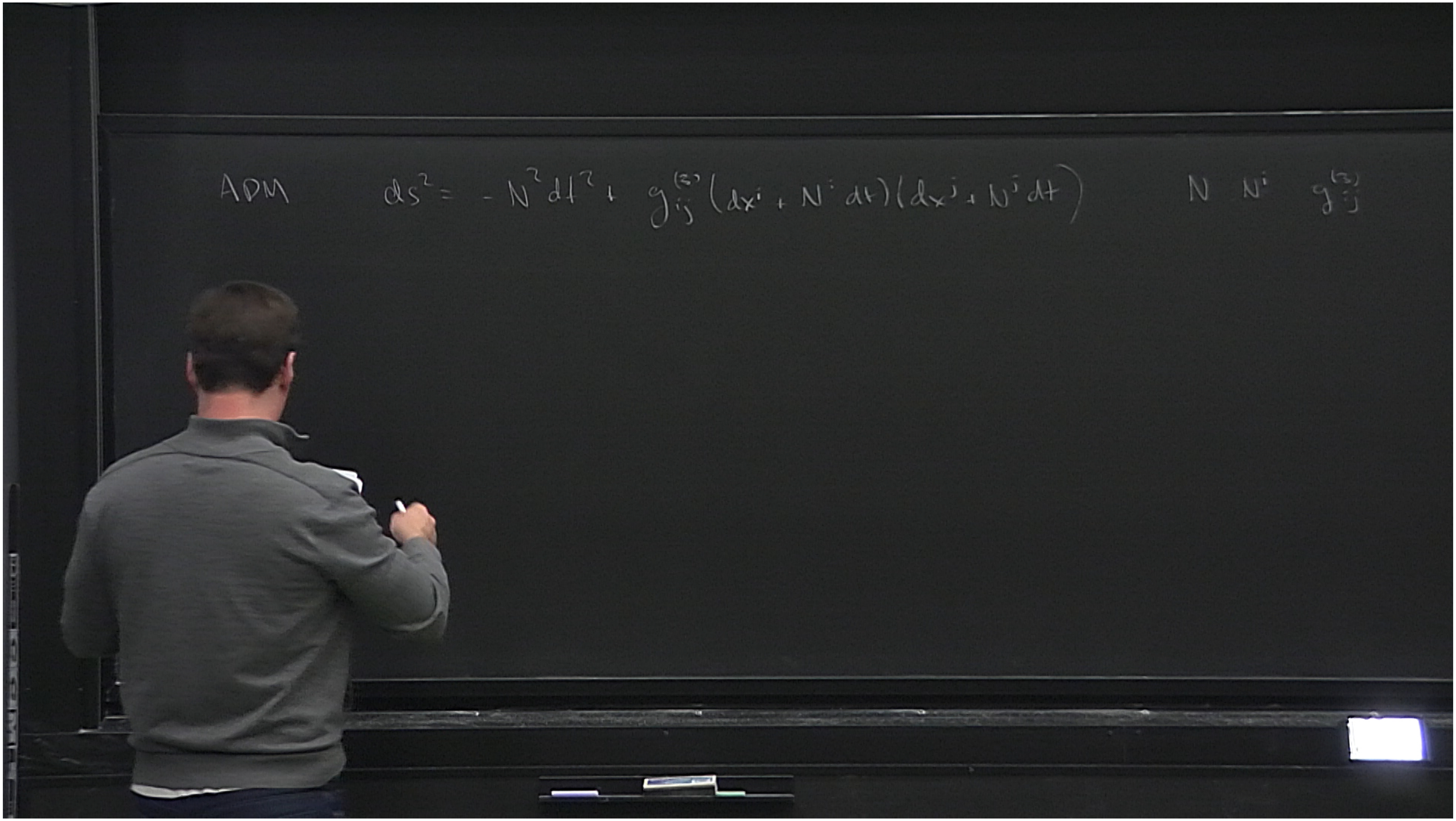
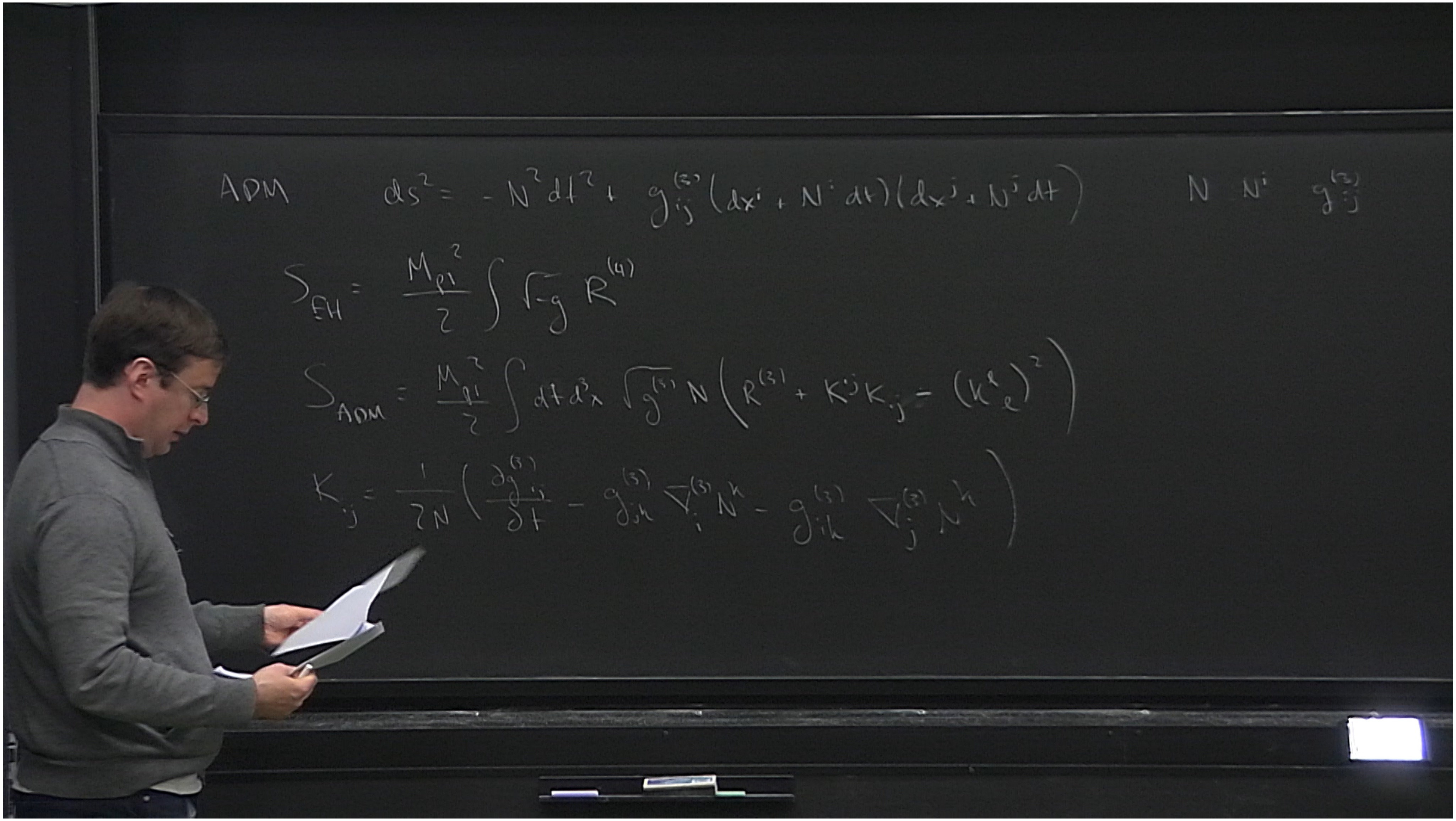
Title: PSI 2017/2018 - Cosmology - Lecture 10

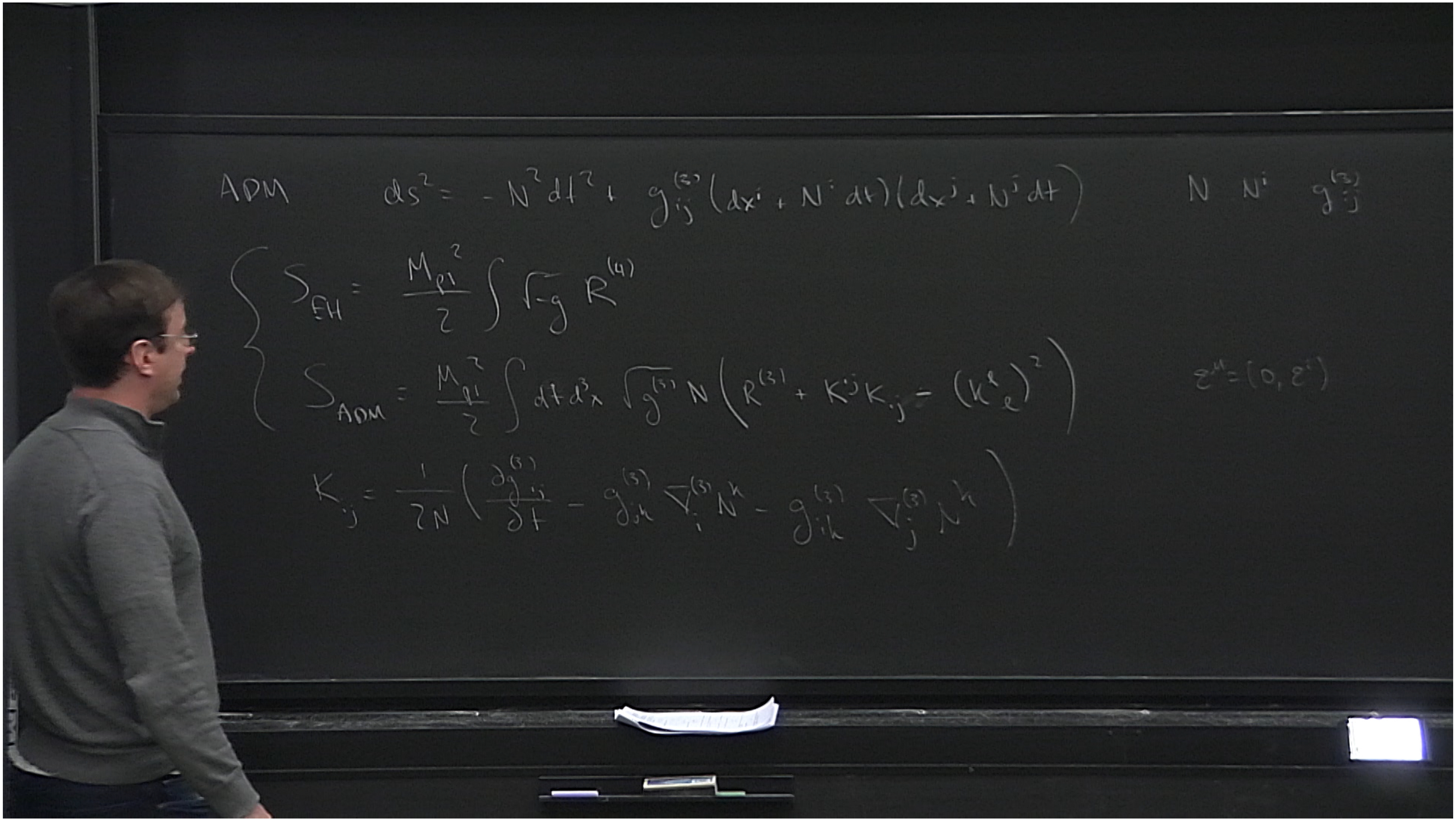
Date: Apr 20, 2018 10:15 AM

URL: http://pirsa.org/18040022

Abstract:


ADM
$ds^2 = -N^2 dt^2 + g^{(3)}_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
$N \quad N^i \quad g^{(3)}_{ij}$


ADM $ds^2 = -N^2 dt^2 + g^{(3)}_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
$N \quad N^i \quad g^{(3)}_{ij}$
$S_{EH} = \frac{M_{pl}^2}{2}\int\sqrt{-g}\,R^{(4)}$
$S_{ADM} = \frac{M_{pl}^2}{2}\int dt\,d^3x\,\sqrt{g^{(3)}}\,N\left(R^{(3)} + K^{ij}K_{ij} - (K^l_l)^2\right)$
$K_{ij} = \frac{1}{2N}\left(\frac{\partial g^{(3)}_{ij}}{\partial t} - g^{(3)}_{jk}\nabla^{(3)}_i N^k - g^{(3)}_{ik}\nabla^{(3)}_j N^k\right)$


ADM $ds^2 = -N^2 dt^2 + g^{(3)}_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
$N \quad N^i \quad g^{(3)}_{ij}$
$S_{EH} = \frac{M_{pl}^2}{2}\int \sqrt{-g}\, R^{(4)}$
$S_{ADM} = \frac{M_{pl}^2}{2}\int dt\, d^3x \sqrt{g^{(3)}}\, N\left(R^{(3)} + K^{ij}K_{ij} - (K^{l}_{l})^2\right)$
$\xi^\mu = (0, \xi^i)$
$K_{ij} = \frac{1}{2N}\left(\frac{\partial g^{(3)}_{ij}}{\partial t} - g^{(3)}_{jk}\nabla^{(3)}_i N^k - g^{(3)}_{ik}\nabla^{(3)}_j N^k\right)$

$g^{(3)}_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$

$N \quad N^i \quad g^{(3)}_{ij}$

$N\left(R^{(3)} + K^{ij}K_{ij} - \left(K^{\ell}{}_{\ell}\right)^2\right)$

$\varepsilon^\mu = (0, \varepsilon^i)$

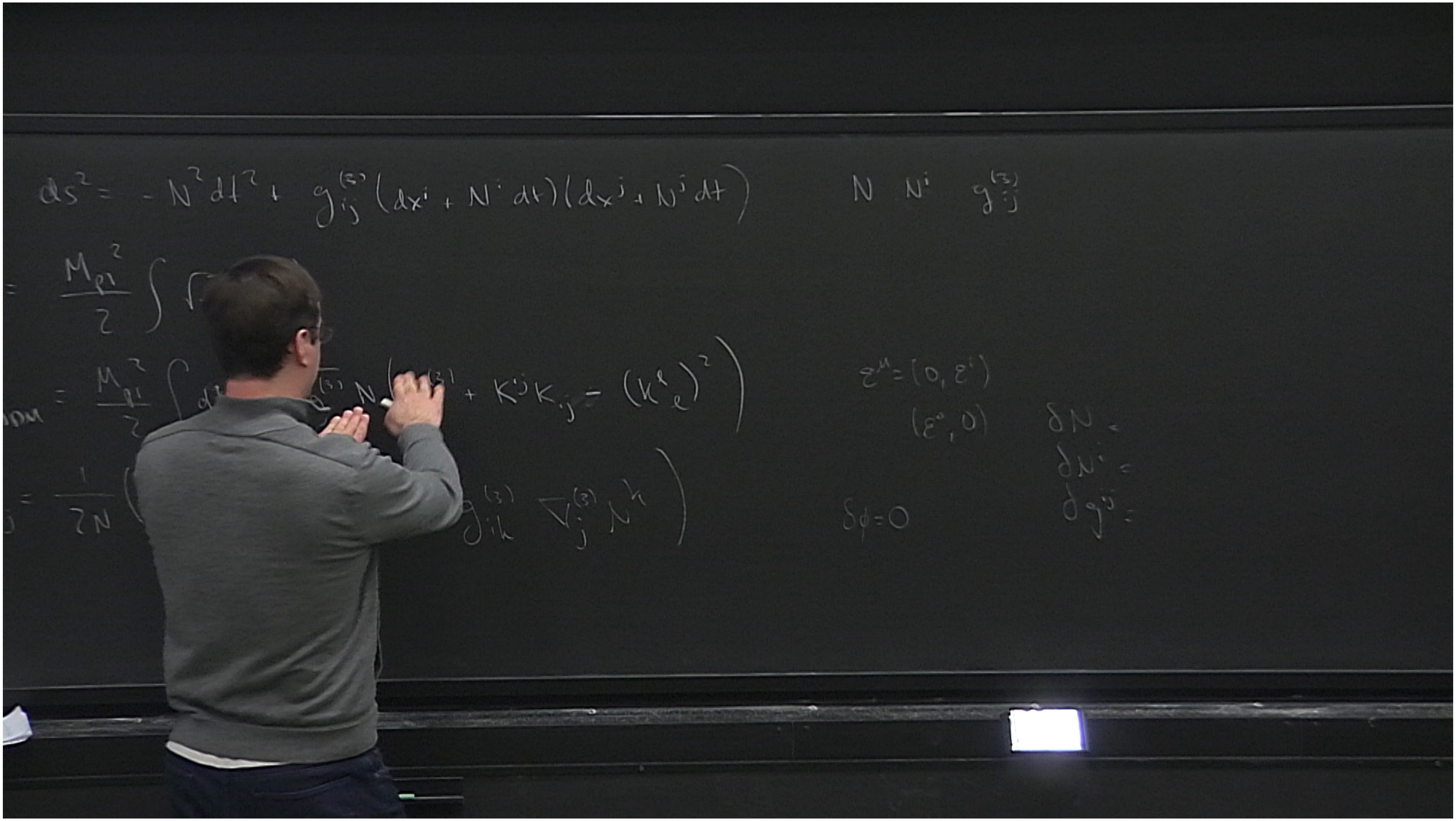
$(\varepsilon^0, 0)$

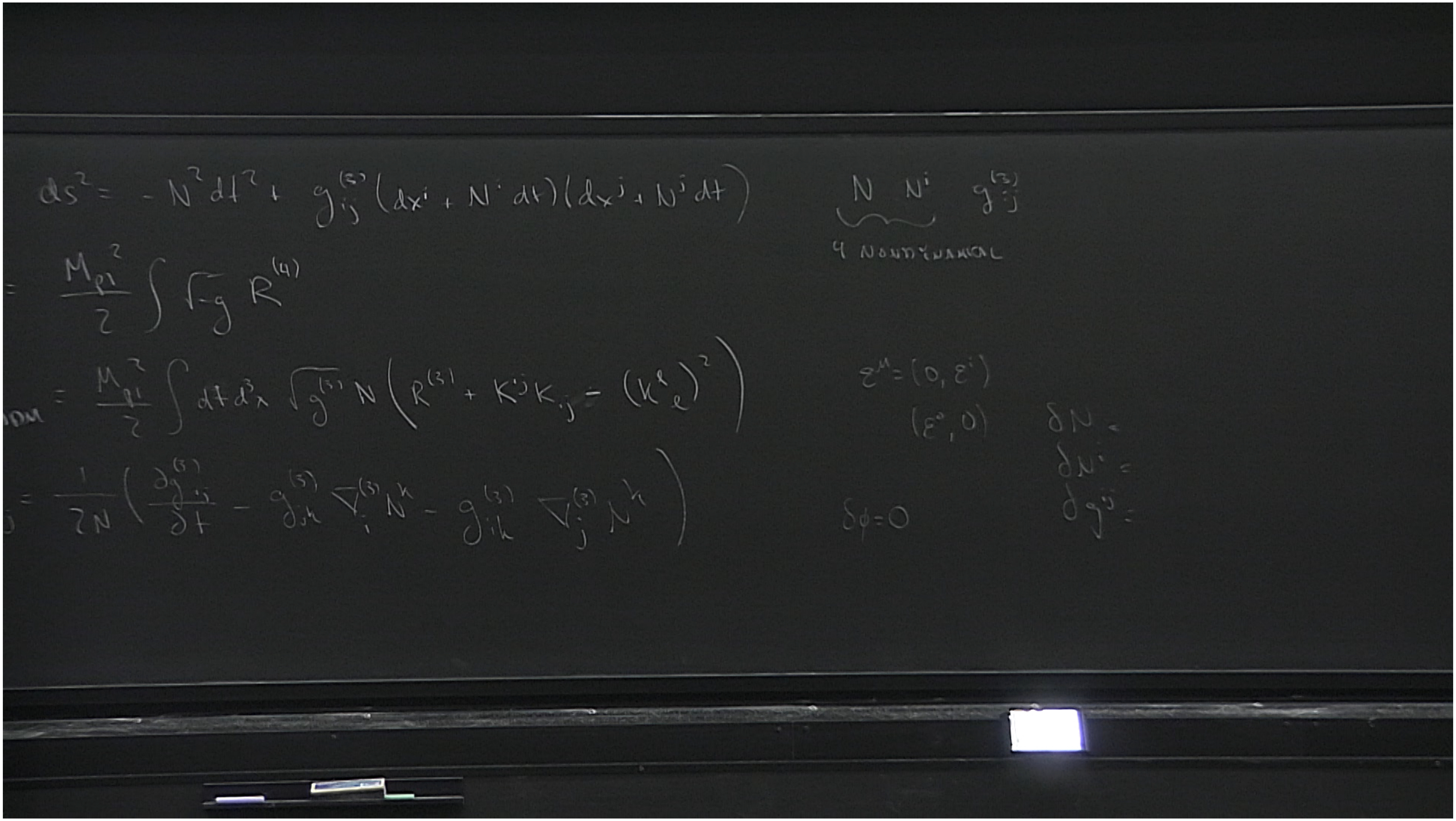
$\delta N =$

$\delta N^i =$

$\delta g^{ij} =$

$\nabla^{(3)}_i N^k - g^{(3)}_{ik} \nabla^{(3)}_j N^k\Big)$


ds² = -N² dt² + g(3)ij (dxi + Ni dt)(dxj + Nj dt)
N Ni g(3)ij
Mpl² / 2 ∫ √
Mpl² / 2 ∫ d N ((3) + Kij Kij − (Kll)²)
1 / 2N (g(3)ik ∇(3)j Nk)
εμ = (0, εi)
(ε0, 0)
δN =
δNi =
δgij =
δφ = 0


