Title: PSI 2017/2018 - Cosmology - Lecture 1

Date: Apr 09, 2018 10:15 AM

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Abstract:

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma}\left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu}\right)$$

$$\boxed{(- + + +)}$$

$$\nabla_\mu T^{\rho_1 \cdots \rho_m}_{\sigma_1 \cdots \sigma_n} = \partial_\mu T^{\rho_1 \cdots \rho_m}_{\sigma_1 \cdots \sigma_n} + \sum_{i=1}^{m} \Gamma^{\rho_i}_{\mu\nu} T^{\rho_1 \cdots \nu \cdots \rho_m}_{\sigma_1 \cdots \sigma_n} - \sum_{j=1}^{n} \Gamma^{\nu}_{\mu\sigma_j} T^{\rho_1 \cdots \rho_m}_{\sigma_1 \cdots \nu \cdots \sigma_n}$$

$$R_{\mu\nu\rho}{}^{\sigma} = \partial_\nu \Gamma^{\sigma}_{\mu\rho} - \partial_\mu \Gamma^{\sigma}_{\nu\rho} + \Gamma^{\lambda}_{\mu\rho} \Gamma^{\sigma}_{\nu\lambda} - \Gamma^{\lambda}_{\nu\rho} \Gamma^{\sigma}_{\mu\lambda}$$

$$\left(\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu\right) T^{\rho_1 \cdots \rho_m}_{\sigma_1 \cdots \sigma_n} = -\sum_{i=1}^{m} R_{\mu\nu\lambda}{}^{\rho_i} T^{\rho_1 \cdots \lambda \cdots \rho_m}_{\sigma_1 \cdots \sigma_n} + \sum_{j=1}^{n} R_{\mu\nu\sigma_j}{}^{\lambda} T^{\rho_1 \cdots \rho_m}_{\sigma_1 \cdots \lambda \cdots \sigma_n}$$

$$R_{\mu\nu} = R_{\mu\rho\nu}{}^{\rho} \qquad R = g^{\mu\nu} R_{\mu\nu}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$S = S_{EH} + S_m$$

$$S_{EH} = \frac{M_{pl}^2}{2} \int d^4x \sqrt{-g}\, R \qquad \left(M_{pl} = (8\pi G)^{-1/2} \right)$$

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S_m}{\delta g^{\mu\nu}}$$

$$G_{\mu\nu} = M_{pl}^{-2} T_{\mu\nu}$$

$$\nabla_\mu f \overset{?}{=} \partial_\mu f \quad \checkmark$$

$$\nabla_\mu \nabla_\nu f \doteq \partial_\mu \partial_\nu f - \Gamma^\rho_{\mu\nu} \partial_\rho f$$

$$\nabla_\mu \nabla^\nu f \doteq g^{\nu\rho} \nabla_\mu \nabla_\rho f \quad \checkmark \qquad \boxed{\nabla_\mu g_{\nu\rho} = 0}$$

$$\nabla_\mu \nabla^\nu f = \nabla_\mu \left(g^{\nu\rho} \nabla_\rho f \right) \checkmark$$

$$\nabla_\mu f \overset{?}{=} \partial_\mu f \quad \checkmark$$

$$\nabla_\mu \nabla_\nu f \overset{?}{=} \nabla$$

$$\nabla_\mu \nabla_\nu f \doteq \partial_\mu \partial_\nu f - \Gamma^\rho_{\mu\nu} \partial_\rho f$$

"METRIC COMPATIBILITY"

$$\nabla_\mu \nabla^\nu f \doteq g^{\nu\rho} \nabla_\mu \nabla_\rho f \quad \checkmark \qquad \nabla_\mu g_{\nu\rho} = 0$$

$$\nabla_\mu \nabla^\nu f = \nabla_\mu \left(g^{\nu\rho} \nabla_\rho f \right) \checkmark$$

$$\nabla_\mu \nabla_\nu f \overset{?}{=} \nabla_\nu \nabla_\mu f \quad \checkmark$$

"TORSION FREENESS"

$$\nabla_\mu \nabla_\nu X^\rho \overset{?}{=} \nabla_\nu \nabla_\mu X^\rho - R_{\mu\nu\sigma}{}^{\rho} X^\sigma$$

DIVERGENCE FORMULA:

$$\nabla_\mu X^\mu = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\, X^\mu \right)$$

$$\nabla_\mu \nabla_\nu f \overset{?}{=} \nabla_\nu \nabla_\mu f \quad \checkmark$$

"TORSION FREENESS"

$$\nabla_\mu \nabla_\nu X^\rho \overset{?}{=} \nabla_\nu \nabla_\mu X^\rho - R_{\mu\nu\sigma}{}^{\rho} X^\sigma$$

ATIBILITY"

