Title: PSI 2017/2018 - Cosmology - Lecture 9

Date: Apr 19, 2018 10:15 AM

URL: http://pirsa.org/18040021

Abstract:

$$g_{\mu\nu}(x,t) = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(x,t)$$

$$\phi(x,t) = \bar{\phi}(t) + \delta\phi(x,t)$$

$$\bar{g}_{\mu\nu}(t) = \begin{pmatrix} -1 & \\ & a(t)^2\delta_{ij} \end{pmatrix}$$

$a(t) \sim$ EXPONENTIAL

$\bar{\phi}(t)$ CHANGING SLOWLY

$$\varepsilon^\mu = (\varepsilon^0, \varepsilon^i)$$

$$\mathcal{L}_\varepsilon \phi = \varepsilon^0 \dot{\bar{\phi}}$$

$$\mathcal{L}_\varepsilon g_{00} = -2\dot{\varepsilon}^0$$

$$\mathcal{L}_\varepsilon g_{0i} = a^2 \dot{\varepsilon}^i - \partial_i \varepsilon^0$$

$$\mathcal{L}_\varepsilon g_{ij} = 2a^2 H \varepsilon^0 \delta_{ij} + a^2(\partial_i \varepsilon_j + \partial_j \varepsilon_i)$$

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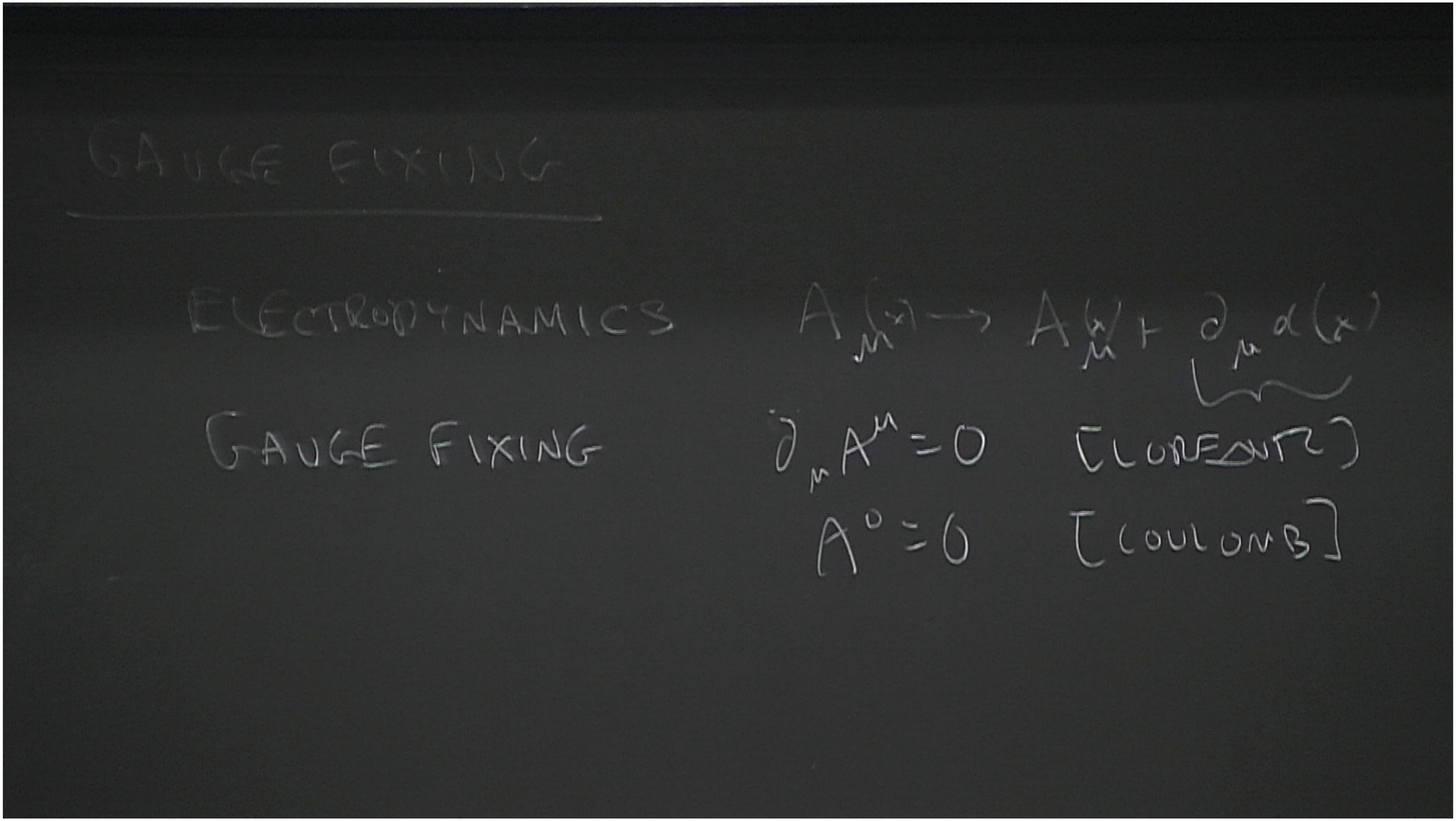
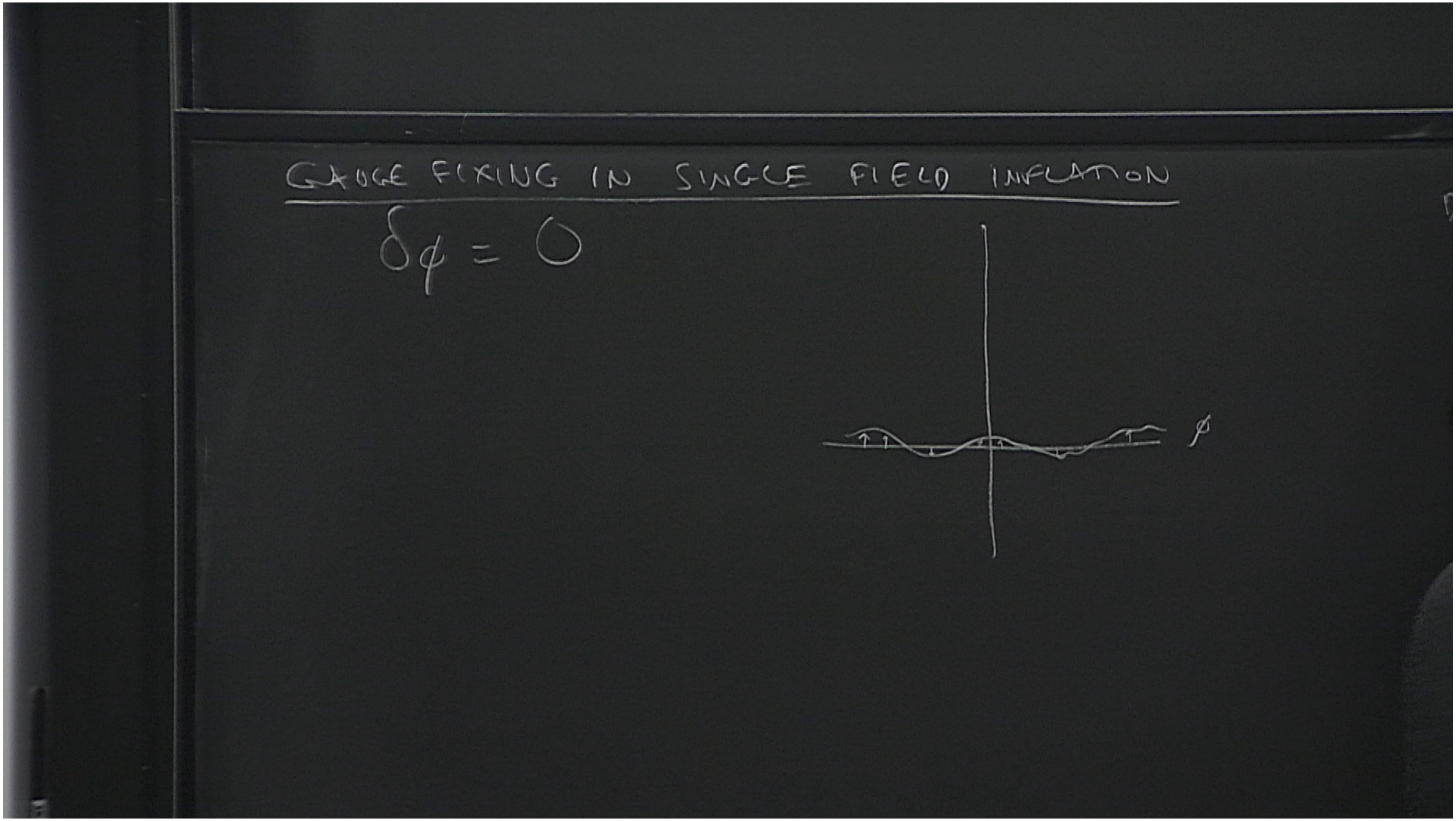
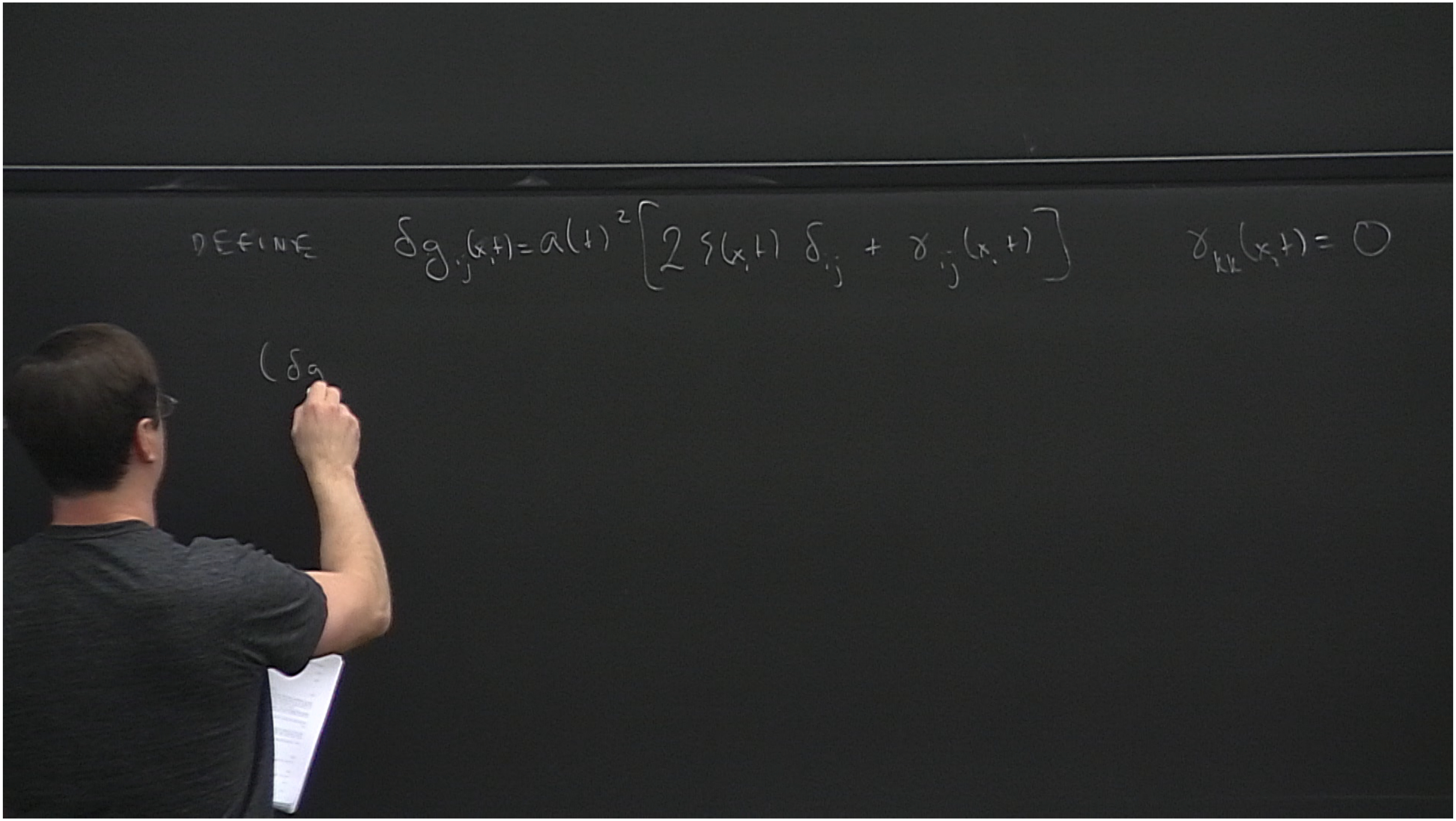
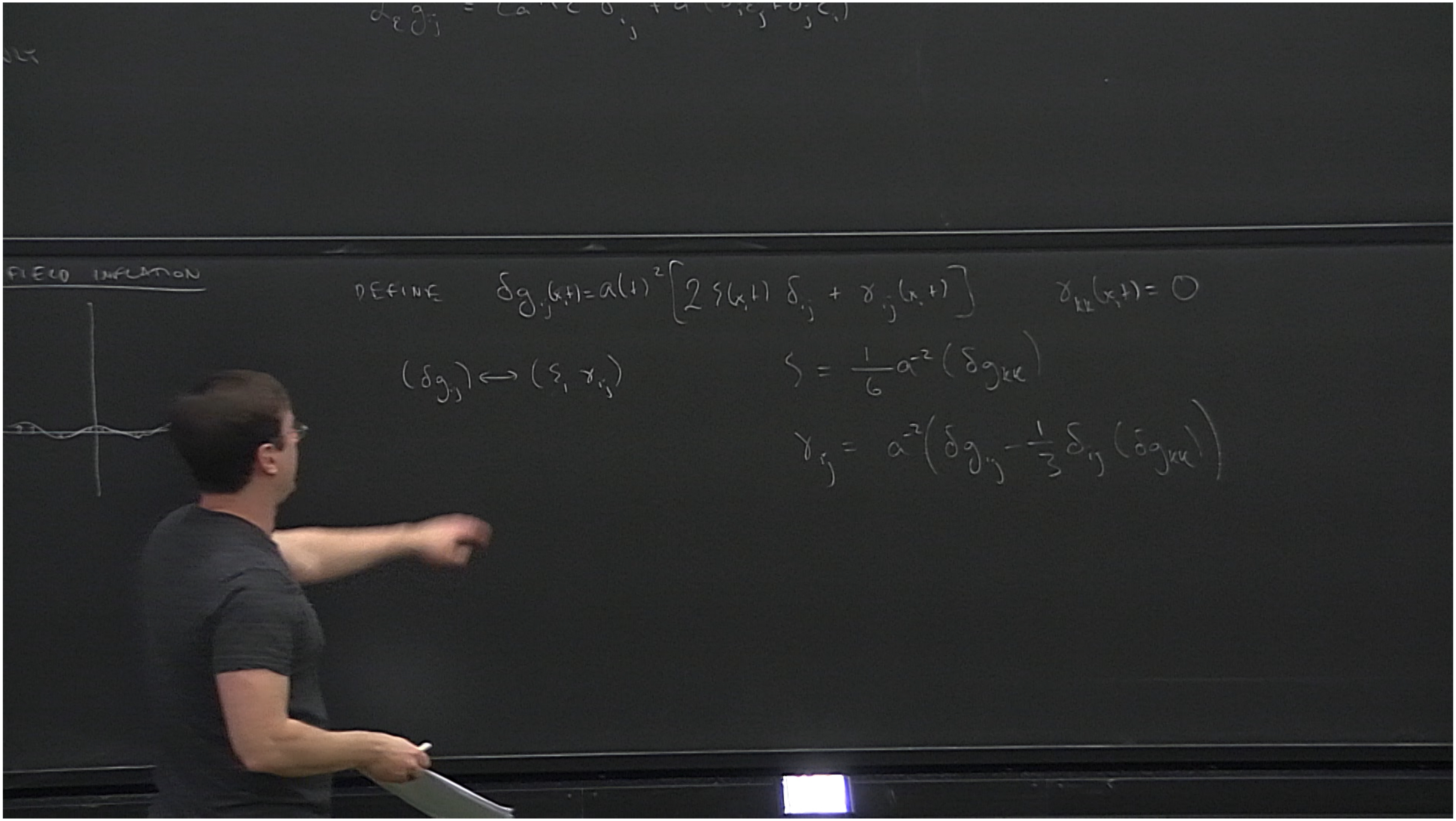
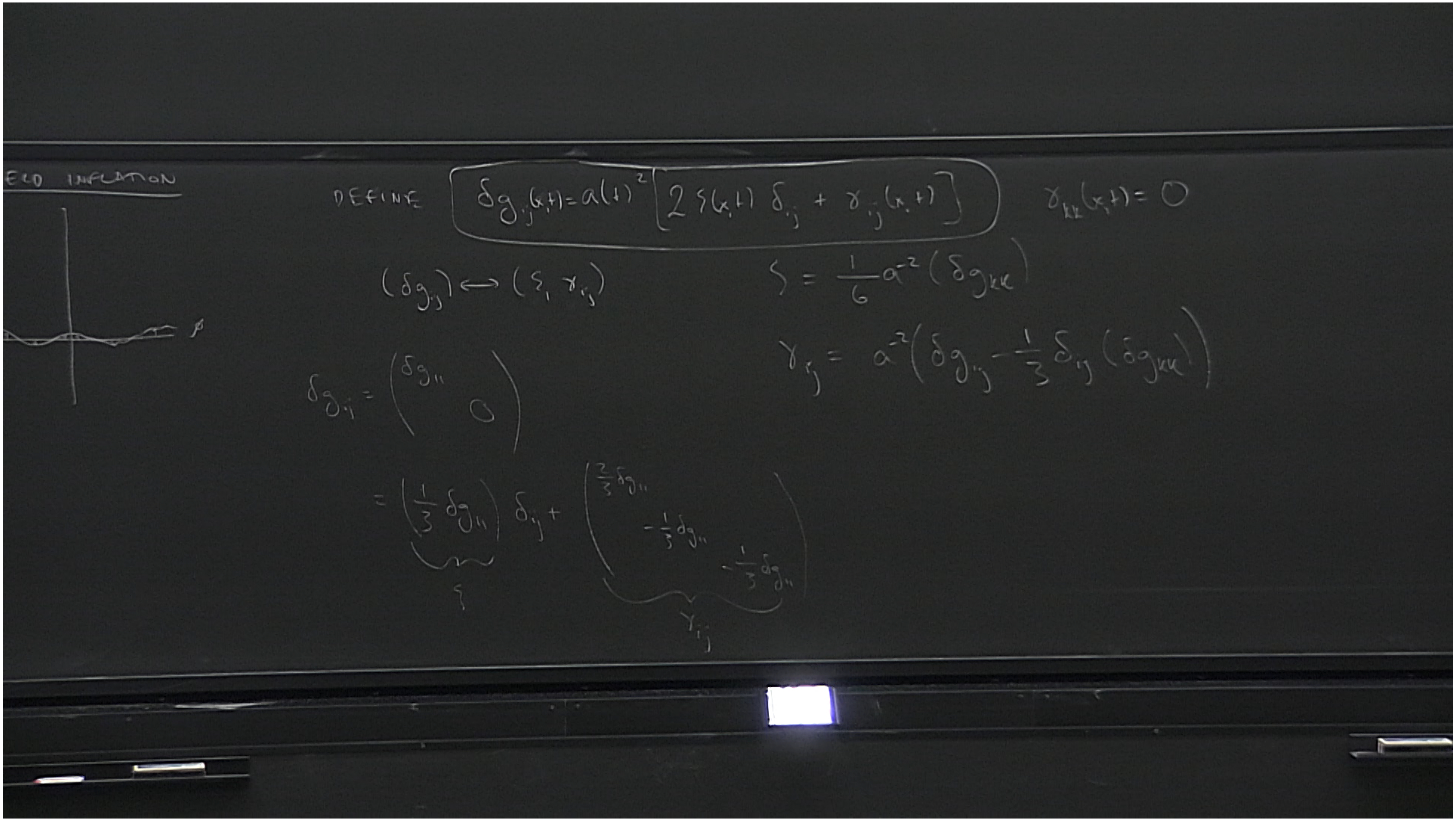

