Title: PSI 2017/2018 - Cosmology - Lecture 5

Date: Apr 13, 2018 10:15 AM

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Abstract:

Standard cosmological model

Kendrick Smith
PSI 2018, Explorations in Cosmology

Standard model of cosmology:

- Background metric is FRW
- Expansion history is ΛCDM
- Initial perturbations are Gaussian random
- Initial perturbations are scalar adiabatic
- Power spectrum of initial perturbations is a power law: $(k^3/2\pi^2)P(k) = \Delta_\zeta^2(k/k_0)^{n_s-1}$

Six parameters:

$h = 0.677 \pm 0.005$	Hubble parameter
$\Omega_\Lambda = 0.691 \pm 0.006$	Dark energy abundance (c.c.)
$\Omega_b = 0.0486 \pm 0.0007$	Baryonic(*) matter abundance
$\Delta_\zeta^2 = (2.11 \pm 0.05) \times 10^{-9}$	Initial power spectrum amplitude
$n_s = 0.967 \pm 0.004$	Spectral index
$\tau = 0.058 \pm 0.012$	CMB optical depth

(*) "Baryons" = protons + neutrons + electrons(!)

Fractional contributions to the energy density today (a=1):

$\Omega_\Lambda = 0.691$ cosmological constant

$\Omega_b = 0.0486$ baryonic matter

$\Omega_\gamma \sim$ (few x 10^{-5}) photons (precise value is HW problem)

$\Omega_\nu \sim$ (few x 10^{-5}) neutrinos (precise value is HW problem)

$\Omega_c = (1 - \Omega_\Lambda - \Omega_b - \Omega_\gamma - \Omega_\nu) \sim 0.26$ cold dark matter

Physical energy densities today (a=1)

$$\rho_i = \frac{\Omega_i}{\rho_{\rm tot}} \qquad \rho_{\rm tot} = \frac{3}{8\pi G} H_0^2 \qquad H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$$

At earlier times, energy density scales as $\rho_\gamma \sim a^{-3}$, $\rho_b \sim a^{-3}$, $\rho_c \sim a^{-3}$, $\rho_\Lambda \sim a^0$. For neutrinos, it is more complicated! (HW problem)

Expansion history follows from integrating Friedmann equation

$$H(t)^2 = \frac{8\pi G}{3} \rho(t)$$

Initial conditions: at early times, the FRW metric and stress-energy have small perturbations.

Metric: $ds^2 = -dt^2 + a(t)^2 e^{2\zeta(x)} dx^2$

where $\zeta(x)$ is a three-dimensional Gaussian random field with power-law power spectrum

$$\frac{k^3}{2\pi^2} P_\zeta(k) = \Delta_\zeta^2 \left(\frac{k}{0.05 \; h \; \text{Mpc}^{-1}} \right)^{n_s - 1}$$

with free parameters

$\Delta_\zeta^2 = (2.11 \pm 0.05) \times 10^{-9}$	Initial power spectrum amplitude
$n_s = 0.967 \pm 0.004$	Spectral index

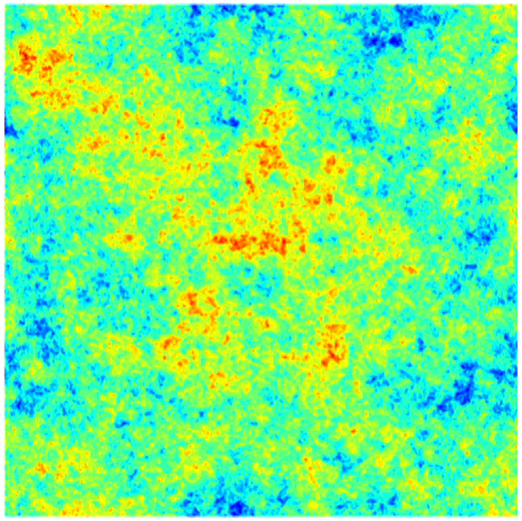
The field $\zeta(x)$ is called the "adiabatic curvature" or the "initial curvature".

Let's interpret this power spectrum:

$$\frac{k^3}{2\pi^2} P_\zeta(k) = \Delta_\zeta^2 \left(\frac{k}{0.05\ h\ \mathrm{Mpc}^{-1}} \right)^{n_s - 1}$$

$$\Delta_\zeta^2 = 2.11 \times 10^{-9}$$
$$n_s = 0.967$$

- Initial perturbations are self-similar (no preferred scale)
- Almost scale-invariant, small trend toward more power on large scales.
- Characteristic size of fluctuations is $\Delta_\zeta \sim (5 \times 10^{-5})$



Initial perturbations are "scalar adiabatic".

- "Scalar" means that there are no gravity wave perturbations in the initial metric. (Some models of inflation predict this, but so far it has not been observed.)

$$ds^2 = -dt^2 + a(t)^2 e^{2\zeta(x)}(\delta_{ij} + h_{ij}(x))$$

absent

- "Adiabatic" is more technical. It means that the ζ field also completely determines the perturbations in the stress-energy tensor, by a universal set of rules which will be explained later!

$$\rho(\mathbf{x}, t) = \bar{\rho}(t)\left(1 + \frac{4}{7}\zeta(\mathbf{x})\right)$$
$$\cdots$$

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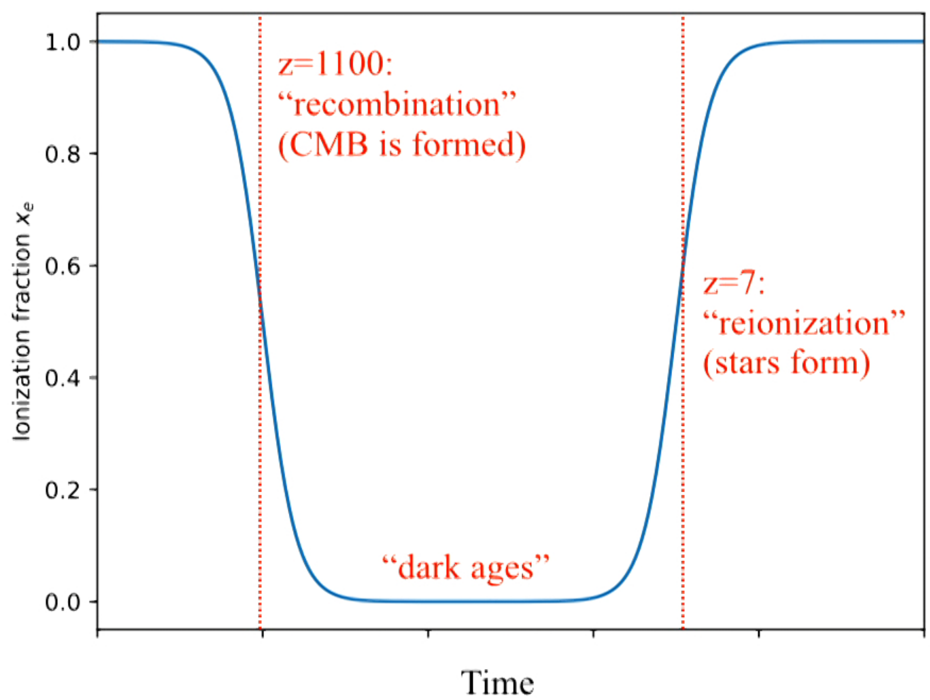
Ionization history of the universe

$x_e(t)$ = electron ionization fraction

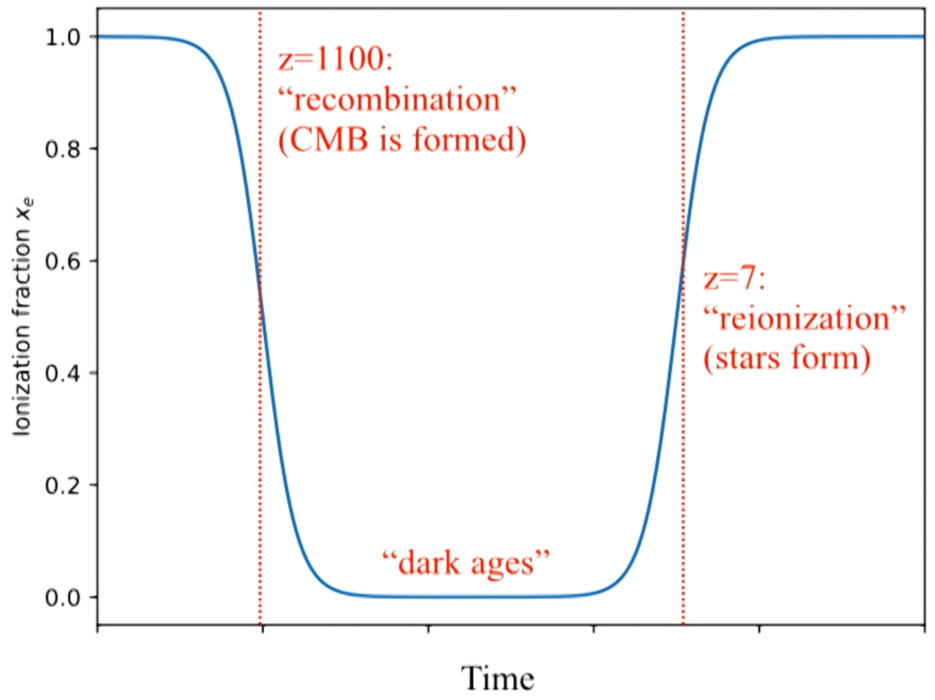
= probability that a random electron in the universe is ionized (rather than being part of an atom)



Ionization history of the universe

τ = CMB optical depth

= probability that a CMB photon emitted at z~1100 scatters from an electron at low redshift, before being observed at z=0.



Ionization history of the universe

τ = CMB optical depth

= probability that a CMB photon emitted at $z \sim 1100$ scatters from an electron at low redshift, before being observed at $z=0$.

Astrophysical nuisance parameter: τ affects the CMB power spectrum.

When fitting cosmological parameters from the CMB, we need to include τ in the fit, and account for uncertainty in τ when assigning errors to other parameters.

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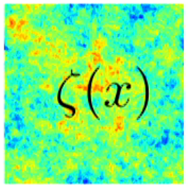
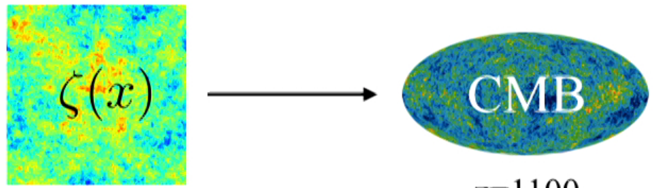


High-z

The standard cosmological model specifies the perturbations at very early times (high-z). They are fairly simple, and parameterized by a Gaussian random field $\zeta(x)$ with a featureless power spectrum.

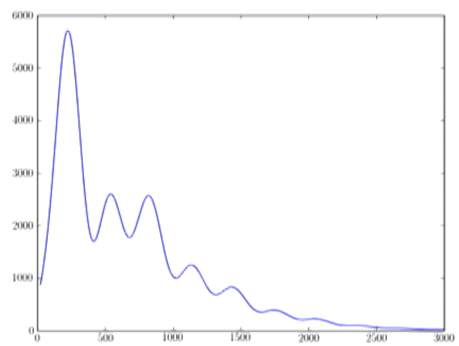


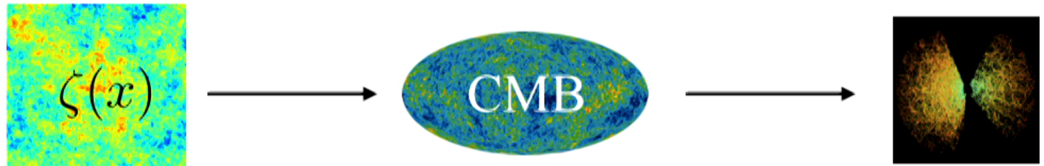
High-z

z=1100

The standard cosmological model specifies the perturbations at very early times (high-z). They are fairly simple, and parameterized by a Gaussian random field ζ(x) with a featureless power spectrum.

As time evolves, the perturbations become more complex. By the time the CMB is formed (z=1100), a lot of physics has been "imprinted" on the power spectrum.





High-z   z=1100   z~1

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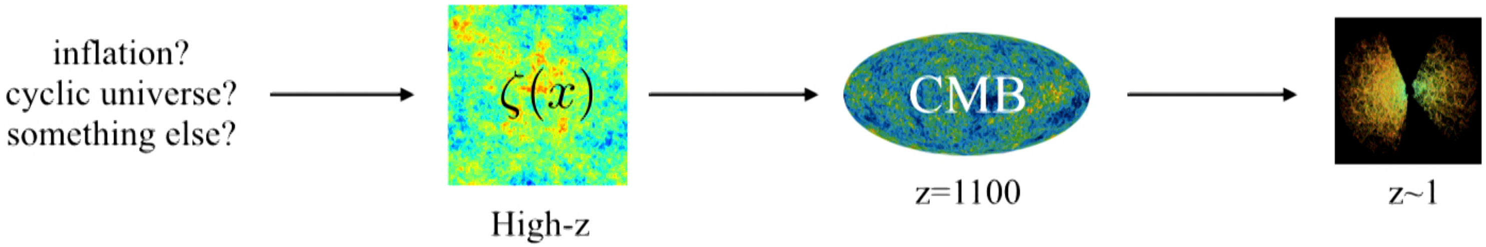
At late times (z~1), nonlinear effects are important and the perturbations are very non-Gaussian.



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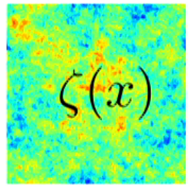
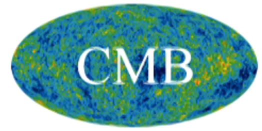
At late times (z~1), nonlinear effects are important and the perturbations are very non-Gaussian.

There are also models for the "early universe", a hypothetical phase preceding the radiation-dominated part of the expansion, which try to explain where the Gaussian field ζ came from!

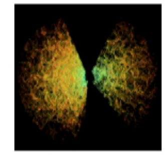
inflation?
cyclic universe?
something else?



High-z



z=1100

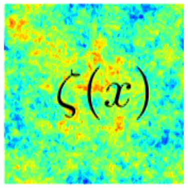


z~1

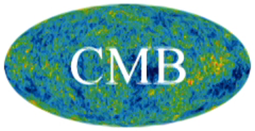
In each of these three stages, different physics is important:

- Early universe: Quantum mechanics in expanding spacetime generates Gaussian perturbations from vacuum
- Formation of the CMB: Linear perturbation theory in a plasma with multiple components (dark matter, baryons, photons, neutrinos) + metric degrees of freedom
- Late times: Gravitational N-body physics. Messy astrophysics! (galaxy formation, star formation, …)

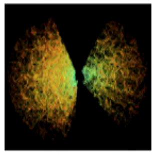
inflation?
cyclic universe?
something else?