$ds^2 = -N^2 dt^2 + g^{(3)}_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
$\underbrace{N \quad N^i}_{\text{4 NONDYNAMICAL}} \quad g^{(3)}_{ij}$
$= \frac{M_{pl}^2}{2}\int \sqrt{-g}\, R^{(4)}$
$_{DM} = \frac{M_{pl}^2}{2}\int dt\, d^3x \sqrt{g^{(3)}}\, N \left(R^{(3)} + K^{ij}K_{ij} - (K^l_l)^2\right)$
$_j = \frac{1}{2N}\left(\frac{\partial g^{(3)}_{ij}}{\partial t} - g^{(3)}_{jk}\nabla^{(3)}_i N^k - g^{(3)}_{ik}\nabla^{(3)}_j N^k\right)$
$\varepsilon^\mu = (0, \varepsilon^i)$
$(\varepsilon^0, 0)$
$\delta N =$
$\delta N^i =$
$\delta g^{ij} =$
$\delta\phi = 0$

$$ds^2 = -N^2 dt^2 + g^{(3)}_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$\underbrace{N \quad N^i}_{4 \text{ NONDYNAMICAL}} \quad \underbrace{g^{(3)}_{ij}}_{6 \text{ DYNAMICAL}}$$

$$= \frac{M_{Pl}^2}{2}\int \sqrt{-g}\, R^{(4)}$$

$$\text{DM} = \frac{M_{Pl}^2}{2}\int dt\, d^3x \sqrt{g^{(3)}}\, N \left(R^{(3)} + K^{ij}K_{ij} - (K^l_l)^2\right)$$

$$j = \frac{1}{2N}\left(\frac{\partial g^{(3)}_{ij}}{\partial t} - g^{(3)}_{jk}\nabla^{(3)}_i N^k - g^{(3)}_{ik}\nabla^{(3)}_j N^k\right)$$

$$\varepsilon^\mu = (0, \varepsilon^i)$$

$$(\varepsilon^0, 0)$$

$$\delta N =$$

$$\delta N^i =$$

$$\delta g^{ij} =$$

$$\delta\phi = 0$$

$(dx^i + N^i dt)(dx^j + N^j dt))$

$\underbrace{N \quad N^i}_{\text{4 NONDYNAMICAL}} \quad \underbrace{g^{(3)}_{ij}}_{\text{6 DYNAMICAL}}$

$\left(R^{(3)} + K^{ij}K_{ij} - (K^l{}_l)^2\right)$

$N^k - g^{(3)}_{ik} \nabla^{(3)}_j N^k)$

$\varepsilon^\mu = (0, \varepsilon^i)$

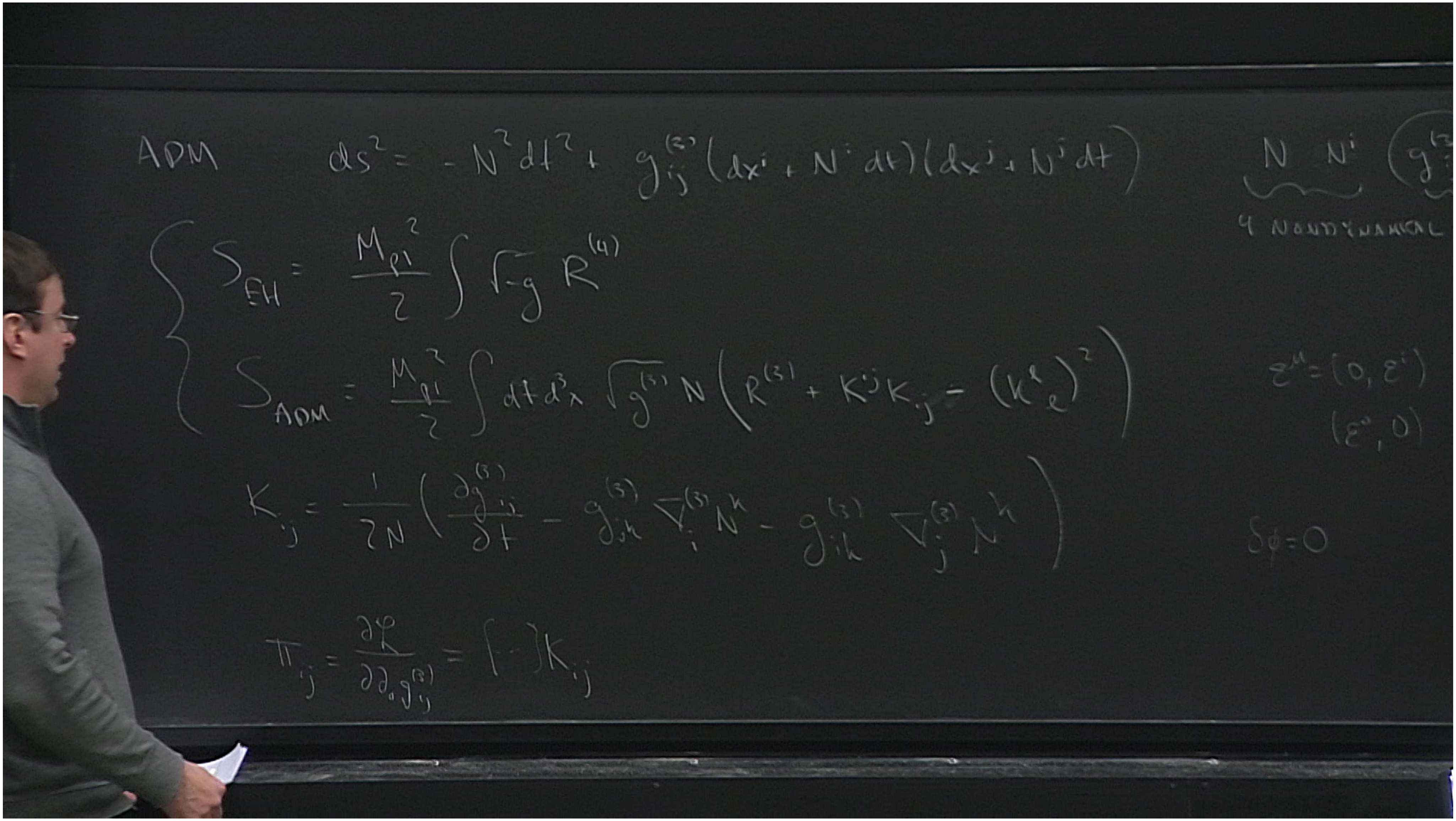
$(\varepsilon^0, 0)$

$\delta N =$

$\delta N^i =$

$\delta g^{ij} =$

$\delta\phi = 0$


ADM
$ds^2 = -N^2 dt^2 + g^{(3)}_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
$N \quad N^i \quad g^{(3)}$
4 NONDYNAMICAL
$S_{EH} = \frac{M_{pl}^2}{2}\int \sqrt{-g}\, R^{(4)}$
$S_{ADM} = \frac{M_{pl}^2}{2}\int dt\, d^3x \sqrt{g^{(3)}}\, N\left(R^{(3)} + K^{ij}K_{ij} - (K^l{}_l)^2\right)$
$K_{ij} = \frac{1}{2N}\left(\frac{\partial g^{(3)}_{ij}}{\partial t} - g^{(3)}_{jk}\nabla^{(3)}_i N^k - g^{(3)}_{ik}\nabla^{(3)}_j N^k\right)$
$\Pi_{ij} = \frac{\partial \mathcal{L}}{\partial \partial_0 g^{(3)}_{ij}} = \{\cdots\} K_{ij}$
$\varepsilon^\mu = (0, \varepsilon^i)$
$(\varepsilon^0, 0)$
$\delta\phi = 0$

ADM $\quad ds^2 = -N^2 dt^2 + g^{(3)}_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$

$\underbrace{N \quad N^i}_{\text{4 NONDYNAMICAL}} \quad \underbrace{g^{(3)}_{ij}}_{\text{6 DYNA}}$

$$\begin{cases} S_{EH} = \dfrac{M_{Pl}^2}{2}\displaystyle\int \sqrt{-g}\, R^{(4)} \\ S_{ADM} = \dfrac{M_{Pl}^2}{2}\displaystyle\int dt\, d^3x \sqrt{g^{(3)}}\, N\left(R^{(3)} + K^{ij}K_{ij} - (K^{\ell}{}_{\ell})^2\right) \end{cases}$$

$$\rightarrow \quad K_{ij} = \frac{1}{2N}\left(\frac{\partial g^{(3)}_{ij}}{\partial t} - g^{(3)}_{jk}\nabla^{(3)}_i N^k - g^{(3)}_{ik}\nabla^{(3)}_j N^k\right)$$

$$\pi_{ij} = \frac{\partial \mathcal{L}}{\partial \partial_0 g^{(3)}_{ij}} = \{\cdot\} K_{ij}$$

$\varepsilon^\mu = (0, \varepsilon^i)$

$(\varepsilon^0, 0)$

$\delta\phi = 0$

δN

δN^i

δg_{ij}

ADM $\quad ds^2 = -N^2 dt^2 + g^{(3)}_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$

$\underbrace{N \quad N^i}_{}$ $\quad g^{(3)}_{ij}$

4 NONDYNAMICAL $\quad$ 6 DYNA…

$$\boxed{S_{EH} = \frac{M_{pl}^2}{2}\int \sqrt{-g}\, R^{(4)}}$$

$$S_{ADM} = \frac{M_{pl}^2}{2}\int dt\, d^3x \sqrt{g^{(3)}}\, N\left(R^{(3)} + K^{ij}K_{ij} - (K^{\ell}{}_{\ell})^2\right)$$

$\varepsilon^{\mu} = (0, \varepsilon^i)$

$(\varepsilon^0, 0)$

δN

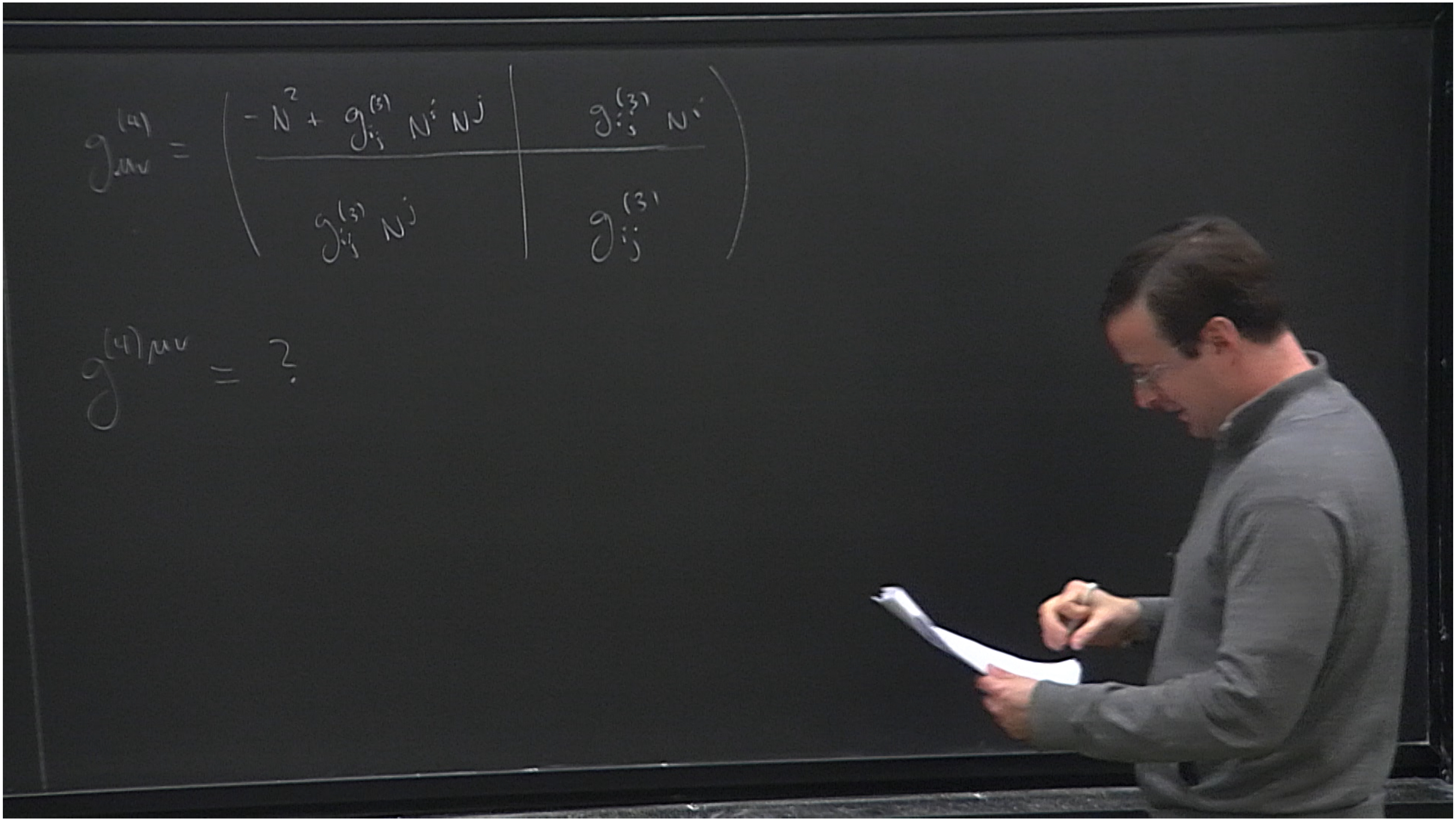
δN^i

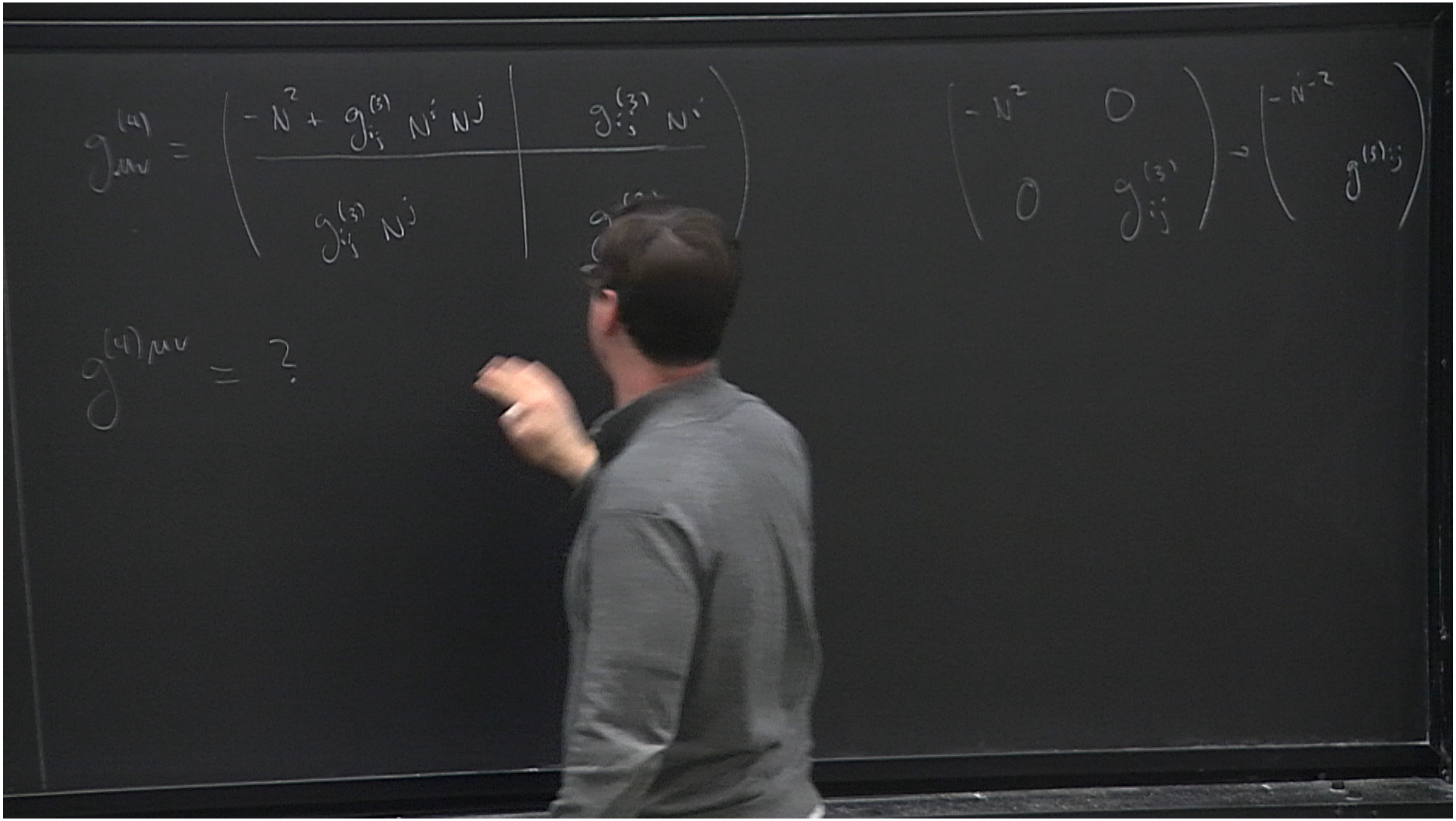
δg…

$$\to K_{ij} = \frac{1}{2N}\left(\frac{\partial g^{(3)}_{ij}}{\partial t} - g^{(3)}_{jk}\nabla^{(3)}_i N^k - g^{(3)}_{ik}\nabla^{(3)}_j N^k\right)$$

$\delta\phi = 0$

$$\pi_{ij} = \frac{\partial \mathcal{L}}{\partial \partial_0 g^{(3)}_{ij}} = (\cdot\cdot)K_{ij}$$


$g^{(4)}_{\mu\nu} = \left(\begin{array}{c|c} -N^2 + g^{(3)}_{ij} N^i N^j & g^{(3)}_{ij} N^i \\ \hline g^{(3)}_{ij} N^j & g^{(3)}_{ij} \end{array}\right)$
$g^{(4)\mu\nu} = ?$


$g^{(4)}_{\mu\nu} = \left(\begin{array}{c|c} -N^2 + g^{(3)}_{ij} N^i N^j & g^{(3)}_{ij} N^i \\ \hline g^{(3)}_{ij} N^j & \end{array} \right)$
$\begin{pmatrix} -N^2 & 0 \\ 0 & g^{(3)}_{ij} \end{pmatrix} \to \begin{pmatrix} -N^{-2} & \\ & g^{(3)ij} \end{pmatrix}$
$g^{(4)\mu\nu} = ?$

$$g^{(4)}_{\mu\nu} = \left(\begin{array}{c|c} -N^2 + g^{(3)}_{ij} N^i N^j & g^{(3)}_{ij} N^i \\ \hline g^{(3)}_{ij} N^j & g^{(3)}_{ij} \end{array}\right)$$

$$\begin{pmatrix} -N^2 & 0 \\ 0 & g^{(3)}_{ij} \end{pmatrix} \to \begin{pmatrix} -N^{-2} & \\ & g^{(3)ij} \end{pmatrix}$$

$$g^{(4)}_{\mu\nu} = \begin{pmatrix} 1 & N^j \\ 0 & \delta_{ij} \end{pmatrix} \begin{pmatrix} -N^2 & \\ & g^{(3)}_{ij} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ N^i & \delta_{ij} \end{pmatrix}$$

ADM $ds^2 = -N^2 dt^2 + g^{(3)}_{ij}(\underbrace{dx^i + N^i dt}_{dx'})(\underbrace{dx^j + N^j dt}_{dx})$

$\underbrace{N \quad N^i}_{\text{4 NONDYNAMICAL}}$ $\underbrace{g^{(3)}_{ij}}_{\text{6 DYNA}}$

$$\boxed{S_{EH} = \frac{M_{Pl}^2}{2}\int \sqrt{-g}\, R^{(4)}}$$

$$S_{ADM} = \frac{M_{Pl}^2}{2}\int dt\, d^3x\, \sqrt{g^{(3)}}\, N\left(R^{(3)} + K^{ij}K_{ij} - (K^l{}_l)^2\right)$$

$\varepsilon^\mu = (0, \varepsilon^i)$

$(\varepsilon^0, 0)$

δN

δN^i

δg

$$\rightarrow K_{ij} = \frac{1}{2N}\left(\frac{\partial g^{(3)}_{ij}}{\partial t} - g^{(3)}_{jk}\nabla^{(3)}_i N^k - g^{(3)}_{ik}\nabla^{(3)}_j N^k\right)$$

$\delta\phi = 0$

$$\pi_{ij} = \frac{\partial \mathcal{L}}{\partial \partial_0 g^{(3)}_{ij}} = \{\cdot\} K_{ij}$$

NAMICAL

$$g^{(4)}_{\mu\nu} = \left(\begin{array}{c|c} -N^2 + g^{(3)}_{ij} N^i N^j & g^{(3)}_{ij} N^i \\ \hline g^{(3)}_{ij} N^j & g^{(3)}_{ij} \end{array}\right)$$

$$\begin{pmatrix} -N^2 & 0 \\ 0 & g^{(3)}_{ij} \end{pmatrix} \to \begin{pmatrix} -N^{-2} & \\ & g^{(3)ij} \end{pmatrix}$$

$$g^{(4)}_{\mu\nu} = \begin{pmatrix} 1 & N^j \\ 0 & \delta_{ij} \end{pmatrix} \begin{pmatrix} -N^2 & \\ & g^{(3)}_{ij} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ N^i & \delta_{ij} \end{pmatrix}$$

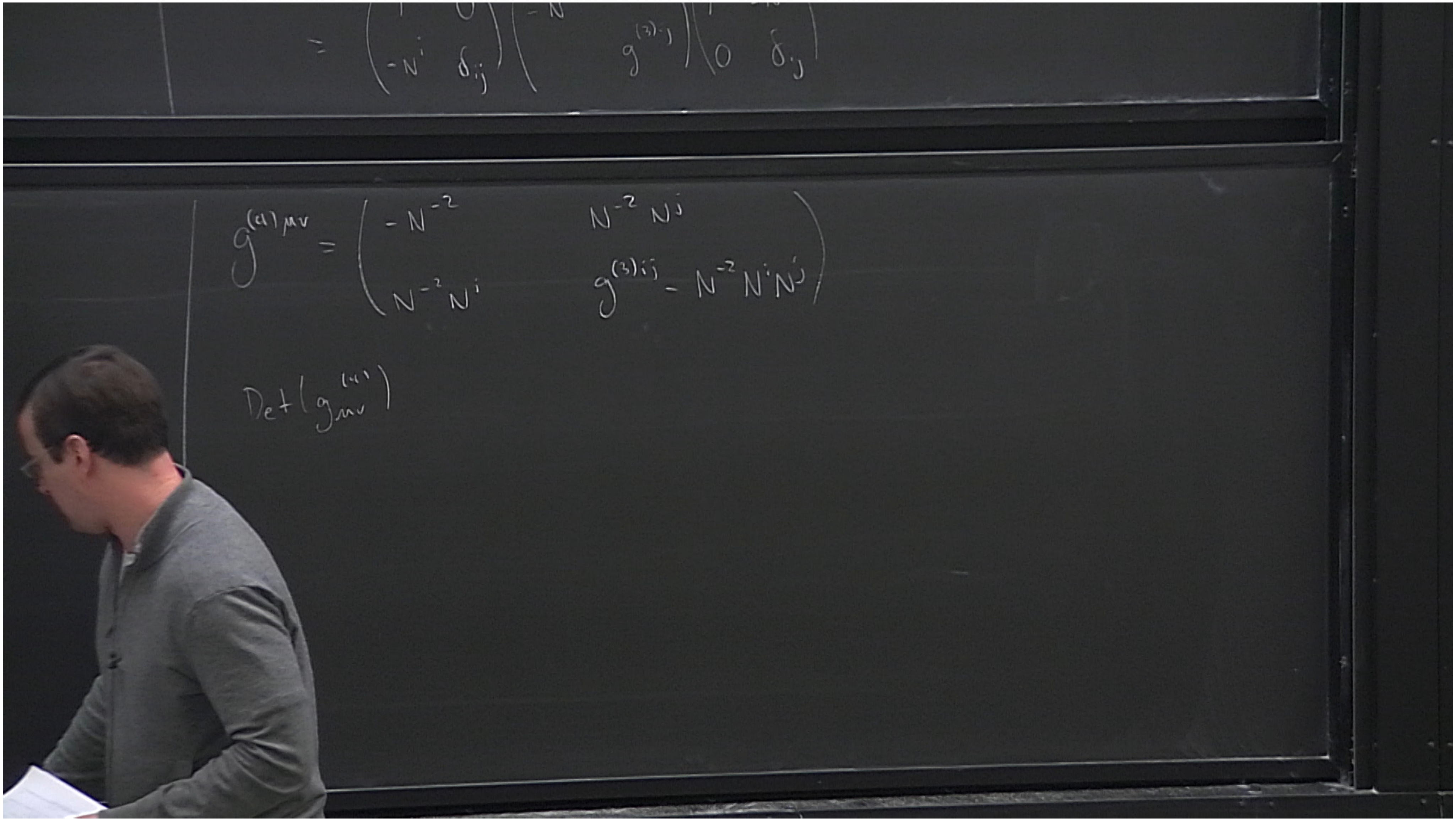
$N =$

$N^i =$

$g^{ij} =$

$$g^{(4)\mu\nu} = \begin{pmatrix} 1 & 0 \\ N^i & \delta_{ij} \end{pmatrix}^{-1} \begin{pmatrix} -N^2 & \\ & g^{(3)}_{ij} \end{pmatrix}^{-1} \begin{pmatrix} 1 & N^j \\ 0 & \delta_{ij} \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 1 & 0 \\ -N^i & \delta_{ij} \end{pmatrix} \begin{pmatrix} -N^{-2} & \\ & g^{(3)ij} \end{pmatrix} \begin{pmatrix} 1 & -N^i \\ 0 & \delta_{ij} \end{pmatrix}$$