GAUGE FIXING
ELECTRODYNAMICS
$A_\mu(x) \to A_\mu(x) + \partial_\mu \alpha(x)$
GAUGE FIXING
$\partial_\mu A^\mu = 0$ [LORENTZ]
$A^0 = 0$ [COULOMB]


GAUGE FIXING IN SINGLE FIELD INFLATION
δφ = 0
φ


DEFINE $\delta g_{ij}(x,t) = a(t)^2 \left[2\zeta(x,t)\,\delta_{ij} + \gamma_{ij}(x,t) \right]$
$\gamma_{kk}(x,t) = 0$
$(\delta g$


FIELD INFLATION
DEFINE $\delta g_{ij}(x,t) = a(t)^2 \left[2\zeta(x,t)\,\delta_{ij} + \gamma_{ij}(x,t) \right]$
$\gamma_{kk}(x,t) = 0$
$(\delta g_{ij}) \longleftrightarrow (\zeta, \gamma_{ij})$
$\zeta = \frac{1}{6} a^{-2} (\delta g_{kk})$
$\gamma_{ij} = a^{-2}\left(\delta g_{ij} - \frac{1}{3}\delta_{ij}(\delta g_{kk})\right)$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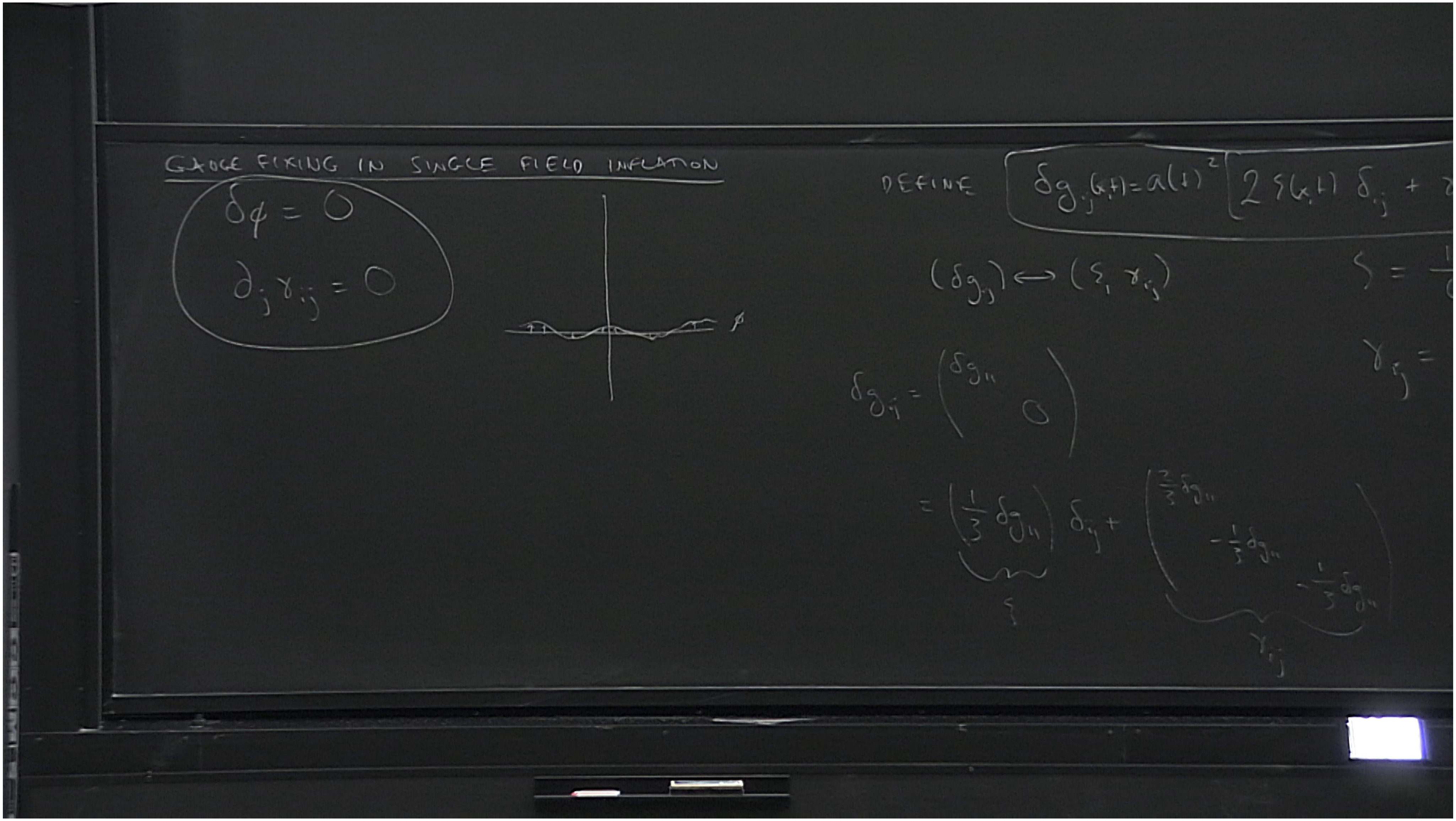
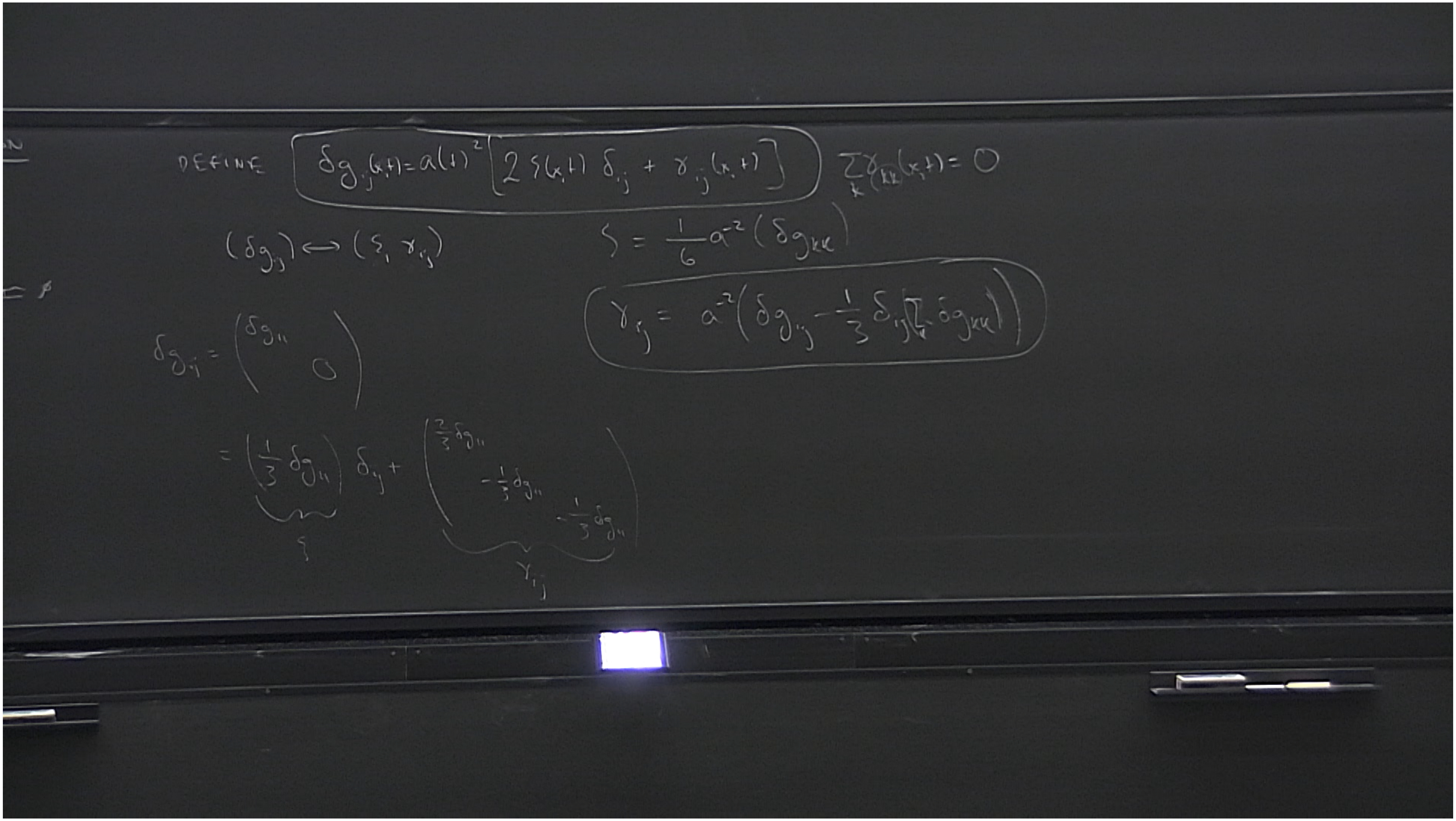
$\zeta = \frac{1}{6} a^{-2} (\delta g_{kk})$
$\gamma_{ij} = a^{-2}\left(\delta g_{ij} - \frac{1}{3}\delta_{ij}(\delta g_{kk})\right)$
ϕ
$\delta g_{ij} = \begin{pmatrix} \delta g_{11} & \\ & 0 \end{pmatrix}$
$= \left(\frac{1}{3}\delta g_{11}\right)\delta_{ij} + \begin{pmatrix} \frac{2}{3}\delta g_{11} & & \\ & -\frac{1}{3}\delta g_{11} & \\ & & -\frac{1}{3}\delta g_{11} \end{pmatrix}$
ζ
γ_{ij}


