Title: PSI 2017/2018 - Quantum Gravity - Lecture 13

Date: Apr 04, 2018 10:15 AM

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Abstract:

Gravity in 4D

- ADM formalism:
  - Canonical analysis $(\Sigma \otimes \mathbb{R})$ spatial metric (q_{ab}, π_{ab})
  - Constraints: $H^{\mu} = \begin{pmatrix} S \\ V^a \end{pmatrix}$; non-polynomial functions (q, π)

No Dirac Program

b)

(q, π)

- LQG formalism.

$SU(2)$-g.s.

- Change of variables $\quad q_{ab} = e^i_a e^j_b \delta_{ij}$
- AB variables $\begin{cases} E \sim \varepsilon \varepsilon e e \\ A \sim \Gamma + \gamma K \end{cases}$

$\gamma \to$ Immirzi p.

- Algebra of constraints

$\boxed{G, H^\mu}$ $\quad$ $\boxed{\text{poly}(E, A)}$ $\quad$ $\boxed{1^{st} \text{ class}}$

Issues w/ $\{S[\alpha], S[\beta]\}$

• Canonical analysis $(\Sigma \otimes \mathbb{R})$ metric

• Constraints: $H^{\mu} = \begin{pmatrix} S \\ V^a \end{pmatrix}$, non-polynomial functions (q, π)

No Dirac Program

• AB variables $\{ A \sim \Gamma + \gamma K$

• Algebra of constraints

$\boxed{G, H^{\mu}}$ $\boxed{\text{poly}(E, A)}$ $\boxed{1^{st}\ \text{class}}$

Issues

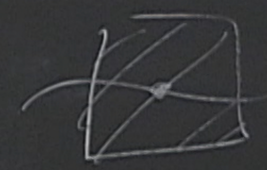
Dirac P. LQG

- $A \to h_{\gamma}$, $E \to E$
- $d\mu_{AL}$ $\langle \psi_{\Gamma_1} | \psi_{\Gamma_2} \rangle$

G – SN basis ON

V^a – s-Knots

Dirac P. LQG

- $A \longrightarrow h_\gamma$, $E \longrightarrow E$ 

- $d\mu_{AL}$ $\langle \Psi_{\Gamma_1} | \Psi_{\Gamma_2} \rangle$

G – SN basis ON

V^a – s-Knots

S – ? $\longleftarrow$ Spinfoams

SF

on-polynomial functions (q, π)

No Dirac Program

- Algebra of constraints

$\boxed{G, H^{\mu}}$ $\boxed{\text{poly}(E, A)}$ $\boxed{1^{st}\text{ class}}$

$\hookrightarrow$ Immirzi p.

Issues w/ $\{S[\alpha], S[\beta]\}$

QG

$\rightarrow E$

ams

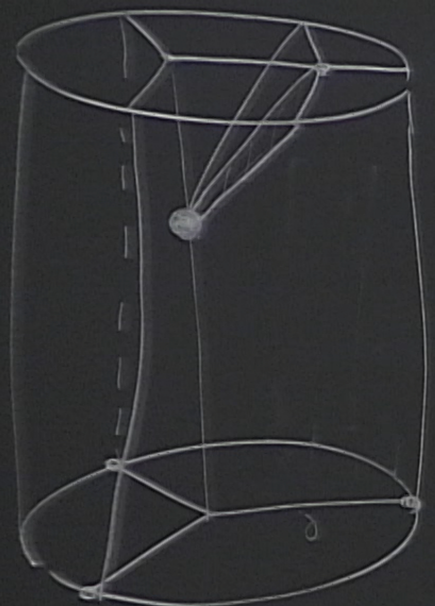
SF: the path integral representation of the dynamics in LQG.

$$P(\Sigma_{final}, \Sigma_{initial}) = \int D[e] D[A] \mu[A,e] \, e^{iS_{GR}[A,e]}$$

how to give sense to this?

SF approach: Reisenberger & Rovelli 1997.

In the spinnetwork basis $\langle s | s' \rangle_{phy}$ can be expressed as a sum over spinnetwork histories.