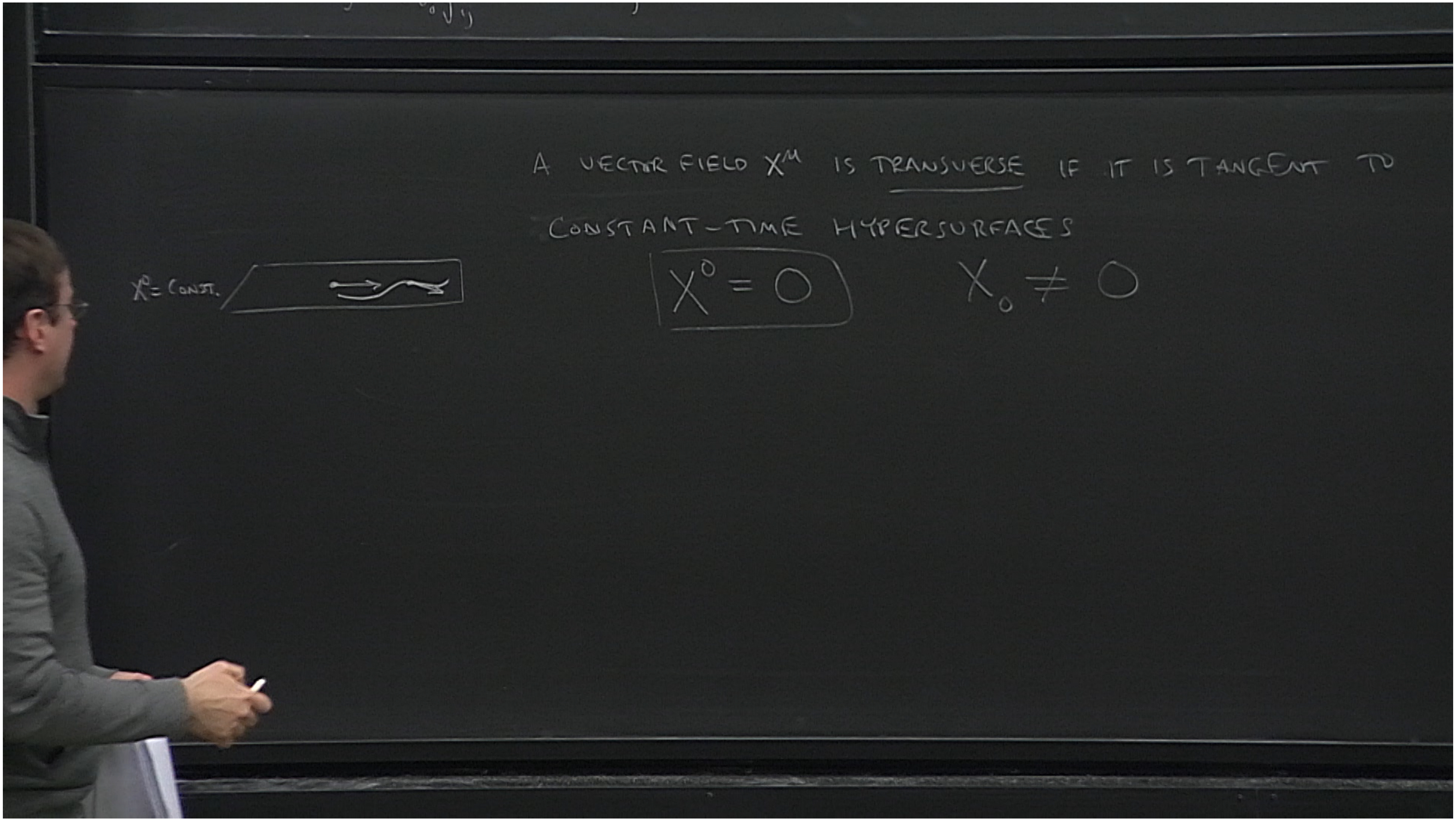

g^(4)μν = (-N^-2   N^-2 N^j ; N^-2 N^i   g^(3)ij - N^-2 N^i N^j)
Det(g^(4)_μν)

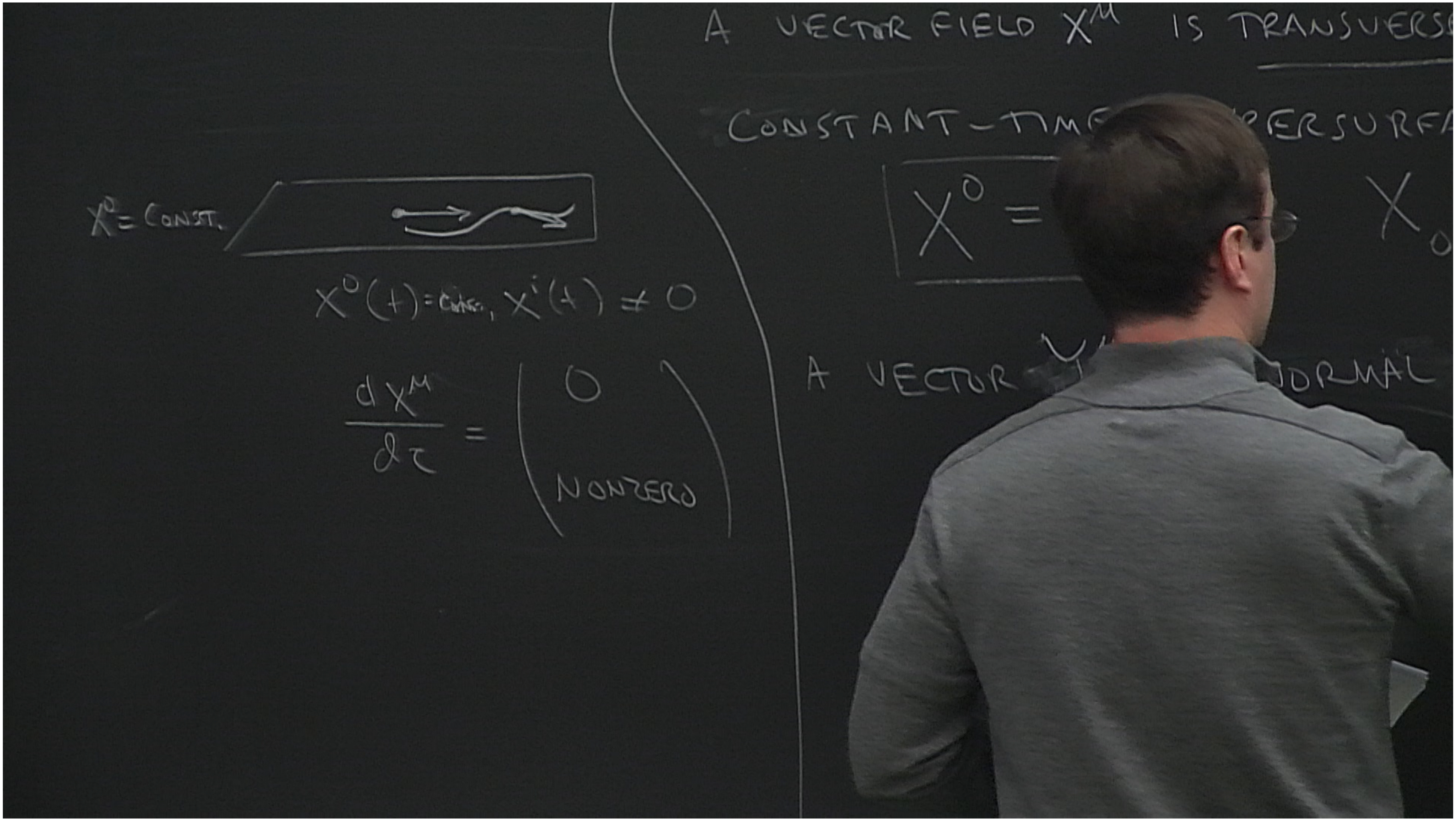
$$= \begin{pmatrix} 1 & 0 \\ -N^i & \delta_{ij} \end{pmatrix} \begin{pmatrix} -N^{-2} & \\ & g^{(3)ij} \end{pmatrix} \begin{pmatrix} 1 & -N^j \\ 0 & \delta_{ij} \end{pmatrix}$$

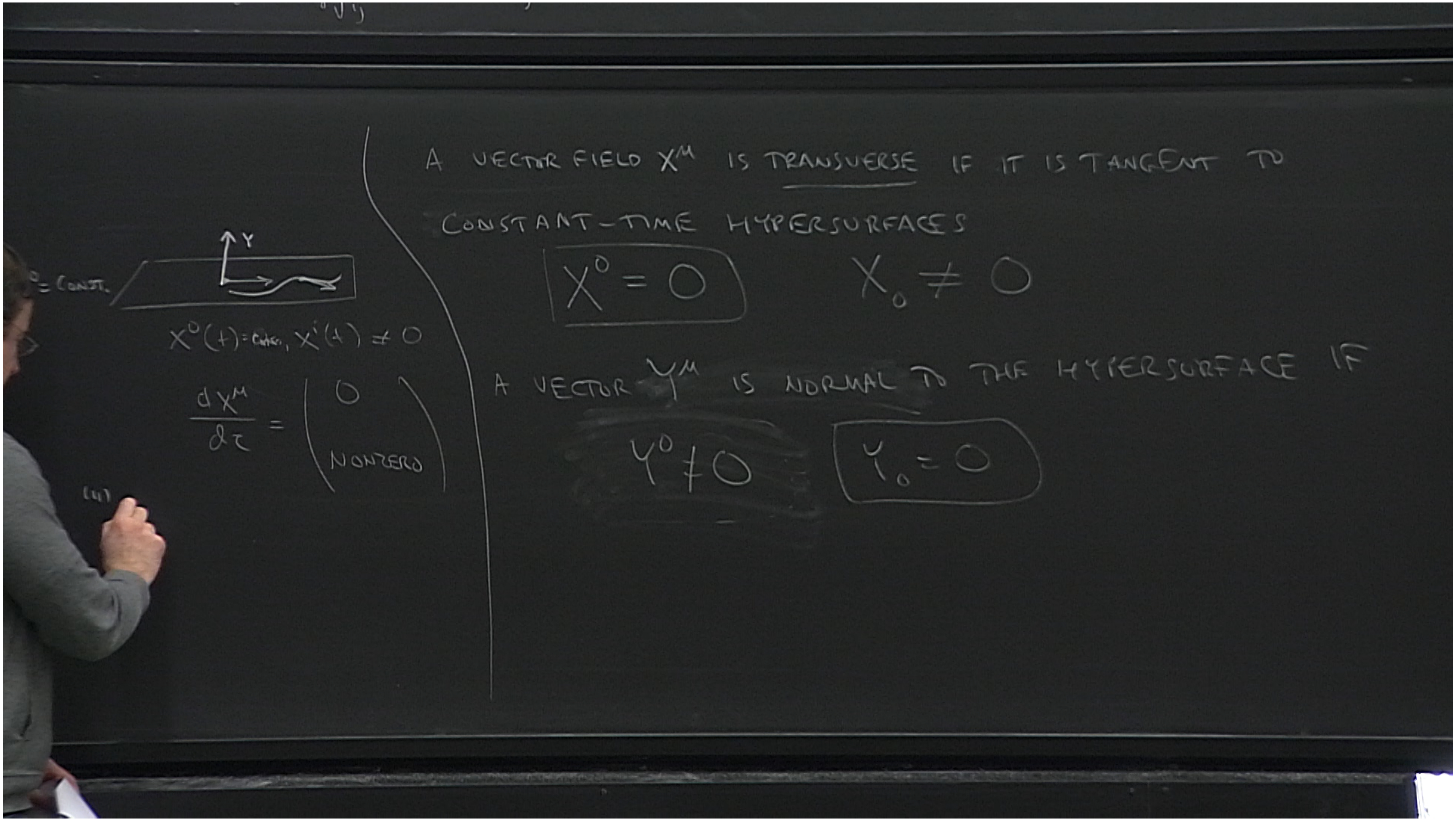
$$g^{(4)\mu\nu} = \begin{pmatrix} -N^{-2} & N^{-2} N^j \\ N^{-2} N^i & g^{(3)ij} - N^{-2} N^i N^j \end{pmatrix}$$

$$\mathrm{Det}\left(g^{(4)}_{\mu\nu}\right) = -N^2 \, \mathrm{Det}\left(g^{(3)}_{ij}\right)$$

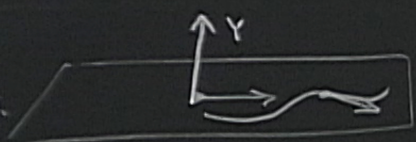
$$\sqrt{-g^{(4)}} = N \sqrt{g^{(3)}}$$


A VECTOR FIELD X^μ IS TRANSVERSE IF IT IS TANGENT TO
CONSTANT-TIME HYPERSURFACES
$X^0 =$ CONST.
$X^0 = 0$
$X_0 \neq 0$


A VECTOR FIELD X^M IS TRANSVERSE
CONSTANT-TIME
X^0 = CONST.
X^0(t) = CONST., X^i(t) ≠ 0
dX^M / dτ = 0 NONZERO
X^0 =
A VECTOR
NORMAL


A VECTOR FIELD X^M IS TRANSVERSE IF IT IS TANGENT TO
CONSTANT-TIME HYPERSURFACES
X^0 = 0
X_0 ≠ 0
A VECTOR Y^M IS NORMAL TO THE HYPERSURFACE IF
Y^0 ≠ 0
Y_0 = 0
= CONST.
Y
X^0(t) = const, X^i(t) ≠ 0
dX^M/dτ =
0
NONZERO

$X^0 = \text{CONST.}$



$X^0(t) = \text{const}, \; X^i(t) \neq 0$

$$\frac{dX^\mu}{d\tau} = \begin{pmatrix} 0 \\ \text{NONZERO} \end{pmatrix}$$

$$0 = g^{(4)}_{\mu\nu} X^\mu Y^\nu = \underbrace{X^\mu}_{(0,\,\cdot)} \underbrace{Y_\mu}_{(Y_0,\,0)}$$

X IS TRANSVERSE

A VECTOR FIELD X^μ IS TRANSVERSE IF IT IS TA

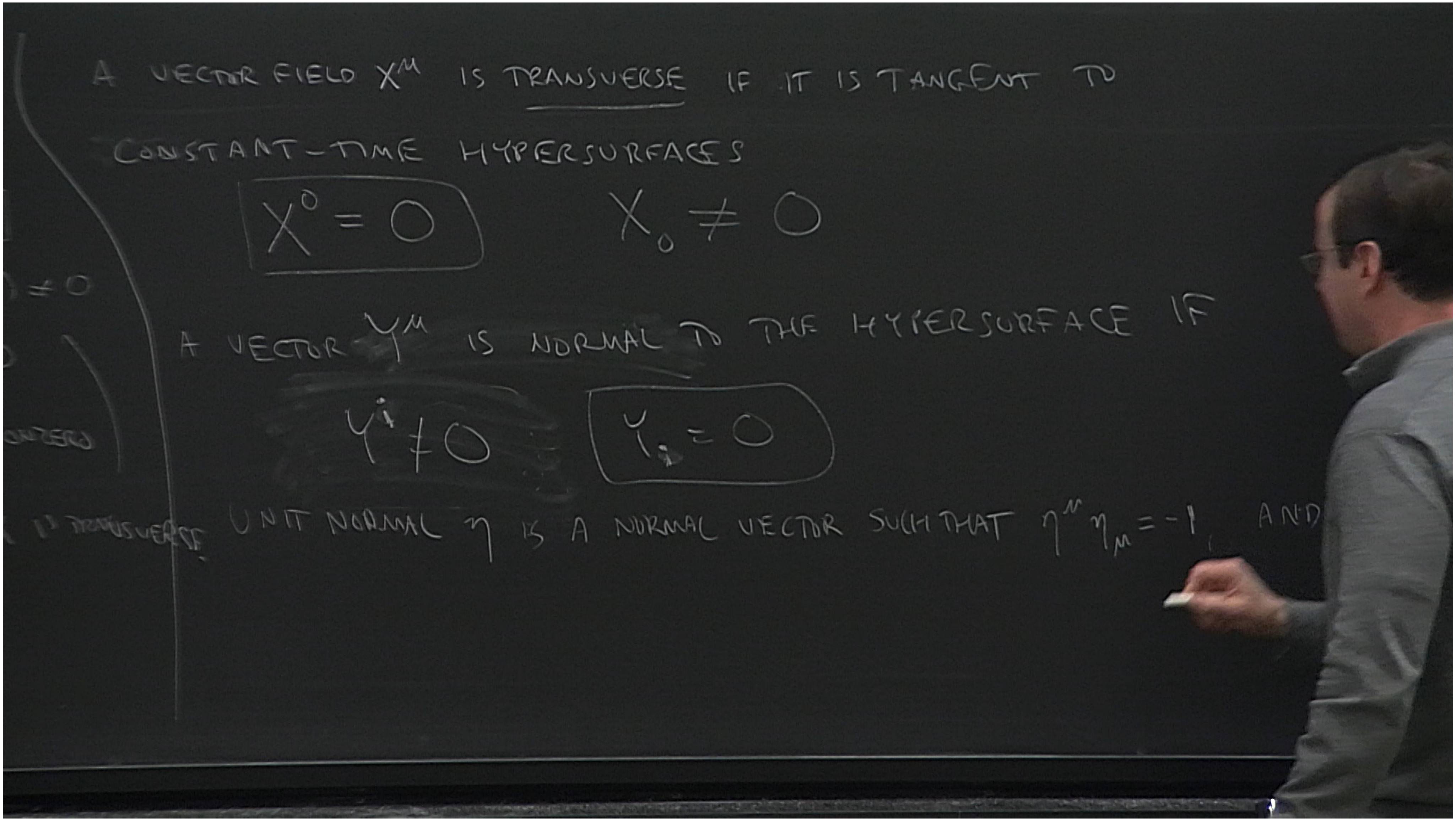
CONSTANT-TIME HYPERSURFACES

$\boxed{X^0 = 0}$ $\qquad X_0 \neq 0$

A VECTOR Y^μ IS NORMAL TO THE HYP

$Y^i \neq 0$ $\qquad \boxed{Y_i = 0}$

UNIT NORMAL η IS A NORMAL VECTOR SUCH


A VECTOR FIELD X^μ IS TRANSVERSE IF IT IS TANGENT TO
CONSTANT-TIME HYPERSURFACES
$\boxed{X^0 = 0}$
$X_0 \neq 0$
A VECTOR Y^μ IS NORMAL TO THE HYPERSURFACE IF
$Y^i \neq 0$
$\boxed{Y_i = 0}$
UNIT NORMAL η IS A NORMAL VECTOR SUCH THAT $\eta^\mu \eta_\mu = -1$ AND