DIVERGENCE FORMULA:

$$\nabla_\mu X^\mu = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\, X^\mu \right)$$

$$\nabla_\mu \sqrt{g}\, \nabla_\rho f) =$$

$$\int d^4x \, (\partial_\mu X^\mu) \stackrel{?}{=} 0 \quad \checkmark$$

$$\int d^4x \, \sqrt{-g}\, (\partial_\mu X^\mu) \stackrel{?}{=} 0 \quad \times$$

$$\int d^4x \, \nabla_\mu X^\mu \stackrel{?}{=} 0 \quad \times$$

$$\boxed{\int d^4x \, \sqrt{-g}\, \nabla_\mu X^\mu \stackrel{?}{=} 0} \quad \checkmark$$

$$\int d^4x \, \sqrt{-g}\, (\nabla_\mu X^\mu)\, f = \int d^4x$$

$$\int d^4x \sqrt{-g}\,(\nabla_\mu X^\mu)\, f = \int d^4x \sqrt{-g}\left[\cancel{\nabla_\mu (f X^\mu)} - X^\mu \nabla_\mu f\right]$$

$$= -\int d^4x \sqrt{-g}\, X^\mu \nabla_\mu f$$

$$\int d^4x \sqrt{-g}\,(\nabla_\mu \phi)(\nabla^\mu \psi) = -\int d^4x \sqrt{-g}\, \phi\,(\nabla^2 \psi) \qquad \nabla^2 \psi = \nabla^\mu$$

$$\int d^4x \sqrt{-g}\left[\nabla_\mu(fX^\mu) - X^\mu \nabla_\mu f\right]$$

$$= -\int d^4x \sqrt{-g}\, X^\mu \nabla_\mu f$$

$$= -\int d^4x \sqrt{-g}\, \phi\,(\nabla^2\psi) \qquad \nabla^2\psi = \nabla^\mu \nabla_\mu \psi$$

$$\int d^4x \sqrt{-g}\,(\nabla_\mu X_\nu)(\nabla^\nu Y^\mu) = -\int d^4x \sqrt{-g}\, X^\nu (\nabla_\mu \nabla_\nu Y^\mu)$$

$$= -\int d^4x \sqrt{-g}\, X^\nu \left(\nabla_\nu \nabla_\mu Y^\mu - R_{\mu\nu\rho}{}^{\mu} Y^\rho\right)$$

$$\int d^4x \sqrt{-g}\,(\nabla_\mu X_\nu)(\nabla^\nu Y^\mu) = -\int d^4x \sqrt{-g}\, X^\nu (\nabla_\mu \nabla_\nu Y^\mu)$$

$$= -\int d^4x \sqrt{-g}\, X^\nu \left(\nabla_\nu \nabla_\mu Y^\mu - R_{\mu\nu\rho}{}^{\mu} Y^\rho\right)$$

$$= -\int d^4x \sqrt{-g} \left[-(\nabla_\nu X^\nu)(\nabla_\mu Y^\mu)\right.$$

$$\int d^4x \sqrt{-g}\,(\nabla_\mu X_\nu)(\nabla^\nu Y^\mu) = -\int d^4x \sqrt{-g}\, X^\nu (\nabla_\mu \nabla_\nu Y^\mu)$$

$$= -\int d^4x \sqrt{-g}\, X^\nu \left(\nabla_\nu \nabla_\mu Y^\mu - R_{\mu\nu\rho}{}^{\mu} Y^\rho\right)$$

$$= -\int d^4x \sqrt{-g}\left[-(\nabla_\nu X^\nu)(\nabla_\mu Y^\mu) - (-R_{\nu\rho})\, Y^\rho\right]$$

$$= \int d^4x \sqrt{-g}\,(\nabla_\nu X^\nu)(\nabla_\mu Y^\mu) - \int d^4x \sqrt{-g}\, R_{\nu\rho} X^\nu Y^\rho$$

SCALAR FIELD ϕ WITH POTENTIAL $V(\phi)$

MINIMALLY COUPLED TO GRAVITY

$$S = \underbrace{\frac{M_{Pl}^2}{2}\int \sqrt{-g}\, R}_{S_{EH}} + \int d^4x \sqrt{-g}\left(-\frac{1}{2}(\nabla^\mu \phi)(\nabla_\mu \phi) - V(\phi)\right)$$

VARY WITH RESPECT TO ϕ $\Rightarrow$ EQUATION OF MOTION

$$\phi(x,t) \to \phi(x,t) + \delta\phi(x,t) \qquad g_{\mu\nu} \to g_{\mu\nu}$$

$$S \to S + \delta S$$

$$\delta S = \int d^4x \sqrt{-g}\left[-(\nabla^\mu \phi)(\nabla_\mu \delta\phi) - V'(\phi)\,\delta\phi\right]$$

$$= \int d^4x \sqrt{-g}\left[\nabla^2\phi - V'(\phi)\right]\delta\phi$$

$$\boxed{\nabla^2\phi = V'(\phi)}$$

EQ OF MOTION

$$\nabla_\mu \nabla^\nu f = \nabla_\mu \left(g^{\nu\rho} \nabla_\rho f \right)$$

$$\nabla_\mu X^\mu = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g}\, X^\mu \right)$$