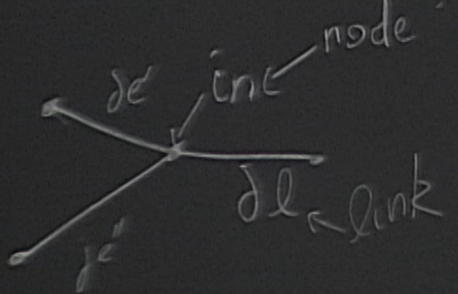


SF: colored 2-complex.

↳ 2 complex: vertices, edges, faces.

coloring c: j_f to each face c.

i_e to each edge.



Standard ansatz for local SF amplitudes

$$K[s] = \sum_{c/\partial c=\Gamma_s} \omega(c) \sum_{j_{f_1}, i_e} \prod_f \underbrace{A_f(j_f)} \prod_e \underbrace{A_e(j_f, i_e)} \prod_v \underbrace{A_v(j_f, i_e)}$$

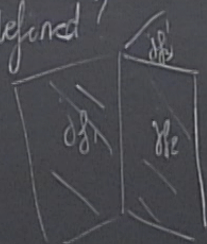
Γ_s : graph

spinnetwork

well defined ?

(Group field theory)

GFT



all the dynamical information is encoded in the vertex amplitudes

$\partial l \sim$ link

$$Z = \int D(e)\, D[A]\, \mu(A)\, e^{i S_{GR}(A,e)} \xrightarrow[\text{can be derived}]{} Z = \sum_{c} \omega(c) \sum_{j_f, i_e} \prod A_f(j_f) \prod A_e(j_f, i_e) \prod_{v} A_v(j_f, i_e)$$

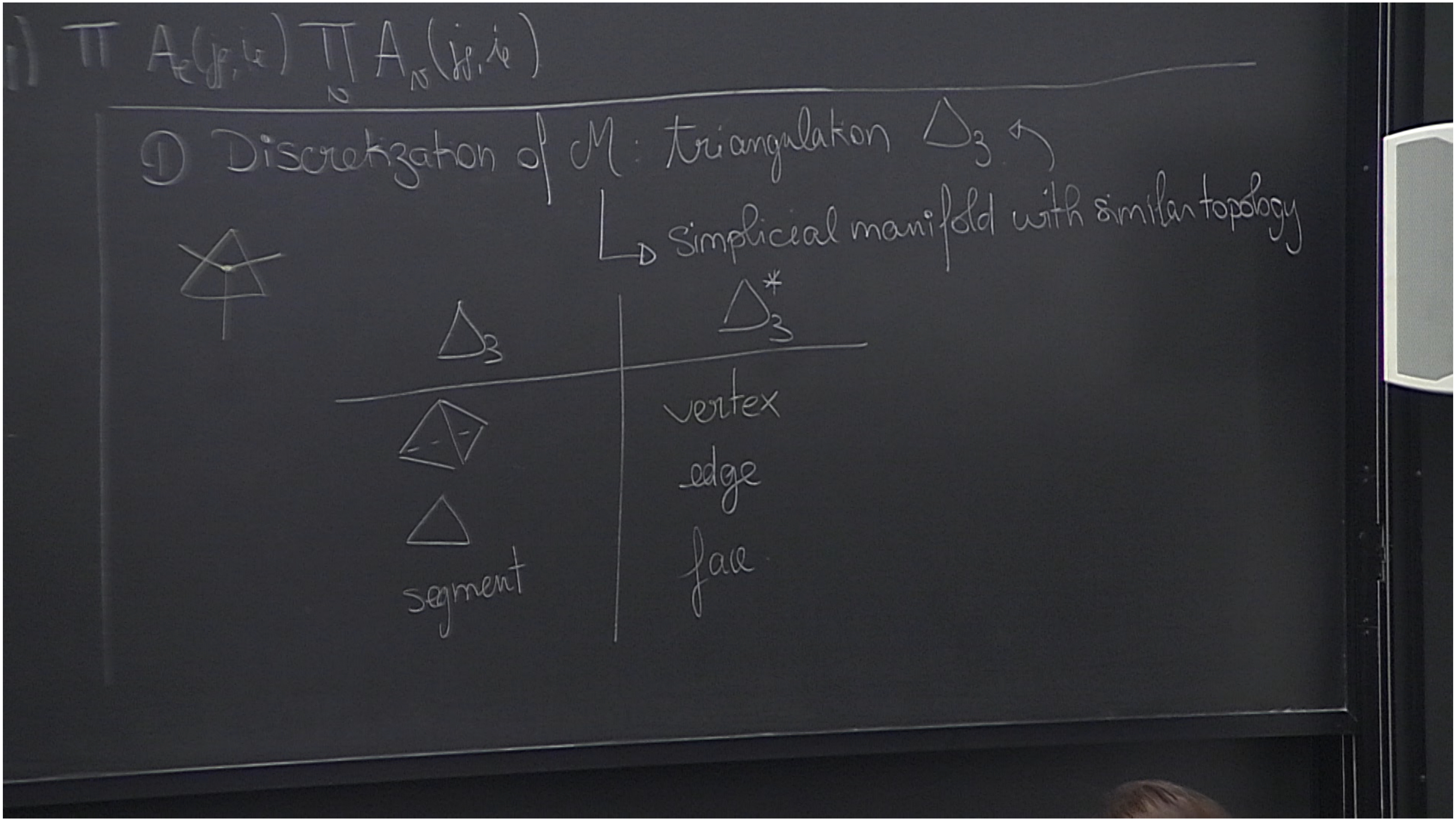
can be derived directly as a regularizat° of path integral?