VECTOR FIELD X^μ IS TRANSVERSE IF IT IS TANGENT TO CONSTANT-TIME HYPERSURFACES

$$\boxed{X^0 = 0} \qquad X_0 \neq 0$$

A VECTOR Y^μ IS NORMAL TO THE HYPERSURFACE IF

$$Y^i \neq 0 \qquad \boxed{Y_i = 0}$$

UNIT NORMAL η IS A NORMAL VECTOR SUCH THAT $\underbrace{\eta^\mu \eta_\mu}_{g^{(4)00}\eta_0\eta_0} = -1$, AND $\eta^0 > 0$

$$\eta_\mu = (- \quad , 0)$$

VECTOR FIELD X^μ IS TRANSVERSE IF IT IS TANGENT TO CONSTANT-TIME HYPERSURFACES

$$\boxed{X^0 = 0} \qquad X_0 \neq 0$$

A VECTOR Y^μ IS NORMAL TO THE HYPERSURFACE IF

$$Y^i \neq 0 \qquad \boxed{Y_i = 0}$$

UNIT NORMAL η IS A NORMAL VECTOR SUCH THAT $\underbrace{\eta^\mu \eta_\mu}_{g^{(4)00}\eta_0\eta_0} = -1$ AND $\eta^0 > 0$

$$\eta_\mu = (-N, 0)$$

$$\eta^\mu = (N^{-1}, -N^{-1}N^i)$$

VECTOR FIELD X^μ IS TRANSVERSE IF IT IS TANGENT TO CONSTANT-TIME HYPERSURFACES

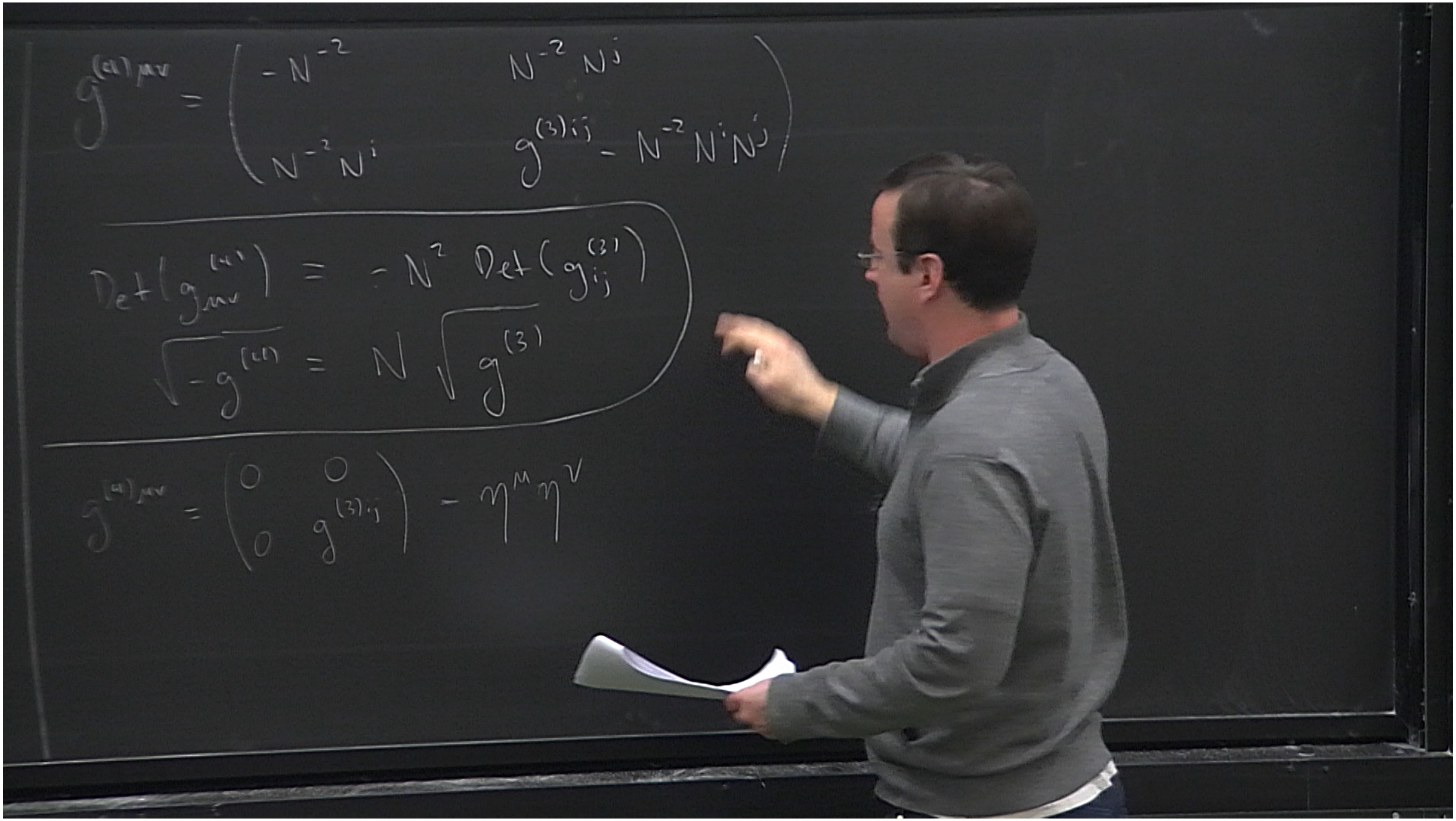
$$\boxed{X^0 = 0} \qquad X_0 \neq 0$$

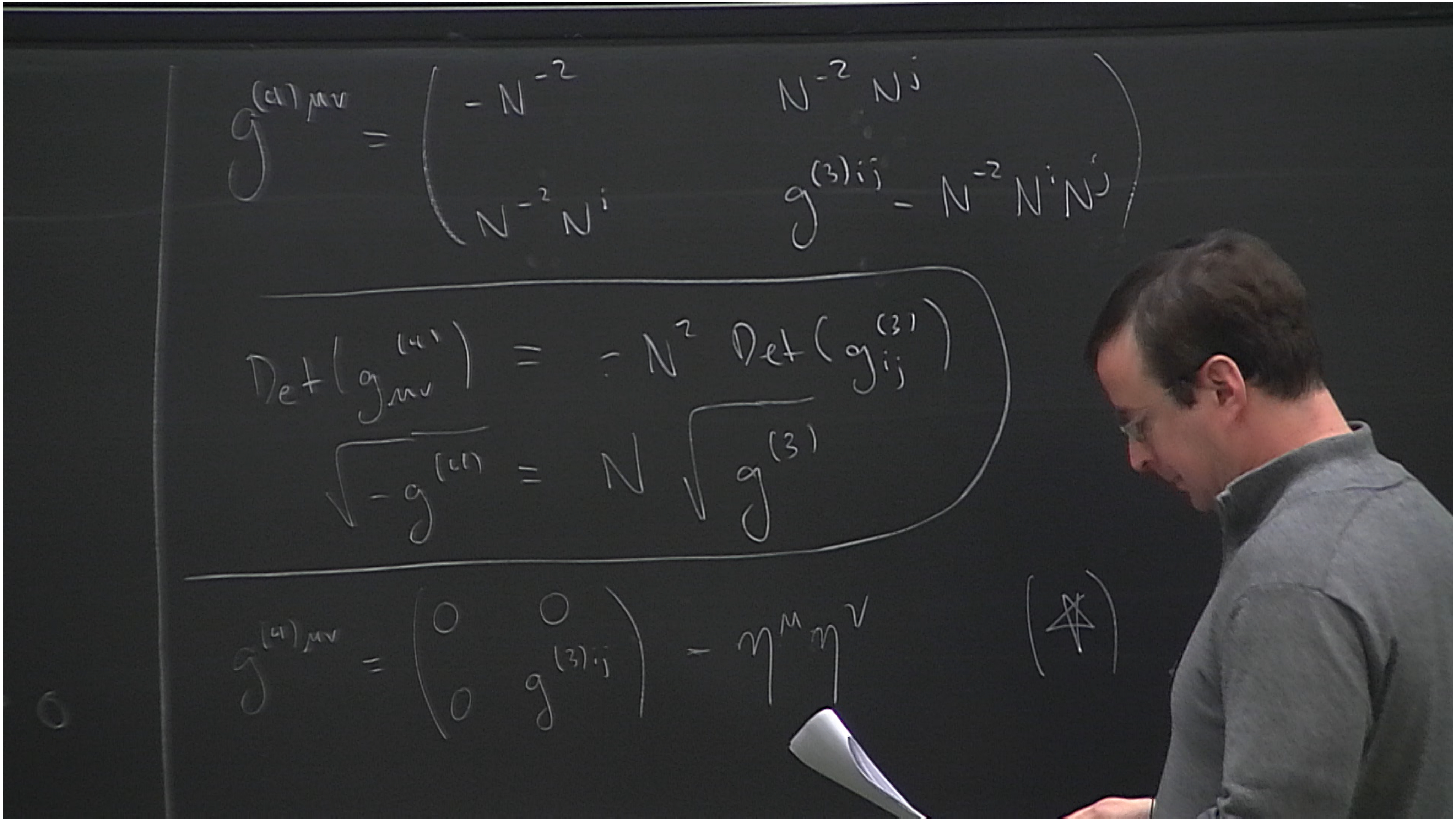
A VECTOR Y^μ IS NORMAL TO THE HYPERSURFACE IF

$$Y^i \neq 0 \qquad \boxed{Y_i = 0}$$

UNIT NORMAL η IS A NORMAL VECTOR SUCH THAT $\underbrace{\eta^\nu \eta_\mu}_{g^{(4)00}\eta_0\eta_0} = -1$, AND $\eta^0 > 0$

$$\boxed{\begin{aligned} \eta_\mu &= (-N, 0) \\ \eta^\mu &= (N^{-1}, -N^{-1}N^i) \end{aligned}}$$


$g^{(4)\mu\nu} = \begin{pmatrix} -N^{-2} & N^{-2}N^j \\ N^{-2}N^i & g^{(3)ij} - N^{-2}N^iN^j \end{pmatrix}$
$\mathrm{Det}(g^{(4)}_{\mu\nu}) = -N^2\,\mathrm{Det}(g^{(3)}_{ij})$
$\sqrt{-g^{(4)}} = N\sqrt{g^{(3)}}$
$g^{(4)\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & g^{(3)ij} \end{pmatrix} - \eta^\mu\eta^\nu$



$g^{(4)\mu\nu} = \begin{pmatrix} -N^{-2} & N^{-2}N^j \\ N^{-2}N^i & g^{(3)ij} - N^{-2}N^iN^j \end{pmatrix}$
$\mathrm{Det}(g^{(4)}_{\mu\nu}) = -N^2\,\mathrm{Det}(g^{(3)}_{ij})$
$\sqrt{-g^{(4)}} = N\sqrt{g^{(3)}}$
$g^{(4)\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & g^{(3)ij} \end{pmatrix} - n^\mu n^\nu$
$(*)$

$$g^{(4)\mu\nu} = \begin{pmatrix} -N^{-2} & N^{-2}N^j \\ N^{-2}N^i & g^{(3)ij} - N^{-2}N^iN^j \end{pmatrix}$$

$$\boxed{\begin{aligned} \mathrm{Det}\left(g^{(4)}_{\mu\nu}\right) &= -N^2\,\mathrm{Det}\left(g^{(3)}_{ij}\right) \\ \sqrt{-g^{(4)}} &= N\sqrt{g^{(3)}} \end{aligned}}$$

$$g^{(4)\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & g^{(3)ij} \end{pmatrix} - \eta^\mu \eta^\nu \qquad (*)$$

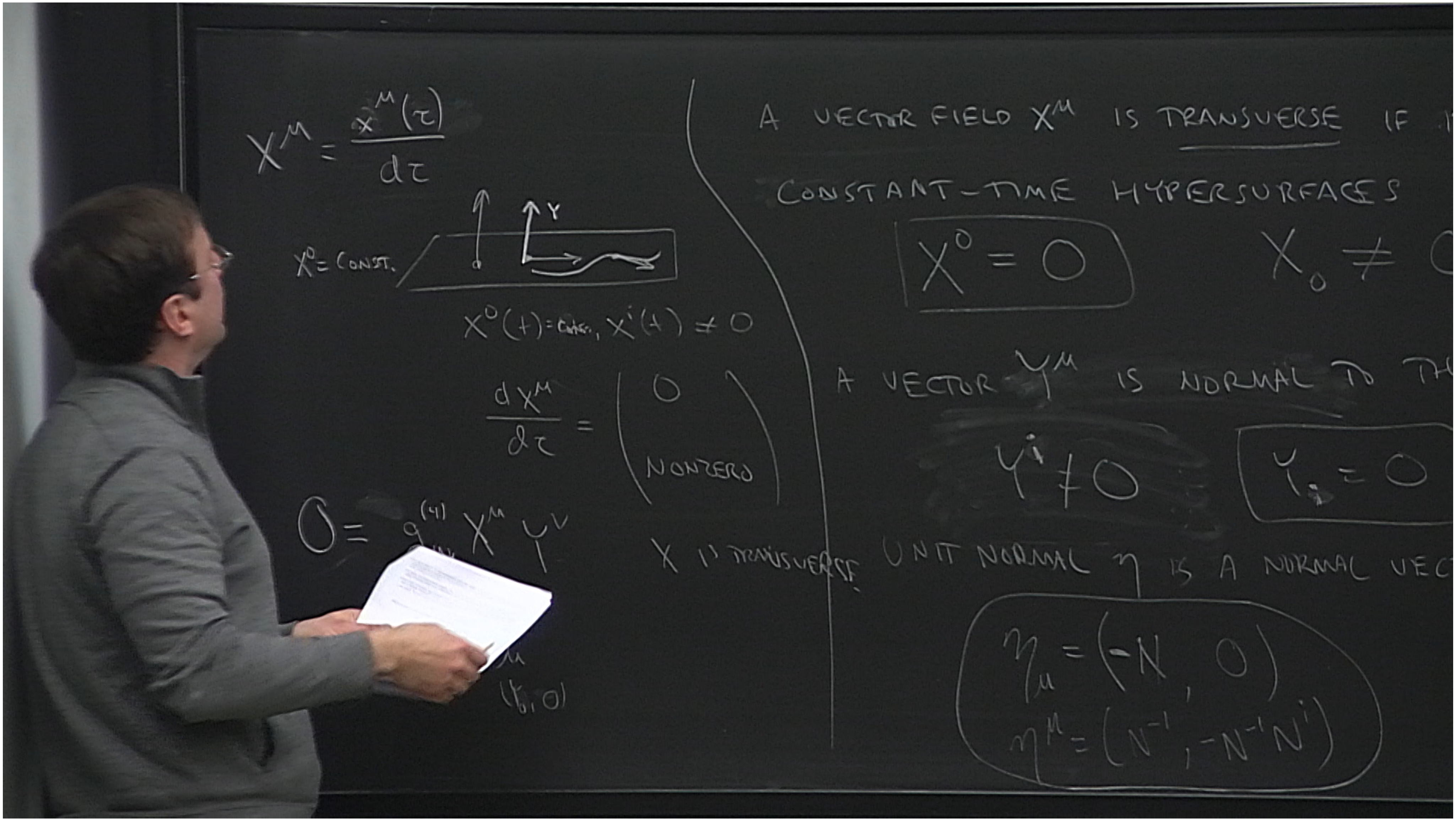
$$= \delta^\mu_i \delta^\nu_j\, g^{(3)ij} - \eta^\mu \eta^\nu$$

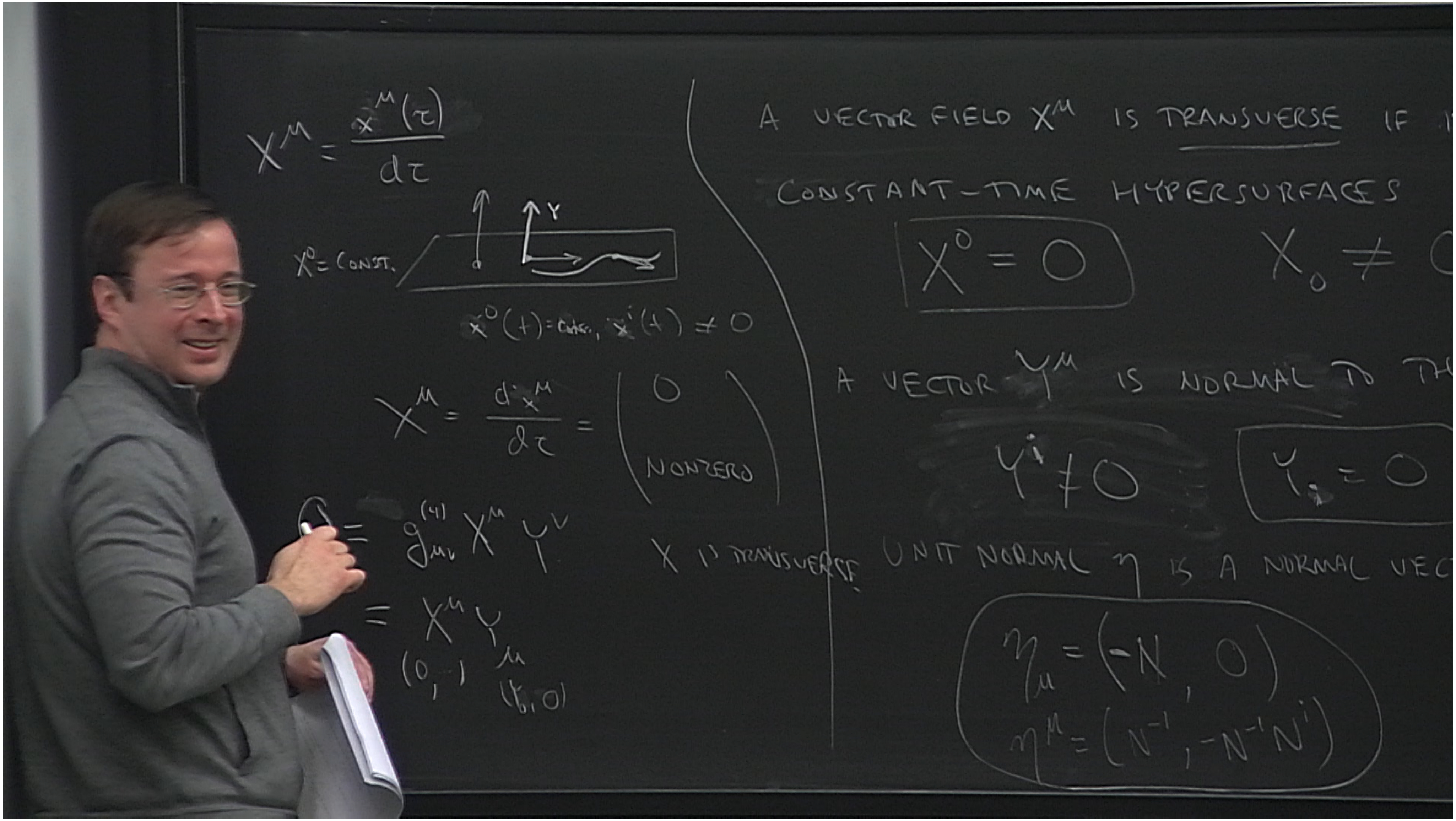
$$g^{(4)\mu\nu} = \begin{pmatrix} -N^{-2} & N^{-2}N^j \\ N^{-2}N^i & g^{(3)ij} - N^{-2}N^iN^j \end{pmatrix}$$

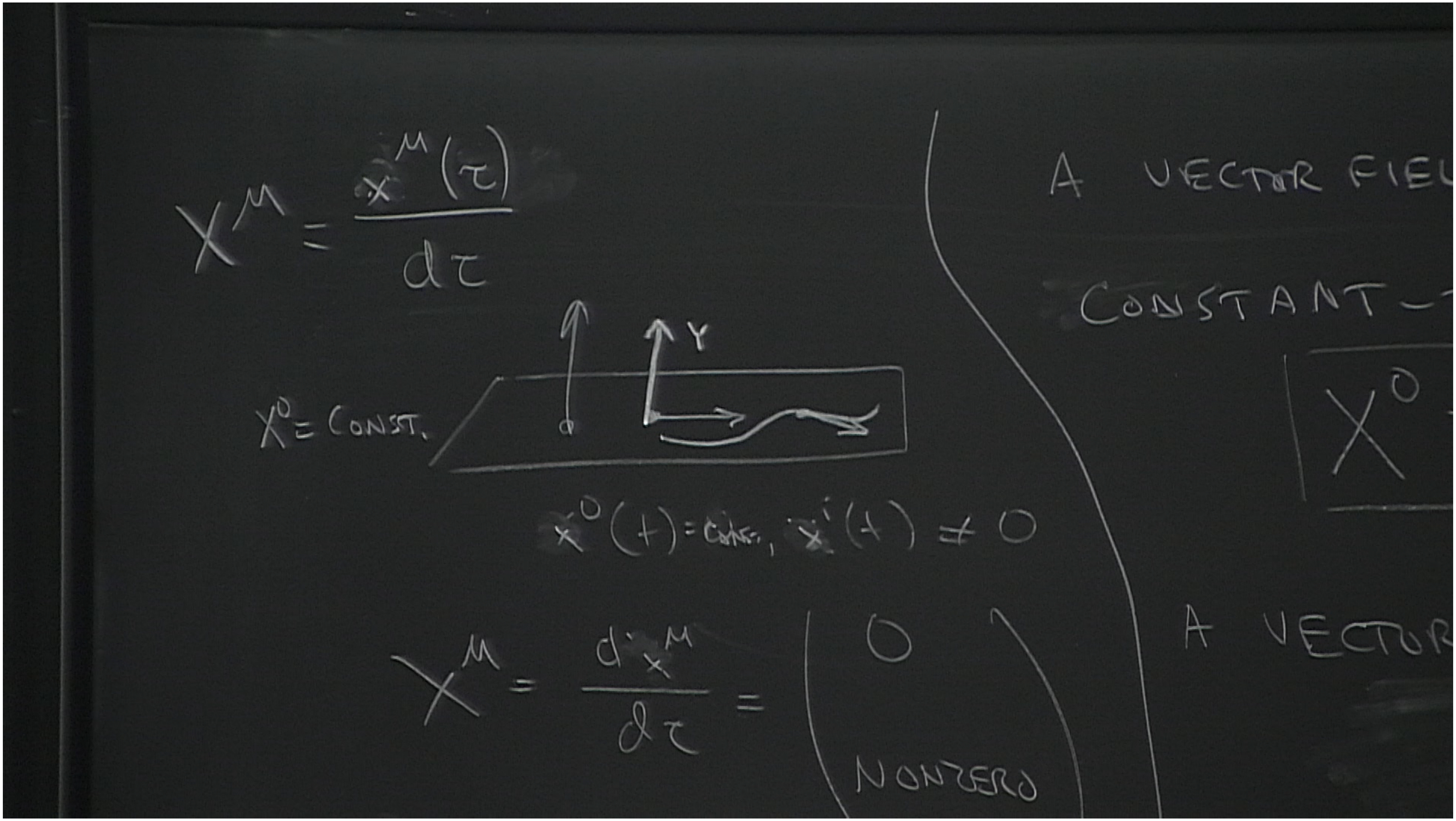
$$\boxed{\begin{aligned} \mathrm{Det}\left(g^{(4)}_{\mu\nu}\right) &= -N^2\, \mathrm{Det}\left(g^{(3)}_{ij}\right) \\ \sqrt{-g^{(4)}} &= N\sqrt{g^{(3)}} \end{aligned}}$$