High-z



z=1100


z~1

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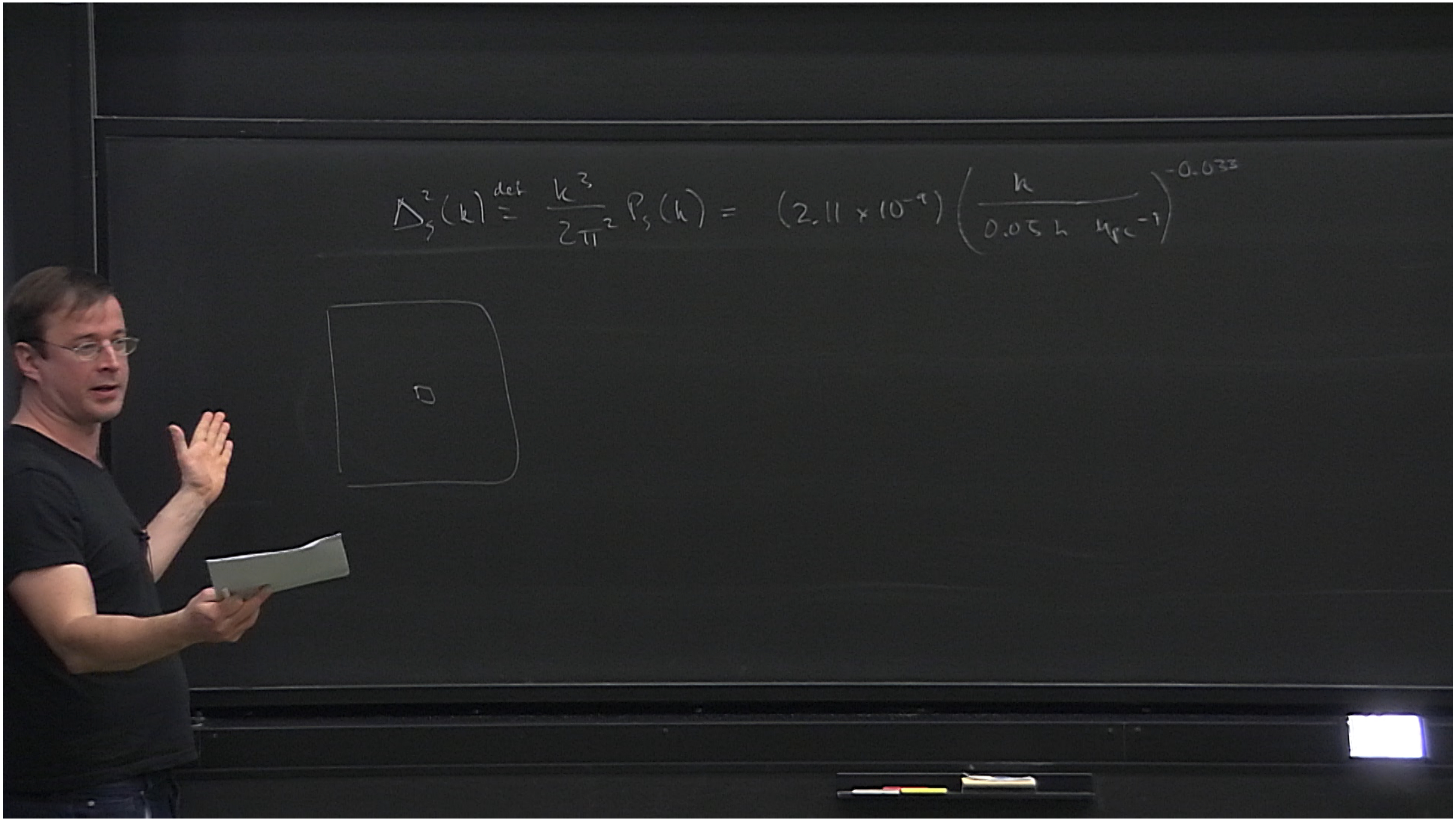
Cosmology is largely concerned with looking for extensions of the 6-parameter standard model.

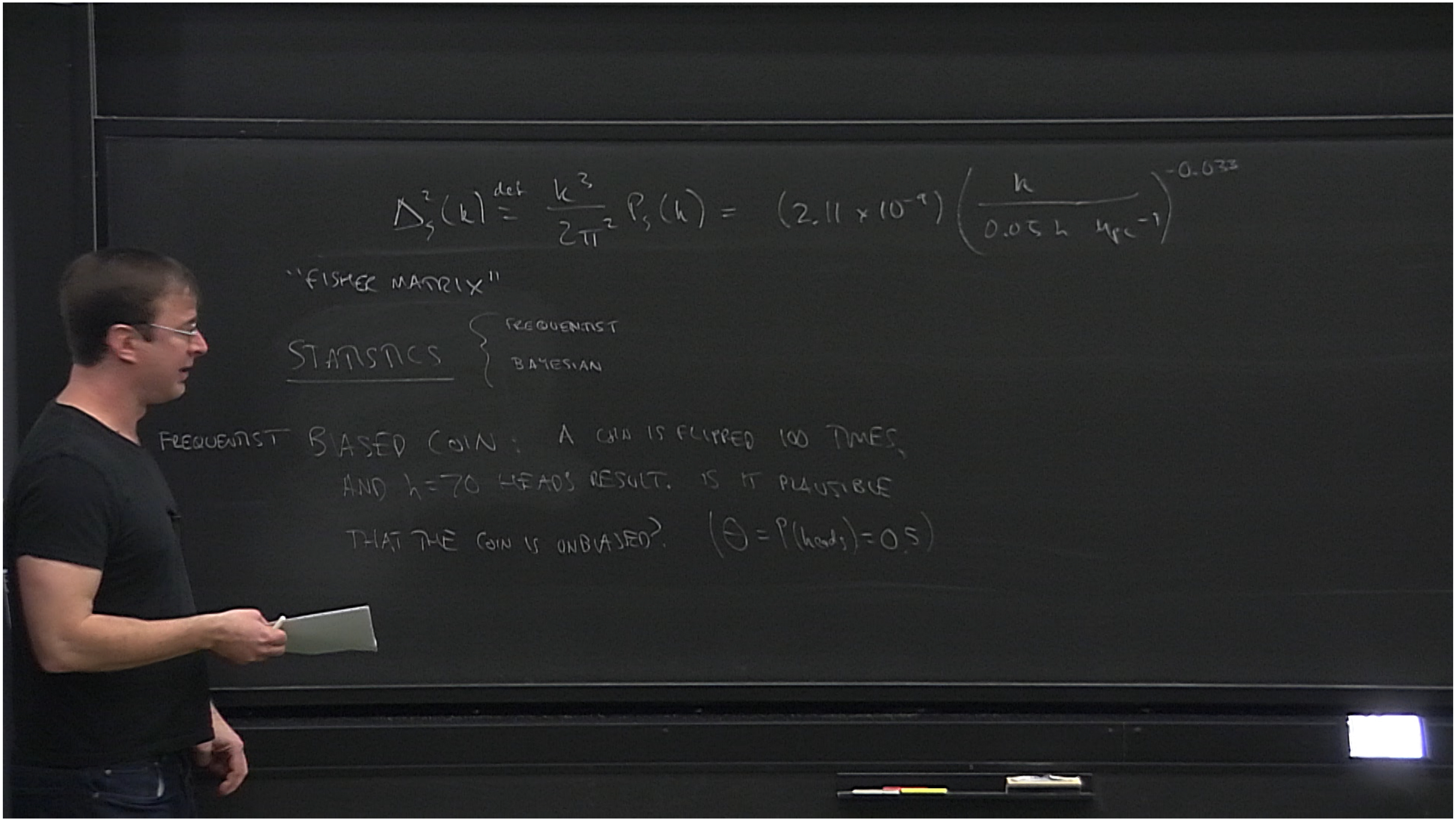
- Non-Gaussian initial conditions
- Non-minimal neutrino mass
- Extra neutrino species or other light relics
- Interacting dark matter
- Nonzero spatial curvature
- Cosmological gravity waves

+ many others!

The standard model includes ingredients which were originally surprises (dark matter, cosmological constant, quantum mechanically generated perturbations).

Can we understand these phenomena in more detail? Will we find new surprises?


$\Delta_s^2(k) \overset{def}{=} \frac{k^3}{2\pi^2} P_s(k) = (2.11 \times 10^{-9}) \left(\frac{k}{0.05\,h\ \mathrm{Mpc}^{-1}}\right)^{-0.033}$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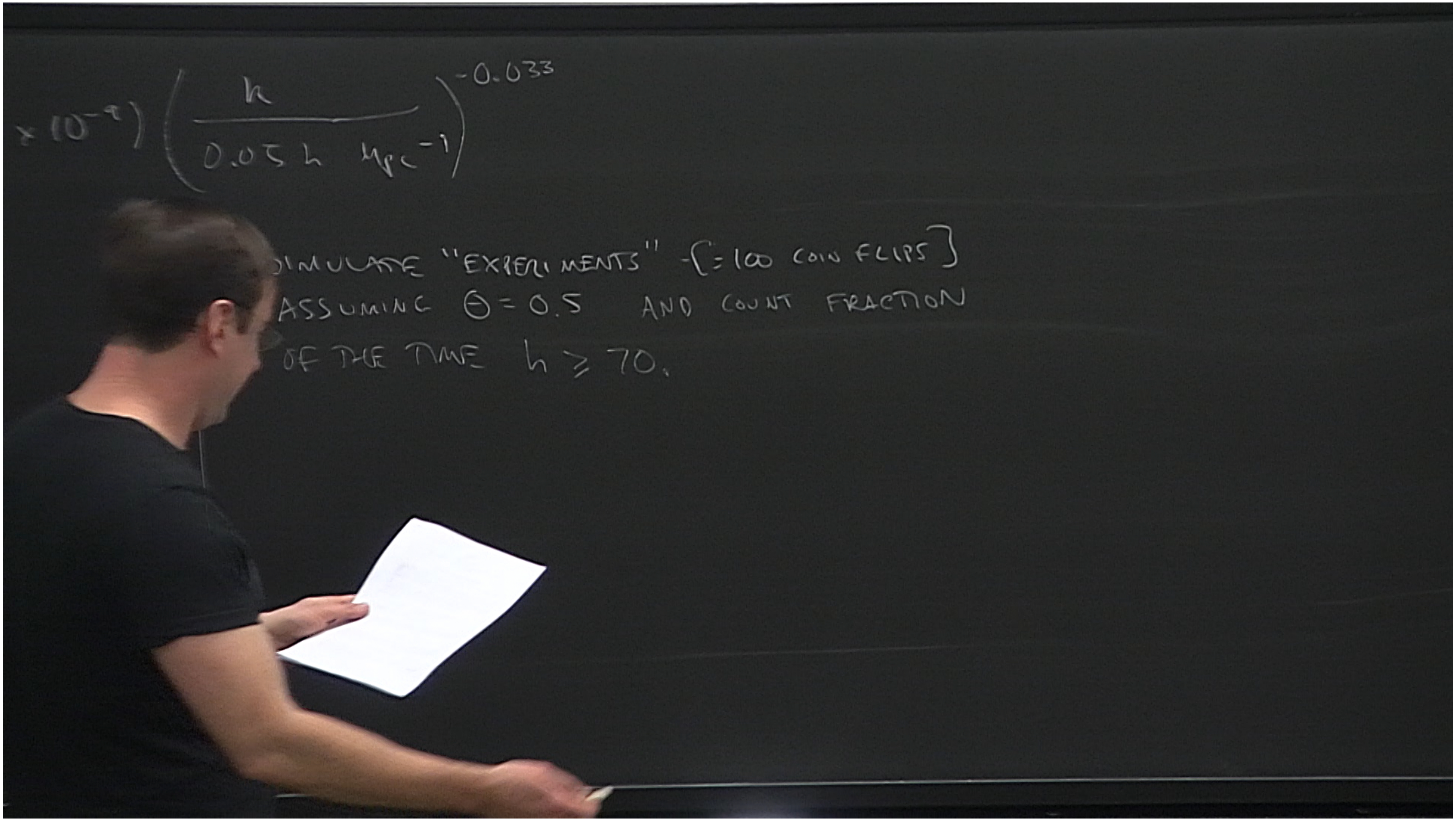
"FISHER MATRIX"
STATISTICS
FREQUENTIST
BAYESIAN
FREQUENTIST
BIASED COIN: A COIN IS FLIPPED 100 TIMES,
AND h=70 HEADS RESULT. IS IT PLAUSIBLE
THAT THE COIN IS UNBIASED? $(\theta = P(\mathrm{heads}) = 0.5)$


IMULATE "EXPERIMENTS" - [= 100 COIN FLIPS]
ASSUMING Θ = 0.5 AND COUNT FRACTION
OF THE TIME h ≥ 70.