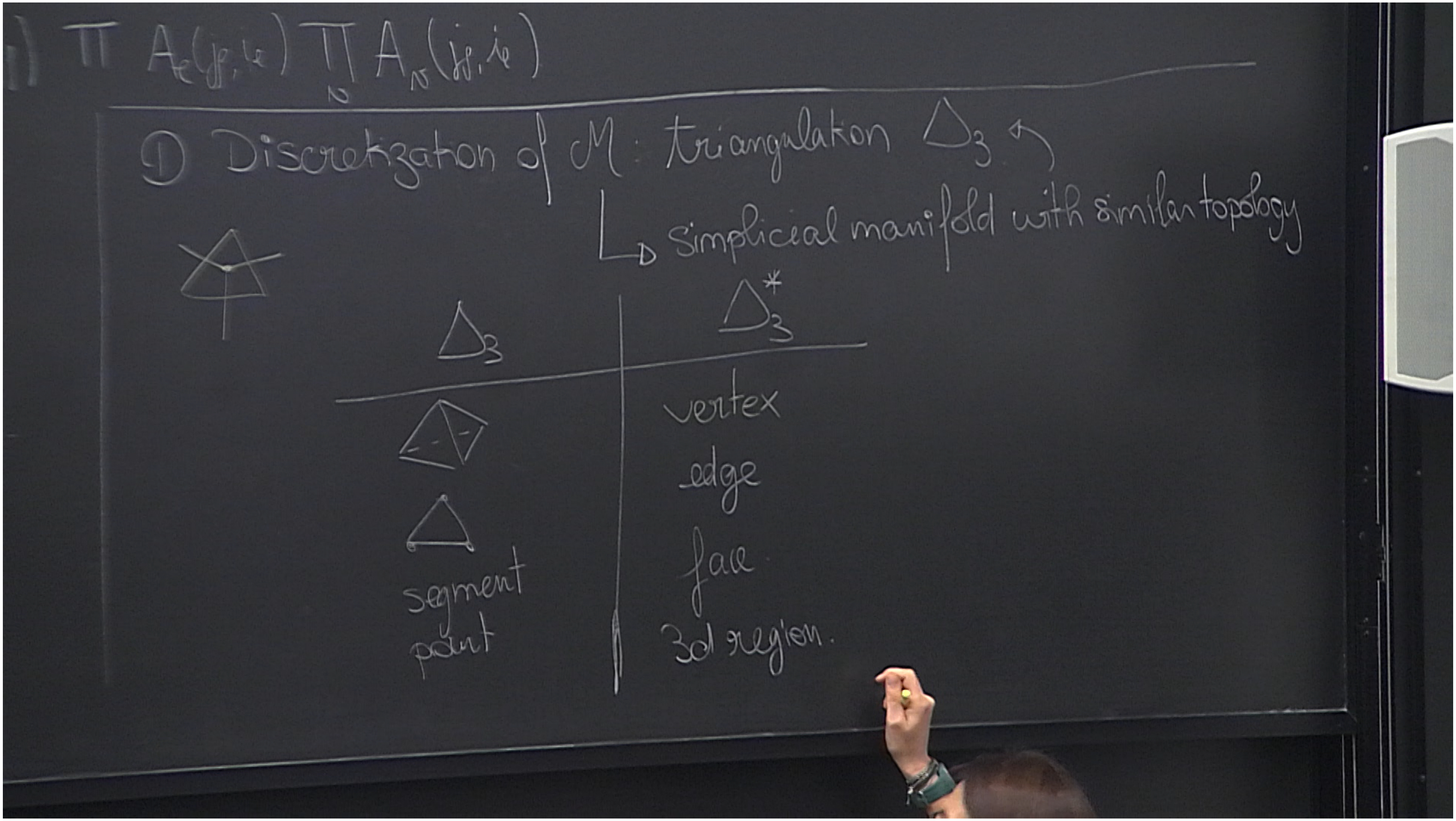
① Discretization of M

3D case: Ponzano-Regge model

$$S[e,\omega] = \int tr(e \wedge F)\, d^3x.$$

Formal path integral for 3d gravity $Z_M = \int de\, d\omega\, e^{iS[e,\omega]}$.


$\prod A_e(j_f, i_e) \prod_v A_v(j_f, i_e)$
① Discretization of M : triangulation Δ_3
