Title: PSI 2017/2018 - Cosmology - Lecture 4

Date: Apr 12, 2018 10:15 AM

URL: http://pirsa.org/18040016

Abstract:

RANDOM FIELDS IN 1D

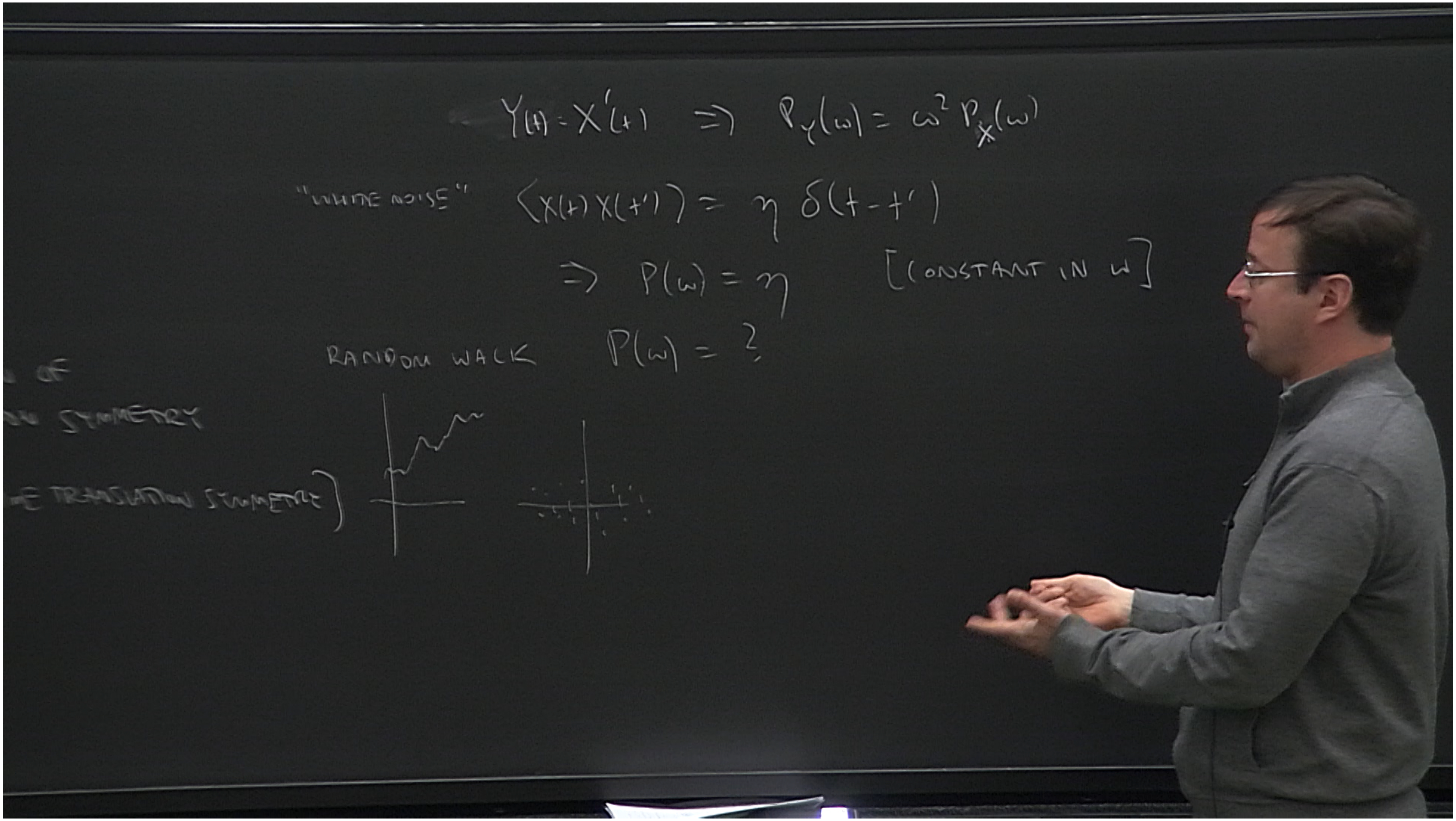
$X(t)$ RANDOM FIELD

$\langle X(t) \rangle = 0$ BY ASSUMPTION

$\langle X(t) X(t') \rangle = S(t-t')$ BY ASSUMPTION OF TIME TRANSLATION SYMMETRY

$\langle \tilde{X}(\omega) \tilde{X}(\omega')^{*} \rangle = P(\omega)(2\pi)\delta(\omega-\omega')$ [TIME TRANSLATION SYMMETRY]

$$P(\omega) = \int dt \, S(t) \, e^{-i\omega t}$$


Y(t) = X'(t) ⇒ P_Y(ω) = ω² P_X(ω)
"WHITE NOISE" ⟨X(t) X(t')⟩ = η δ(t − t')
⇒ P(ω) = η
[CONSTANT IN ω]
RANDOM WALK P(ω) = ?
SYMMETRY
TRANSLATION SYMMETRY

$$Y(t) = X'(t) \Rightarrow P_Y(\omega) = \omega^2 P_X(\omega)$$

"WHITE NOISE" $\langle X(t)\, X(t') \rangle = \eta\, \delta(t - t')$

$$\Rightarrow P(\omega) = \eta \qquad [\text{CONSTANT IN } \omega]$$

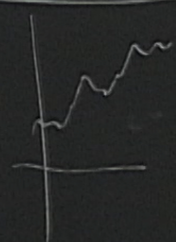
... OF

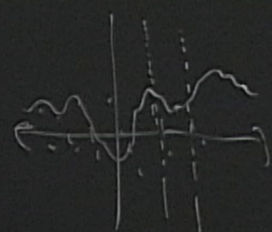
...ON SYMMETRY

...E TRA... SYMMETRY]

RANDOM WALK

$$P(\omega) = \omega^{-2} \eta$$





$P(\omega) \propto \omega^{-1}$ "SCALE INVARIANT"

$P_Y(\omega) = \omega^2 P_X(\omega)$

$\eta\, \delta(t - t')$

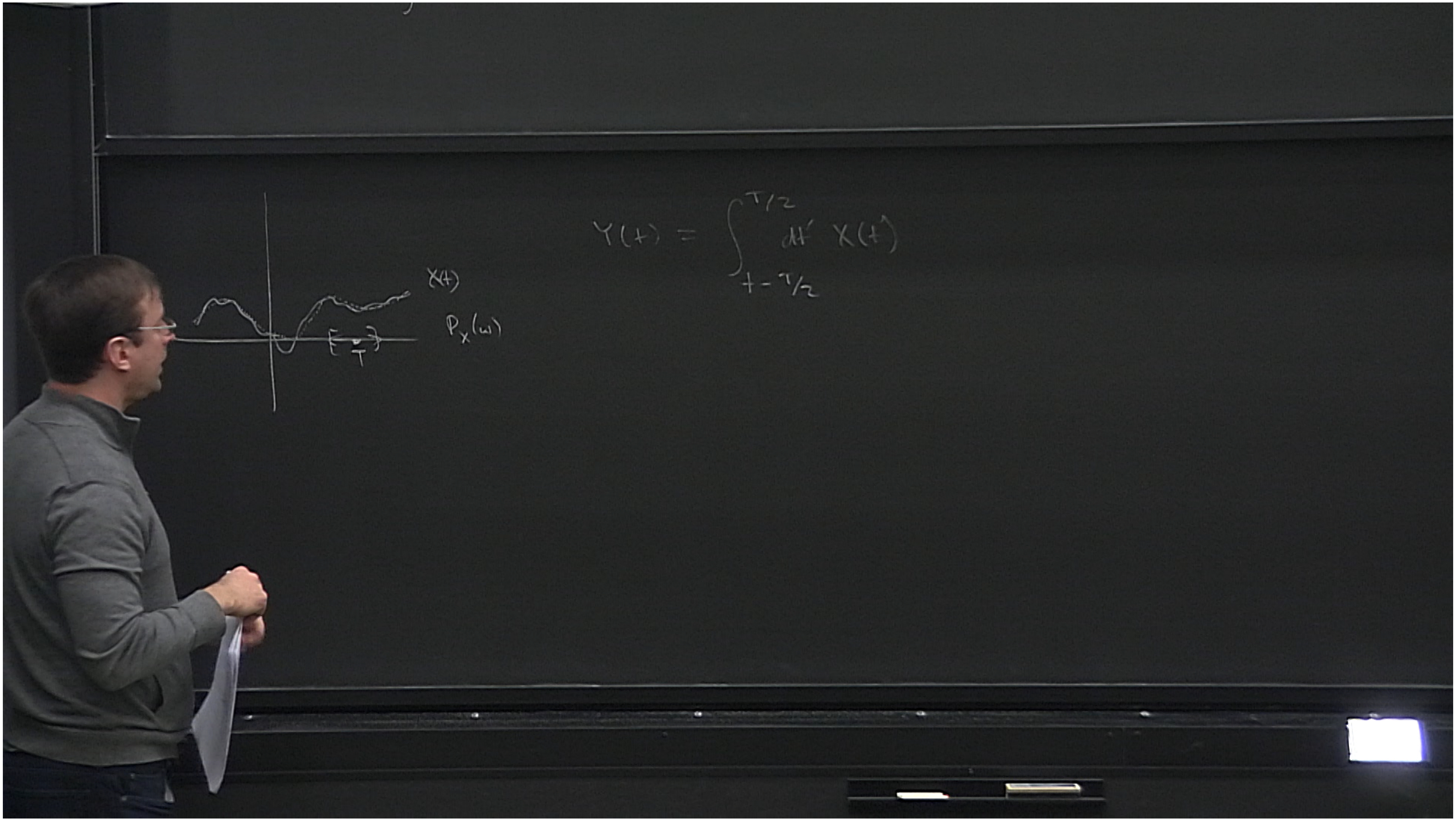
η [CONSTANT IN ω]

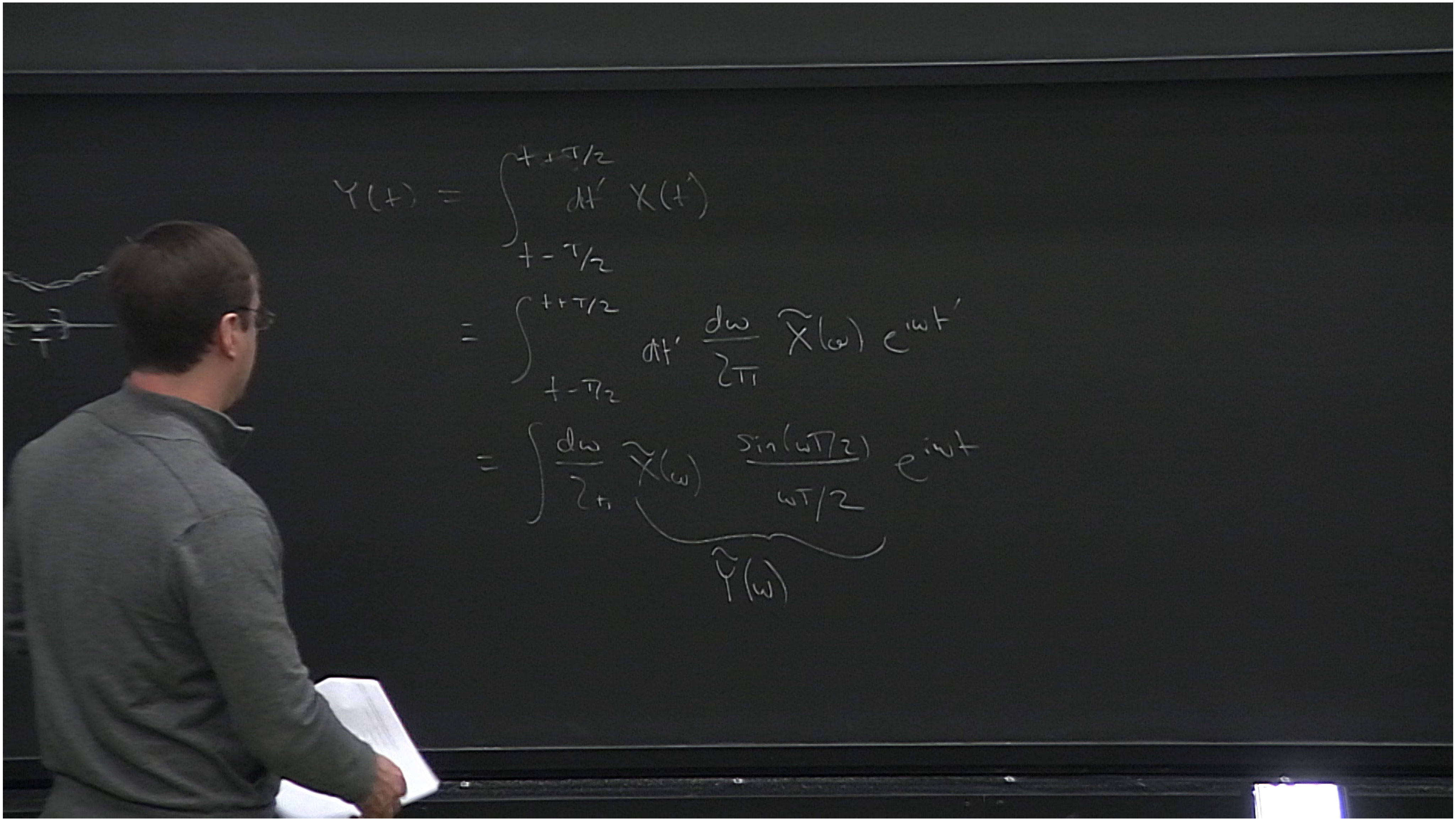
$\omega^{-2}\eta$

$X(t) =$ WHITE NOISE

$Y(t) = \int_{-\infty}^{t} dt'\, X(t')$ IS A RANDOM WALK

$(\omega) \propto \omega^{-1}$ "SCALE INVARIANT" [HW]


X(t)
P_x(ω)
Y(t) = ∫_{t-T/2}^{T/2} dt' X(t')

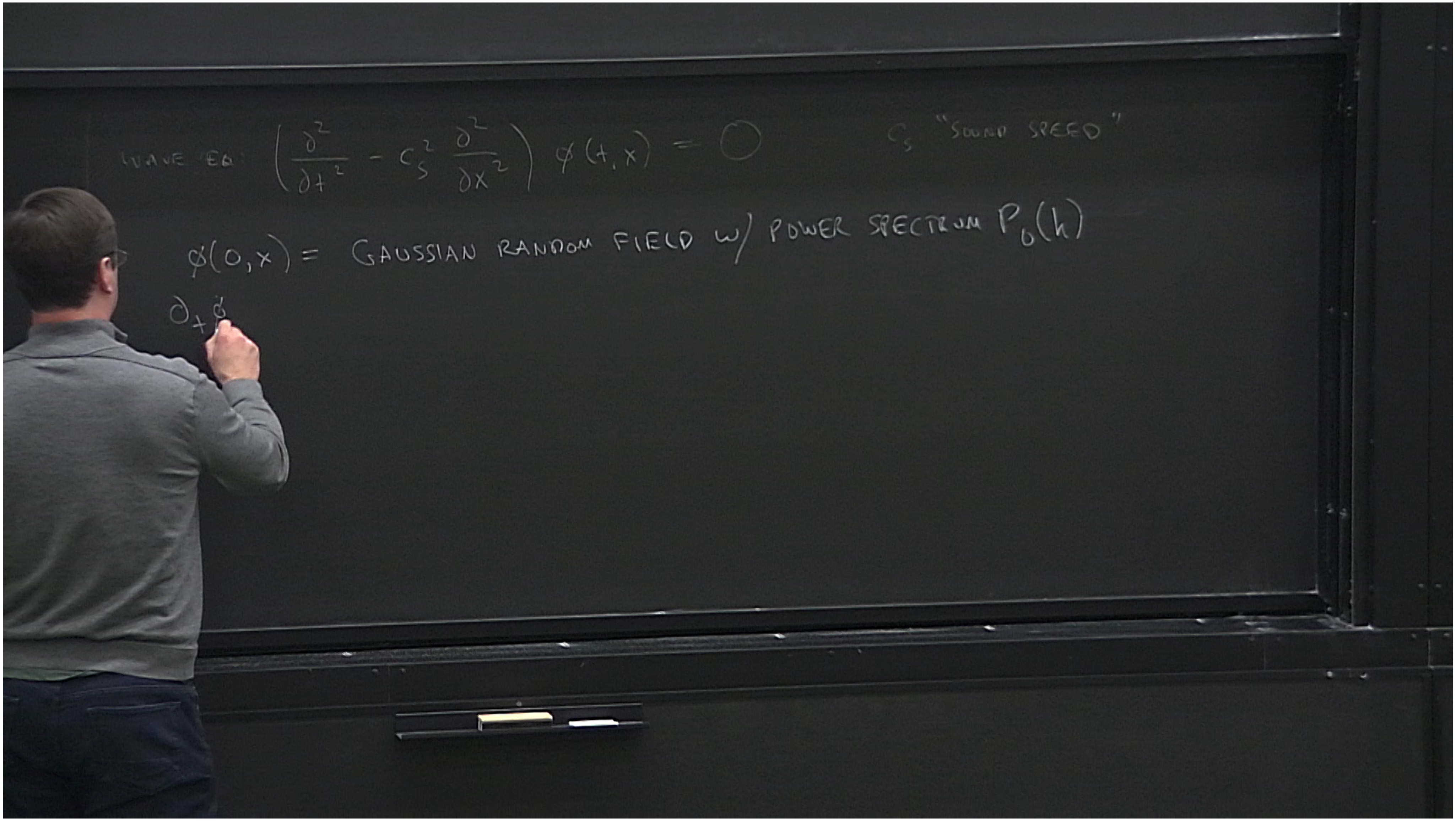

$Y(t) = \int_{t-T/2}^{t+T/2} dt' \, X(t')$
$= \int_{t-T/2}^{t+T/2} dt' \, \frac{d\omega}{2\pi} \tilde{X}(\omega) \, e^{i\omega t'}$
$= \int \frac{d\omega}{2\pi} \underbrace{\tilde{X}(\omega) \, \frac{\sin(\omega T/2)}{\omega T/2}}_{\tilde{Y}(\omega)} e^{i\omega t}$

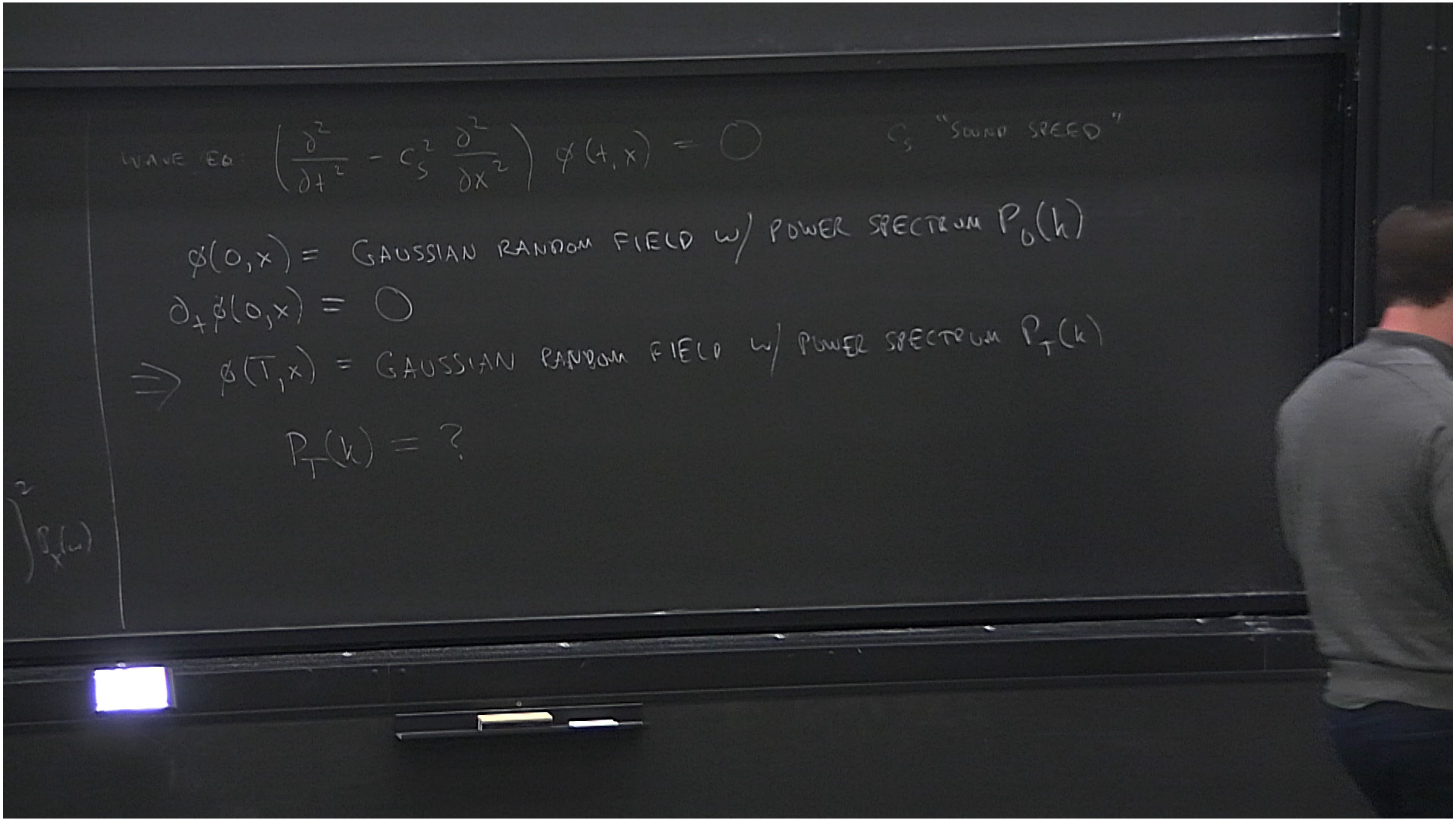
$$Y(t) = \int_{t-T/2}^{t+T/2} dt' \, X(t')$$

$$= \int_{t-T/2}^{t+T/2} dt' \, \frac{d\omega}{2\pi} \tilde{X}(\omega) \, e^{i\omega t'}$$

$$= \int \frac{d\omega}{2\pi} \underbrace{\tilde{X}(\omega) \, \frac{\sin(\omega T/2)}{\omega T/2}}_{\tilde{Y}(\omega)} \, e^{i\omega t}$$