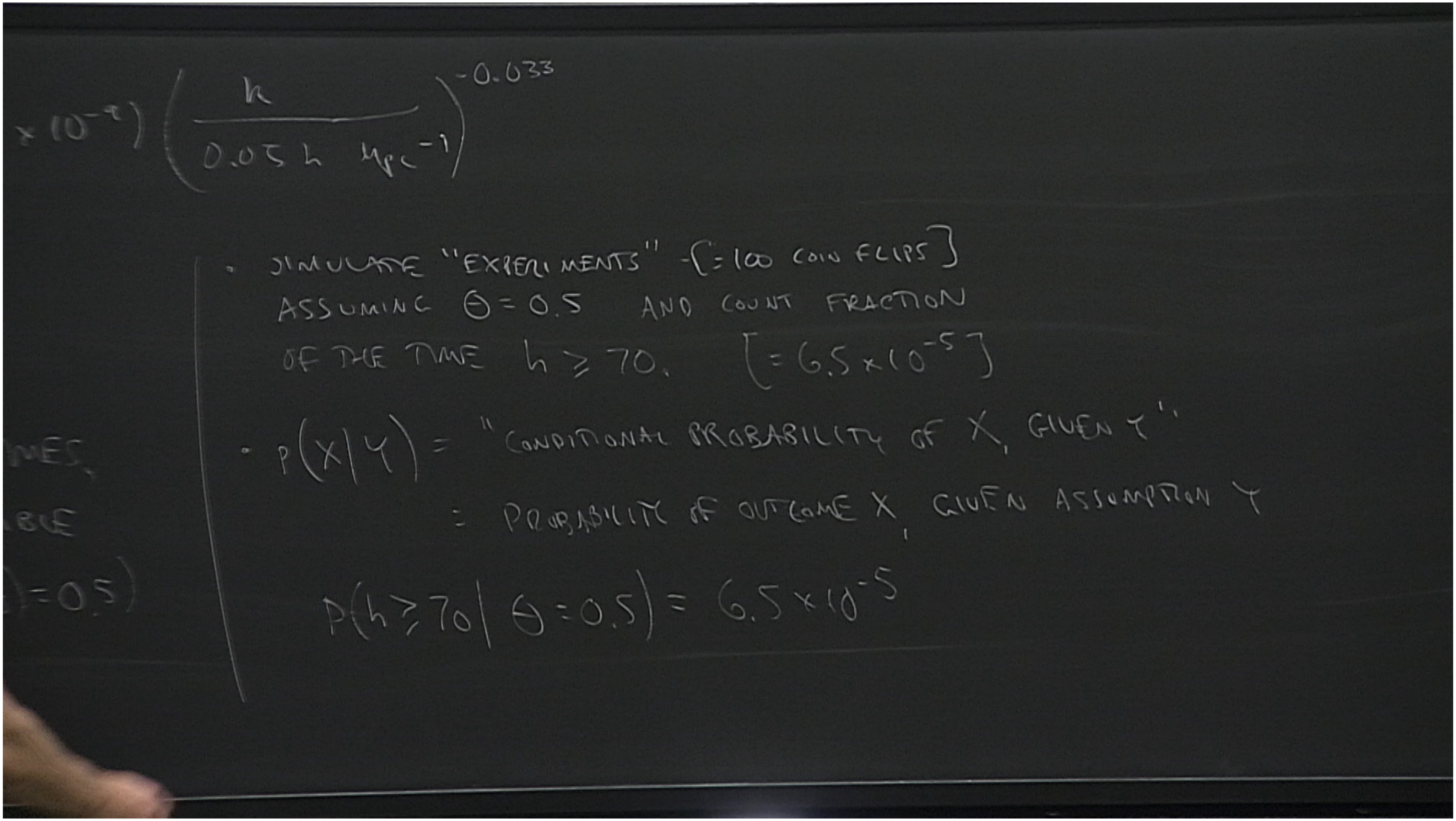
$$\times 10^{-9}) \left(\frac{h}{0.05\,h\ \mathrm{Mpc}^{-1}} \right)^{-0.033}$$

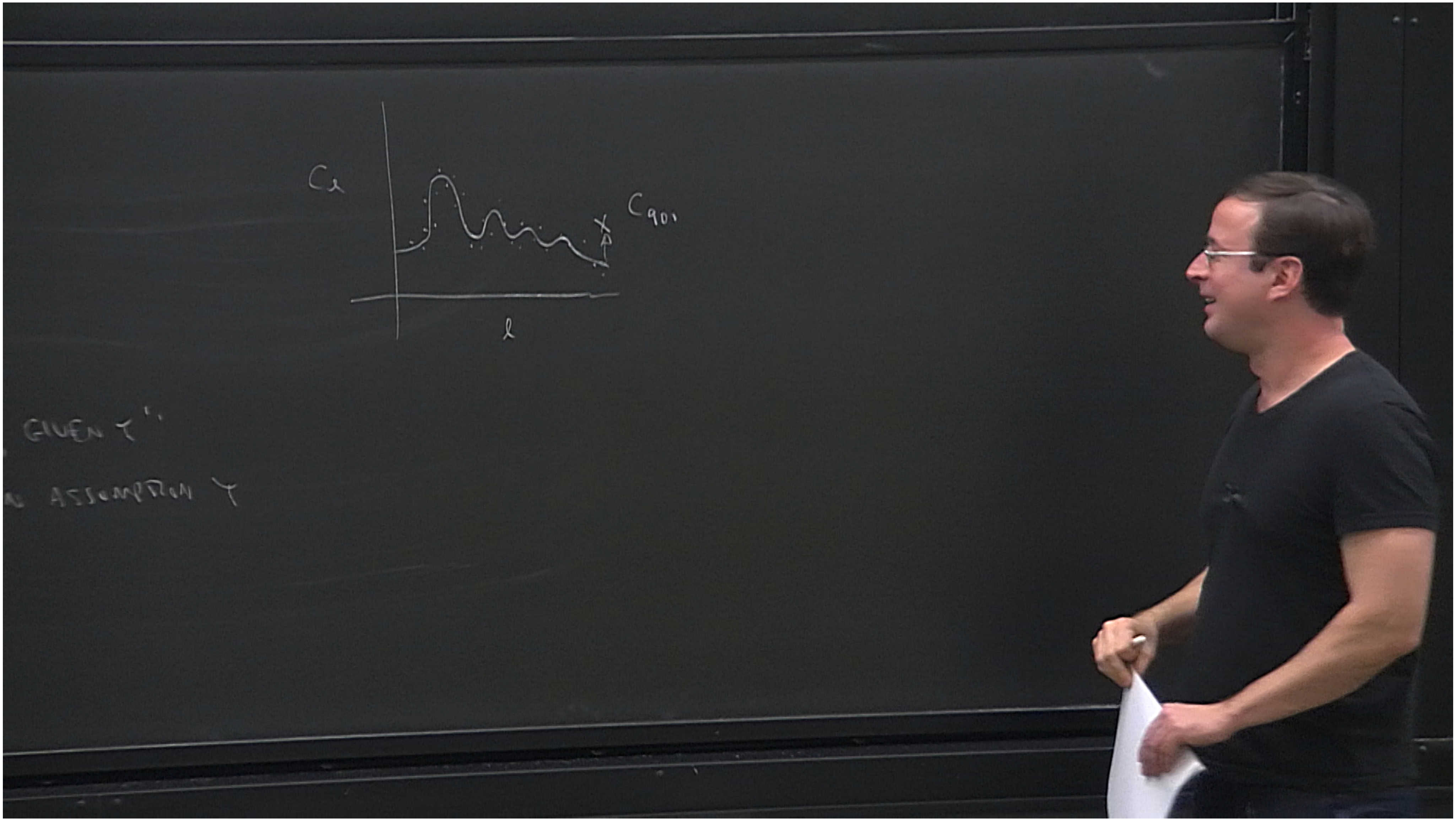
MES,

BLE

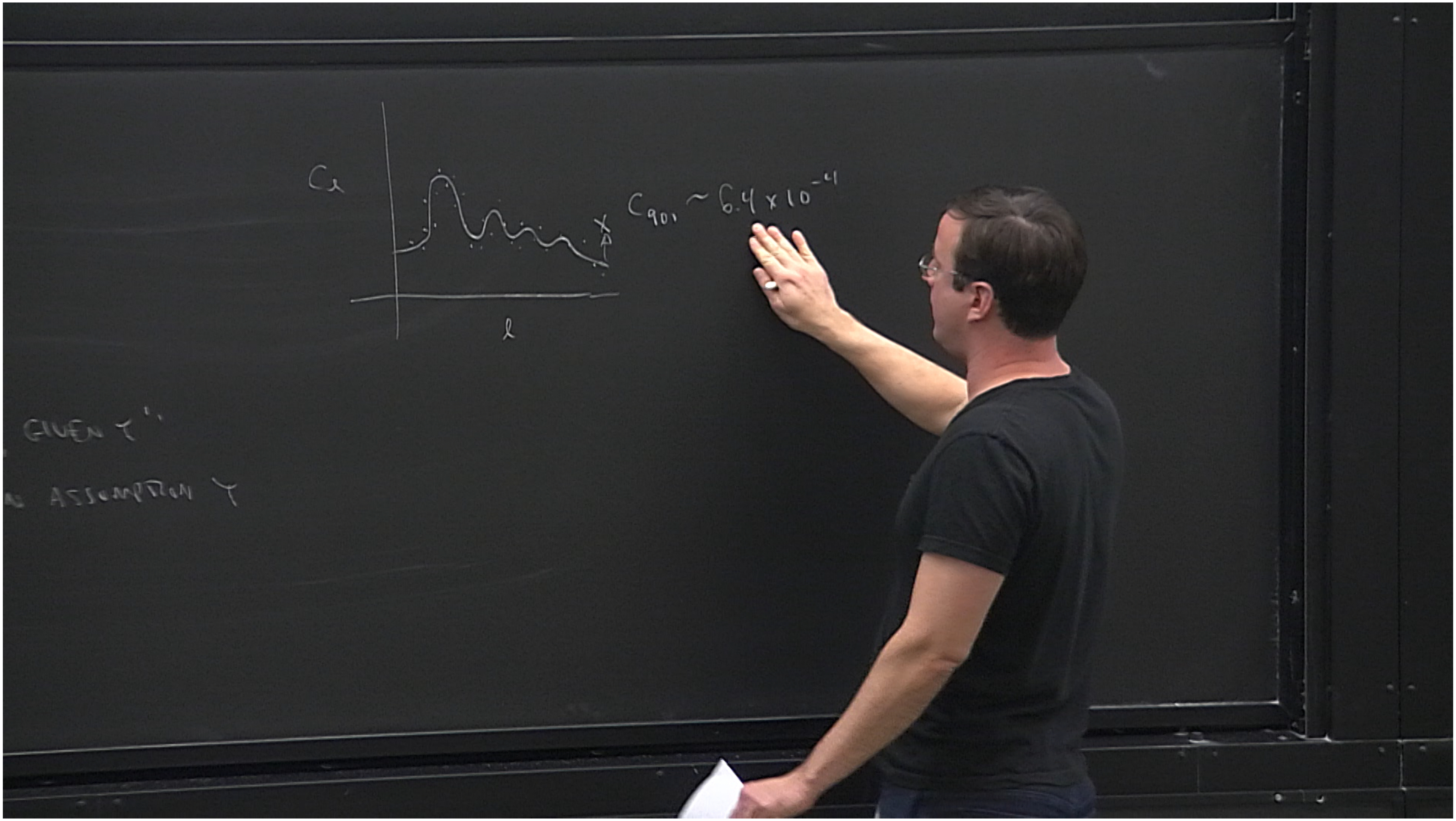
$) = 0.5)$

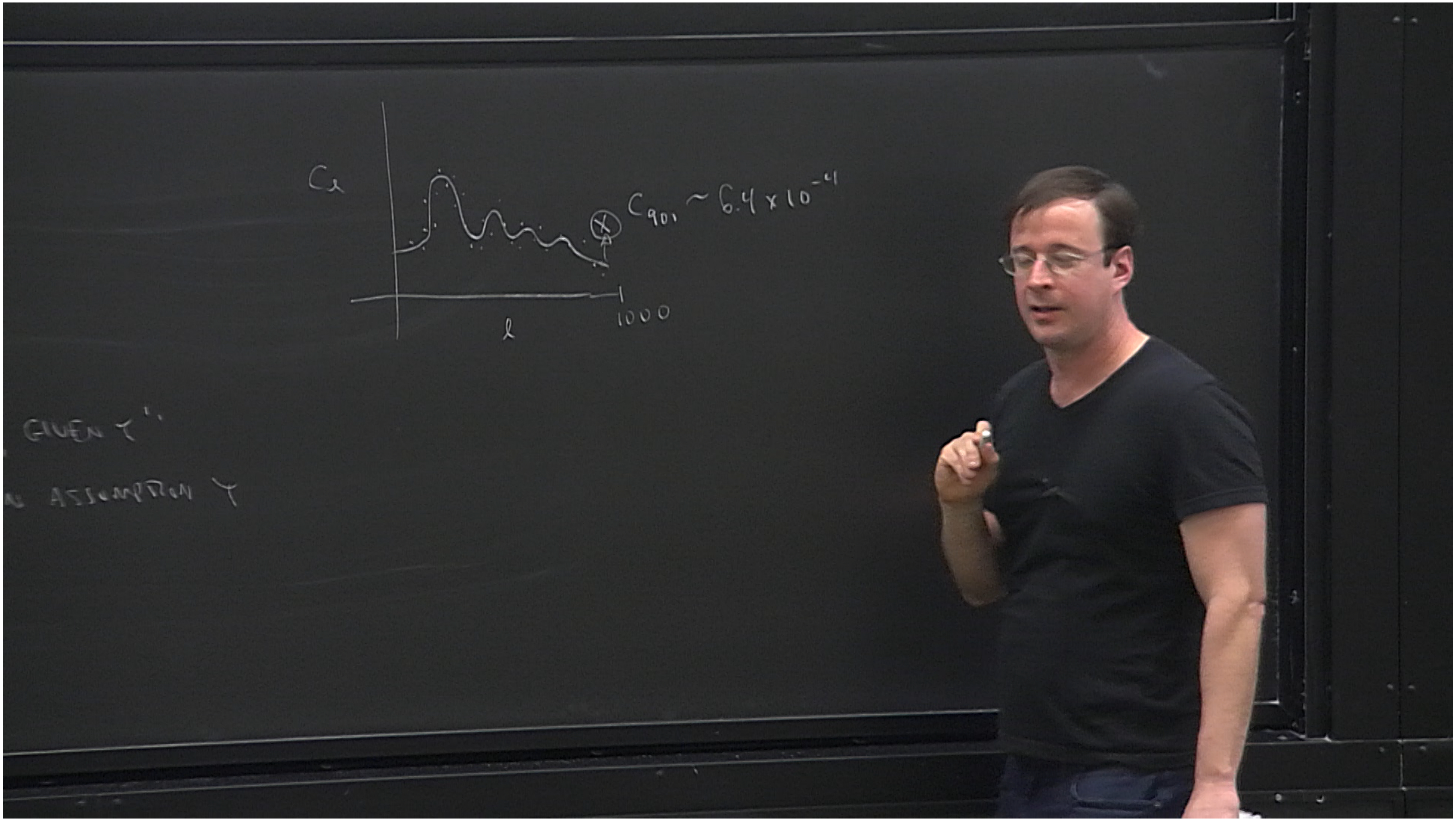
SIMULATE "EXPERIMENTS" - [= 100 COIN FLIPS]
ASSUMING $\Theta = 0.5$ AND COUNT FRACTION
OF THE TIME $h \geq 70$. $[= 6.5 \times 10^{-5}]$


$\times 10^{-9}) \left(\frac{h}{0.05\, h \ \mathrm{Mpc}^{-1}} \right)^{-0.033}$
• SIMULATE "EXPERIMENTS" - [= 100 COIN FLIPS]
ASSUMING $\Theta = 0.5$ AND COUNT FRACTION
OF THE TIME $h \geq 70$. [$= 6.5 \times 10^{-5}$]
• $P(X|Y)$ = "CONDITIONAL PROBABILITY OF X, GIVEN Y"
= PROBABILITY OF OUTCOME X, GIVEN ASSUMPTION Y
$P(h \geq 70 \mid \Theta = 0.5) = 6.5 \times 10^{-5}$

