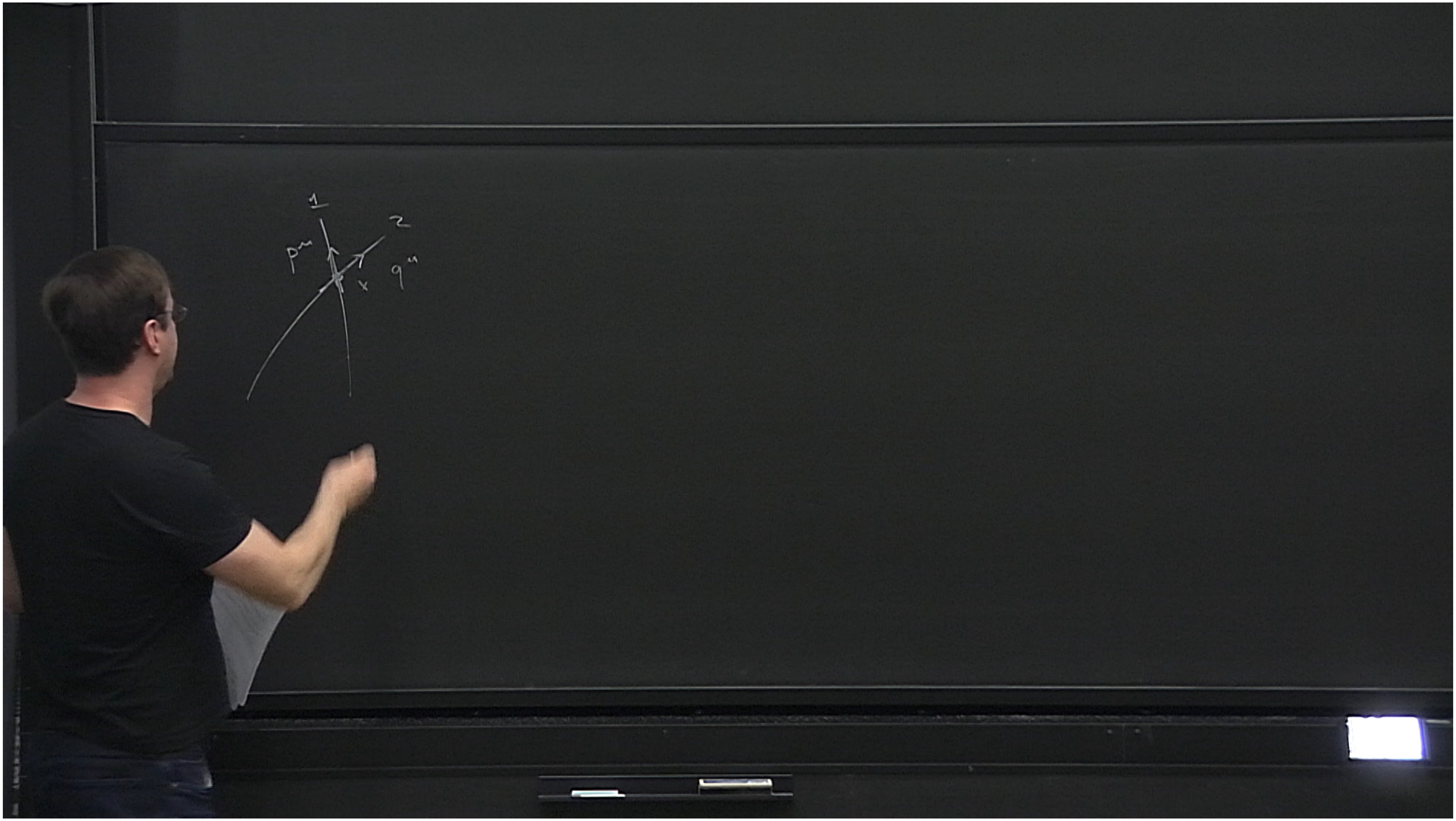
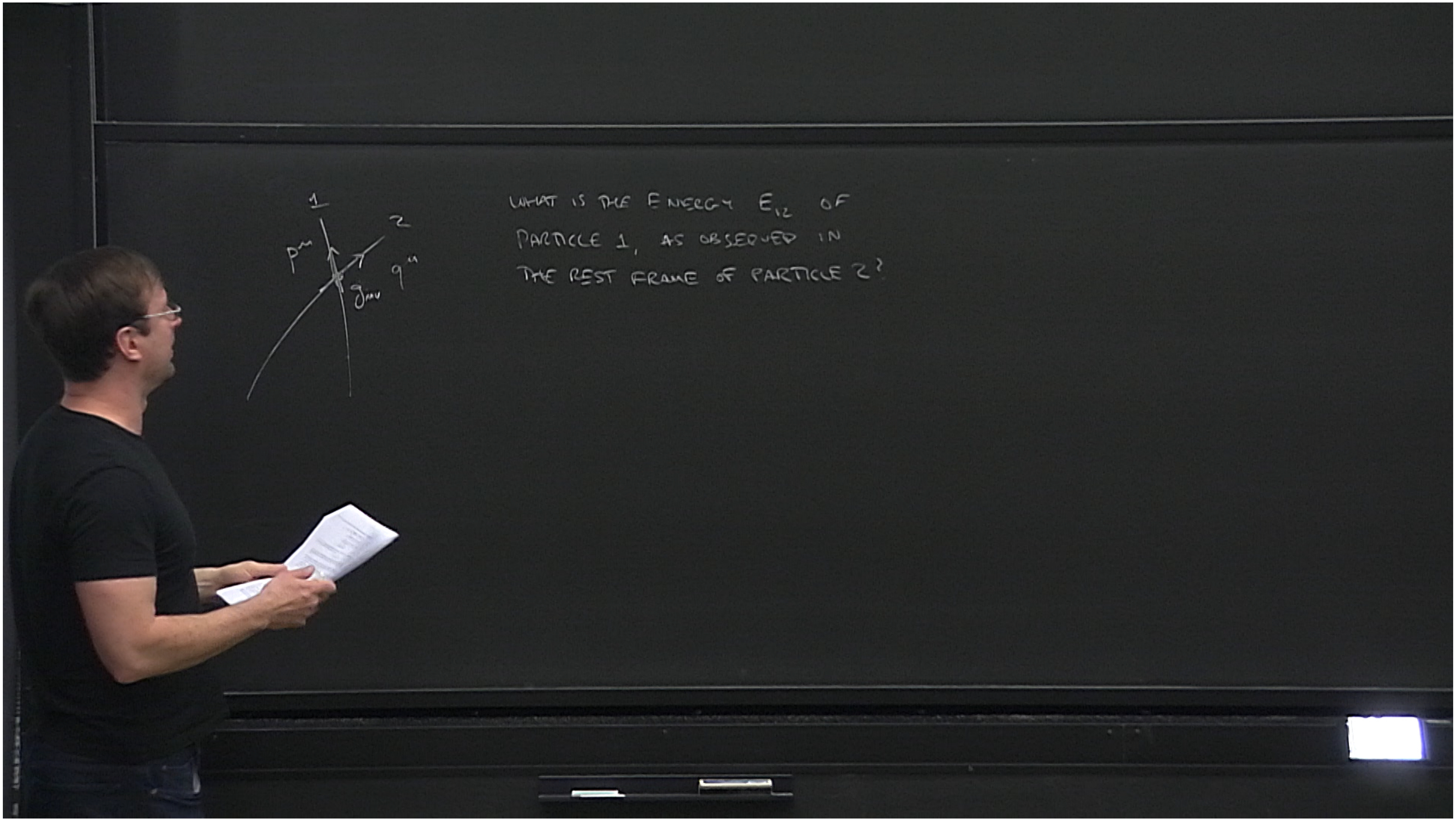
Title: PSI 2017/2018 - Cosmology - Lecture 8

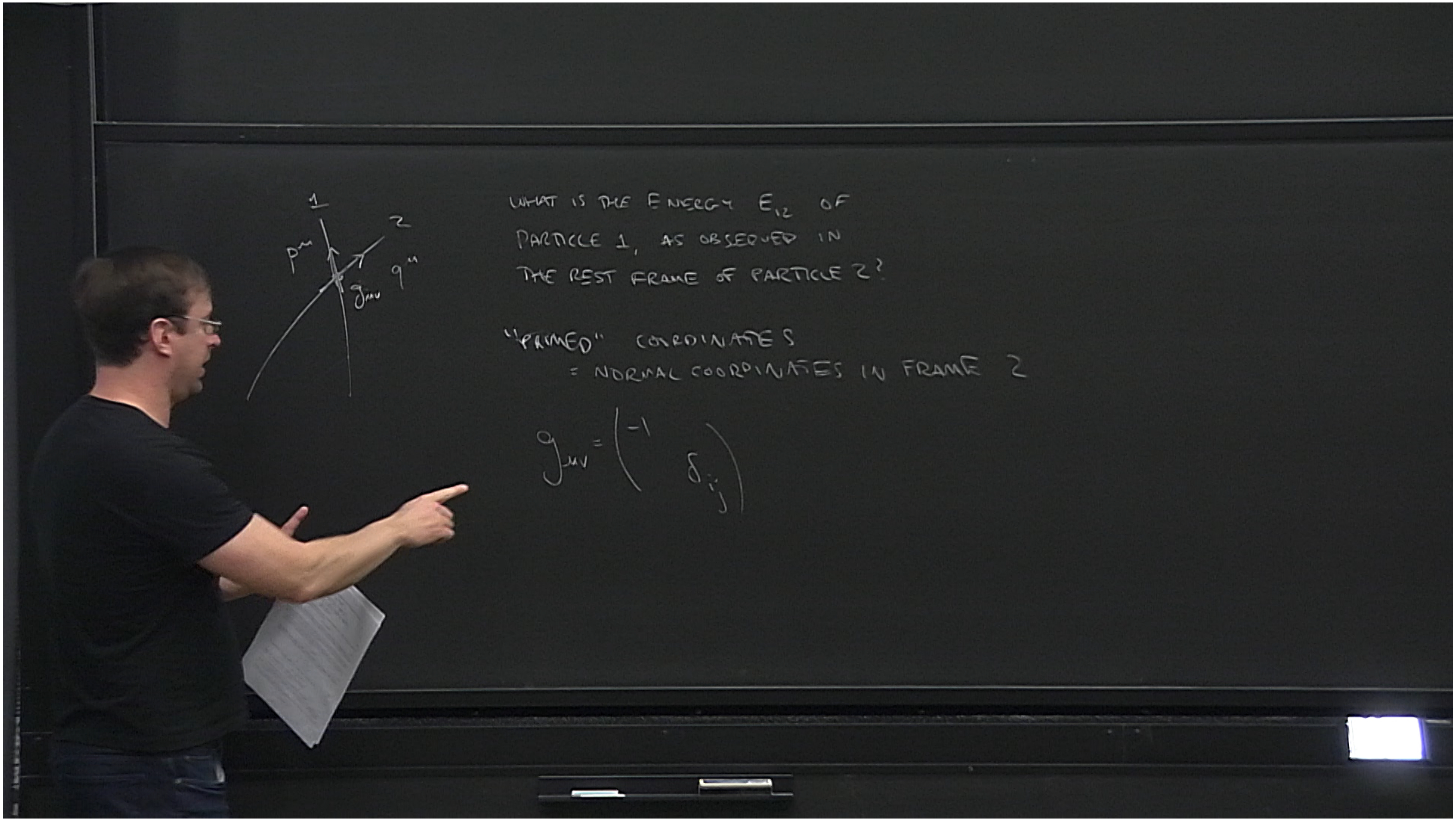
Date: Apr 18, 2018 10:15 AM

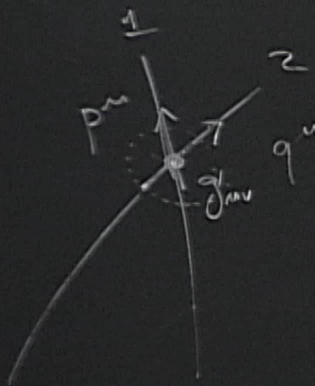
URL: http://pirsa.org/18040020

Abstract:


1
2
p^μ
q^μ
x


1
2
p^μ
q^μ
$g_{\mu\nu}$
WHAT IS THE ENERGY E_{12} OF
PARTICLE 1, AS OBSERVED IN
THE REST FRAME OF PARTICLE 2?


1
2
p^μ
q^μ
g_μν
WHAT IS THE ENERGY E_12 OF
PARTICLE 1, AS OBSERVED IN
THE REST FRAME OF PARTICLE 2?
"PRIMED" COORDINATES
= NORMAL COORDINATES IN FRAME 2
g_μν = (-1 δ_ij)



WHAT IS THE ENERGY (E_{12}) OF PARTICLE 1, AS OBSERVED IN THE REST FRAME OF PARTICLE 2?

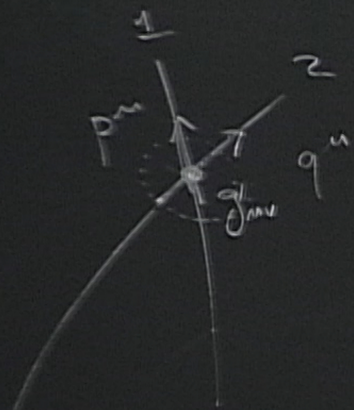
"PRIMED" COORDINATES

= NORMAL COORDINATES IN FRAME 2

$$g'_{\mu\nu} = \begin{pmatrix} -1 & \\ & \delta_{ij} \end{pmatrix}$$

$$q'^{\mu} = (1, 0, 0, 0)$$

$$p'^{\mu} = (E_{12}, \dots)$$



WHAT IS THE ENERGY (E_{12}) OF PARTICLE 1, AS OBSERVED IN THE REST FRAME OF PARTICLE 2?

$E_{12} =$

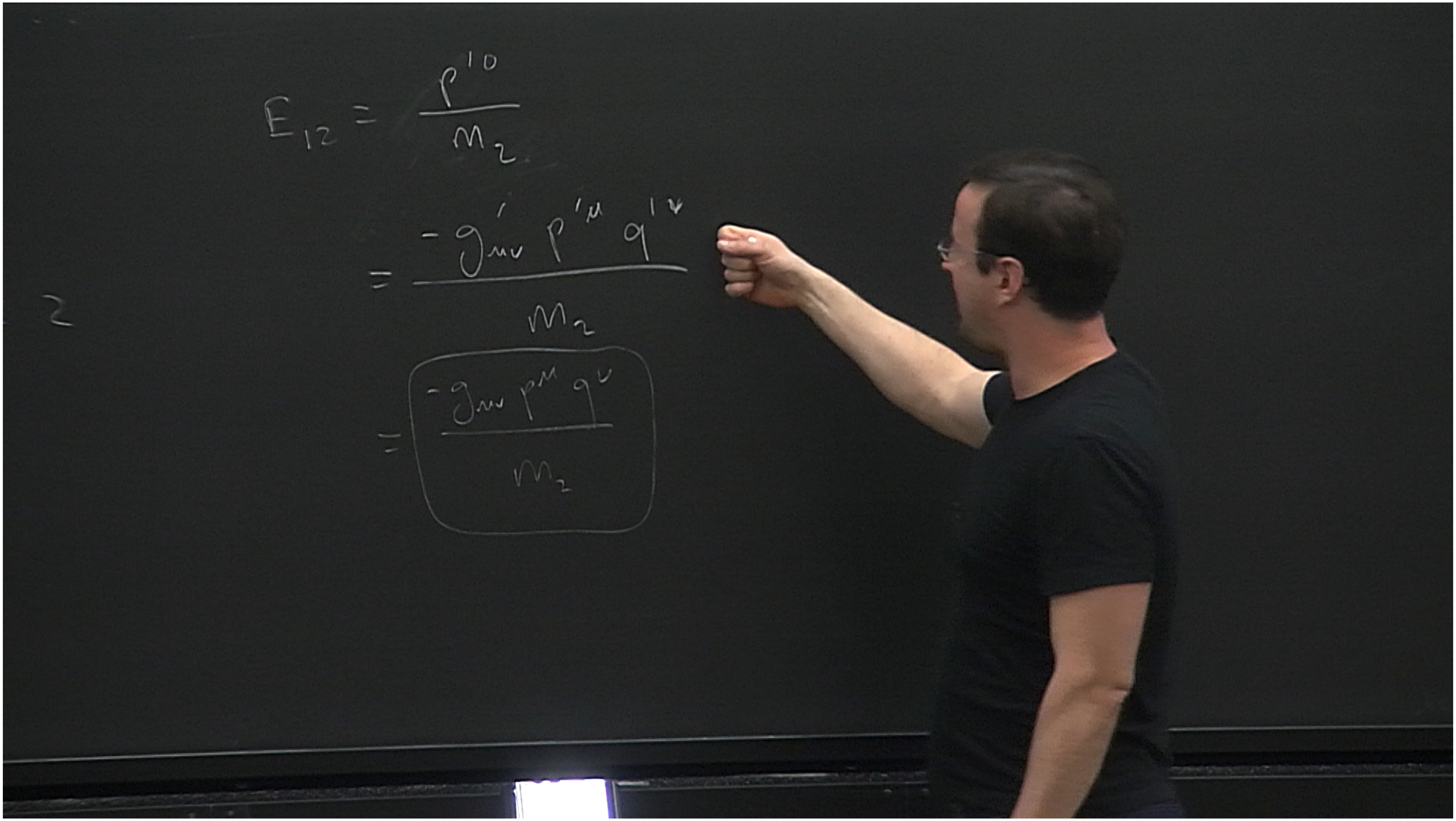
"PRIMED" COORDINATES

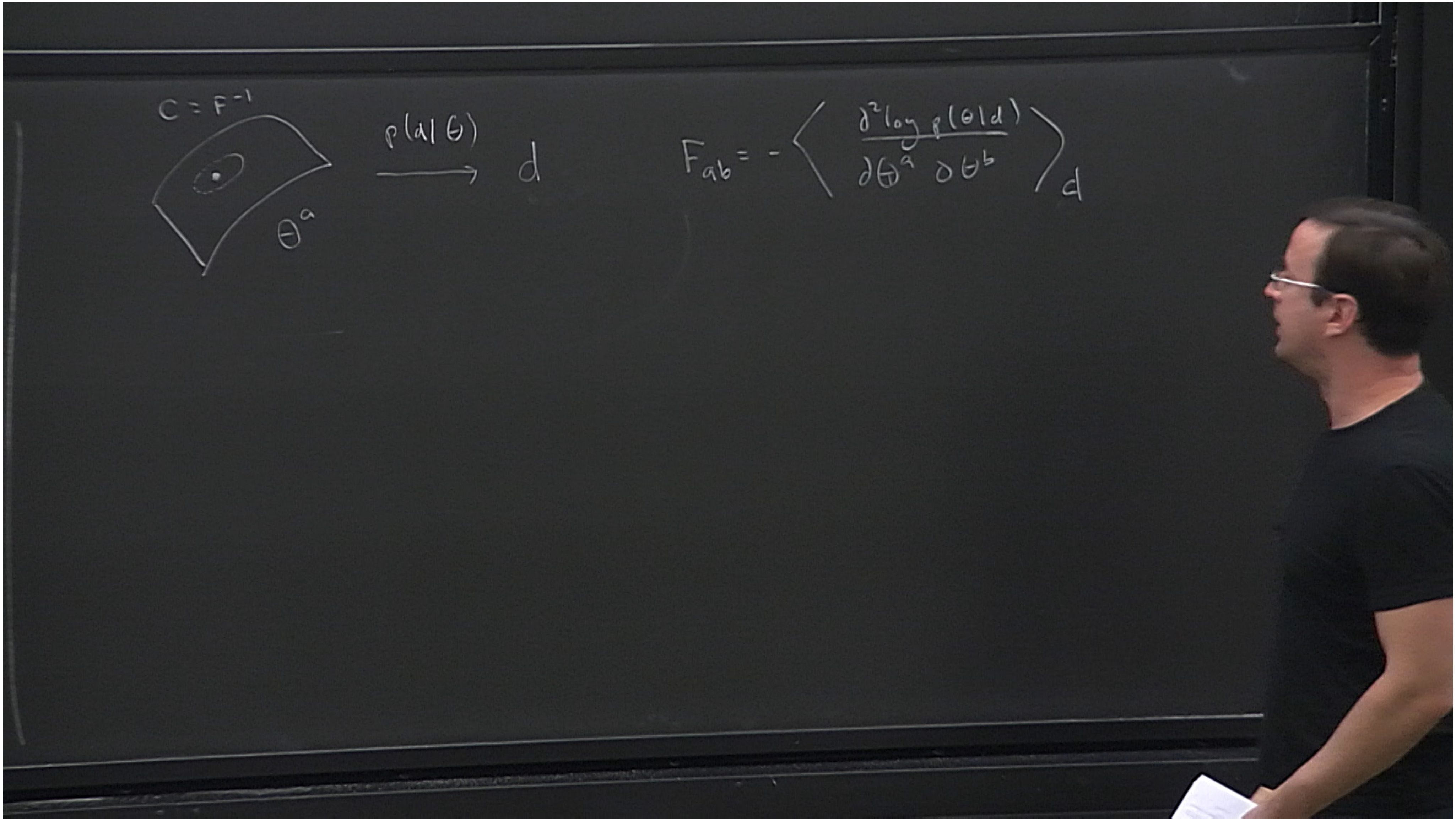
= NORMAL COORDINATES IN FRAME 2

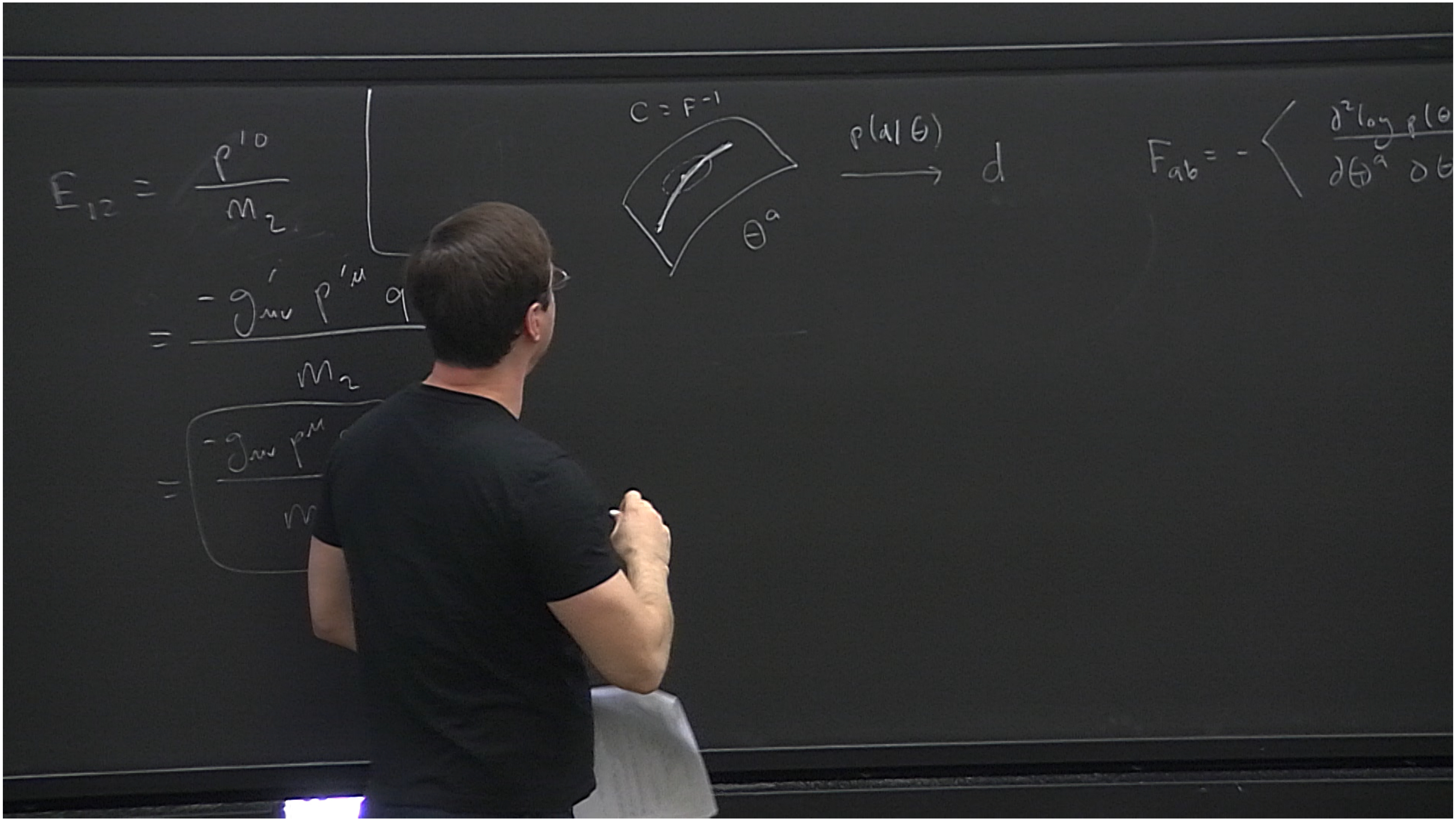
$$g'_{\mu\nu} = \begin{pmatrix} -1 & \\ & \delta_{ij} \end{pmatrix}$$