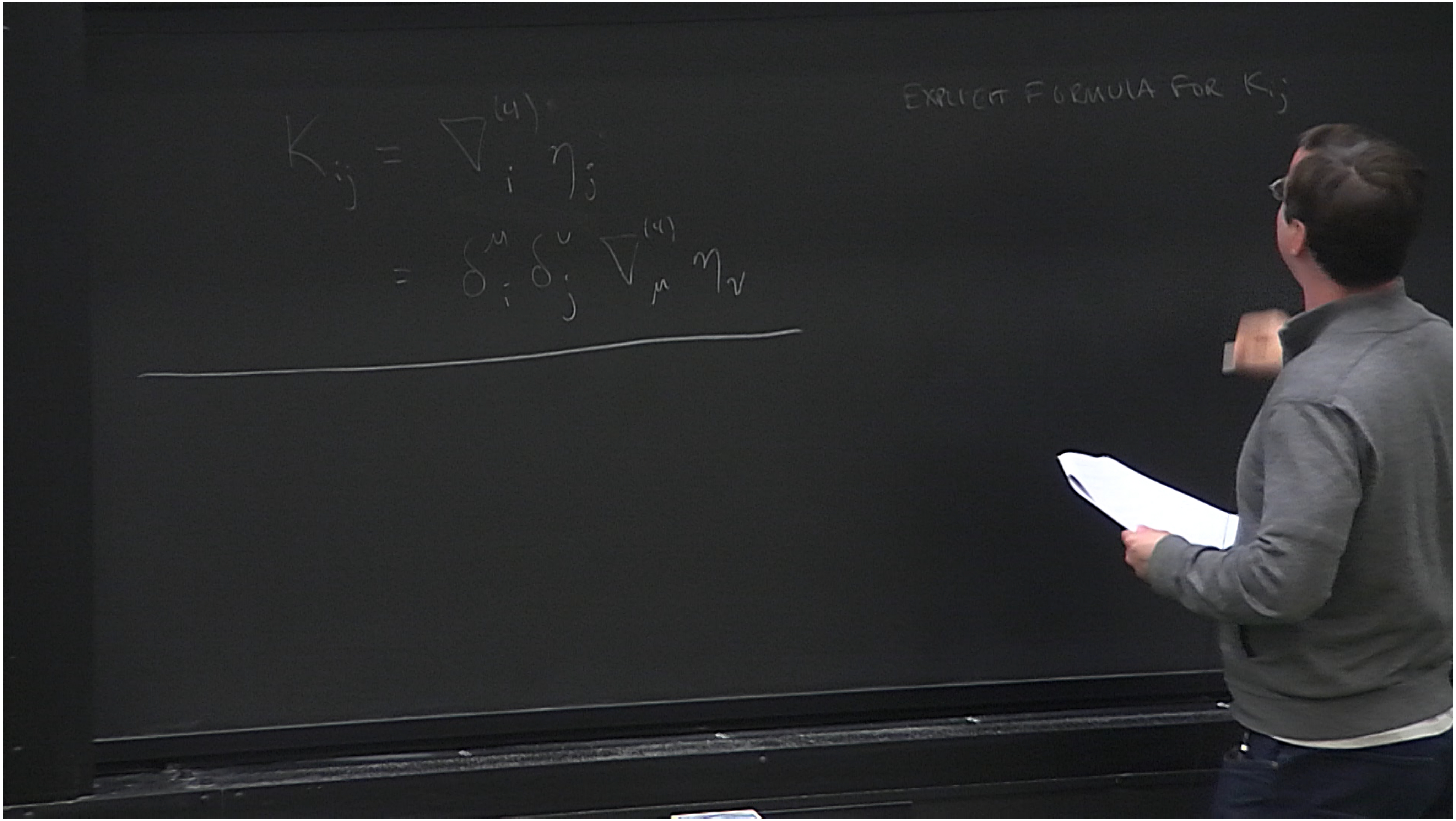
$$g^{(4)\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & g^{(3)ij} \end{pmatrix} - \eta^\mu \eta^\nu \qquad (*)$$

$$= \delta^\mu_i \delta^\nu_j\, g^{(3)ij} - \eta^\mu \eta^\nu$$


X^μ = dx^μ(τ) / dτ
X⁰ = CONST.
Y
X⁰(t) = const., X^i(t) ≠ 0
dX^μ / dτ = (0, NONZERO)
0 = g^(4)_μν X^μ Y^ν
X IS TRANSVERSE
A VECTOR FIELD X^μ IS TRANSVERSE IF
CONSTANT-TIME HYPERSURFACES
X⁰ = 0
X₀ ≠ 0
A VECTOR Y^μ IS NORMAL TO TH
Y^i ≠ 0
Y_i = 0
UNIT NORMAL η IS A NORMAL VEC
η_μ = (-N, 0)
η^μ = (N⁻¹, -N⁻¹N^i)


X^μ = x^μ(τ) / dτ
X^0 = CONST.
Y
x^0(t) = const., x^i(t) ≠ 0
X^μ = dx^μ / dτ = (0 NONZERO)
= g^(4)_μν X^μ Y^ν
= X^μ Y_μ
(0, ...)
(Y_0, 0)
X IS TRANSVERSE
A VECTOR FIELD X^μ IS TRANSVERSE IF
CONSTANT-TIME HYPERSURFACES
X^0 = 0
X_0 ≠
A VECTOR Y^μ IS NORMAL TO THE
Y^i ≠ 0
Y_i = 0
UNIT NORMAL η IS A NORMAL VEC
η_μ = (-N, 0)
η^μ = (N^-1, -N^-1 N^i)


$X^\mu = \frac{x^\mu(\tau)}{d\tau}$
$X^0 = $ CONST.
Y
$x^0(t) = $ CONST., $x^i(t) \neq 0$
$X^\mu = \frac{d x^\mu}{d\tau} = $
0
NONZERO
A VECTOR FIEL
CONSTANT
X^0
A VECTOR

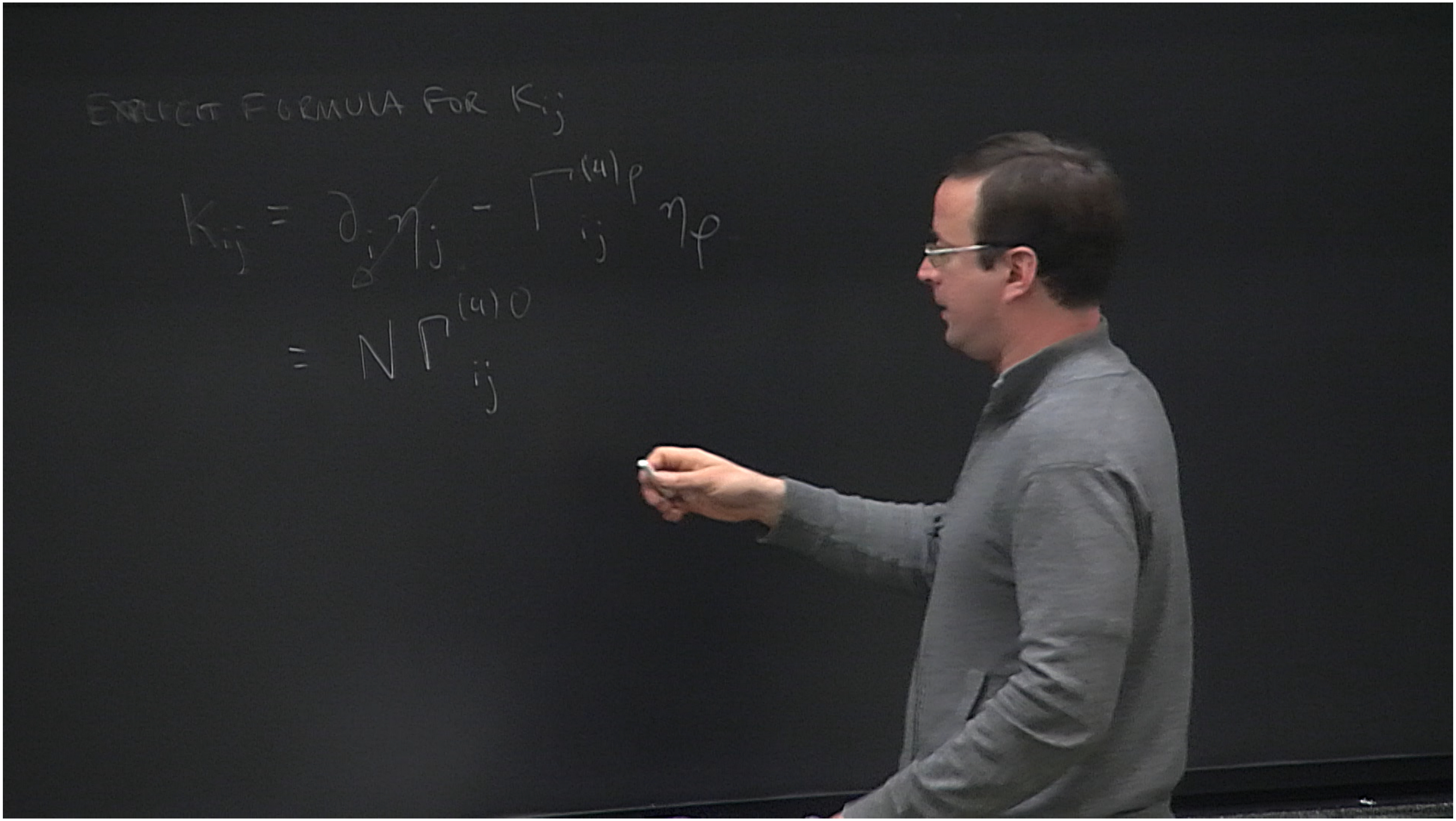

EXPLICIT FORMULA FOR K_{ij}
$K_{ij} = \nabla^{(4)}_i \eta_j$
$= \delta^\mu_i \delta^\nu_j \nabla^{(4)}_\mu \eta_\nu$

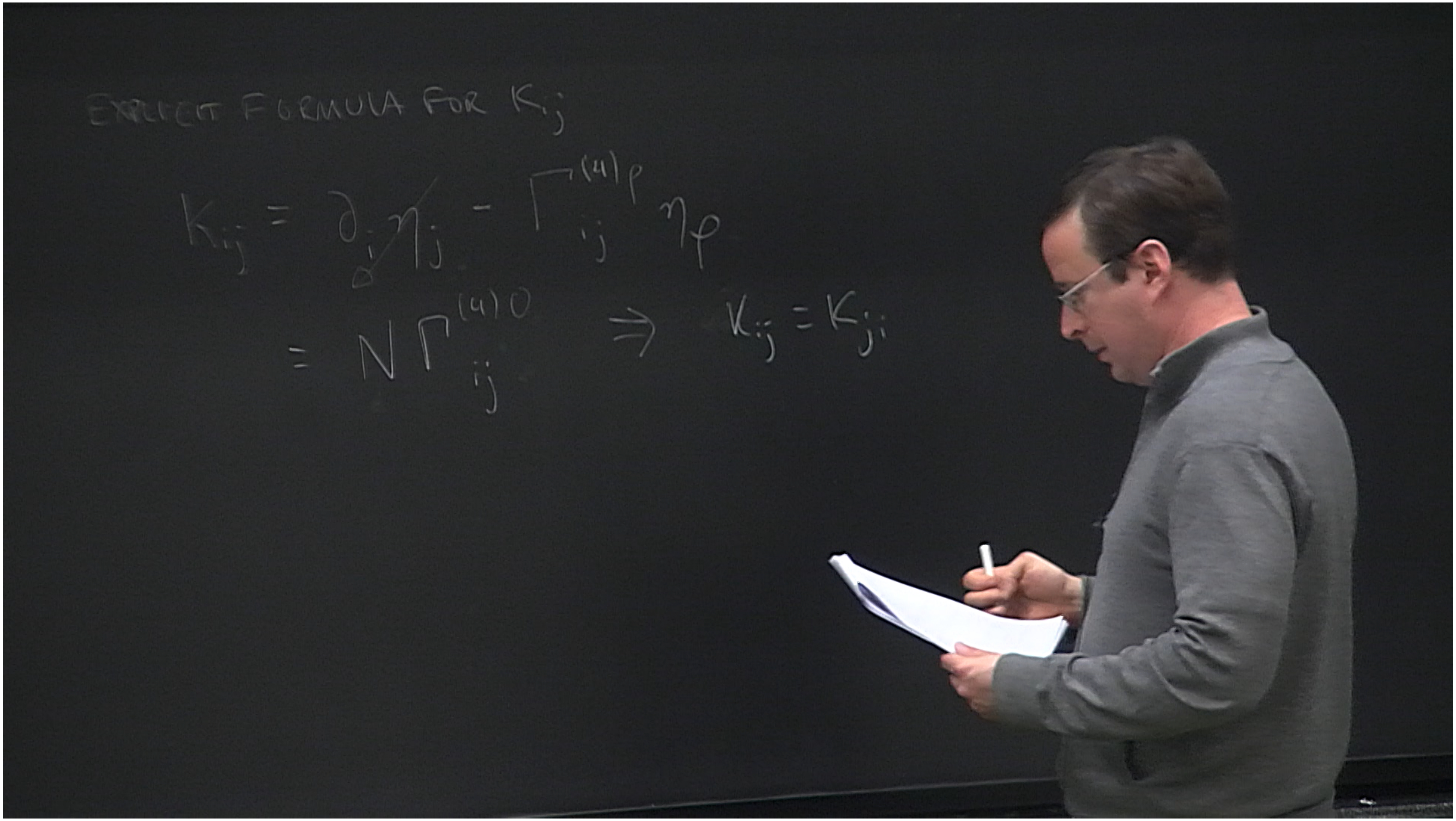
$$K_{ij} = \nabla^{(4)}_i \eta_j$$

$$= \delta^\mu_i \delta^\nu_j \nabla^{(4)}_\mu \eta_\nu$$

EXPLICIT FORMULA FOR K_{ij}

$$K_{ij} = \partial_i \eta_j - \Gamma^{(4)\rho}_{ij} \eta_\rho$$


EXPLICIT FORMULA FOR K_{ij}
$K_{ij} = \partial_i \eta_j - \Gamma^{(4)\rho}_{ij} \eta_\rho$
$= N \Gamma^{(4)0}_{ij}$


EXPLICIT FORMULA FOR K_{ij}
$K_{ij} = \partial_i \eta_j - \Gamma^{(4)\rho}_{ij} \eta_\rho$
$= N \Gamma^{(4)0}_{ij} \Rightarrow K_{ij} = K_{ji}$

EXPLICIT FORMULA FOR K_{ij}

$$K_{ij} = \cancel{\partial_i \eta_j} - \Gamma^{(4)\rho}_{ij}\,\eta_\rho$$

$$= N\,\Gamma^{(4)0}_{ij} \quad \Rightarrow \quad K_{ij} = K_{ji}$$

$$= \cdots \qquad (\mathrm{HW})$$

$$= \boxed{\frac{1}{2N}\left(\frac{\partial g^{(3)}_{ij}}{\partial t} - g^{(3)}_{jk}\,\nabla^{(3)}_i N^k - g^{(3)}_{ik}\,\nabla^{(3)}_j N^k\right)}$$

$$\Gamma^{(4)\rho}{}_{ij} = \frac{1}{2} g^{(4)\rho\sigma}\left(\partial_i g^{(4)}_{j\sigma} + \partial_j g^{(4)}_{i\sigma} - \partial_\sigma g^{(4)}_{ij}\right)$$

$$= \frac{1}{2}\left(\delta^\rho_k \delta^\sigma_\ell g^{(3)k\ell} - \eta^\rho \eta^\sigma\right)\left(\partial_i g^{(4)}_{j\sigma} + \partial_j g^{(4)}_{i\sigma} - \partial_\sigma g^{(4)}_{ij}\right)$$

$$= \frac{1}{2}\delta^\rho_k \, g^{(3)k\ell}\left(\partial_i g^{(3)}_{j\ell} + \partial_j g^{(3)}_{i\ell} - \partial_\ell g^{(3)}_{ij}\right) + [X_{ij}]\eta^\rho$$

$$\Gamma^{(4)\rho}_{ij} = \frac{1}{2} g^{(4)\rho\sigma}\left(\partial_i g^{(4)}_{j\sigma} + \partial_j g^{(4)}_{i\sigma} - \partial_\sigma g^{(4)}_{ij}\right)$$

$$= \frac{1}{2}\left(\delta^\rho_k \delta^\sigma_\ell\, g^{(3)k\ell} - \eta^\rho \eta^\sigma\right)\left(\partial_i g^{(4)}_{j\sigma} + \partial_j g^{(4)}_{i\sigma} - \partial_\sigma g^{(4)}_{ij}\right)$$

$$= \frac{1}{2}\delta^\rho_k\, g^{(3)k\ell}\left(\partial_i g^{(3)}_{j\ell} + \partial_j g^{(3)}_{i\ell} - \partial_\ell g^{(3)}_{ij}\right) + \left[X_{ij}\right]\eta^\rho$$

$$= \delta^\rho_k\, \Gamma^{(3)k}_{ij} + X_{ij}\eta^\rho$$

$$\Gamma^{(4)\rho}_{ij} = \frac{1}{2} g^{(4)\rho\sigma}\left(\partial_i g^{(4)}_{j\sigma} + \partial_j g^{(4)}_{i\sigma} - \partial_\sigma g^{(4)}_{ij}\right)$$

$$= \frac{1}{2}\left(\delta^\rho_k \delta^\sigma_l g^{(3)kl} - \eta^\rho \eta^\sigma\right)\left(\partial_i g^{(4)}_{j\sigma} + \partial_j g^{(4)}_{i\sigma} - \partial_\sigma g^{(4)}_{ij}\right)$$

$$= \frac{1}{2}\delta^\rho_k \; g^{(3)kl}\left(\partial_i g^{(3)}_{jl} + \partial_j g^{(3)}_{il} - \partial_l g^{(3)}_{ij}\right) + [X_{ij}]\eta^\rho$$

$$X_{ij} = K_{ij}$$

$$= \delta^\rho_k \Gamma^{(3)k}_{ij} + K_{ij}\eta^\rho$$

$$\Gamma^{(4)\rho}_{ij} = \frac{1}{2} g^{(4)\rho\sigma}\left(\partial_i g^{(4)}_{j\sigma} + \partial_j g^{(4)}_{i\sigma} - \partial_\sigma g^{(4)}_{ij}\right)$$

$$= \frac{1}{2}\left(\delta^\rho_k \delta^\sigma_\ell g^{(3)k\ell} - \eta^\rho \eta^\sigma\right)\left(\partial_i g^{(4)}_{j\sigma} + \partial_j g^{(4)}_{i\sigma} - \partial_\sigma g^{(4)}_{ij}\right)$$

$$= \frac{1}{2}\delta^\rho_k \; g^{(3)k\ell}\left(\partial_i g^{(3)}_{j\ell} + \partial_j g^{(3)}_{i\ell} - \partial_\ell g^{(3)}_{ij}\right) + [X_{ij}]\eta^\rho$$

$X_{ij} = K_{ij}$

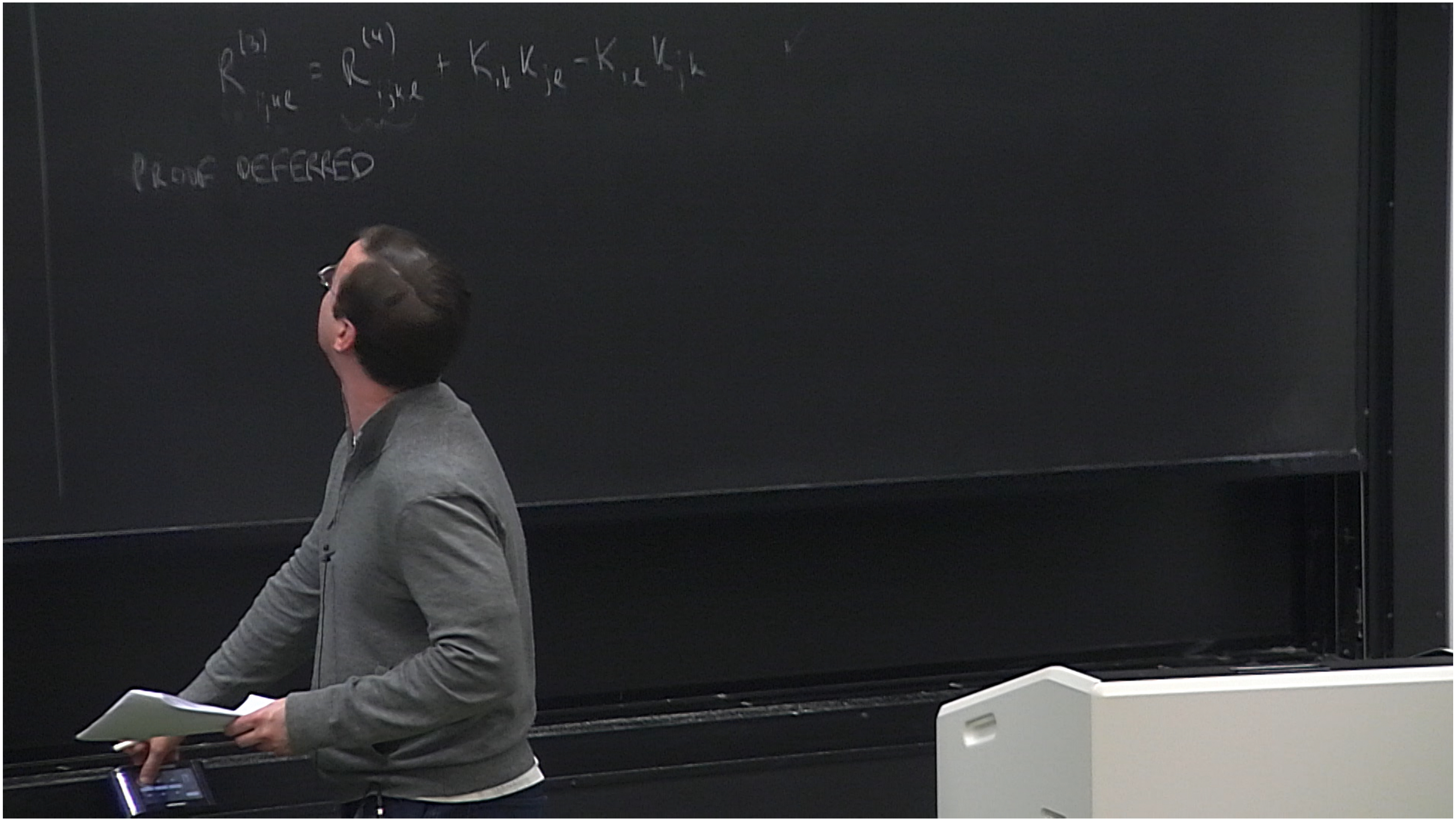
(OMITTED)