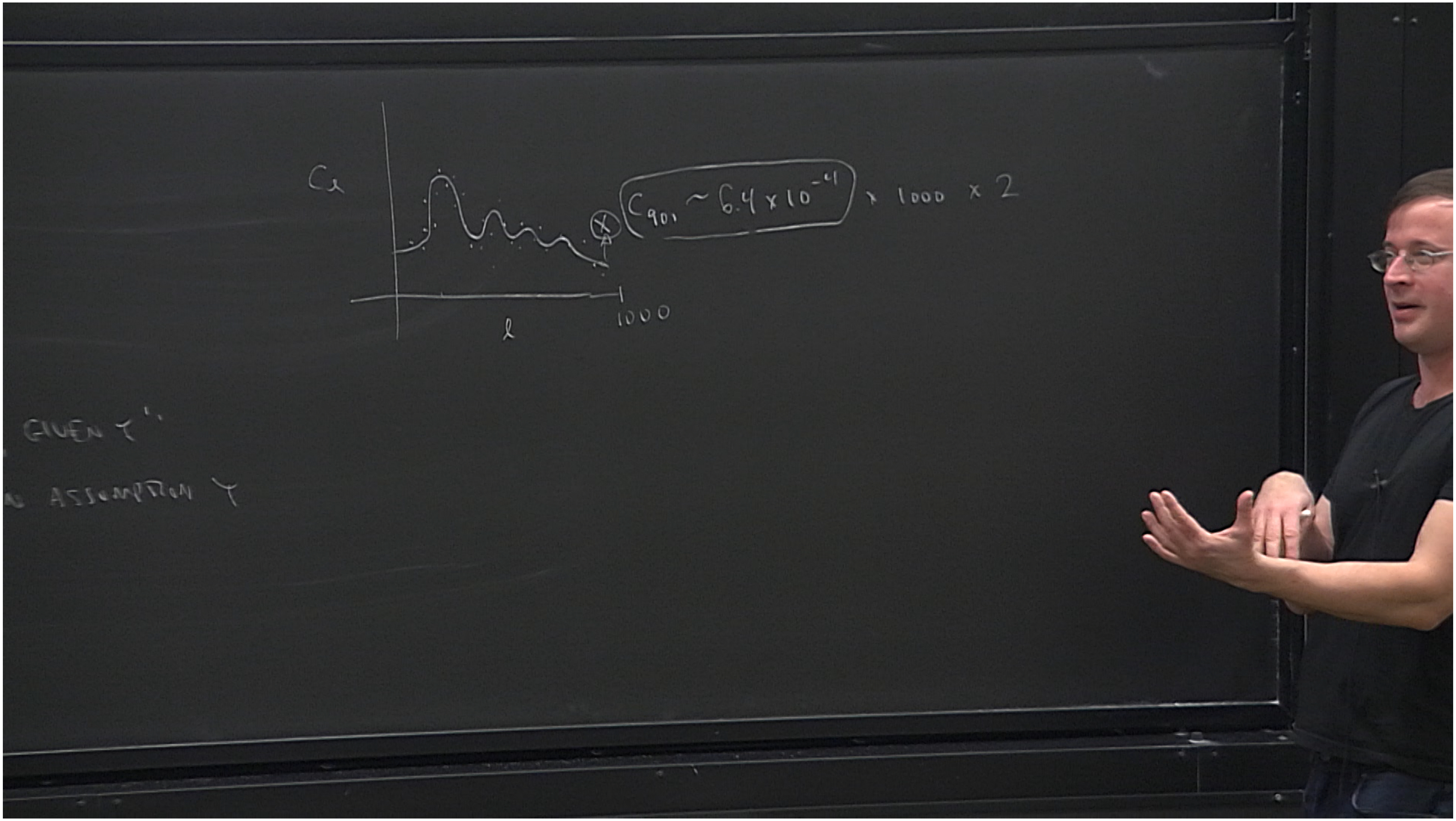


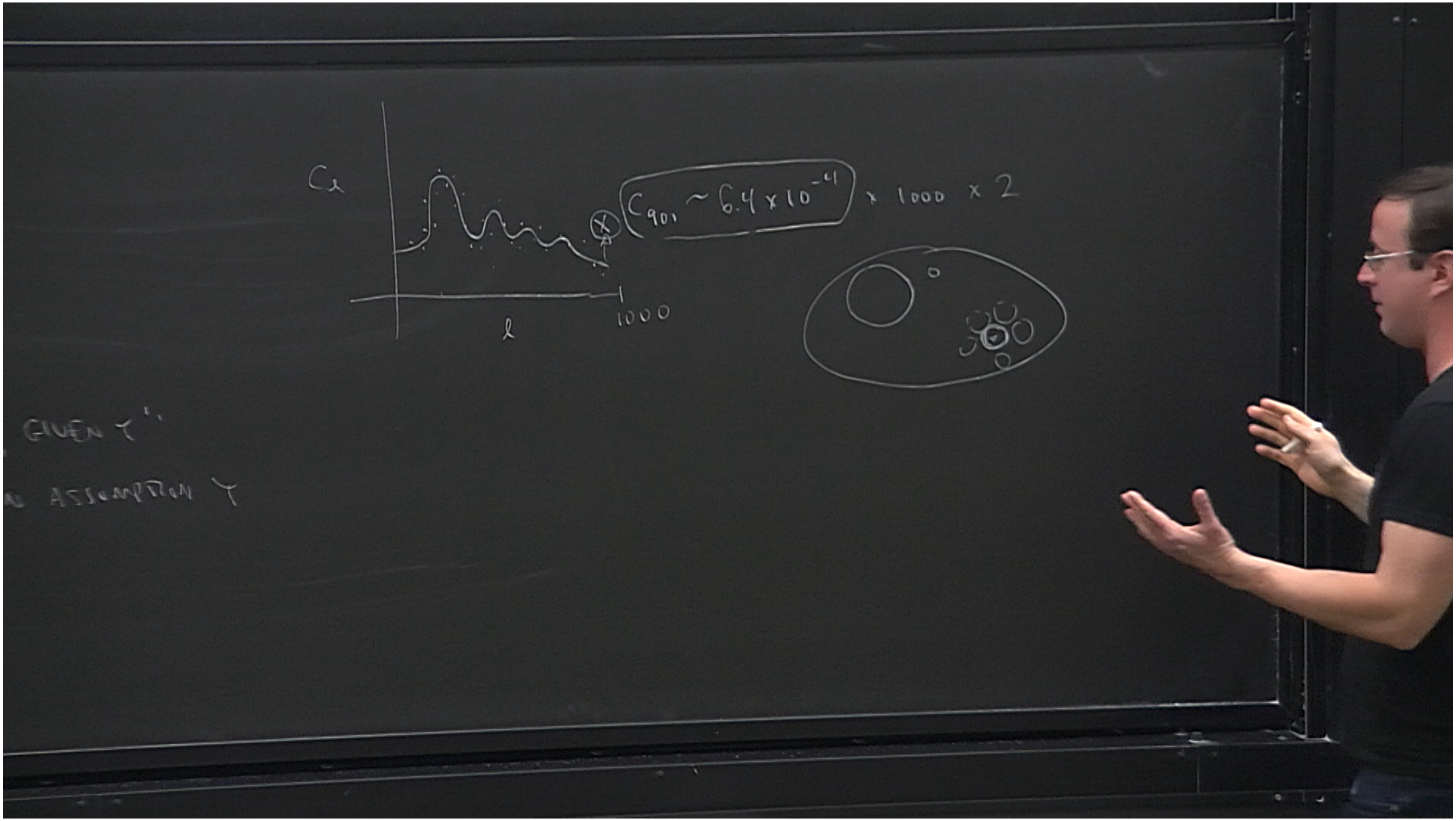
C_ℓ
C_{90}
ℓ
GIVEN
ASSUMPTION

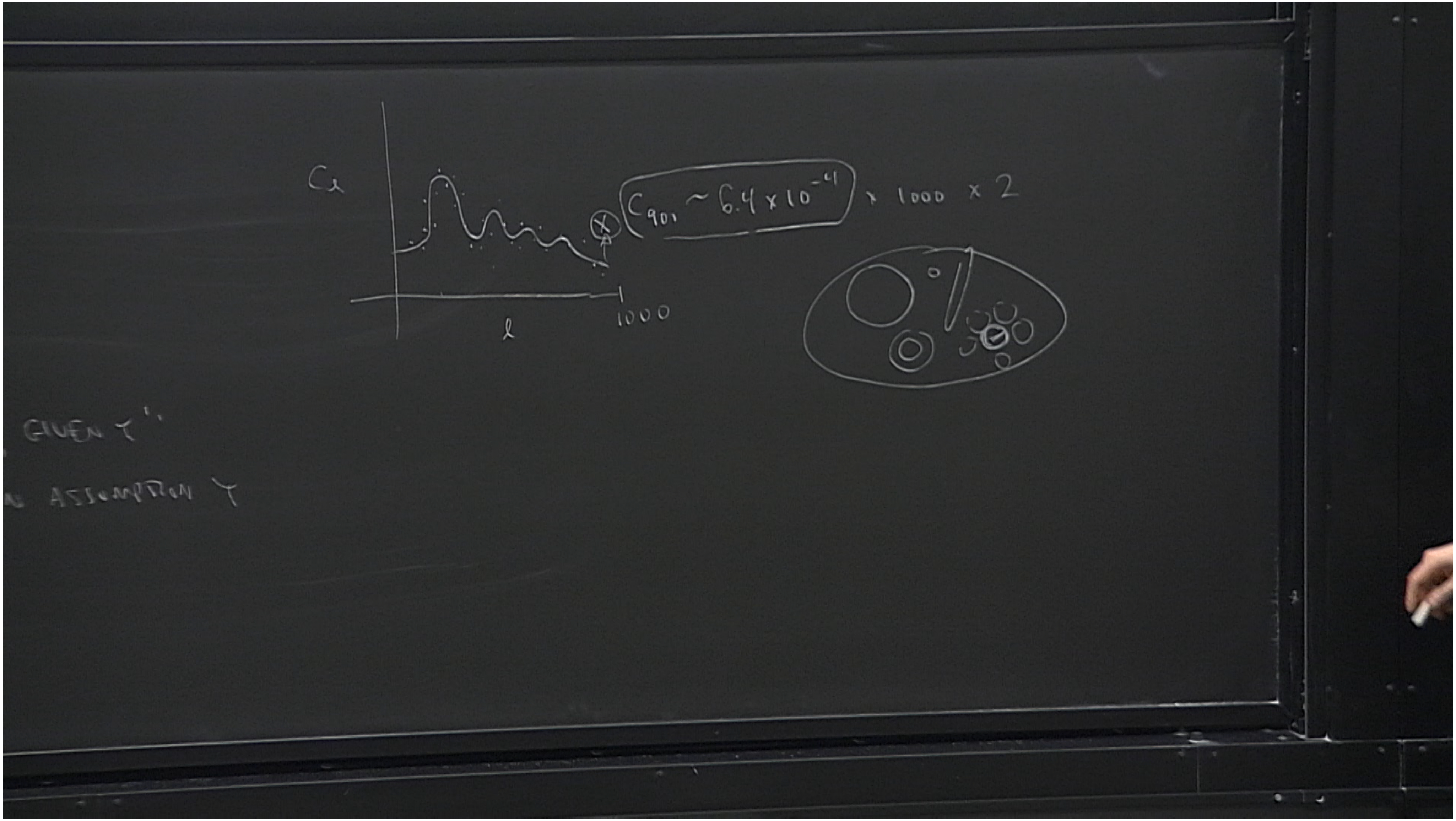

C_ℓ
ℓ
$C_{901} \sim 6.4 \times 10^{-4}$
GIVEN
ASSUMPTION

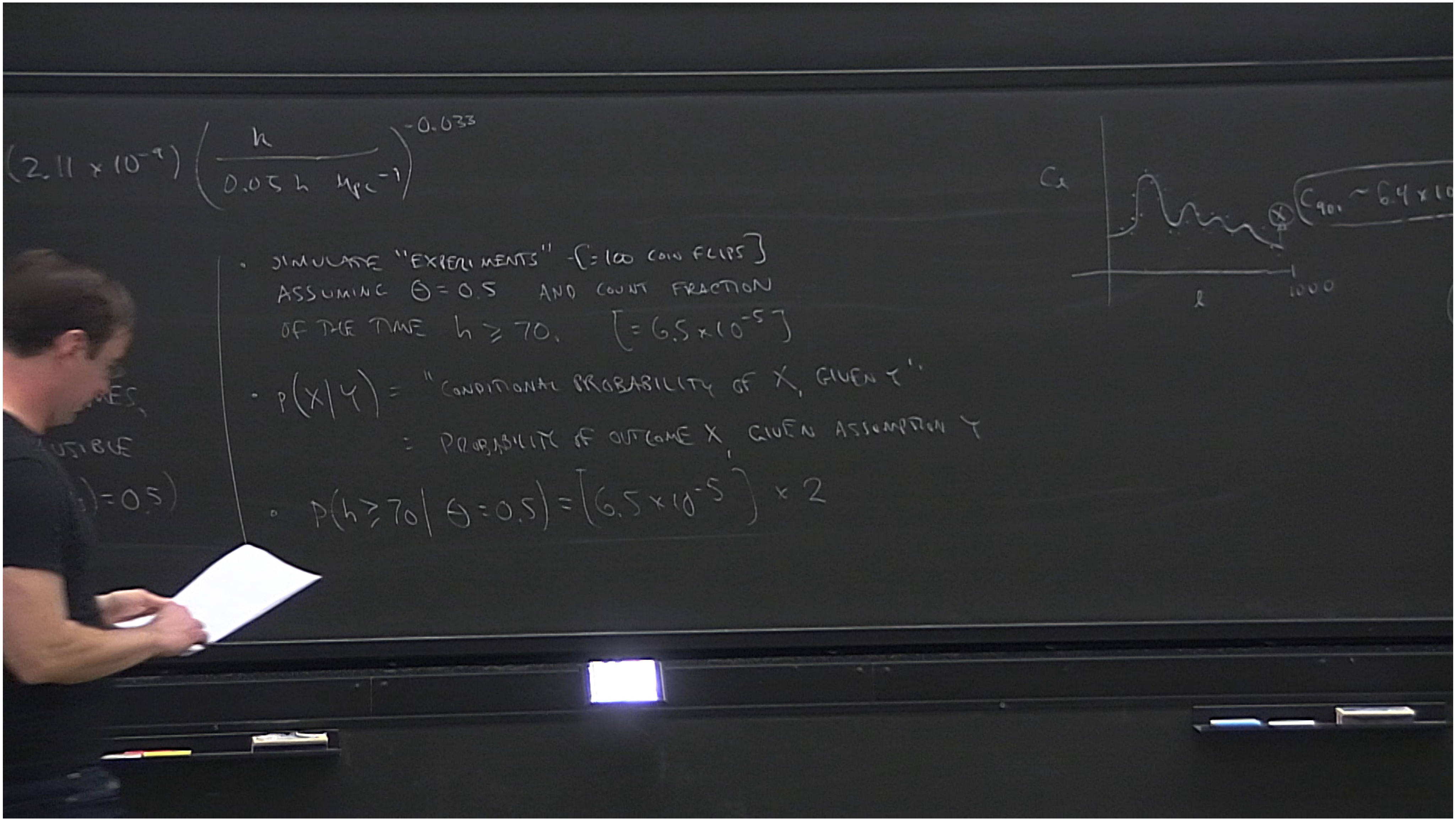


C_ℓ
$C_{901} \sim 6.4 \times 10^{-4}$
1000
ℓ
GIVEN
ASSUMPTION

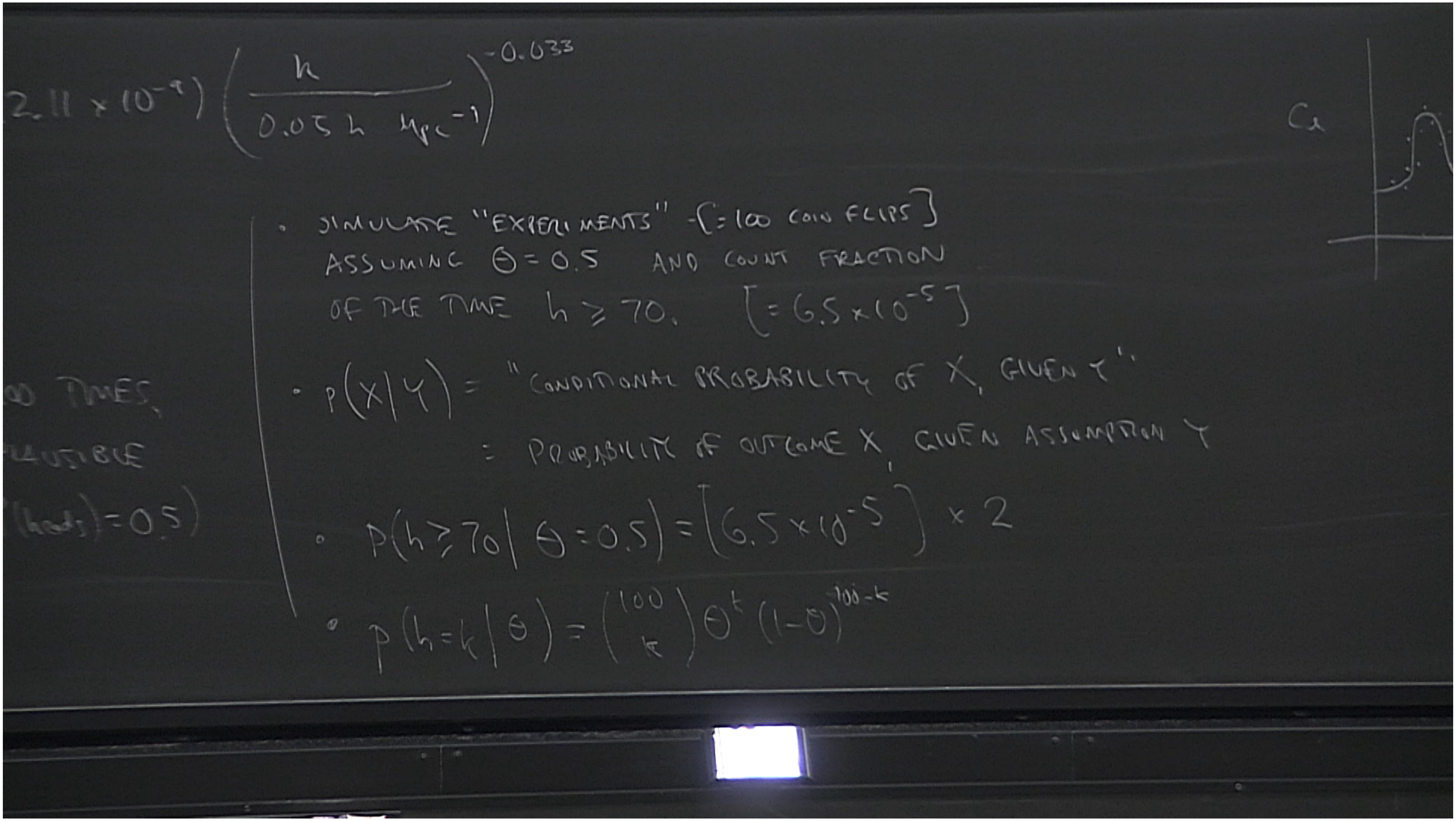

C_l
l
1000
C_901 ~ 6.4 x 10^-4 x 1000 x 2
GIVEN
ASSUMPTION


Cℓ
C901 ~ 6.4 x 10^-4 x 1000 x 2
ℓ
1000
GIVEN
ASSUMPTION


C_ℓ
ℓ
1000
