Title: PSI 2017/2018 - Relativistic Quantum Information - Lecture 13

Date: Apr 05, 2018 09:00 AM

URL: http://pirsa.org/18040006

Abstract:

Let's talk thermodynamics for a minute

Passivity of thermal states

Thermal states are passive: No unitary operations can have a negative energy cost

For arbitrary unitary operations,

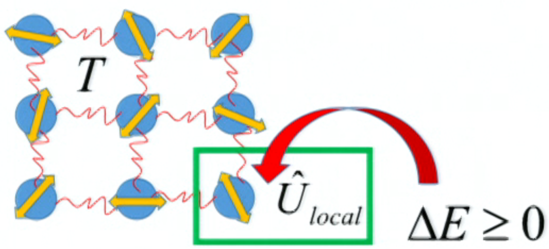
$$\hat{\rho}' = \hat{U}\hat{\rho}(\beta)\hat{U}^\dagger$$

$$\Delta E = Tr\left[\hat{H}\hat{\rho}'\right] - Tr\left[\hat{H}\hat{\rho}(\beta)\right] \geq 0$$

Pusz and Woronowicz (1978)

This implies stability of thermal equilibrium state under unitary disturbance.

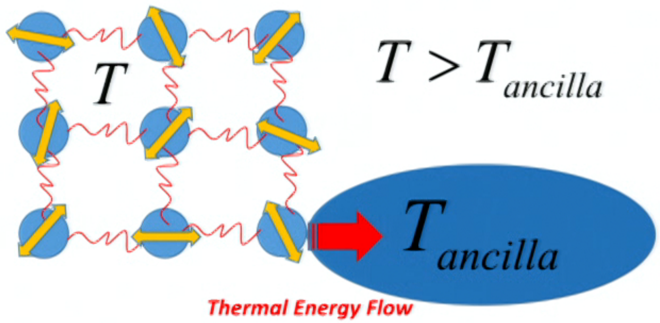
In particular local operations



Passivity of thermal states

How about generic operations? CPTP maps? We know how!

The map on the system resulting from coupling a colder ancilla $\rho_T \to \Gamma[\rho_T]$



$$\Delta E = \mathrm{Tr}[H\rho_T] - \mathrm{Tr}\Big(H\Gamma[\rho_T]\Big) < 0$$

Is this local?

Passivity of thermal states

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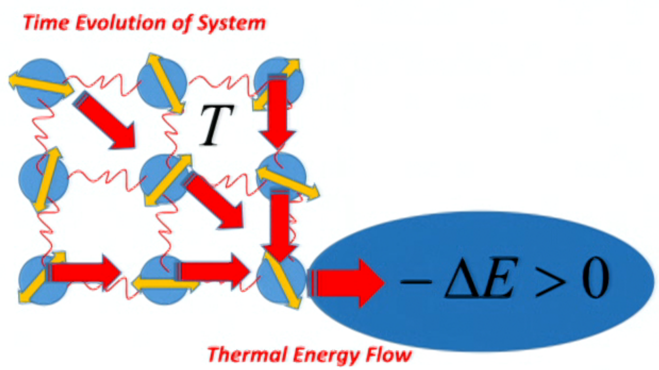
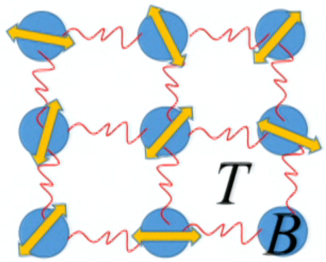


$$\Delta E = \mathrm{Tr}[H\rho_T] - \mathrm{Tr}\Big(H\Gamma[\rho_T]\Big) < 0$$

Usually non-local!

Passivity of thermal states

Can we extract energy purely locally by using a colder ancilla?



$$T_{hand} < T$$

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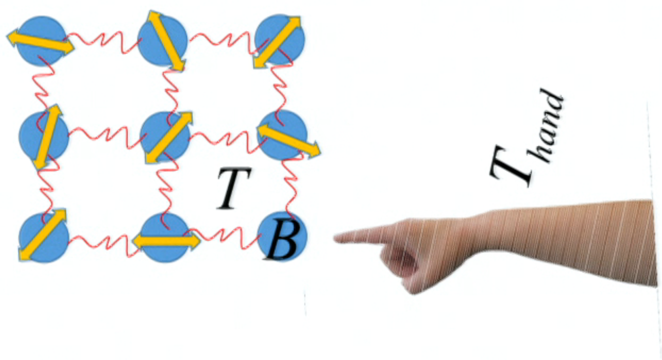


$$T_{hand} < T$$

Will the hand get burnt?

