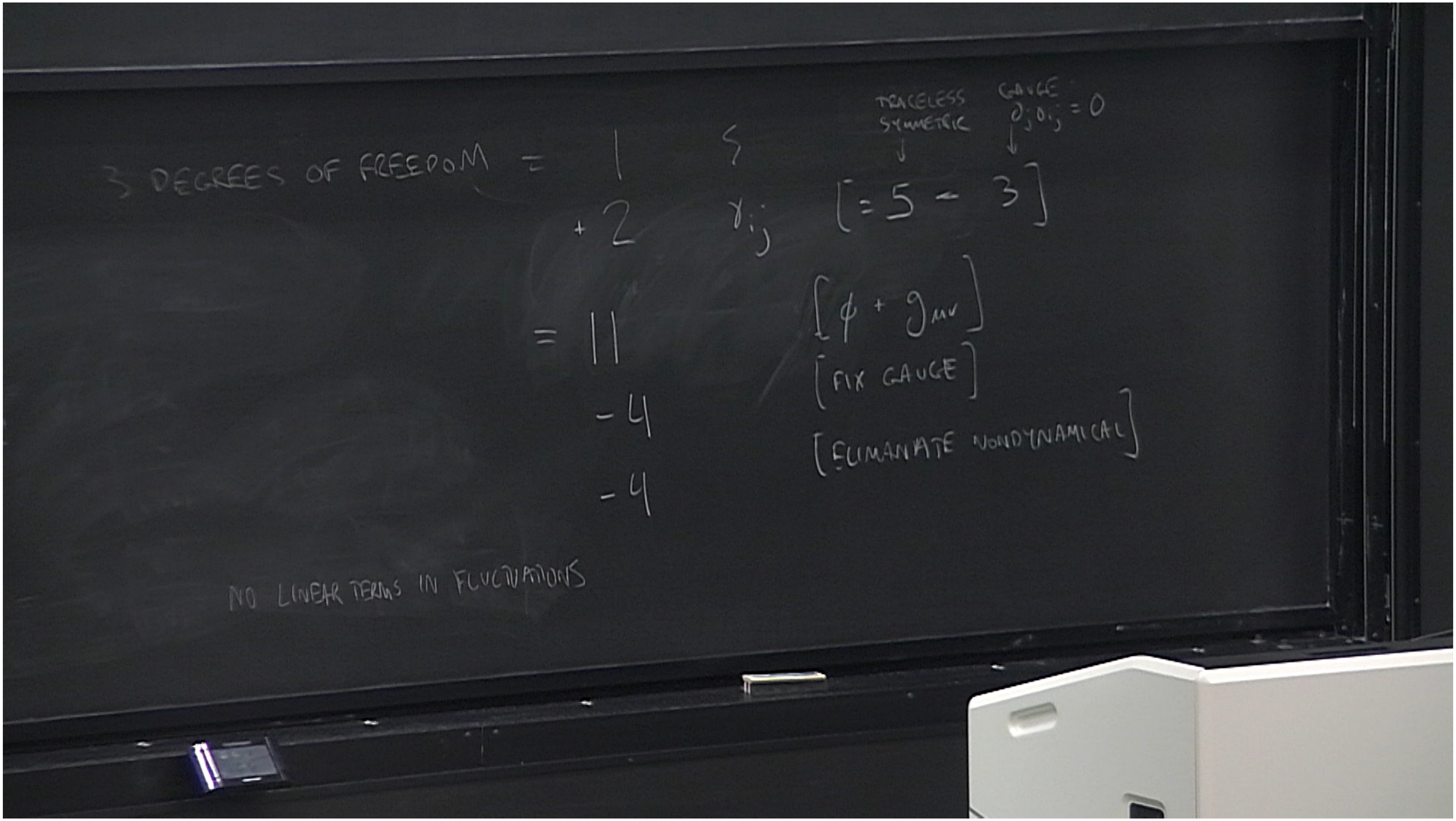
$$= \dots$$

$$= \frac{1}{2}\int d^4x\ \frac{\dot\phi^2}{H^2}\left[a^3\dot\zeta^2 - a(\partial_i\zeta)(\partial_i\zeta)\right]$$

$$+ \frac{M_{pl}^2}{8}\int d^4x\left[a^3\dot\gamma_{ij}\dot\gamma_{ij} - a(\partial_k\gamma_{ij})(\partial_k\gamma_{ij})\right]$$

TO SECOND ORDER IN FLUCTUATIONS!


3 DEGREES OF FREEDOM = 1 ζ
+ 2 γ_ij
= 11
- 4
TRACELESS SYMMETRIC
GAUGE: ∂_j γ_ij = 0
[= 5 - 3]
[φ + g_μν]
[FIX GAUGE]


TRACELESS SYMMETRIC
GAUGE
$\partial_j \gamma_{ij} = 0$
3 DEGREES OF FREEDOM = 1 ζ
+ 2 γ_{ij}
[= 5 - 3]
= 11
- 4
- 4
[ϕ + $g_{\mu\nu}$]
[FIX GAUGE]
[ELIMANATE NONDYNAMICAL]
NO LINEAR TERMS IN FLUCTUATIONS