$C_{901} \sim 6.4 \times 10^{-4}$ x 1000 x 2
GIVEN
ASSUMPTION



SIMULATE "EXPERIMENTS" [= 100 COIN FLIPS]
ASSUMING Θ = 0.5 AND COUNT FRACTION
OF THE TIME h ≥ 70. [= 6.5 × 10^-5]
p(X|Y) = "CONDITIONAL PROBABILITY OF X, GIVEN Y"
= PROBABILITY OF OUTCOME X, GIVEN ASSUMPTION Y
P(h ≥ 70 | Θ = 0.5) = [6.5 × 10^-5] × 2
1000


2.11 × 10^-9) (h / 0.05 h Mpc^-1)^-0.033
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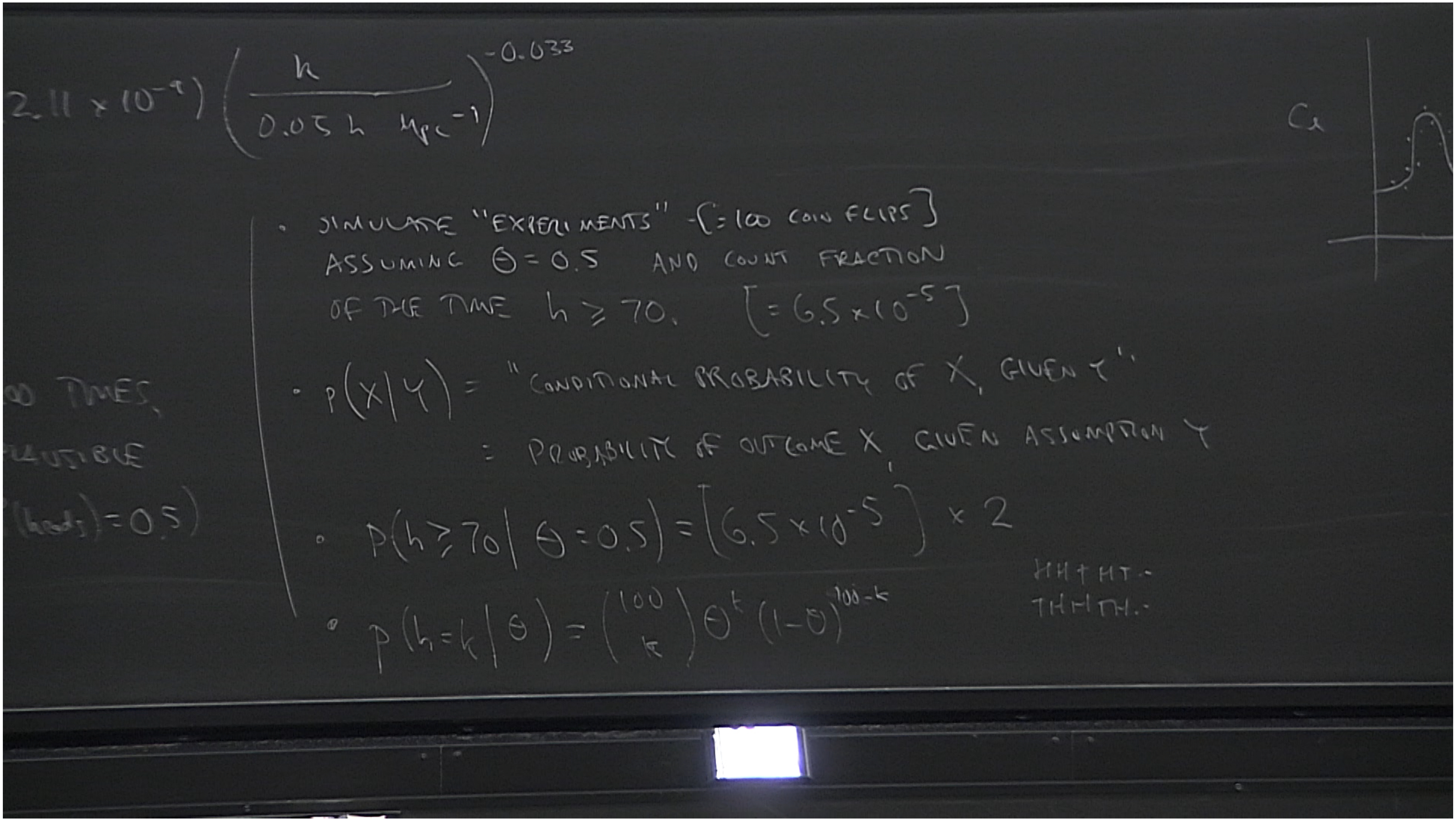
• p(h = k | Θ) = (100 k) Θ^k (1-Θ)^100-k
00 TIMES,
PLAUSIBLE
(heads) = 0.5)