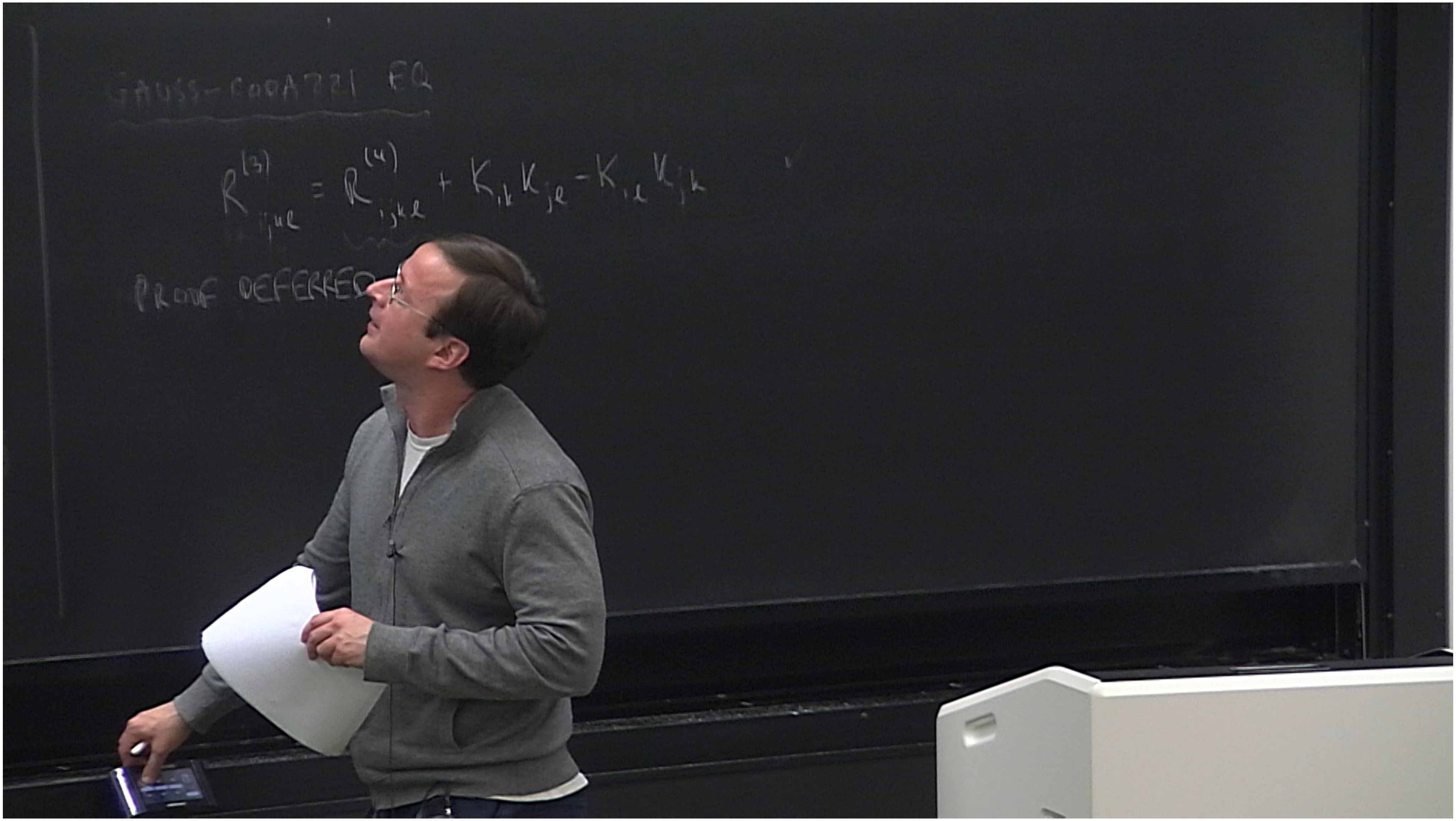
$$= \delta^\rho_k \Gamma^{(3)k}_{ij} + K_{ij}\eta^\rho$$

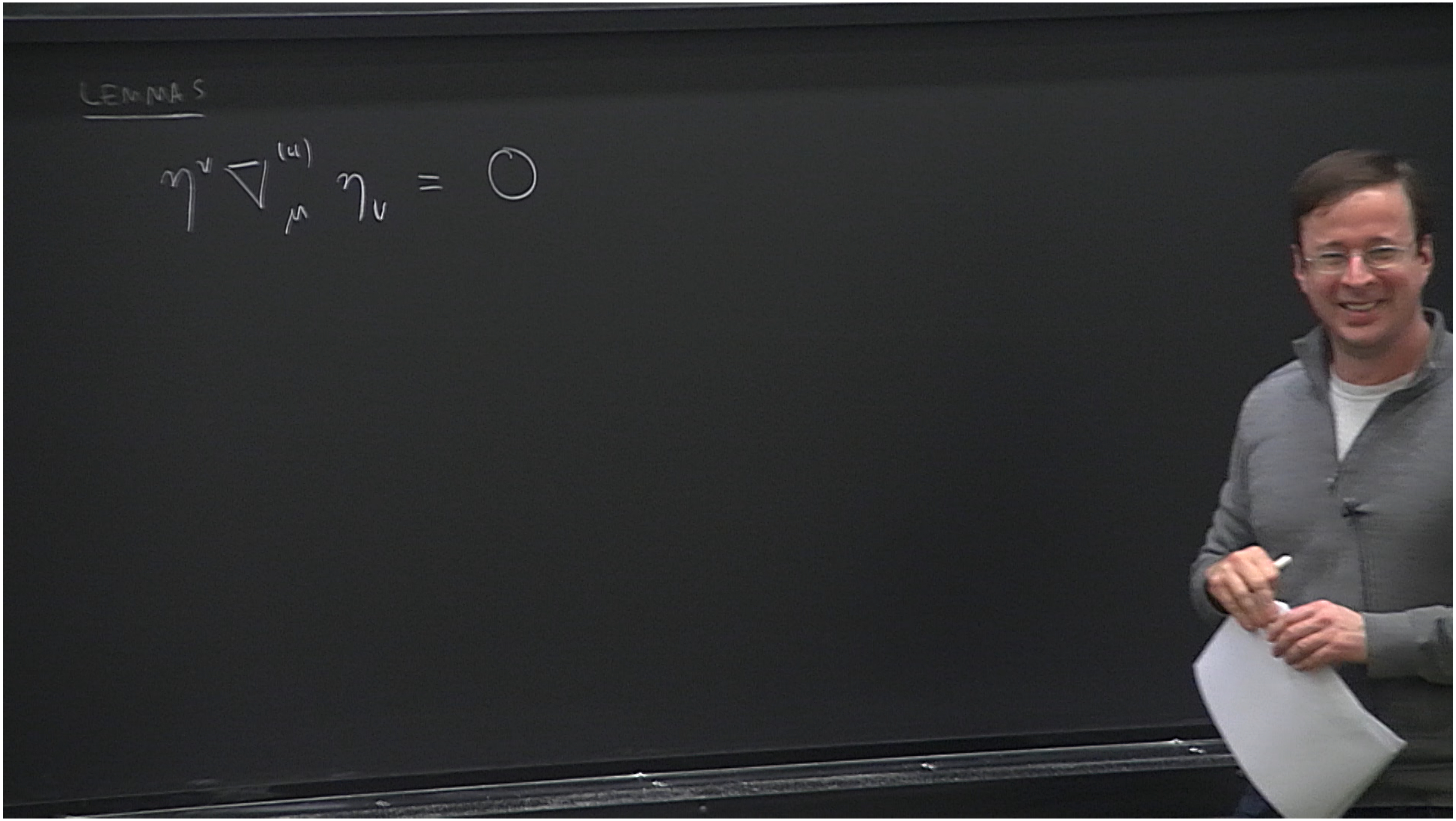
$$= \frac{1}{2} \left(\partial_k \partial_l g \quad - \underbrace{\eta \eta}_{} \right) \left(\partial_i g_{j\sigma} \right.$$

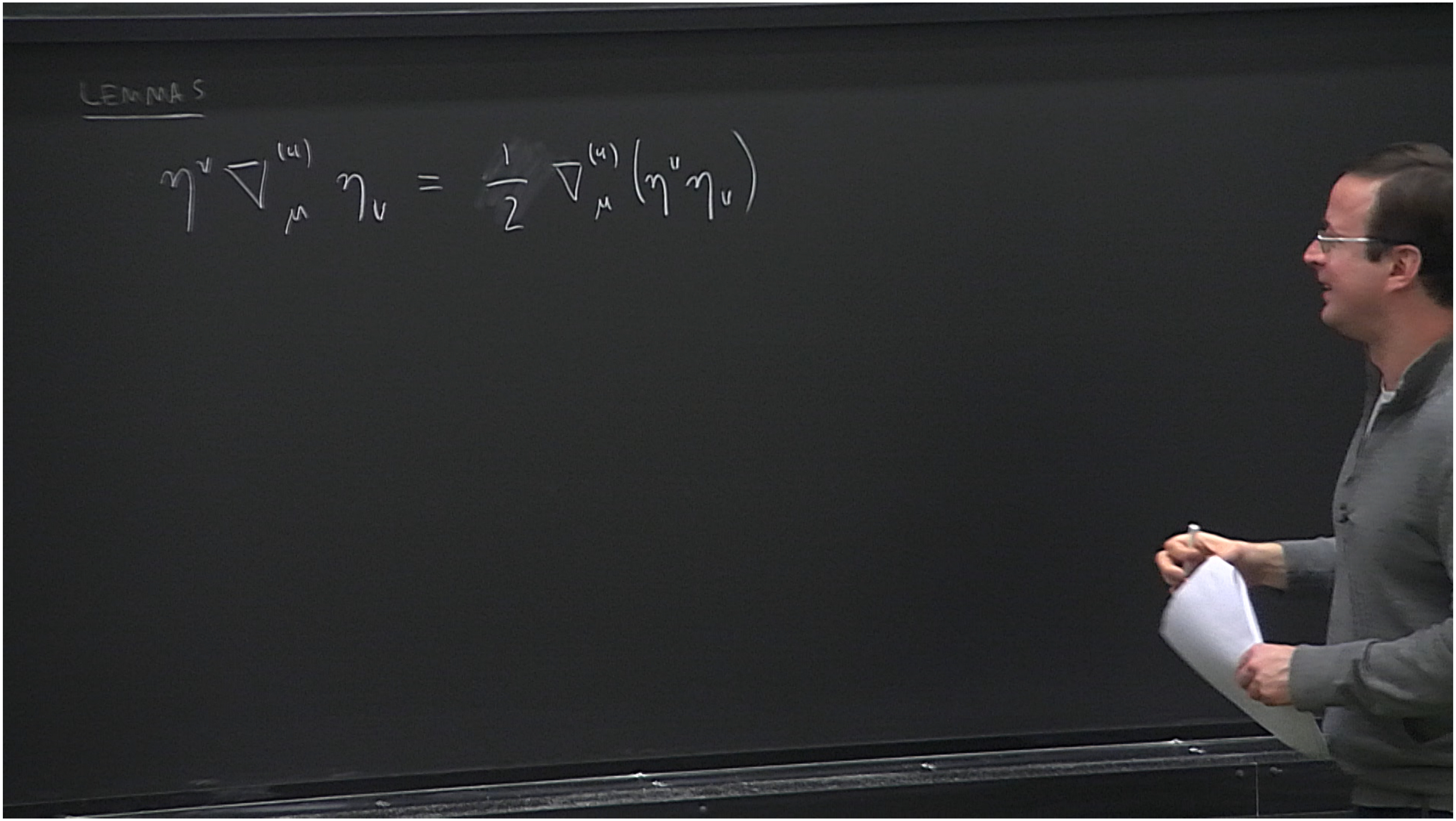
$$= \frac{1}{2} \delta^\rho_k \, g^{(3)kl} \left(\partial_i g^{(3)}_{jl} + \partial_j g^{(3)}_{il} - \partial_l g \right.$$

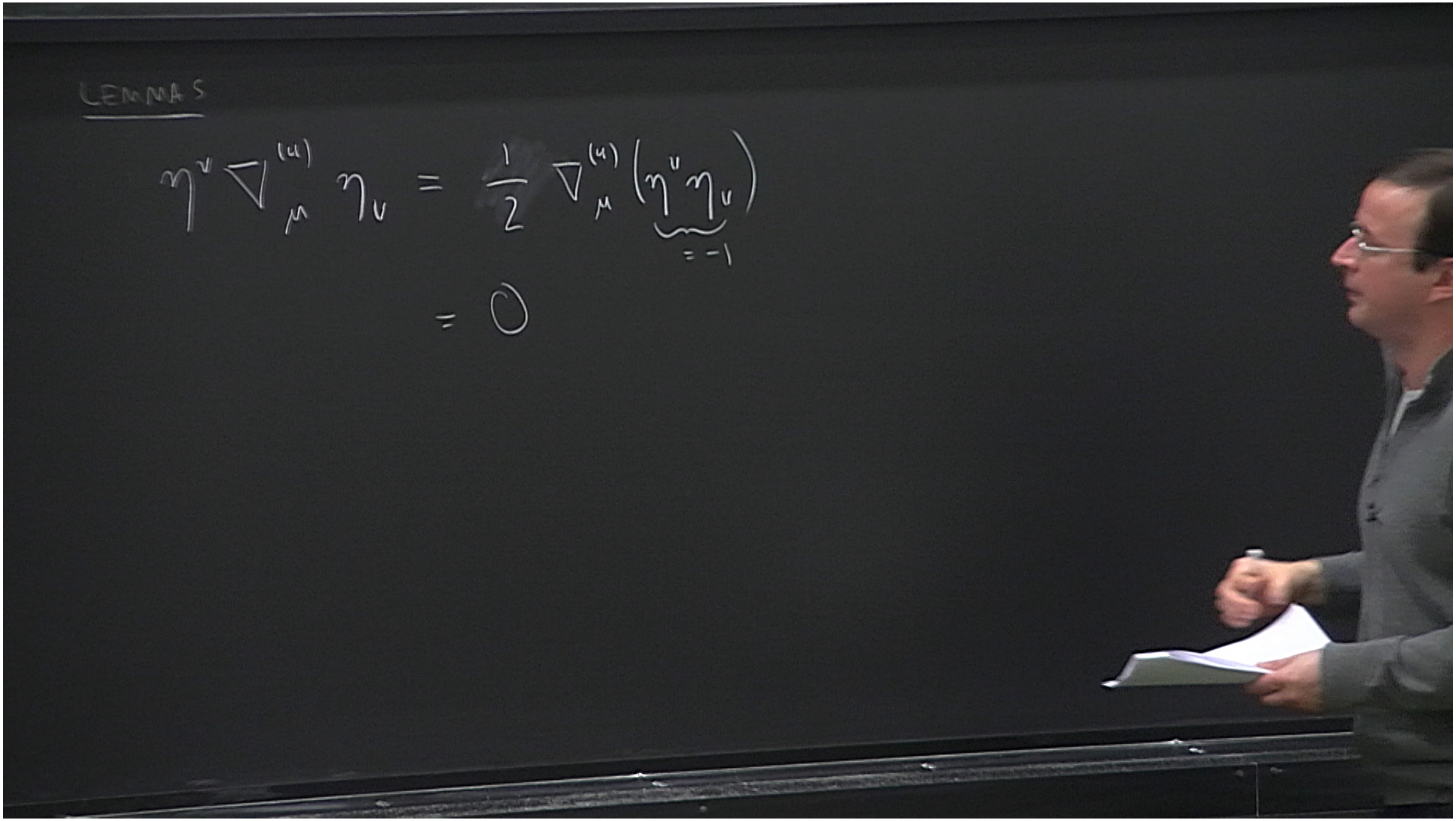
$$\boxed{\Gamma^{(4)\rho}_{ij} = \delta^\rho_k \, \Gamma^{(3)k}_{ij} + K_{ij} \, \eta^\rho}$$

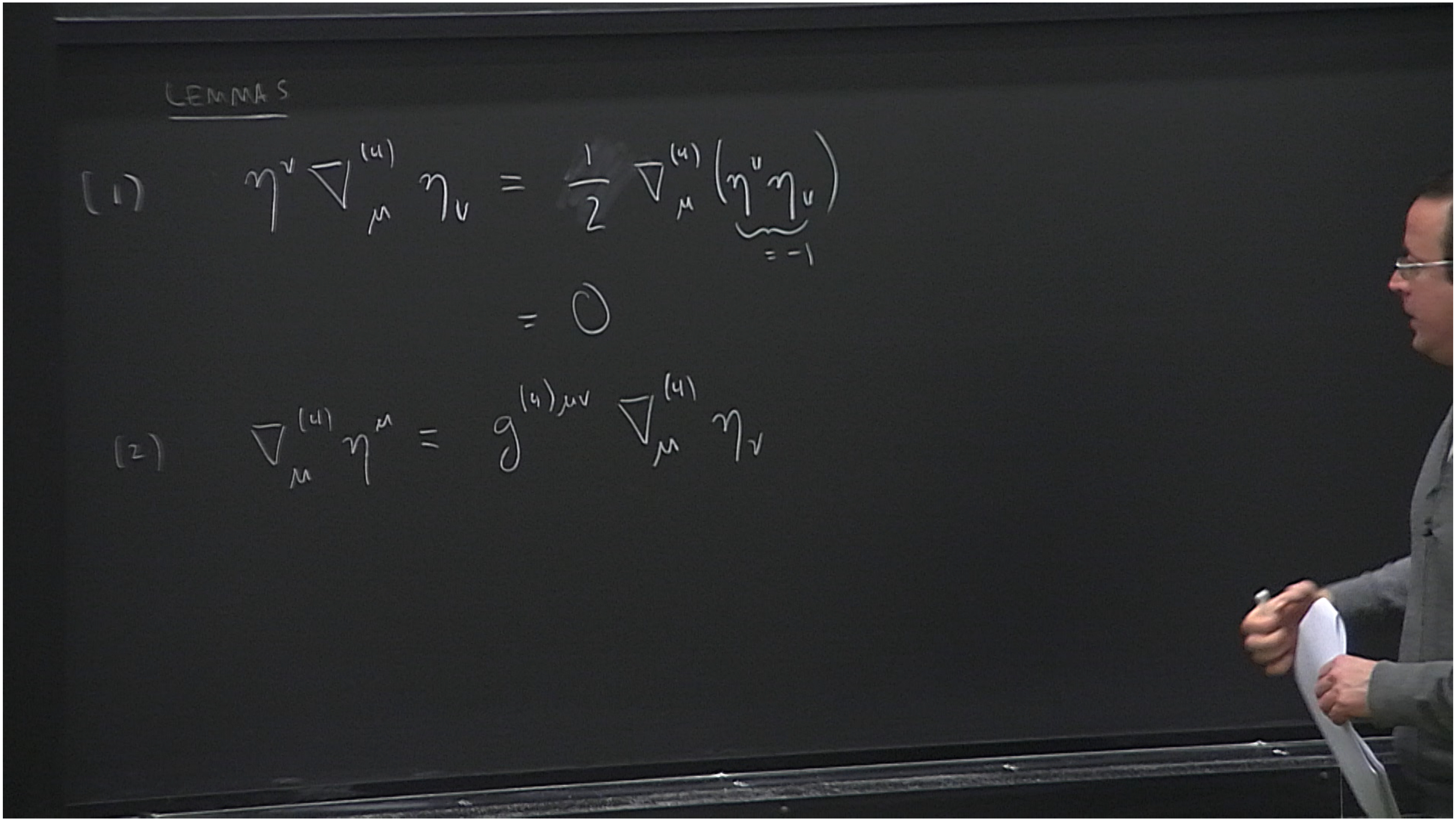

$R^{(3)}_{ijkl} = R^{(4)}_{ijkl} + K_{ik}K_{jl} - K_{il}K_{jk}$
PROOF DEFERRED

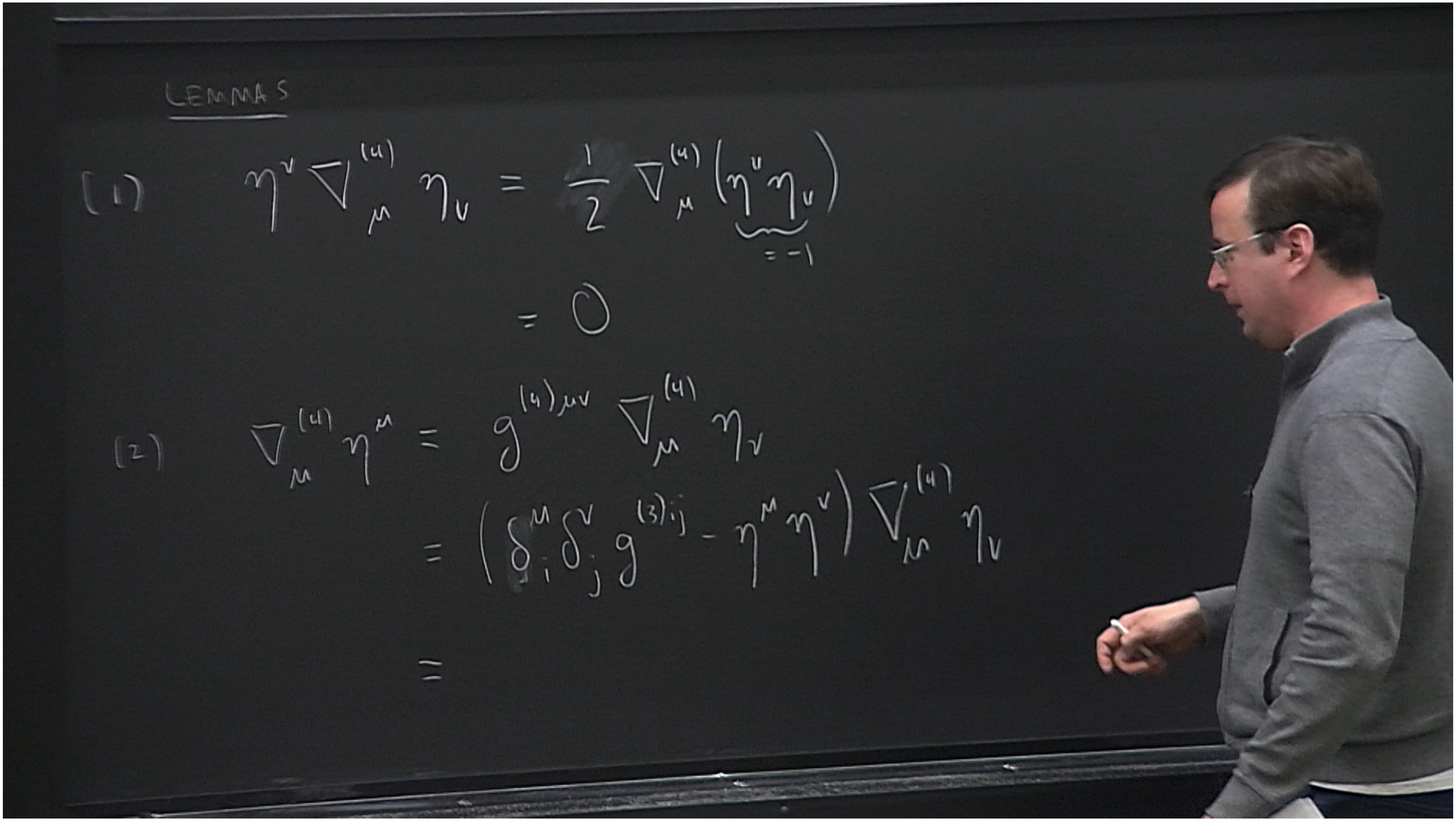

GAUSS-CODAZZI EQ
$R^{(3)}_{ijkl} = R^{(4)}_{ijkl} + K_{ik}K_{jl} - K_{il}K_{jk}$
PROOF DEFERRED


LEMMAS
$\eta^\nu \nabla^{(u)}_\mu \eta_\nu = 0$


LEMMAS
$\eta^\nu \nabla^{(u)}_\mu \eta_\nu = \frac{1}{2} \nabla^{(u)}_\mu (\eta^\nu \eta_\nu)$


LEMMAS
$\eta^\nu \nabla^{(u)}_\mu \eta_\nu = \frac{1}{2} \nabla^{(u)}_\mu (\underbrace{\eta^\nu \eta_\nu}_{=-1})$
$= 0$