GAUGE FIXING IN SINGLE FIELD INFLATION
δφ = 0
∂j γij = 0
DEFINE
δgij(x,t) = a(t)² [2 ζ(x,t) δij +
(δgij) ⟷ (ζ, γij)


DEFINE
δg_ij(x,t) = a(t)^2 [2 ζ(x,t) δ_ij + γ_ij(x,t)]
Σ_k γ_kk(x,t) = 0
(δg_ij) ⟷ (ζ, γ_ij)
ζ = 1/6 a^-2 (δg_kk)
γ_ij = a^-2 (δg_ij − 1/3 δ_ij Σ_k δg_kk)
δg_ij = (δg_11 0)
= (1/3 δg_11) δ_ij + (2/3 δg_11, −1/3 δg_11, −1/3 δg_11)
ζ
γ_ij

$\left(\delta g_{\mu\nu}(x,t),\ \delta\phi(x,t)\right)$ ARBITRARY

$\Rightarrow$ THERE EXISTS A GAUGE TRANSFORMATION ε^{μ}

SUCH THAT $\delta\phi = \underbrace{\partial_j \gamma_{ij}} = 0$

$$\delta\phi \to \delta\phi + \varepsilon_0 \dot{\phi}$$

$$\delta g_{ij} \to (\delta g_{ij}) + 2a^2 H \varepsilon^0 \delta_{ij} + a^2(\partial_i \varepsilon_j + \partial_j \varepsilon_i)$$

$$\gamma_{ij} \to \gamma_{ij} + \partial_i \varepsilon_j + \partial_j \varepsilon_i - \frac{2}{3}(\partial_k \varepsilon_k)\delta_{ij}$$

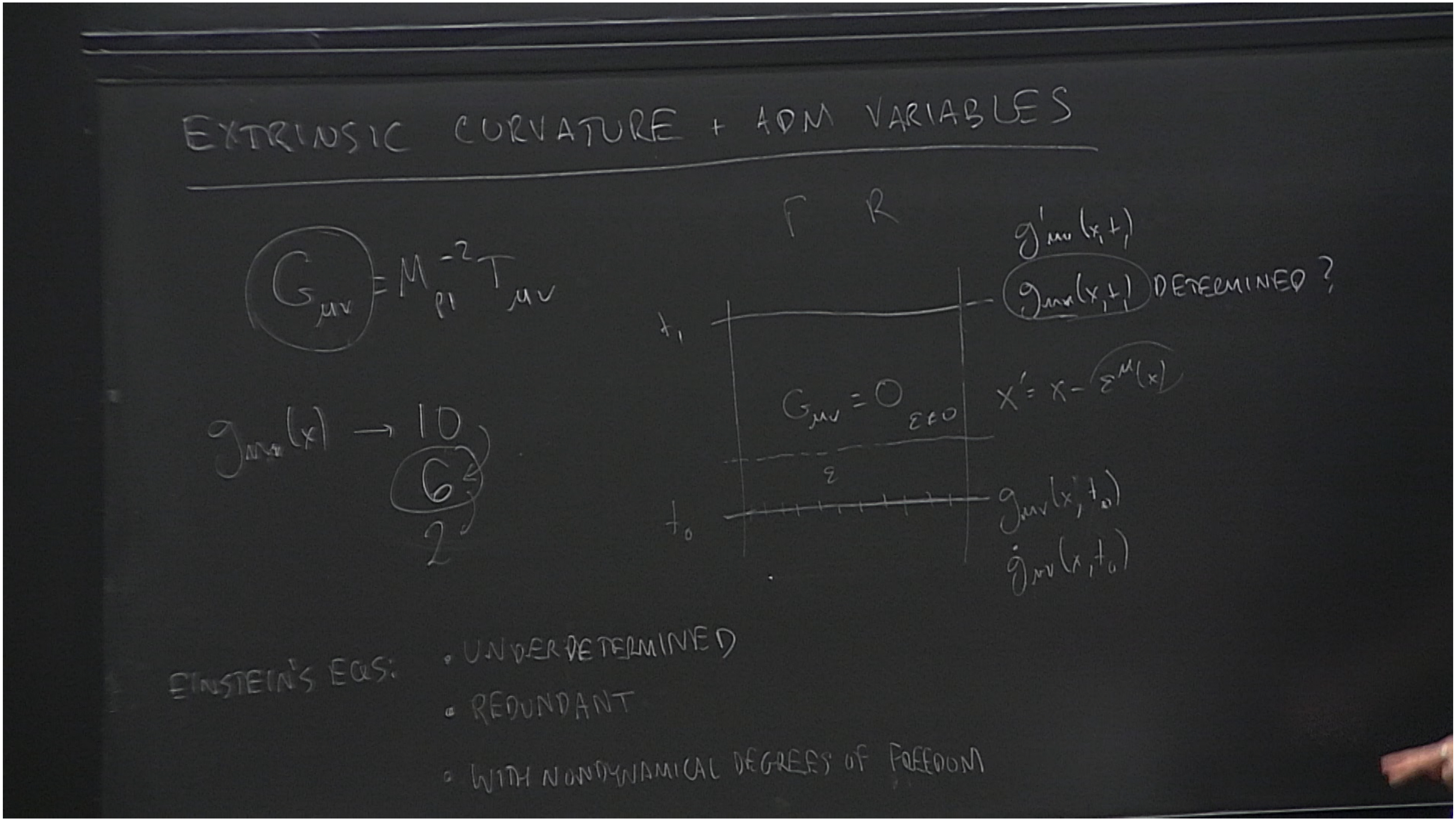
$$\partial_j \gamma_{ij} \to \partial_j \gamma_{ij} + \partial^2 \varepsilon_i + \frac{1}{3}\partial_i \partial_j \varepsilon^j$$