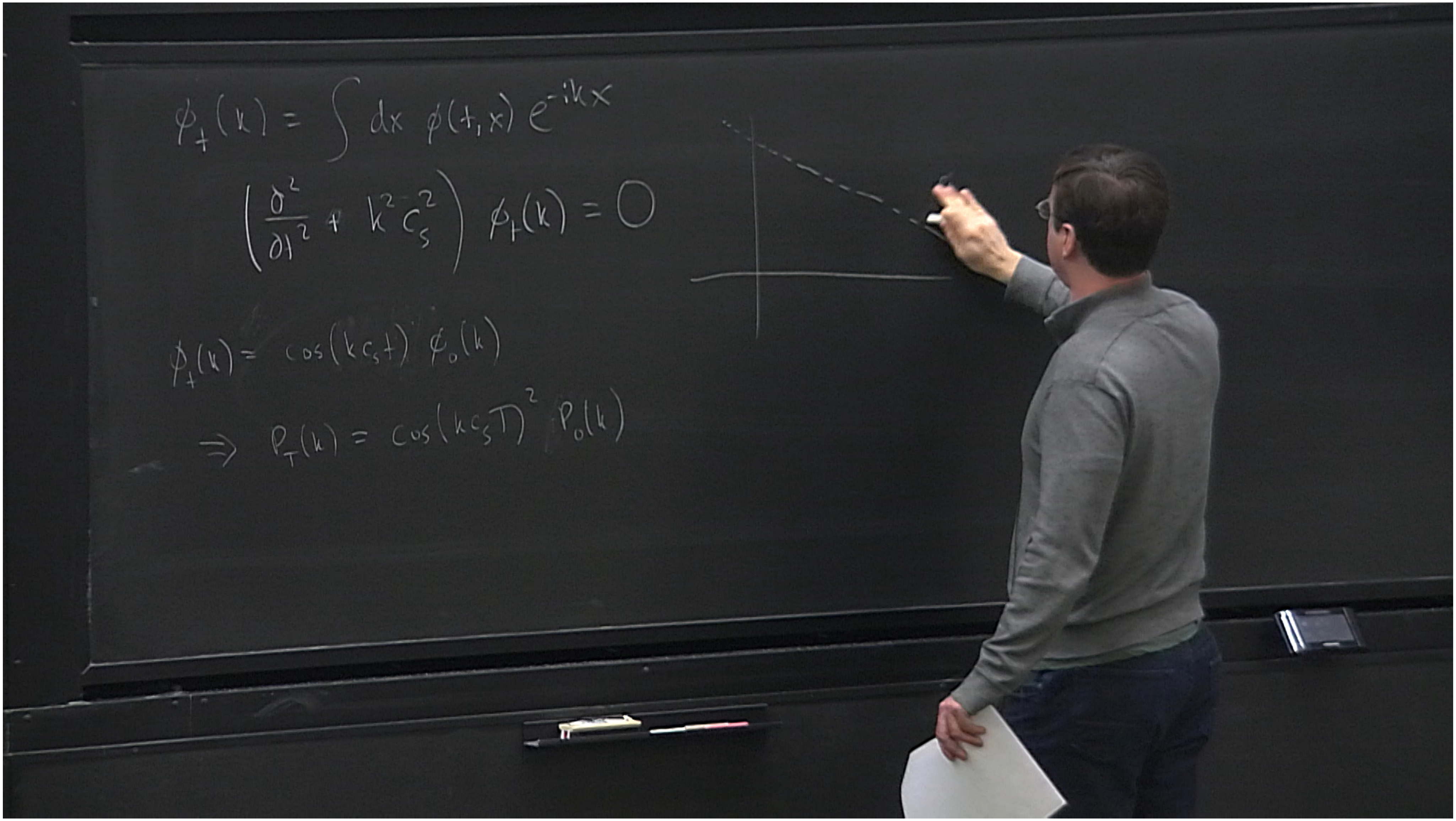
$$\tilde{Y}(\omega) = \frac{\sin(\omega T/2)}{\omega T/2} \tilde{X}(\omega) \quad \Rightarrow \quad P_Y(\omega) = \left[\frac{\sin(\omega T/2)}{\omega T/2}\right]^2 P_X(\omega)$$

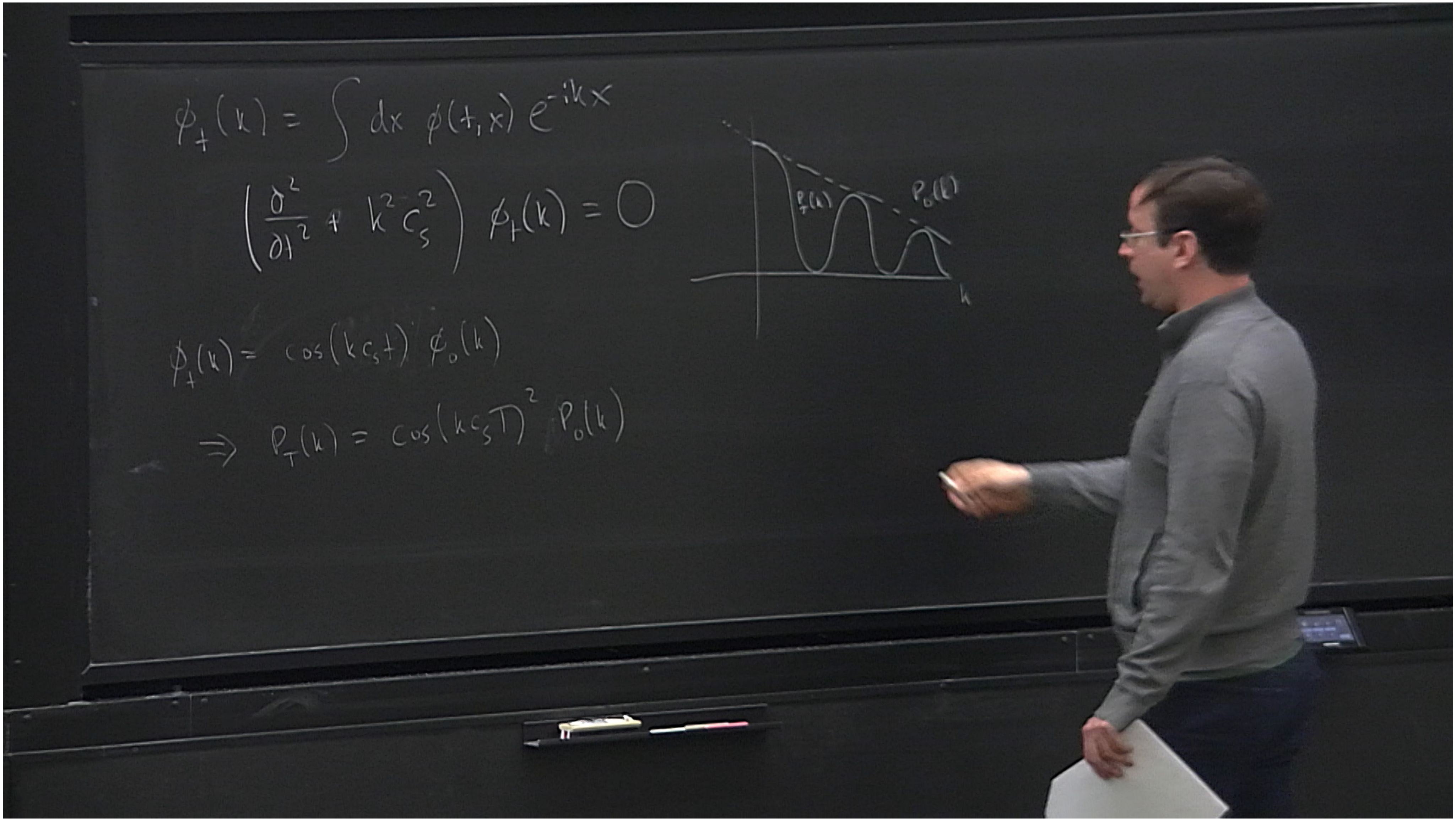

WAVE EQ: $\left(\frac{\partial^2}{\partial t^2} - c_s^2 \frac{\partial^2}{\partial x^2}\right) \phi(t,x) = 0$
c_s "SOUND SPEED"
$\phi(0,x)$ = GAUSSIAN RANDOM FIELD w/ POWER SPECTRUM $P_0(k)$
$\partial_t \phi$


WAVE EQ: $\left(\frac{\partial^2}{\partial t^2} - c_s^2 \frac{\partial^2}{\partial x^2}\right) \phi(t,x) = 0$
c_s "SOUND SPEED"
$\phi(0,x) =$ GAUSSIAN RANDOM FIELD w/ POWER SPECTRUM $P_0(k)$
$\partial_t \phi(0,x) = 0$
$\Rightarrow \phi(T,x) =$ GAUSSIAN RANDOM FIELD w/ POWER SPECTRUM $P_T(k)$
$P_T(k) = ?$

$$\phi_t(k) = \int dx \, \phi(t,x)\, e^{-ikx}$$

$$\left(\frac{\partial^2}{\partial t^2} + k^2 c_s^2\right)\phi_t(k) = 0$$

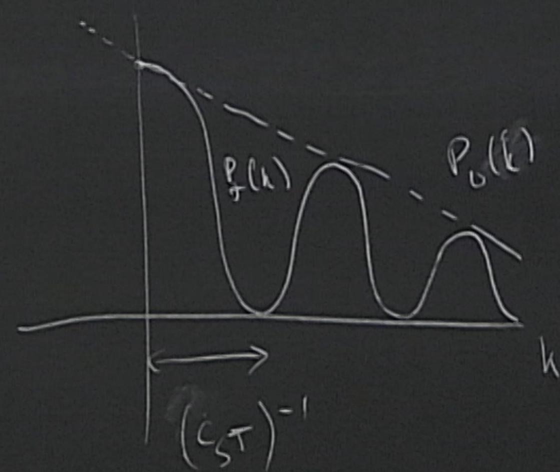

$\phi_t(k) = \int dx \, \phi(t,x) \, e^{-ikx}$
$\left(\frac{\partial^2}{\partial t^2} + k^2 c_s^2\right) \phi_t(k) = 0$
$\phi_t(k) = \cos(k c_s t) \, \phi_0(k)$
$\Rightarrow P_T(k) = \cos(k c_s T)^2 \, P_0(k)$



$\phi_t(k) = \int dx\, \phi(t,x)\, e^{-ikx}$
$\left(\frac{\partial^2}{\partial t^2} + k^2 c_s^2\right) \phi_t(k) = 0$
$\phi_t(k) = \cos(k c_s t)\, \phi_0(k)$
$\Rightarrow P_T(k) = \cos(k c_s T)^2\, P_0(k)$
$P_T(k)$
$P_0(k)$
k

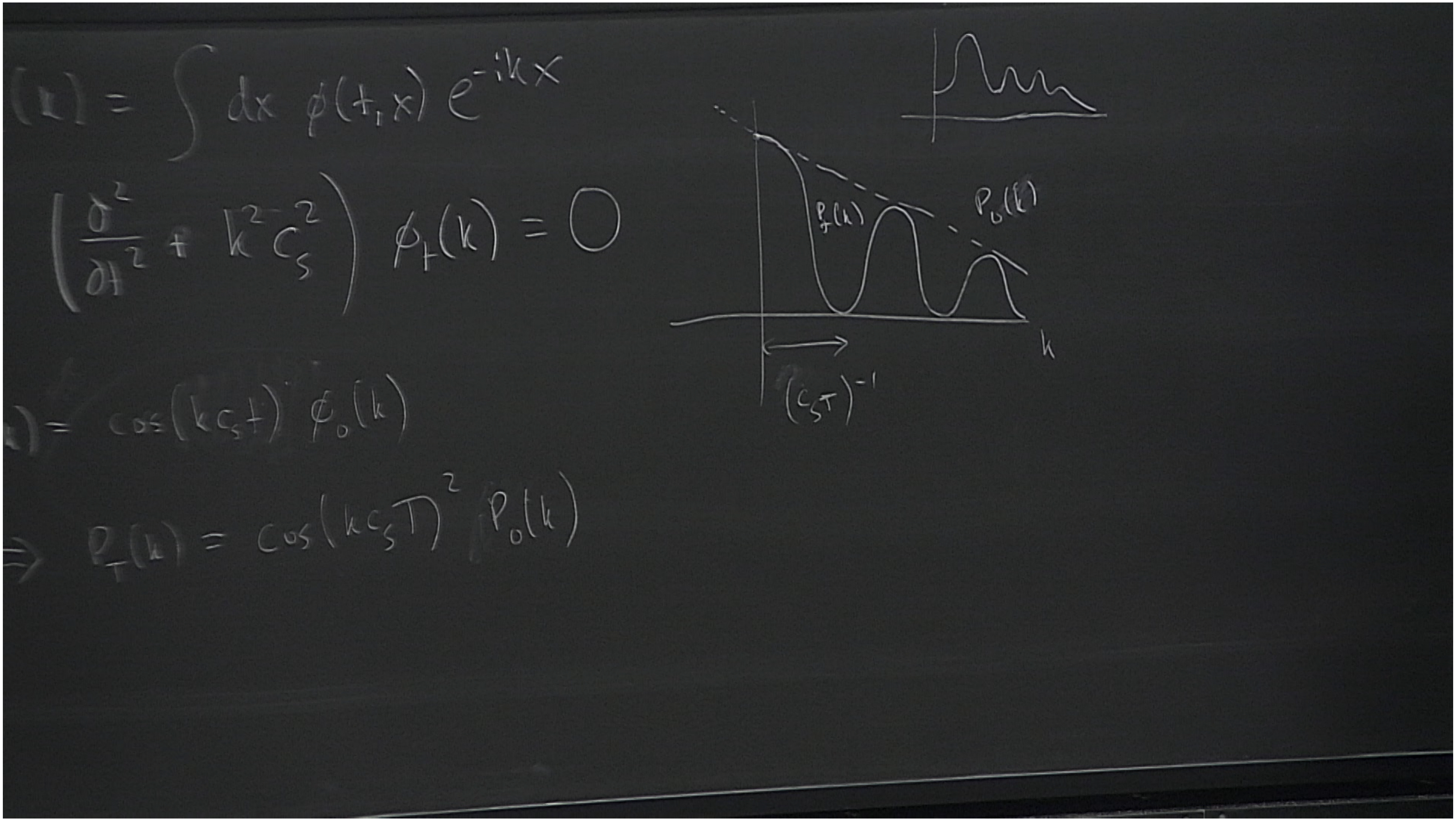
$$(k) = \int dx \; \phi(t,x)\, e^{-ikx}$$

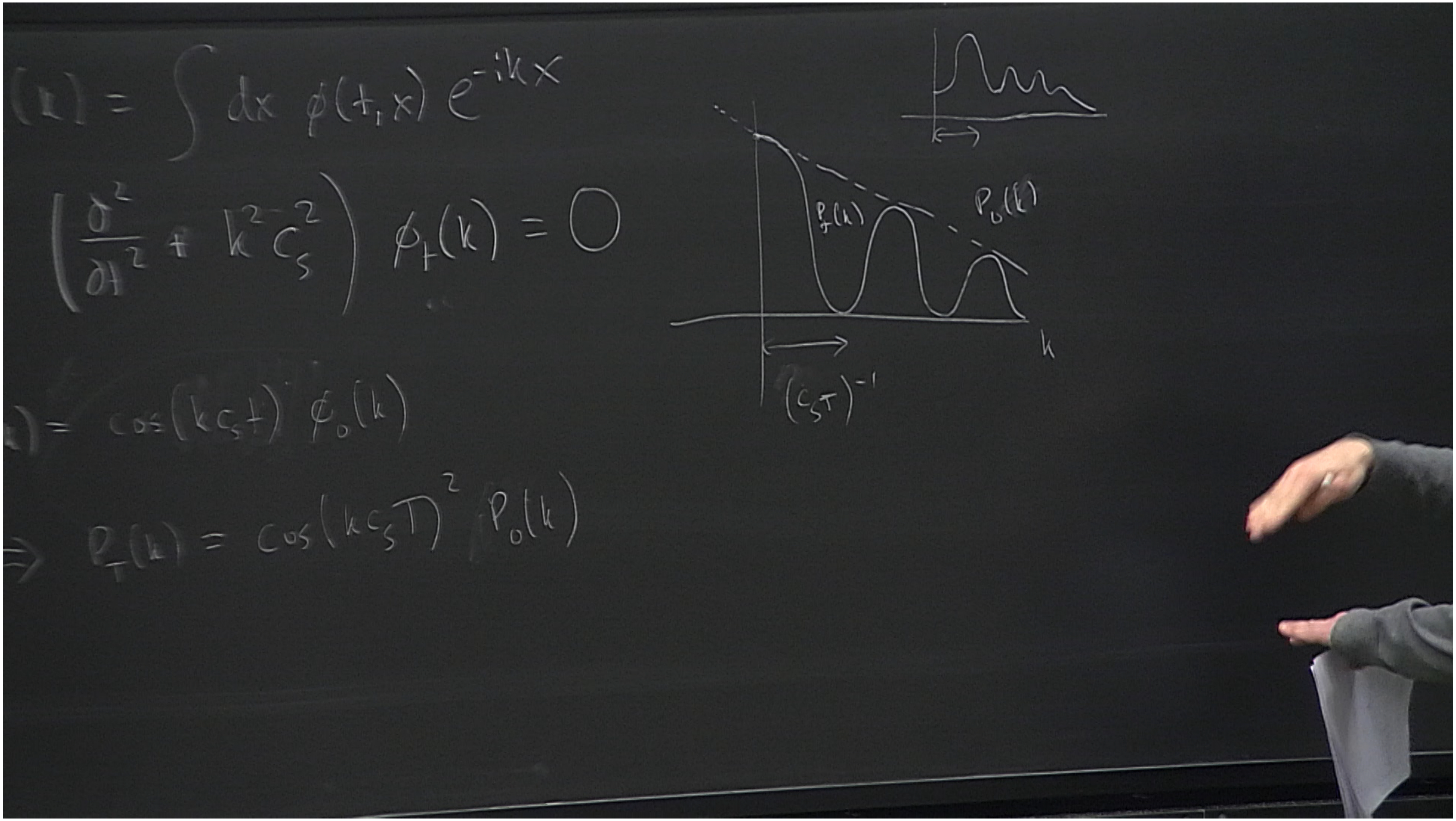
$$\left(\frac{\partial^2}{\partial t^2} + k^2 c_s^2\right) \phi_t(k) = 0$$