LEMMAS
(1) $\eta^{\nu} \nabla^{(4)}_{\mu} \eta_{\nu} = \frac{1}{2} \nabla^{(4)}_{\mu} (\underbrace{\eta^{\nu}\eta_{\nu}}_{=-1})$
$= 0$
(2) $\nabla^{(4)}_{\mu} \eta^{\mu} = g^{(4)\mu\nu} \nabla^{(4)}_{\mu} \eta_{\nu}$


LEMMAS
(1) $\eta^\nu \nabla^{(4)}_\mu \eta_\nu = \frac{1}{2} \nabla^{(4)}_\mu (\underbrace{\eta^\nu \eta_\nu}_{=-1})$
$= 0$
(2) $\nabla^{(4)}_\mu \eta^\mu = g^{(4)\mu\nu} \nabla^{(4)}_\mu \eta_\nu$
$= (\delta^\mu_i \delta^\nu_j g^{(3)ij} - \eta^\mu \eta^\nu) \nabla^{(4)}_\mu \eta_\nu$
$=$

LEMMAS

(1) $$\eta^\nu \nabla^{(4)}_\mu \eta_\nu = \frac{1}{2} \nabla^{(4)}_\mu (\underbrace{\eta^\nu \eta_\nu}_{=-1})$$

$$= 0$$

(2) $$\nabla^{(4)}_\mu \eta^\mu = g^{(4)\mu\nu} \nabla^{(4)}_\mu \eta_\nu$$

$$= \left(\delta^\mu_i \delta^\nu_j g^{(3)ij} - \eta^\mu \eta^\nu \right) \nabla^{(4)}_\mu \eta_\nu$$

$$= \boxed{g^{(3)ij} K_{ij}}$$

(3) $\left(\overset{(4)}{\nabla}_\nu \eta^\mu\right)\left(\overset{(4)}{\nabla}_\mu \eta^\nu\right) = \overset{(4)}{g}{}^{\mu\sigma} \overset{(4)}{g}{}^{\nu\rho} \left(\overset{(4)}{\nabla}_\mu \eta_\nu\right)\left(\overset{(4)}{\nabla}_\rho \eta_\sigma\right)$

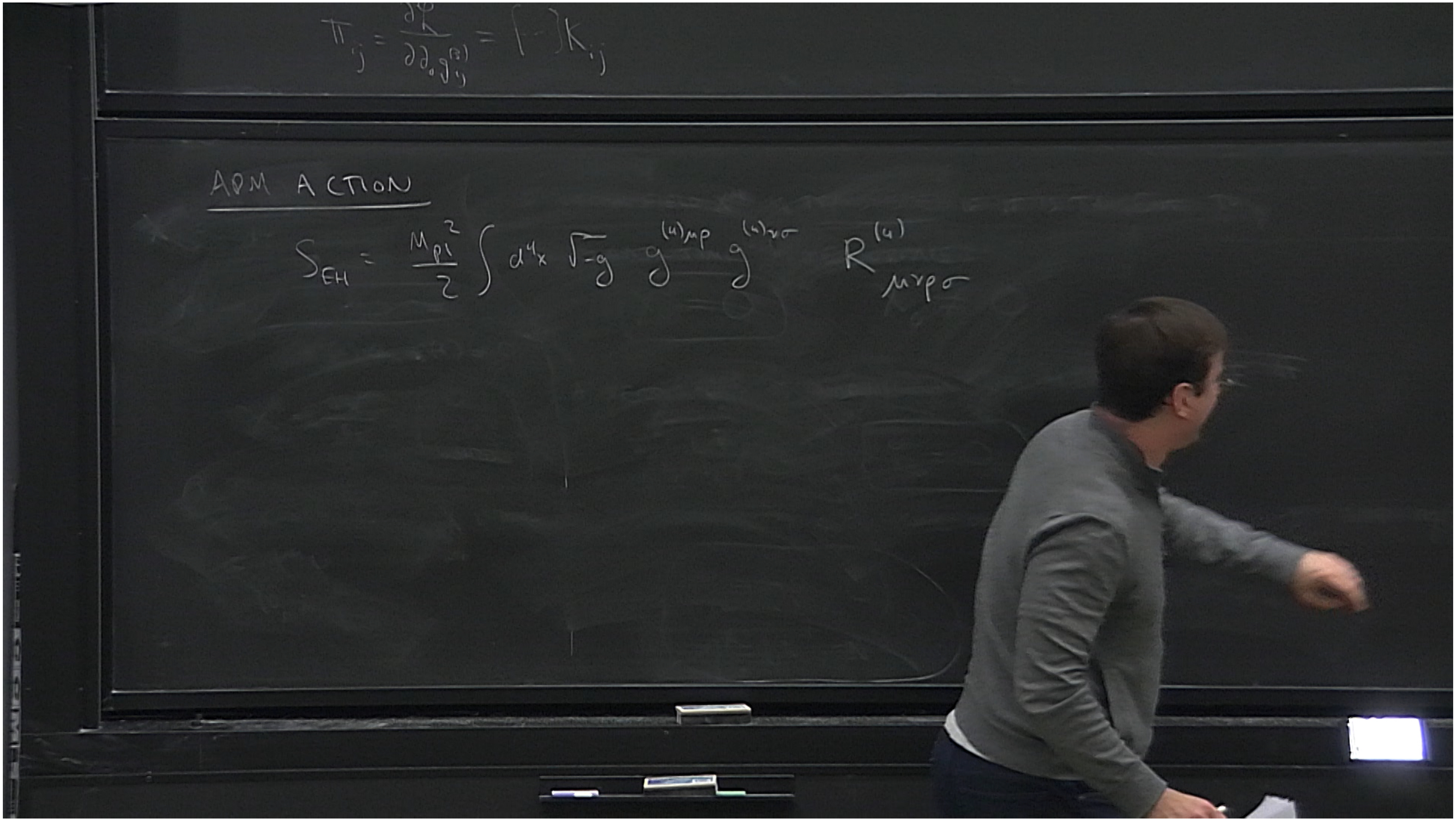
$= \cdots$

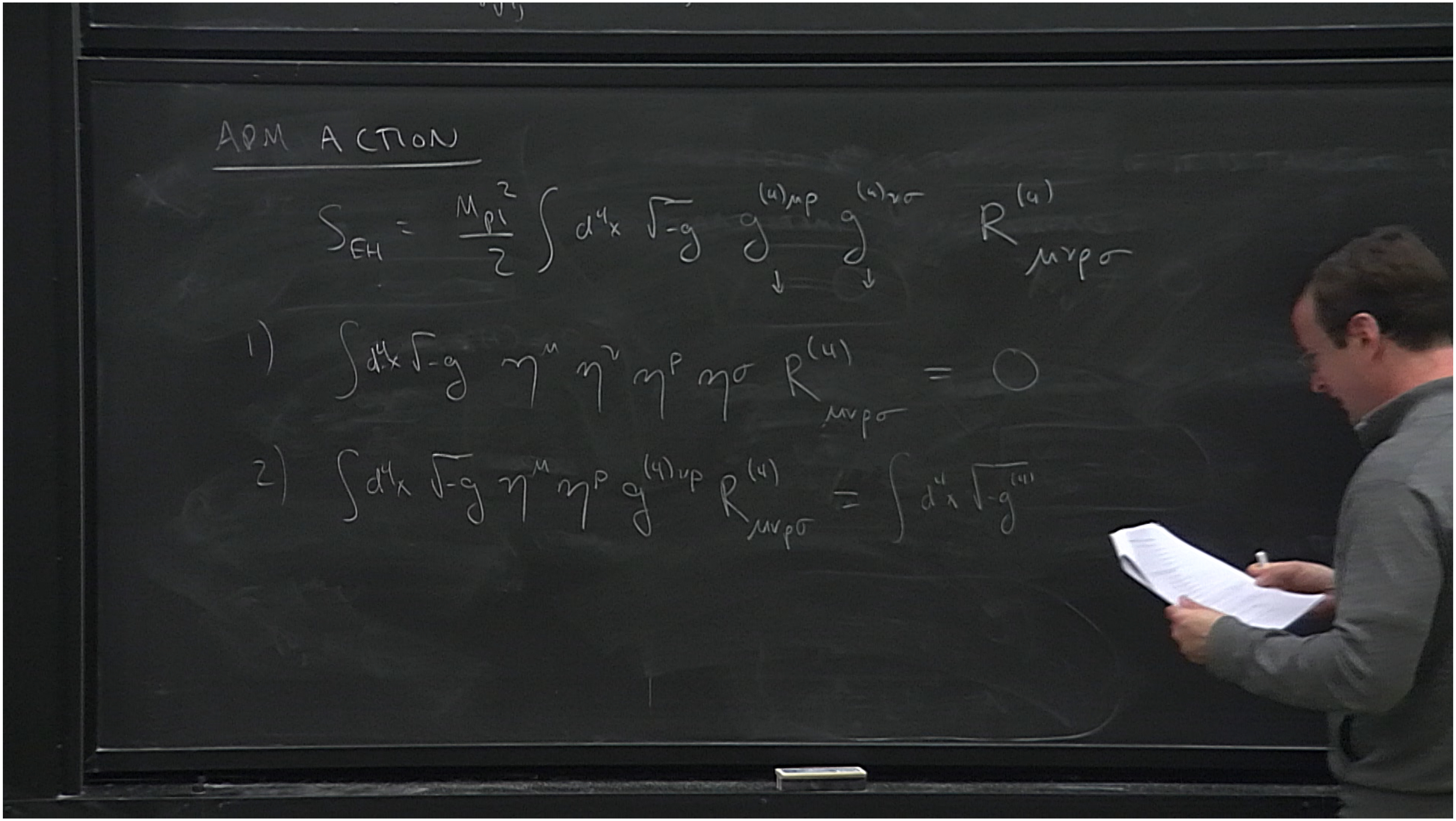
$= \overset{(3)}{g}{}^{ik} \overset{(3)}{g}{}^{jk} K_{ij} K_{kl}$

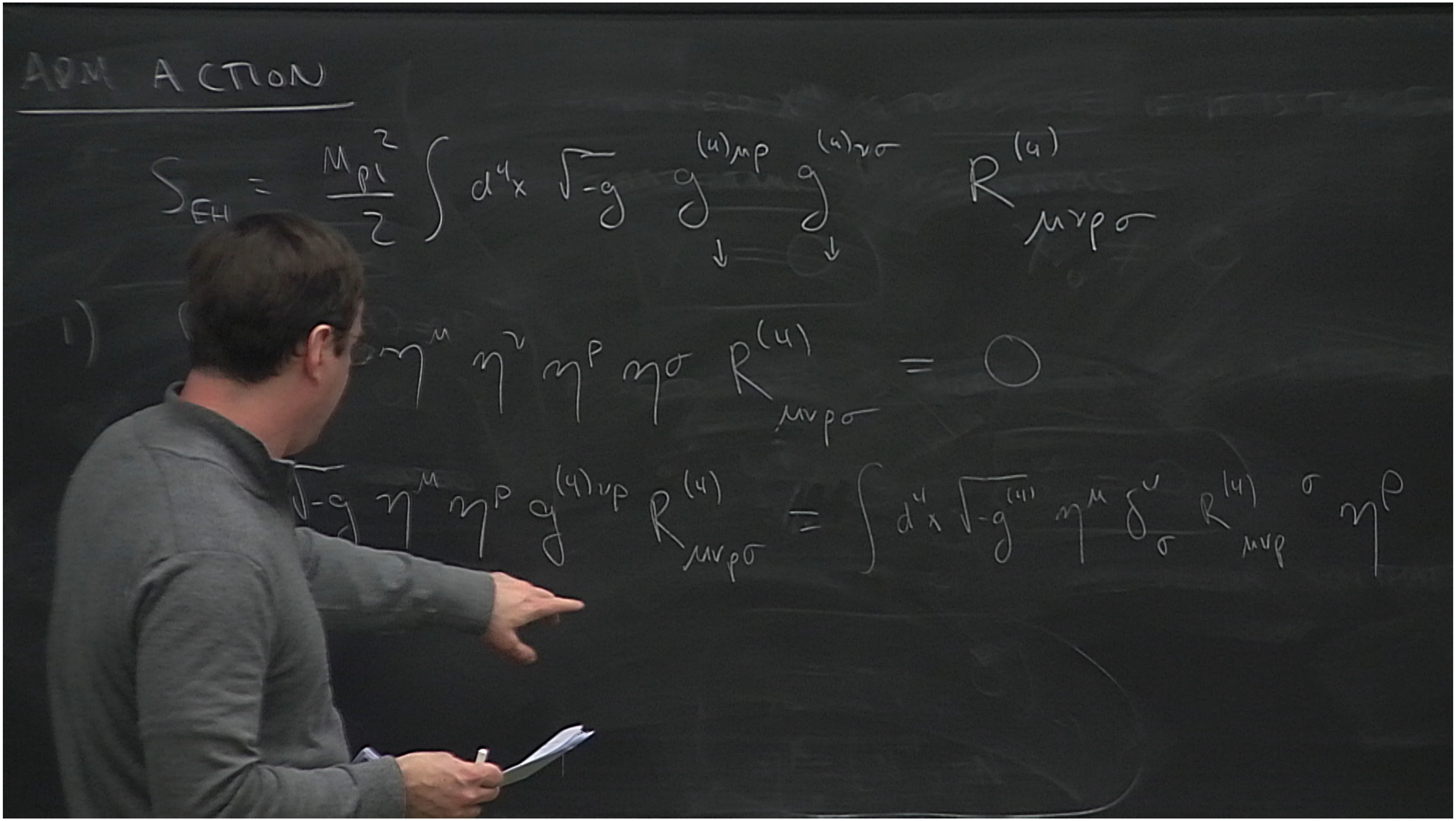
(3) $$\left(\overset{(4)}{\nabla}_\nu \eta^\mu\right)\left(\overset{(4)}{\nabla}_\mu \eta^\nu\right) = g^{(4)\mu\sigma} g^{(4)\nu\rho}\left(\overset{(4)}{\nabla}_\mu \eta_\nu\right)\left(\overset{(4)}{\nabla}_\rho \eta_\sigma\right)$$

$$= \cdots$$

$$= g^{(3)ik} g^{(3)jl} K_{ij} K_{kl}$$


$\pi_{ij} = \frac{\partial \mathcal{L}}{\partial \partial_0 g^{(3)}_{ij}} = [\ldots] K_{ij}$
ADM ACTION
$S_{EH} = \frac{M_{pl}^2}{2} \int d^4x \sqrt{-g}\, g^{(4)\mu\rho} g^{(4)\nu\sigma} R^{(4)}_{\mu\nu\rho\sigma}$


ADM ACTION
$S_{EH} = \frac{M_{pl}^2}{2} \int d^4x \sqrt{-g}\, g^{(4)\mu\rho} g^{(4)\nu\sigma} R^{(4)}_{\mu\nu\rho\sigma}$
1) $\int d^4x \sqrt{-g}\, \eta^{\mu} \eta^{\nu} \eta^{\rho} \eta^{\sigma} R^{(4)}_{\mu\nu\rho\sigma} = 0$
2) $\int d^4x \sqrt{-g}\, \eta^{\mu} \eta^{\rho} g^{(4)\nu\rho} R^{(4)}_{\mu\nu\rho\sigma} = \int d^4x \sqrt{-g^{(4)}}$


ADM ACTION
$S_{EH} = \frac{M_{Pl}^2}{2}\int d^4x \sqrt{-g}\, g^{(4)\mu\rho}\, g^{(4)\nu\sigma}\, R^{(4)}_{\mu\nu\rho\sigma}$
1)
$n^\mu n^\nu n^\rho n^\sigma R^{(4)}_{\mu\nu\rho\sigma} = 0$
$\sqrt{-g}\, n^\mu n^\rho g^{(4)\nu\rho} R^{(4)}_{\mu\nu\rho\sigma} = \int d^4x \sqrt{-g^{(4)}}\, n^\mu \delta^\nu_\sigma R^{(4)}{}_{\mu\nu\rho}{}^{\sigma} n^\rho$

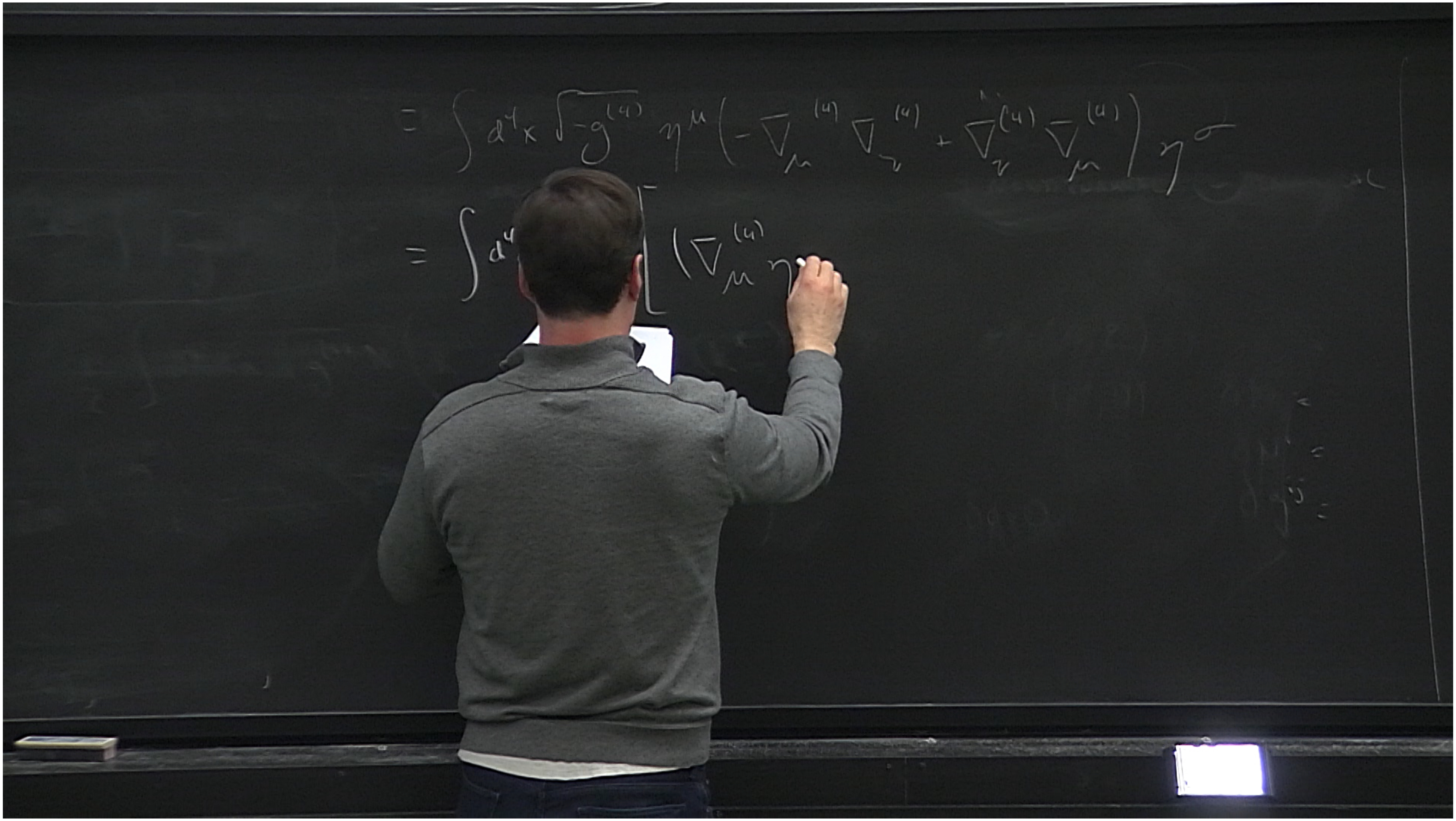
ACTION

$$S_{EH} = \frac{M_{Pl}^2}{2}\int d^4x \sqrt{-g}\; g^{(4)\mu\rho}\, g^{(4)\nu\sigma}\, R^{(4)}_{\mu\nu\rho\sigma}$$

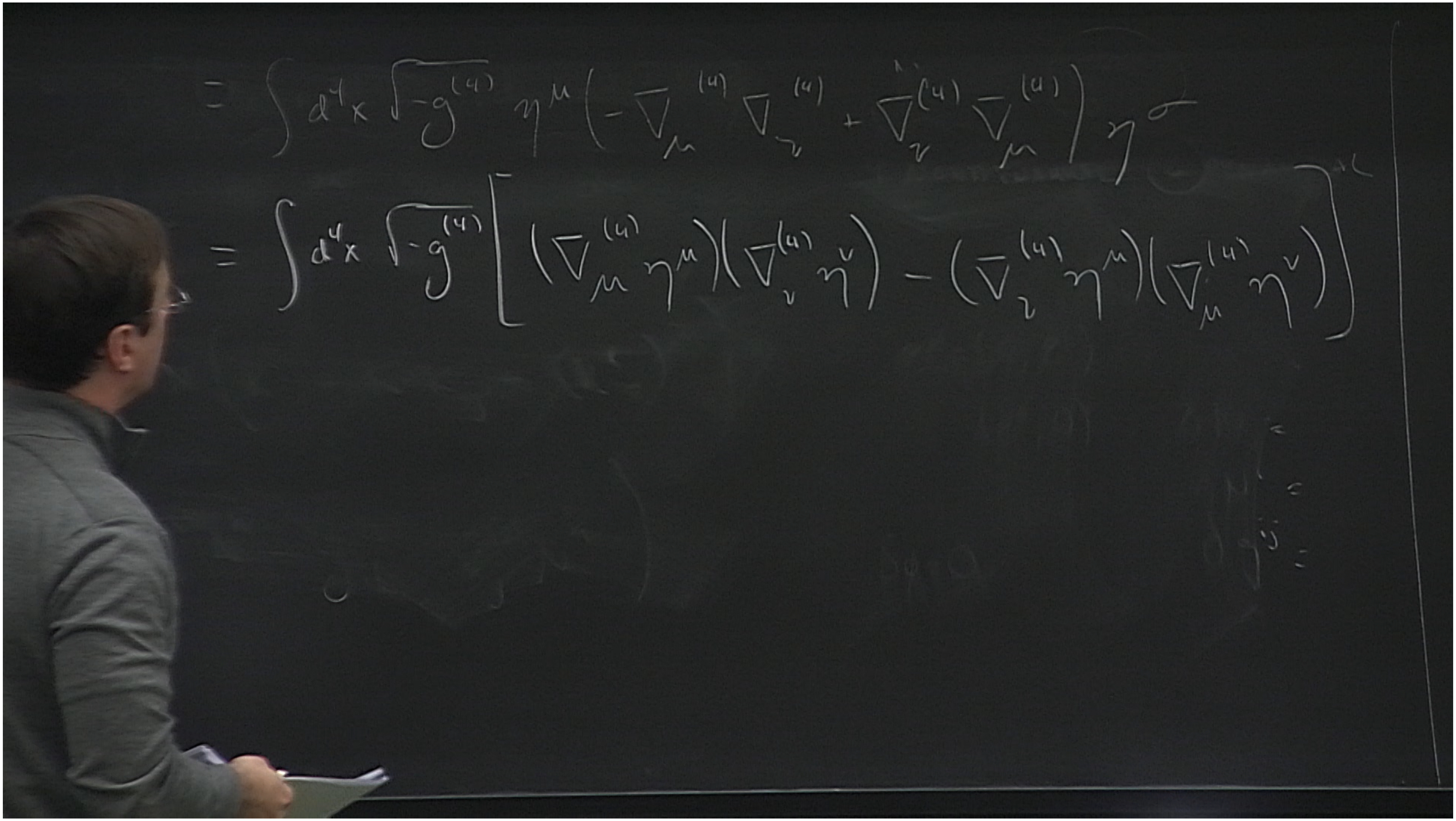
$$\int d^4x \sqrt{-g}\; \eta^\mu \eta^\nu \eta^\rho \eta^\sigma\, R^{(4)}_{\mu\nu\rho\sigma} = 0$$

$$\int d^4x \sqrt{-g}\; \eta^\mu \eta^\rho\, g^{(4)\nu\sigma}\, R^{(4)}_{\mu\nu\rho\sigma} = \int d^4x \sqrt{-g^{(4)}}\; \eta^\mu\, \delta^\nu_\sigma\, R^{(4)}{}_{\mu\nu\rho}{}^{\sigma}\, \eta^\rho$$

$$= \int d^4x \sqrt{-g^{(4)}}\; \delta^\nu_\sigma \left(-\nabla^{(4)}_\mu \nabla^{(4)}_\nu + \nabla^{(4)}_\nu \nabla^{(4)}_\mu\right)\eta^\sigma$$