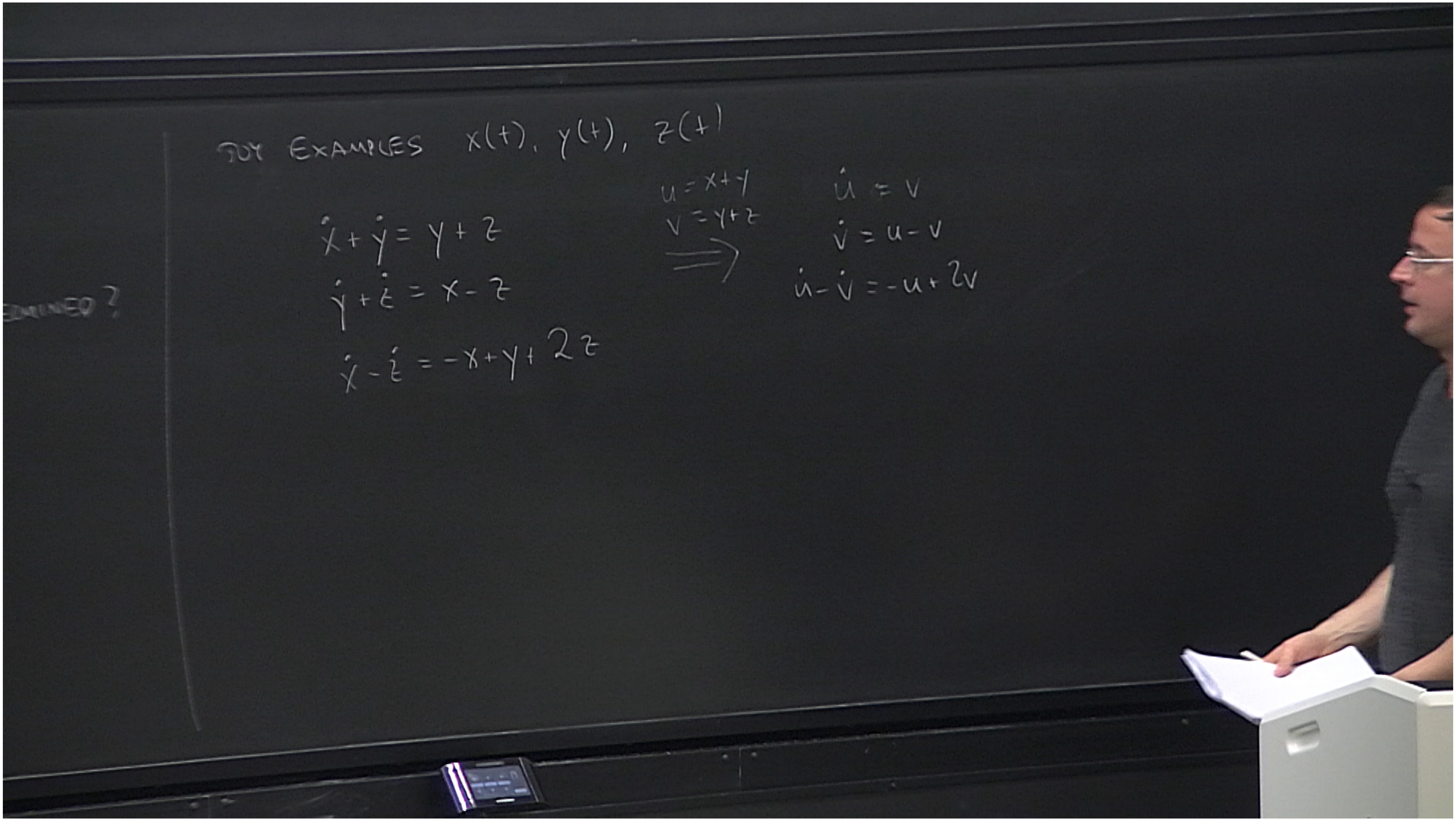
WANT TO SOLVE:

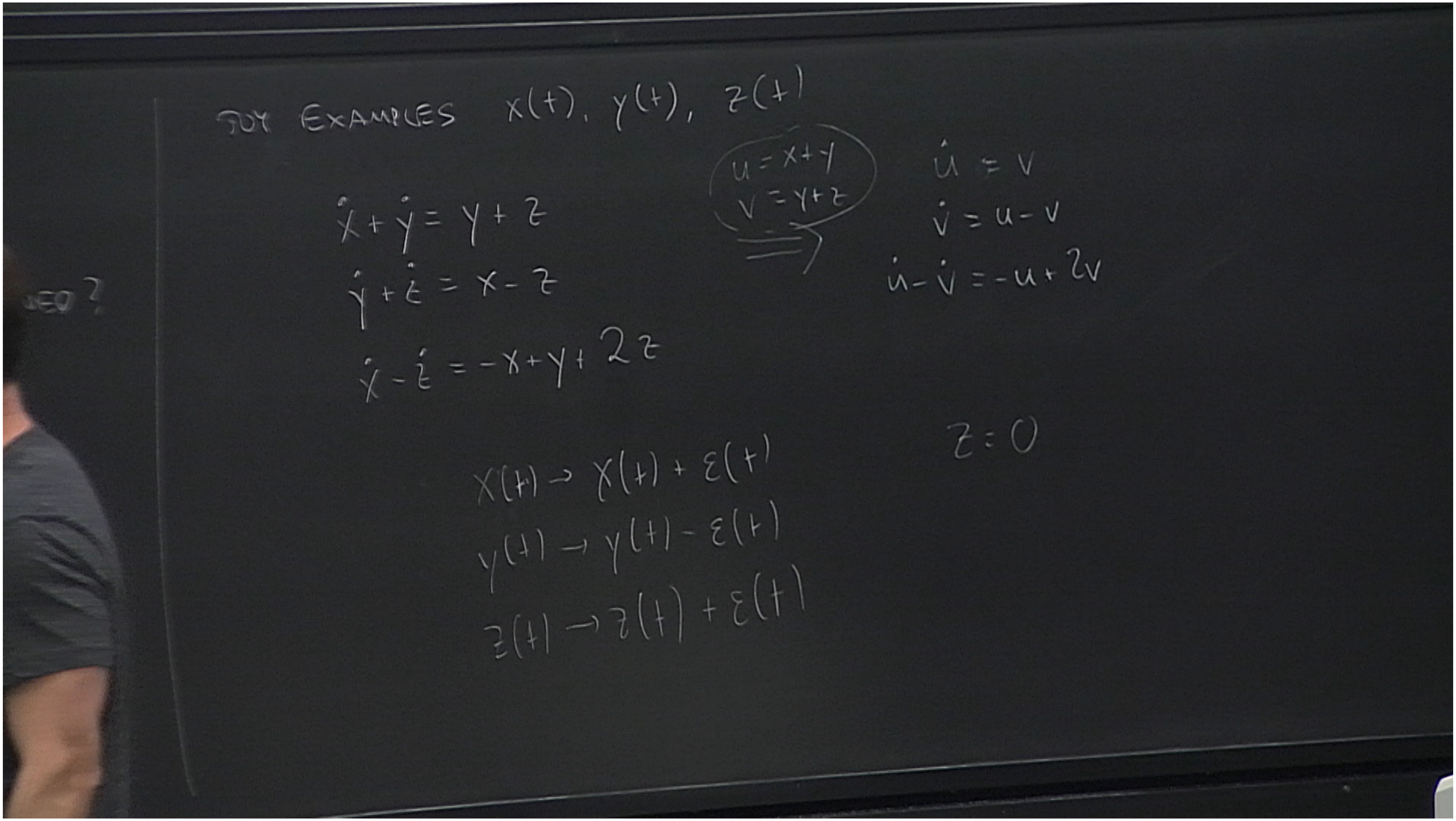
$$\left.\begin{aligned} \dot{\phi}\,\varepsilon_0 &= S_0 \\ \partial^2 \varepsilon_i + \frac{1}{3}\partial_i \partial_j \varepsilon_j &= S_i \end{aligned}\right\}$$