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}$$

$$\phi \to \phi \qquad g^{\mu\nu} \to g^{\mu\nu} + \delta g^{\mu\nu}$$

$$\delta S_m = \int d^4x \sqrt{-g} \left(-\frac{1}{2} T_{\mu\nu} \right) \delta g^{\mu\nu}$$

$$\delta(\nabla_\mu \phi) = \delta(\partial_\mu \phi) = 0$$

$$\delta(\nabla^\mu \phi) = \delta(g^{\mu\nu} \nabla_\nu \phi) = (\delta g^{\mu\nu}) \nabla_\nu \phi$$

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}$$

$$\phi \to \phi \qquad g^{\mu\nu} \to g^{\mu\nu} + \delta g^{\mu\nu}$$

$$\delta S_m = \int d^4x \sqrt{-g}\left(-\frac{1}{2}T_{\mu\nu}\right)\delta g^{\mu\nu}$$

$$\delta(\nabla_\mu \phi) = \delta($$

$$\delta(\nabla^\mu \phi) = \delta(g$$

$$\delta(\sqrt{-g}) =$$

$$\delta(\nabla_\mu \phi) = \delta(\partial_\mu \phi) = 0$$

$$\delta(\nabla^\mu \phi) = \delta(g^{\mu\nu} \nabla_\nu \phi) = (\delta g^{\mu\nu}) \nabla_\nu \phi$$

$$\delta \mathrm{Det}(A) =$$

$$\delta(\sqrt{-g}) =$$

$$\delta(\nabla_\mu \phi) = \delta(\partial_\mu \phi) = 0$$

$$\delta(\nabla^\mu \phi) = \delta(g^{\mu\nu} \nabla_\nu \phi) = (\delta g^{\mu\nu}) \nabla_\nu \phi$$

$$\delta \mathrm{Det}(A) = \mathrm{De}$$

$$\delta(\sqrt{-g}) = \delta(\det(-g)^{1/2})$$

$$= \frac{1}{2} \det(-g)^{-1/2} \det(-g)\, \mathrm{Tr}\left[-g^{-1} \delta g\right] (-1) \quad \star$$

$$\delta(\nabla_\mu \phi) = \delta(\partial_\mu \phi) = 0$$

$$\delta(\nabla^\mu \phi) = \delta(g^{\mu\nu} \nabla_\nu \phi) = (\delta g^{\mu\nu}) \nabla_\nu \phi$$

$$\delta(\sqrt{-g}) = \delta(\det(-g)^{1/2})$$

$$= \frac{1}{2} \det(-g)^{-1/2} \det(-g) \, \mathrm{Tr}\left[-g^{-1} \delta g\right] (-1)$$

$$= \frac{1}{2} (-g)^{-1/2} (-g) \left[-g^{\mu\nu} \delta g_{\mu\nu}\right] (-1)$$

$$\delta \mathrm{Det}(A) = \mathrm{Det}(A)\, \mathrm{Tr}\left(A^{-1}\delta A\right)$$

"JACOBI'S IDENTITY"