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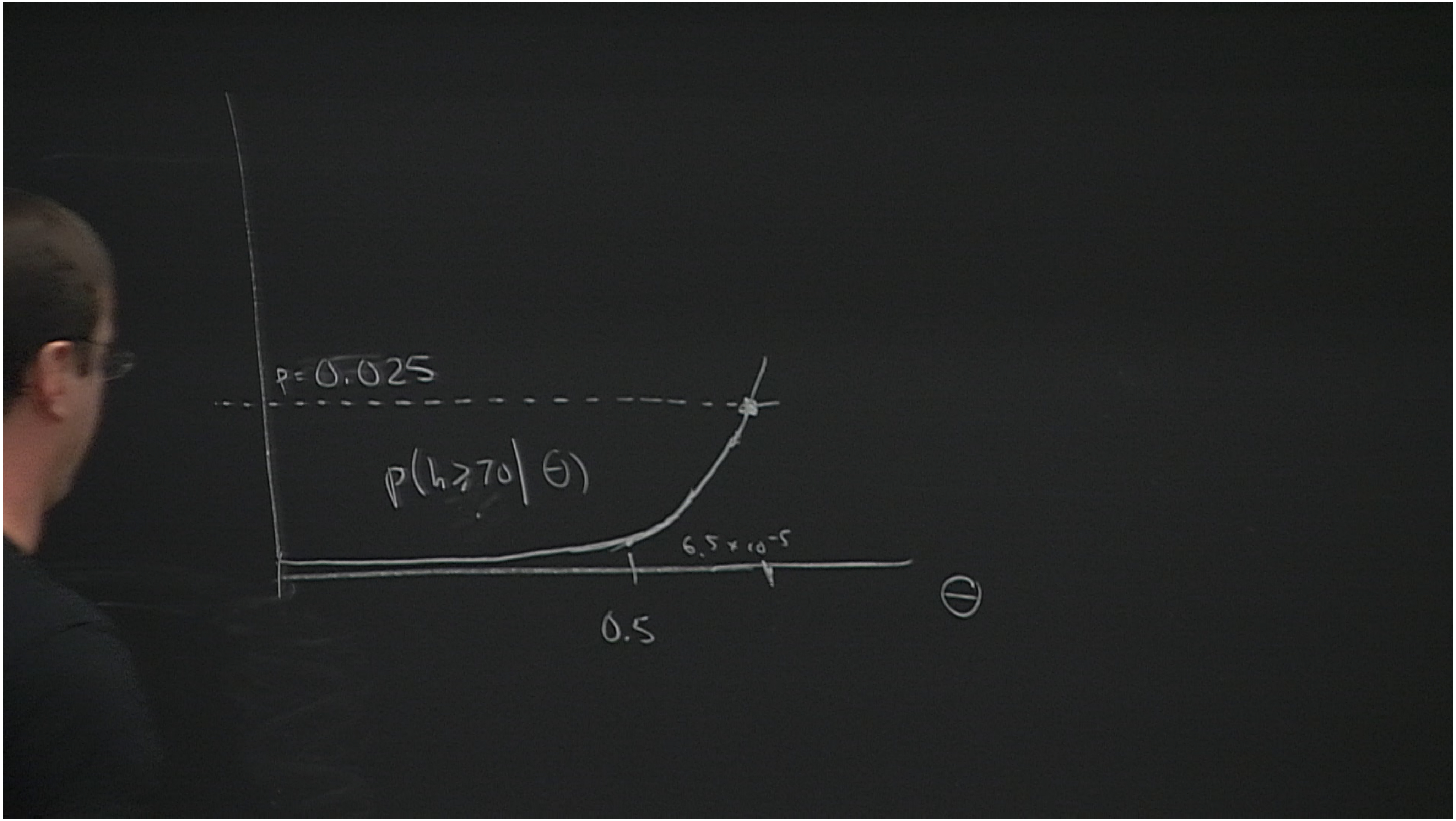
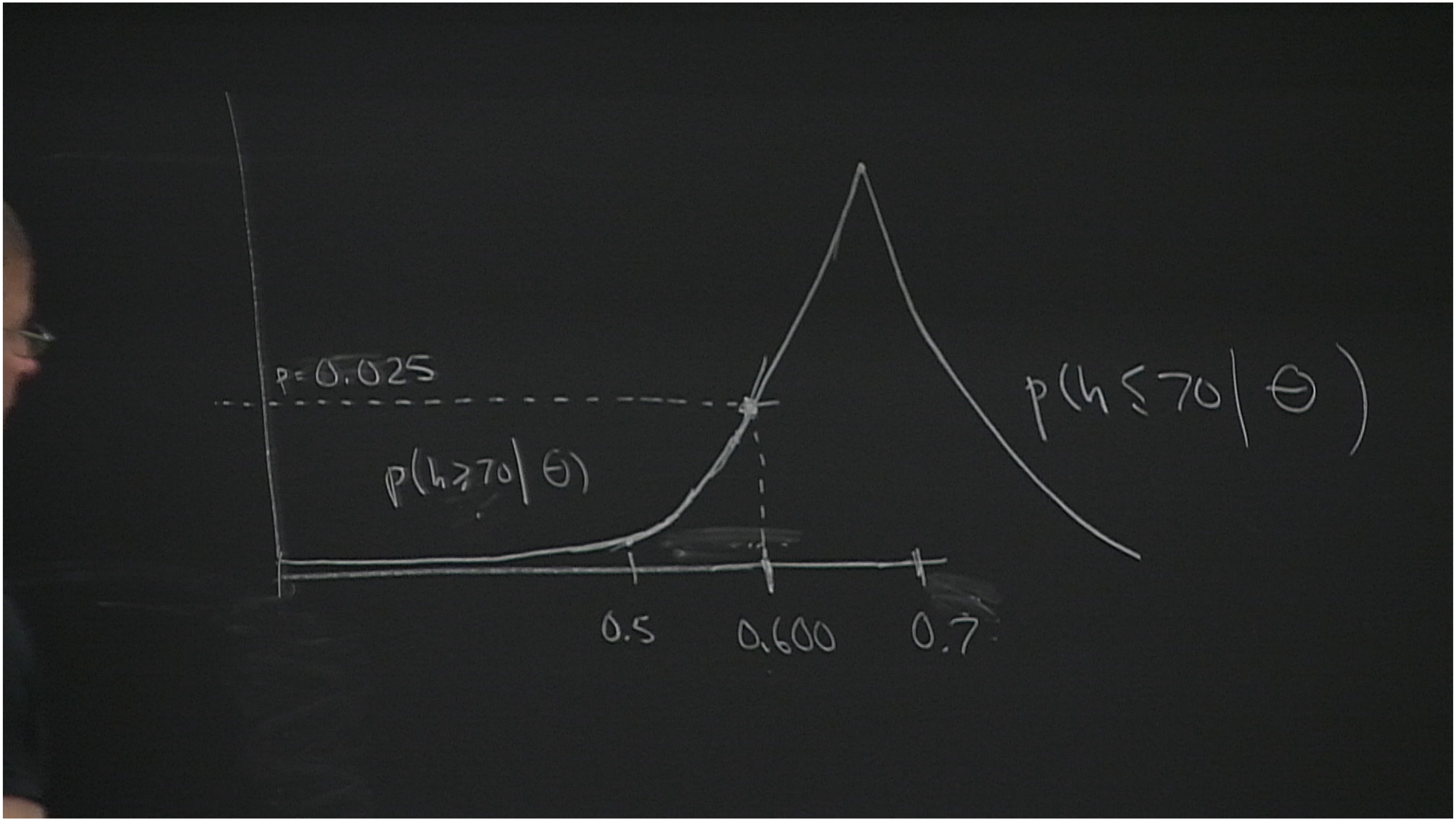
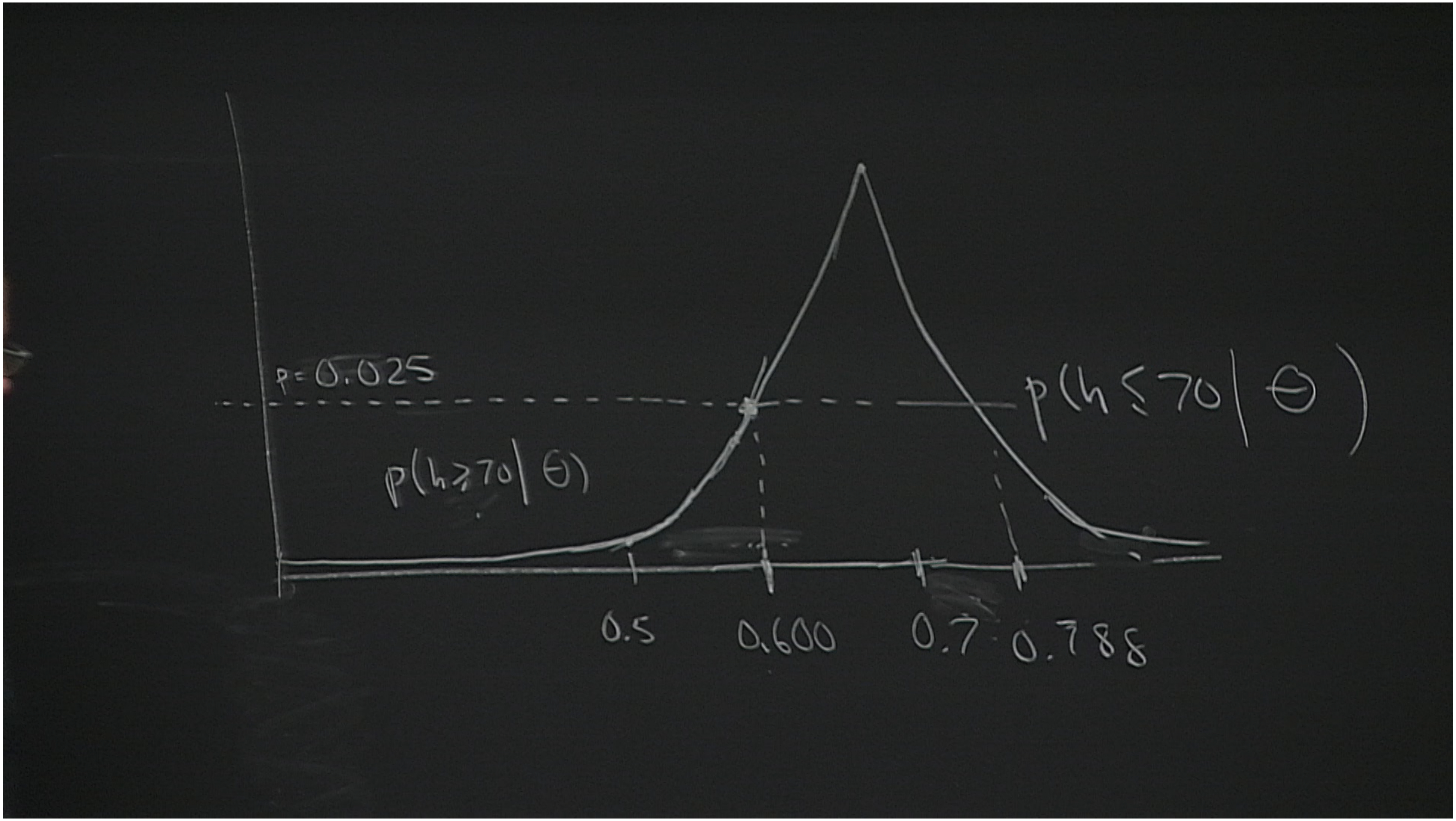
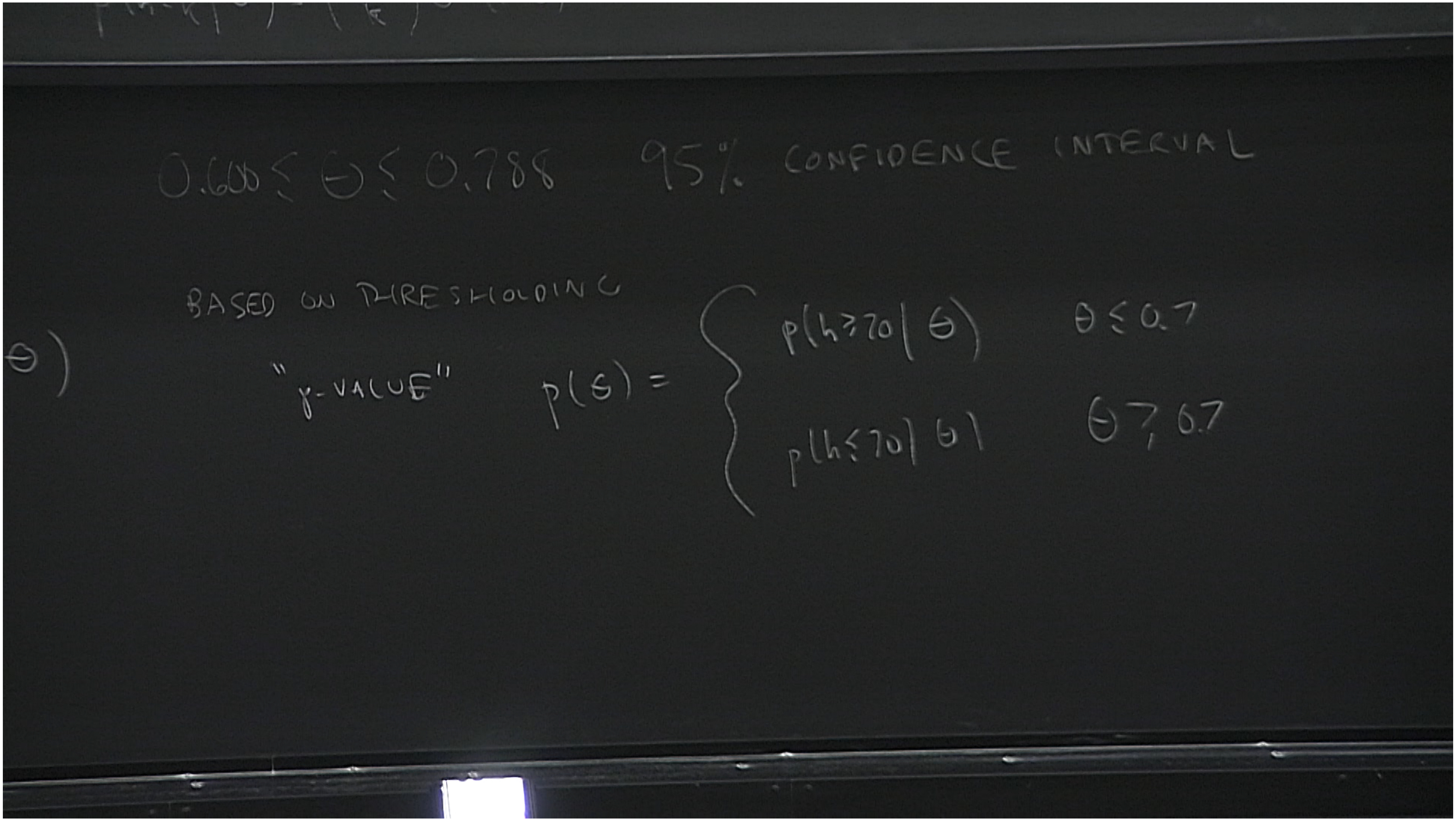
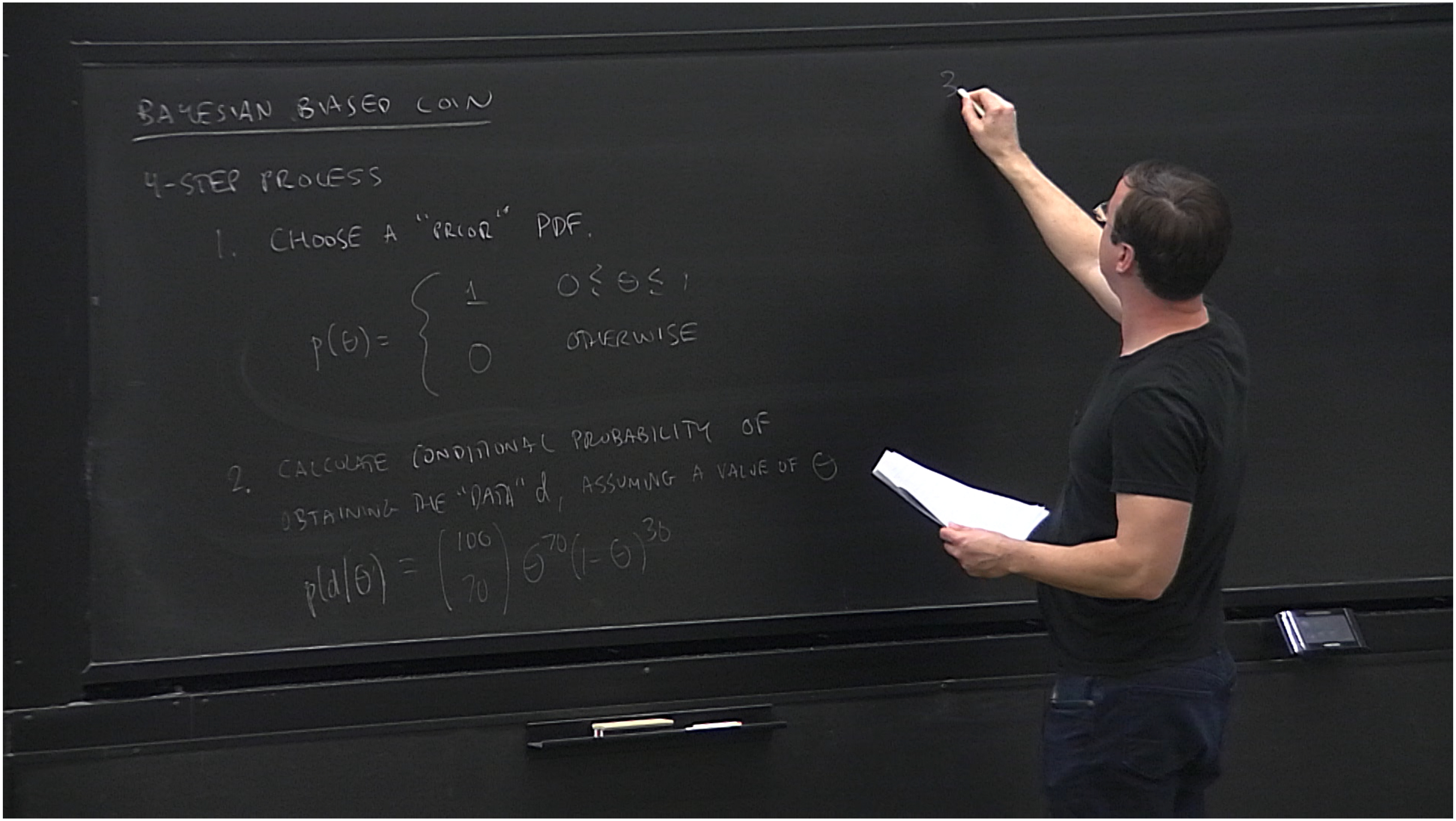
p(X|Y) = "CONDITIONAL PROBABILITY OF X, GIVEN Y"
= PROBABILITY OF OUTCOME X, GIVEN ASSUMPTION Y
p(h ≥ 70 | Θ = 0.5) = [6.5 × 10^-5] × 2
p(h = k | Θ) = (100 k) Θ^k (1 − Θ)^(100−k)
HH + HT ...
THHTH ...