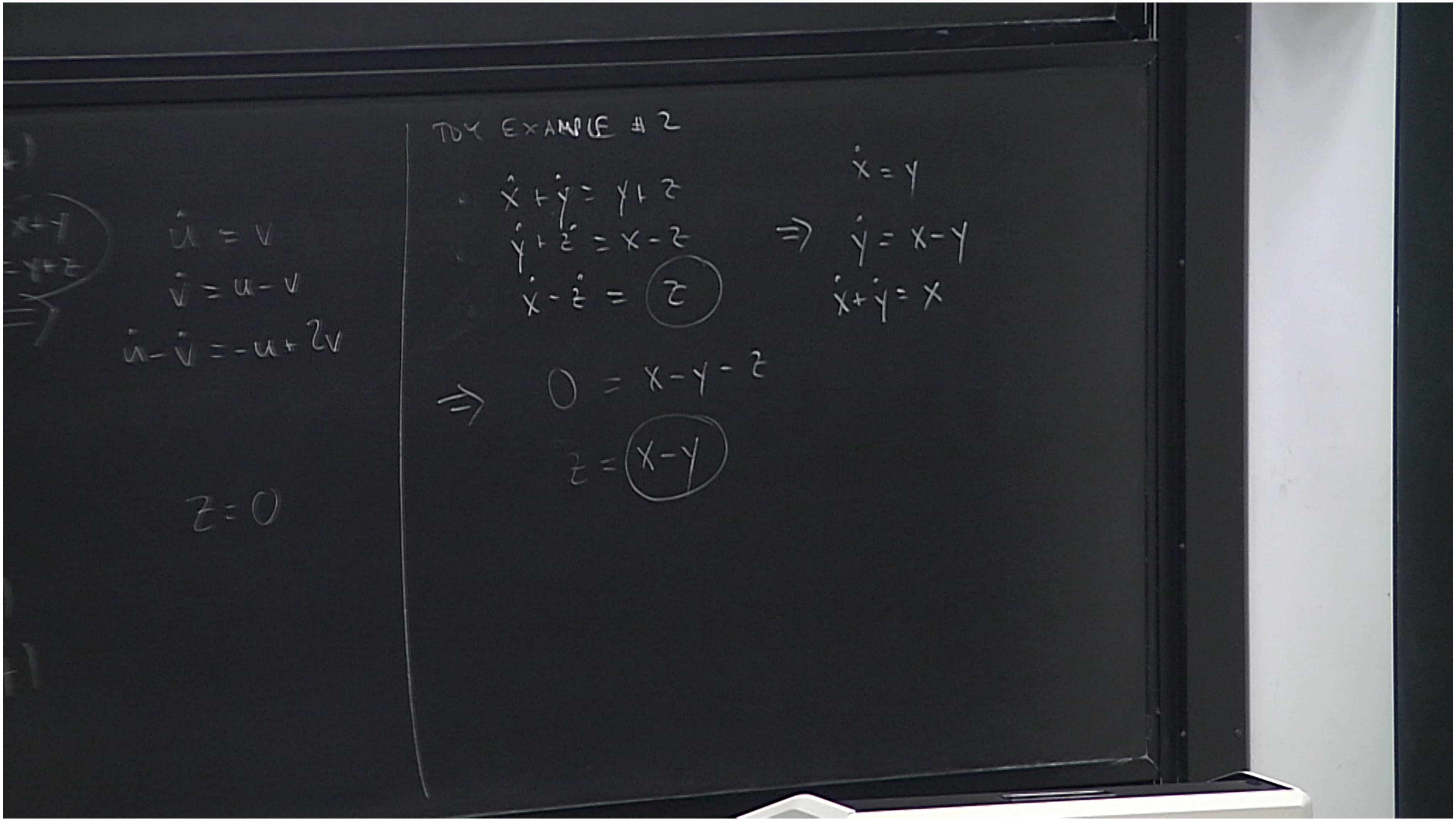
"SOURCES" (S_0, S_i)