$$\left(\delta g^{\mu\nu}\right) \nabla_\mu \phi$$

$$= \frac{1}{2}(-g)^{1/2}\left[g_{\mu\nu}\,\delta g^{\mu\nu}\right](-1)$$

$$= \sqrt{-g}\left(-\frac{1}{2} g_{\mu\nu}\,\delta g^{\mu\nu}\right)$$

$$\left.\right](-1)$$

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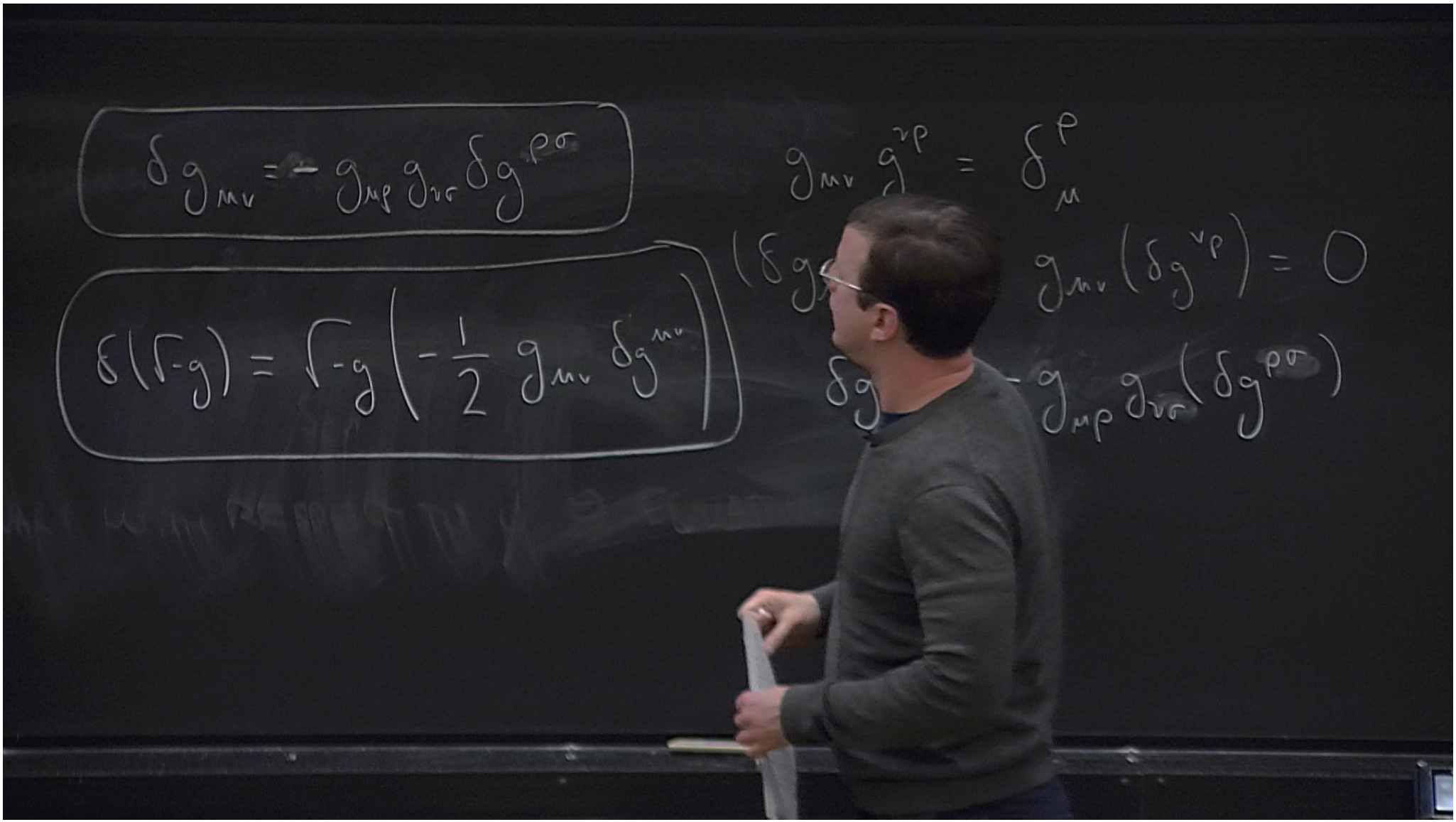
$\delta g_{\mu\nu} = - g_{\mu\rho} g_{\nu\sigma} \delta g^{\rho\sigma}$
$\delta(\sqrt{-g}) = \sqrt{-g}\left(-\frac{1}{2} g_{\mu\nu} \delta g^{\mu\nu}\right)$
$g_{\mu\nu} g^{\nu\rho} = \delta^{\rho}_{\mu}$
$g_{\mu\nu}(\delta g^{\nu\rho}) = 0$
δg $- g_{\mu\rho} g_{\nu\sigma}(\delta g^{\rho\sigma})$

$$= \frac{1}{2}\det(-g)^{-1/2}\det(-g)\,\mathrm{Tr}$$

$$= \frac{1}{2}(-g)^{-1/2}(-g)\int -g^{\mu\nu}$$

$$S_m = \int d^4x\sqrt{-g}\left(-\frac{1}{2}(\nabla^\mu\phi)(\nabla_\mu\phi) - V(\phi)\right) \qquad g^{\mu\nu} \to g^{\mu\nu} + \delta g^{\mu\nu}$$

$$\delta S_m = \int d^4x\,\sqrt{-g}\left(-\frac{1}{2}g_{\mu\nu}\,\delta g^{\mu\nu}\right)\left(-\frac{1}{2}(\nabla^\mu\phi)(\nabla_\mu\phi) - V(\phi)\right)$$

$$+ \int d^4x\,\sqrt{-g}\left(-\frac{1}{2}(\delta g^{\mu\nu})(\nabla_\nu\phi)(\nabla_\mu\phi)\right)$$

$$= \int d^4x\,\sqrt{-g}\left[\frac{1}{4}g_{\mu\nu}(\nabla^\rho\phi)(\nabla_\rho\phi) + \frac{1}{2}V(\phi)g_{\mu\nu} - \frac{1}{2}(\nabla_\mu\phi)(\nabla_\nu\phi)\right]\delta g^{\mu\nu}$$

$$= \frac{1}{2}\det(-g)^{-1/2}\det(-g)\,\mathrm{Tr}$$

$$= \frac{1}{2}(-g)^{-1/2}(-g)\left(-g^{\mu\nu}\right.$$

$$S_m = \int d^4x\sqrt{-g}\left(-\frac{1}{2}(\nabla^\mu\phi)(\nabla_\mu\phi) - V(\phi)\right) \qquad g^{\mu\nu} \to g^{\mu\nu} + \delta g^{\mu\nu}$$

$$\delta S_m = \int d^4x\,\sqrt{-g}\left(-\frac{1}{2}g_{\mu\nu}\,\delta g^{\mu\nu}\right)\left(-\frac{1}{2}(\nabla^\mu\phi)(\nabla_\mu\phi) - V(\phi)\right)$$