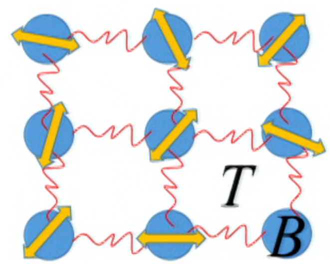
Strong Local Passivity of thermal states

Can we extract energy purely locally by using a colder ancilla?

Will the hand get burnt?

Answer: Not in general!

In fact the hand can get cooler!



Strong Local Passivity of thermal states

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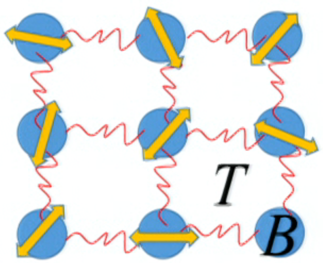
If the ground state of the system **contains** max-rank **entanglement** (not necessarily maximal entanglement):

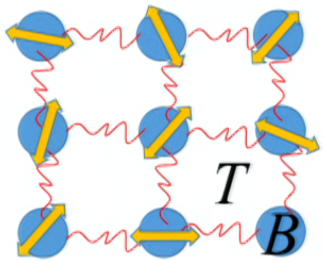


There exists a temperature $T^* > 0$

For $T < T^*$ It is **not possible** to extract work from the system through **any** local CPTP map.

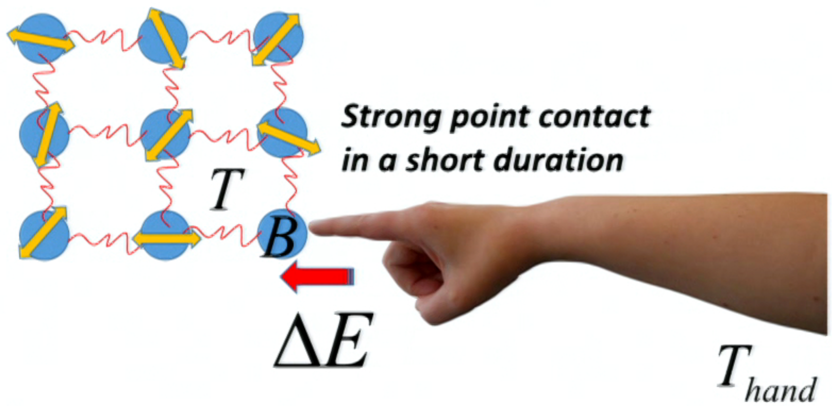
Two Qubits : M. Frey, K. Gerlach, M. Hotta, J. Phys. A: Math. Theor. 46, 455304 (2013).
General Theorem: M. Frey, K. Funo, M. Hotta, Phys. Rev. E90, 012127 (2014).

Strong Local Passivity of thermal states



Hot system at temperature lower than T_*

Strong Local Passivity of thermal states



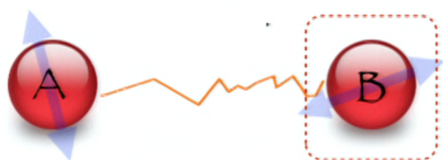
Strong Local Passivity!

Breaking Strong Local Passivity
Quantum Energy Teleportation

We can use ground state entanglement as a resource for local energy extraction!

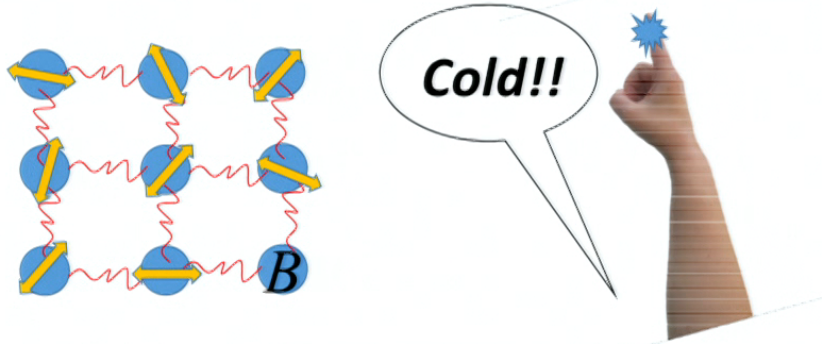
If we assist local operations with classical communication (LOCC) it is possible to extract energy with local operations.