↳ simplicial manifold with similar topology
Δ_3
Δ_3^*
vertex
edge
face
segment



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Δ_3
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vertex
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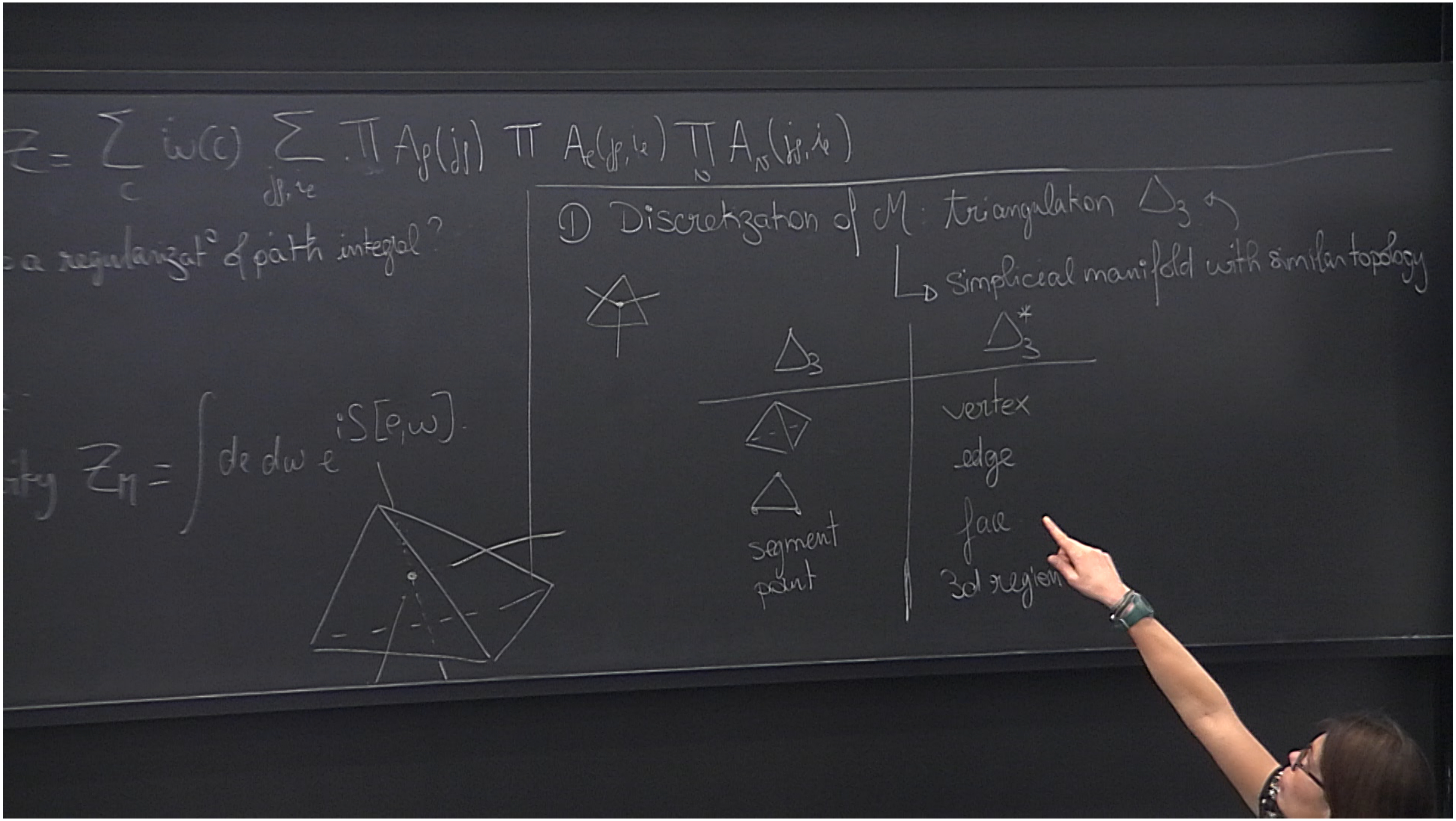
point
3d region.