$$+ \int d^4x\,\sqrt{-g}\left(-\frac{1}{2}(\delta g^{\mu\nu})(\nabla_\nu\phi)(\nabla_\mu\phi)\right)$$

$$= \int d^4x\,\sqrt{-g}\,\underbrace{\left[\frac{1}{4}g_{\mu\nu}(\nabla^\rho\phi)(\nabla_\rho\phi) + \frac{1}{2}V(\phi)g_{\mu\nu} - \frac{1}{2}(\nabla_\mu\phi)(\nabla_\nu\phi)\right]}_{-\frac{1}{2}T_{\mu\nu}}\,\delta g^{\mu\nu}$$

$$\sqrt{-g}\,\mathrm{Tr}\left[-g^{-1}\delta g\right](-1) = \sqrt{-g}\left(-\frac{1}{2}g_{\mu\nu}\delta g^{\mu\nu}\right)$$

$$\left[-g^{\mu\nu}\delta g\right](-1)$$

$$\boxed{T_{\mu\nu} = (\nabla_\mu\phi)(\nabla_\nu\phi) - \frac{1}{2}(\nabla^\rho\phi)(\nabla_\rho\phi)\,g_{\mu\nu} - V(\phi)\,g_{\mu\nu}}$$

$$= \frac{1}{2}\det(-g)^{-1/2}\det(-g)\,\mathrm{Tr}\left(-g^{-1}\right.$$

$$= \frac{1}{2}(-g)^{-1/2}(-g)\int -g^{\mu\nu}\,\delta g$$

$$S_m = \int d^4x\sqrt{-g}\,\overbrace{\left(-\frac{1}{2}(\nabla^\mu\phi)(\nabla_\mu\phi) - V(\phi)\right)}^{\mathcal{L}_m} \qquad g^{\mu\nu} \to g^{\mu\nu} + \delta g^{\mu\nu}$$

$$\delta S_m = \int d^4x\sqrt{-g}\left(-\frac{1}{2}g_{\mu\nu}\,\delta g^{\mu\nu}\right)\left(-\frac{1}{2}(\nabla^\mu\phi)(\nabla_\mu\phi) - V(\phi)\right)$$

$$+ \int d^4x\sqrt{-g}\left(-\frac{1}{2}(\delta g^{\mu\nu})(\nabla_\nu\phi)(\nabla_\mu\phi)\right)$$

$$= \int d^4x\sqrt{-g}\,\underbrace{\left[\frac{1}{4}g_{\mu\nu}(\nabla^\rho\phi)(\nabla_\rho\phi) + \frac{1}{2}V(\phi)g_{\mu\nu} - \frac{1}{2}(\nabla_\mu\phi)(\nabla_\nu\phi)\right]}_{-\frac{1}{2}T_{\mu\nu}}\,\delta g^{\mu\nu}$$

T_μ

$$\sqrt{-g}\left(-\frac{1}{2} g_{\mu\nu}\,\delta g^{\mu\nu}\right)$$

$$\boxed{T_{\mu\nu} = (\nabla_\mu \phi)(\nabla_\nu \phi) \underbrace{- \frac{1}{2}(\nabla^\rho \phi)(\nabla_\rho \phi)\, g_{\mu\nu}}_{\mathcal{L}_m\, g_{\mu\nu}} - V(\phi)\, g_{\mu\nu}}$$

$$G_{\mu\nu} = \frac{1}{M_{pl}^2} T_{\mu\nu}$$

EQUATION OF MOTION

$$\nabla_\mu G^{\mu\nu} = 0$$

$$\boxed{T_{\mu\nu} = (\nabla_\mu \phi)(\nabla_\nu \phi) \underbrace{- \frac{1}{2}(\nabla^\rho \phi)(\nabla_\rho \phi)\, g_{\mu\nu} - V(\phi)\, g_{\mu\nu}}_{\mathcal{L}_m\, g_{\mu\nu}}}$$