$$q'^{\mu} = (m_2, 0, 0, 0)$$

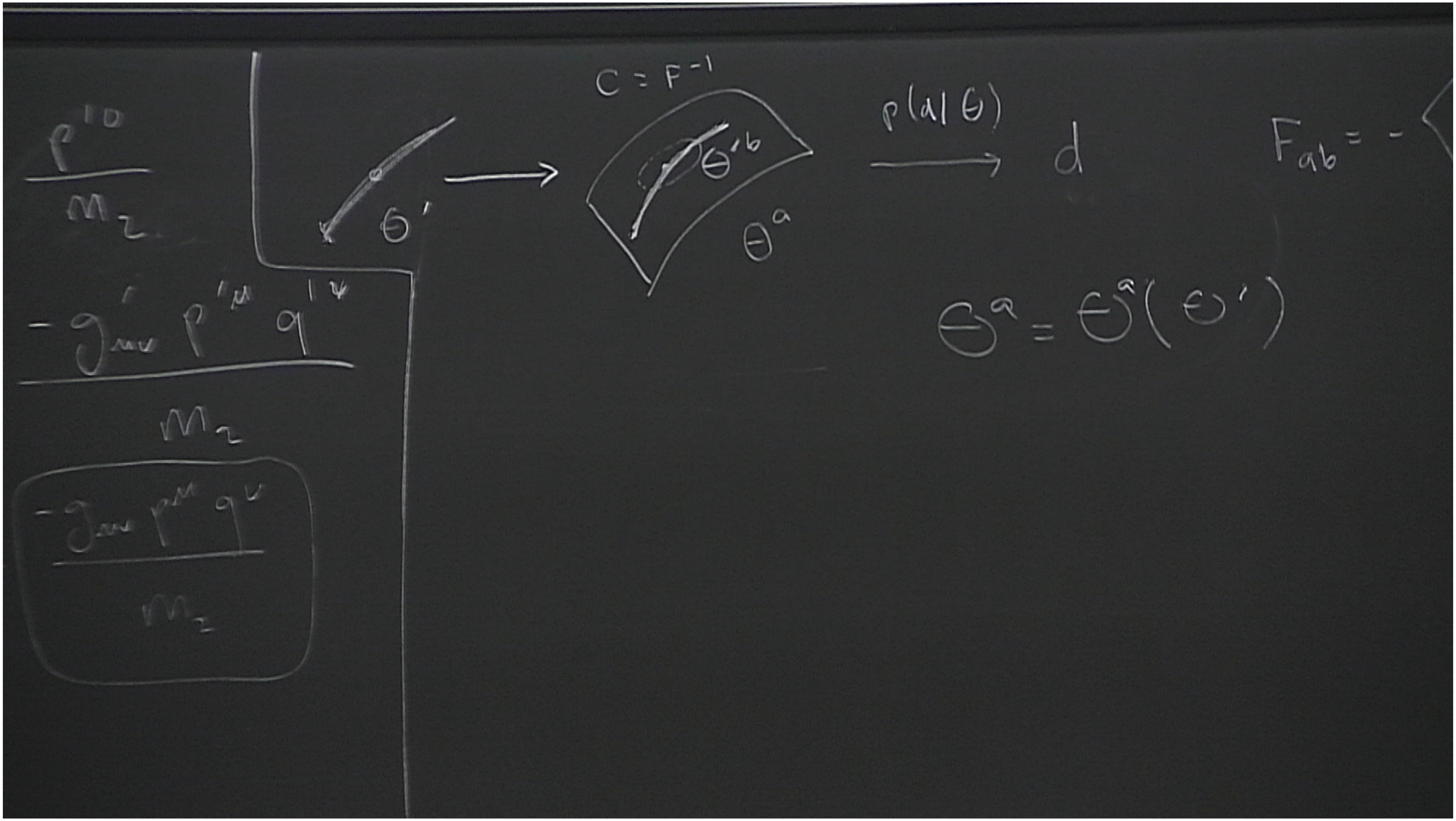
$$p'^{\mu} = (E_{12}, \dots)$$

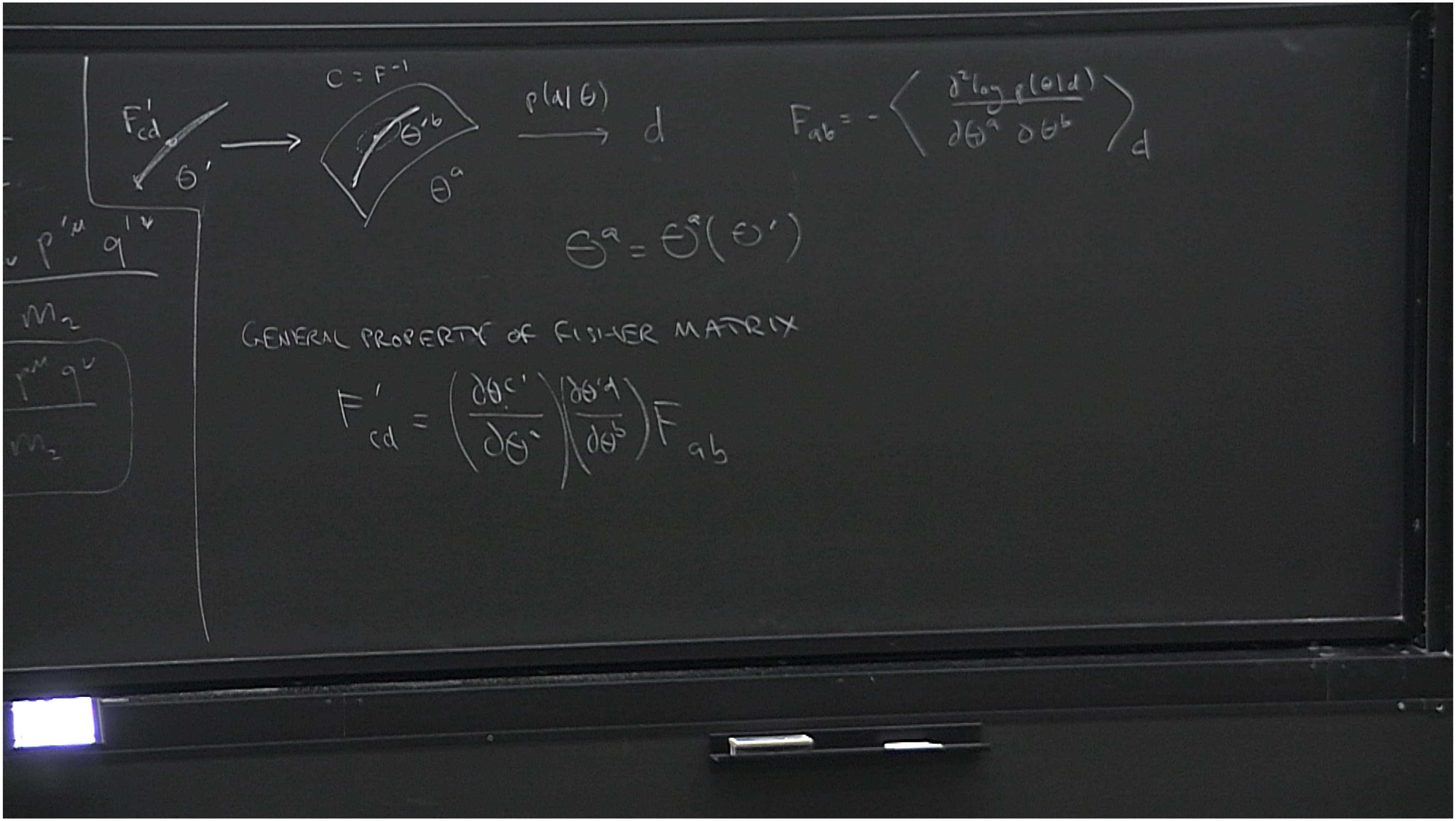

$E_{12} = \frac{p'^0}{m_2}$
$= \frac{-g'_{\mu\nu} p'^{\mu} q'^{\nu}}{m_2}$
$= \boxed{\frac{-g_{\mu\nu} p^{\mu} q^{\nu}}{m_2}}$

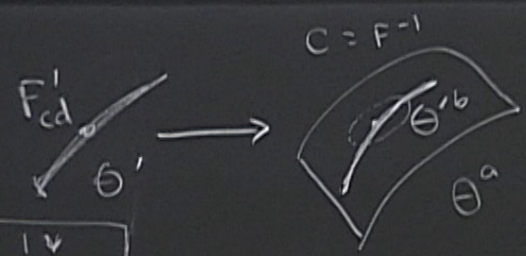

$C = F^{-1}$
$p(d|\theta)$
d
θ^a
$F_{ab} = -\left\langle \frac{\partial^2 \log p(\theta|d)}{\partial\theta^a \partial\theta^b} \right\rangle_d$



C = F^-1
p(d|θ)
d
θ^a


$\frac{p'^0}{m_2}$
$\frac{-g'_{\mu\nu} p'^\mu q'^\nu}{m_2}$
$\frac{-g_{\mu\nu} p^\mu q^\nu}{m_2}$
θ'
$C = F^{-1}$
θ'^b
θ^a
$p(d|\theta)$
d
$F_{ab} = -$
$\theta^a = \theta^a(\theta')$


$C = F^{-1}$
F'_{cd}
Θ'
Θ'^b
Θ^a
$p(d|\theta)$
d
$F_{ab} = -\left\langle \frac{\partial^2 \log p(\theta|d)}{\partial\theta^a \partial\theta^b} \right\rangle_d$
$\Theta^a = \Theta^a(\Theta')$
GENERAL PROPERTY OF FISHER MATRIX
$F'_{cd} = \left(\frac{\partial\theta^{c'}}{\partial\theta^a}\right)\left(\frac{\partial\theta^{d'}}{\partial\theta^b}\right) F_{ab}$
m_2
$p^\mu \; q^\nu$
m_2



$$\xrightarrow{p(d|\theta)} d$$

$$F_{ab} = -\left\langle \frac{\partial^2 \log p(\theta|d)}{\partial\theta^a\,\partial\theta^b} \right\rangle_d$$

$$\theta^a = \theta^a(\theta')$$

$p'^\mu \quad q'^\nu$

m_2

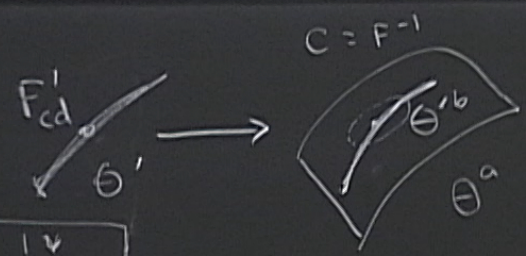
$p^\mu \quad q^\nu$

m_2

GENERAL PROPERTY OF FISHER MATRIX

$$F'_{cd} = \left(\frac{\partial\theta^{c'}}{\partial\theta^a}\right)\left(\frac{\partial\theta'^d}{\partial\theta^b}\right)F_{ab}$$

SAME TRANSFORMATION LAW AS METRIC IN GR



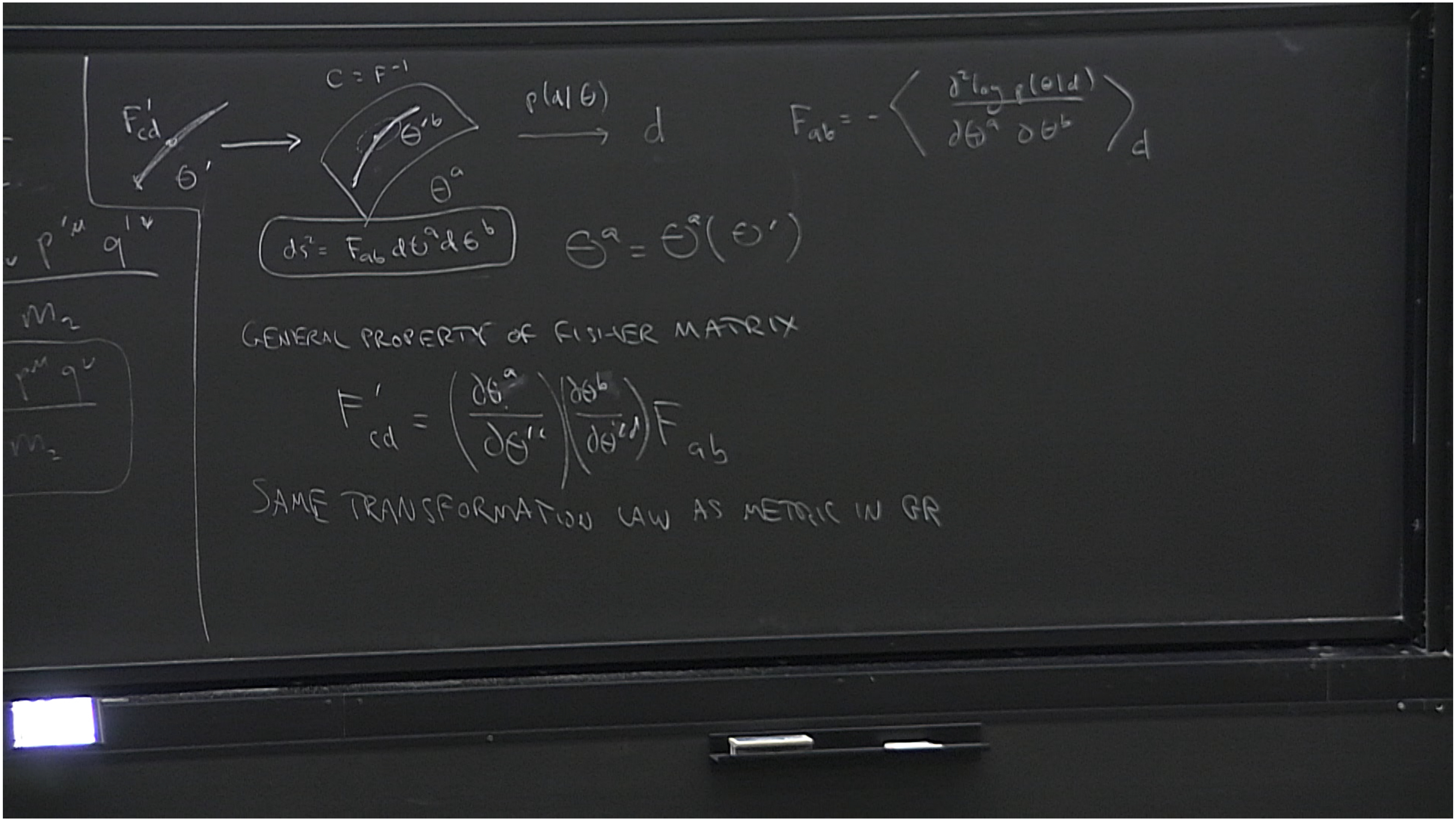
$$\xrightarrow{p(d|\theta)} d \qquad F_{ab} = -\left\langle \frac{\partial^2 \log p(\theta|d)}{\partial\theta^a\,\partial\theta^b} \right\rangle_d$$

$$\theta^a = \theta^a(\theta')$$

GENERAL PROPERTY OF FISHER MATRIX

$$F'_{cd} = \left(\frac{\partial\theta^a}{\partial\theta'^c}\right)\left(\frac{\partial\theta^b}{\partial\theta'^d}\right)F_{ab}$$

SAME TRANSFORMATION LAW AS METRIC IN GR


$C = F^{-1}$
F'_{cd}
θ'
θ'^b
θ^a
$p(d|\theta)$
d
$F_{ab} = -\left\langle \frac{\partial^2 \log p(\theta|d)}{\partial\theta^a \partial\theta^b} \right\rangle_d$
$ds^2 = F_{ab}\, d\theta^a d\theta^b$
$\theta^a = \theta^a(\theta')$
GENERAL PROPERTY OF FISHER MATRIX
$F'_{cd} = \left(\frac{\partial\theta^a}{\partial\theta'^c}\right)\left(\frac{\partial\theta^b}{\partial\theta'^d}\right) F_{ab}$
SAME TRANSFORMATION LAW AS METRIC IN GR

$$\boxed{ds^2 = F_{ab}\, d\theta^a d\theta^b}$$

$$\Theta^a = \Theta^a(\Theta')$$

GENERAL PROPERTY OF FISHER MATRIX

$$F'_{cd} = \left(\frac{\partial\theta^a}{\partial\theta'^c}\right)\left(\frac{\partial\theta^b}{\partial\theta'^d}\right) F_{ab}$$

SAME TRANSFORMATION LAW AS METRIC IN GR

Θ^a

$$\boxed{ds^2 = F_{ab}(\theta)\, d\theta^a d\theta^b}$$

$$\Theta^a = \Theta^a(\Theta')$$

GENERAL PROPERTY OF FISHER MATRIX

$$F'_{cd} = \left(\frac{\partial\theta^a}{\partial\theta'^c}\right)\left(\frac{\partial\theta^b}{\partial\theta'^d}\right) F_{ab}$$

SAME TRANSFORMATION LAW AS METRIC IN GR

$$\xrightarrow{p(d|\theta)} d \qquad F_{ab} = -\left\langle \frac{\partial^2 l_0}{\partial\theta \ldots} \right.$$

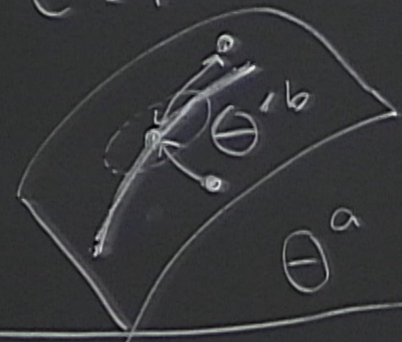
$$\boxed{ds^2 = F_{ab}\, d\theta^a d\theta^b} \qquad \theta^a = \theta^a(\theta')$$

GENERAL PROPERTY OF FISHER MATRIX

$$F'_{cd} = \left(\frac{\partial\theta^a}{\partial\theta'^c}\right)\left(\frac{\partial\theta^b}{\partial\theta'^d}\right) F_{ab}$$

SAME TRANSFORMATION LAW AS METRIC IN GR

$C = F^{-1}$



$$p(d|\theta) \longrightarrow d$$

$$F_{ab} = -\left\langle \frac{\partial^2 l}{\partial\theta \ldots} \right.$$

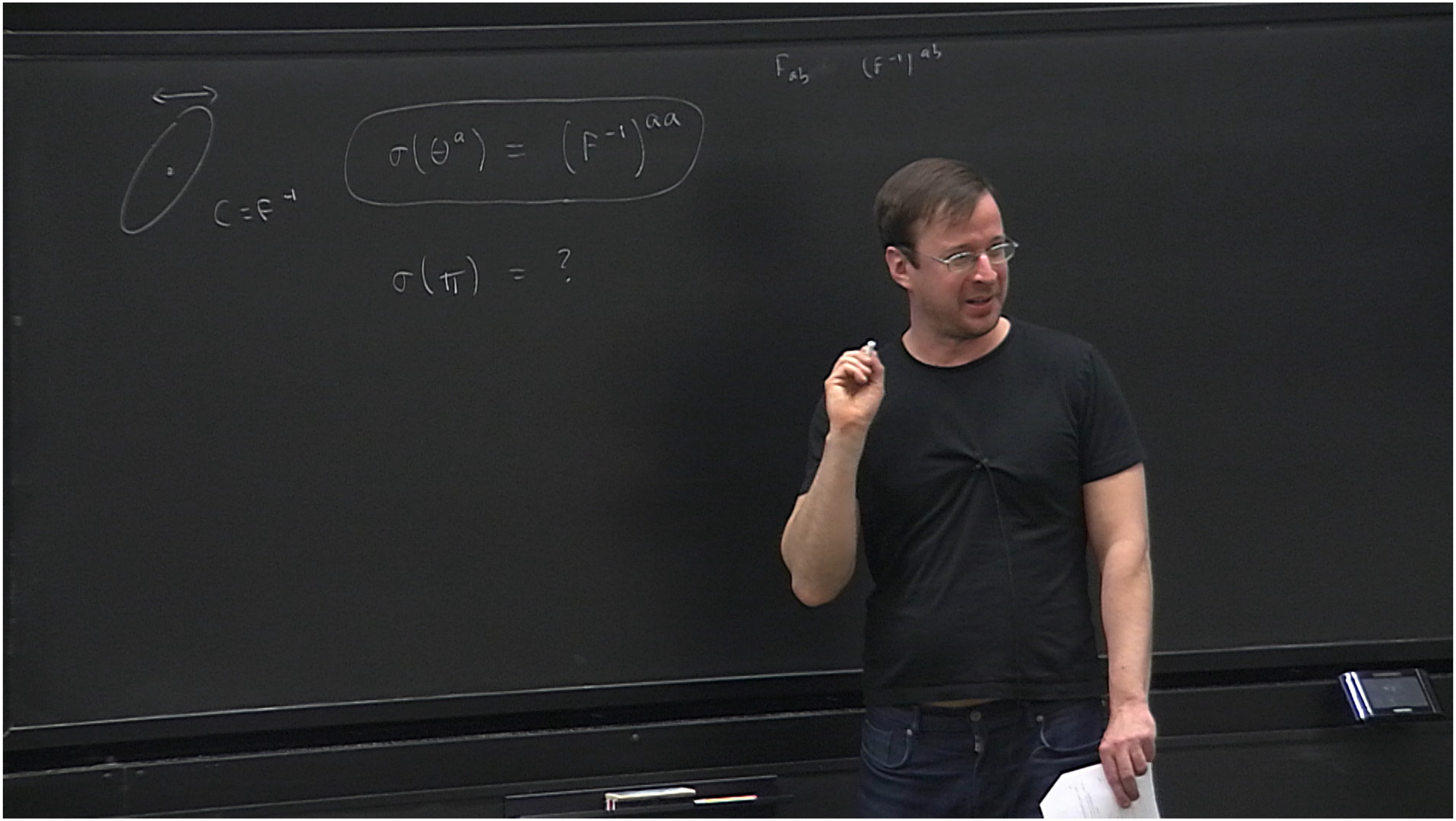
$$\boxed{ds^2 = F_{ab}\, d\theta^a d\theta^b}$$

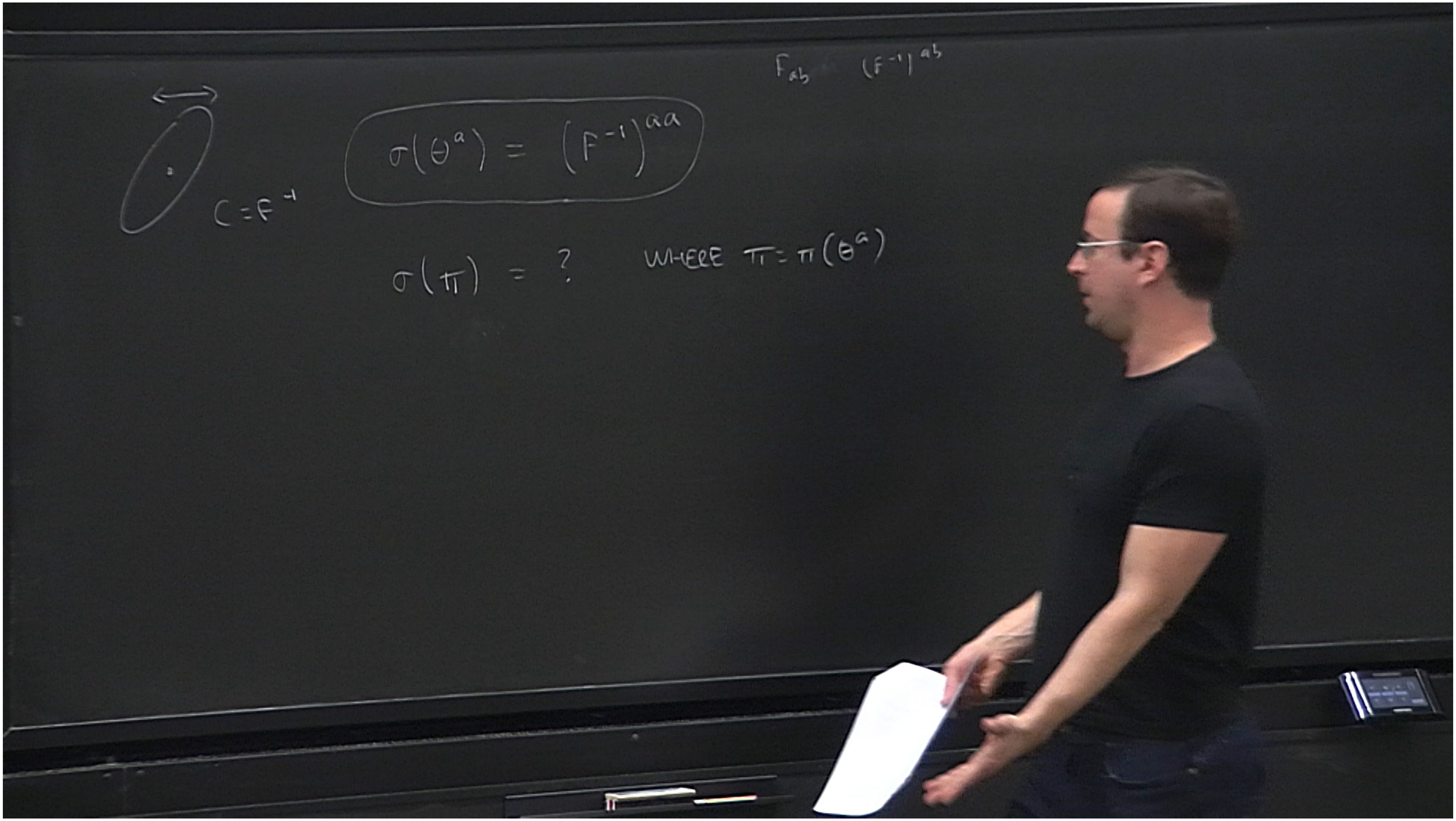
$$\theta^a = \theta^a(\theta')$$

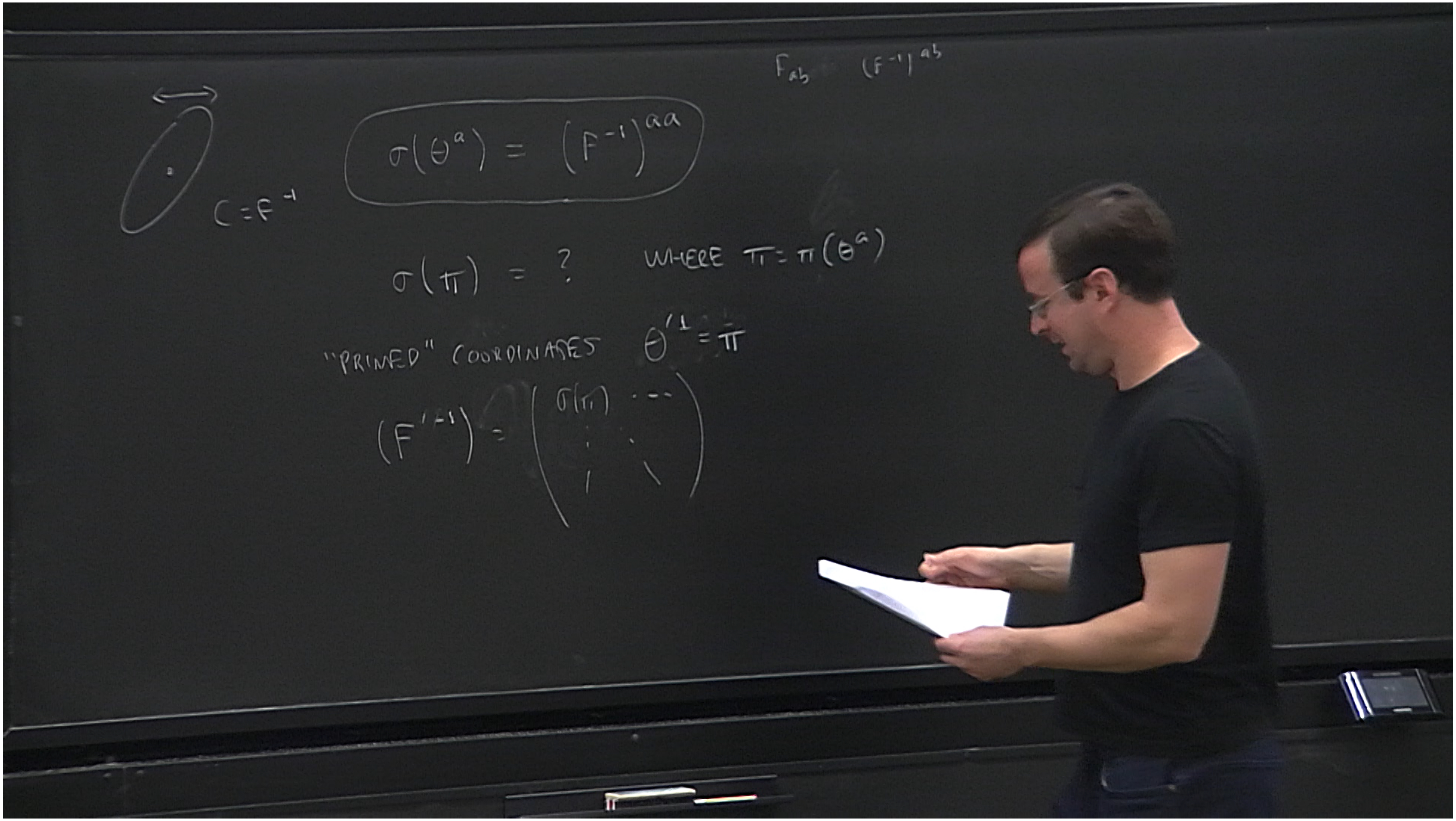
GENERAL PROPERTY OF FISHER MATRIX

$$F'_{cd} = \left(\frac{\partial\theta^a}{\partial\theta'^c}\right)\left(\frac{\partial\theta^b}{\partial\theta'^d}\right) F_{ab}$$

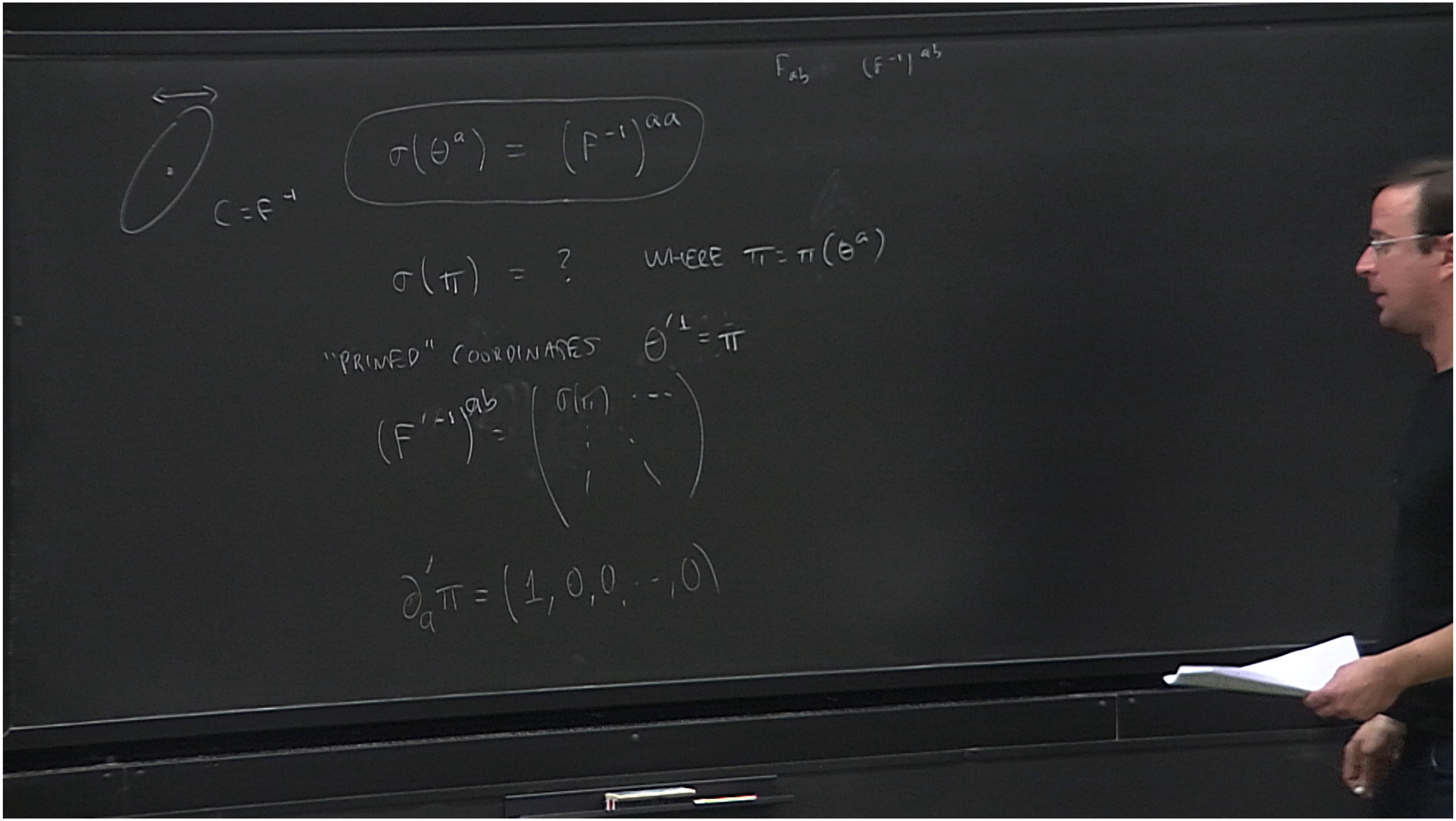
SAME TRANSFORMATION LAW AS METRIC IN GR


F_{ab} $(F^{-1})^{ab}$
$\sigma(\theta^a) = (F^{-1})^{aa}$
$C = F^{-1}$
$\sigma(\pi) = ?$