Z = ∑ ω(c) ∑ Π A_f(j_f) Π A_e(j_f, i_e) Π A_v(j_f, i_e)
a regularizat° of path integral?
Z_M = ∫ de dw e^{iS[e,ω]}
① Discretization of M : triangulation Δ_3
↳ simplicial manifold with similar topology
Δ_3
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vertex
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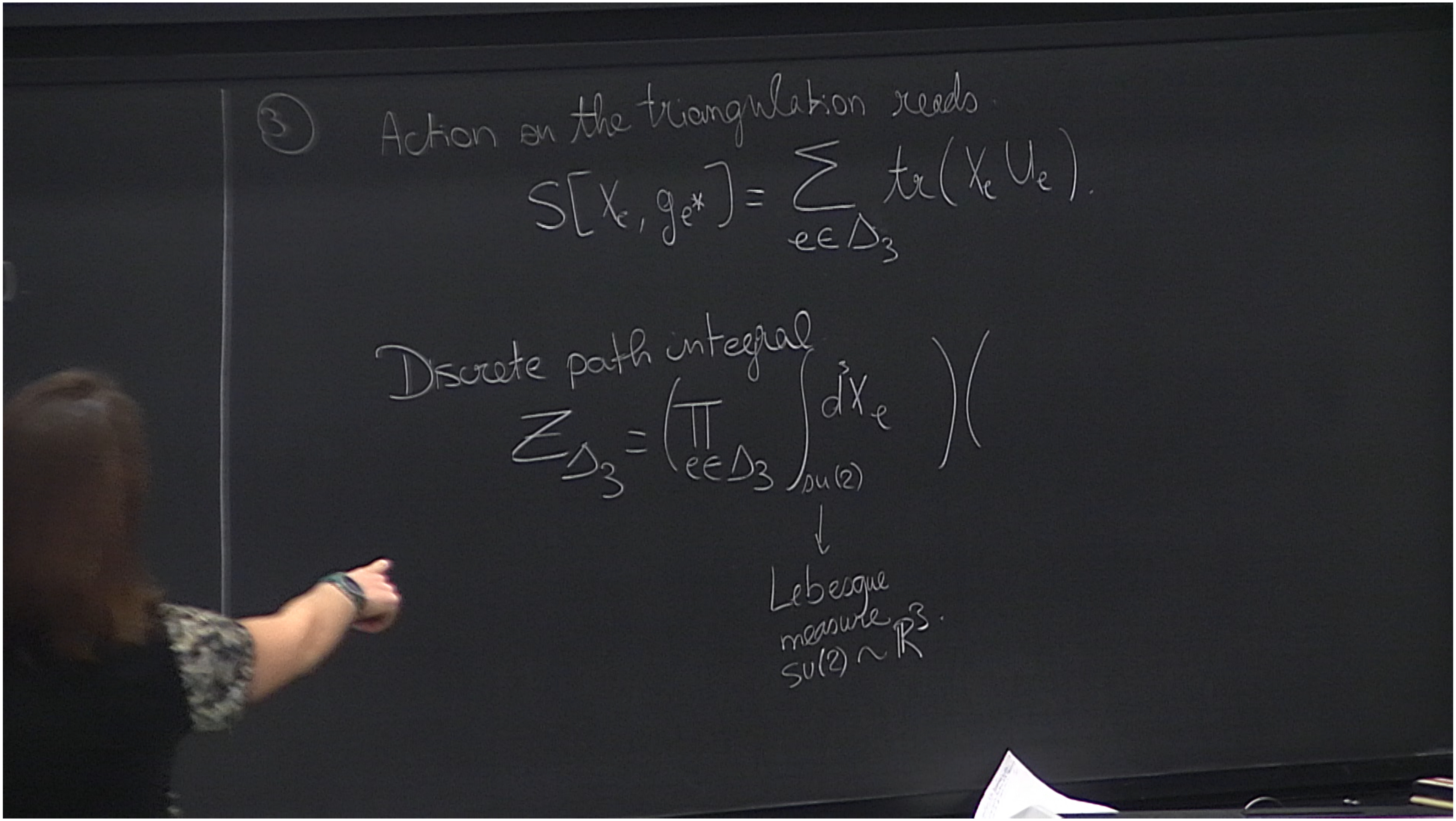
point