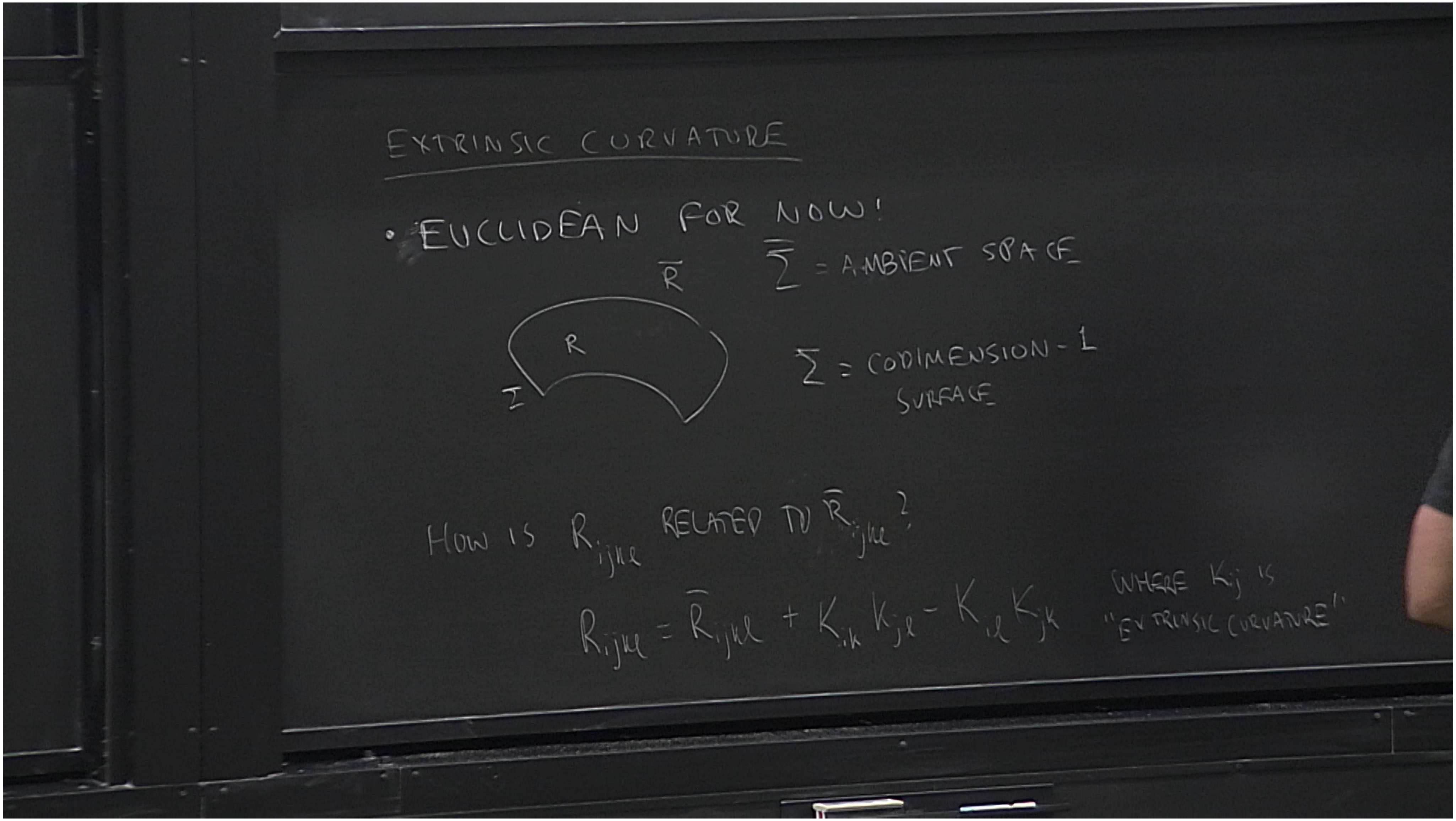
$(\varepsilon_0, \varepsilon_i)$

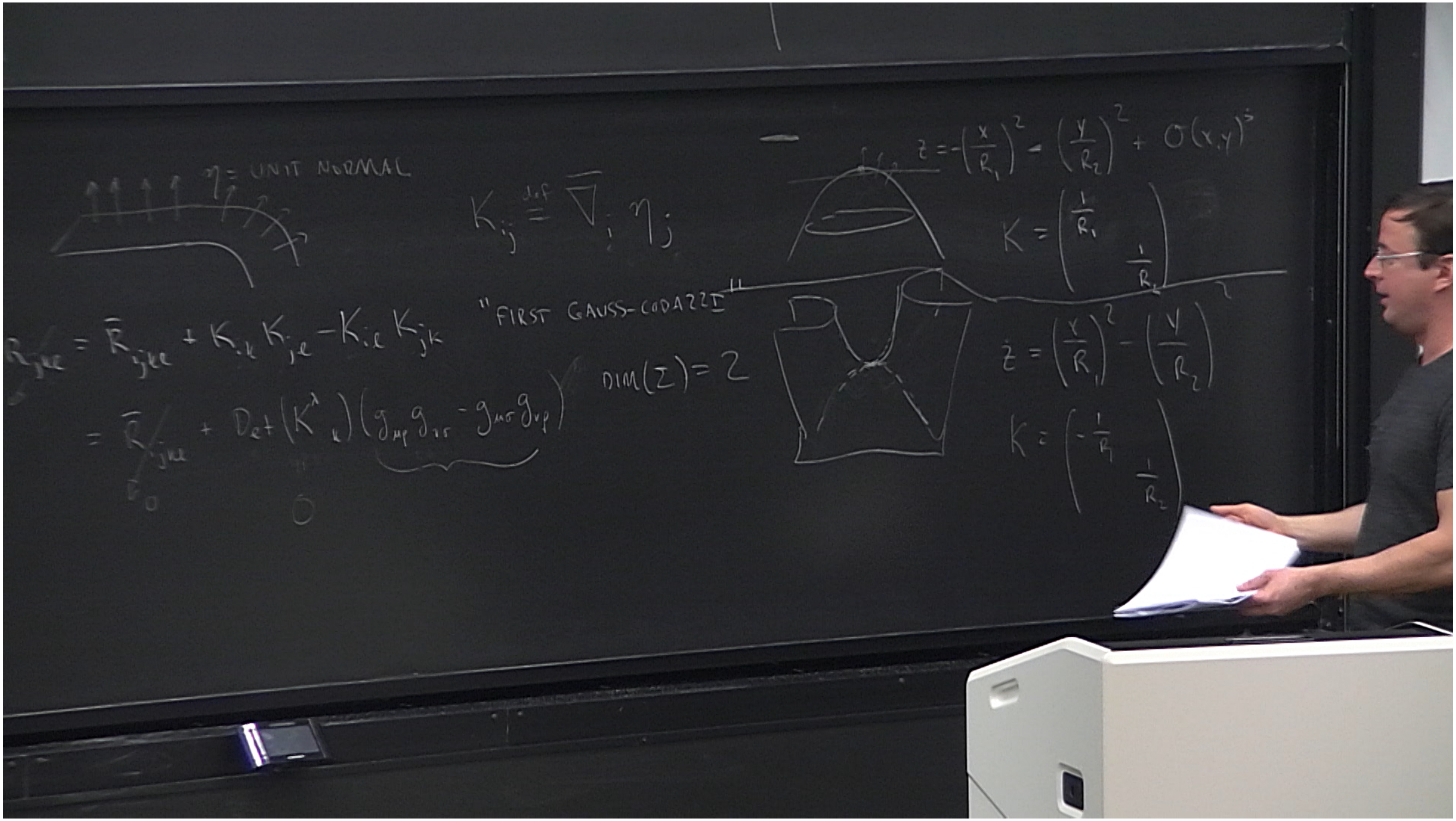

EXTRINSIC CURVATURE + ADM VARIABLES
$G_{\mu\nu} = M_{Pl}^{-2} T_{\mu\nu}$
$g_{\mu\nu}(x) \rightarrow 10$
6
2
F R
t_1
t_0
$G_{\mu\nu} = 0$
$\varepsilon \neq 0$
ε
$g'_{\mu\nu}(x, t_1)$
$g_{\mu\nu}(x, t_1)$ DETERMINED ?
$x' = x - \varepsilon^{\mu}(x)$
$g_{\mu\nu}(x, t_0)$
$\dot{g}_{\mu\nu}(x, t_0)$
EINSTEIN'S EQS:
• UNDERDETERMINED
• REDUNDANT
• WITH NONDYNAMICAL DEGREES OF FREEDOM


TOY EXAMPLES x(t), y(t), z(t)
$\dot{x} + \dot{y} = y + z$
$\dot{y} + \dot{z} = x - z$
$\dot{x} - \dot{z} = -x + y + 2z$
$u = x + y$
$v = y + z$
$\Longrightarrow$
$\dot{u} = v$
$\dot{v} = u - v$
$\dot{u} - \dot{v} = -u + 2v$


TOY EXAMPLES x(t), y(t), z(t)
$\dot{x} + \dot{y} = y + z$
$\dot{y} + \dot{z} = x - z$
$\dot{x} - \dot{z} = -x + y + 2z$
u = x + y
v = y + z
$\dot{u} = v$
$\dot{v} = u - v$
$\dot{u} - \dot{v} = -u + 2v$
z = 0
$x(t) \to x(t) + \varepsilon(t)$
$y(t) \to y(t) - \varepsilon(t)$
$z(t) \to z(t) + \varepsilon(t)$


TOY EXAMPLE # 2
ẋ + ẏ = y + z
ẏ + ż = x − z
ẋ − ż = z
⇒ 0 = x − y − z
z = x − y
ẋ = y
⇒ ẏ = x − y
ẋ + ẏ = x
u̇ = v
v̇ = u − v
u̇ − v̇ = −u + 2v
z = 0


EXTRINSIC CURVATURE
• EUCLIDEAN FOR NOW!
$\bar{R}$
$\bar{\Sigma}$ = AMBIENT SPACE
R
Σ
Σ = CODIMENSION - 1 SURFACE
HOW IS R_{ijkl} RELATED TO $\bar{R}_{ijkl}$?
$R_{ijkl} = \bar{R}_{ijkl} + K_{ik} K_{jl} - K_{il} K_{jk}$
WHERE K_{ij} IS "EXTRINSIC CURVATURE"


η = UNIT NORMAL
K_ij ≝ ∇̄_i η_j
"FIRST GAUSS-CODAZZI"
R_ijkl = R̄_ijkl + K_ik K_jl − K_il K_jk
= R̄_ijkl + Det(K^λ_κ)(g_μρ g_νσ − g_μσ g_νρ)
DIM(Σ) = 2
z = −(x/R_1)^2 − (y/R_2)^2 + O(x,y)^3
K = (1/R_1, 1/R_2)
z = (x/R_1)^2 − (y/R_2)^2
K = (−1/R_1, 1/R_2)

BACK TO $(-+++)$ SIGNATURE

Σ DEFINED BY $x^0 = \text{CONSTANT}$

ADM VARIABLES: $\quad ds^2 = -N^2 dt^2 + g^{(3)}_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$