F_{ab} $(F^{-1})^{ab}$
$C = F^{-1}$
$\sigma(\theta^a) = (F^{-1})^{aa}$
$\sigma(\pi) = ?$ WHERE $\pi = \pi(\theta^a)$



F_{ab} $(F^{-1})^{ab}$
$\sigma(\theta^a) = (F^{-1})^{aa}$
$C = F^{-1}$
$\sigma(\pi) = ?$ WHERE $\pi = \pi(\theta^a)$
"PRIMED" COORDINATES $\theta'^1 = \pi$
$(F'^{-1}) = \begin{pmatrix} \sigma(\pi) & \cdots \\ \vdots & \ddots \end{pmatrix}$


F_{ab} $(F^{-1})^{ab}$
$C = F^{-1}$
$\sigma(\theta^a) = (F^{-1})^{aa}$
$\sigma(\pi) = ?$ WHERE $\pi = \pi(\theta^a)$
"PRIMED" COORDINATES $\theta'^1 = \pi$
$(F'^{-1})^{ab} = \begin{pmatrix} \sigma(\pi) & \cdots \\ \vdots & \ddots \end{pmatrix}$
$\partial'_a \pi = (1, 0, 0, \ldots, 0)$

F_{ab} $\quad (F^{-1})^{ab}$

$\sigma(\pi)$

$C = F^{-1}$

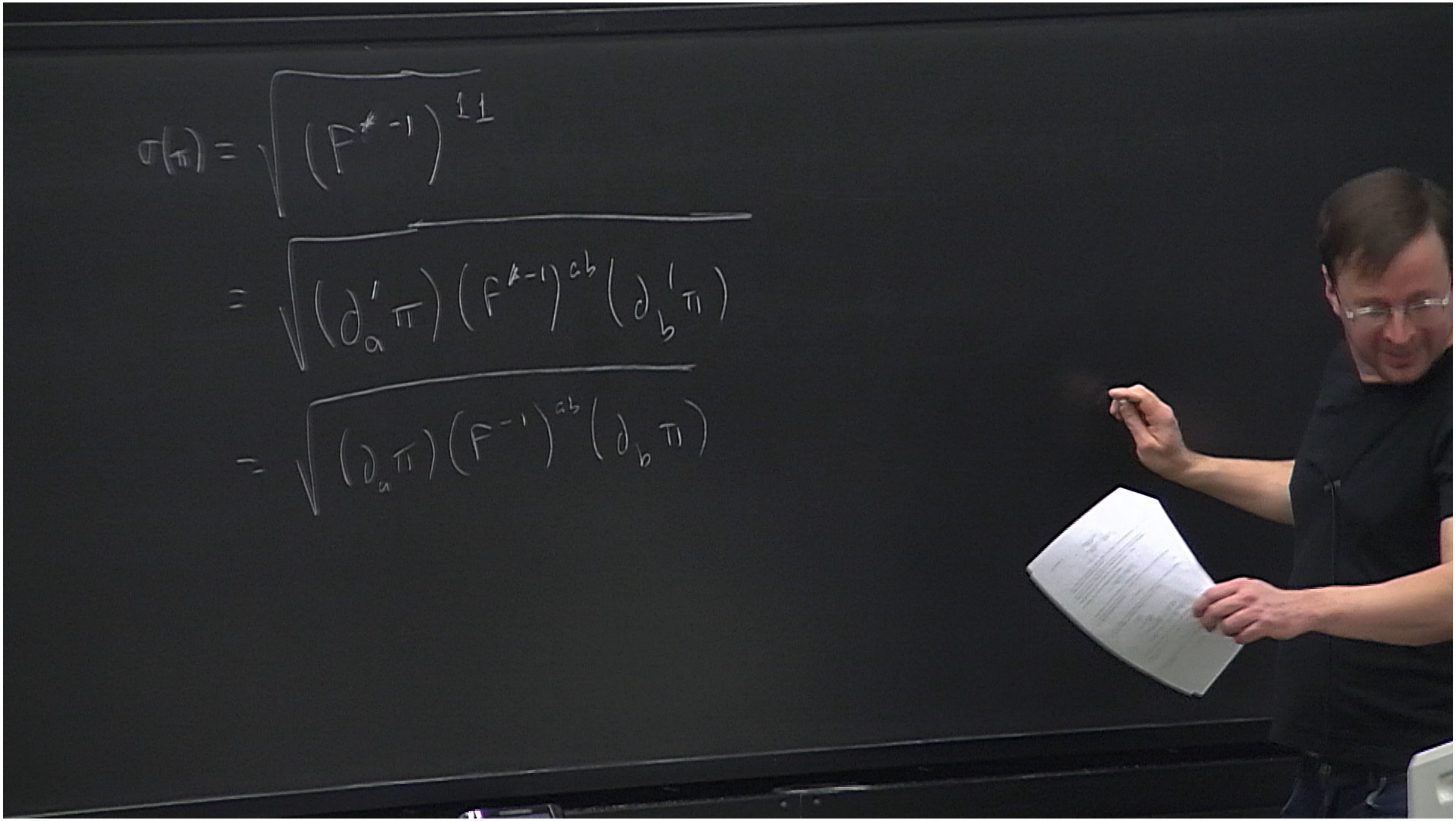
$$\boxed{\sigma(\theta^a) = \sqrt{(F^{-1})^{aa}}}$$

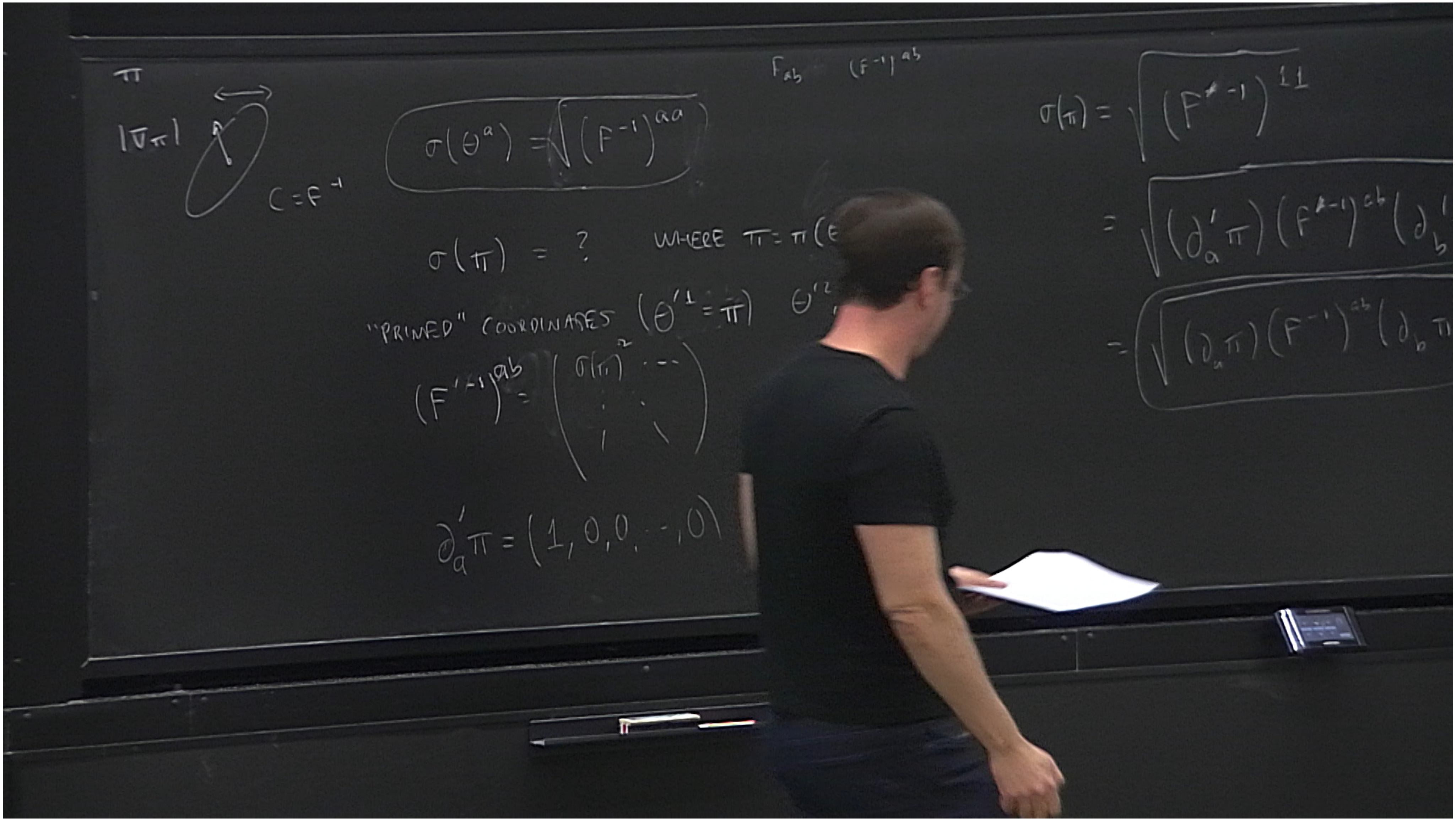
$\sigma(\pi) = ?$ WHERE $\pi = \pi(\theta^a)$

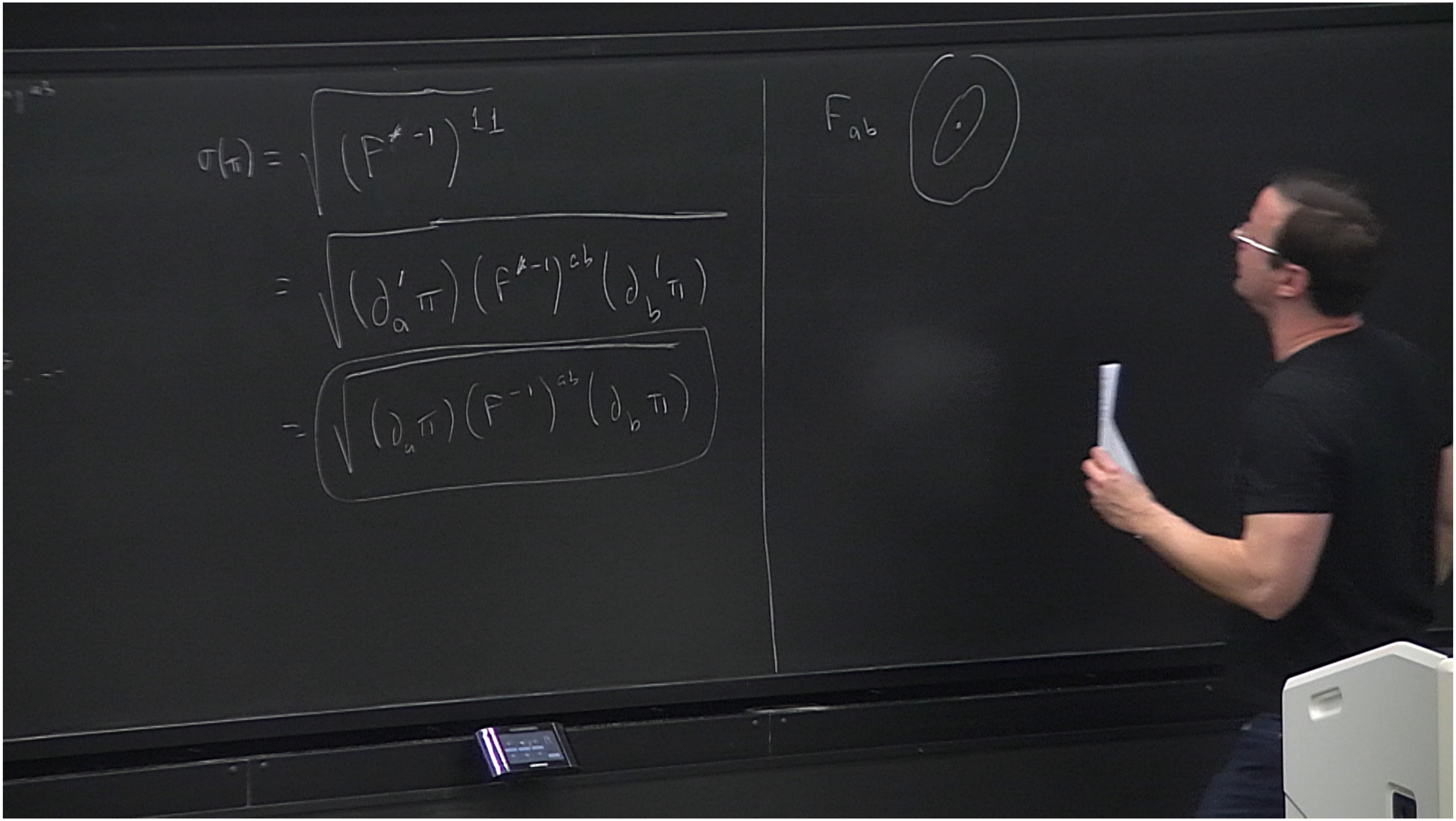
"PRIMED" COORDINATES $(\theta'^1 = \pi)$ $\quad \theta'^2, \theta'^3, \ldots$

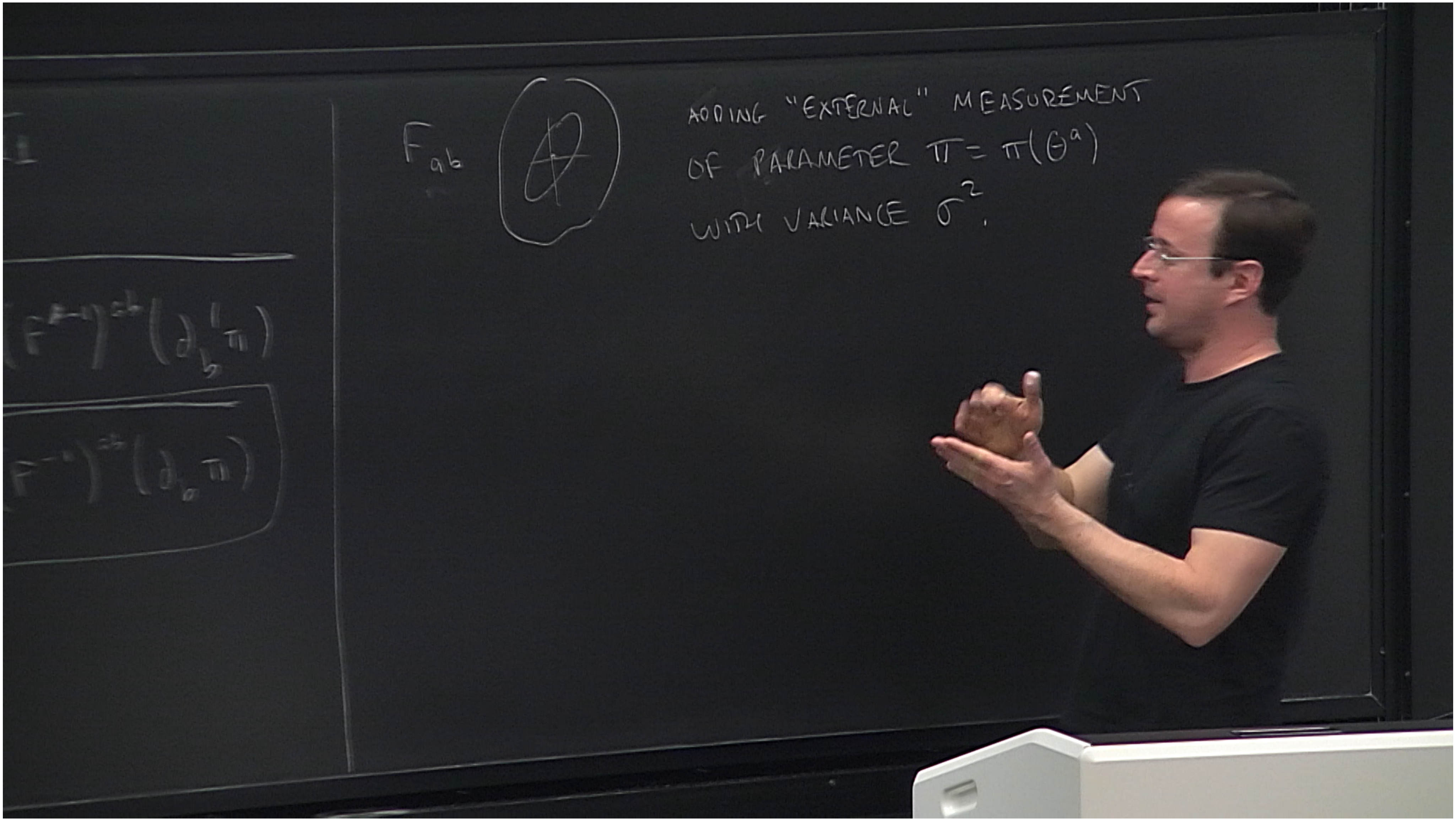
$$(F'^{-1})^{ab} = \begin{pmatrix} \sigma(\pi)^2 & \cdots \\ \vdots & \ddots \end{pmatrix}$$

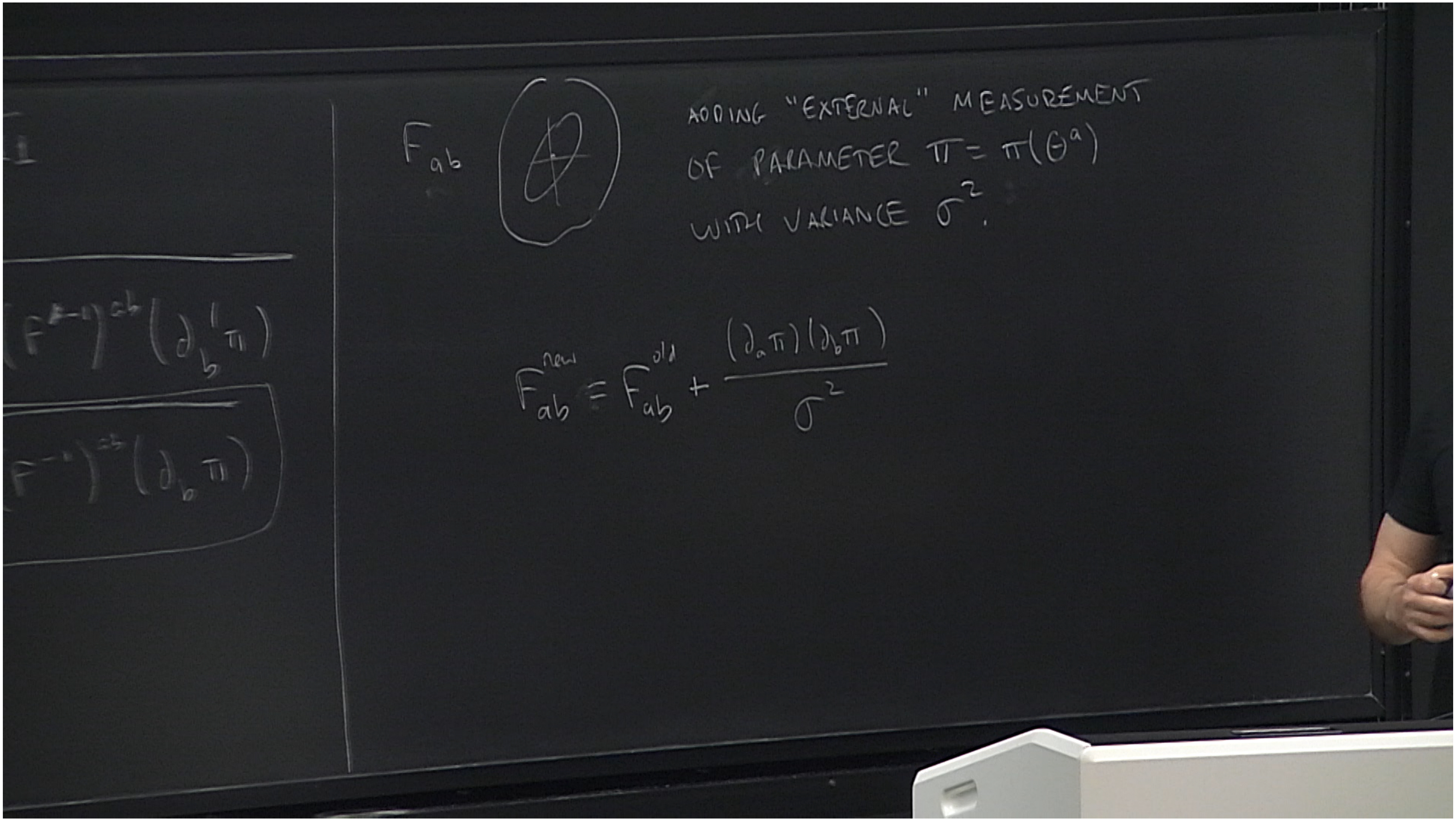
$$\partial'_a \pi = (1, 0, 0, \ldots, 0)$$

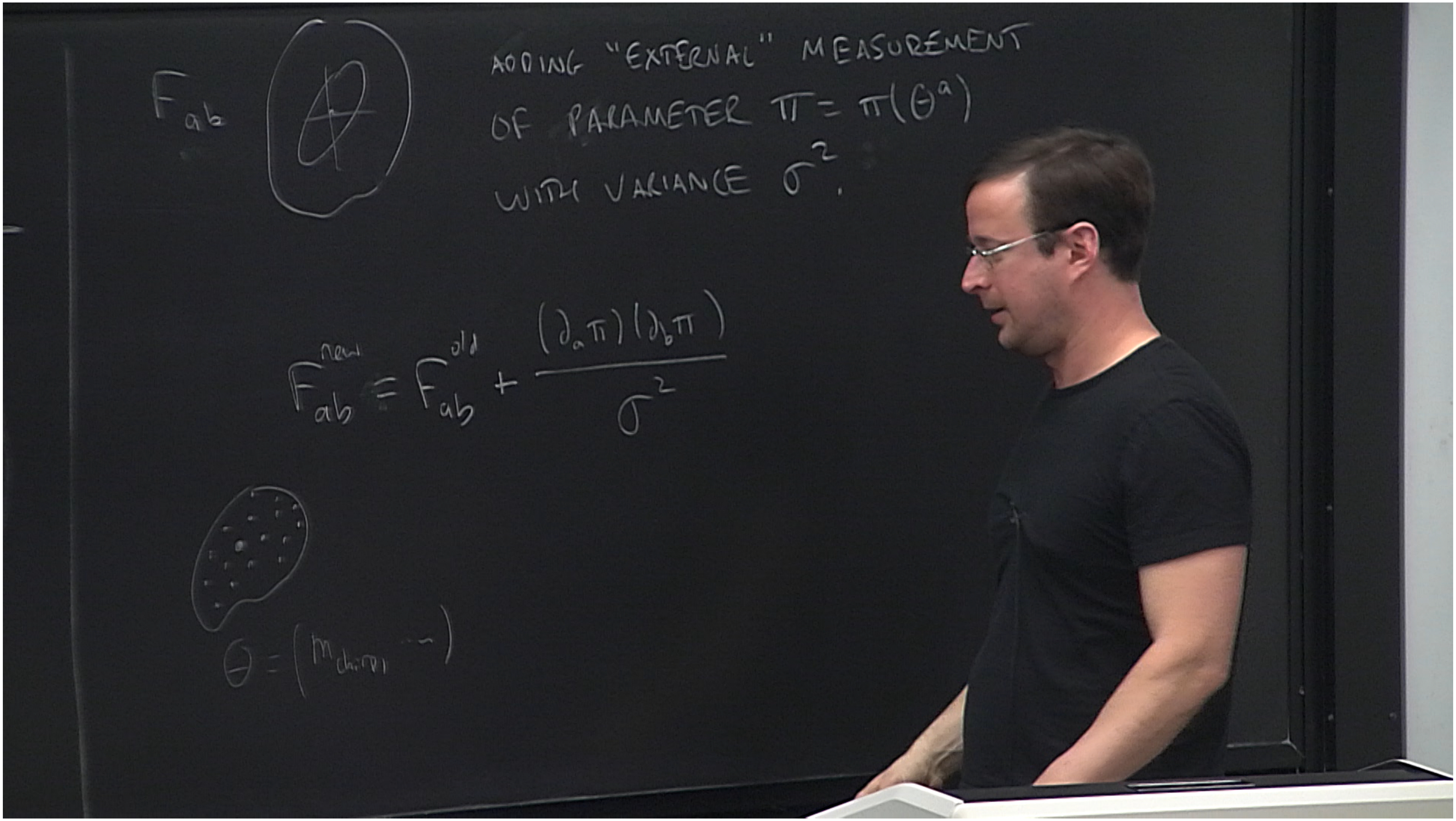

$\sigma(\pi) = \sqrt{\left(F^{*-1}\right)^{11}}$
$= \sqrt{(\partial'_a \pi)\left(F^{*-1}\right)^{ab}(\partial'_b \pi)}$
$= \sqrt{(\partial_a \pi)\left(F^{-1}\right)^{ab}(\partial_b \pi)}$

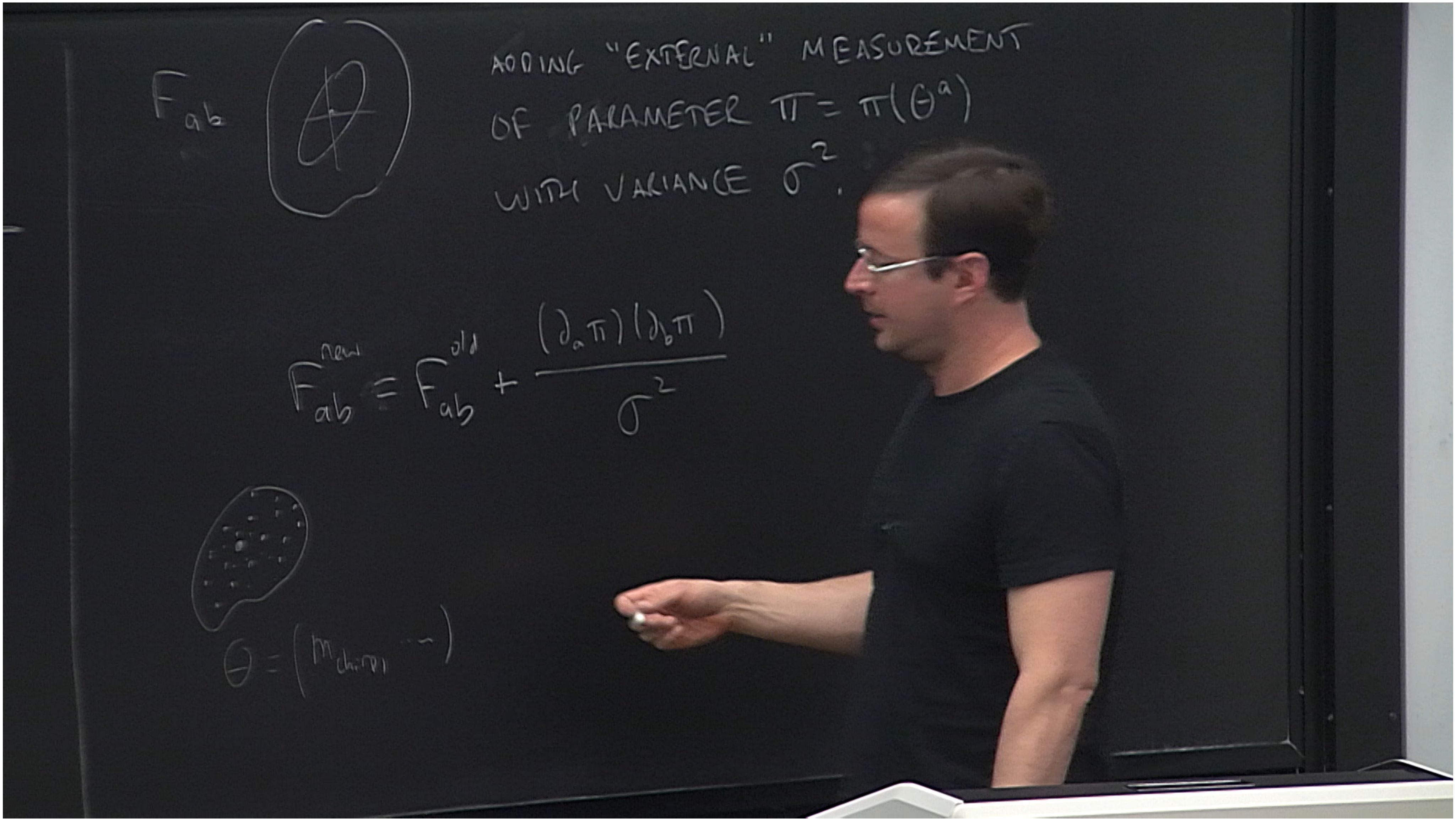

σ(θ^a) = √(F^-1)^aa
C = F^-1
σ(π) = ?   WHERE π = π(θ
"PRIMED" COORDINATES (θ'1 = π)
∂'a π = (1, 0, 0, ..., 0)
σ(π) = √(F'^-1)^11
= √(∂'a π)(F'^-1)^ab(∂'b
= √(∂a π)(F^-1)^ab(∂b π