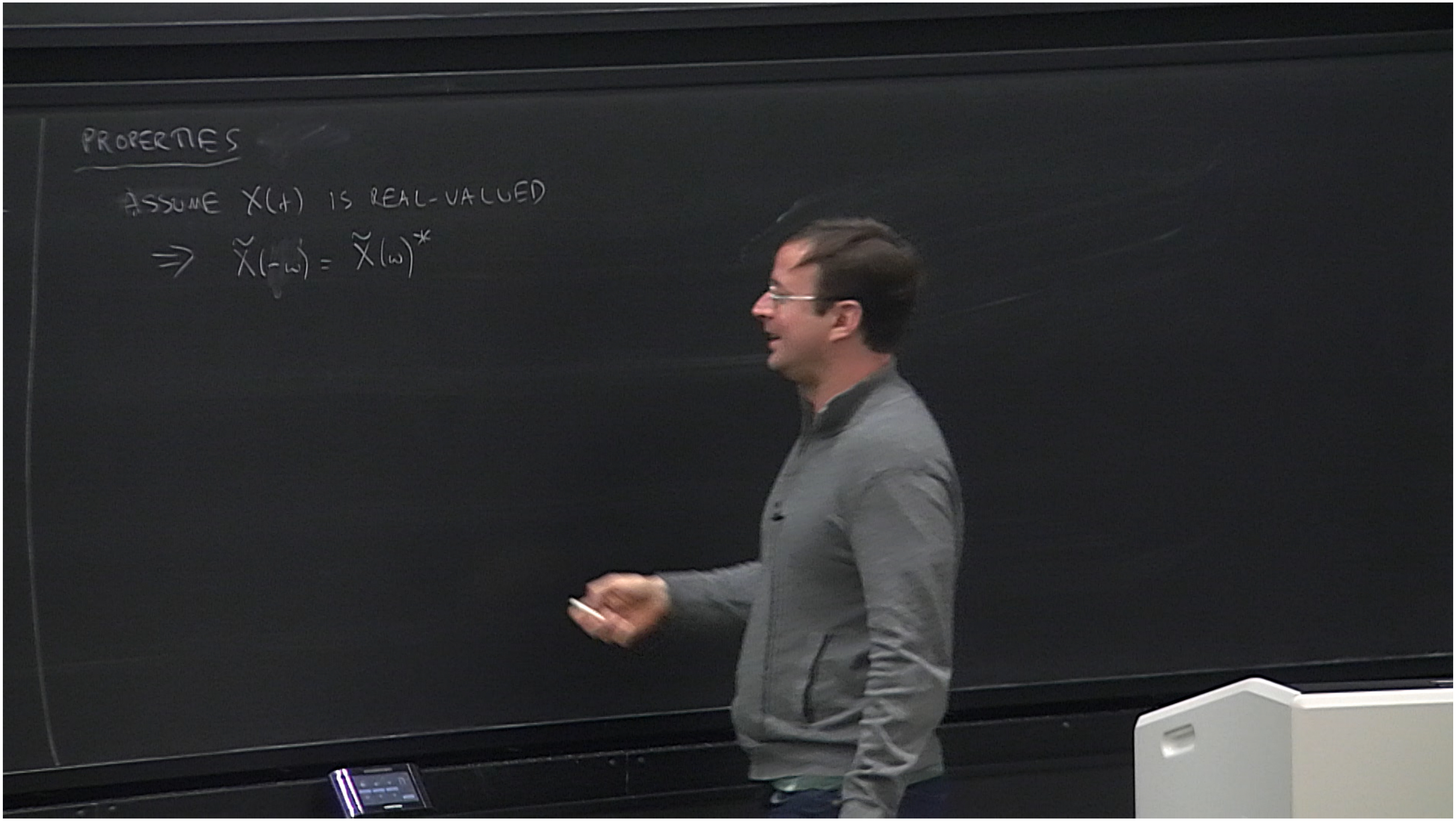
$$k) = \cos(k c_s t)\, \phi_0(k)$$

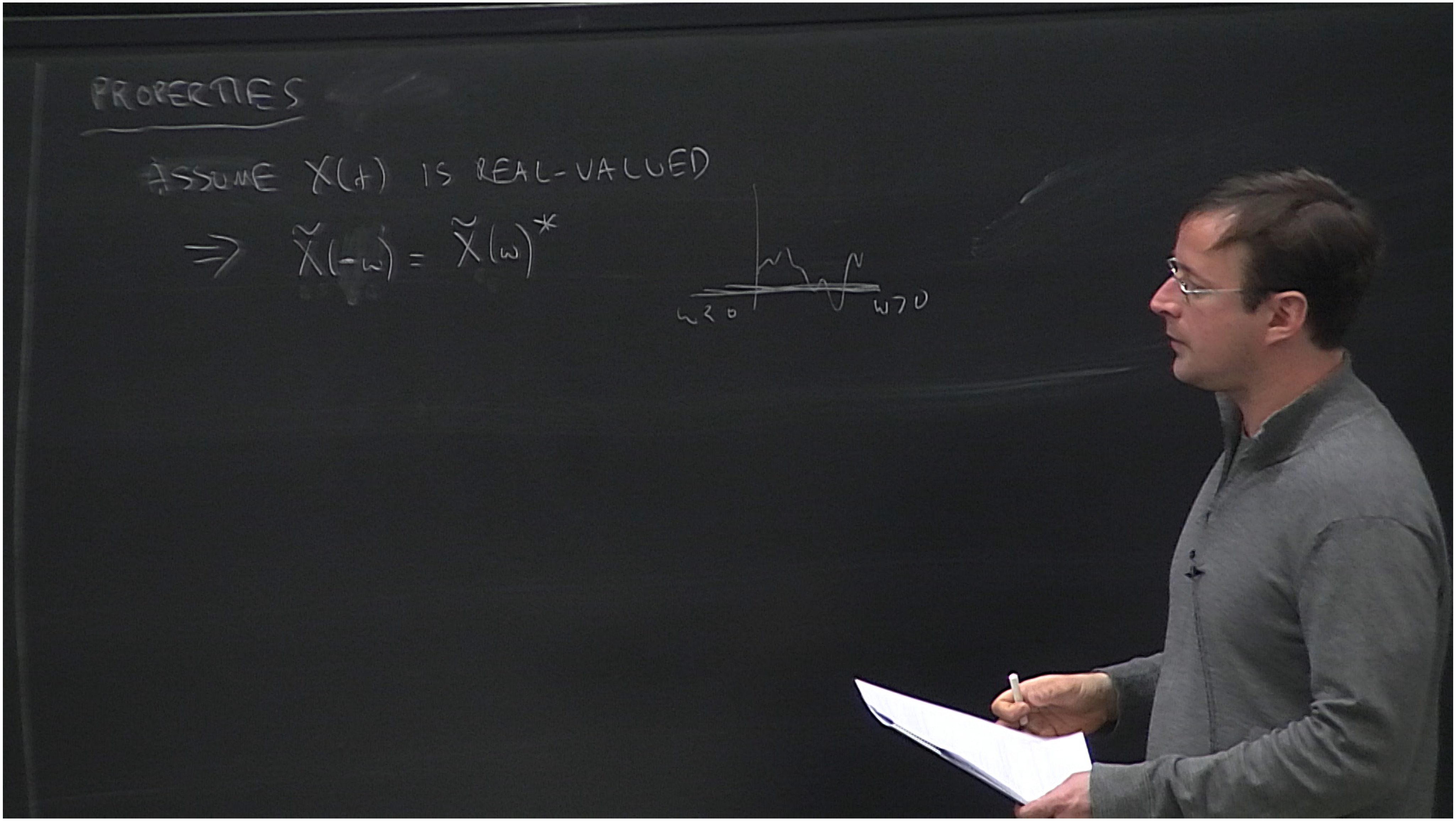
$$\Rightarrow P_T(k) = \cos(k c_s T)^2\, P_0(k)$$

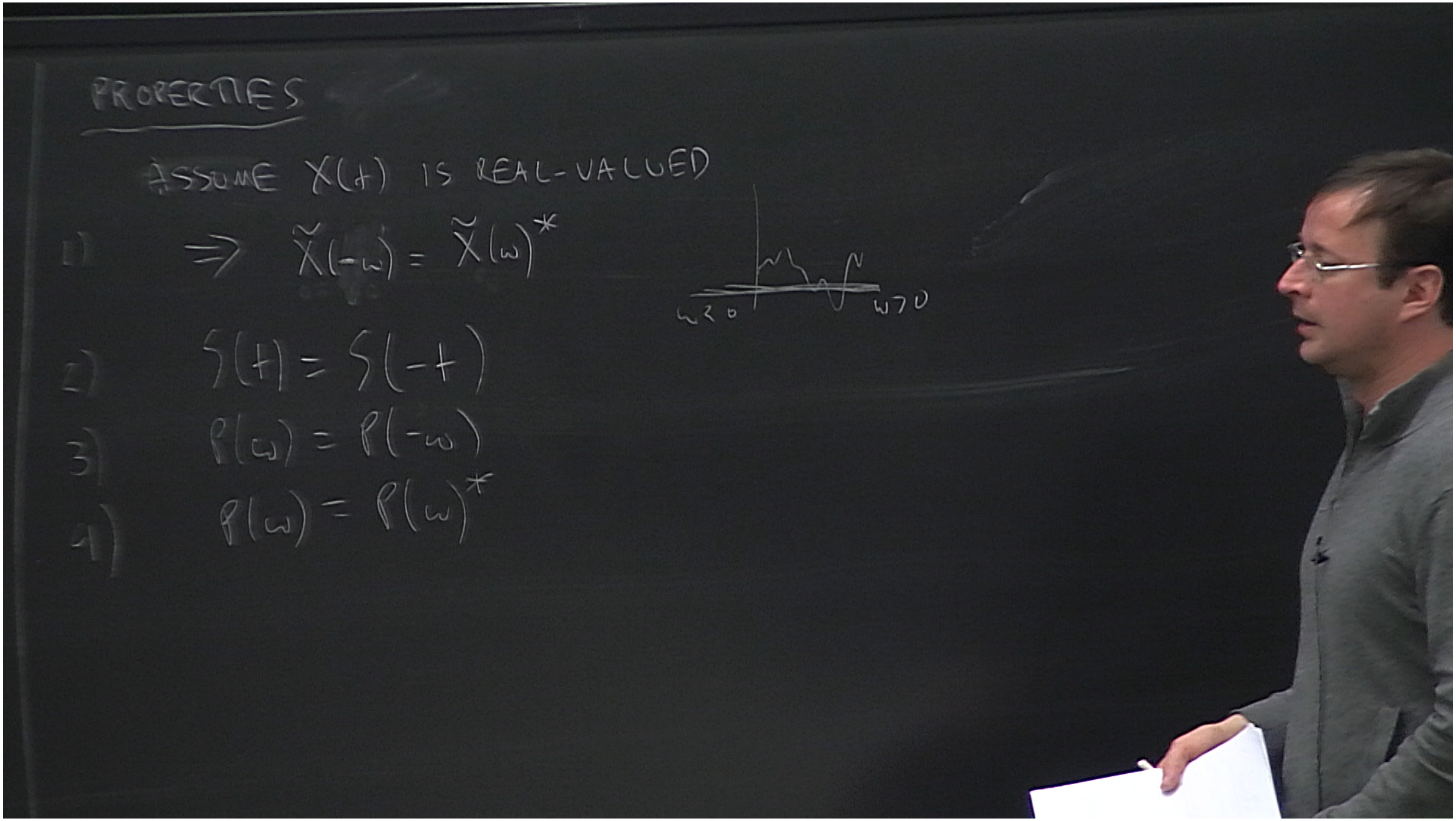

$(k) = \int dx\ \phi(t,x)\, e^{-ikx}$
$\left(\frac{\partial^2}{\partial t^2} + k^2 c_s^2\right) \phi_t(k) = 0$
$k) = \cos(k c_s t)\, \phi_0(k)$
$\Rightarrow P_T(k) = \cos(k c_s T)^2\, P_0(k)$
$P_T(k)$
$P_0(k)$
k
$(c_s T)^{-1}$

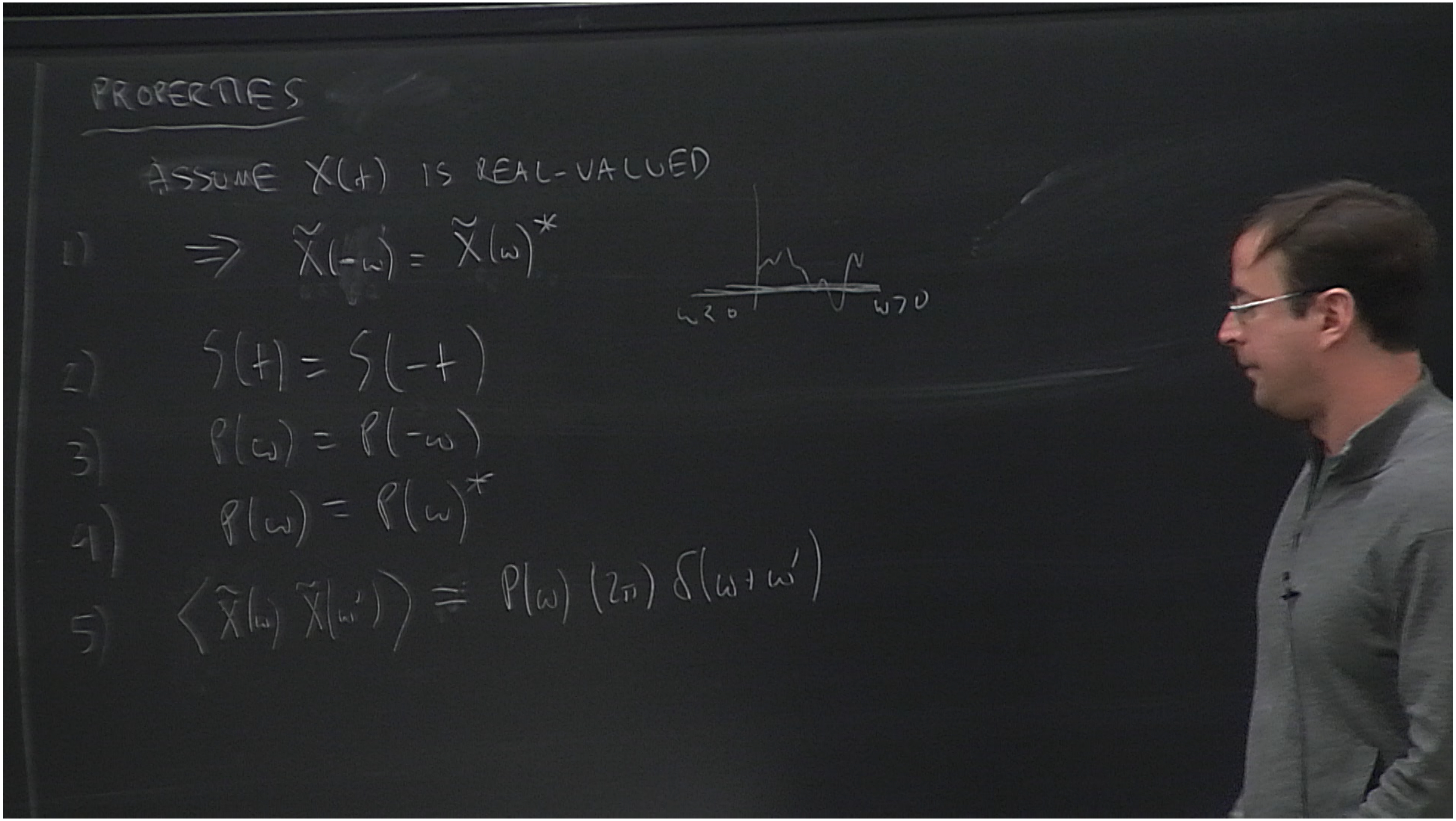


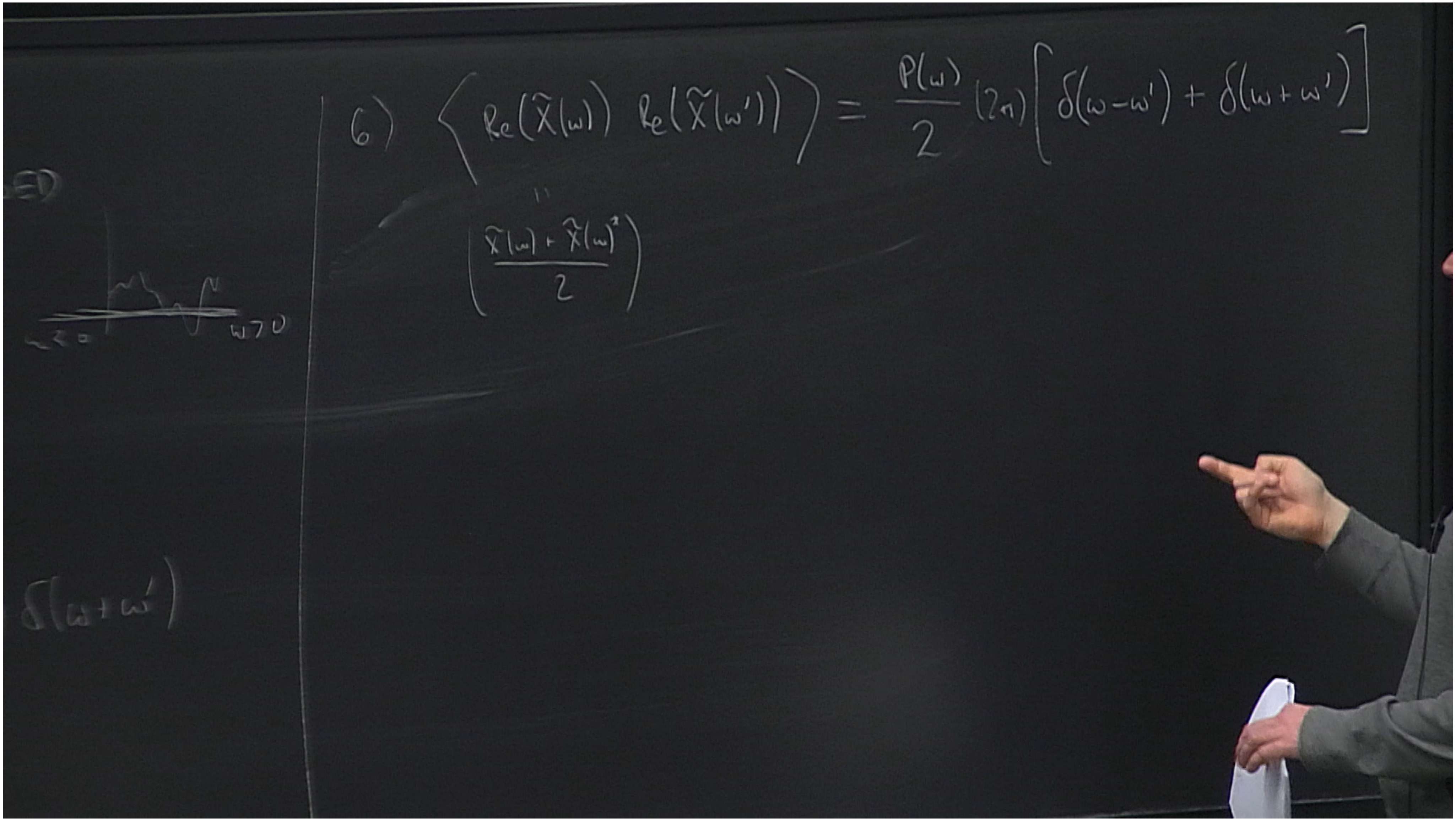
$(k) = \int dx \, \phi(t,x) \, e^{-ikx}$
$\left(\frac{\partial^2}{\partial t^2} + k^2 c_s^2\right) \phi_t(k) = 0$
$k) = \cos(k c_s t) \, \phi_0(k)$
$\Rightarrow P_T(k) = \cos(k c_s T)^2 \, P_0(k)$
$P_T(k)$
$P_0(k)$
k
$(c_s T)^{-1}$


PROPERTIES
ASSUME X(t) IS REAL-VALUED
$\Rightarrow \tilde{X}(-\omega) = \tilde{X}(\omega)^*$

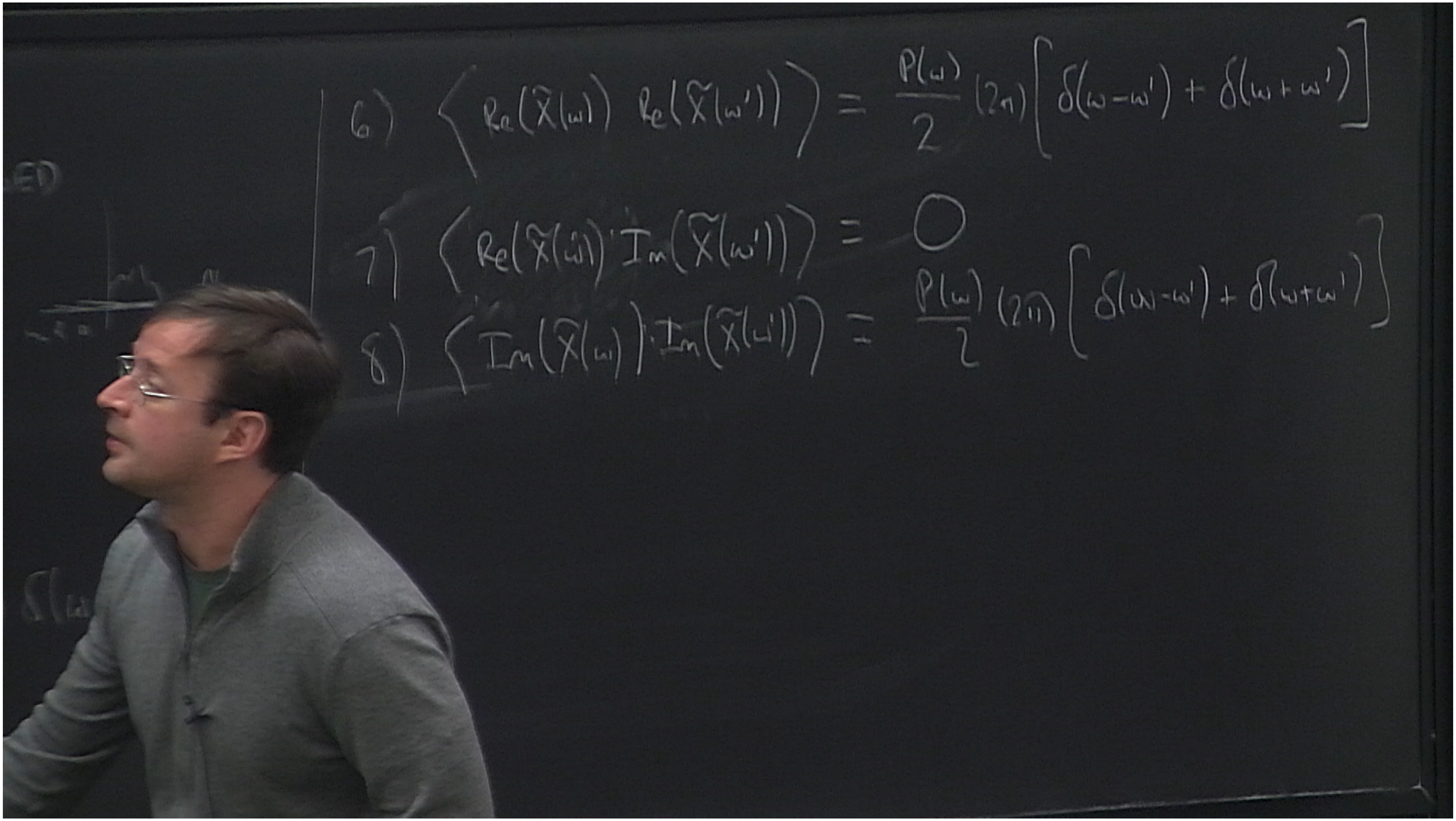

PROPERTIES
ASSUME X(t) IS REAL-VALUED
$\Rightarrow \tilde{X}(-\omega) = \tilde{X}(\omega)^*$
$\omega < 0$
$\omega > 0$