Even from the ground state



Strong Local Passivity of thermal states

Even if $T_{hand} < T$***, energy is transferred from hand to B!***



$$+\Delta E \geq 0 \qquad -\Delta E \leq 0$$

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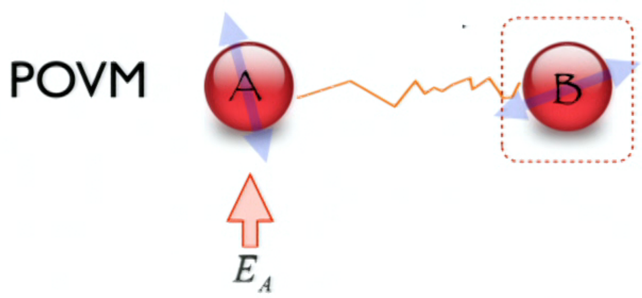

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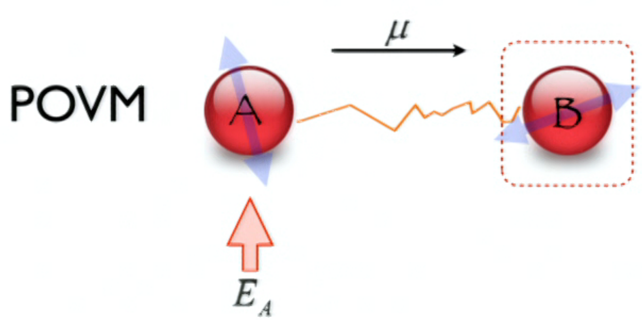

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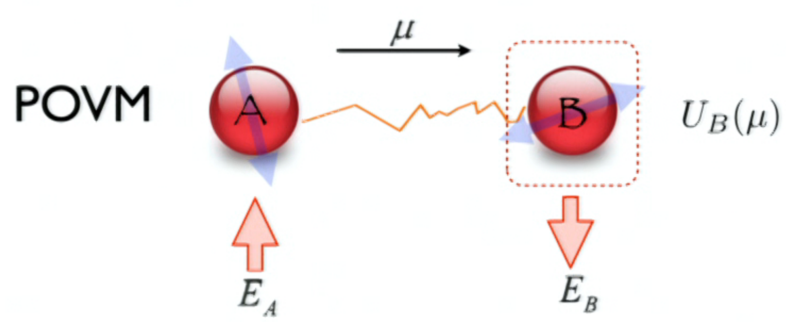


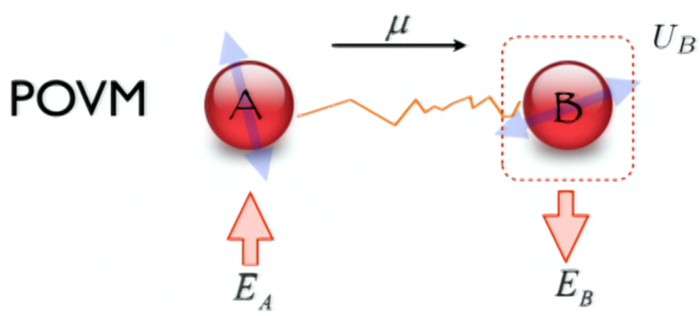
Breaking Strong Local Passivity
Quantum Energy Teleportation

Intuition:

Because of ground state entanglement, the measurement in A provides information about fluctuations in B. "Unlocking zero-point fluctuations at a cost"

extract energy with local operations assisted by the information in A.

Energy does not travel from A to B. Information does.



MINIMAL QET MODEL

Consider two qubits A and B and the following Hamiltonian: $\hat{H} = \hat{H}_A + \hat{H}_B + \hat{V}$

where: $\hat{H}_A = h\,\hat{\sigma}_z^A + f(h,k)\mathbb{1}$; $\hat{H}_B = h\,\hat{\sigma}_z^B + f(h,k)\mathbb{1}$; $\hat{V} = 2\left[k\,\sigma_x^A\sigma_x^B + \frac{k^2}{h^2} f(h,k)\mathbb{1}\right]$

if we pick $f(h,k) = \frac{h^2}{\sqrt{h^2+k^2}} \Rightarrow \langle g|\hat{H}_A|g\rangle = \langle g|\hat{H}_B|g\rangle = \langle g|\hat{V}|g\rangle = 0$ & $\hat{H}$ is non-negative

$$|g\rangle = \frac{1}{\sqrt{2}}\left(\sqrt{1-\frac{f(h,k)}{h}}\,|1\rangle_A|1\rangle_B - \sqrt{1+\frac{f(h,k)}{h}}\,|0\rangle_A|0\rangle_B\right)$$