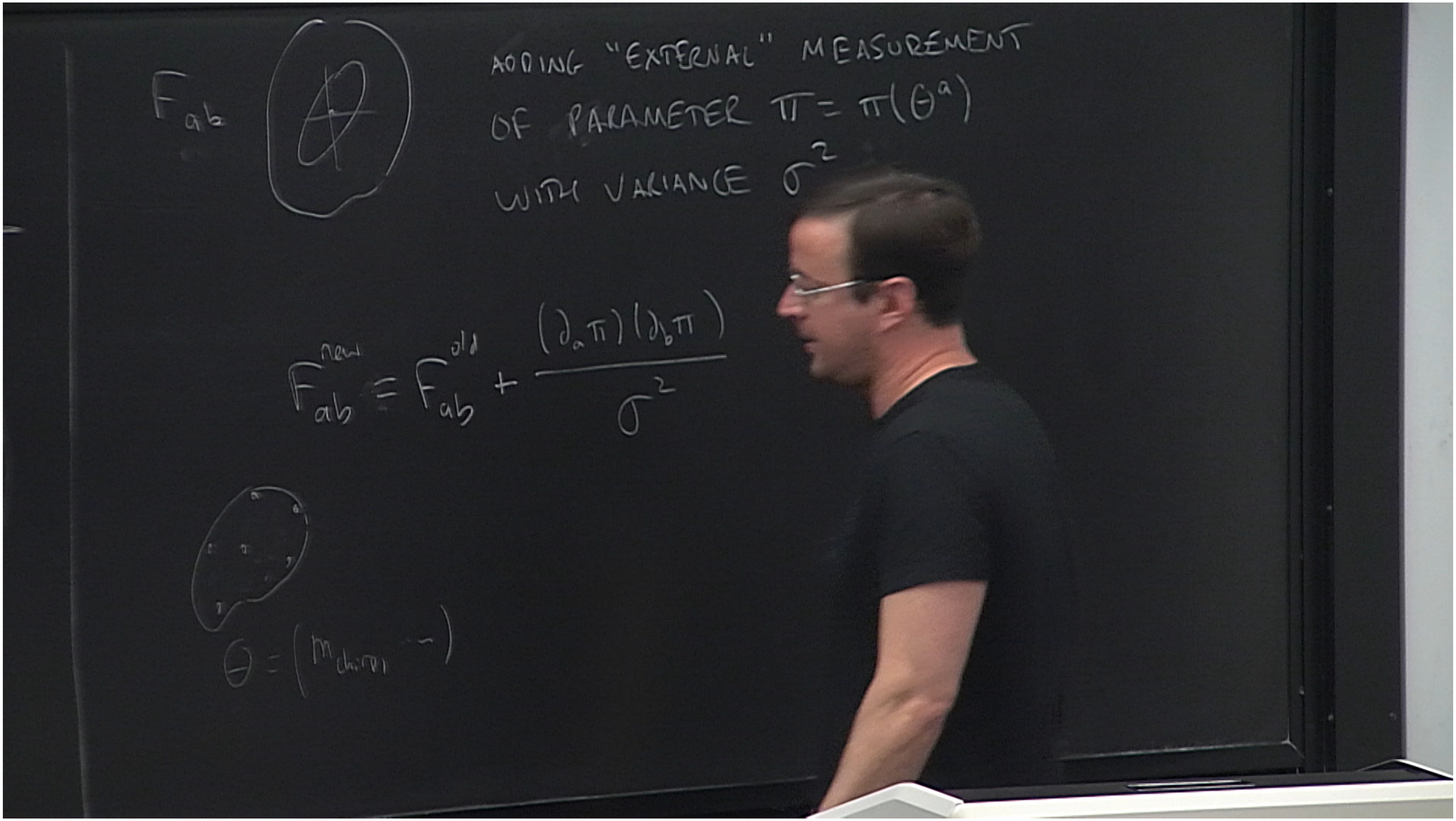

$\sigma(\pi) = \sqrt{(F^{*-1})^{11}}$
$= \sqrt{(\partial'_a \pi)(F^{*-1})^{ab}(\partial'_b \pi)}$
$= \sqrt{(\partial_a \pi)(F^{-1})^{ab}(\partial_b \pi)}$
F_{ab}


F_{ab}
ADDING "EXTERNAL" MEASUREMENT
OF PARAMETER $\pi = \pi(\theta^a)$
WITH VARIANCE σ^2.

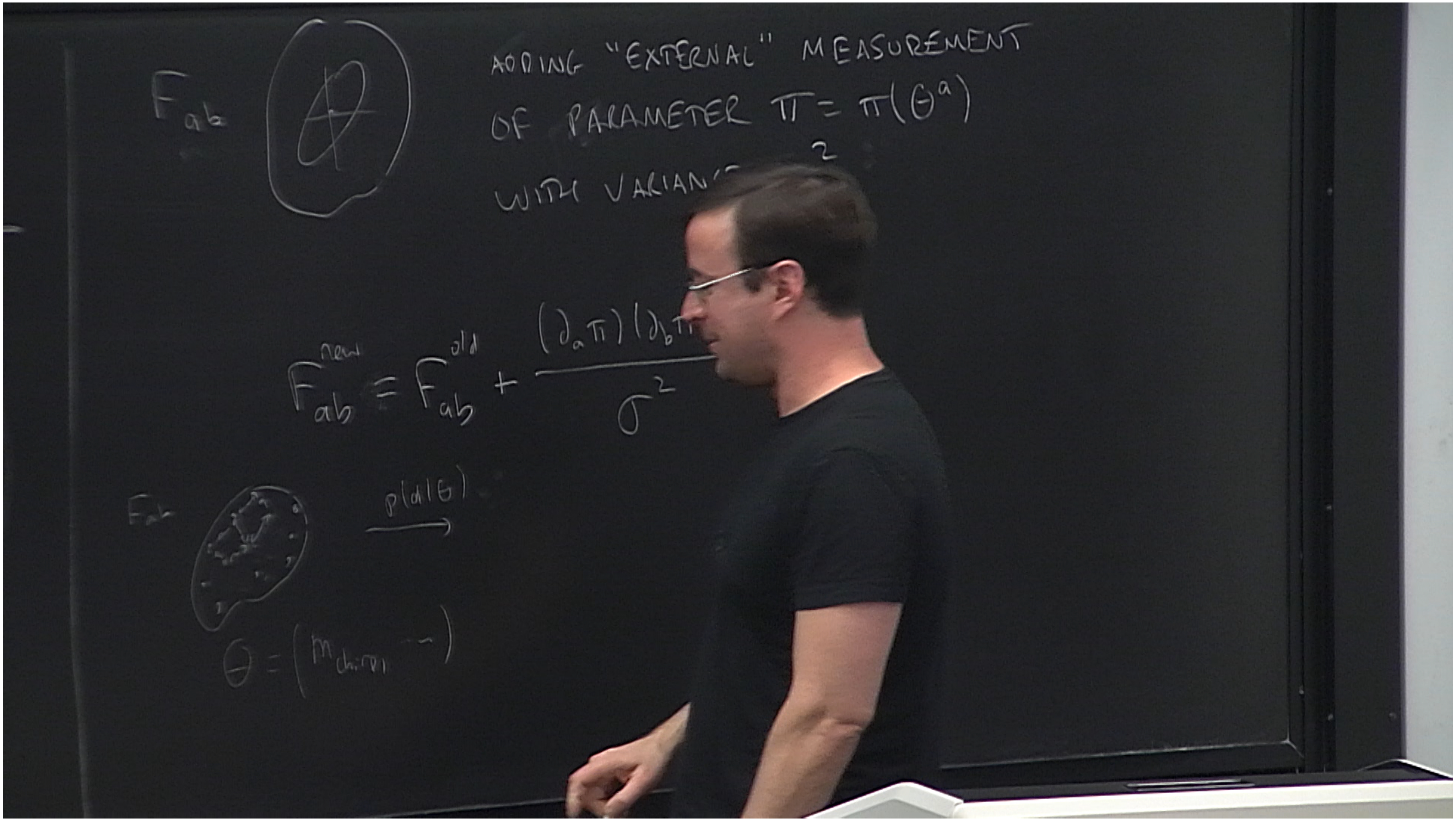

F_{ab}
ADDING "EXTERNAL" MEASUREMENT
OF PARAMETER $\pi = \pi(\theta^a)$
WITH VARIANCE σ^2.
$F_{ab}^{new} = F_{ab}^{old} + \frac{(\partial_a \pi)(\partial_b \pi)}{\sigma^2}$


F_{ab}
ADDING "EXTERNAL" MEASUREMENT
OF PARAMETER $\pi = \pi(\theta^a)$
WITH VARIANCE σ^2.
$F_{ab}^{new} = F_{ab}^{old} + \frac{(\partial_a \pi)(\partial_b \pi)}{\sigma^2}$
$\theta = (m_{chirp} \dots)$

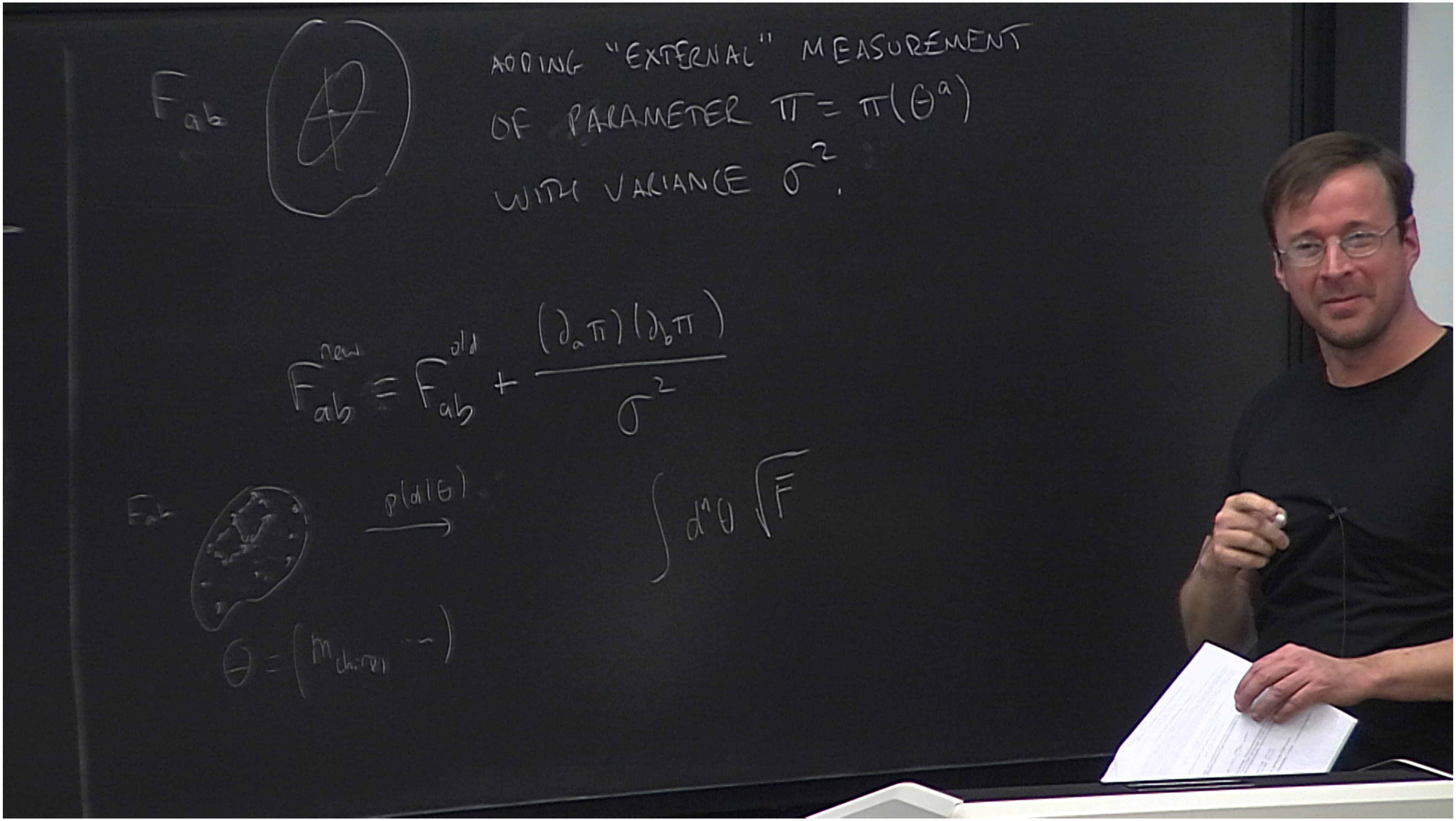

ADDING "EXTERNAL" MEASUREMENT
OF PARAMETER $\pi = \pi(\theta^a)$
WITH VARIANCE σ^2.
F_{ab}
$F_{ab}^{new} = F_{ab}^{old} + \frac{(\partial_a \pi)(\partial_b \pi)}{\sigma^2}$
$\theta = (m_{chirp} \ldots)$

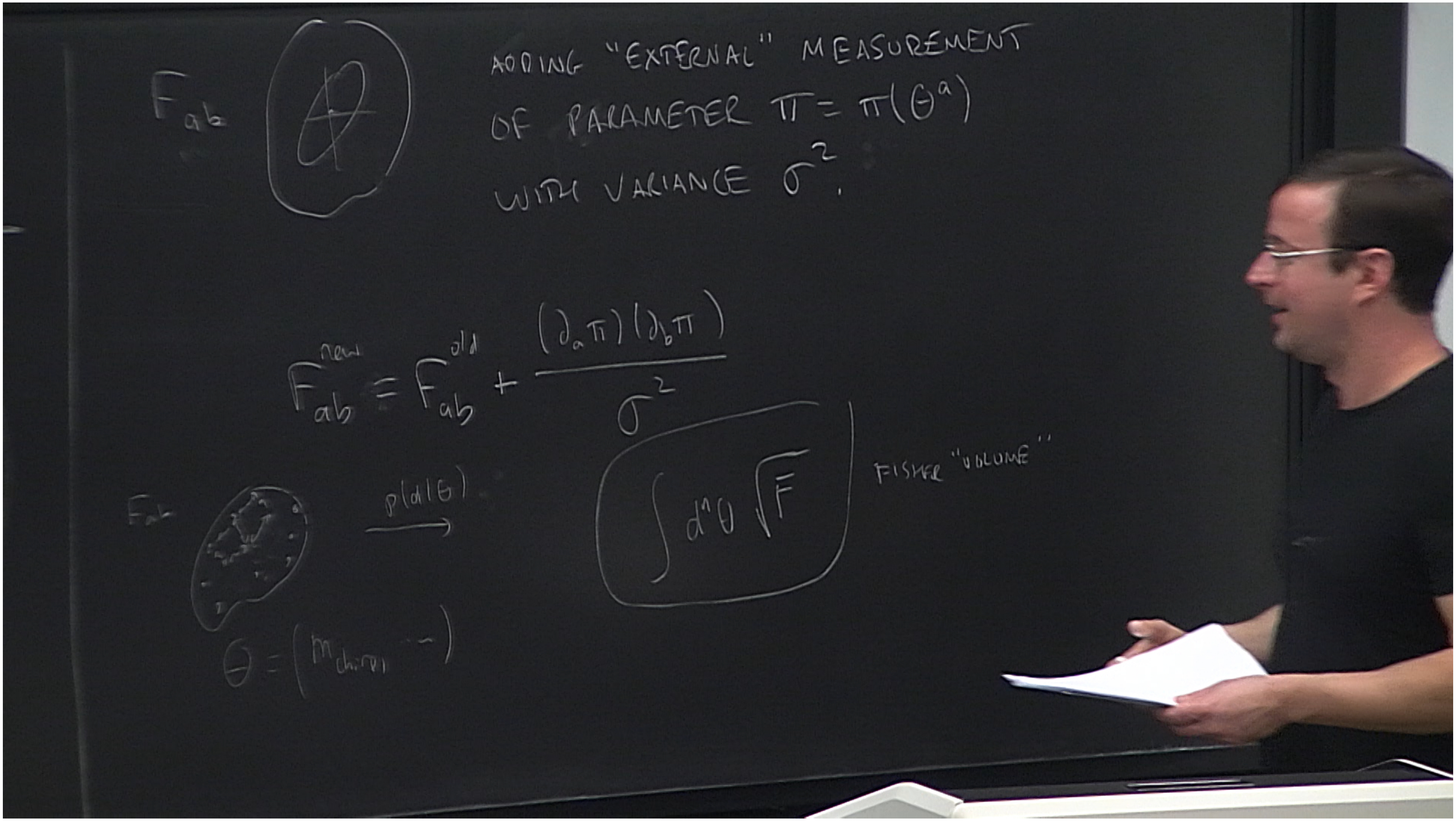

ADDING "EXTERNAL" MEASUREMENT
OF PARAMETER $\pi = \pi(\theta^a)$
WITH VARIANCE σ^2
F_{ab}
$F_{ab}^{new} = F_{ab}^{old} + \frac{(\partial_a \pi)(\partial_b \pi)}{\sigma^2}$
$\theta = (m_{chirp} \ldots)$

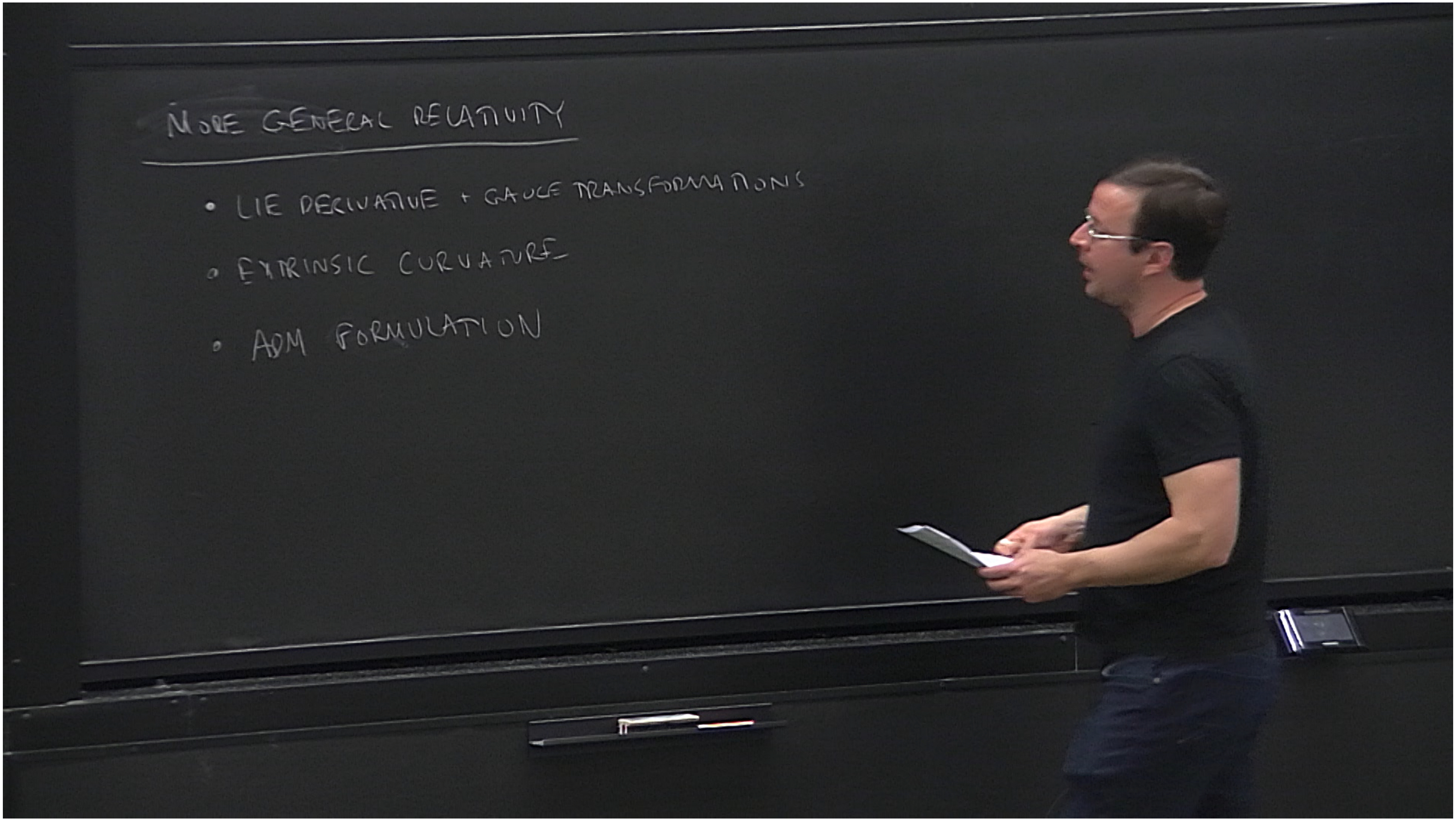

ADDING "EXTERNAL" MEASUREMENT
OF PARAMETER $\pi = \pi(\theta^a)$
WITH VARIANCE σ^2.
F_{ab}
$F_{ab}^{new} = F_{ab}^{old} + \frac{(\partial_a \pi)(\partial_b \pi)}{\sigma^2}$
$\Theta = (m_{chirp} \dots)$

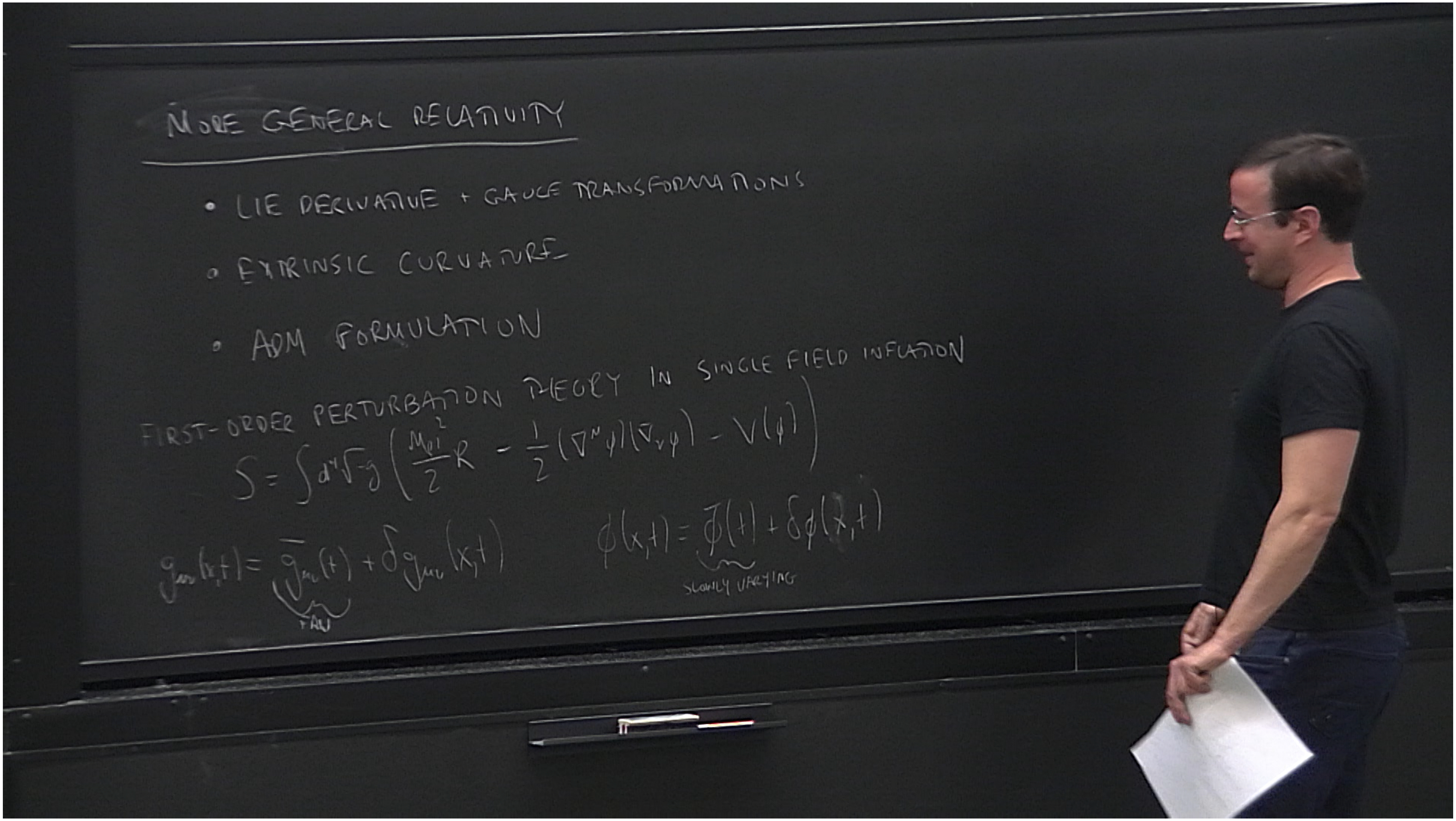


ADDING "EXTERNAL" MEASUREMENT
OF PARAMETER π = π(Θ^a)
F_ab
p(d|Θ)
Θ = (m_chirp ...)


F_ab
ADDING "EXTERNAL" MEASUREMENT
OF PARAMETER π = π(θ^a)
WITH VARIANCE σ^2.
F_ab^new = F_ab^old + (∂_a π)(∂_b π) / σ^2
p(d|θ)
∫ d^n θ √F
θ = (m_chirp ...)