$$G_{\mu\nu} = \frac{1}{M_{pl}^2} T_{\mu\nu}$$

EQUATION OF MOTION

$$\nabla_\mu G^{\mu\nu} = 0$$

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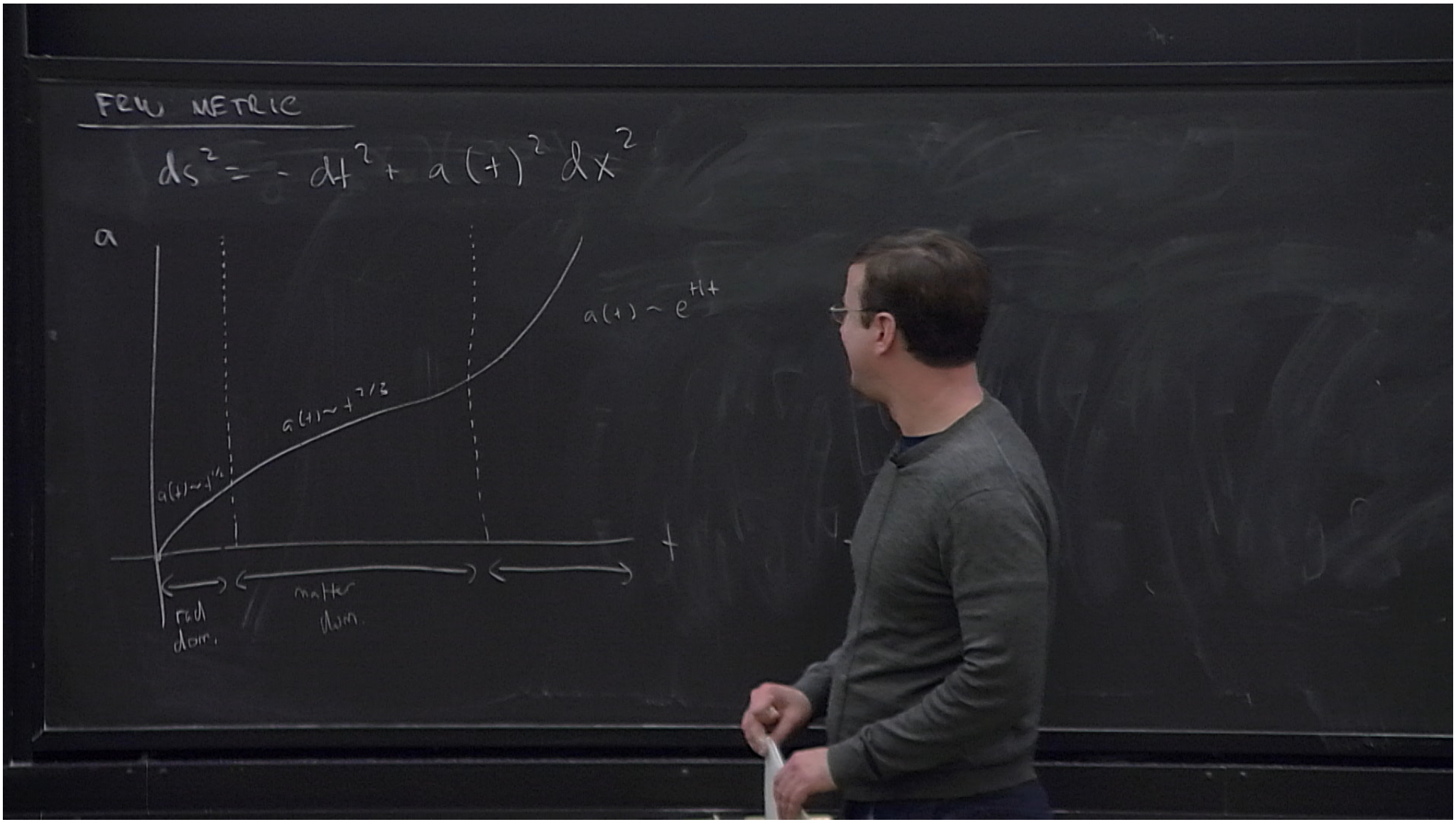
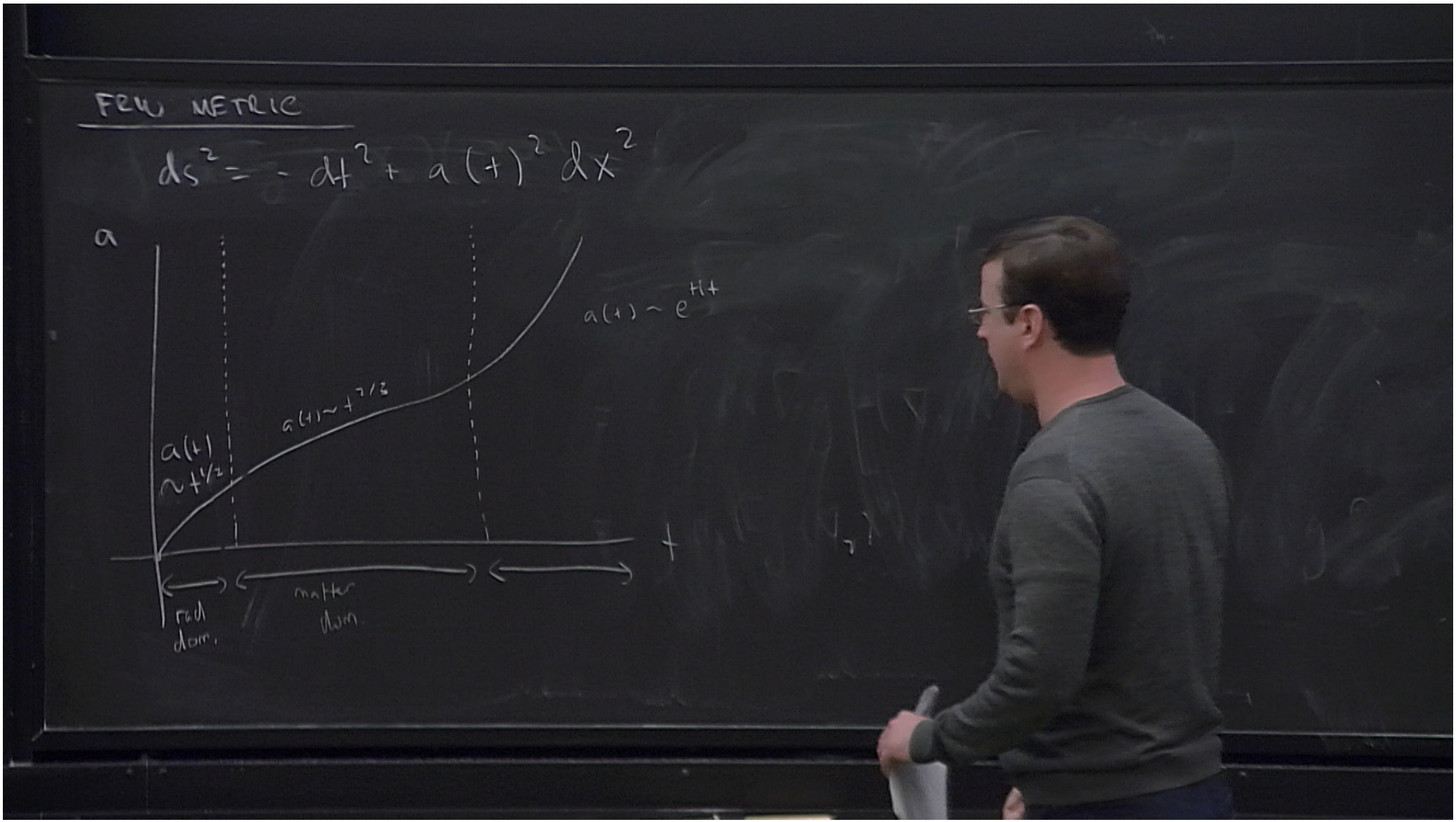