$= \int d^4x \sqrt{-g^{(4)}}\, \eta^\mu \left(-\nabla^{(4)}_\mu \nabla^{(4)}_\nu + \nabla^{(4)}_\nu \nabla^{(4)}_\mu\right) \eta^\sigma$
$= \int d^4 \,\Big[(\nabla^{(4)}_\mu \eta$


$= \int d^4x \sqrt{-g^{(4)}}\, \eta^\mu \left(-\nabla^{(4)}_\mu \nabla^{(4)}_\nu + \nabla^{(4)}_\nu \nabla^{(4)}_\mu\right)\eta^\nu$
$= \int d^4x \sqrt{-g^{(4)}} \left[(\nabla^{(4)}_\mu \eta^\mu)(\nabla^{(4)}_\nu \eta^\nu) - (\nabla^{(4)}_\nu \eta^\mu)(\nabla^{(4)}_\mu \eta^\nu)\right]$

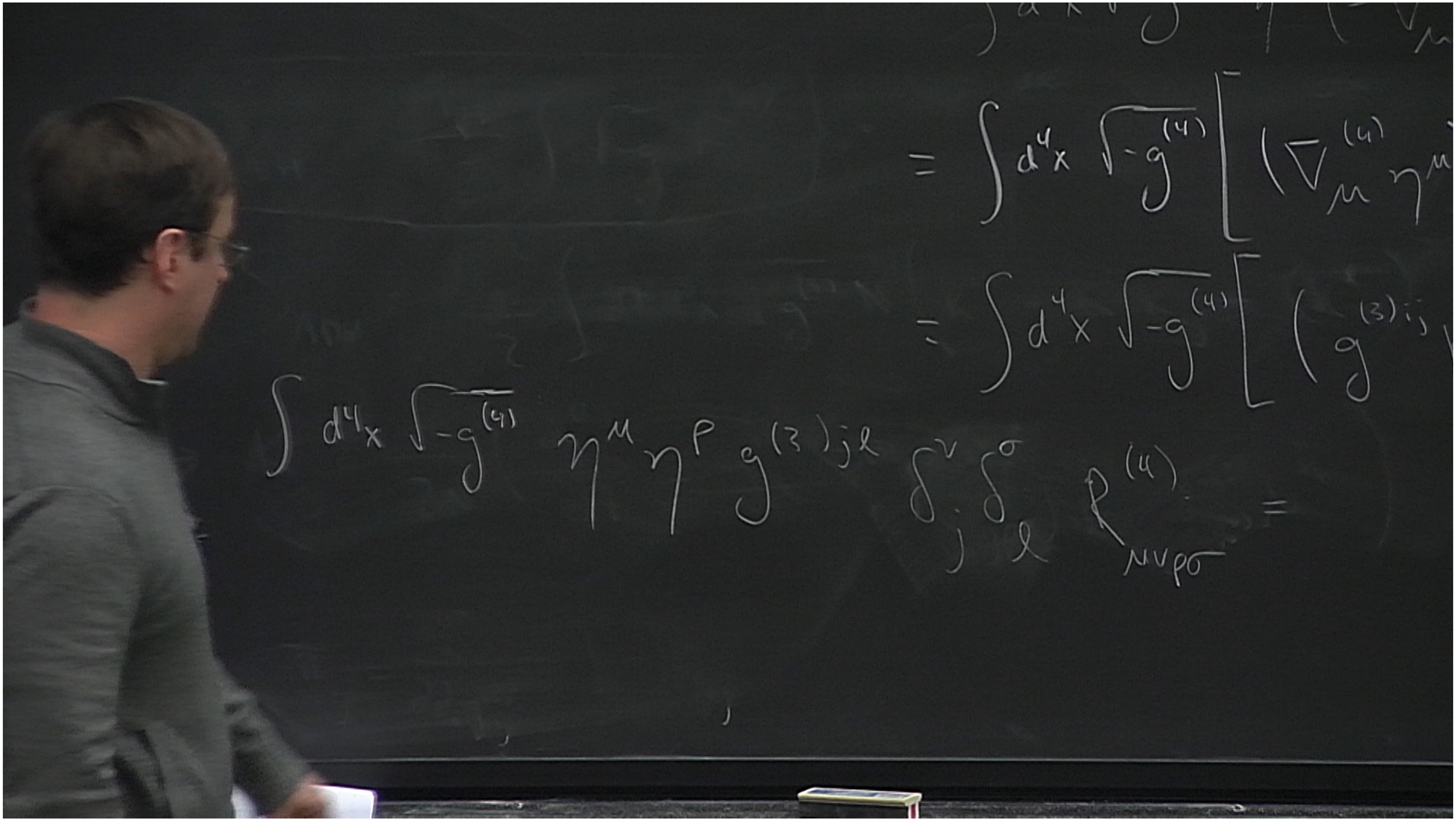
$$= \int d^4x \sqrt{-g^{(4)}}\, \eta^\mu \left(-\nabla^{(4)}_\mu \nabla^{(4)}_\nu + \nabla^{(4)}_\nu \nabla^{(4)}_\mu \right) \eta^\nu$$

$$= \int d^4x \sqrt{-g^{(4)}} \left[(\nabla^{(4)}_\mu \eta^\mu)(\nabla^{(4)}_\nu \eta^\nu) - (\nabla^{(4)}_\nu \eta^\mu)(\nabla^{(4)}_\mu \eta^\nu) \right]$$

$$= \int d^4x \sqrt{-g^{(4)}}\, \eta^\mu \left(-\nabla^{(4)}_\mu \nabla^{(4)}_\nu + \nabla^{(4)}_\nu \nabla^{(4)}_\mu \right) \eta^\nu$$

$$= \int d^4x \sqrt{-g^{(4)}} \left[\left(\nabla^{(4)}_\mu \eta^\mu\right)\left(\nabla^{(4)}_\nu \eta^\nu\right) - \left(\nabla^{(4)}_\nu \eta^\mu\right)\left(\nabla^{(4)}_\mu \eta^\nu\right) \right]$$

$$= \int d^4x \sqrt{-g^{(4)}} \left[\left(g^{(3)ij} K_{ij}\right)^2 - \left(g^{(3)ik} g^{(3)jl} K_{ij} K_{kl}\right) \right]$$


$= \int d^4x \sqrt{-g^{(4)}} \Big[(\nabla^{(4)}_\mu \eta^\mu$
$= \int d^4x \sqrt{-g^{(4)}} \Big[(g^{(3)ij}$
$\int d^4x \sqrt{-g^{(4)}} \; \eta^\mu \eta^\rho g^{(3)jl} \delta^\nu_j \delta^\sigma_l R^{(4)}_{\mu\nu\rho\sigma} =$

$$= \int d^4x \sqrt{-g^{(4)}} \Big[(\nabla^{(4)}_\mu \eta^\mu)(\nabla^{(4)}_\nu \eta^\nu)$$

$$= \int d^4x \sqrt{-g^{(4)}} \Big[\left(g^{(3)ij} K_{ij}\right)^2 -$$

$$\int d^4x \sqrt{-g^{(4)}} \; \eta^\mu \eta^\rho g^{(3)jl} \delta^\nu_j \delta^\sigma_l R^{(4)}_{\mu\nu\rho\sigma} = \text{SAME}$$

$$= \int d^4x \sqrt{-g^{(4)}} \Big[\big(\nabla^{(4)}_\mu$$

$$= \int d^4x \sqrt{-g^{(4)}} \Big[\big($$

$$\int d^4x \sqrt{-g^{(4)}}\, \eta^\mu \eta^\rho g^{(3)jl} \delta^\nu_j \delta^\sigma_l R^{(4)}_{\mu\nu\rho\sigma}$$

$$\int d^4x \sqrt{-g}\, g^{(4)\mu\rho} \eta^\nu \eta^\sigma_{;} R^{(4)}_{\mu\nu\rho\sigma} = \text{SAME}$$

$$= \int d^4x \sqrt{-g^{(4)}}\, \eta^\mu \left(-\nabla^{(4)}_\mu \nabla^{(4)}_\nu + \nabla^{(4)}_\nu \nabla^{(4)}_\mu \right.$$

$$= \int d^4x \sqrt{-g^{(4)}} \left[(\nabla^{(4)}_\mu \eta^\mu)(\nabla^{(4)}_\nu \eta^\nu) - (\nabla^{(4)}_\nu \right.$$

$$= \int d^4x \sqrt{-g^{(4)}} \left[\left(g^{(3):ij} K_{;j}\right)^2 - \left(g^{(3):ik}\, g^{(3)} \right. \right.$$

$$\int d^4x \sqrt{-g^{(4)}}\, \eta^\mu \eta^\rho g^{(3)jl} \delta^\nu_j \delta^\sigma_l R^{(4)}_{\mu\nu\rho\sigma} = \text{SAME BY (1)}$$

$$\int d^4x \sqrt{-g}\, g^{(4)\mu\rho} \eta^\nu \eta^\sigma R^{(4)}_{\mu\nu\rho\sigma} = \text{SAME} \quad R^{(4)}_{\mu\nu\rho\sigma} = R^{(4)}_{\nu\mu\sigma\rho}$$

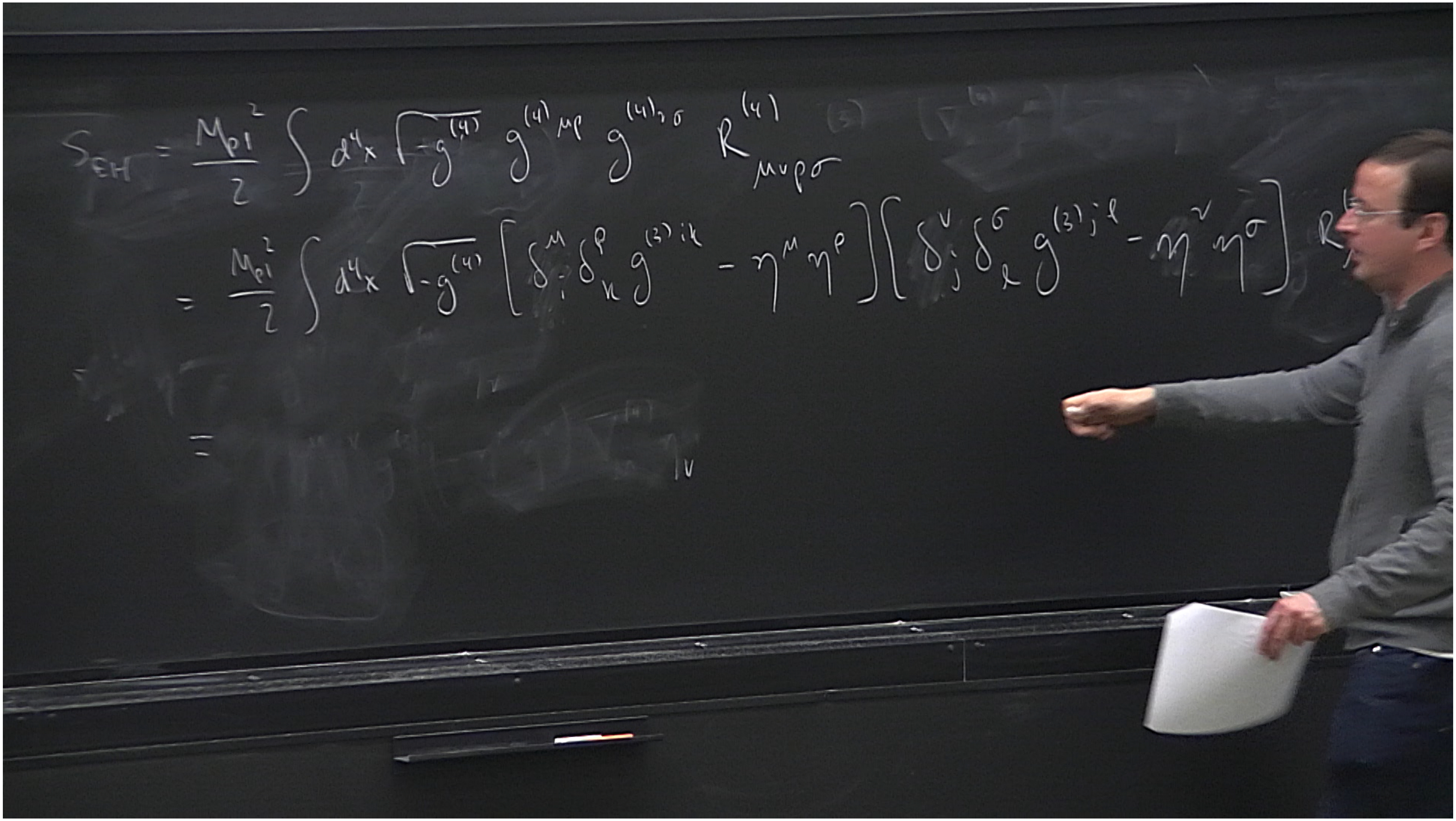
$$= \int d^4x \sqrt{-g^{(4)}}\; \eta^\mu \left(-\nabla^{(4)}_\mu \nabla^{(4)}_\nu + \nabla^{(4)}_\nu \nabla^{(4)}_\mu \right.$$

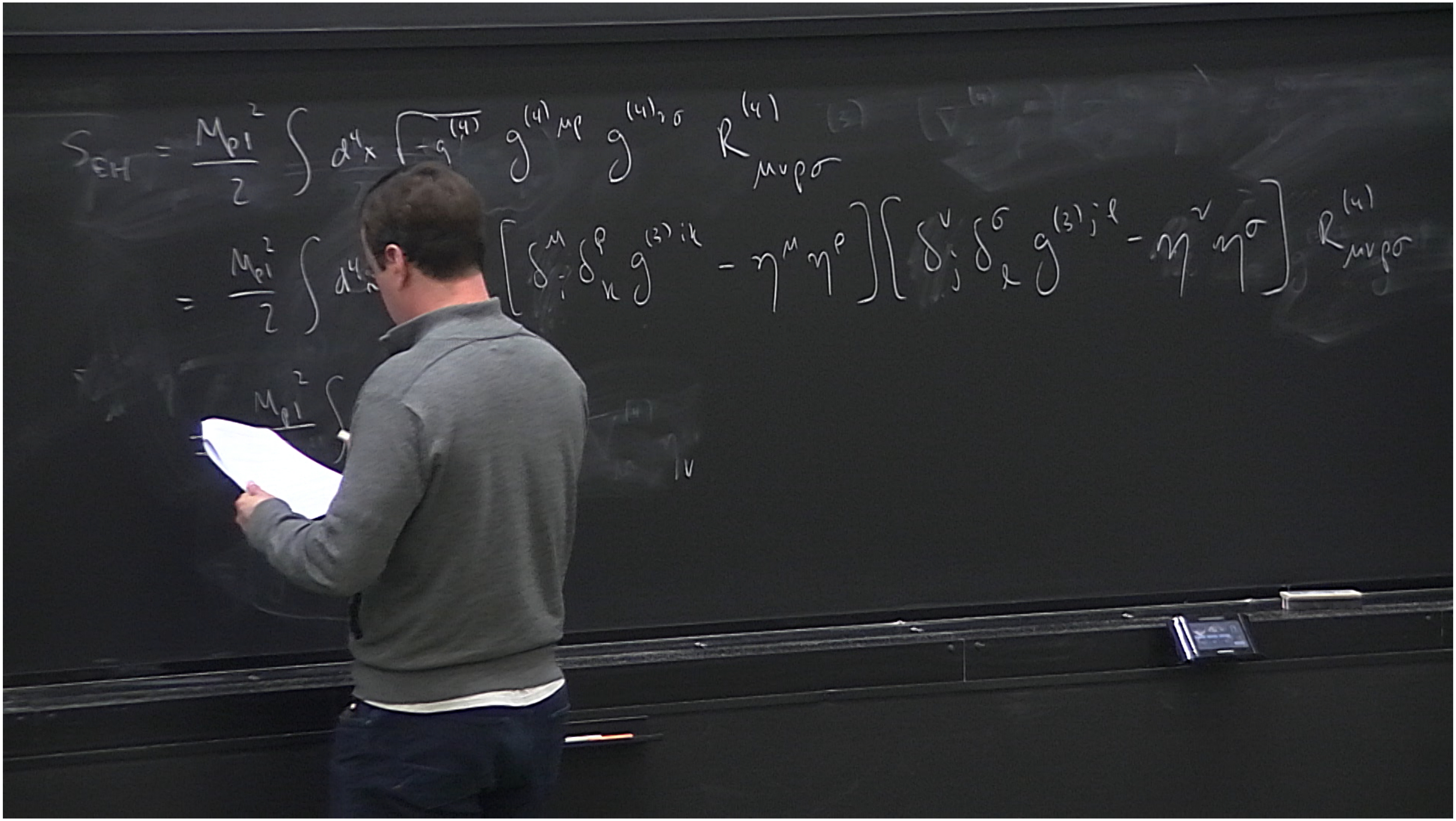
$$= \int d^4x \sqrt{-g^{(4)}} \Big[(\nabla^{(4)}_\mu \eta^\mu)(\nabla^{(4)}_\nu \eta^\nu) - (\nabla^{(4)}_\nu$$

$$= \int d^4x \sqrt{-g^{(4)}} \Big[\left(g^{(3)ij} K_{ij}\right)^2 - \left(g^{(3)ik}\, g^{(3)}\right.$$

$$\int d^4x \sqrt{-g^{(4)}}\; \eta^\mu \eta^\rho g^{(3)jl} \delta^\nu_j \delta^\sigma_l R^{(4)}_{\mu\nu\rho\sigma} = \text{SAME BY (1)}$$

$$\int d^4x \sqrt{-g}\; g^{(4)\mu\rho} \eta^\nu \eta^\sigma R^{(4)}_{\mu\nu\rho\sigma} = \text{SAME SINCE } R^{(4)}_{\mu\nu\rho\sigma} = R^{(4)}_{\nu\mu\sigma\rho}$$


$S_{EH} = \frac{M_{pl}^2}{2} \int d^4x \sqrt{-g^{(4)}}\, g^{(4)\mu\rho}\, g^{(4)\nu\sigma}\, R^{(4)}_{\mu\nu\rho\sigma}$
$= \frac{M_{pl}^2}{2} \int d^4x \sqrt{-g^{(4)}} \left[\delta^\mu_i \delta^\rho_k g^{(3)ik} - n^\mu n^\rho\right]\left[\delta^\nu_j \delta^\sigma_l g^{(3)jl} - n^\nu n^\sigma\right] R$
$=$



$S_{EH} = \frac{M_{pl}^2}{2} \int d^4x \sqrt{-g^{(4)}}\, g^{(4)\mu\rho}\, g^{(4)\nu\sigma}\, R^{(4)}_{\mu\nu\rho\sigma}$
$= \frac{M_{pl}^2}{2} \int d^4x \left[\delta^\mu_i \delta^\rho_k g^{(3)ik} - \eta^\mu \eta^\rho\right]\left[\delta^\nu_j \delta^\sigma_l g^{(3)jl} - \eta^\nu \eta^\sigma\right] R^{(4)}_{\mu\nu\rho\sigma}$
$= \frac{M_{pl}^2}{2} \int$

$$= \frac{M_{pl}^2}{2} \int d^4x \sqrt{-g^{(4)}}\, g^{(4)\mu\rho}\, g^{(4)\nu\sigma}\, R^{(4)}_{\mu\nu\rho\sigma}$$

$$= \frac{M_{pl}^2}{2} \int d^4x \sqrt{-g^{(4)}} \left[\delta^{\mu}_{i} \delta^{\rho}_{k} g^{(3)ik} - \eta^{\mu}\eta^{\rho} \right] \left[\delta^{\nu}_{j} \delta^{\sigma}_{l} g^{(3)jl} - \eta^{\nu}\eta^{\sigma} \right] R^{(4)}_{\mu\nu\rho\sigma}$$

$$= \frac{M_{pl}^2}{2} \int d^4x \sqrt{-g^{(4)}} \Big[g^{(3)ik}\, g^{(3)jl}\, R^{(3)}_{ijkl} -$$

$$S_{EH} = \frac{M_{Pl}^2}{2}\int d^4x \sqrt{-g^{(4)}}\, g^{(4)\mu\rho}\, g^{(4)\nu\sigma}\, R^{(4)}_{\mu\nu\rho\sigma}$$

$$= \frac{M_{Pl}^2}{2}\int d^4x \sqrt{-g^{(4)}}\left[\delta^\mu_i\,\delta^\rho_k\, g^{(3)ik} - \eta^\mu\eta^\rho\right]\left[\delta^\nu_j\,\delta^\sigma_l\, g^{(3)jl} - \eta^\nu\eta^\sigma\right] R^{(4)}_{\mu\nu\rho\sigma}$$

$$= \cdots = \frac{M_{Pl}^2}{2}\int d^4x \sqrt{-g^{(4)}}\left[g^{(3)ik}\, g^{(3)jl}\, R^{(3)}_{ijkl} - \left(g^{(3)ij}K_{ij}\right)^2 + \left(g^{(3)ik}\, g^{(3)jl} K_{ij}K_{kl}\right)\right]$$

$$= \frac{M_{Pl}^2}{2}\int d^4x \sqrt{-g^{(3)}}\, N\left[R^{(3)} + K^{ij}K_{ij} - \left(K^l{}_l\right)^2\right]$$

ADM