ADDING "EXTERNAL" MEASUREMENT
OF PARAMETER $\pi = \pi(\theta^a)$
WITH VARIANCE σ^2.
F_{ab}
$F^{new}_{ab} = F^{old}_{ab} + \frac{(\partial_a \pi)(\partial_b \pi)}{\sigma^2}$
$\int d^n\theta \sqrt{F}$
FISHER "VOLUME"
$p(d|\theta)$
$\theta = (m_{chirp}, \ldots)$


MORE GENERAL RELATIVITY
• LIE DERIVATIVE + GAUGE TRANSFORMATIONS
• EXTRINSIC CURVATURE
• ADM FORMULATION

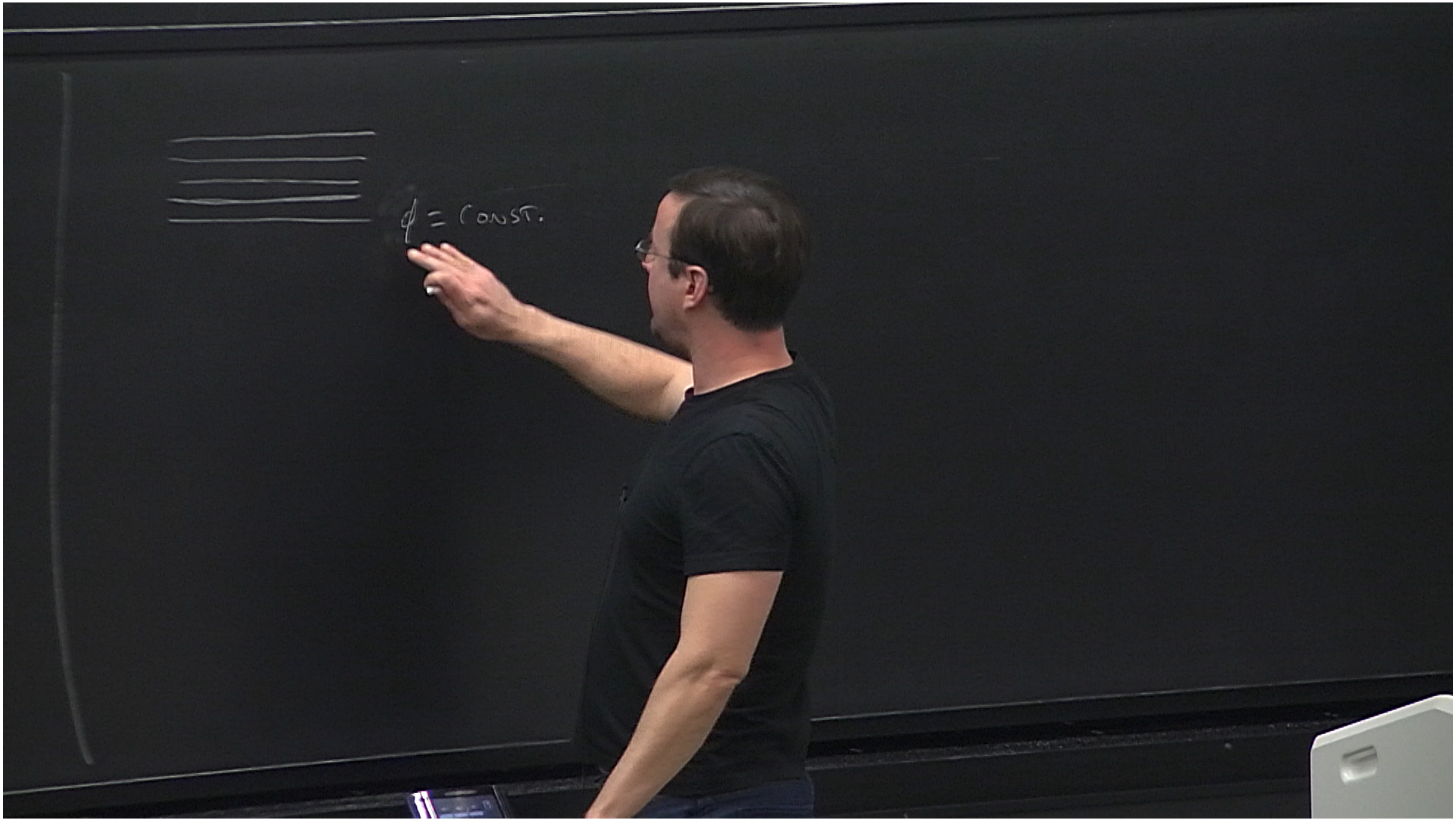

MORE GENERAL RELATIVITY
• LIE DERIVATIVE + GAUGE TRANSFORMATIONS
• EXTRINSIC CURVATURE
• ADM FORMULATION
FIRST-ORDER PERTURBATION THEORY IN SINGLE FIELD INFLATION
$S = \int d^4x \sqrt{-g} \left(\frac{M_{pl}^2}{2} R - \frac{1}{2} (\nabla^\mu \phi)(\nabla_\mu \phi) - V(\phi) \right)$
$g_{\mu\nu}(x,t) = \underbrace{\bar{g}_{\mu\nu}(t)}_{FRW} + \delta g_{\mu\nu}(x,t)$
$\phi(x,t) = \underbrace{\bar{\phi}(t)}_{\text{SLOWLY VARYING}} + \delta\phi(x,t)$

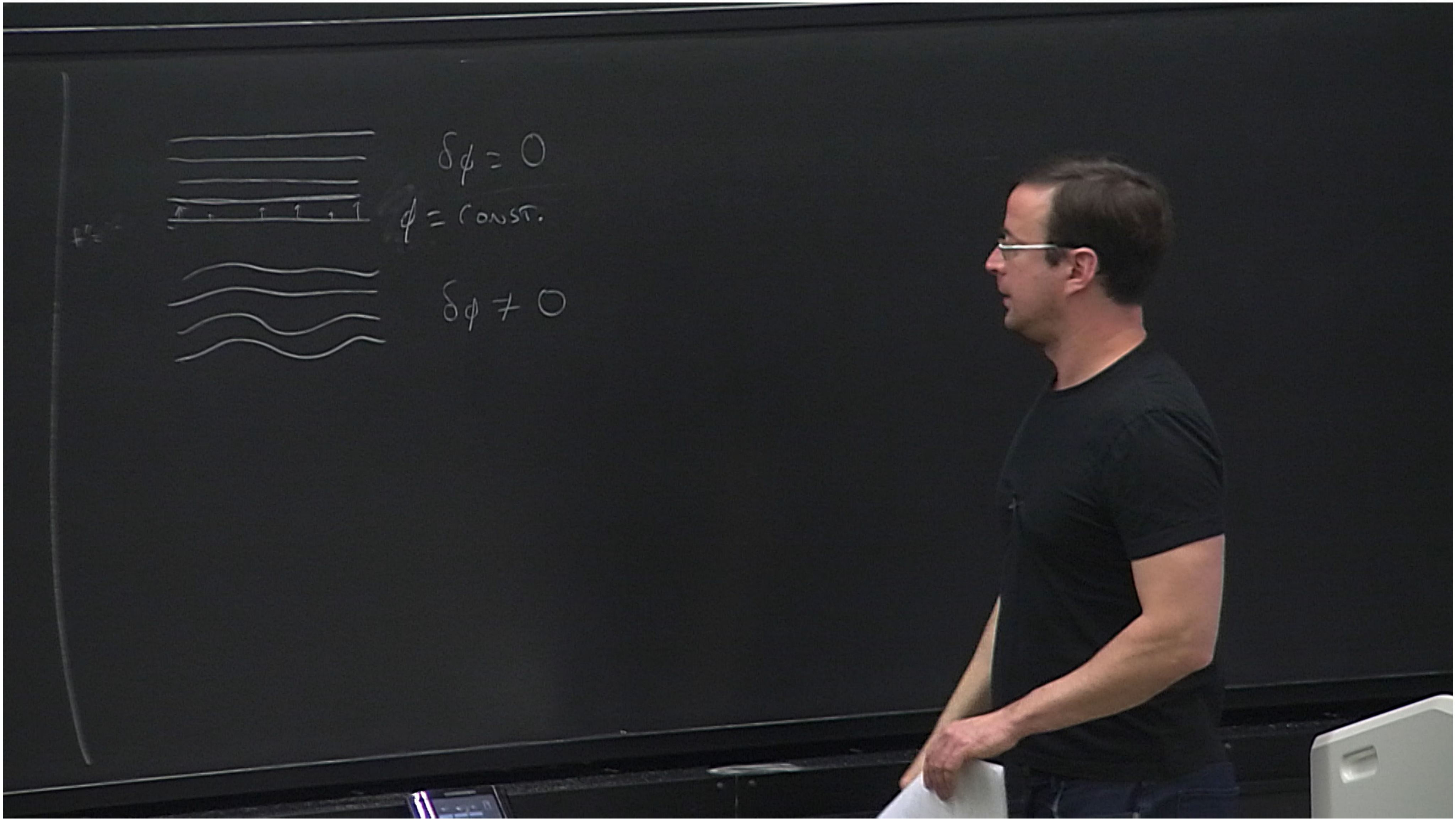
- LIE DERIVATIVE + GAUGE TRANSFORMATIONS
- EXTRINSIC CURVATURE
- ADM FORMULATION

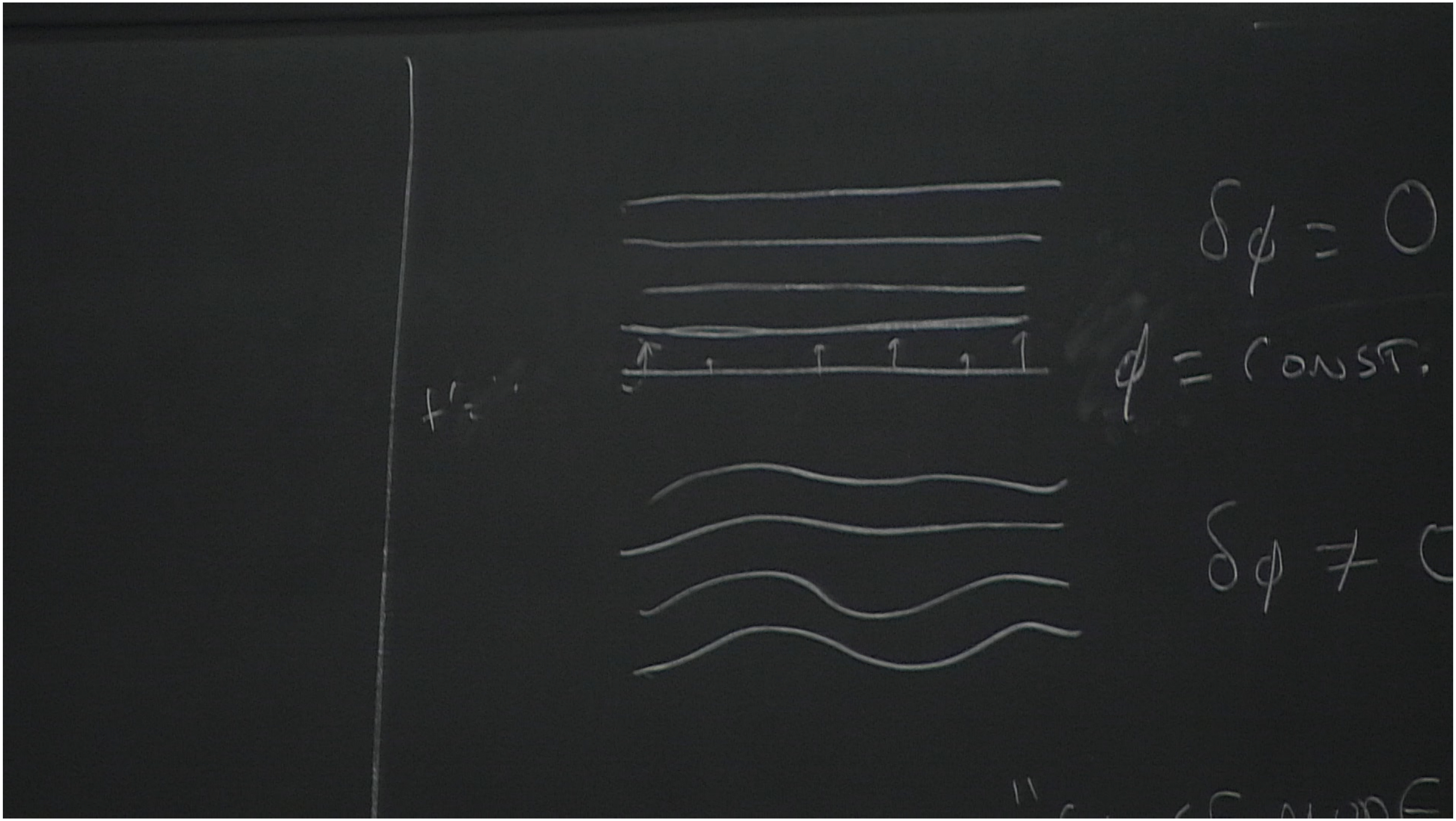
FIRST-ORDER PERTURBATION THEORY IN SINGLE FIELD INFLATION

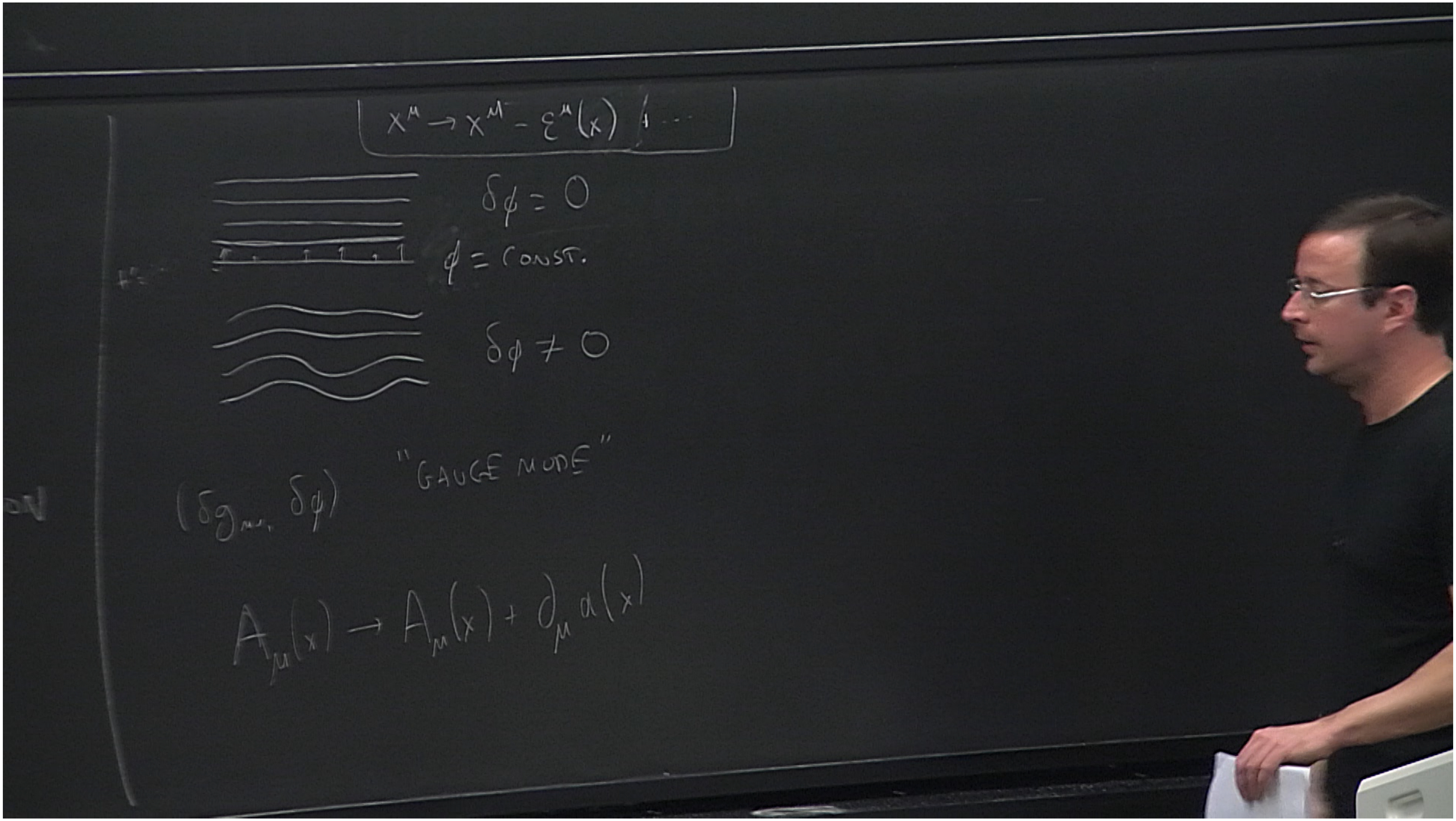
$$S = \int d^4x \sqrt{-g} \left(\frac{M_{pl}^2}{2} R - \frac{1}{2} (\nabla^\mu \phi)(\nabla_\nu \phi) - V(\phi) \right)$$

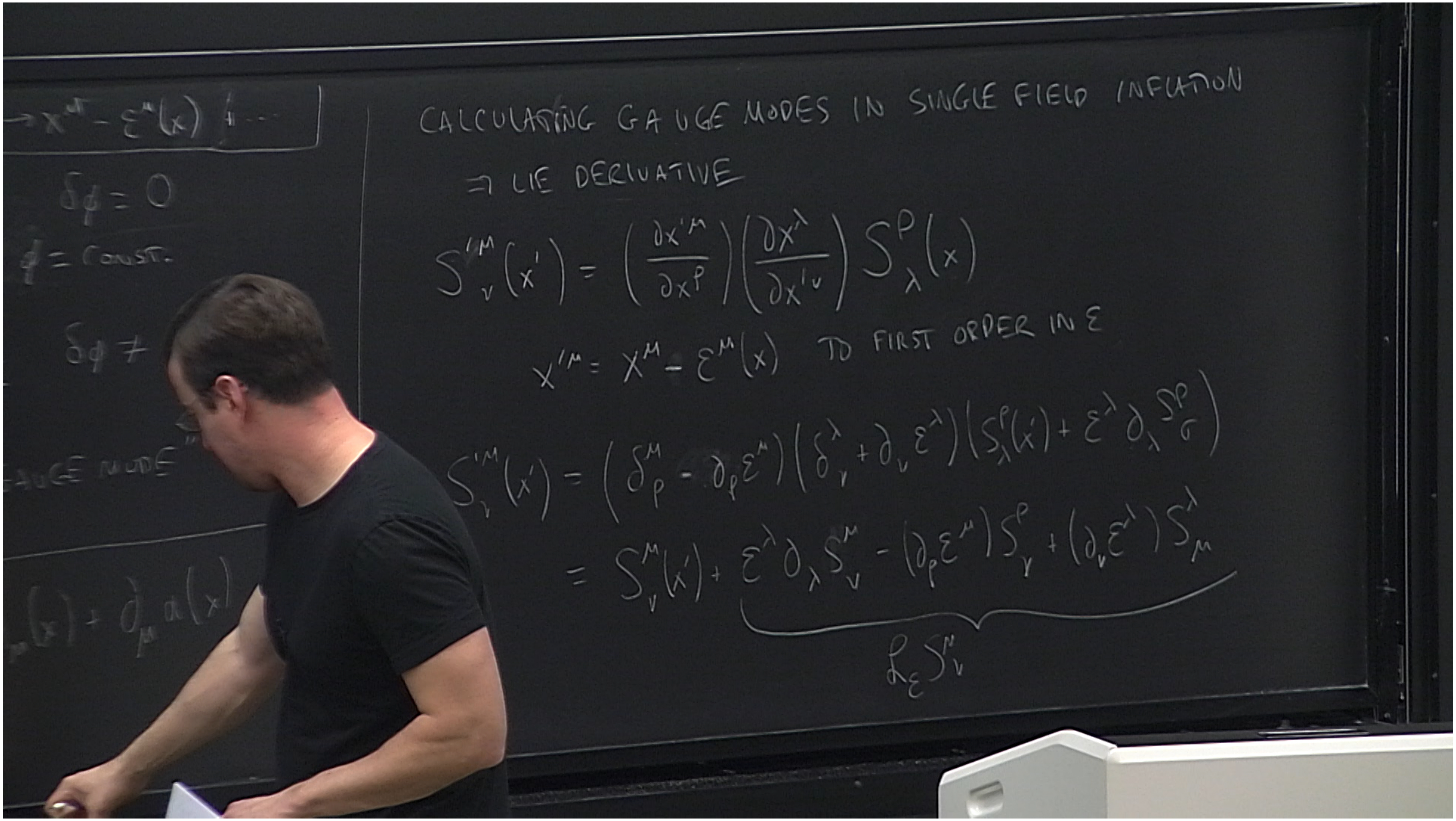
$$g_{\mu\nu}(x,t) = \underbrace{\bar{g}_{\mu\nu}(t)}_{\text{FRW}} + \delta g_{\mu\nu}(x,t) \qquad \phi(x,t) = \underbrace{\bar{\phi}(t)}_{\text{SLOWLY VARYING}} + \delta\phi(x,t)$$


ϕ = CONST.


δφ = 0
φ = CONST.
δφ ≠ 0


δφ = 0
φ = CONST.
δφ ≠ 0


$x^\mu \to x^\mu - \varepsilon^\mu(x) + \ldots$
$\delta\phi = 0$
$\phi = $ CONST.
$\delta\phi \neq 0$
$(\delta g_{\mu\nu}, \delta\phi)$
"GAUGE MODE"
$A_\mu(x) \to A_\mu(x) + \partial_\mu \alpha(x)$


CALCULATING GAUGE MODES IN SINGLE FIELD INFLATION
⇒ LIE DERIVATIVE
$S'^{\mu}{}_{\nu}(x') = \left(\frac{\partial x'^{\mu}}{\partial x^{\rho}}\right)\left(\frac{\partial x^{\lambda}}{\partial x'^{\nu}}\right) S^{\rho}{}_{\lambda}(x)$
$x'^{\mu} = x^{\mu} - \varepsilon^{\mu}(x)$ TO FIRST ORDER IN ε
$S'^{\mu}{}_{\nu}(x') = (\delta^{\mu}_{\rho} - \partial_\rho \varepsilon^{\mu})(\delta^{\lambda}_{\nu} + \partial_\nu\varepsilon^\lambda)(S^\rho_\lambda(x') + \varepsilon^\lambda \partial_\lambda S^\rho_\sigma)$
$= S^\mu_\nu(x') + \underbrace{\varepsilon^\lambda\partial_\lambda S^\mu_\nu - (\partial_\rho\varepsilon^\mu) S^\rho_\nu + (\partial_\nu \varepsilon^\lambda) S^\lambda_\mu}_{\mathcal{L}_\varepsilon S^\mu_\nu}$
$\to x^{\mu} - \varepsilon^{\mu}(x)$
$\delta\phi = 0$
ϕ = CONST.
$\delta\phi \neq$
GAUGE MODE
$(x) + \partial_\mu \alpha(x)$

$\dot\phi = 0$

$\phi = $ CONST.

$\delta\phi \neq 0$

AUGE MODE"

$(x) + \partial_\mu \alpha(x)$

⇒ LIE DERIVATIVE

$$S'^{\mu}_{\nu}(x') = \left(\frac{\partial x'^{\mu}}{\partial x^{\rho}}\right)\left(\frac{\partial x^{\lambda}}{\partial x'^{\nu}}\right) S^{\rho}_{\lambda}(x)$$

$x'^{\mu} = x^{\mu} - \varepsilon^{\mu}(x)$ TO FIRST ORDER IN ε

$$S'^{\mu}_{\nu}(x') = \left(\delta^{\mu}_{\rho} - \partial_{\rho}\varepsilon^{\mu}\right)\left(\delta^{\lambda}_{\nu} + \partial_{\nu}\varepsilon^{\lambda}\right)\left(S^{\rho}_{\lambda}(x') + \varepsilon^{\lambda}\partial_{\lambda} S^{\rho}_{\sigma}\right)$$

$$= S^{\mu}_{\nu}(x') + \underbrace{\varepsilon^{\lambda}\partial_{\lambda} S^{\mu}_{\nu} - (\partial_{\rho}\varepsilon^{\mu}) S^{\rho}_{\nu} + (\partial_{\nu}\varepsilon^{\lambda}) S^{\lambda}_{\mu}}_{\mathcal{L}_{\varepsilon} S^{\mu}_{\nu}}$$

$$\varepsilon = \varepsilon^{\mu}(x)$$

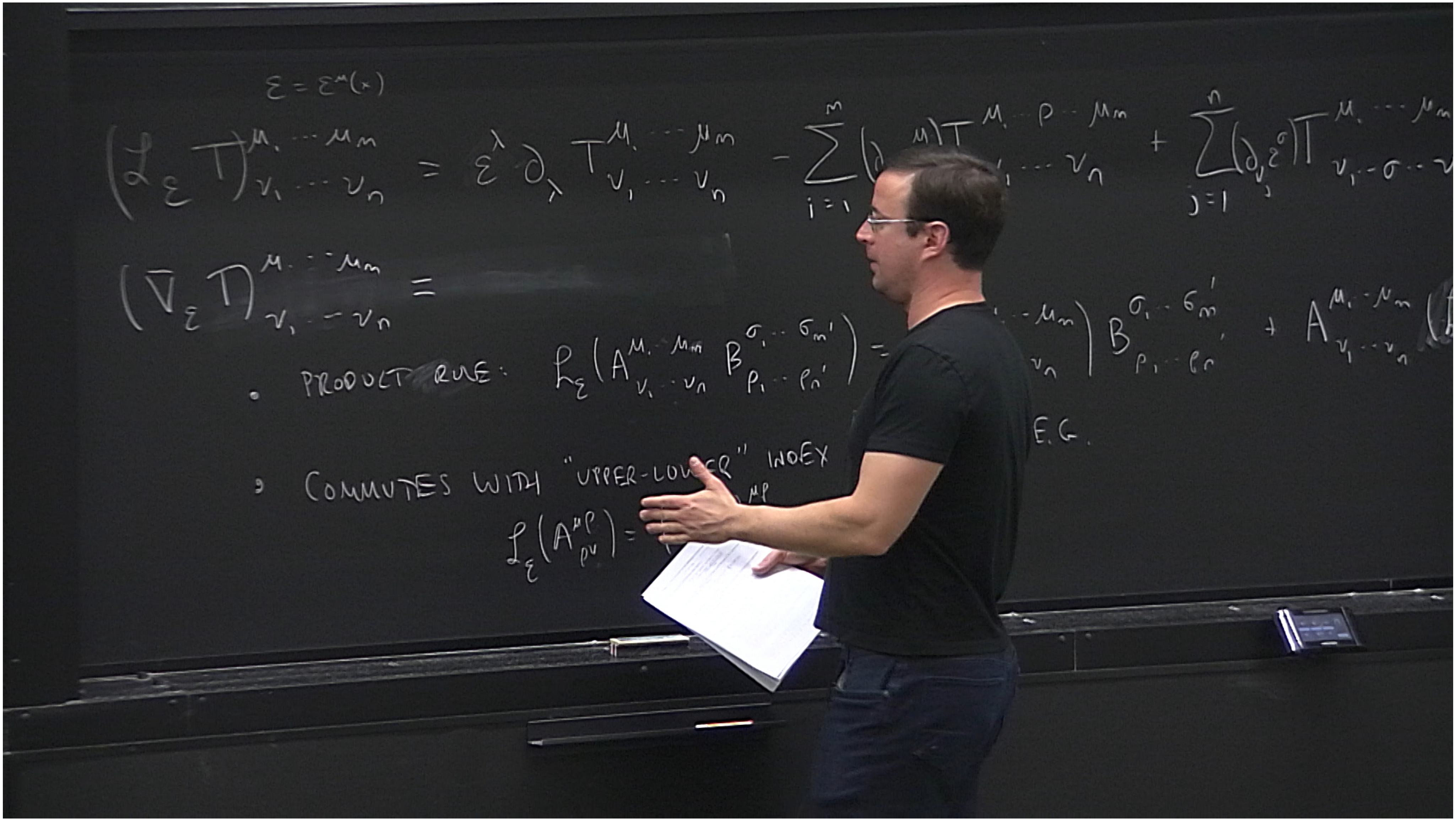
$$\mathcal{L}_{\varepsilon} T^{\mu_1 \cdots \mu_m}{}_{\nu_1 \cdots \nu_n} = \varepsilon^{\lambda} \partial_{\lambda} T^{\mu_1 \cdots \mu_m}{}_{\nu_1 \cdots \nu_n} - \sum_{i=1}^{m} (\partial_{\rho} \varepsilon^{\mu_i}) T^{\mu_1 \cdots \rho \cdots \mu_m}{}_{\nu_1 \cdots \nu_n} + \sum_{j=1}^{n} (\partial_{\nu_j} \varepsilon^{\sigma}) T^{\mu_1 \cdots \mu_m}{}_{\nu_1 \cdots \sigma \cdots \nu_n}$$

$\xi = \xi^\mu(x)$

$$(\mathcal{L}_\xi T)^{\mu_1 \cdots \mu_m}{}_{\nu_1 \cdots \nu_n} = \xi^\lambda \partial_\lambda T^{\mu_1 \cdots \mu_m}{}_{\nu_1 \cdots \nu_n} - \sum_{i=1}^{m} (\partial_\rho \xi^{\mu_i}) T^{\mu_1 \cdots \rho \cdots \mu_m}{}_{\nu_1 \cdots \nu_n} + \sum_{j=1}^{n} (\partial_{\nu_j} \xi^\sigma) T^{\mu_1 \cdots \mu_m}{}_{\nu_1 \cdots \sigma \cdots \nu_n}$$

- $(\mathcal{L}_\xi T)$ IS A TENSOR
- PRODUCT RULE: $\mathcal{L}_\xi (A^{\mu_1 \cdots \mu_m}{}_{\nu_1 \cdots \nu_n} B^{\sigma_1 \cdots \sigma_{m'}}{}_{\rho_1 \cdots \rho_{n'}}) = (\mathcal{L}_\xi A^{\mu_1 \cdots \mu_m}{}_{\nu_1 \cdots \nu_n}) B^{\sigma_1 \cdots \sigma_{m'}}{}_{\rho_1 \cdots \rho_{n'}} + A^{\mu_1 \cdots \mu_m}{}_{\nu_1 \cdots \nu_n} (\mathcal{L}_\xi B^{\sigma_1 \cdots \sigma_{m'}}{}_{\rho_1 \cdots \rho_{n'}})$
- COMMUTES WITH "UPPER-LOWER" INDEX CONTRACTIONS, E.G.

$$\mathcal{L}_\xi (A^{\mu\rho}{}_{\rho\nu}) = (\mathcal{L}_\xi A)^{\mu\rho}{}_{\rho\nu}$$


ε = ε^μ(x)
PRODUCT RULE:
COMMUTES WITH "UPPER-LOWER" INDEX
E.G.