FRW METRIC
$ds^2 = -dt^2 + a(t)^2 dx^2$
a
$a(t) \sim e^{Ht}$
$a(t) \sim t^{2/3}$
$a(t) \sim t^{1/2}$
t
rad dom.
matter dom.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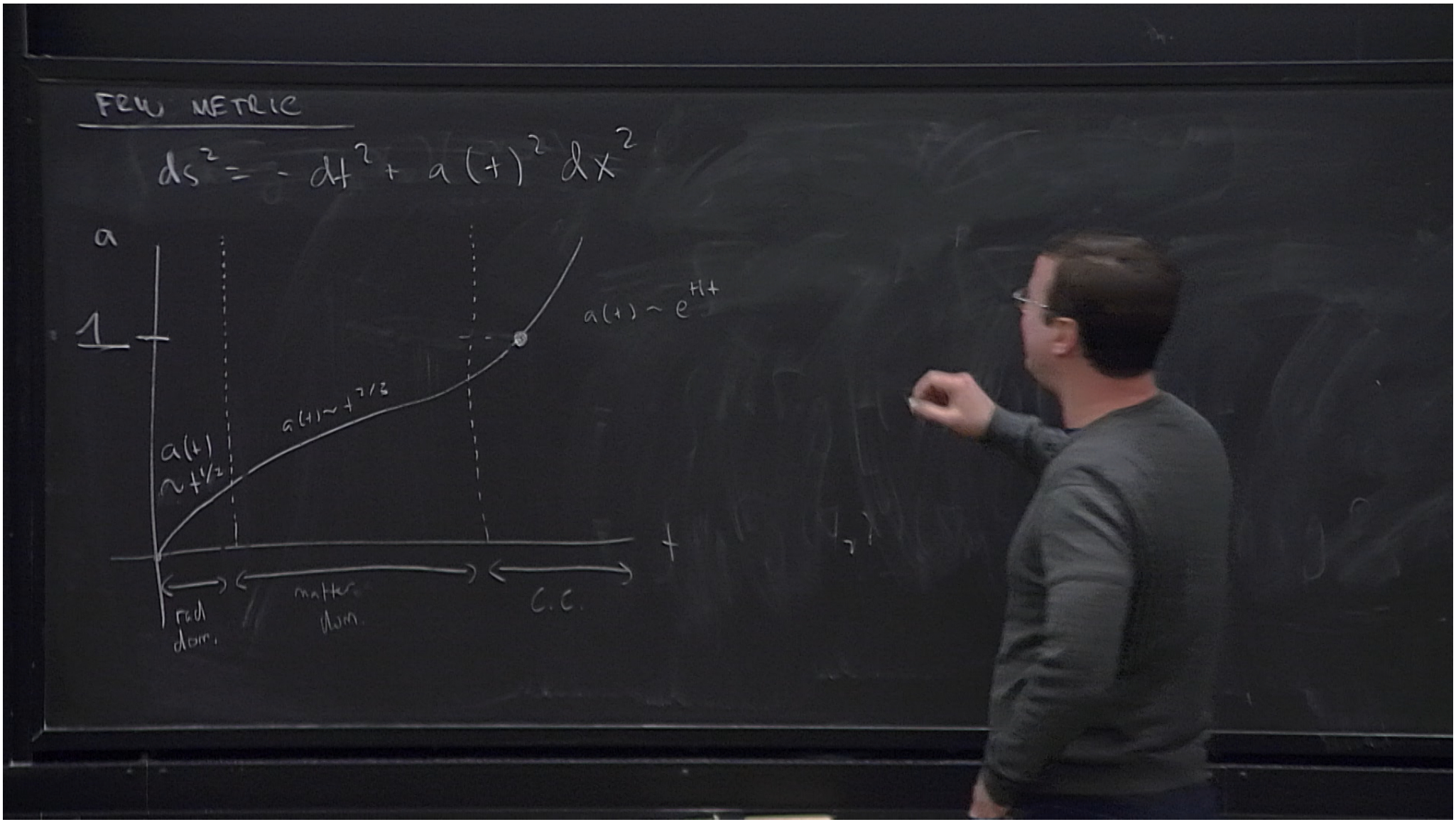
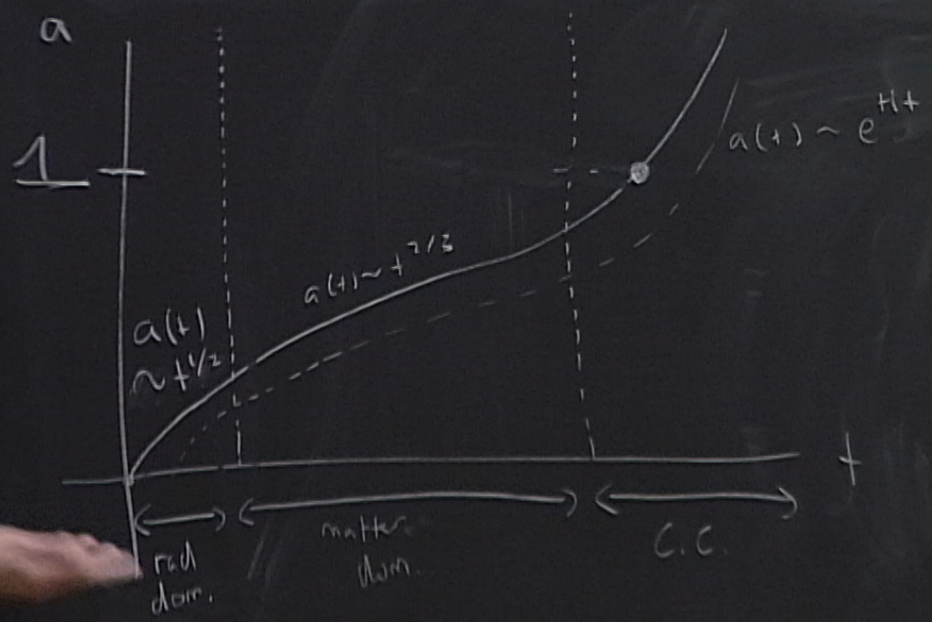