$\sigma_z^\nu|0\rangle_\nu = -|0\rangle_\nu$

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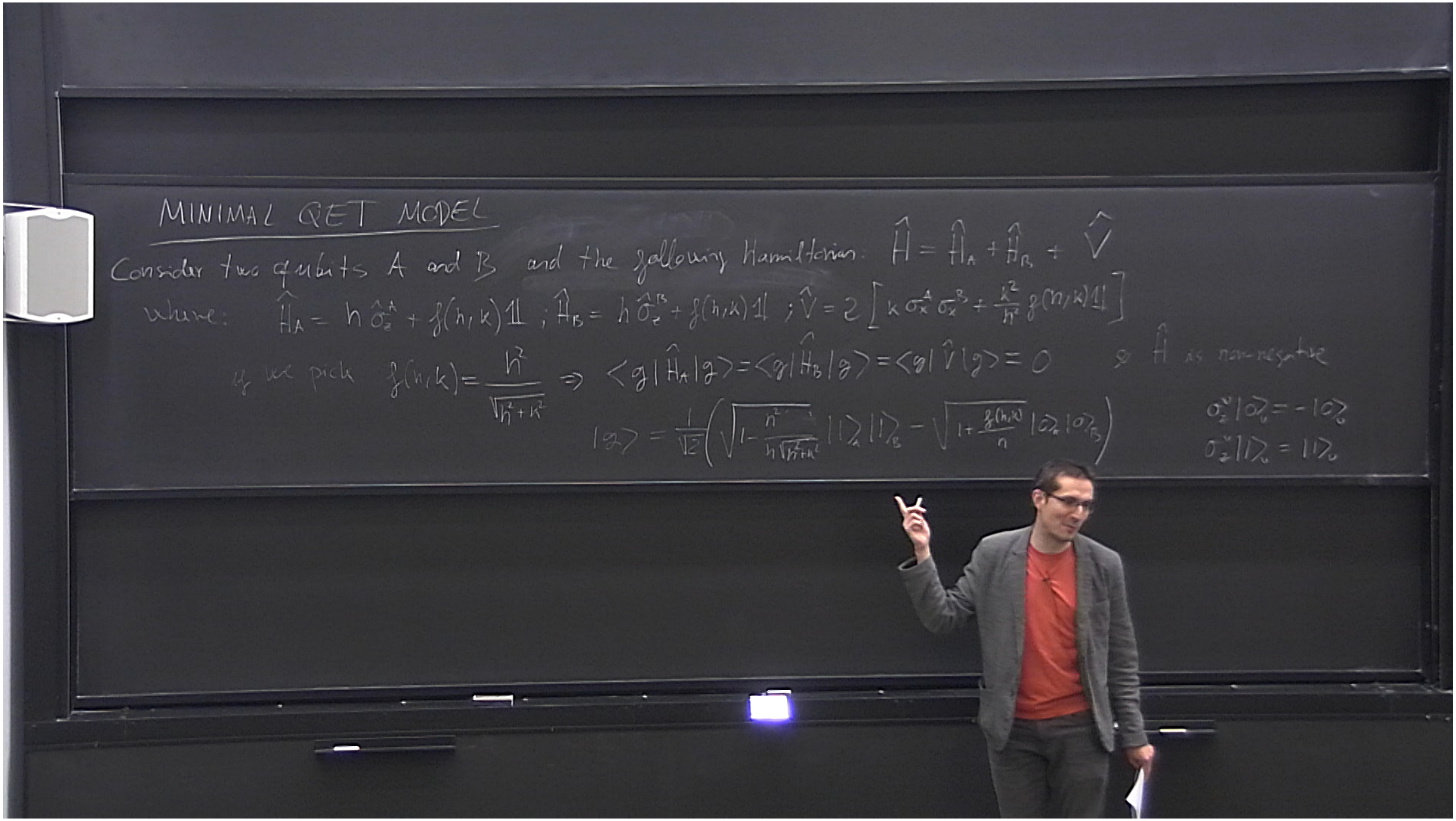
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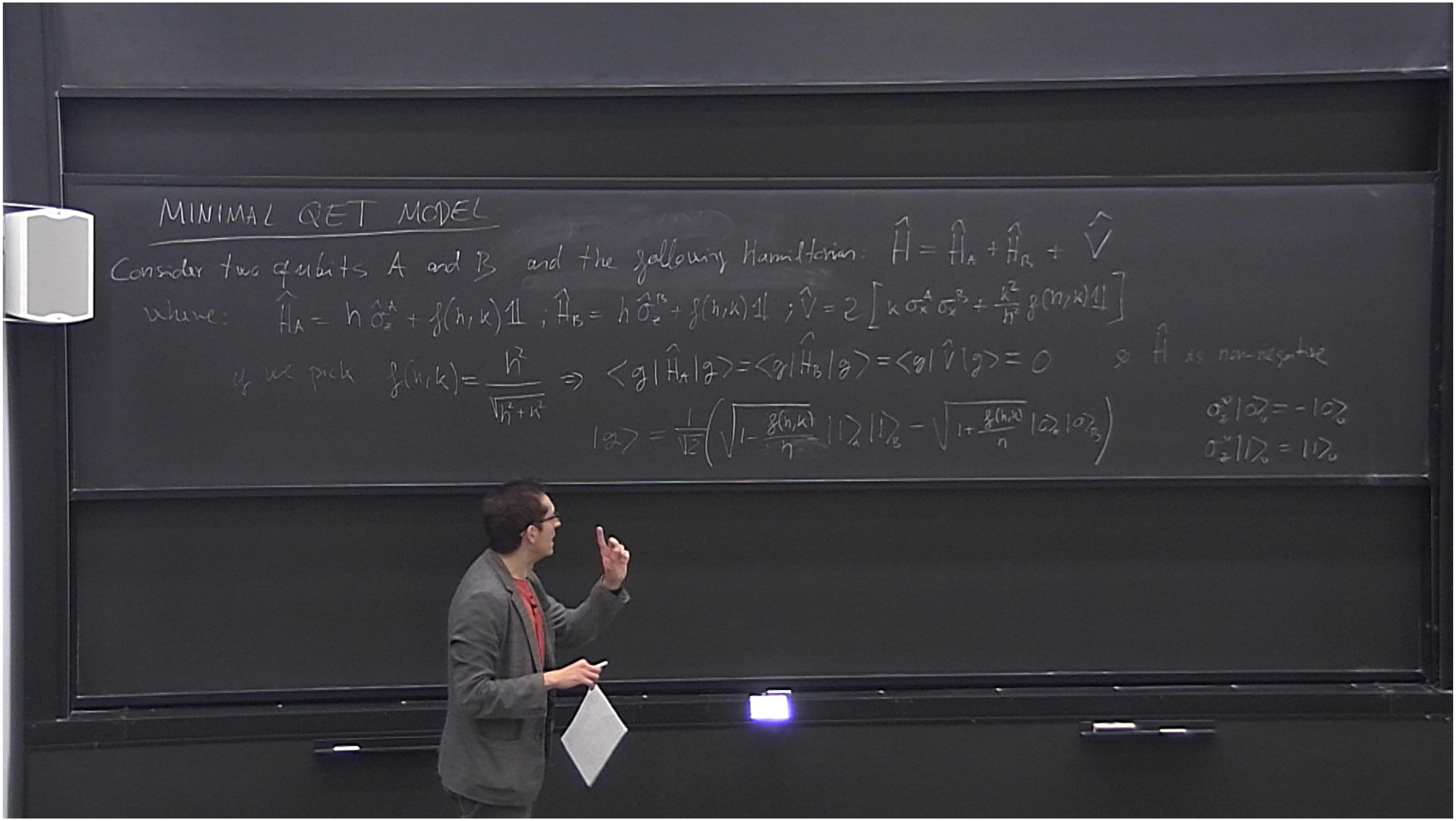
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$\sigma_z^\nu|0$

$\sigma_z^\nu|1$

The protocol

1. We start from the ground state $|0\rangle$
2. Alice carries out a PVM of σ_x^A. (Repeated many times this will have an average energy cost $E_A > 0$)