$\varepsilon = \varepsilon^{\mu}(x)$

$$(\mathcal{L}_{\varepsilon} T)^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \nu_n} = \varepsilon^{\lambda} \partial_{\lambda} T^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \nu_n} - \sum_{i=1}^{m} (\partial_{\rho} \varepsilon^{\mu_i}) T^{\mu_1 \cdots \rho \cdots \mu_m}_{\nu_1 \cdots \nu_n} + \sum_{j=1}^{n} (\partial_{\nu_j} \varepsilon^{\sigma}) T^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \sigma \cdots}$$

$$(\nabla_{\varepsilon} T)^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \nu_n} = \varepsilon^{\lambda} \partial_{\lambda} T^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \nu_n} + \Gamma$$

- PRODUCT RULE: $\mathcal{L}_{\varepsilon}\left(A^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \nu_n} B^{\sigma_1 \cdots \sigma_{m'}}_{\rho_1 \cdots \rho_{n'}}\right) = \left(\mathcal{L}_{\varepsilon} A^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \nu_n}\right) B^{\sigma_1 \cdots \sigma_{m'}}_{\rho_1 \cdots \rho_{n'}} + A^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \nu_n} ($

- COMMUTES WITH "UPPER-LOWER" INDEX CONTRACTIONS, E.G.

$$\mathcal{L}_{\varepsilon}\left(A^{\mu\rho}_{\rho\nu}\right) = \left(\mathcal{L}_{\varepsilon} A\right)^{\mu\rho}_{\rho\nu}$$

$$\varepsilon = \varepsilon^{\mu}(x)$$

$$\left(\mathcal{L}_{\varepsilon} T\right)^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \nu_n} = \varepsilon^{\lambda} \partial_{\lambda} T^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \nu_n} - \sum_{i=1}^{m} \left(\partial_{\rho} \varepsilon^{\mu_i}\right) T^{\mu_1 \cdots \rho \cdots \mu_m}_{\nu_1 \cdots \nu_n} + \sum_{j}^{n} \left(\partial_{\nu_j} \varepsilon^{\sigma}\right) T^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \sigma \cdots \nu_n}$$

$$\left(\nabla_{\varepsilon} T\right)^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \nu_n} = \varepsilon^{\lambda} \partial_{\lambda} T^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \nu_n} + \sum_{i} \left(\varepsilon^{\lambda} \Gamma^{\mu_i}_{\lambda \rho}\right) T^{\mu_1 \cdots \rho \cdots \mu_n}_{\nu_1 \cdots \nu_n} - \sum$$

- PRODUCT RULE: $\mathcal{L}_{\varepsilon}\left(A^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \nu_n} B^{\sigma_1 \cdots \sigma_{m'}}_{\rho_1 \cdots \rho_{n'}}\right) = \left(\mathcal{L}_{\varepsilon} A^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \nu_n}\right) B^{\sigma_1 \cdots \sigma_{m'}}_{\rho_1 \cdots \rho_{n'}} + A^{\mu_1 \cdots}_{\nu_1 \cdots \nu_n} ($

- COMMUTES WITH "UPPER-LOWER" INDEX CONTRACTIONS, E.G.

$$\mathcal{L}_{\varepsilon}\left(A^{\mu\rho}{}_{\rho\nu}\right) = \left(\mathcal{L}_{\varepsilon} A\right)^{\mu\rho}{}_{\rho\nu}$$

$\xi = \xi^\mu(x)$

$$(\mathcal{L}_\xi T)^{\mu_1\cdots\mu_m}_{\nu_1\cdots\nu_n} = \xi^\lambda \partial_\lambda T^{\mu_1\cdots\mu_m}_{\nu_1\cdots\nu_n} - \sum_{i=1}^{m} (\partial_\rho \xi^{\mu_i}) T^{\mu_1\cdots\rho\cdots\mu_m}_{\nu_1\cdots\nu_n} + \sum_{j}^{n} (\partial_{\nu_j} \xi^\sigma) T^{\mu_1\cdots\mu_m}_{\nu_1\cdots\sigma\cdots\nu_n}$$

$$(\nabla_\xi T)^{\mu_1\cdots\mu_m}_{\nu_1\cdots\nu_n} = \xi^\lambda \partial_\lambda T^{\mu_1\cdots\mu_m}_{\nu_1\cdots\nu_n} + \sum_i (\xi^\lambda \Gamma^{\mu_i}_{\lambda\rho}) T^{\mu_1\cdots\rho\cdots\mu_m}_{\nu_1\cdots\nu_n} - \sum (\xi^\lambda \Gamma^{\sigma}_{\lambda\nu_j}) T^{\mu_1\cdots\mu_m}_{\nu_1\cdots\sigma\cdots\nu_n}$$

- PRODUCT RULE: $\mathcal{L}_\xi \left(A^{\mu_1\cdots\mu_m}_{\nu_1\cdots\nu_n} B^{\sigma_1\cdots\sigma_{m'}}_{\rho_1\cdots\rho_{n'}}\right) = \left(\mathcal{L}_\xi A^{\mu_1\cdots\mu_m}_{\nu_1\cdots\nu_n}\right) B^{\sigma_1\cdots\sigma_{m'}}_{\rho_1\cdots\rho_{n'}} + A^{\mu_1\cdots\mu_m}_{\nu_1\cdots\nu_n} \left(\mathcal{L}_\xi B^{\sigma_1\cdots\sigma_{m'}}_{\rho_1\cdots\rho_{n'}}\right)$

- COMMUTES WITH "UPPER-LOWER" INDEX CONTRACTIONS, E.G.

$$\mathcal{L}_\xi \left(A^{\mu\rho}_{\rho\nu}\right) = \left(\mathcal{L}_\xi A\right)^{\mu\rho}_{\rho\nu}$$

$$\xi = \xi^\mu(x)$$

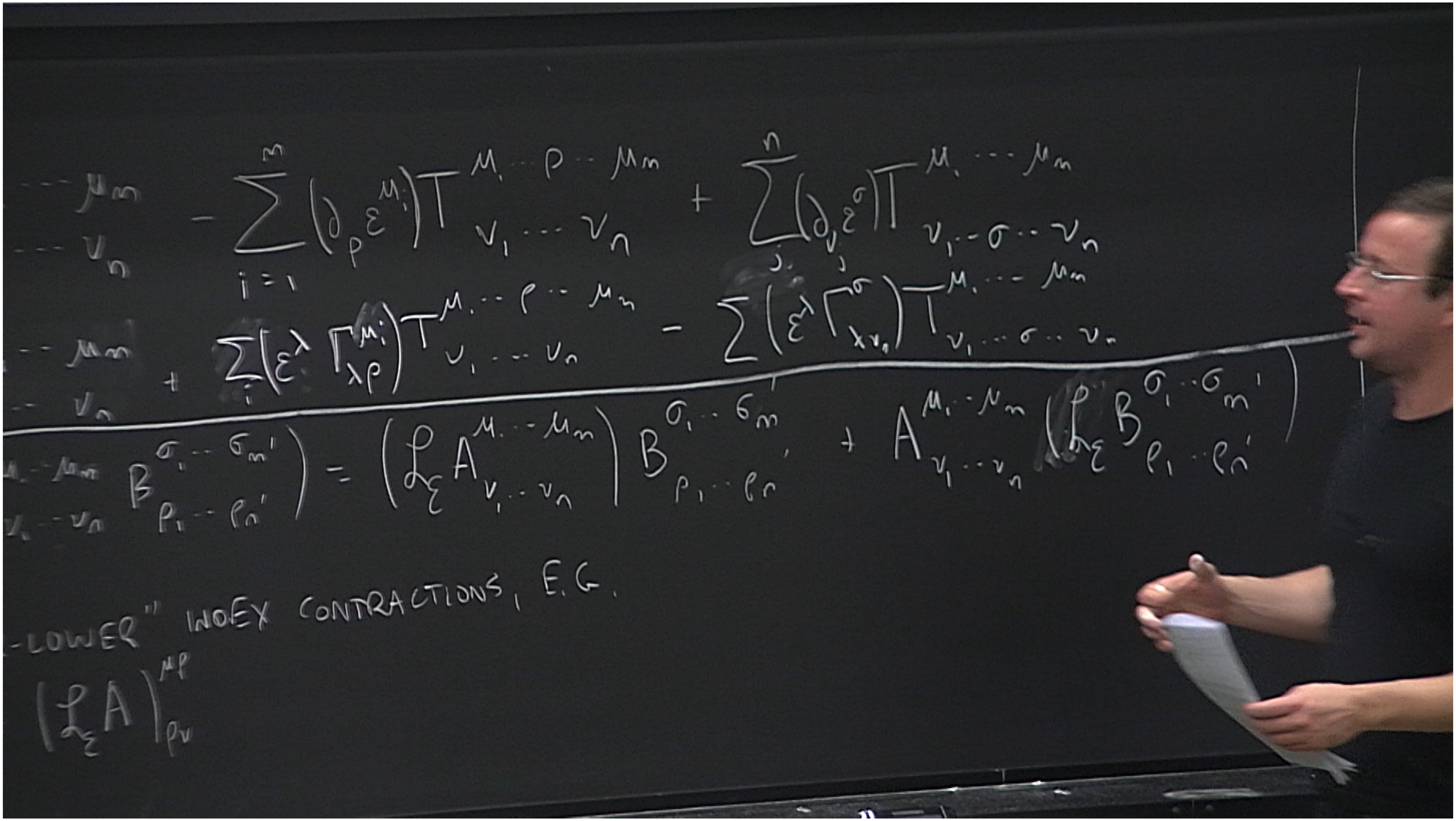
$$\left(\mathcal{L}_\xi T\right)^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \nu_n} = \xi^\lambda \partial_\lambda T^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \nu_n} - \sum_{i=1}^{m} \left(\partial_\rho \xi^{\mu_i}\right) T^{\mu_1 \cdots \rho \cdots \mu_m}_{\nu_1 \cdots \nu_n}$$

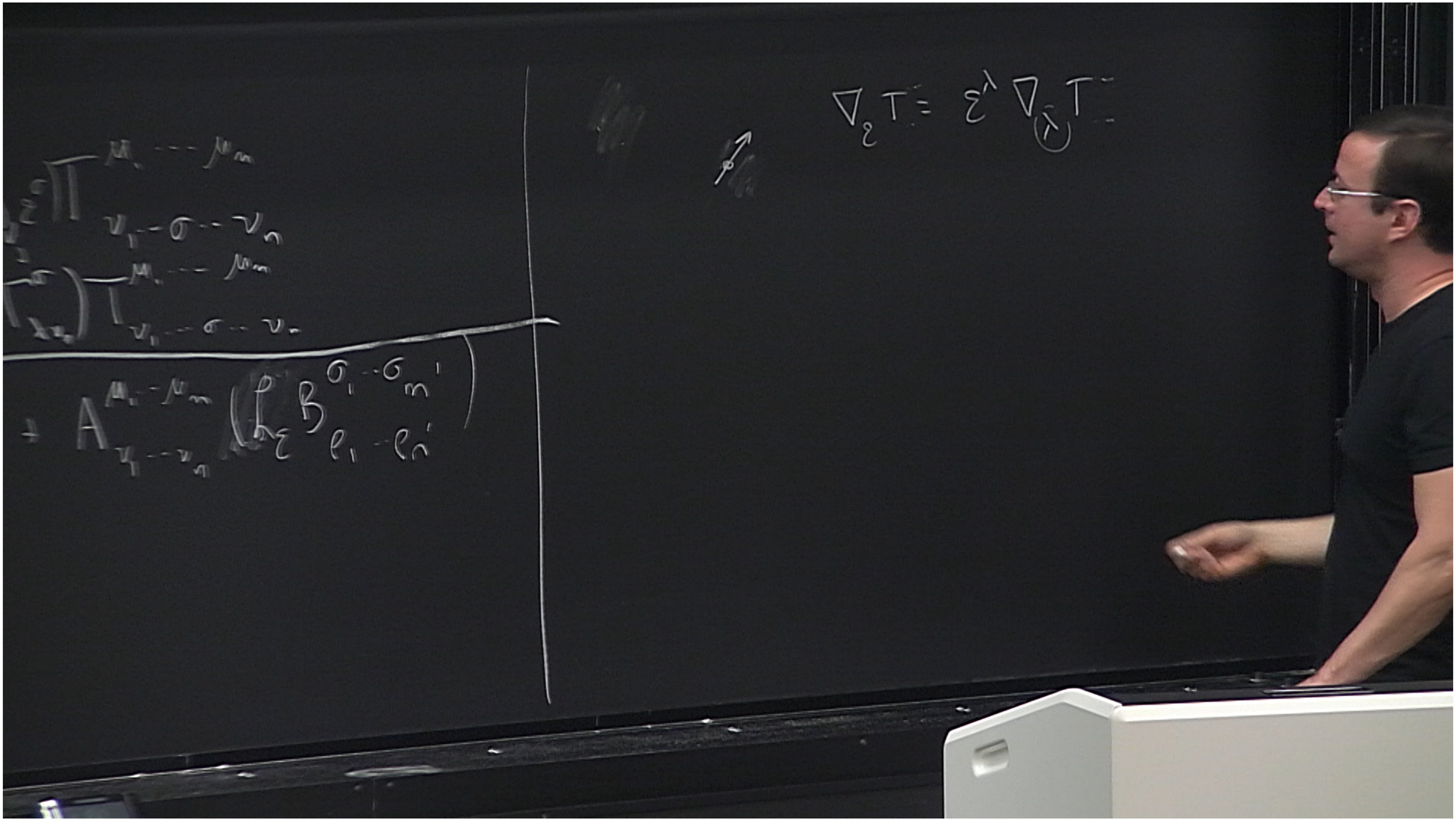
$$\left(\nabla_\xi T\right)^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \nu_n} = \xi^\lambda \partial_\lambda T^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \nu_n} + \sum_i \left(\xi^\lambda \Gamma^{\mu_i}_{\lambda\rho}\right) T^{\mu_1 \cdots \rho \cdots \mu_m}_{\nu_1 \cdots \nu_n}$$

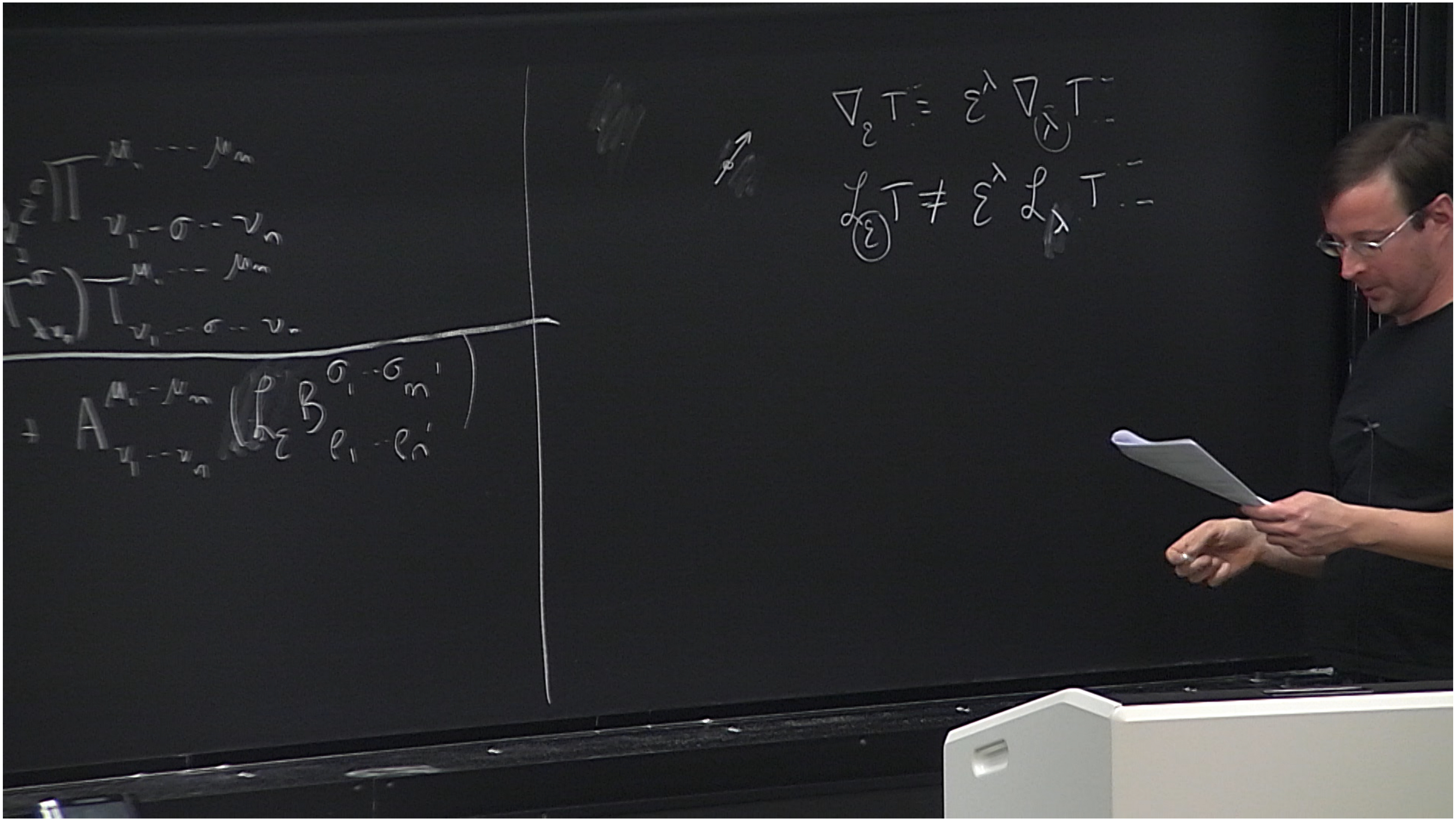
- PRODUCT RULE: $\mathcal{L}_\xi \left(A^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \nu_n} B^{\sigma_1 \cdots \sigma_{m'}}_{\rho_1 \cdots \rho_{n'}}\right) = \left(\mathcal{L}_\xi A^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \nu_n}\right) B$

- COMMUTES WITH "UPPER-LOWER" INDEX CONTRACTIONS, E.G.

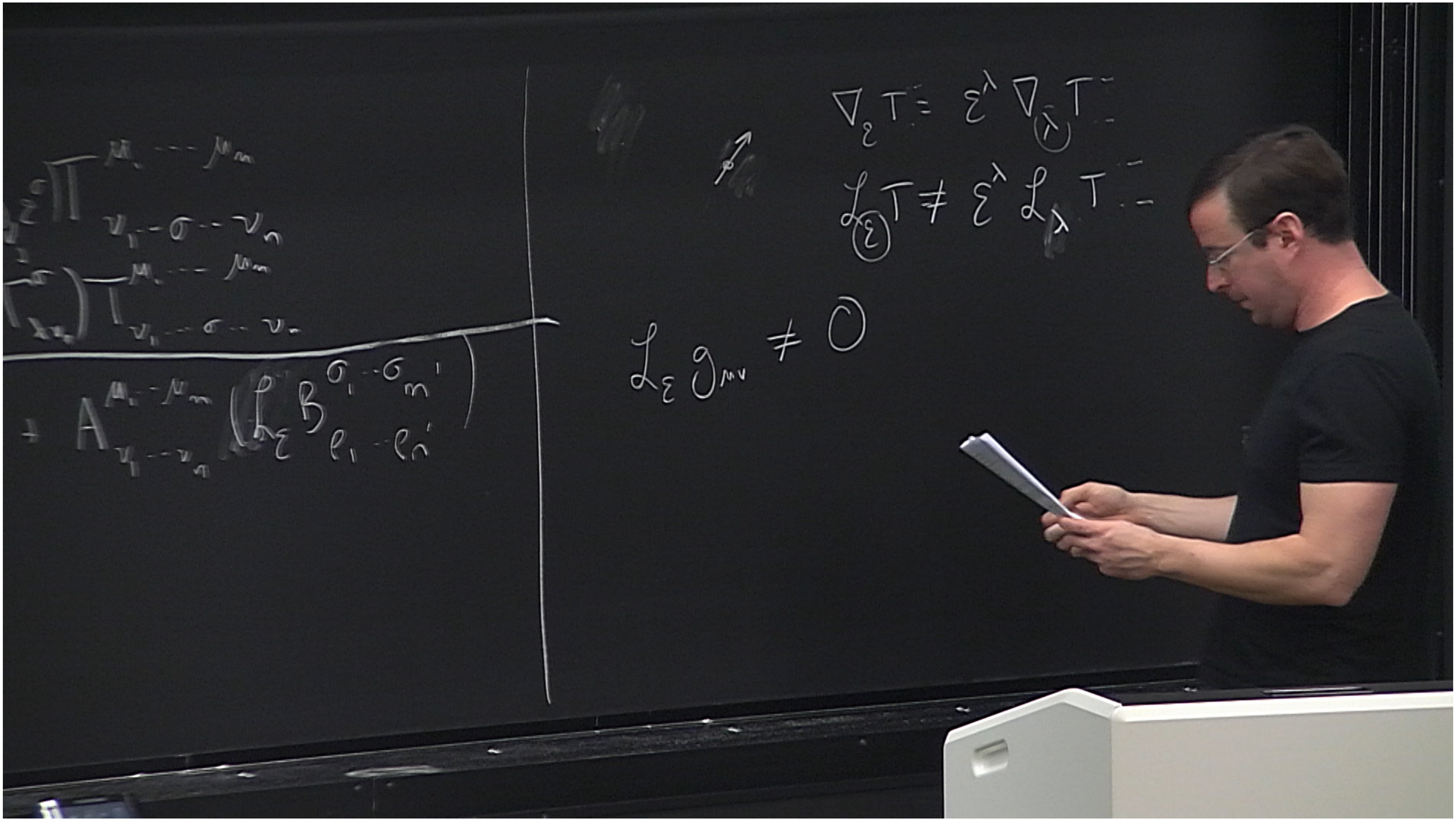
$$\mathcal{L}_\xi \left(A^{\mu\rho}_{\rho\nu}\right) = \left(\mathcal{L}_\xi A\right)^{\mu\rho}_{\rho\nu}$$

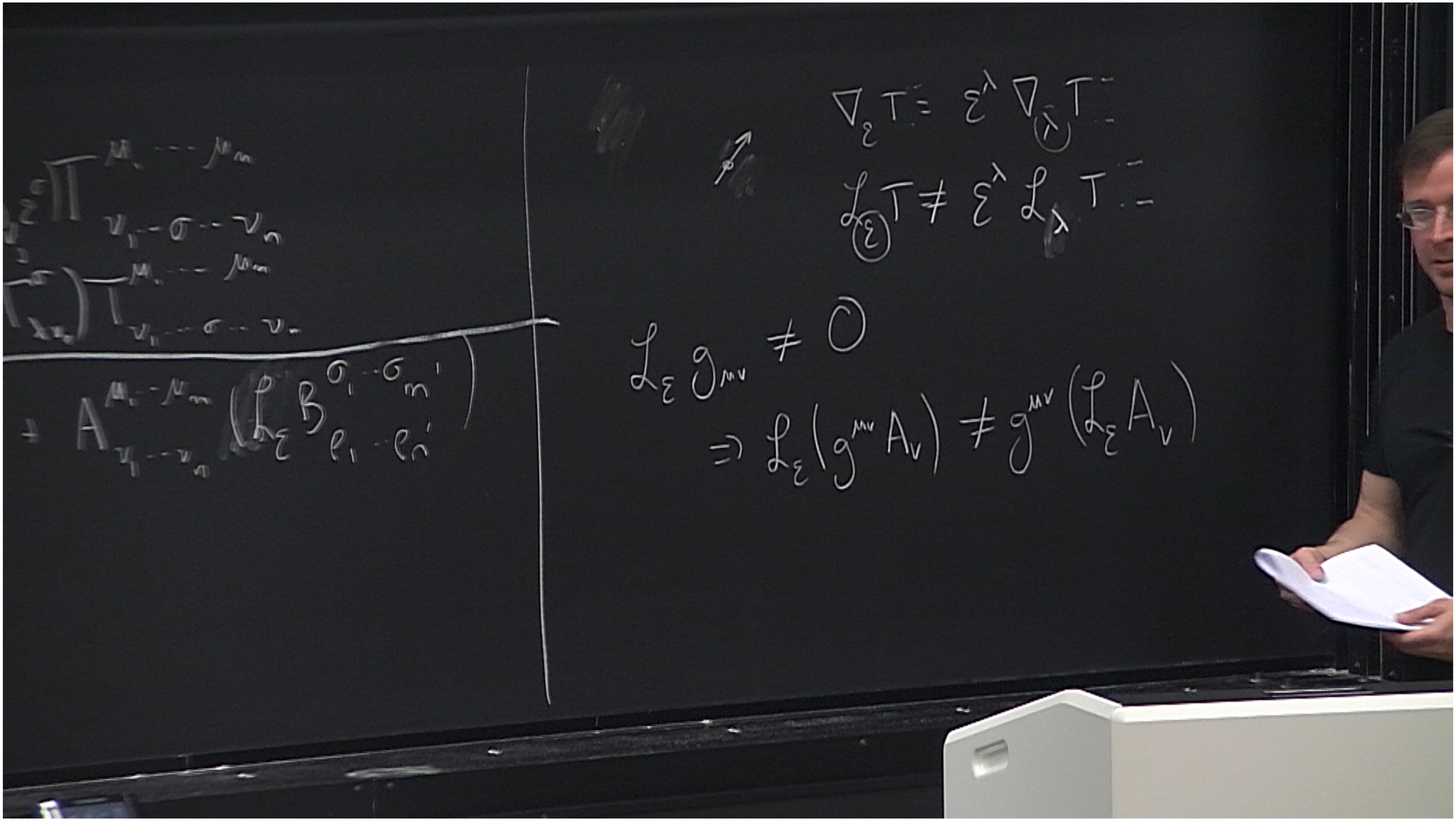

$- \sum_{i=1}^{m} (\partial_\rho \varepsilon^{\mu_i}) T^{\mu_1 \cdots \rho \cdots \mu_m}{}_{\nu_1 \cdots \nu_n} + \sum_{j}^{n} (\partial_{\nu_j} \varepsilon^{\sigma}) T^{\mu_1 \cdots \mu_m}{}_{\nu_1 \cdots \sigma \cdots \nu_n}$
$+ \sum_i (\varepsilon^\lambda \Gamma^{\mu_i}_{\lambda\rho}) T^{\mu_1 \cdots \rho \cdots \mu_m}{}_{\nu_1 \cdots \nu_n} - \sum (\varepsilon^\lambda \Gamma^{\sigma}_{\lambda \nu_i}) T^{\mu_1 \cdots \mu_m}{}_{\nu_1 \cdots \sigma \cdots \nu_n}$
$B^{\sigma_1 \cdots \sigma_{m'}}_{\rho_1 \cdots \rho_{n'}} \Big) = \left(\mathcal{L}_\xi A^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \nu_n}\right) B^{\sigma_1 \cdots \sigma_{m'}}_{\rho_1 \cdots \rho_{n'}} + A^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \nu_n} \left(\mathcal{L}_\xi B^{\sigma_1 \cdots \sigma_{m'}}_{\rho_1 \cdots \rho_{n'}}\right)$
-LOWER" INDEX CONTRACTIONS, E.G.
$(\mathcal{L}_\xi A)^{\mu\rho}{}_{\rho\nu}$