PROPERTIES
ASSUME X(t) IS REAL-VALUED
1) ⇒ $\tilde{X}(-\omega) = \tilde{X}(\omega)^*$
ω<0
ω>0
2) $S(t) = S(-t)$
3) $P(\omega) = P(-\omega)$
4) $P(\omega) = P(\omega)^*$

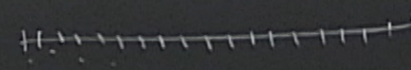

PROPERTIES
ASSUME $X(t)$ IS REAL-VALUED
1) $\Rightarrow \tilde{X}(-\omega) = \tilde{X}(\omega)^*$
$\omega < 0$
$\omega > 0$
2) $S(t) = S(-t)$
3) $P(\omega) = P(-\omega)$
4) $P(\omega) = P(\omega)^*$
5) $\langle \tilde{X}(\omega) \tilde{X}(\omega') \rangle = P(\omega)(2\pi)\delta(\omega + \omega')$



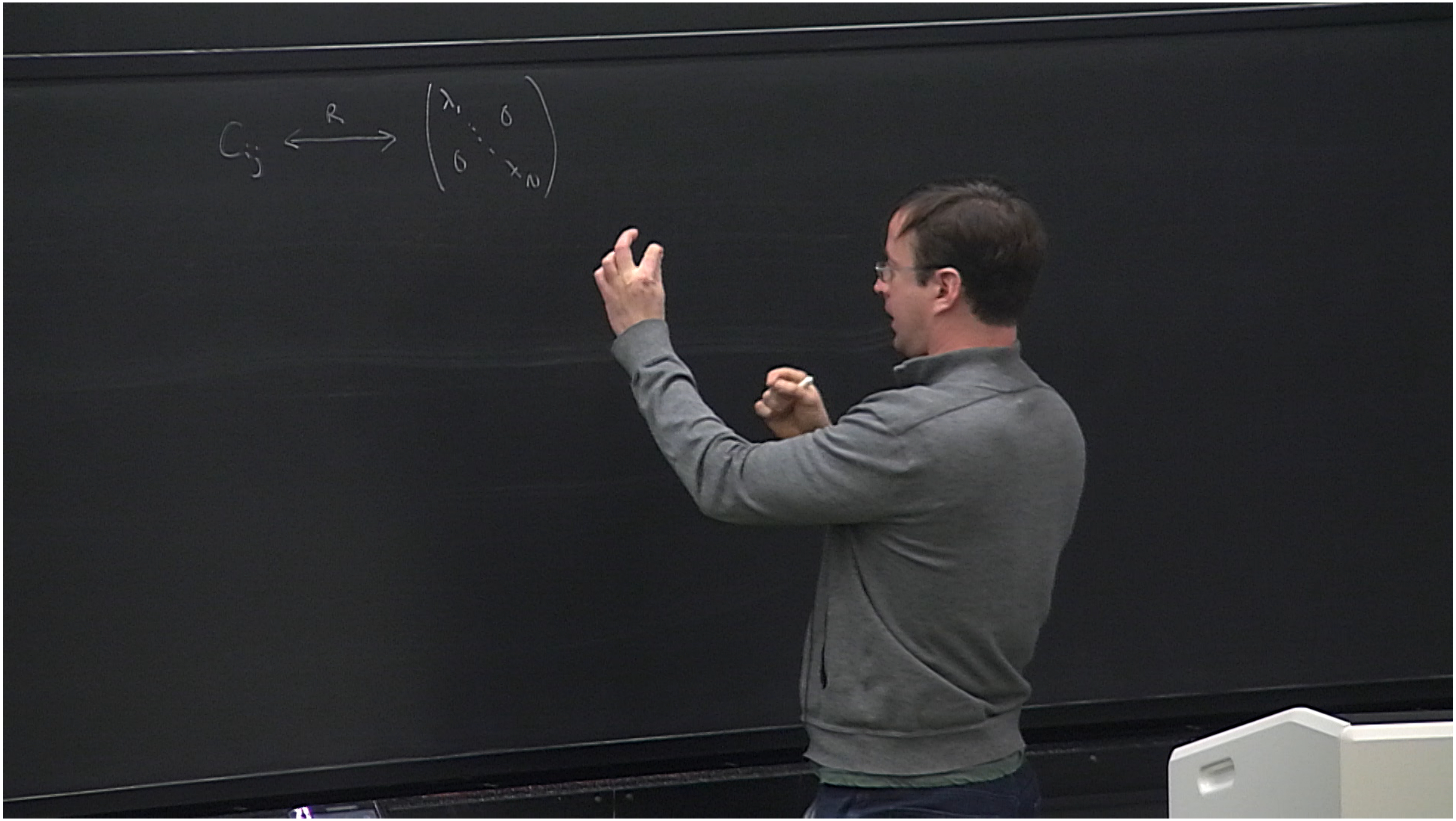
6) $\left\langle \mathrm{Re}(\tilde{X}(\omega))\ \mathrm{Re}(\tilde{X}(\omega'))\right\rangle = \frac{P(\omega)}{2}(2\pi)\left[\delta(\omega-\omega') + \delta(\omega+\omega')\right]$
$\left(\frac{\tilde{X}(\omega)+\tilde{X}(\omega)^*}{2}\right)$
$\omega<0$
$\omega>0$
$\delta(\omega+\omega')$

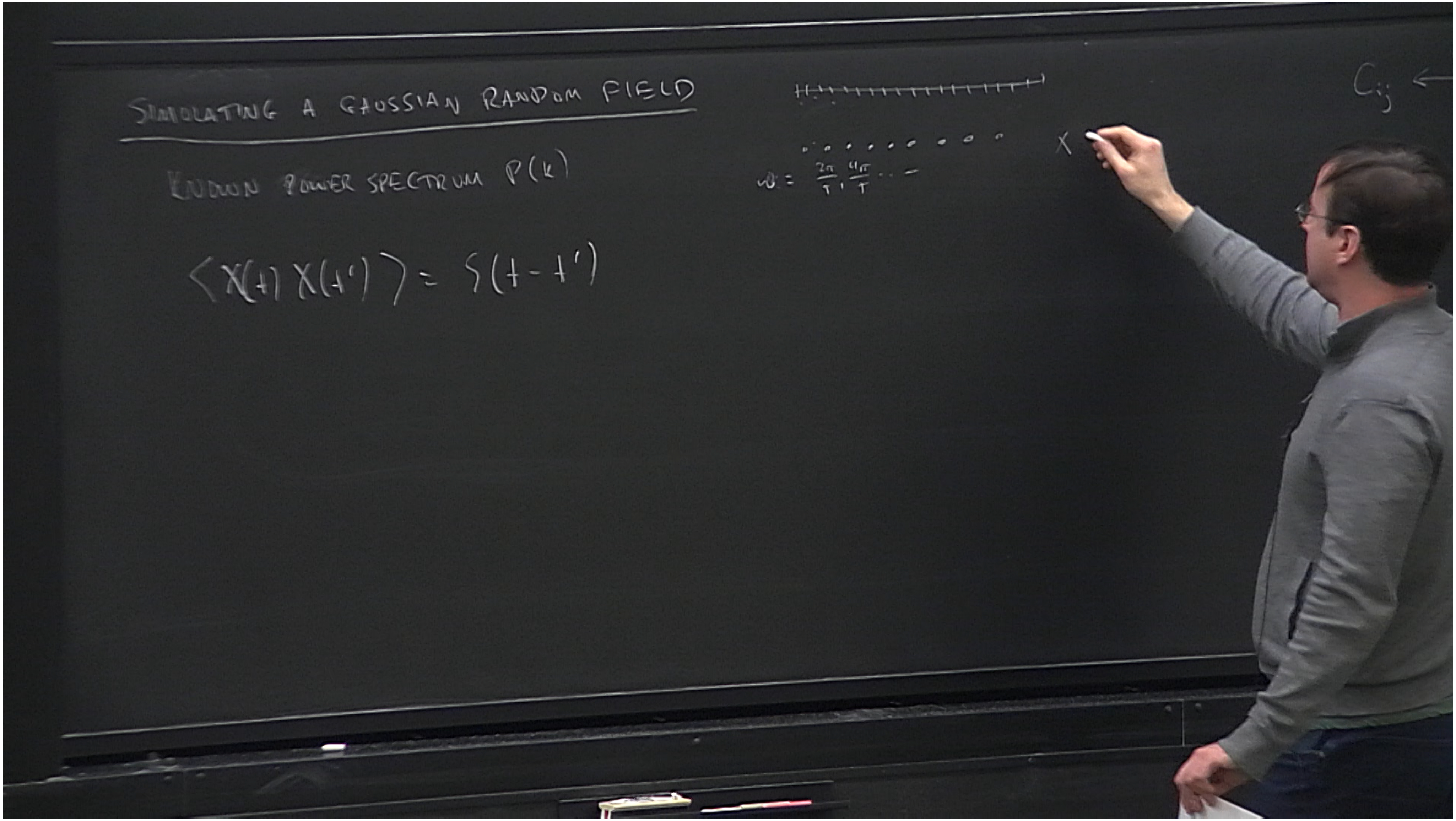

6) $\left\langle Re(\tilde{X}(\omega))\, Re(\tilde{X}(\omega')) \right\rangle = \frac{P(\omega)}{2}(2\pi)\left[\delta(\omega-\omega') + \delta(\omega+\omega')\right]$
7) $\left\langle Re(\tilde{X}(\omega)) \cdot Im(\tilde{X}(\omega')) \right\rangle = 0$
8) $\left\langle Im(\tilde{X}(\omega)) \cdot Im(\tilde{X}(\omega')) \right\rangle = \frac{P(\omega)}{2}(2\pi)\left[\delta(\omega-\omega') + \delta(\omega+\omega')\right]$

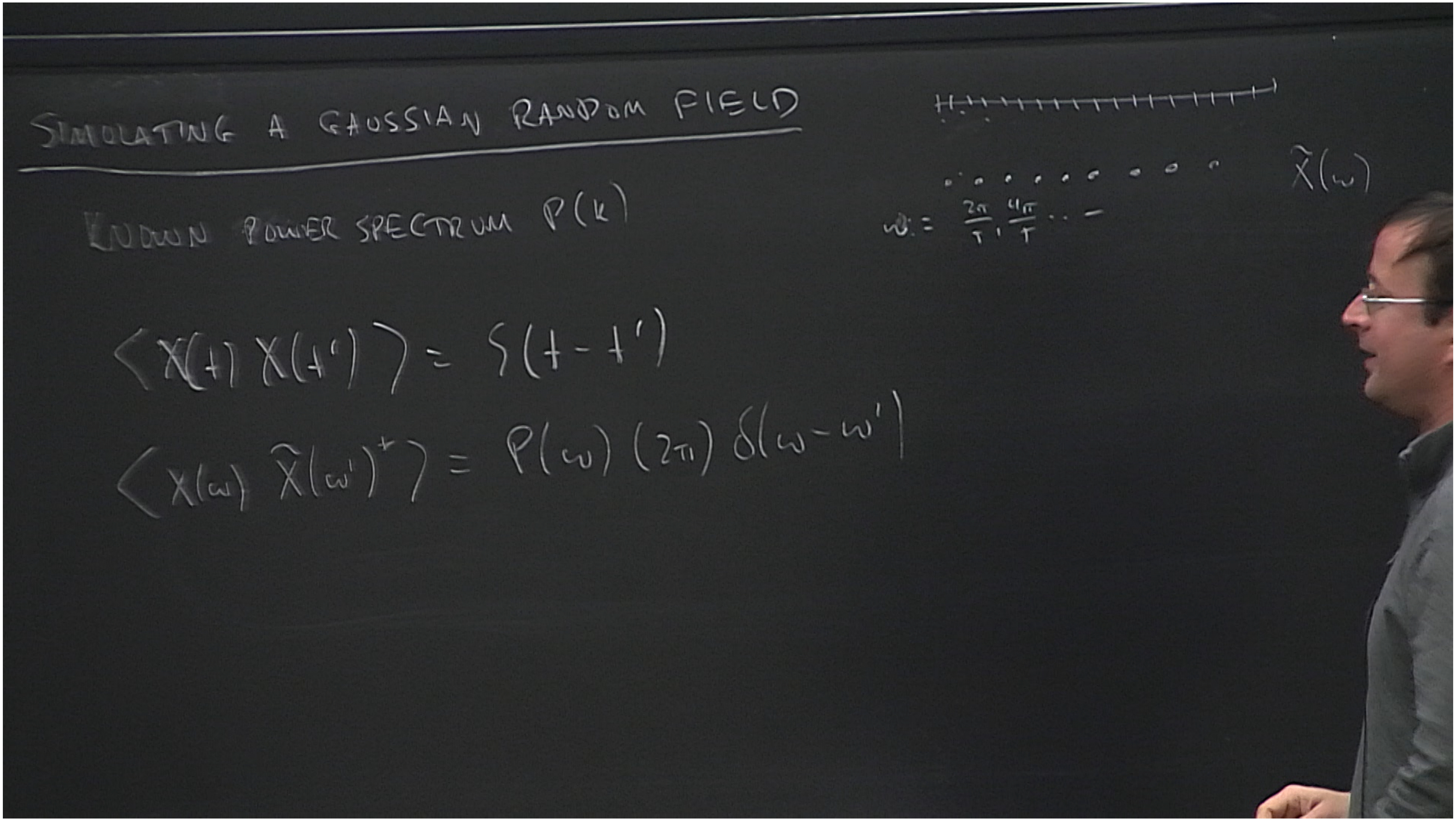
SIMULATING A GAUSSIAN RANDOM FIELD



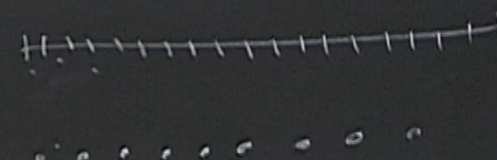
KNOWN POWER SPECTRUM $P(k)$


$C_{ij} \xleftrightarrow{R} \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_N \end{pmatrix}$


SIMULATING A GAUSSIAN RANDOM FIELD
KNOWN POWER SPECTRUM P(k)
⟨X(t) X(t')⟩ = S(t - t')
ω = 2π/T, 4π/T, ...
X
C_ij ←


SIMULATING A GAUSSIAN RANDOM FIELD
KNOWN POWER SPECTRUM $P(k)$
$\langle X(t)\, X(t') \rangle = \xi(t-t')$
$\langle \tilde{X}(\omega)\, \tilde{X}(\omega')^{*} \rangle = P(\omega)\,(2\pi)\,\delta(\omega-\omega')$
$\omega_i = \frac{2\pi}{T}, \frac{4\pi}{T} \ldots$
$\tilde{X}(\omega)$