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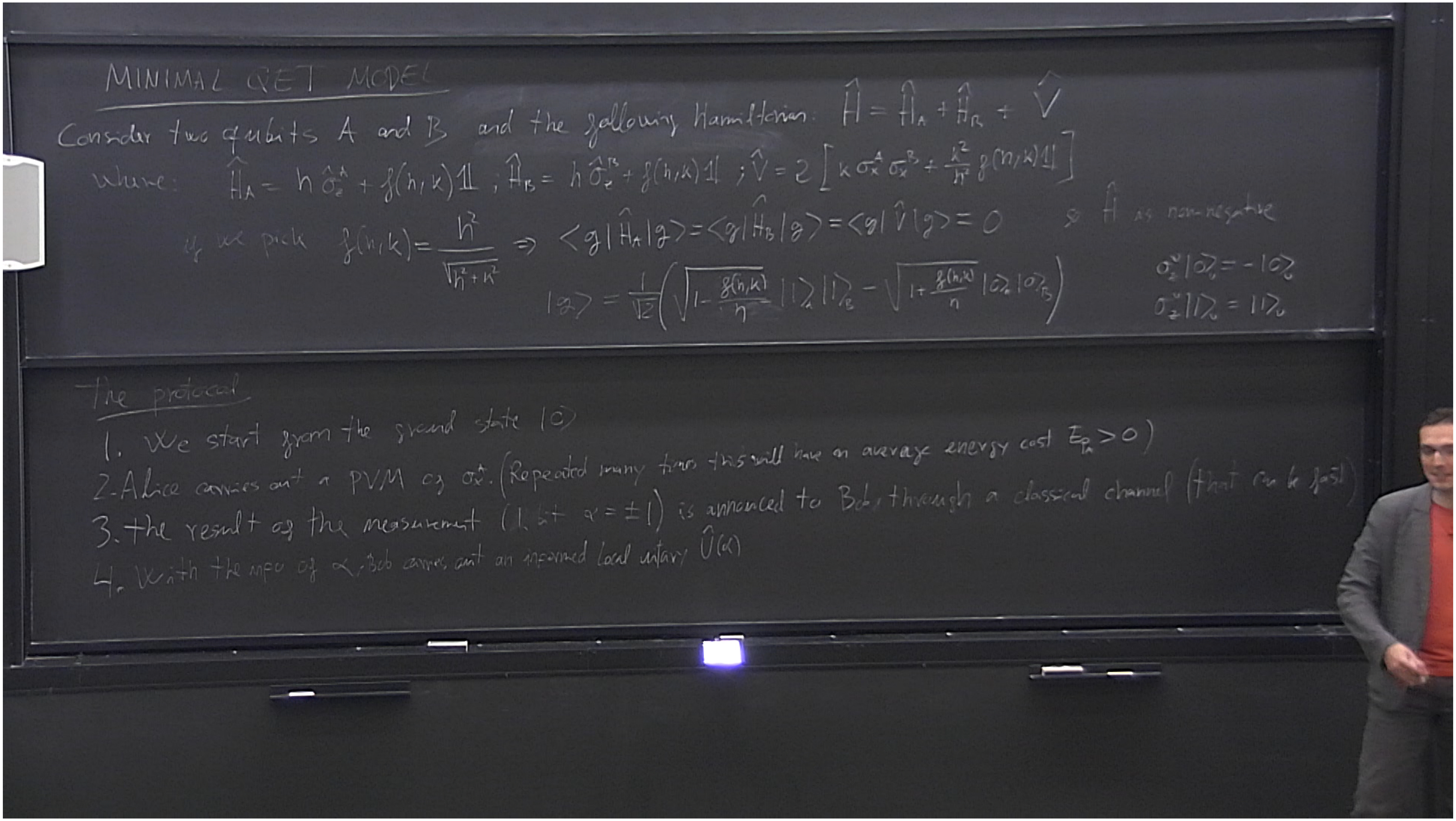
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if we pick f(h,k) = h² / √(h²+k²) ⇒ <g| Ĥ_A |g> = <g| Ĥ_B |g> = <g| V̂ |g> = 0
so Ĥ is non-negative
|g> = 1/√2 (√(1 − f(h,k)/h) |1>_A |1>_B − √(1 + f(h,k)/h) |0>_A |0>_B)