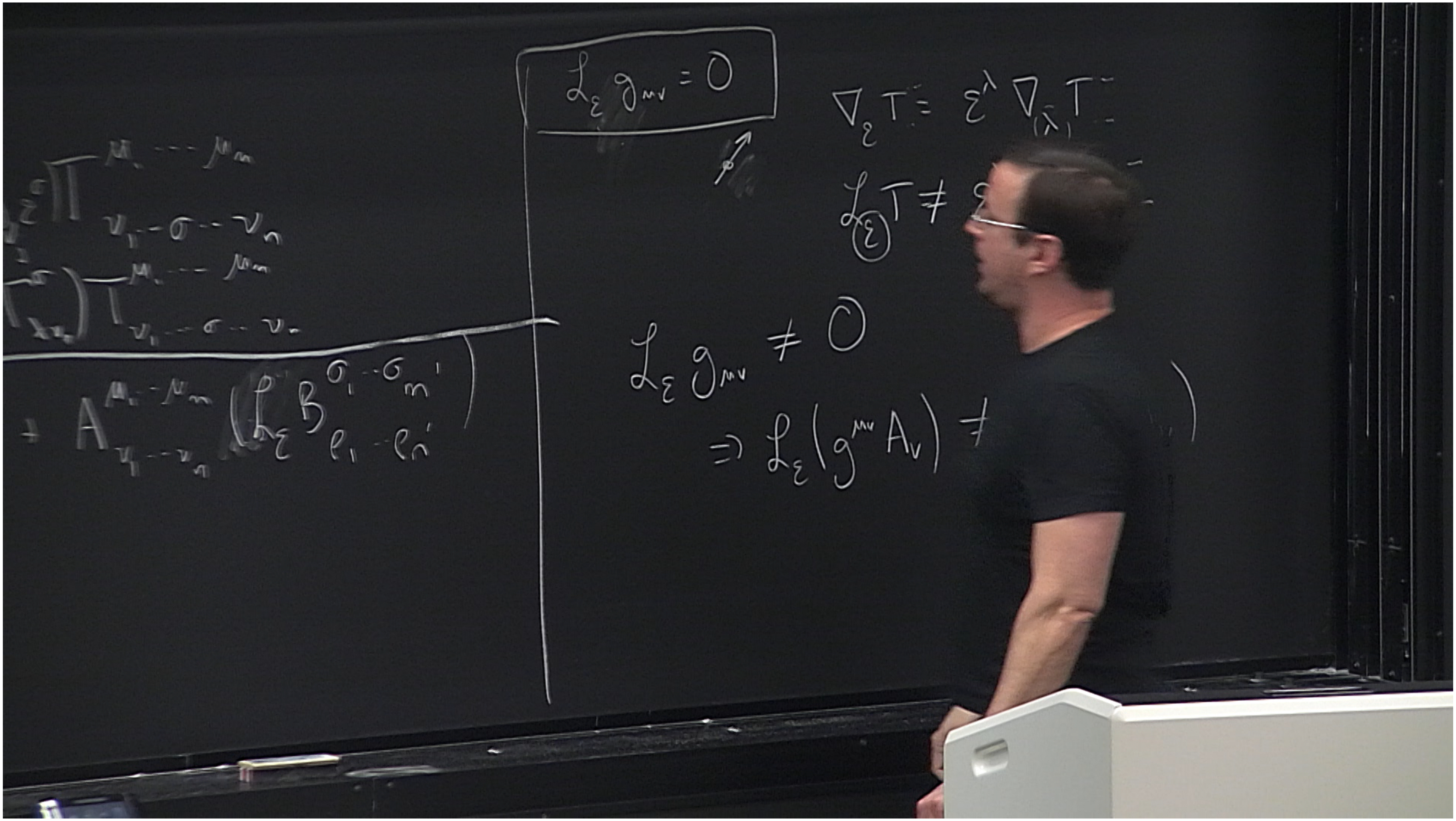

$\nabla_\xi T^{\cdots}_{\cdots} = \xi^\lambda \nabla_\lambda T^{\cdots}_{\cdots}$

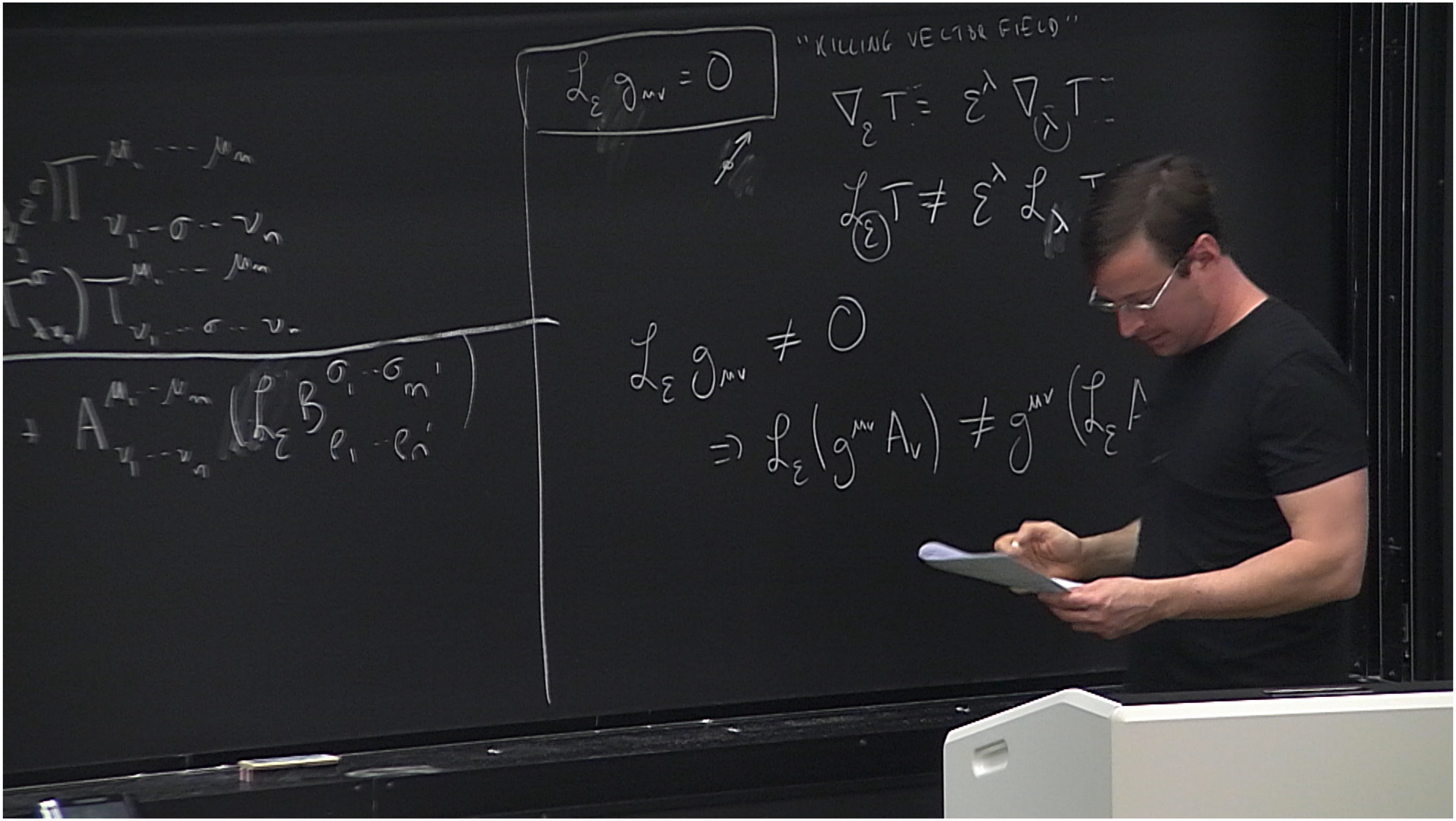


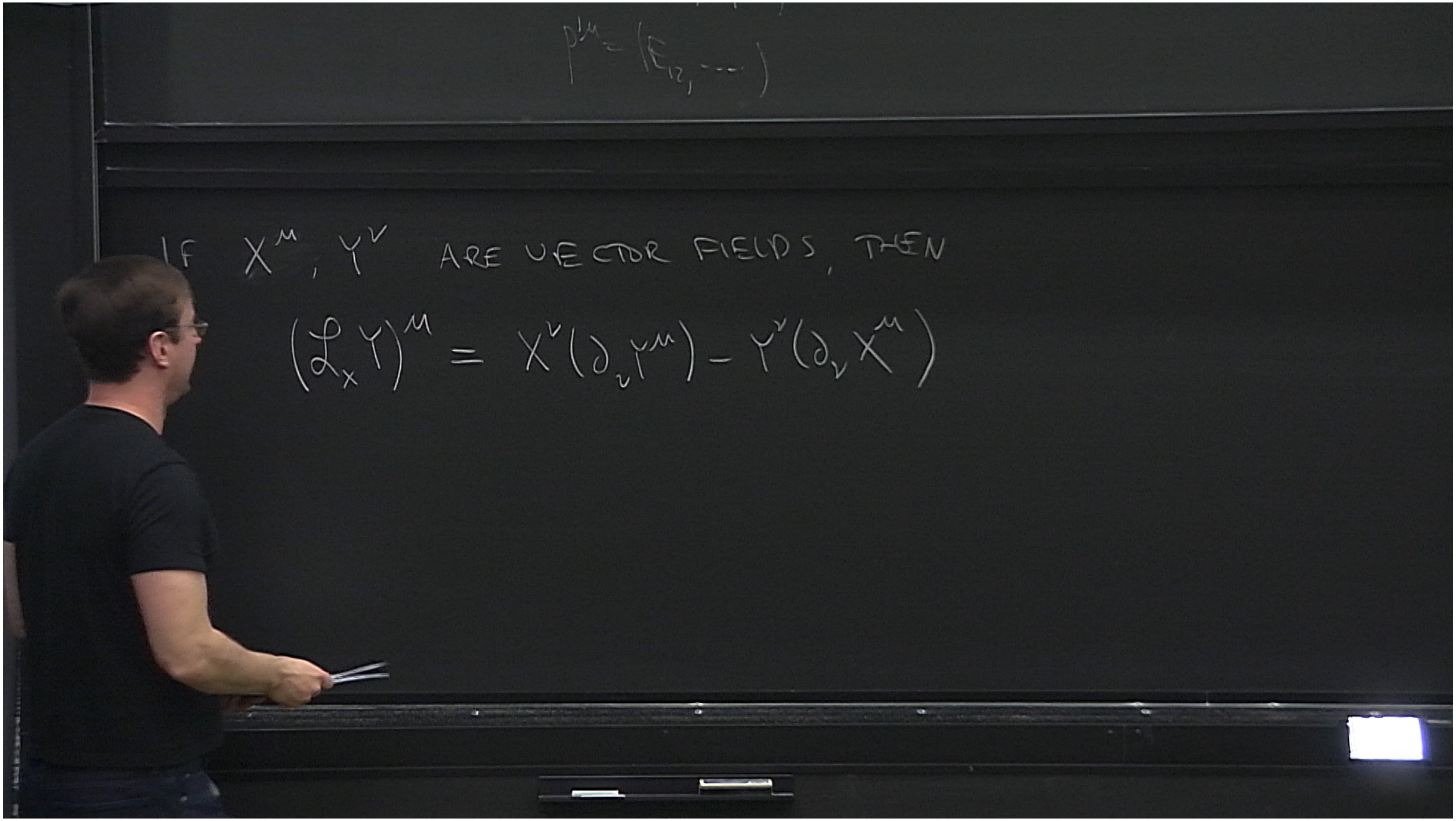
$\nabla_\varepsilon T = \varepsilon^\lambda \nabla_\lambda T$
$\mathcal{L}_\varepsilon T \neq \varepsilon^\lambda \mathcal{L}_\lambda T$

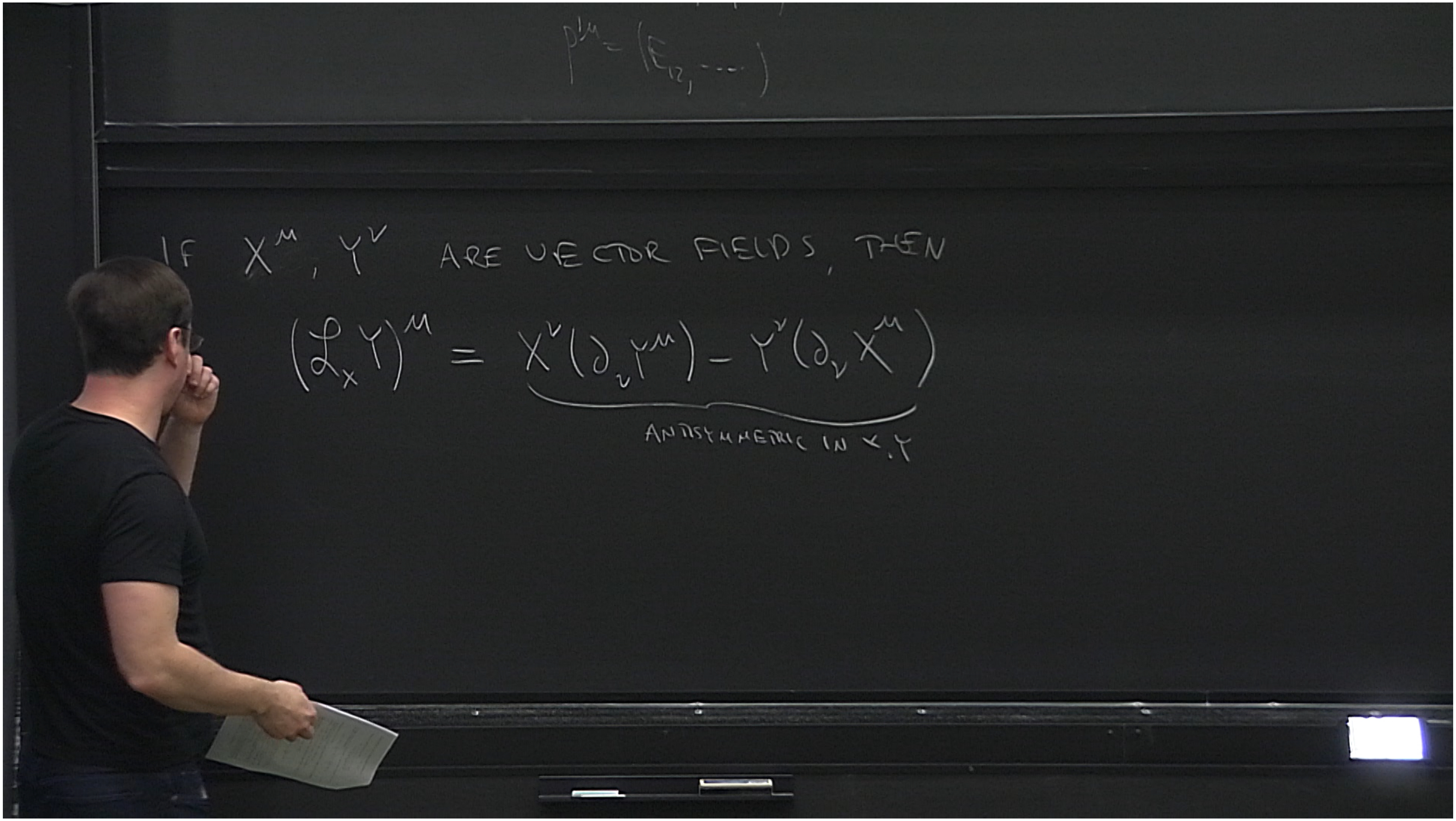

$\nabla_\varepsilon T = \varepsilon^\lambda \nabla_\lambda T$
$\mathcal{L}_\varepsilon T \neq \varepsilon^\lambda \mathcal{L}_\lambda T$
$\mathcal{L}_\varepsilon g_{\mu\nu} \neq 0$


$\nabla_\xi T^{\cdots}_{\cdots} = \xi^\lambda \nabla_\lambda T^{\cdots}_{\cdots}$
$\mathcal{L}_\xi T \neq \xi^\lambda \mathcal{L}_\lambda T^{\cdots}_{\cdots}$
$\mathcal{L}_\xi g_{\mu\nu} \neq 0$
$\Rightarrow \mathcal{L}_\xi (g^{\mu\nu} A_\nu) \neq g^{\mu\nu} (\mathcal{L}_\xi A_\nu)$
$+ A^{\mu_1 \cdots \mu_m}_{\nu_1 \cdots \nu_n} (\mathcal{L}_\xi B^{\sigma_1 \cdots \sigma_{n'}}_{\rho_1 \cdots \rho_{n'}})$


$\mathcal{L}_\varepsilon g_{\mu\nu} = 0$
$\nabla_\varepsilon T = \varepsilon^\lambda \nabla_{(\lambda)} T$
$\mathcal{L}_\varepsilon T \neq$
$\mathcal{L}_\varepsilon g_{\mu\nu} \neq 0$
$\Rightarrow \mathcal{L}_\varepsilon (g^{\mu\nu} A_\nu) \neq$


"KILLING VECTOR FIELD"
$\boxed{\mathcal{L}_\xi g_{\mu\nu} = 0}$
$\nabla_\xi T = \xi^\lambda \nabla_\lambda T$
$\mathcal{L}_\xi T \neq \xi^\lambda \mathcal{L}_\lambda T$
$\mathcal{L}_\xi g_{\mu\nu} \neq 0$
$\Rightarrow \mathcal{L}_\xi (g^{\mu\nu} A_\nu) \neq g^{\mu\nu} (\mathcal{L}_\xi A$


$p'^\mu = (E_{12}, \ldots)$
IF X^μ, Y^ν ARE VECTOR FIELDS, THEN
$(\mathcal{L}_X Y)^\mu = X^\nu(\partial_\nu Y^\mu) - Y^\nu(\partial_\nu X^\mu)$

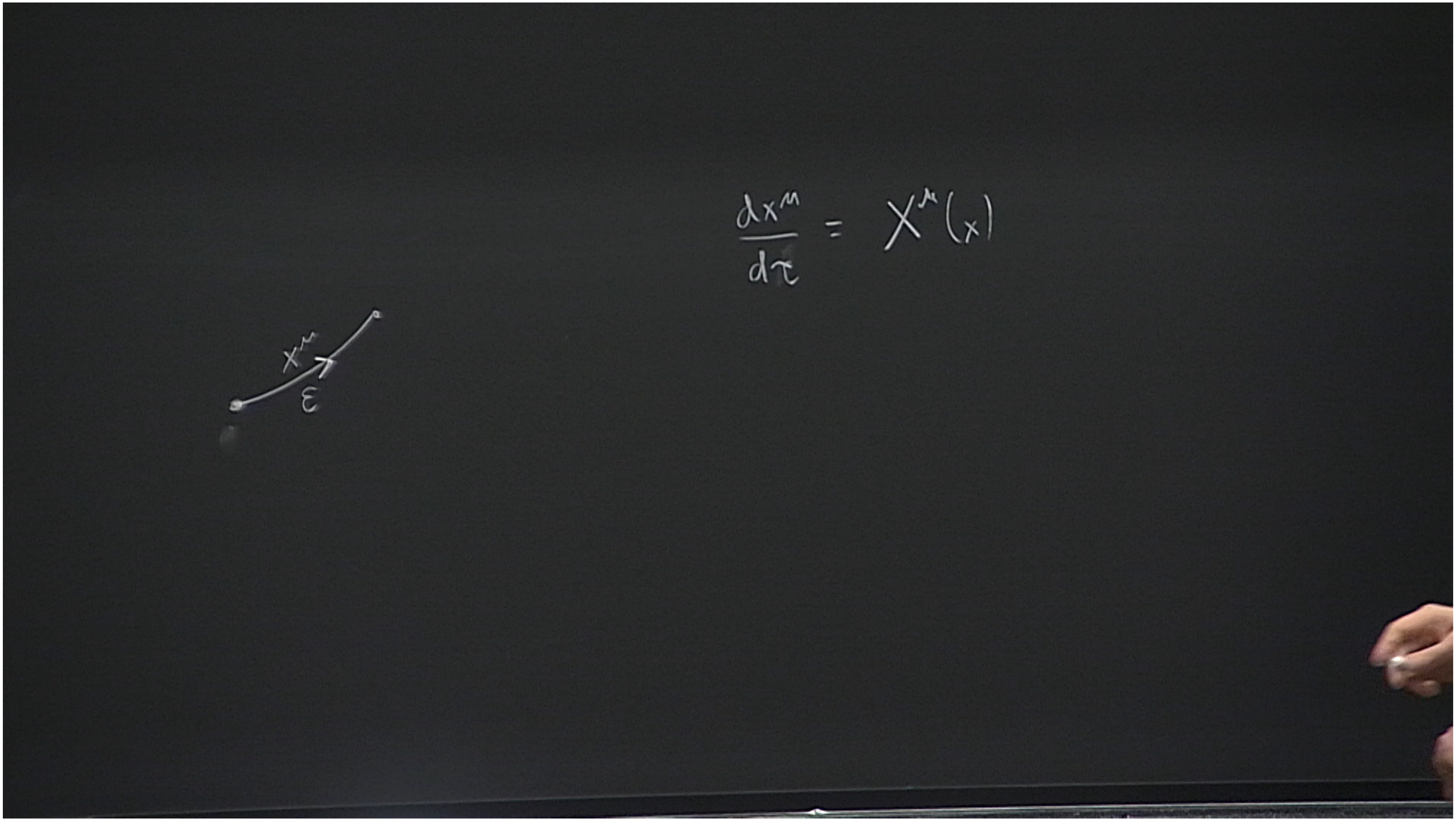

$p'^{\mu} = (E_{12}, \dots)$
IF X^{μ}, Y^{ν} ARE VECTOR FIELDS, THEN
$(\mathcal{L}_X Y)^{\mu} = \underbrace{X^{\nu}(\partial_{\nu} Y^{\mu}) - Y^{\nu}(\partial_{\nu} X^{\mu})}_{\text{ANTISYMMETRIC IN } X, Y}$

$p'^{\mu} = (E_{12}, \ldots)$

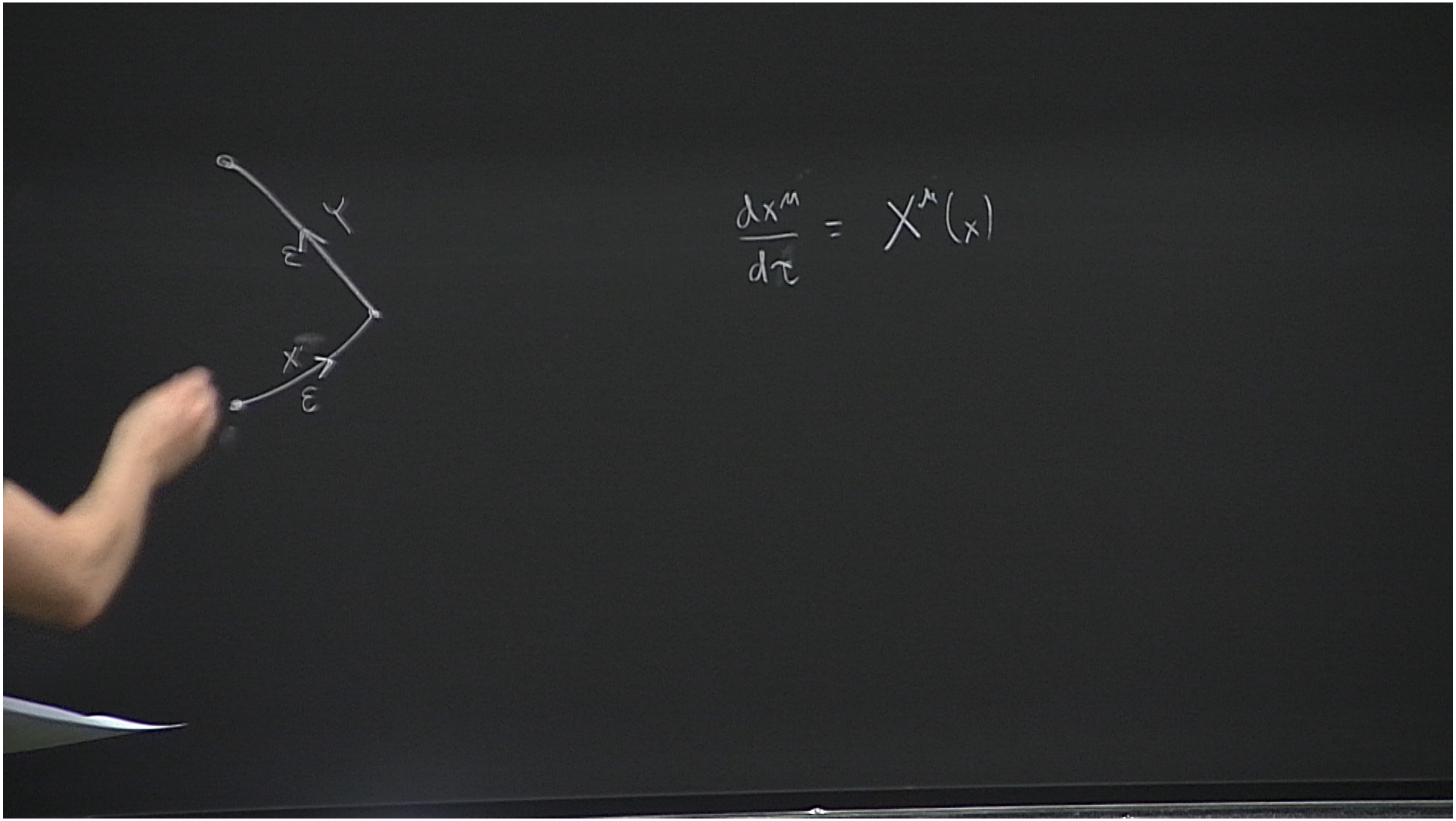
IF X^{μ}, Y^{ν} ARE VECTOR FIELDS, THEN

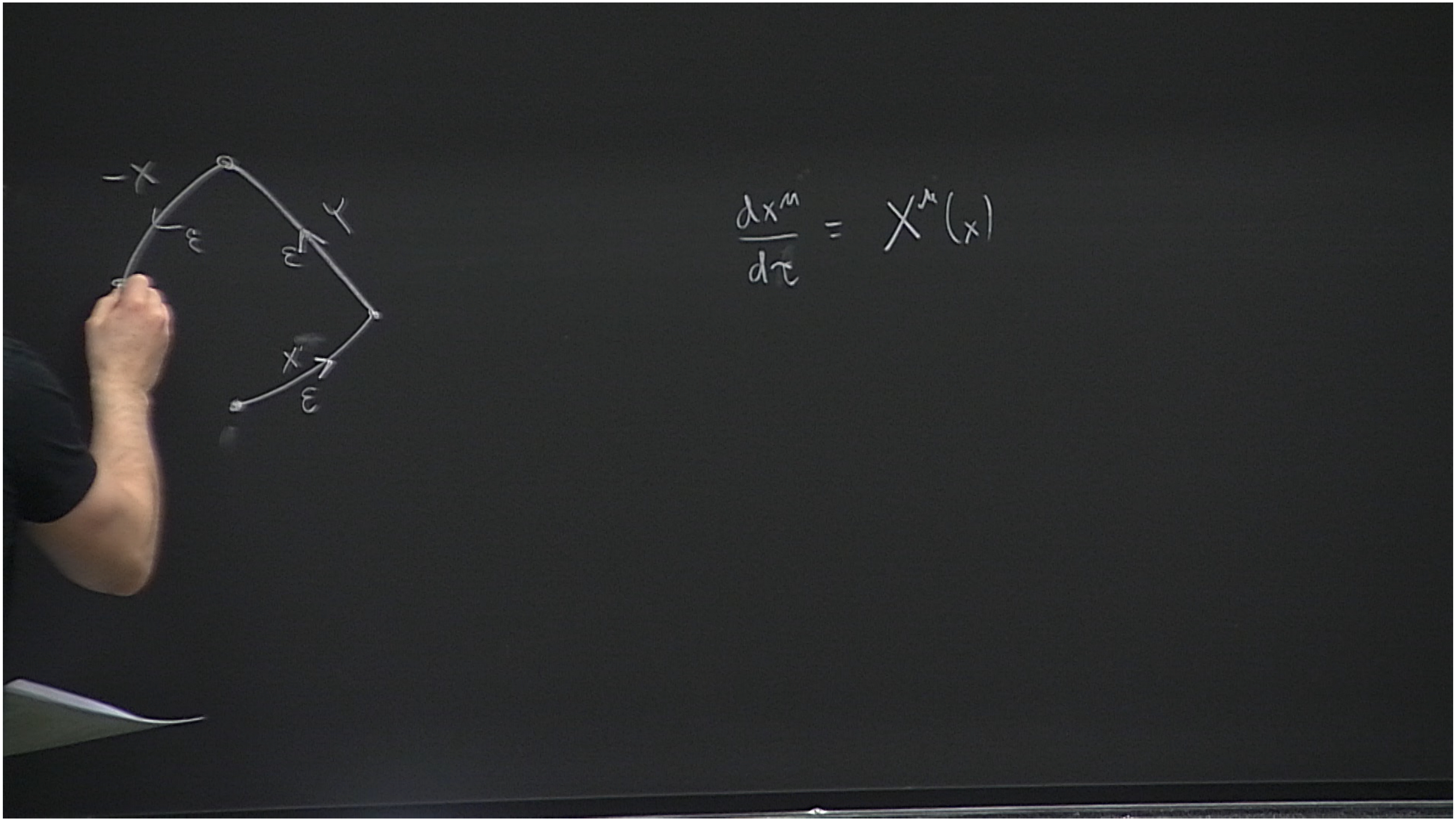
$$(\mathcal{L}_X Y)^{\mu} = \underbrace{X^{\nu}(\partial_{\nu} Y^{\mu}) - Y^{\nu}(\partial_{\nu} X^{\mu})}_{\text{ANTISYMMETRIC IN } X,Y}$$