SIMULATING A GAUSSIAN RANDOM FIELD



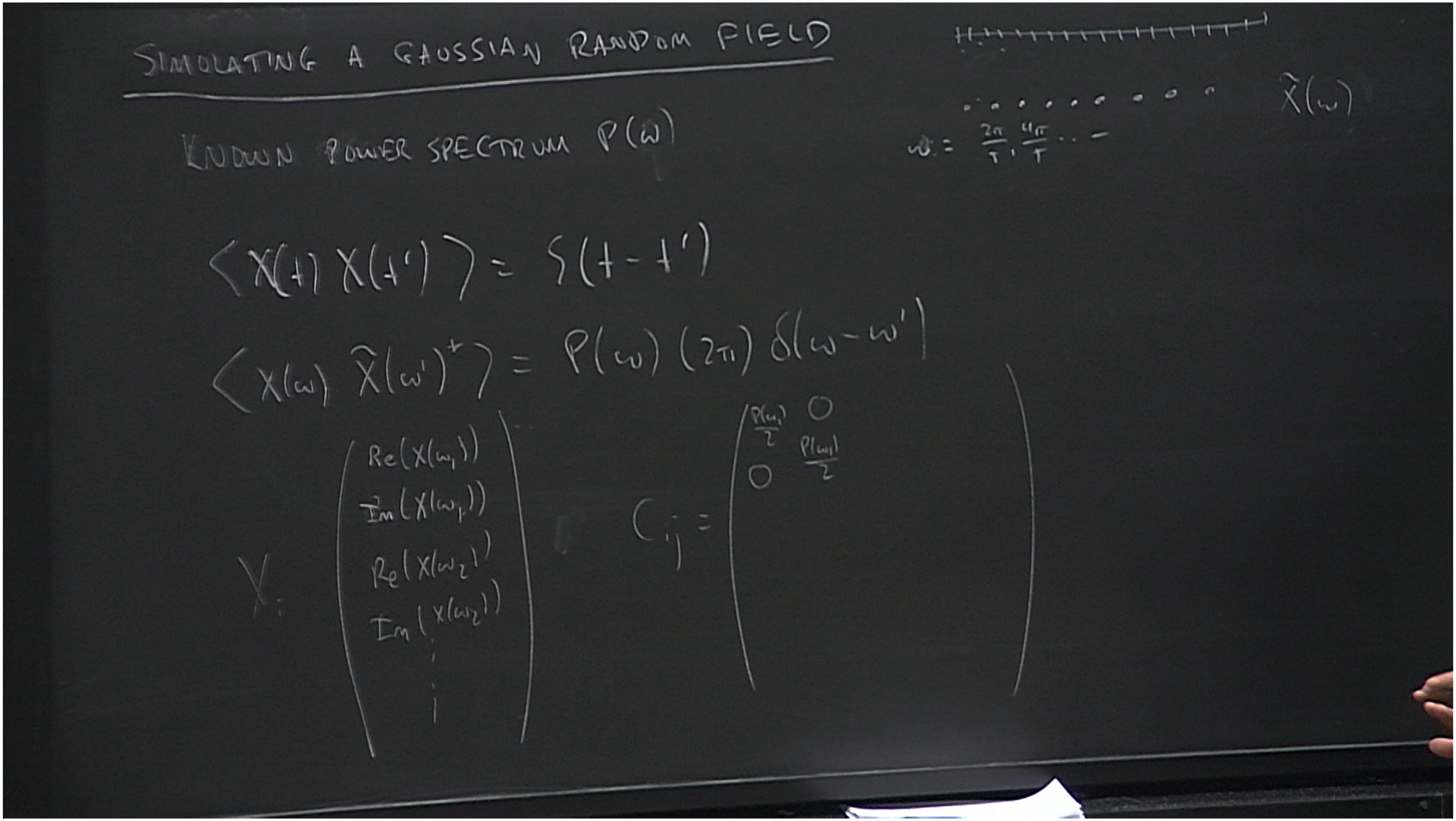
$\tilde{X}(\omega)$

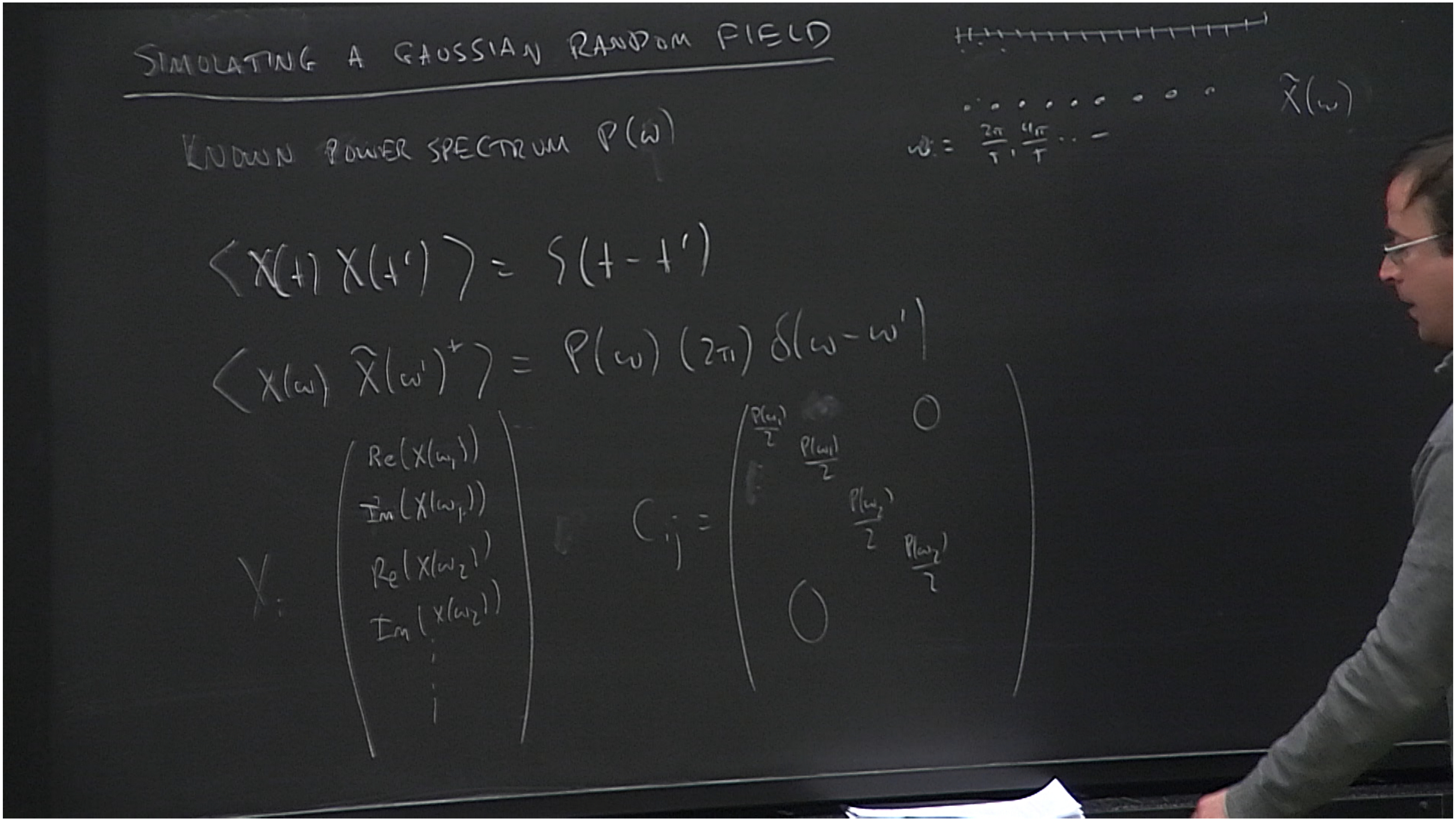
KNOWN POWER SPECTRUM $P(\omega)$

$\omega := \frac{2\pi}{T}, \frac{4\pi}{T} \ldots$ —

$$\langle X(t)\, X(t') \rangle = S(t - t')$$

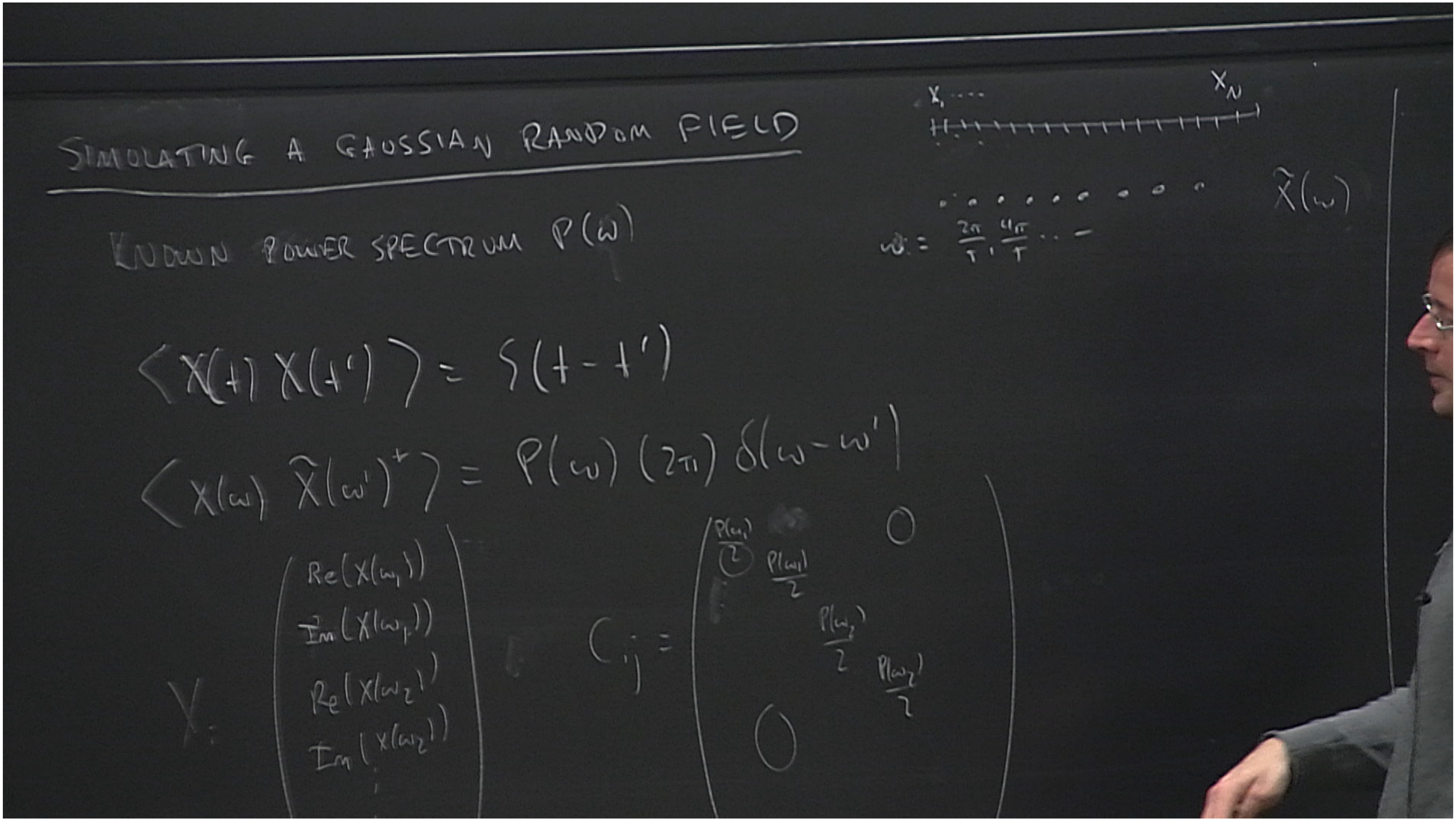
$$\langle X(\omega)\, \tilde{X}(\omega')^{*} \rangle = P(\omega)\,(2\pi)\,\delta(\omega - \omega')$$


SIMULATING A GAUSSIAN RANDOM FIELD
KNOWN POWER SPECTRUM $P(\omega)$
$\omega_i = \frac{2\pi}{T}, \frac{4\pi}{T} \ldots$
$\tilde{X}(\omega)$
$\langle X(t) X(t') \rangle = S(t-t')$
$\langle X(\omega) \tilde{X}(\omega')^{+} \rangle = P(\omega)(2\pi)\delta(\omega-\omega')$
X_i
$\begin{pmatrix} Re(X(\omega_1)) \\ Im(X(\omega_1)) \\ Re(X(\omega_2)) \\ Im(X(\omega_2)) \\ \vdots \end{pmatrix}$
$C_{ij} = \begin{pmatrix} \frac{P(\omega_1)}{2} & 0 \\ 0 & \frac{P(\omega_1)}{2} \end{pmatrix}$


SIMULATING A GAUSSIAN RANDOM FIELD
KNOWN POWER SPECTRUM $P(\omega)$
$\langle X(t)\, X(t') \rangle = \delta(t - t')$
$\langle \tilde{X}(\omega)\, \tilde{X}(\omega')^{*} \rangle = P(\omega)\,(2\pi)\,\delta(\omega - \omega')$
$\tilde{X}(\omega)$
$\omega = \frac{2\pi}{T}, \frac{4\pi}{T}, \dots$
$X_i = \begin{pmatrix} \mathrm{Re}(X(\omega_1)) \\ \mathrm{Im}(X(\omega_1)) \\ \mathrm{Re}(X(\omega_2)) \\ \mathrm{Im}(X(\omega_2)) \\ \vdots \end{pmatrix}$
$C_{ij} = \begin{pmatrix} \frac{P(\omega_1)}{2} & & & & 0 \\ & \frac{P(\omega_1)}{2} & & & \\ & & \frac{P(\omega_2)}{2} & & \\ & & & \frac{P(\omega_2)}{2} & \\ 0 & & & & \end{pmatrix}$

$$\langle \tilde{X}(\omega)\, \tilde{X}(\omega')^{*} \rangle = P(\omega)\,(2\pi)\,\delta(\omega-\omega')$$

$$\begin{pmatrix} \mathrm{Re}(X(\omega_1)) \\ \mathrm{Im}(X(\omega_1)) \\ \mathrm{Re}(X(\omega_2)) \\ \mathrm{Im}(X(\omega_2)) \\ \vdots \end{pmatrix}
\qquad
C_{ij} = \begin{pmatrix} \frac{P(\omega_1)}{2} & & & 0 \\ & \frac{P(\omega_1)}{2} & & \\ & & \frac{P(\omega_2)}{2} & \\ 0 & & & \frac{P(\omega_2)}{2} \end{pmatrix}$$


SIMULATING A GAUSSIAN RANDOM FIELD
KNOWN POWER SPECTRUM $P(\omega)$
$\langle X(t) X(t') \rangle = S(t-t')$
$\langle X(\omega) \tilde{X}(\omega')^* \rangle = P(\omega)(2\pi)\delta(\omega-\omega')$
$X_1 \cdots X_N$
$\tilde{X}(\omega)$
$\omega: = \frac{2\pi}{T}, \frac{4\pi}{T} \cdots$
$X:$
$\mathrm{Re}(X(\omega_1))$
$\mathrm{Im}(X(\omega_1))$
$\mathrm{Re}(X(\omega_2))$
$\mathrm{Im}(X(\omega_2))$
$C_{ij} =$
$\frac{P(\omega_1)}{2}$
$\frac{P(\omega_1)}{2}$
$\frac{P(\omega_2)}{2}$
$\frac{P(\omega_2)}{2}$
0
0

RANDOM FIELDS IN N DIMENSIONS

- ASSUME TRANSLATION INVARIANCE + ROTATIONAL SYMMETRY

CORRELATION FUNCTION $\langle \phi(\vec{x})\,\phi(\vec{y}) \rangle = \xi(\vec{x}-\vec{y})$

RANDOM FIELDS IN N DIMENSIONS

- ASSUME TRANSLATION INVARIANCE + ROTATIONAL SYMMETRY

CORRELATION FUNCTION $\langle \phi(\vec{x})\, \phi(\vec{y}) \rangle = \xi(|\vec{x} - \vec{y}|)$

RANDOM FIELDS IN N DIMENSIONS

- ASSUME TRANSLATION INVARIANCE + ROTATIONAL SYMMETRY

FUNCTION OF 1 VARIABLE

CORRELATION FUNCTION $\langle \phi(\vec{x}) \phi(\vec{y}) \rangle = \xi(|\vec{x}-\vec{y}|)$

POWER SPECTRUM $\langle \phi(\vec{k}) \phi(\vec{k}')^* \rangle = P(|k|) (2\pi)^n \delta^n(\vec{k}-\vec{k}')$

RANDOM FIELDS IN N DIMENSIONS

- ASSUME TRANSLATION INVARIANCE + ROTATIONAL SYMMETRY

CORRELATION FUNCTION $\langle \phi(\vec{x})\, \phi(\vec{y}) \rangle = \xi(|\vec{x} - \vec{y}|)$ FUNCTION OF 1 VARIABLE

POWER SPECTRUM $\langle \phi(\vec{k})\, \phi(\vec{k}')^{*} \rangle = P(\vec{k})\, (2\pi)^{n}\, \delta^{n}(\vec{k} - \vec{k}')$

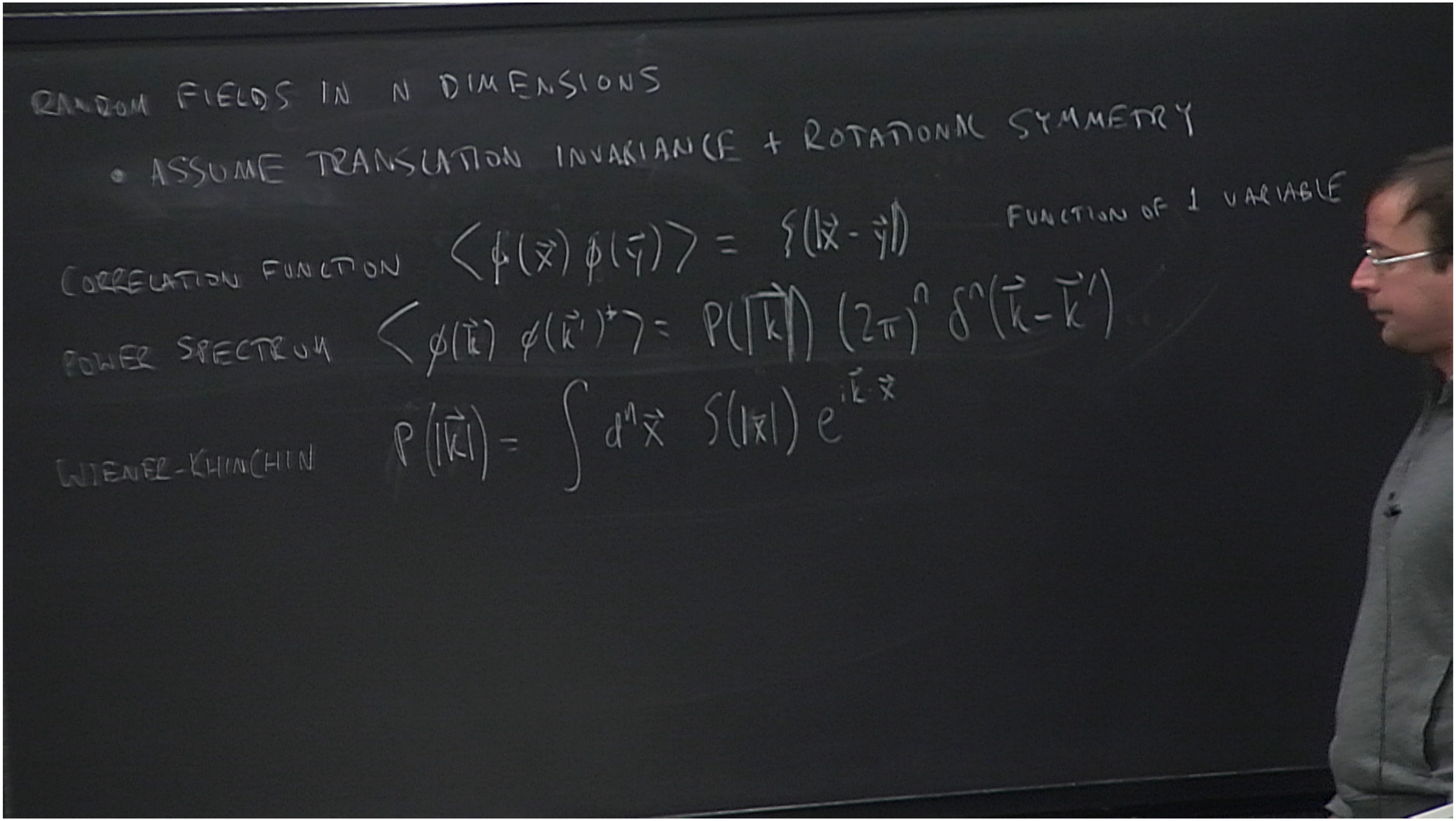
RANDOM FIELDS IN N DIMENSIONS

- ASSUME TRANSLATION INVARIANCE + ROTATIONAL SYMMETRY

FUNCTION OF 1 VARIABLE

CORRELATION FUNCTION $\langle \phi(\vec{x})\, \phi(\vec{y}) \rangle = \xi(|\vec{x} - \vec{y}|)$

POWER SPECTRUM $\langle \phi(\vec{k})\, \phi(\vec{k}')^{*} \rangle = P(|\vec{k}|)\, (2\pi)^n\, \delta^n(\vec{k} - \vec{k}')$


RANDOM FIELDS IN N DIMENSIONS
• ASSUME TRANSLATION INVARIANCE + ROTATIONAL SYMMETRY
FUNCTION OF 1 VARIABLE
CORRELATION FUNCTION $\langle \phi(\vec{x}) \phi(\vec{y}) \rangle = \xi(|\vec{x} - \vec{y}|)$
POWER SPECTRUM $\langle \phi(\vec{k}) \phi(\vec{k}')^* \rangle = P(|\vec{k}|) (2\pi)^n \delta^n(\vec{k} - \vec{k}')$
WIENER-KHINCHIN $P(|\vec{k}|) = \int d^n\vec{x} \; \xi(|\vec{x}|) e^{i\vec{k}\cdot\vec{x}}$

RANDOM FIELDS IN N DIMENSIONS

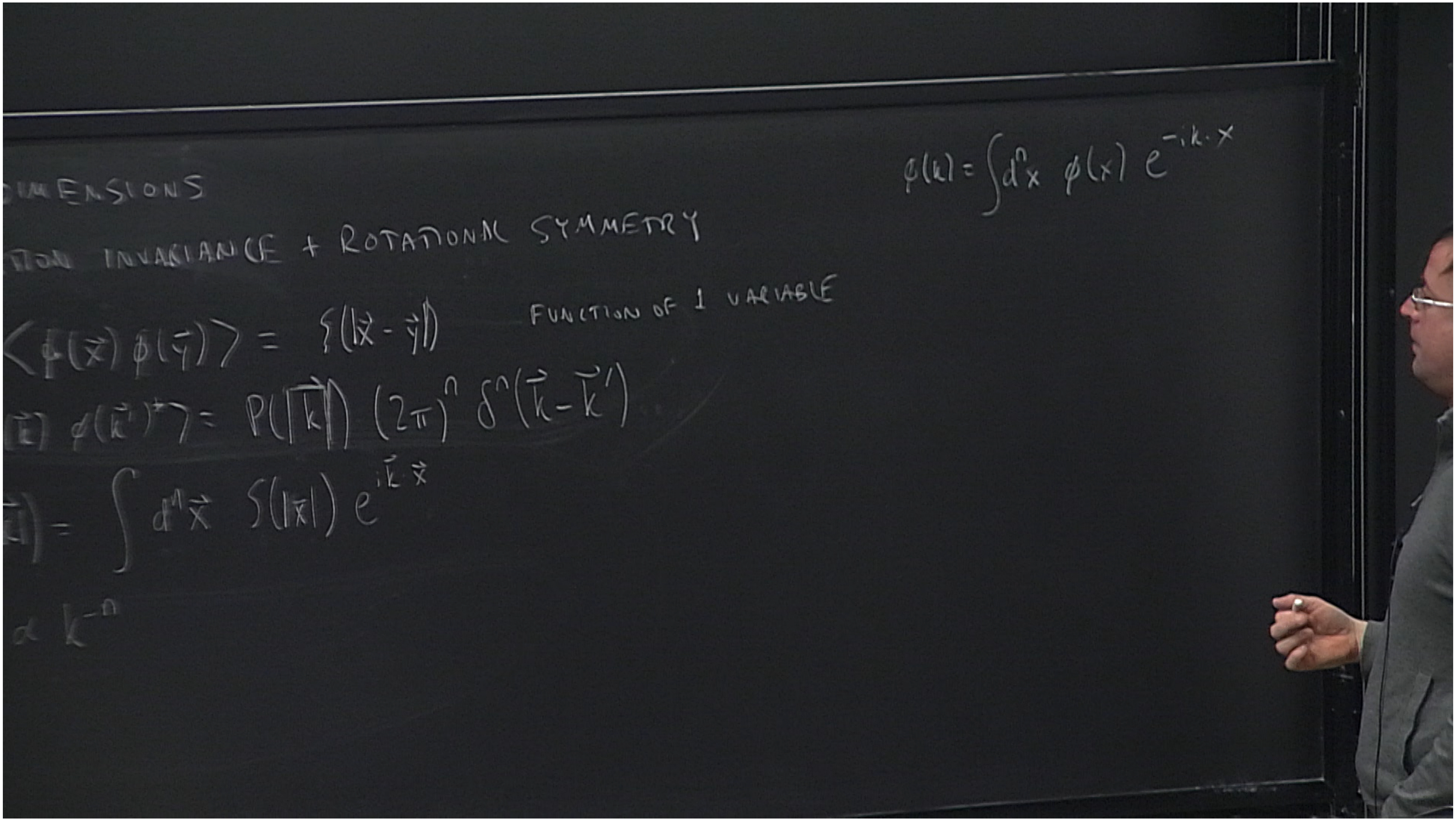
- ASSUME TRANSLATION INVARIANCE + ROTATIONAL SYMMETRY