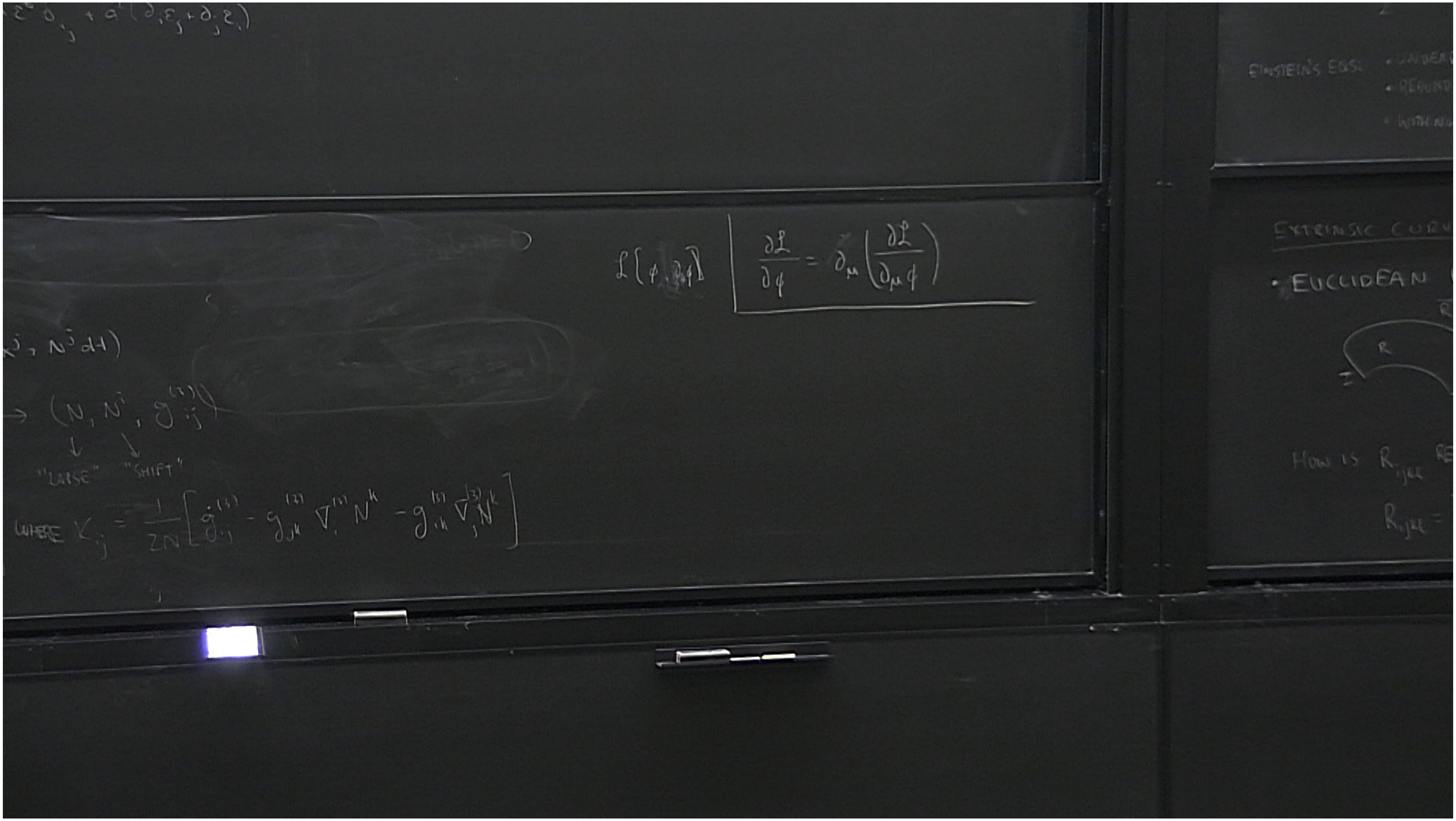
CHANGE OF VARIABLES $\quad g^{(4)}_{\mu\nu} = (g_{00}, g_{0i}, g^{(3)}_{ij}) \longleftrightarrow (N, N^i, g^{(3)}_{ij})$

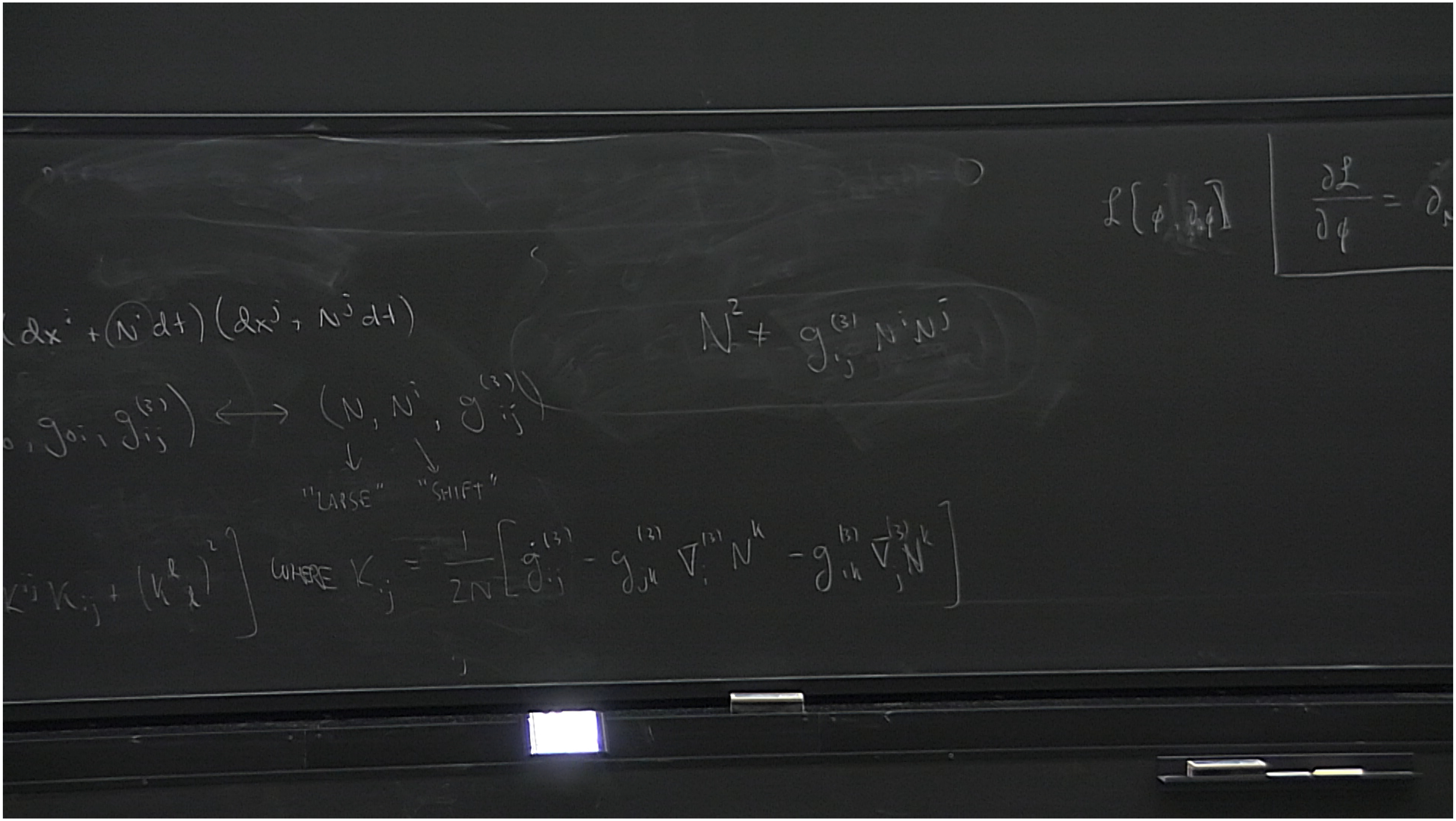
"LAPSE" (N) "SHIFT" (N^i)

$$S_{EH} = \frac{M_{pl}^2}{2}\int d^4x \sqrt{-g^{(4)}}\, R^{(4)}$$

$$S_{ADM} = \frac{M_{pl}^2}{2}\int d^4x \sqrt{-g^{(3)}}\, N\left[R^{(3)} + K^{ij}K_{ij} + (K^l{}_l)^2\right]$$

WHERE $K_{ij} = \frac{1}{2N}\left[\dot{g}^{(3)}_{ij} - g^{(3)}_{jk}\nabla^{(3)}_i N^k - g^{(3)}_{ik}\nabla^{(3)}_j N^k\right]$


EINSTEIN'S EQS.
"LAPSE"
"SHIFT"
WHERE
EXTRINSIC CURV
EUCLIDEAN
How is



$\mathcal{L}[\phi, \partial\phi]$
$\frac{\partial \mathcal{L}}{\partial \phi} = \partial$
$(dx^i + N^i dt)(dx^j + N^j dt)$
$N^2 + g^{(3)}_{ij} N^i N^j$
$g_{0i}, g^{(3)}_{ij}) \longleftrightarrow (N, N^i, g^{(3)}_{ij})$
"LAPSE"
"SHIFT"
$K^{ij} K_{ij} + (K^l{}_l)^2]$ WHERE $K_{ij} = \frac{1}{2N}\left[\dot{g}^{(3)}_{ij} - g^{(3)}_{jk} \nabla^{(3)}_i N^k - g^{(3)}_{ik} \nabla^{(3)}_j N^k\right]$