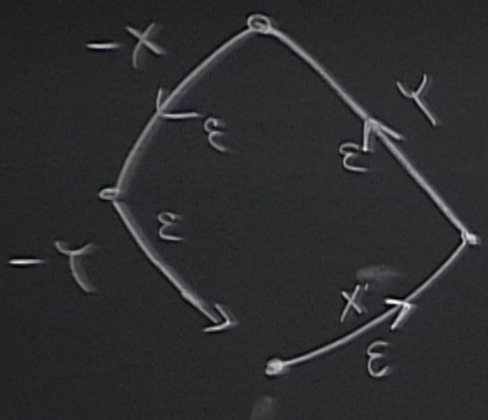
$$\stackrel{\text{def}}{=} [X, Y]^{\mu}$$



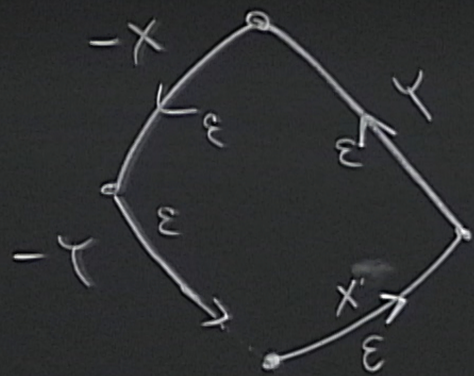
$$\frac{dx^\mu}{d\tau} = X^\mu(x)$$
x^μ
ε


$\frac{dx^\mu}{d\tau} = X^\mu(x)$


$-X$
Y
ε
ε
X'
ε
$\frac{dx^\mu}{d\tau} = X^\mu(x)$

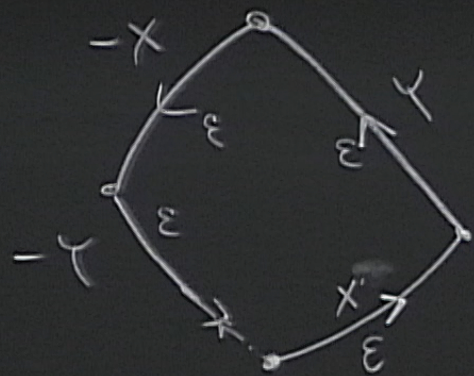


$$\frac{dx^\mu}{d\tau} = X^\mu(x)$$



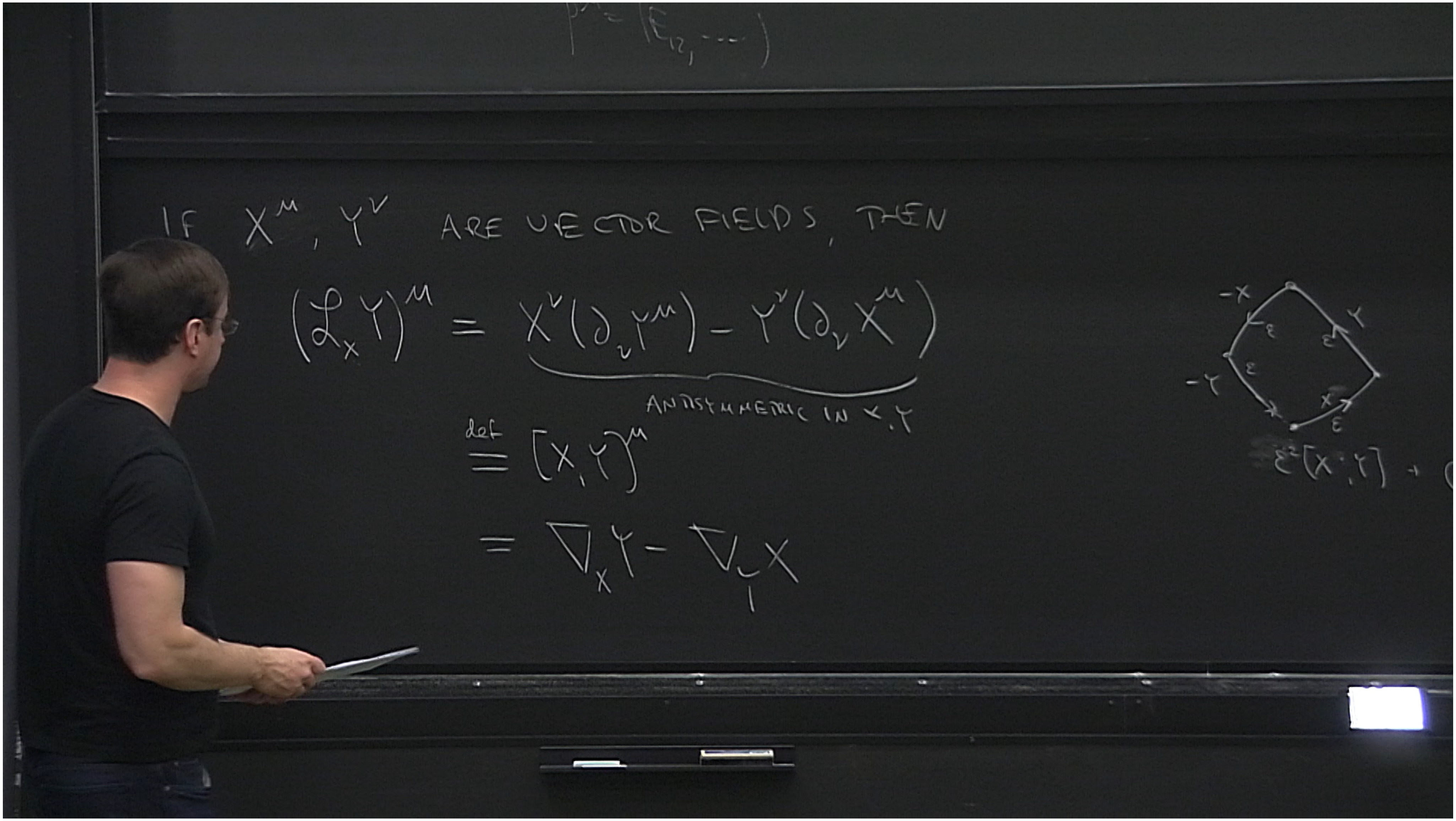
$$\varepsilon^2[X,Y] + \mathcal{O}(\varepsilon^3)$$

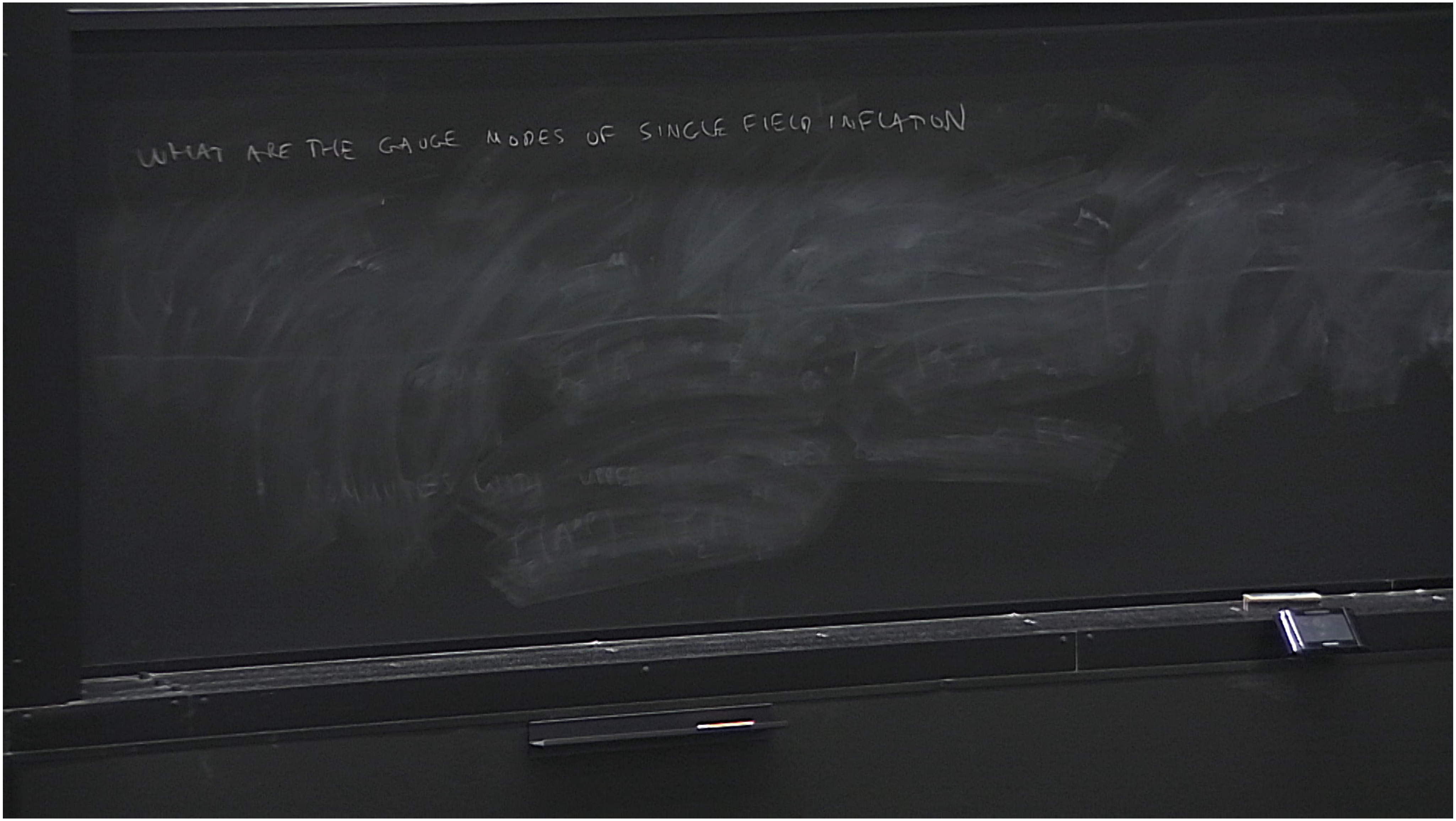
$$\frac{dx^\mu}{d\tau} = X^\mu(x)$$

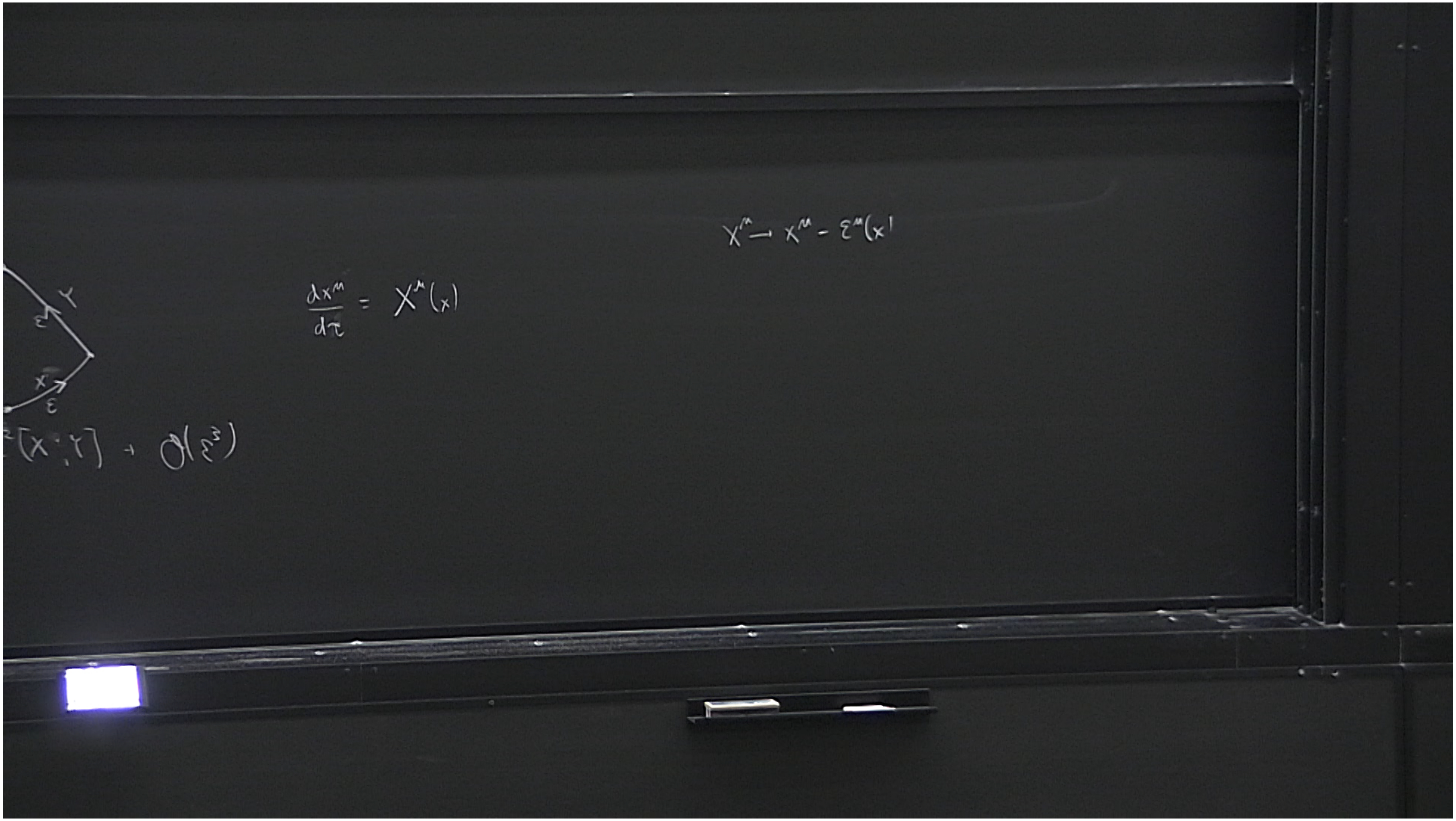


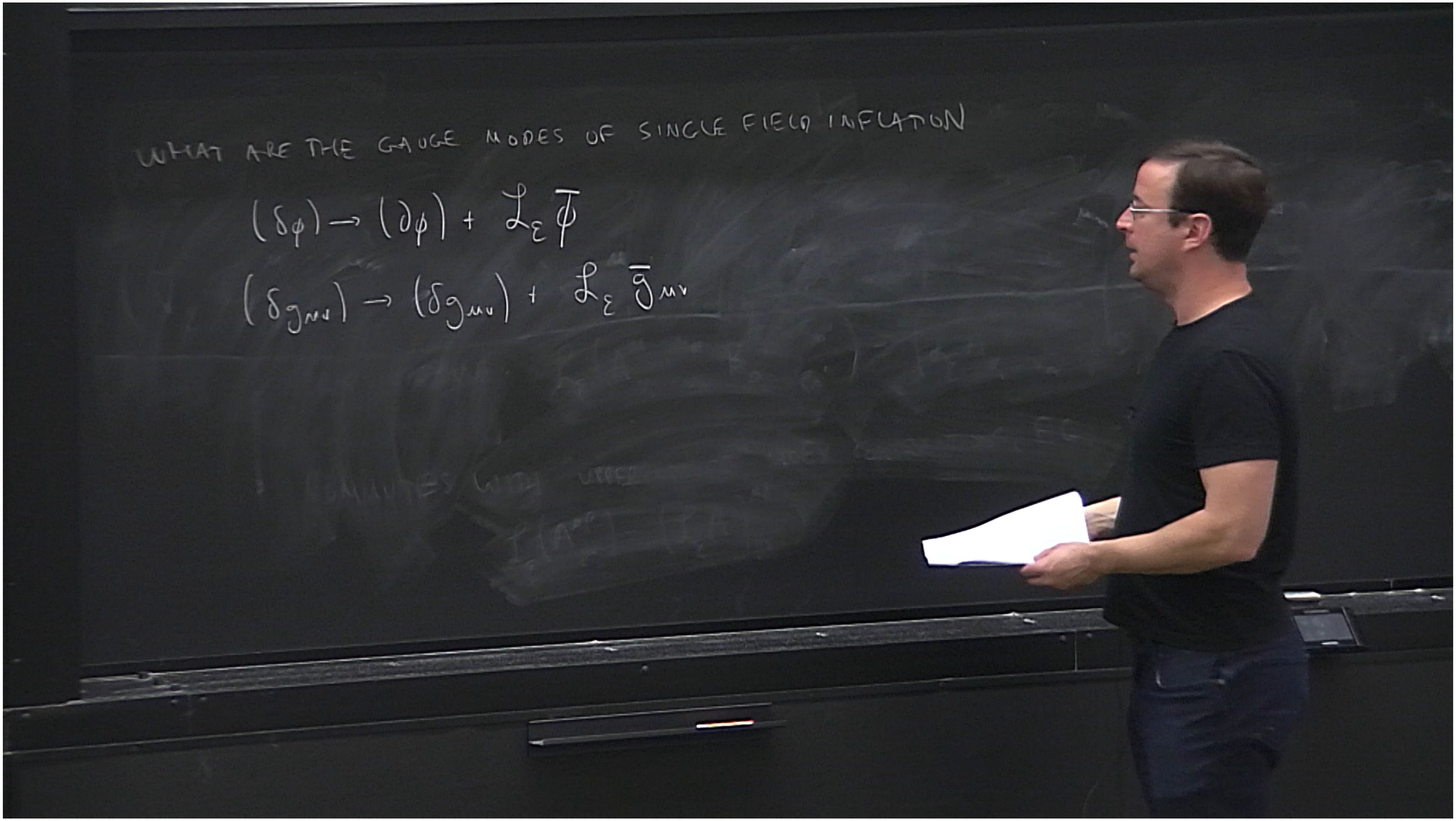
$$\varepsilon^2 [X, Y] + \mathcal{O}(\varepsilon^3)$$

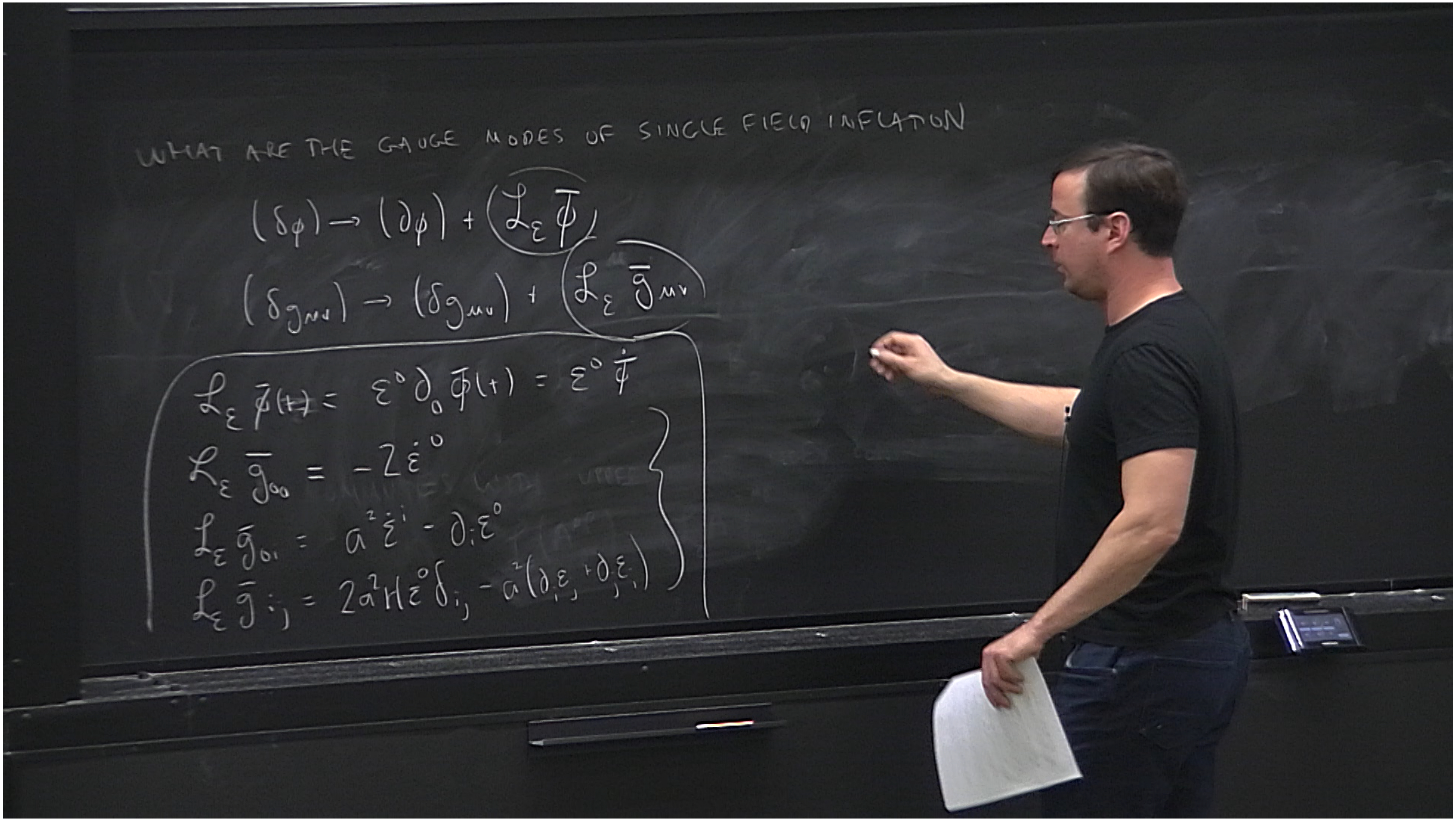
$$\frac{dx^\mu}{d\tau} = X^\mu(x)$$

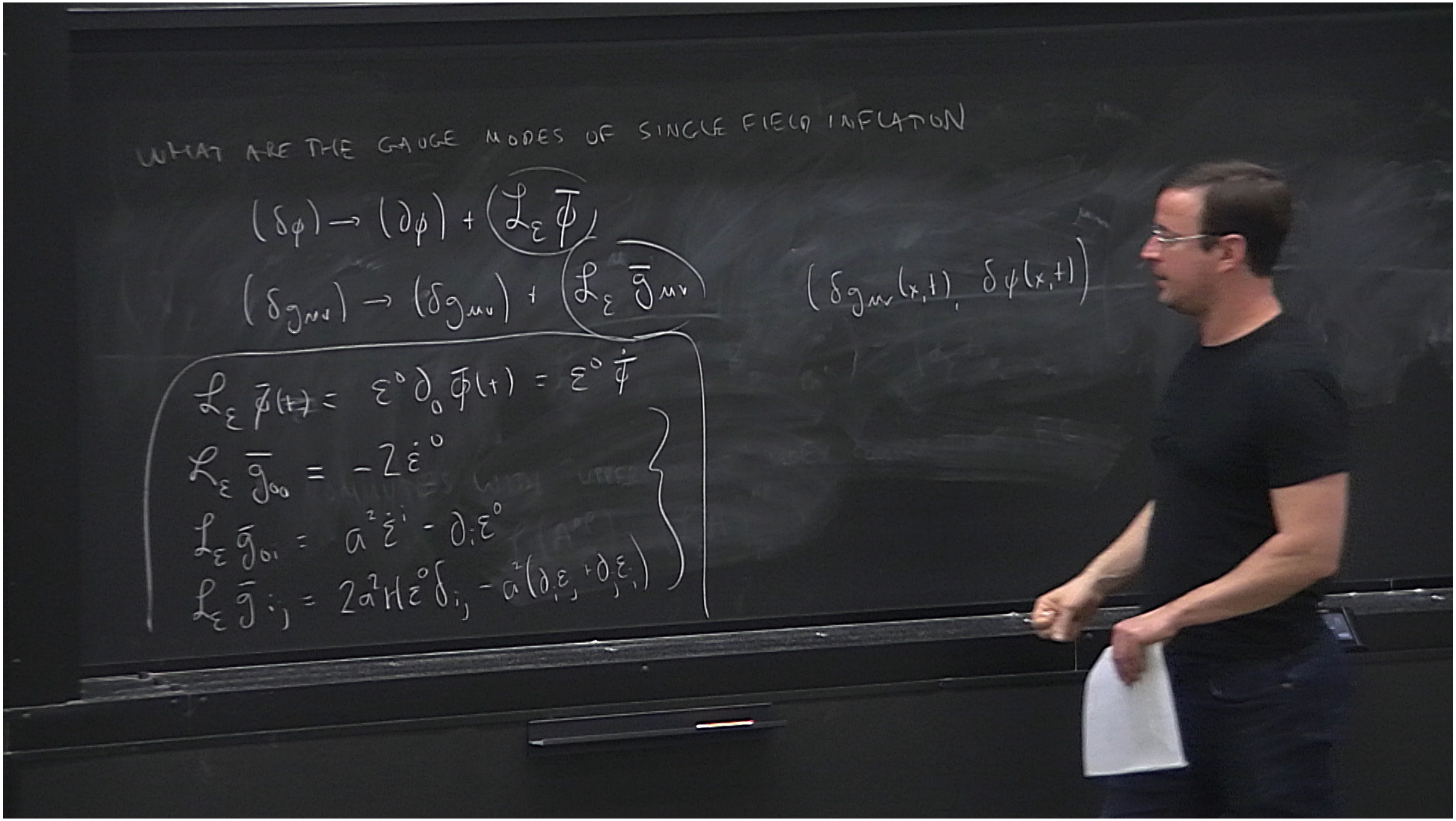

IF X^μ, Y^ν ARE VECTOR FIELDS, THEN
$(\mathcal{L}_X Y)^\mu = X^\nu(\partial_\nu Y^\mu) - Y^\nu(\partial_\nu X^\mu)$
ANTISYMMETRIC IN X, Y
$\overset{\text{def}}{=} [X,Y]^\mu$
$= \nabla_X Y - \nabla_Y X$
$-X$
Y
$-Y$
X
ε
$\varepsilon^2[X,Y]$


WHAT ARE THE GAUGE MODES OF SINGLE FIELD INFLATION


$X^\mu \to X^\mu - \varepsilon^\mu(x)$
$\frac{dx^\mu}{d\tau} = X^\mu(x)$
Y
ε
X
ε
$[X, Y] + \mathcal{O}(\varepsilon^3)$


WHAT ARE THE GAUGE MODES OF SINGLE FIELD INFLATION
$(\delta\phi) \to (\delta\phi) + \mathcal{L}_\xi \bar{\Phi}$
$(\delta g_{\mu\nu}) \to (\delta g_{\mu\nu}) + \mathcal{L}_\xi \bar{g}_{\mu\nu}$


WHAT ARE THE GAUGE MODES OF SINGLE FIELD INFLATION
$(\delta\phi) \to (\delta\phi) + \mathcal{L}_\varepsilon \bar{\phi}$
$(\delta g_{\mu\nu}) \to (\delta g_{\mu\nu}) + \mathcal{L}_\varepsilon \bar{g}_{\mu\nu}$
$\mathcal{L}_\varepsilon \bar{\phi}(t) = \varepsilon^0 \partial_0 \bar{\phi}(t) = \varepsilon^0 \dot{\bar{\phi}}$
$\mathcal{L}_\varepsilon \bar{g}_{00} = -2\dot{\varepsilon}^0$
$\mathcal{L}_\varepsilon \bar{g}_{0i} = a^2 \dot{\varepsilon}^i - \partial_i \varepsilon^0$
$\mathcal{L}_\varepsilon \bar{g}_{ij} = 2a^2 H \varepsilon^0 \delta_{ij} - a^2(\partial_i \varepsilon_j + \partial_j \varepsilon_i)$


WHAT ARE THE GAUGE MODES OF SINGLE FIELD INFLATION
$(\delta\phi) \to (\delta\phi) + \mathcal{L}_\varepsilon \bar{\phi}$
$(\delta g_{\mu\nu}) \to (\delta g_{\mu\nu}) + \mathcal{L}_\varepsilon \bar{g}_{\mu\nu}$
$(\delta g_{\mu\nu}(x,t),\ \delta\phi(x,t))$
$\mathcal{L}_\varepsilon \bar{\phi} = \varepsilon^0 \partial_0 \bar{\phi}(t) = \varepsilon^0 \dot{\bar{\phi}}$
$\mathcal{L}_\varepsilon \bar{g}_{00} = -2\dot{\varepsilon}^0$
$\mathcal{L}_\varepsilon \bar{g}_{0i} = a^2 \dot{\varepsilon}^i - \partial_i \varepsilon^0$
$\mathcal{L}_\varepsilon \bar{g}_{ij} = 2a^2 H \varepsilon^0 \delta_{ij} - a^2(\partial_i \varepsilon_j + \partial_j \varepsilon_i)$