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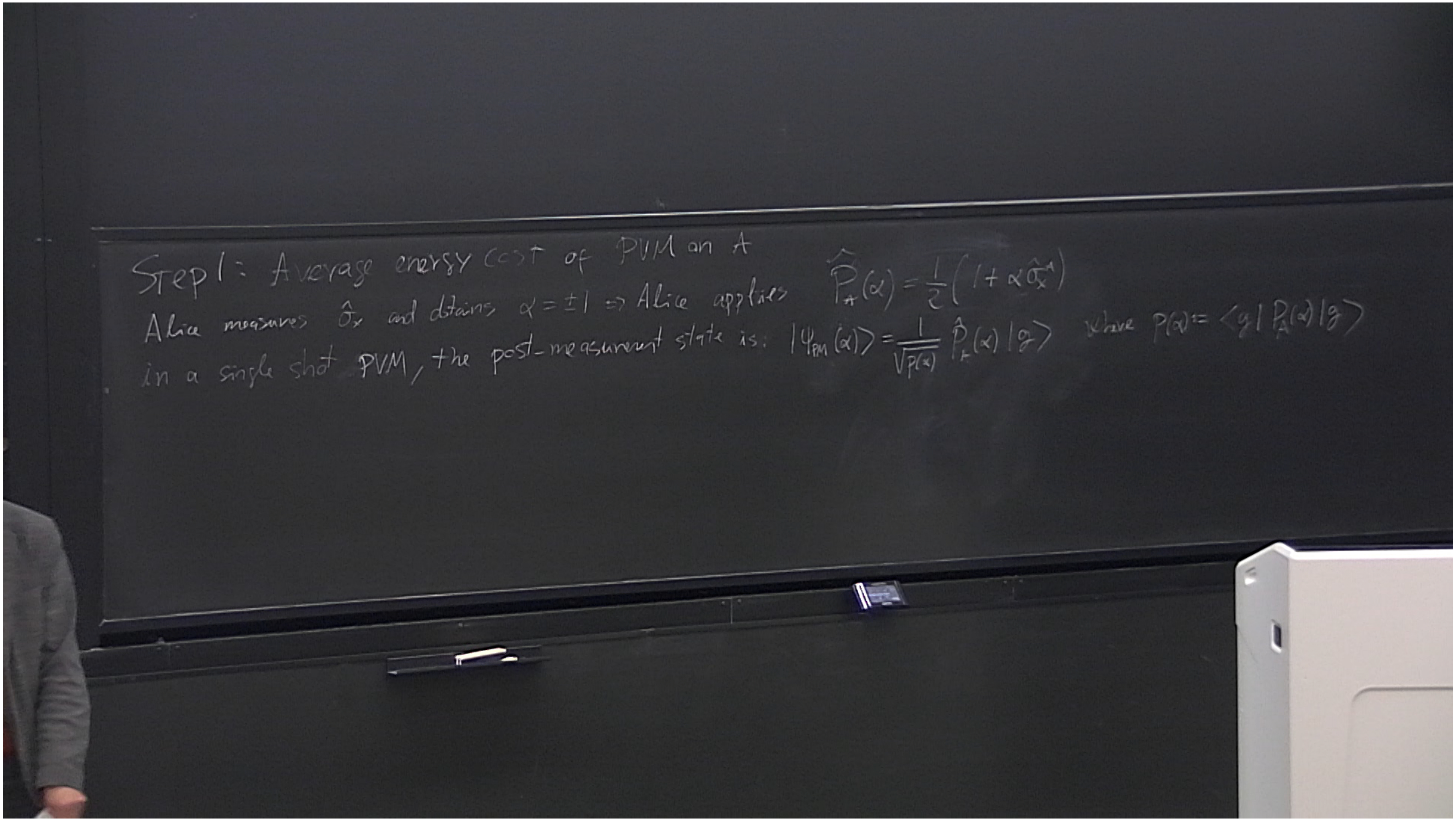
3. the result of the measurement (1 bit $\alpha = \pm 1$) is announced to Bob through a classical channel (that can be fast)
4. With the info of α, Bob carries out an informed local unitary $\hat{U}(\alpha)$. After many repetitions of this protocol, the average energy cost of Bob's unitary will be negative