FRW METRIC
$ds^2 = -dt^2 + a(t)^2 dx^2$
a
1
t
$a(t) \sim e^{Ht}$
$a(t) \sim t^{2/3}$
$a(t) \sim t^{1/2}$
rad dom.
matter dom.
C.C.

FRW METRIC

$$ds^2 = -dt^2 + a(t)^2 dx^2 \quad [K=0]$$



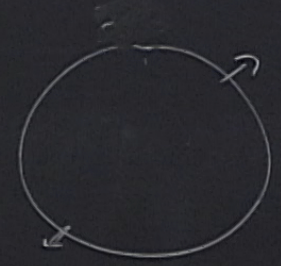
SYMMETRIES?

- SPATIAL ROTATIONS
- SPATIAL TRANSLATIONS
- ✗ TIME TRANSLATIONS

SYMMETRIES?

- SPATIAL ROTATIONS
- SPATIAL TRANSLATIONS
- ✗ TIME TRANSLATIONS
- ✗ BOOSTS $\Leftrightarrow$ UNIVERSE HAS A PREFERRED REST FRAME

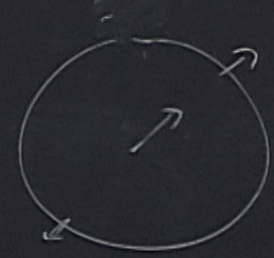
T_0 2.726 K



SYMMETRIES?

- SPATIAL ROTATIONS
- SPATIAL TRANSLATIONS

T_0 2.726 K



$T_0 + 3\,mK$

$T_0 - 3\,mK$

$10^{-3}\,c$

X TIME TRANSLATIONS

X BOOSTS $\Longleftrightarrow$ UNIVERSE HAS A PREFERRED REST FRAME