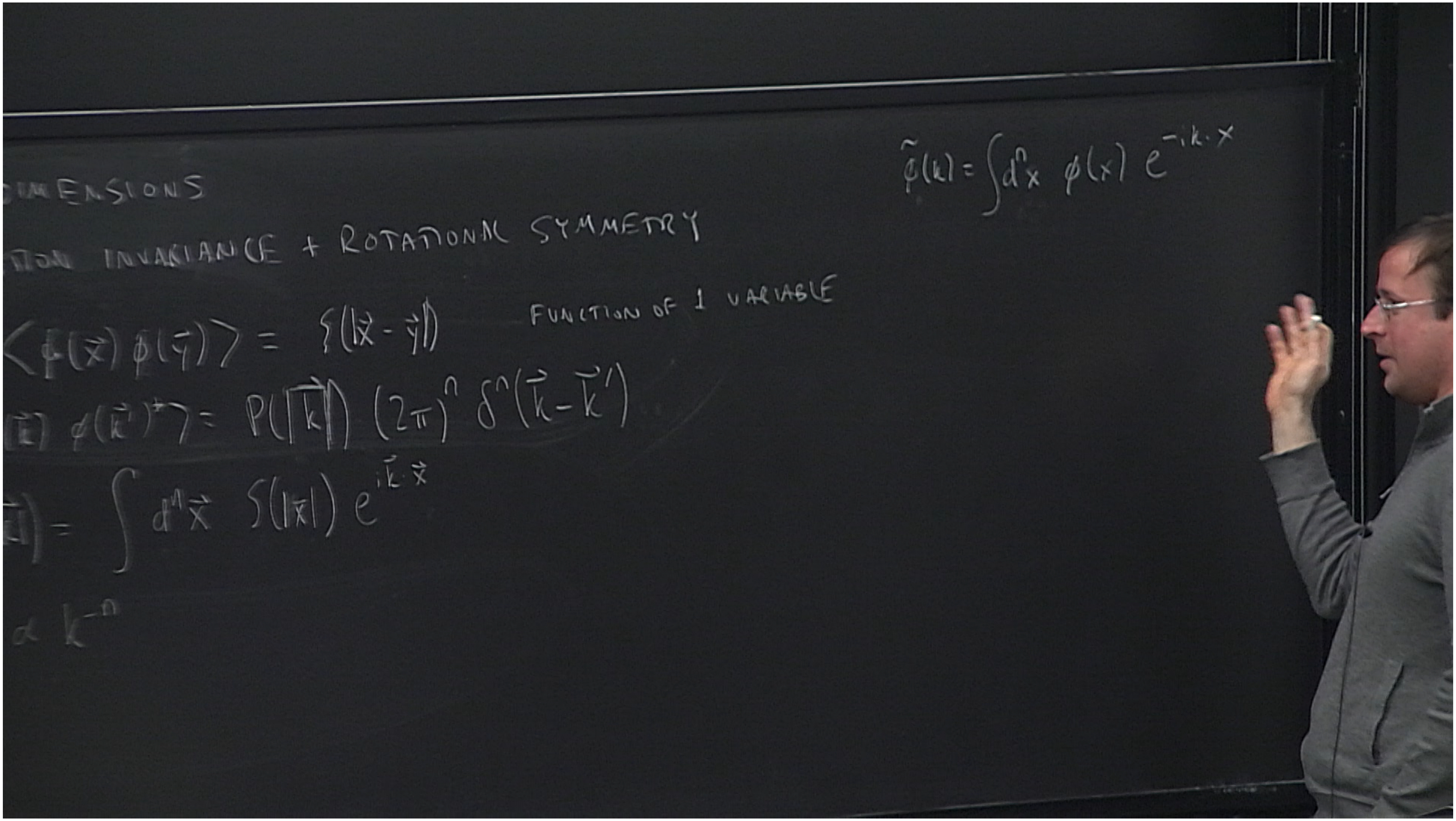
CORRELATION FUNCTION $\langle \phi(\vec{x})\, \phi(\vec{y}) \rangle = \xi(|\vec{x} - \vec{y}|)$ FUNCTION OF 1 VARIABLE

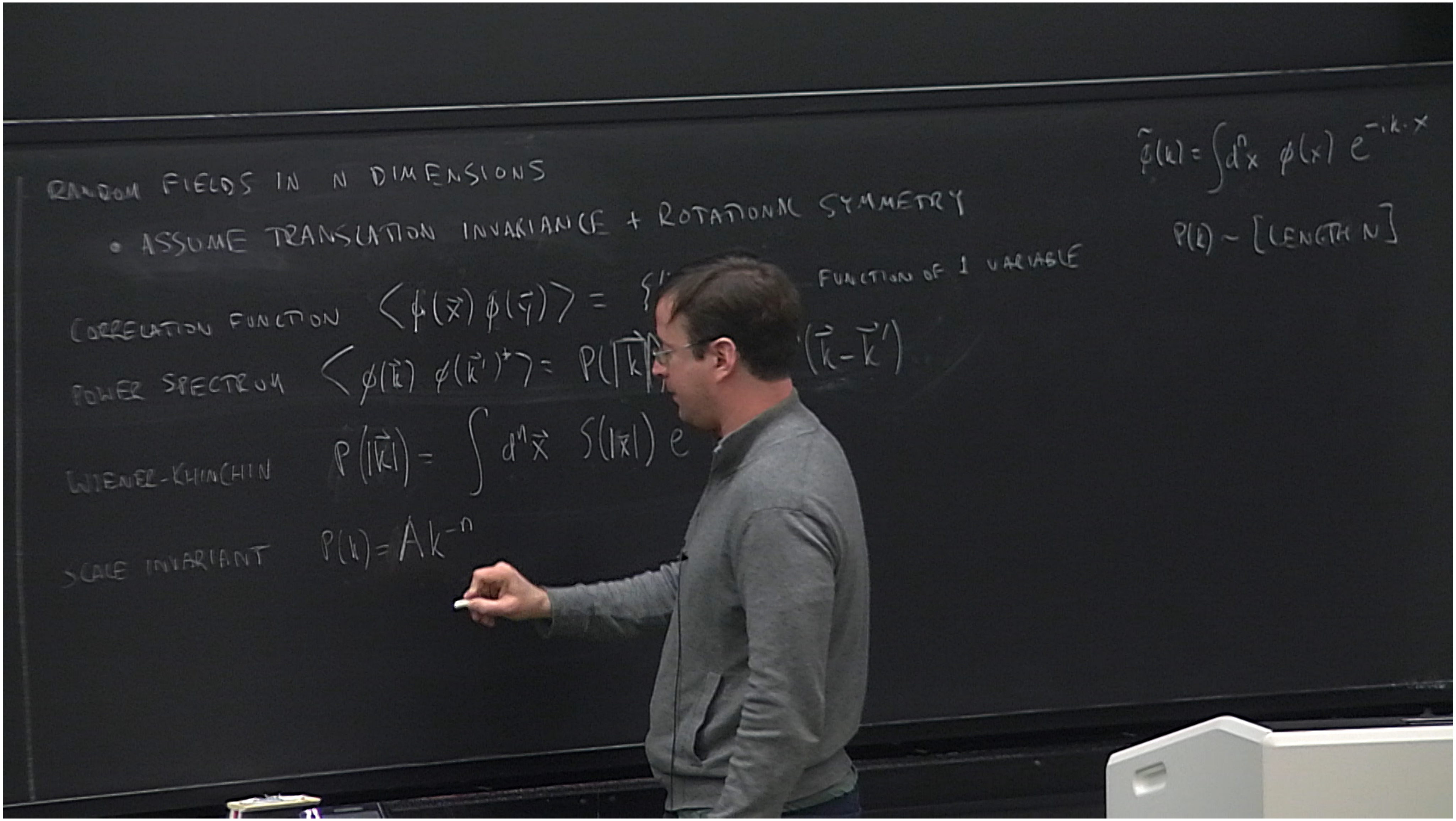
POWER SPECTRUM $\langle \phi(\vec{k})\, \phi(\vec{k}')^* \rangle = P(|\vec{k}|)\, (2\pi)^n\, \delta^n(\vec{k} - \vec{k}')$

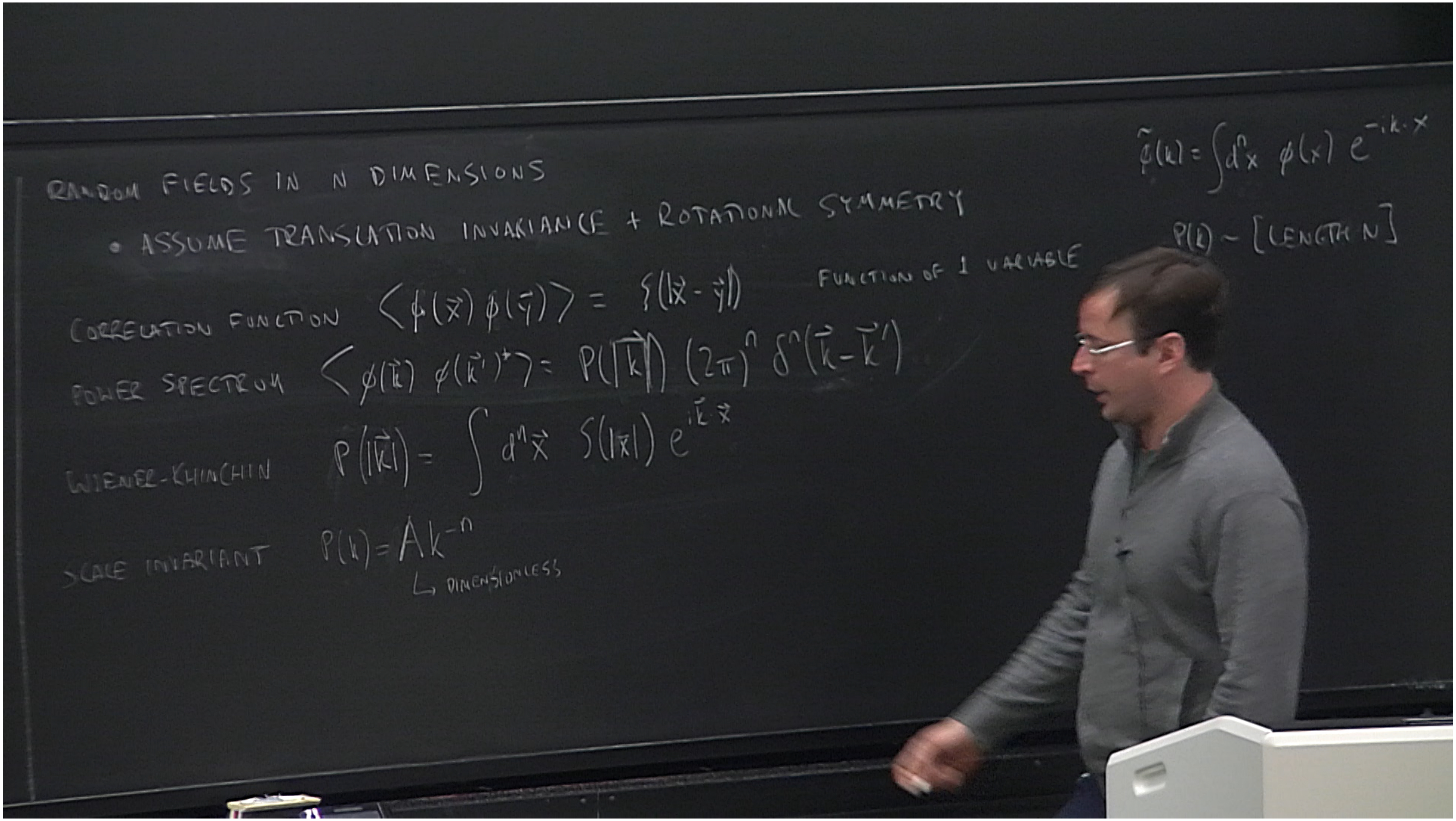
WIENER-KHINCHIN $P(|\vec{k}|) = \int d^n\vec{x}\; \xi(|\vec{x}|)\, e^{i\vec{k}\cdot\vec{x}}$

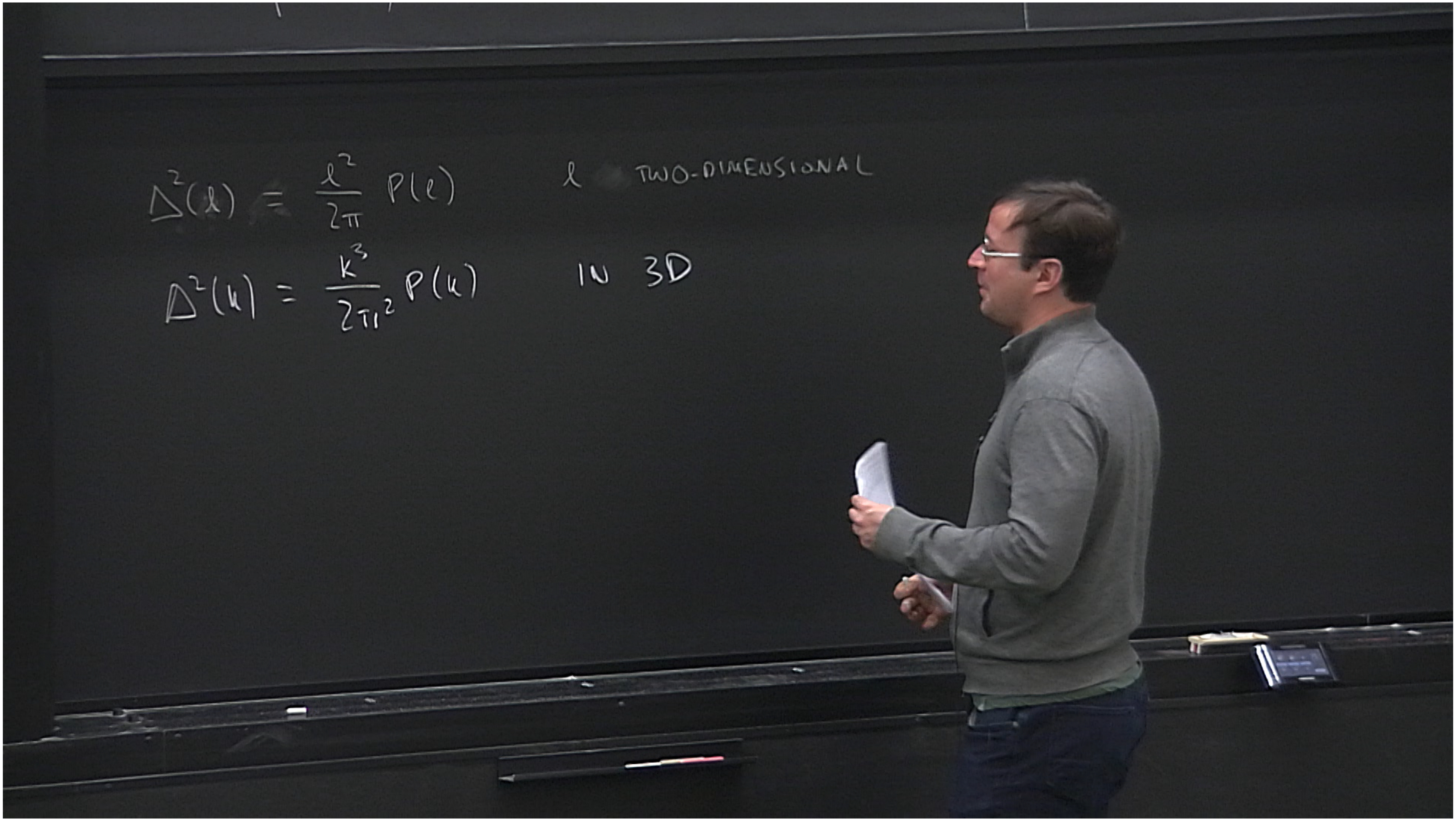
SCALE INVARIANT $P(k) \propto k^{-n}$


IMENSIONS
INVARIANCE + ROTATIONAL SYMMETRY
$\langle \phi(\vec{x})\, \phi(\vec{y}) \rangle = \xi(|\vec{x}-\vec{y}|)$
FUNCTION OF 1 VARIABLE
$\phi(\vec{k}')^* \rangle = P(|\vec{k}|)\,(2\pi)^n\,\delta^n(\vec{k}-\vec{k}')$
$= \int d^n\vec{x}\; \xi(|\vec{x}|)\, e^{i\vec{k}\cdot\vec{x}}$
$\propto k^{-n}$
$\phi(k) = \int d^n x\; \phi(x)\, e^{-ik\cdot x}$


IMENSIONS
TION INVARIANCE + ROTATIONAL SYMMETRY
FUNCTION OF 1 VARIABLE
$\langle \phi(\vec{x}) \phi(\vec{y}) \rangle = \xi(|\vec{x}-\vec{y}|)$
$\phi(\vec{k}) \phi(\vec{k}')^* \rangle = P(|\vec{k}|) (2\pi)^n \delta^n(\vec{k}-\vec{k}')$
$) = \int d^n\vec{x} \; \xi(|\vec{x}|) e^{i\vec{k}\cdot\vec{x}}$
$\propto k^{-n}$
$\tilde{\phi}(k) = \int d^n x \; \phi(x) e^{-ik\cdot x}$


RANDOM FIELDS IN N DIMENSIONS
• ASSUME TRANSLATION INVARIANCE + ROTATIONAL SYMMETRY
FUNCTION OF 1 VARIABLE
CORRELATION FUNCTION
POWER SPECTRUM
WIENER-KHINCHIN
SCALE INVARIANT
P(k) = Ak^{-n}
P(k) ~ [LENGTH N]


RANDOM FIELDS IN N DIMENSIONS
• ASSUME TRANSLATION INVARIANCE + ROTATIONAL SYMMETRY
FUNCTION OF 1 VARIABLE
CORRELATION FUNCTION $\langle \phi(\vec{x})\,\phi(\vec{y}) \rangle = \xi(|\vec{x}-\vec{y}|)$
POWER SPECTRUM $\langle \phi(\vec{k})\,\phi(\vec{k}')^{*} \rangle = P(|\vec{k}|)\,(2\pi)^n\,\delta^n(\vec{k}-\vec{k}')$
WIENER-KHINCHIN $P(|\vec{k}|) = \int d^n\vec{x}\;\xi(|\vec{x}|)\,e^{i\vec{k}\cdot\vec{x}}$
SCALE INVARIANT $P(k) = A k^{-n}$
DIMENSIONLESS
$\tilde{\phi}(k) = \int d^n x\;\phi(x)\,e^{-ik\cdot x}$
$P(k) \sim [\text{LENGTH}^{\,N}]$