② Discretization of the variables

$$e \longrightarrow X_e = \int_e e \in su(2).$$

$$\omega \longrightarrow g_{e^*} = P\, e^{\int_{e^*} \omega} \in SU(2).$$

$$F(\omega) \longrightarrow U_{e\, f^*} = \overrightarrow{\prod_{\substack{e^* \subset e \\ e^* \in f^*}}} g_{e^*}^{\varepsilon(e, e^*)}$$


3
Action on the triangulation reads.
$S[X_e, g_{e^*}] = \sum_{e \in \Delta_3} tr(X_e U_e)$.
Discrete path integral
$Z_{\Delta_3} = \left(\prod_{e \in \Delta_3} \int_{su(2)} d^3X_e \right)\Big($
Lebesgue measure
$su(2) \sim \mathbb{R}^3$.

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$$S[X_e, g_{e^*}] = \sum_{e\in\Delta_3} tr(X_e U_e).$$

Discrete path integral

$$Z_{\Delta_3} = \left(\prod_{e\in\Delta_3}\int_{su(2)} d^3X_e\right)\left(\prod_{e^*\in\Delta_3^*}\int_{SU(2)} dg_{e^*}\right) e^{iS[X_e,g_{e^*}]}.$$

$\downarrow$ Lebesgue measure $su(2) \sim \mathbb{R}^3$.

$\downarrow$ Haar measure

S — ? ⟵ Spinfoams

as a sum over spinnetwork histories.

- integration over the X_e variables
  and using $\int dX_e \, e^{i\,tr(X_e U_e)} = \delta(U_e) = \sum_j d_j \, tr(D^j(U_e))$

$\delta(U_e)$: over SO(3)

$=$: Peter-Weyl th.

D^j: Wigner matrices $\langle g | j m n \rangle = D^j_{mn}(g)$

$$\left(\delta_{SO(3)}(U) = \delta_{SU(2)}(g) + \delta_{SU(2)}(-g) \right)$$

$$Z_{\Delta_3} = \sum_{\{j\}} \left(\prod_{e^* \in \Delta_3^*} \int dg_{e^*} \right) \prod_{e \in \Delta_3} tr\left(D^{j_e}(U_e) \right)$$

$\int D(e)\, D[A]\, \mu(A)\, e^{iS_{GR}(A,e)} \longrightarrow Z = \sum_{c} \omega(c) \sum_{j_l, i_e} \prod A_f(j_l) \prod A_e(j_l, i_e) \prod_{\sigma} A_\sigma(j_l, i_e)$

can be derived directly as a regularizat° of path integral?