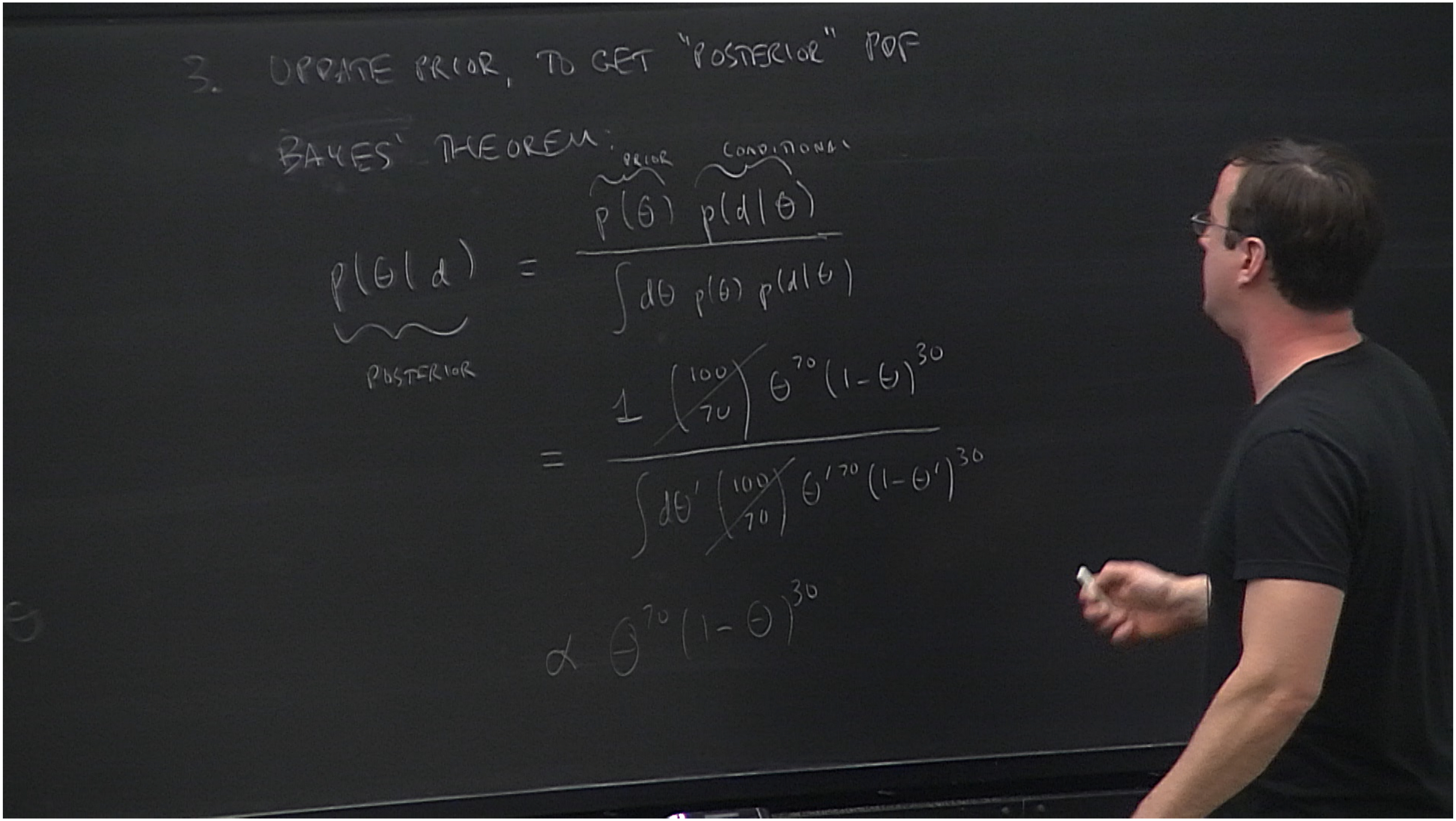

p = 0.025
p(h≥70 | Θ)
6.5×10⁻⁵
0.5
Θ

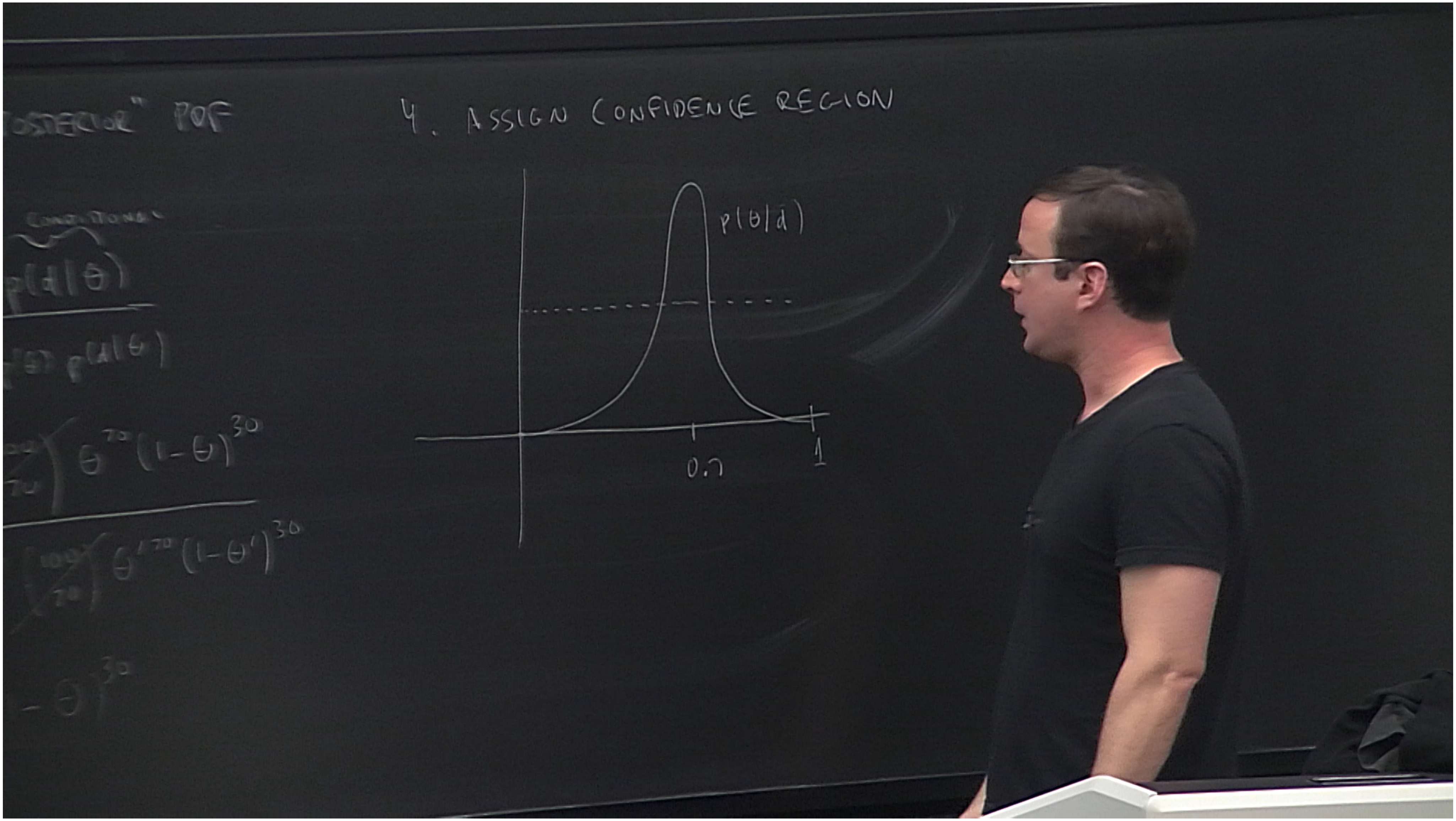

p=0.025
p(h≥70|θ)
p(h≤70|θ)
0.5
0.600
0.7

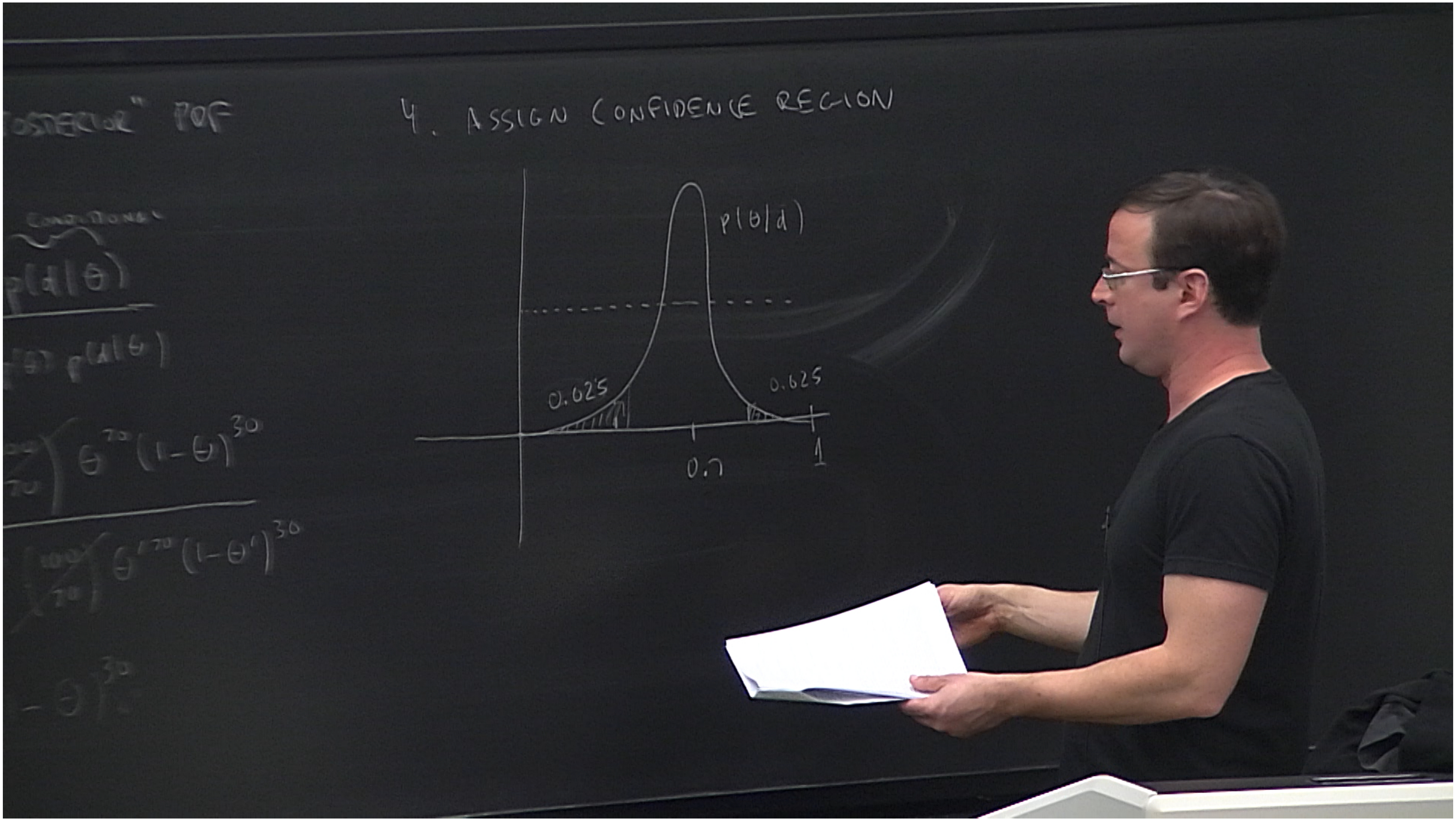

p=0.025
p(h≥70|θ)
p(h≤70|θ)
0.5
0.600
0.7
0.788


0.600 ≤ θ ≤ 0.788 95% CONFIDENCE INTERVAL
BASED ON THRESHOLDING
"p-VALUE" p(θ) =
p(h≥70|θ) θ≤0.7
p(h≤70|θ) θ≥0.7

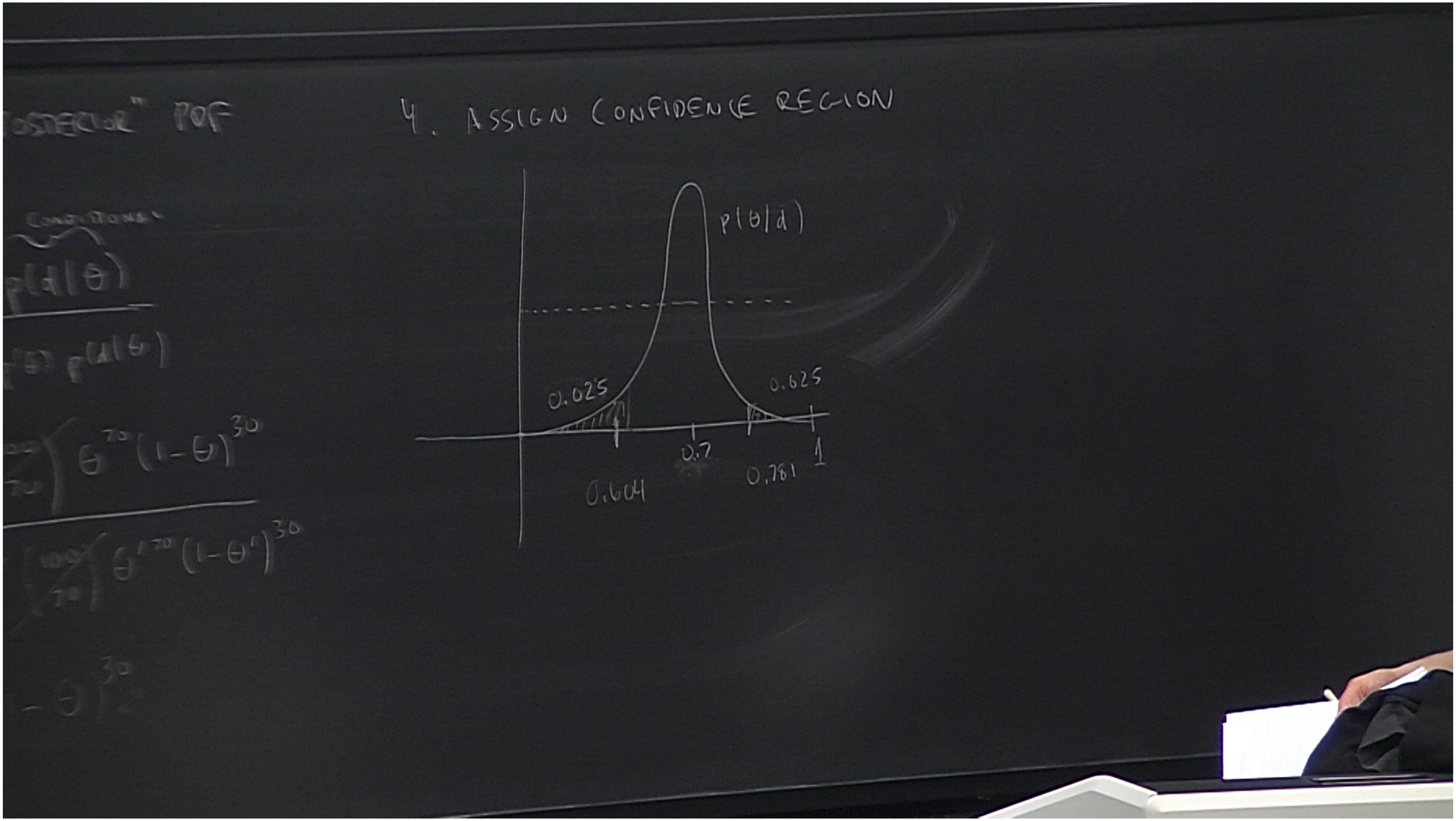

BAYESIAN BIASED COIN
4-STEP PROCESS
1. CHOOSE A "PRIOR" PDF.
p(θ) = { 1   0 ≤ θ ≤ 1
         0   OTHERWISE
2. CALCULATE CONDITIONAL PROBABILITY OF
OBTAINING THE "DATA" d, ASSUMING A VALUE OF θ
p(d|θ) = (100 choose 70) θ^70 (1-θ)^30
3.