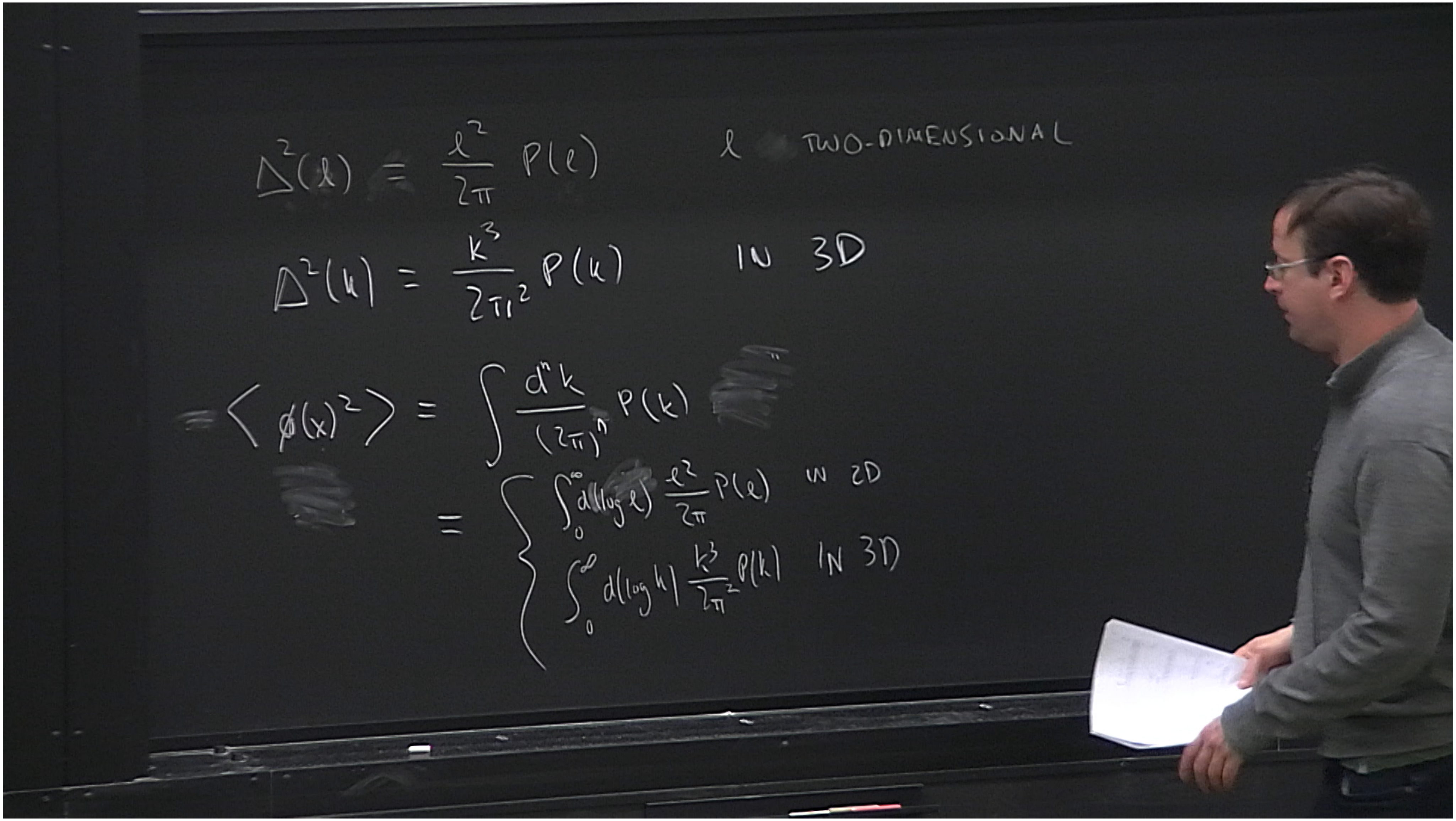

$\Delta^2(\ell) = \frac{\ell^2}{2\pi} P(\ell)$
ℓ TWO-DIMENSIONAL
$\Delta^2(k) = \frac{k^3}{2\pi^2} P(k)$
IN 3D

$$\Delta^2(\ell) = \frac{\ell^2}{2\pi} P(\ell)$$ ℓ TWO-DIMENSIONAL

$$\Delta^2(k) = \frac{k^3}{2\pi^2} P(k)$$ IN 3D

$$\langle \phi(x)^2 \rangle = \int \frac{d^n k}{(2\pi)^n} P(k)$$

$$\xi(0)$$



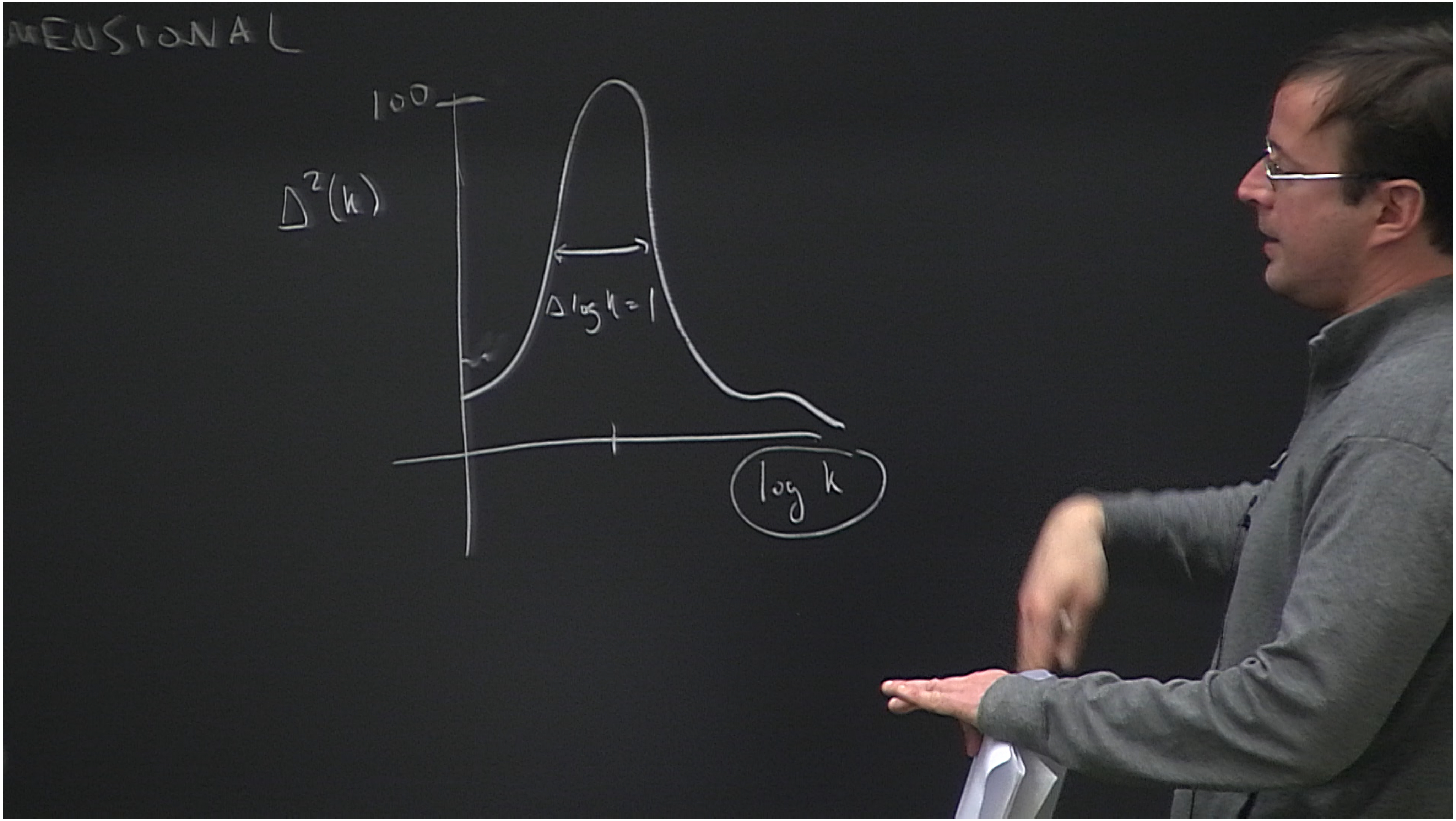
$\Delta^2(\ell) = \frac{\ell^2}{2\pi} P(\ell)$
ℓ TWO-DIMENSIONAL
$\Delta^2(k) = \frac{k^3}{2\pi^2} P(k)$
IN 3D
$\langle \phi(x)^2 \rangle = \int \frac{d^n k}{(2\pi)^n} P(k)$
$= \int_0^\infty d(\log \ell) \frac{\ell^2}{2\pi} P(\ell)$
IN 2D
$\int_0^\infty d(\log k) \frac{k^3}{2\pi^2} P(k)$
IN 3D

$$\Delta^2(\ell) = \frac{\ell^2}{2\pi} P(\ell) \qquad \ell \text{ TWO-DIMENSIONAL}$$

$$\Delta^2(k) = \frac{k^3}{2\pi^2} P(k) \qquad \text{IN 3D}$$

$$\langle \phi(x)^2 \rangle = \int \frac{d^n k}{(2\pi)^n} P(k)$$

$$= \begin{cases} \displaystyle\int_0^\infty d(\log \ell)\, \overbrace{\frac{\ell^2}{2\pi} P(\ell)}^{\Delta^2(\ell)} & \text{IN 2D} \\[2ex] \displaystyle\int_0^\infty d(\log k)\, \underbrace{\frac{k^3}{2\pi^2} P(k)}_{\Delta^2(k)} & \text{IN 3D} \end{cases}$$



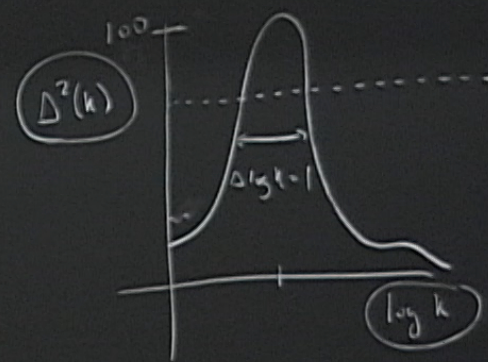
MENSIONAL
100
$\Delta^2(k)$
$\Delta \log k = 1$
$\log k$

$$\Delta^2(\ell) = \frac{\ell^2}{2\pi} P(\ell)$$

ℓ TWO-DIMENSIONAL

$$\Delta^2(k) = \frac{k^3}{2\pi^2} P(k)$$

IN 3D



$$\langle \phi(x)^2 \rangle = \int \frac{d^n k}{(2\pi)^n} P(k)$$

$$= \begin{cases} \int_0^\infty d(\log \ell)\, \overbrace{\frac{\ell^2}{2\pi} P(\ell)}^{\Delta^2(\ell)} & \text{IN 2D} \\ \int_0^\infty d(\log k)\, \underbrace{\frac{k^3}{2\pi^2} P(k)}_{\Delta^2(k)} & \text{IN 3D} \end{cases}$$

Random field slideshow

Kendrick Smith
PSI 2018, Explorations in Cosmology

Reminder: the correlation function ζ(r) and the power spectrum P(k) contain the same information, and are alternate ways of parameterizing the two-point statistics of a random field.

$$\langle \phi(\mathbf{x})\phi(\mathbf{y})\rangle = \zeta(|\mathbf{x}-\mathbf{y}|) \qquad \text{(definition of } \zeta(r))$$

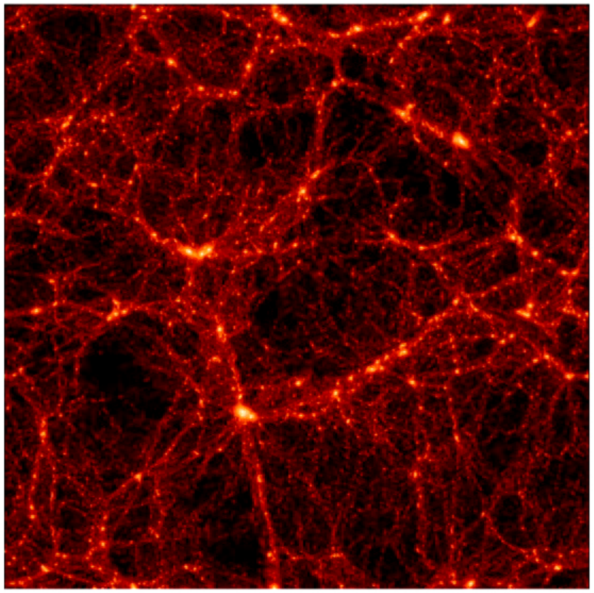
$$\langle \phi(\mathbf{k})\phi(\mathbf{k})^*\rangle = P(k)(2\pi)^n\delta^n(\mathbf{k}-\mathbf{k}') \qquad \text{(definition of P(k))}$$

$$\zeta(r) = \int \frac{d^n\mathbf{k}}{(2\pi)^n} P(k) e^{i\mathbf{k}\cdot\mathbf{r}} \qquad \text{(Wiener-Khinchin theorem)}$$

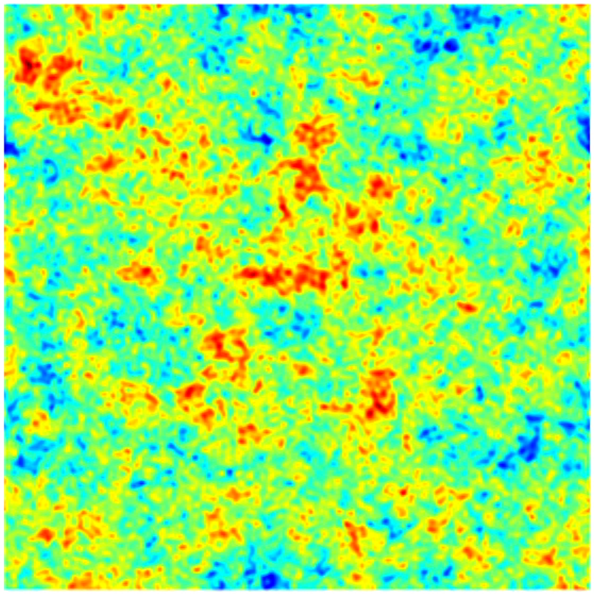
In these slides, we'll use the power spectrum P(k).

Reminder: for a Gaussian field, the statistics of the field are completely determined by the power spectrum P(k).

For a non-Gaussian field, this is not true!



cosmological density
field at z=0

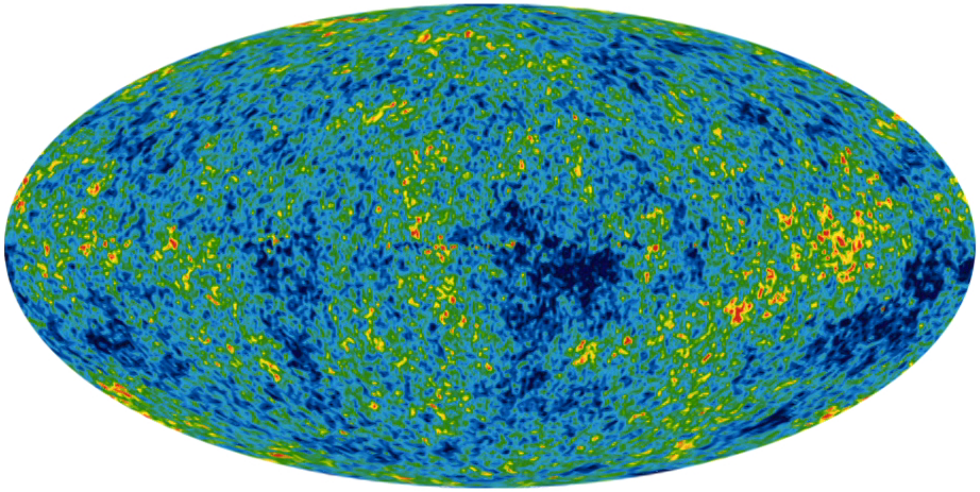


Gaussian field with
same power spectrum

In cosmology, some fields are Gaussian and some are not.
In general, Gaussianity tends to be a good approximation either:

- at early times
- on large scales (i.e. Fourier modes with small k)

We'll explain later why this is true! The CMB is nearly perfectly Gaussian (to 0.1% or better) since it is formed early (z=1100).

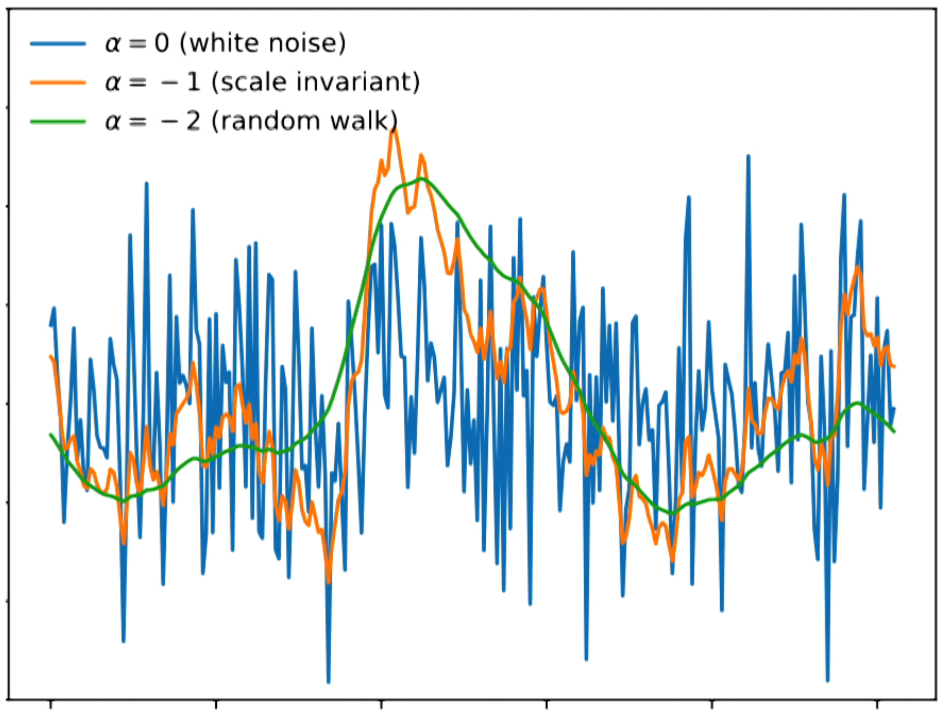


Reminder: a Gaussian field $\phi(x)$ may be simulated as follows.

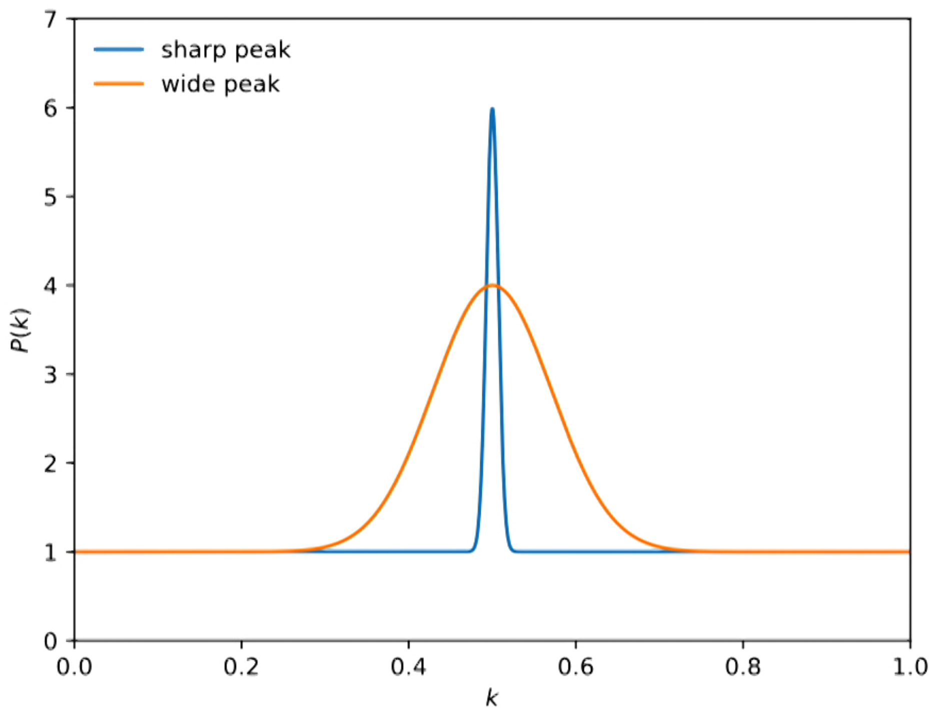
- For each Fourier mode k, initialize the real and complex parts of $\phi(k)$ to random Gaussian numbers with variance $P(k)/2$.
- Detail: don't forget $\phi(-k) = \phi(k)^*$
- Take the Fourier transform $\phi(k) \to \phi(x)$

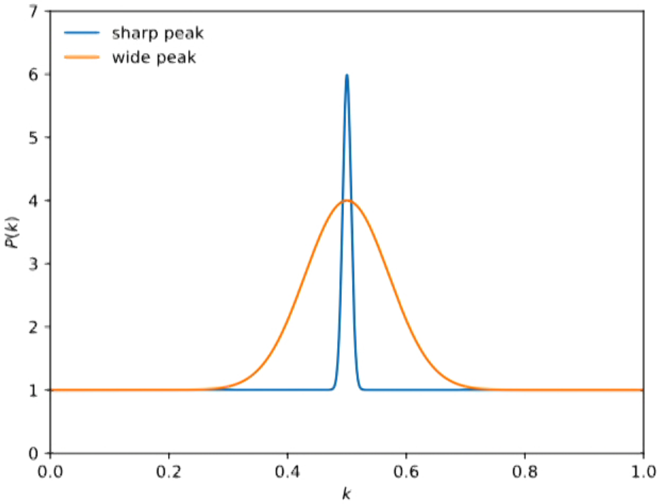
Let's look at some examples, to get some intuition for what the power spectrum represents.

One-dimensional Gaussian random fields with power-law spectra $P(\omega) \propto \omega^{\alpha}$

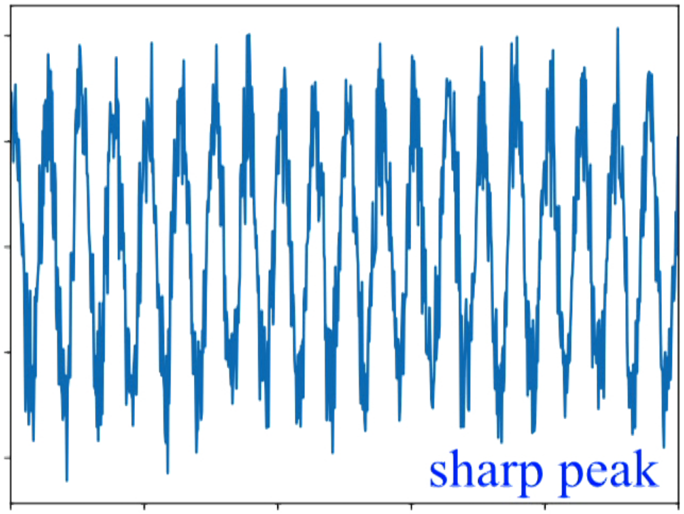


Next example: one-dimensional Gaussian random field, whose power spectrum is a constant (white noise), plus a peak which can be either "sharp" or "wide" in k.