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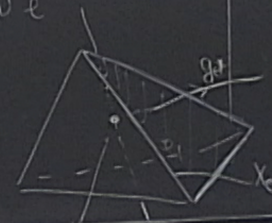

① Discretization of M: triangulation Δ_3

↳ simplicial manifold with similar topology

Δ_3	Δ_3^*
	vertex
	edge e^*
segment e	face f^*
point	3d region

$F(\omega) \longrightarrow U_{f^*} = \prod_{e^* \subset e,\ e \subset f^*} g_{e^*}^{\epsilon(e,e^*)}$

Discrete path integral

$$Z_{\Delta_3} = \Big(\prod_{e \in \Delta_3} \int_{su(2)} dX_e \Big)\Big(\prod_{e^* \in \Delta_3^*} \int_{SU(2)} dg_{e^*}\Big)\, e^{S(X_e, g_{e^*})}$$

$\downarrow$ Lebesgue measure $su(2) \sim \mathbb{R}^3$

$\downarrow$ Haar measure

$\delta_{SO(3)}(U) = \delta_{SU(2)}(g) + \delta_{SU(2)}(-g)$ (on $SO(3)$; Peter-Weyl th.)

$\hookrightarrow$ Wigner matrices

$$\langle g | j m n \rangle = D^{j}_{mn}(g)$$

$$Z_{\Delta_3} = \sum_{\{j\}} \left(\prod_{e^* \in \Delta_3^*} \int dg_{e^*} \right) \prod_{e \in \Delta_3} \mathrm{tr}\left(D^{j_e}(U_e) \right)$$

each g_{e^*} is in 3 U_e : $\mathrm{tr}\left(D^{j}(\ldots g_{e^*} \ldots) \right) \to \int dg_{e^*} D^{j_1}_{m_1 n_1}(g) D^{j_2}_{m_2 n_2}(g) D^{j_3}_{m_3 n_3}(g) = (-1)^{j_1+j_2+j_3} \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \begin{pmatrix} j_1 & j_2 & j_3 \\ n_1 & n_2 & n_3 \end{pmatrix}$

$\int_{SU(2)} dg$  $=$ 

$\hookrightarrow$ for a given tetrahedron $\xrightarrow{SU(2)\text{ integration}}$ $\underbrace{\text{1 intertwiner / face}}$

by combining the 4 intertwiners

$\downarrow$

6j symbol

$$Z_{\Delta_3} = \sum_{\{j_e\}} \prod_{e} (-1)^{2j_e} d_{j_e} \prod_{t} \begin{Bmatrix} j_1^{(t)} & j_2^{(t)} & j_3^{(t)} \\ j_4^{(t)} & j_5^{(t)} & j_6^{(t)} \end{Bmatrix}_N$$

each g_{e^*} is in 3 . Ue. : $tr\left(D(\ldots g_{e^*} \ldots)\right) = (-1) \begin{pmatrix} \\ m_1 m_2 m_3 \end{pmatrix}\begin{pmatrix} \\ n_1 n_2 n_3 \end{pmatrix}$

↳ for a given tetrahedron $\xrightarrow{SU(2)\text{ integration}}$ 1 intertwiner / face

$$\begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix} \underset{j \to \infty}{\sim} \frac{1}{\sqrt{12\pi V}} \cos\left(S_R + \frac{\pi}{4}\right)$$

↓

Regge action

discrete gravity

by combining the 4 intertwiners

↓

6j symbol

$\Delta_3 \quad \{j\} \quad e^* \in \Delta_3^* \quad g \quad e \in \Delta_3$

each g_{e^*} is in

4D gravity

$$S_{\text{Plebanski}}[B,\omega,\lambda] = \int B^{IJ} \wedge F_{IJ} - \frac{1}{2}\lambda_{IJKL} B^{KL} \wedge B^{IJ}$$

$$\left(B = \pm e^I \wedge e^J\right)$$

issue
simplicity
constraints
2^{nd} class
↳ implement
at the quantum level

→ Barett-Crane

→ EPRL - FK model.

Marseille

$P(\Sigma_{fin}$

how to
SF a

In th