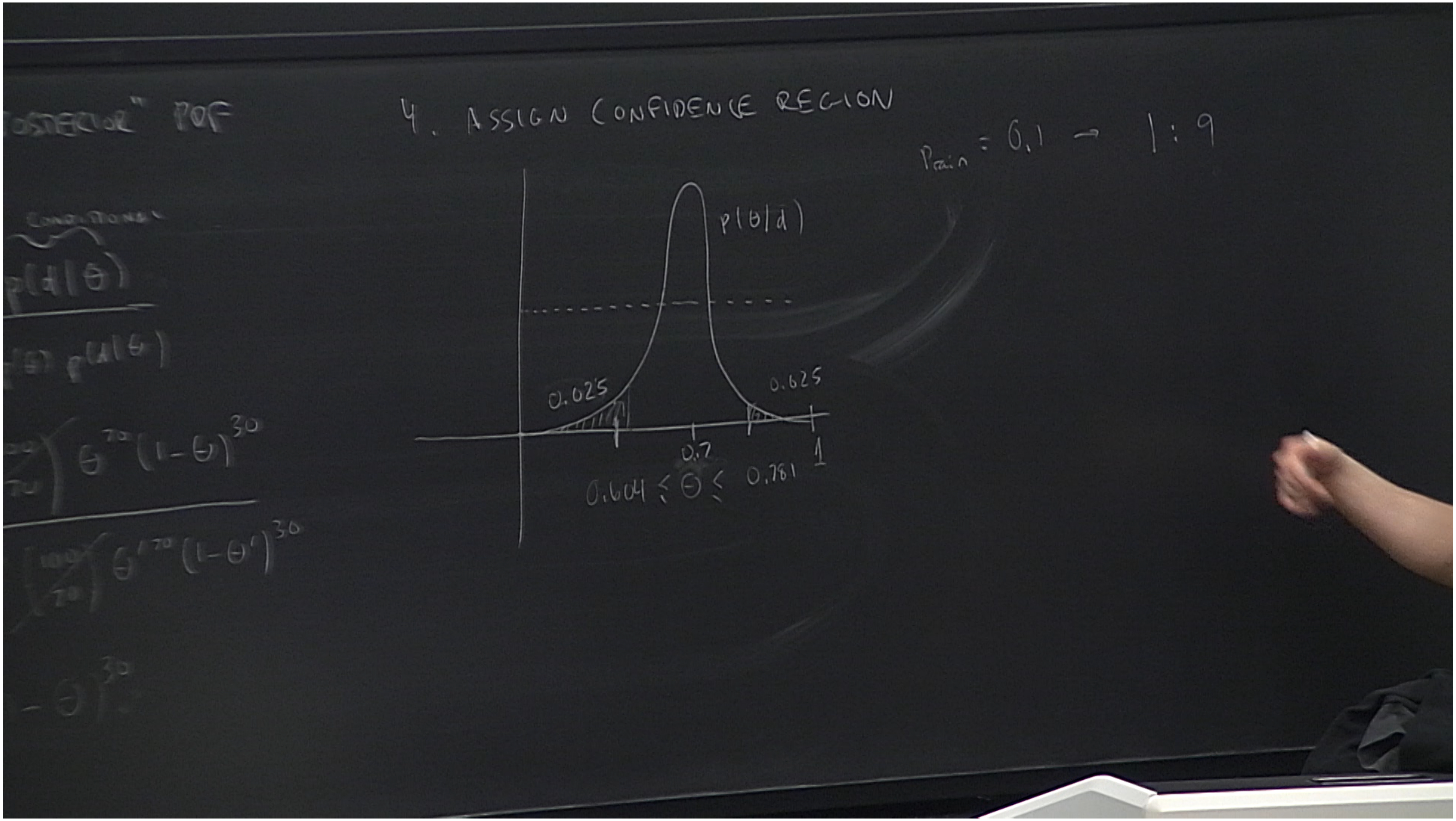

3. UPDATE PRIOR, TO GET "POSTERIOR" PDF
BAYES' THEOREM:
PRIOR
CONDITIONAL
p(θ|d) = p(θ) p(d|θ) / ∫dθ p(θ) p(d|θ)
POSTERIOR
= 1 (100 70) θ^70 (1−θ)^30 / ∫dθ' (100 70) θ'^70 (1−θ')^30
∝ θ^70 (1−θ)^30
θ

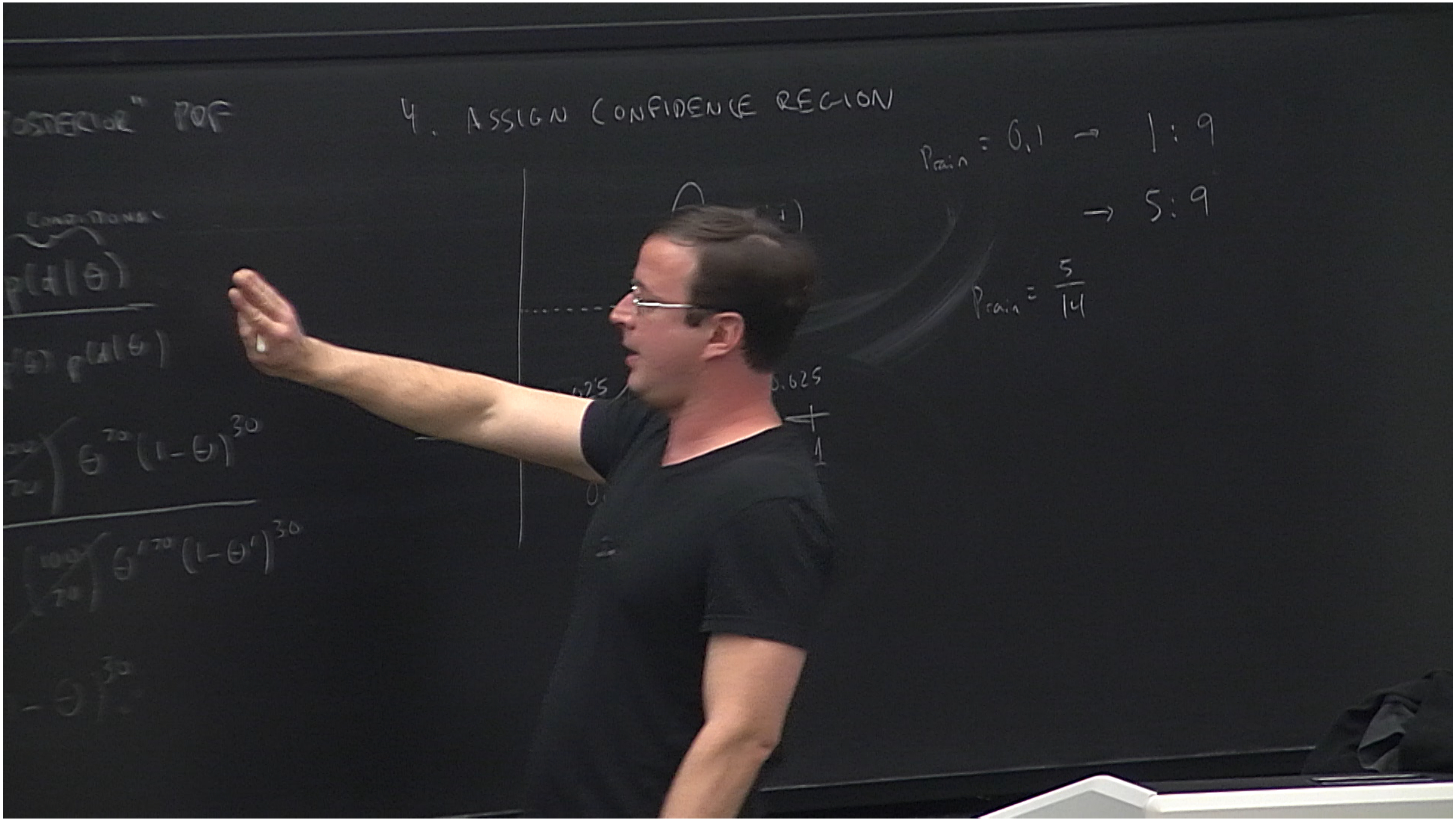

"POSTERIOR" PDF
4. ASSIGN CONFIDENCE REGION
p(θ|d)
0.7
1

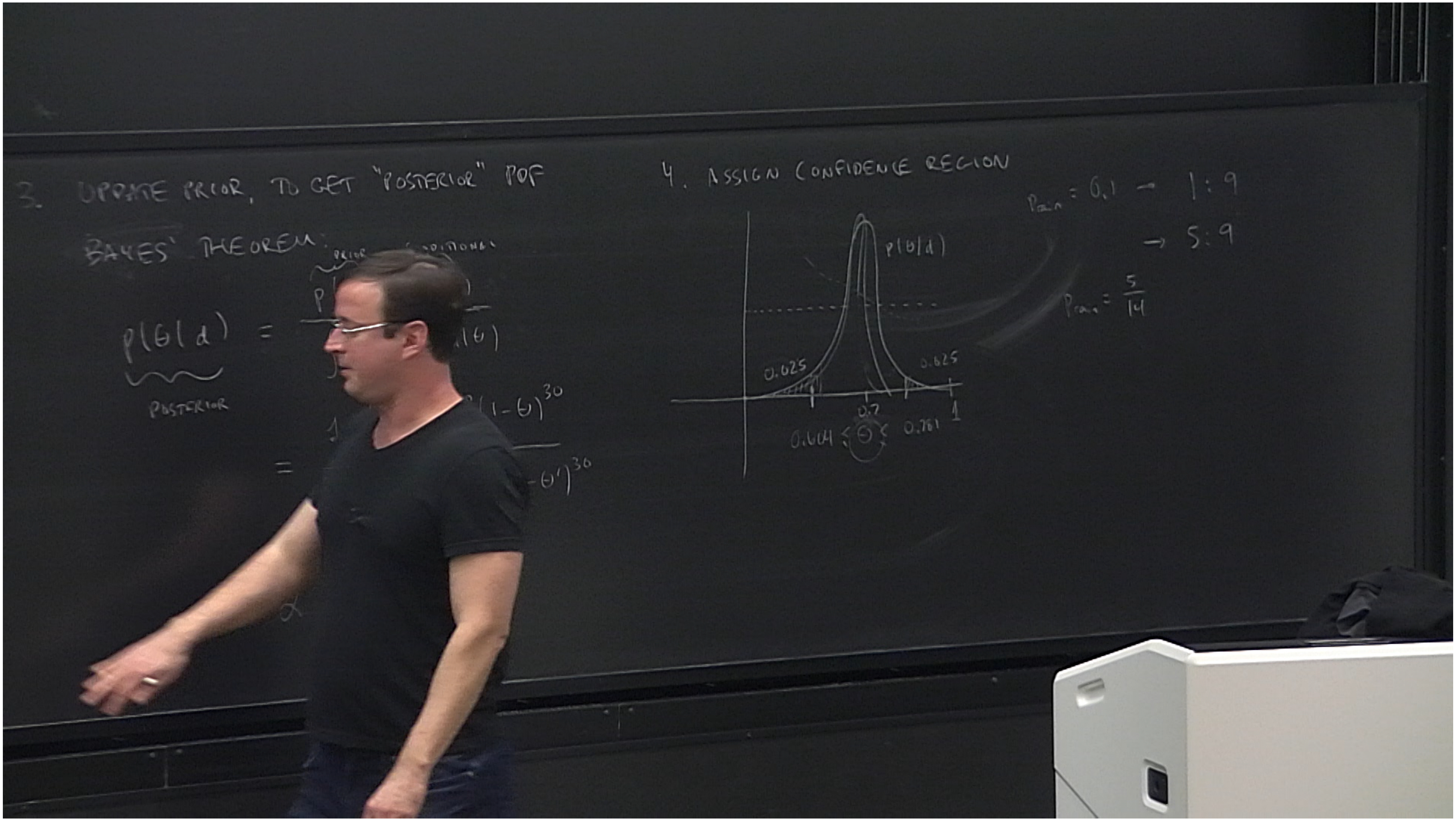


4. ASSIGN CONFIDENCE REGION
p(θ|d)
0.025
0.025
0.7
1


4. ASSIGN CONFIDENCE REGION
p(θ|d)
0.025
0.025
0.604
0.7
0.781
1


4. ASSIGN CONFIDENCE REGION
p(θ|d)
0.025
0.625
0.7
1
0.604 ≤ θ ≤ 0.781
P rain = 0.1 → 1 : 9
PDF


4. ASSIGN CONFIDENCE REGION
P_rain = 0.1 → 1 : 9
→ 5 : 9
P_rain = 5/14


3. UPDATE PRIOR, TO GET "POSTERIOR" PDF
BAYES' THEOREM:
POSTERIOR
4. ASSIGN CONFIDENCE REGION
p(θ|d)
0.025
0.025
0.7
1
→ 5 : 9