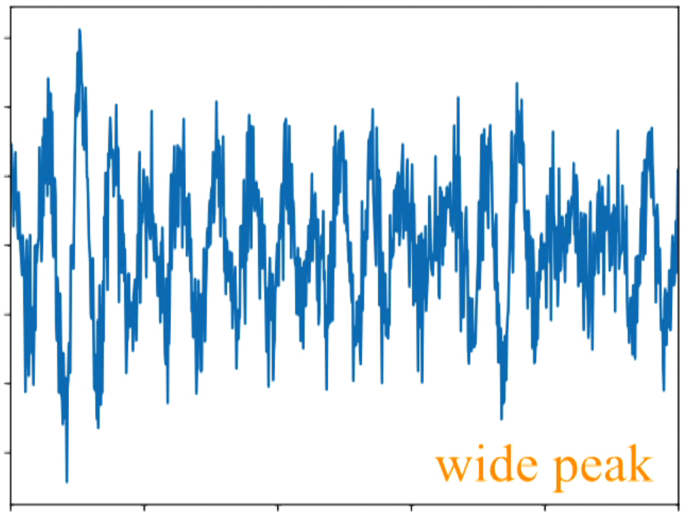




sharp peak
wide peak
P(k)
k

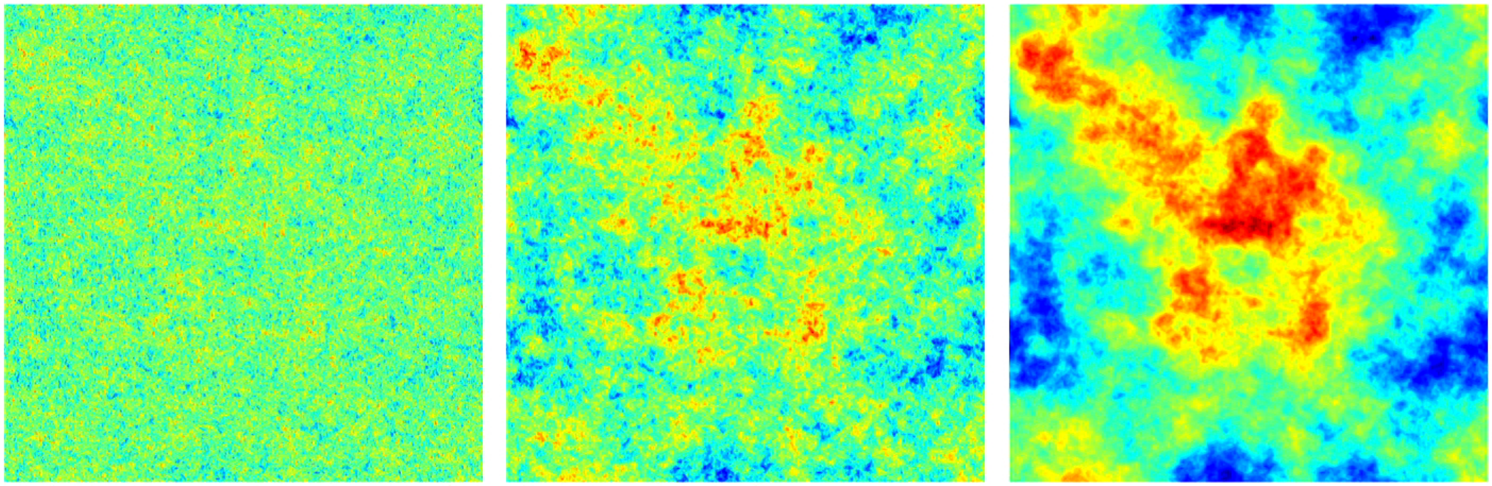


sharp peak



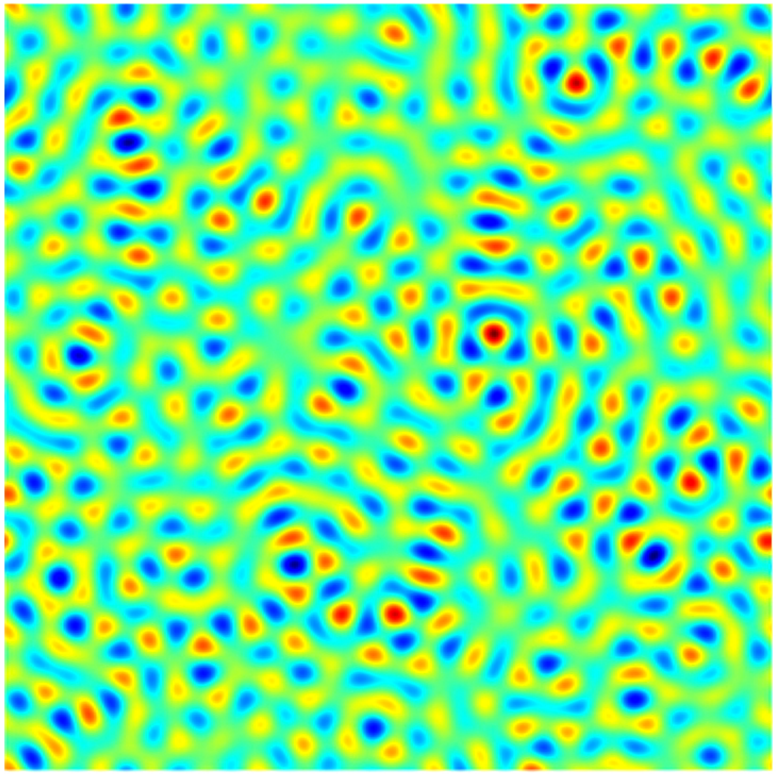
wide peak

Two-dimensional Gaussian random fields with power-law spectra $P(l) \propto l^{\alpha}$



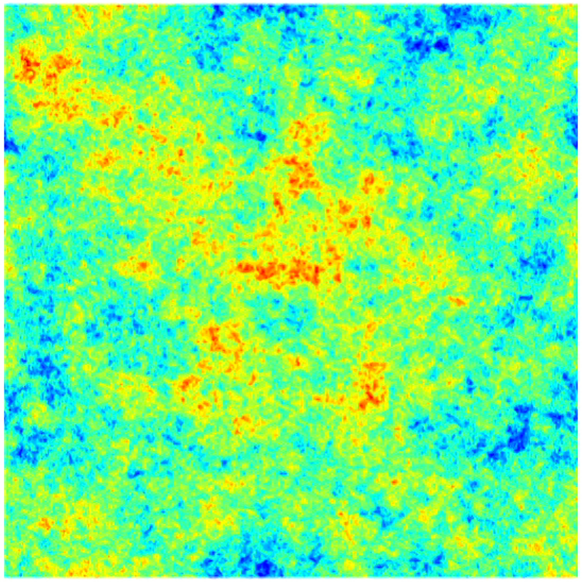
$\alpha = -1$ white noise | $\alpha = -2$ scale invariant | $\alpha = -3$ "red" spectrum

Delta function power spectrum: $P(l) \propto \delta(l-l_0)$

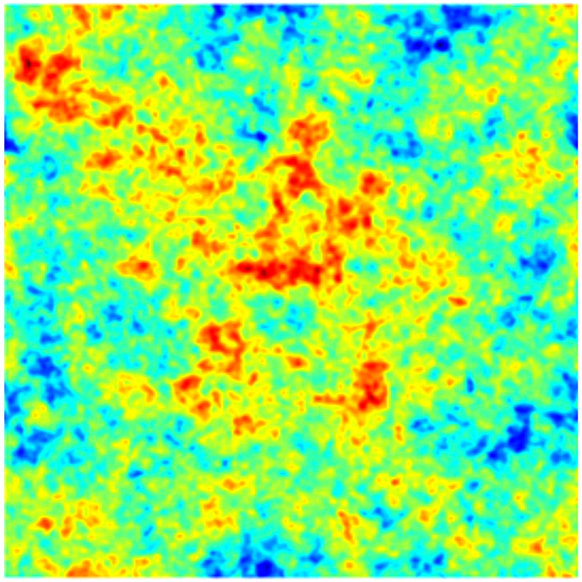


Well-defined characteristic scale

Putting a "cutoff" in the power spectrum removes power on small scales.

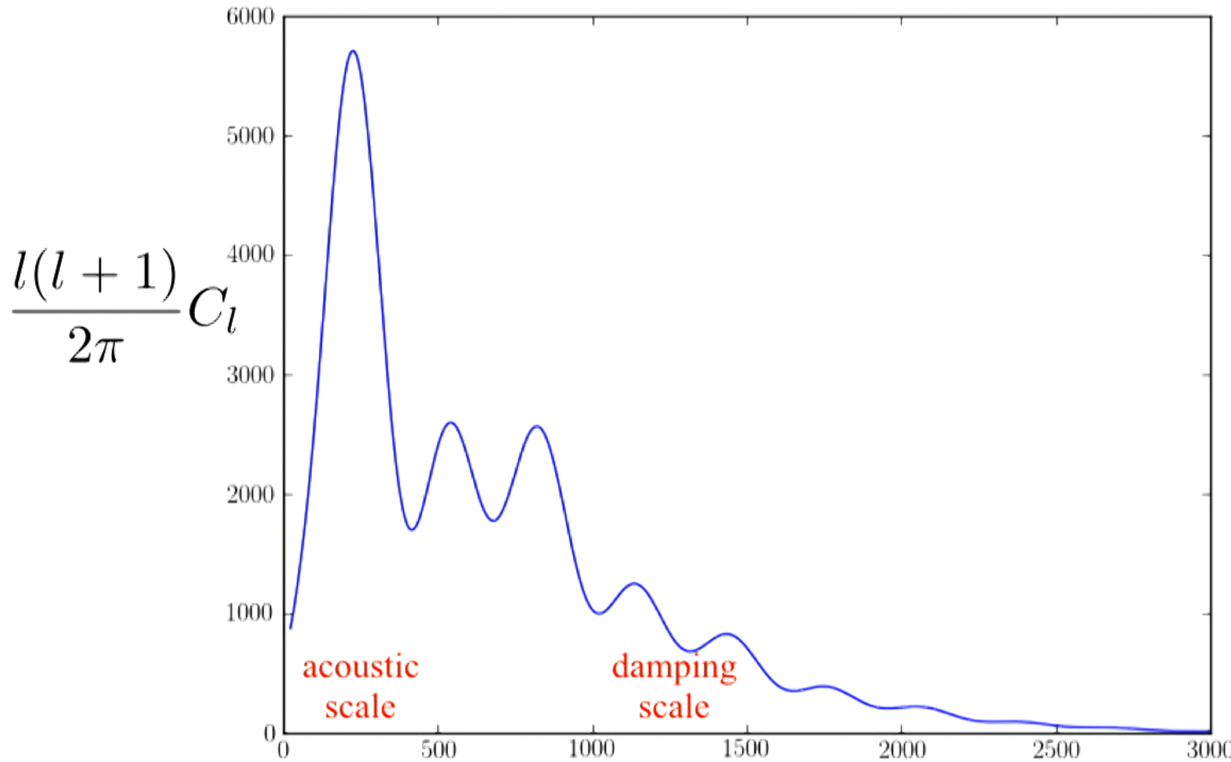


Scale invariant power spectrum: $P(l) \propto l^{-2}$



Scale invariant power spectrum with cutoff: $P(l) \propto l^{-2} \exp(-(l/l_0)^2)$

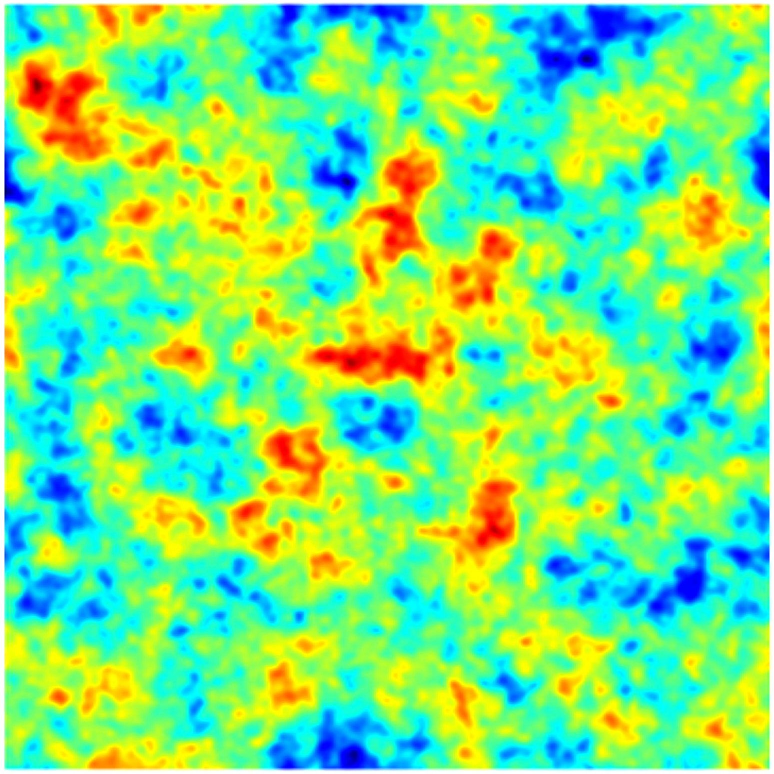
CMB power spectrum



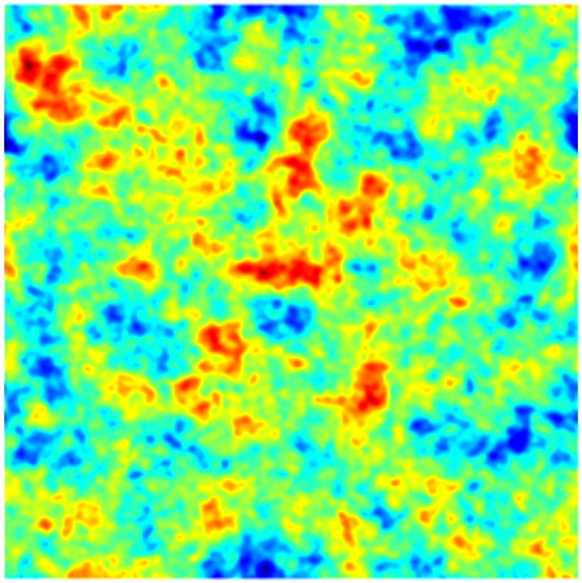
Angular wavenumber l

Calculated numerically!

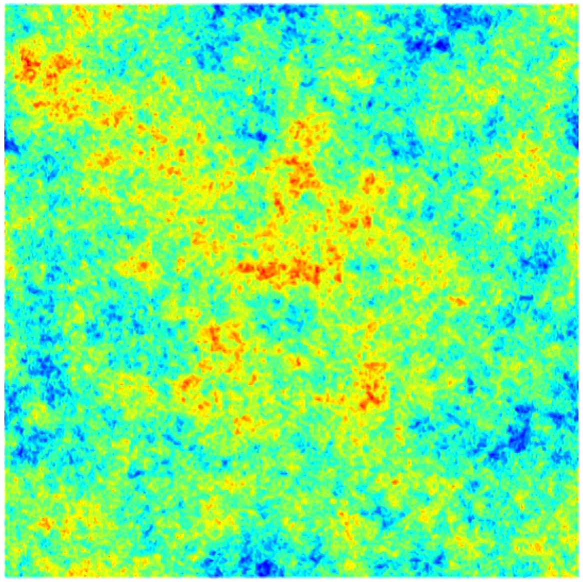
A Gaussian map with the CMB power spectrum.



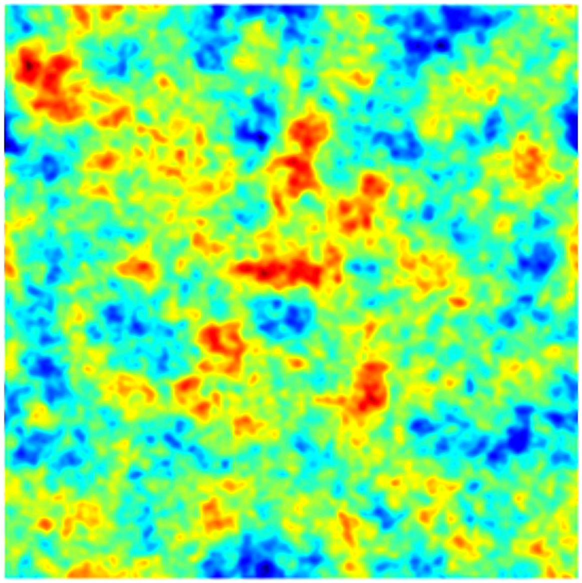
Since the CMB is a Gaussian field (as far as we know), this is a perfect simulation of the true CMB!



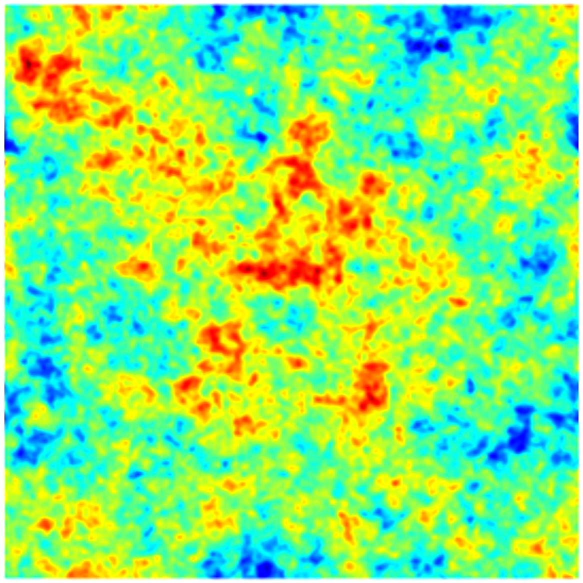
CMB power spectrum



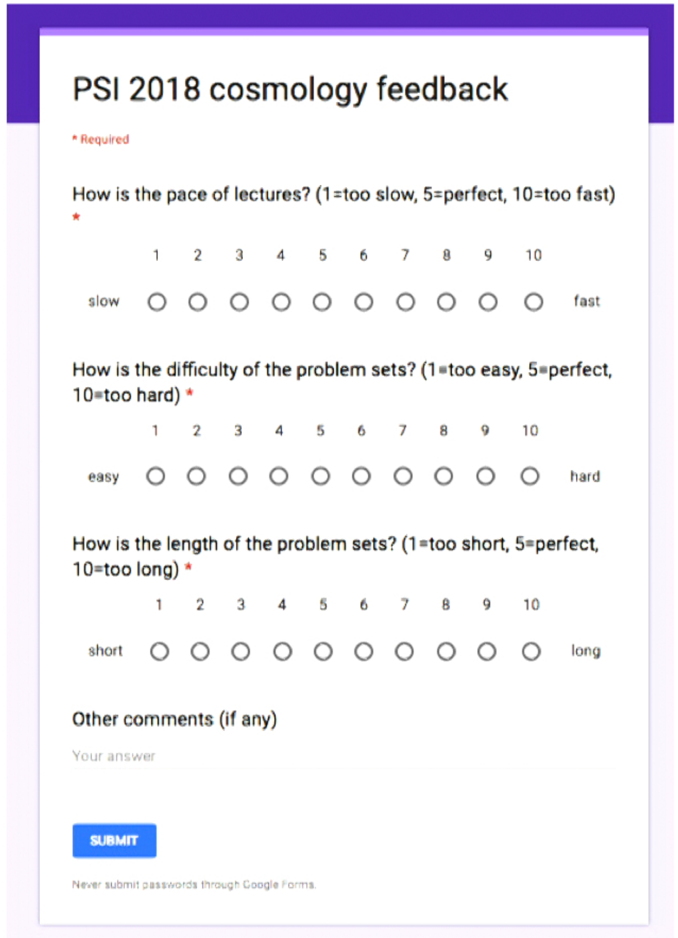
Scale invariant power spectrum



CMB power spectrum



Scale invariant power spectrum with cutoff: $C(l) \propto l^{-2} \exp(-(l/l_0)^2)$



PSI 2018 cosmology feedback

* Required

How is the pace of lectures? (1=too slow, 5=perfect, 10=too fast) *

	1	2	3	4	5	6	7	8	9	10	
slow	○	○	○	○	○	○	○	○	○	○	fast

How is the difficulty of the problem sets? (1=too easy, 5=perfect, 10=too hard) *

	1	2	3	4	5	6	7	8	9	10	
easy	○	○	○	○	○	○	○	○	○	○	hard

How is the length of the problem sets? (1=too short, 5=perfect, 10=too long) *

	1	2	3	4	5	6	7	8	9	10	
short	○	○	○	○	○	○	○	○	○	○	long

Other comments (if any)

Your answer

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(url posted on psi wiki)