Step 1: Average energy cost of PVM on A
Alice measures $\hat{\sigma}_x$ and obtains $\alpha = \pm 1 \Rightarrow$ Alice applies $\hat{P}_A(\alpha) = \frac{1}{2}(1 + \alpha \hat{\sigma}_x^A)$
in a single shot PVM, the post-measurement state is: $|\psi_{PM}(\alpha)\rangle = \frac{1}{\sqrt{p(\alpha)}} \hat{P}_A(\alpha)|g\rangle$ where $p(\alpha) := \langle g|\hat{P}_A(\alpha)|g\rangle$

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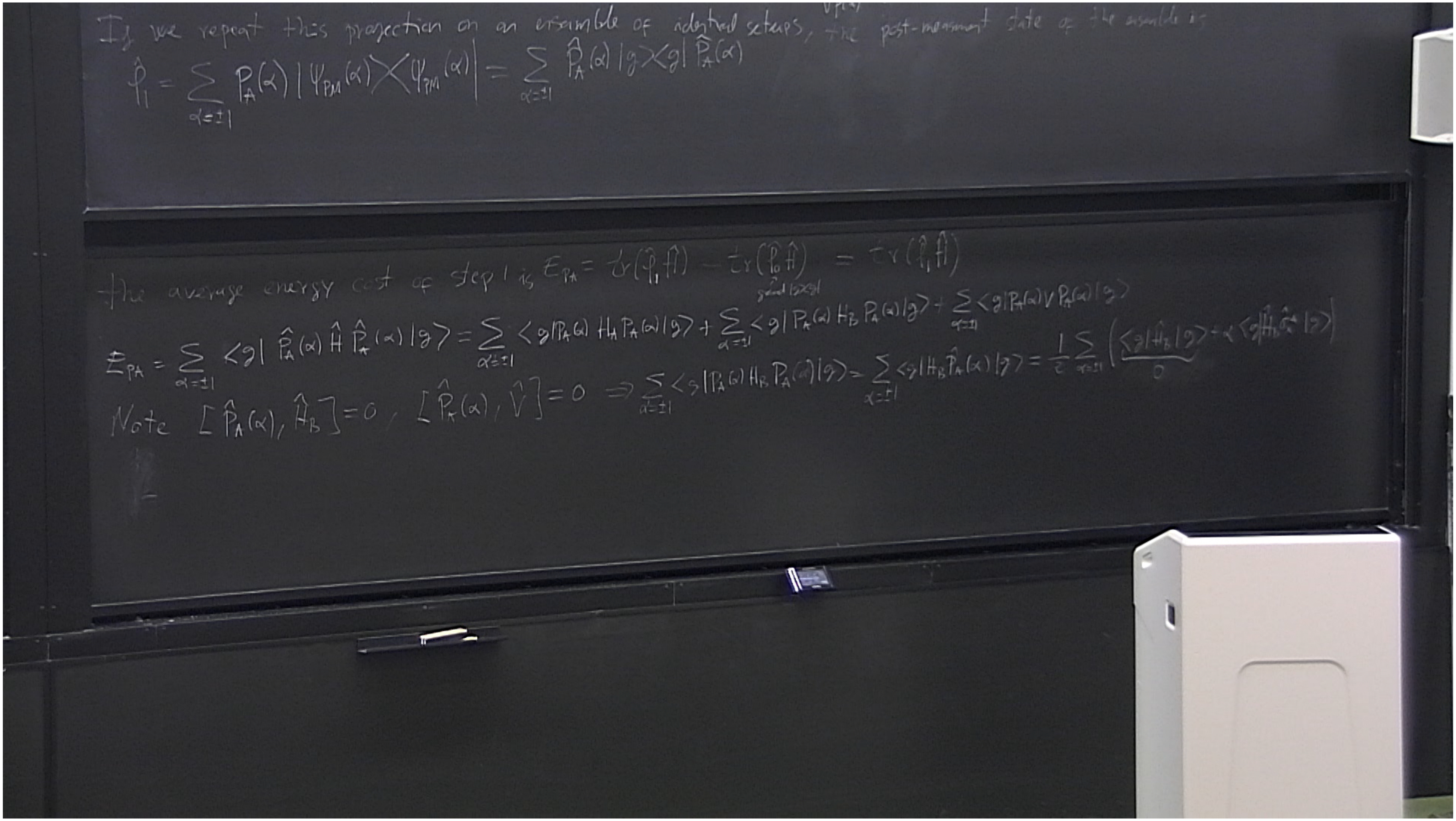
$$\hat{\rho}_1 = \sum_{\alpha=\pm 1} p_A(\alpha)\,|\psi_{PM}(\alpha)\rangle\langle\psi_{PM}(\alpha)| = \sum_{\alpha=\pm 1}\hat{P}_A(\alpha)|g\rangle\langle g|\hat{P}_A(\alpha)$$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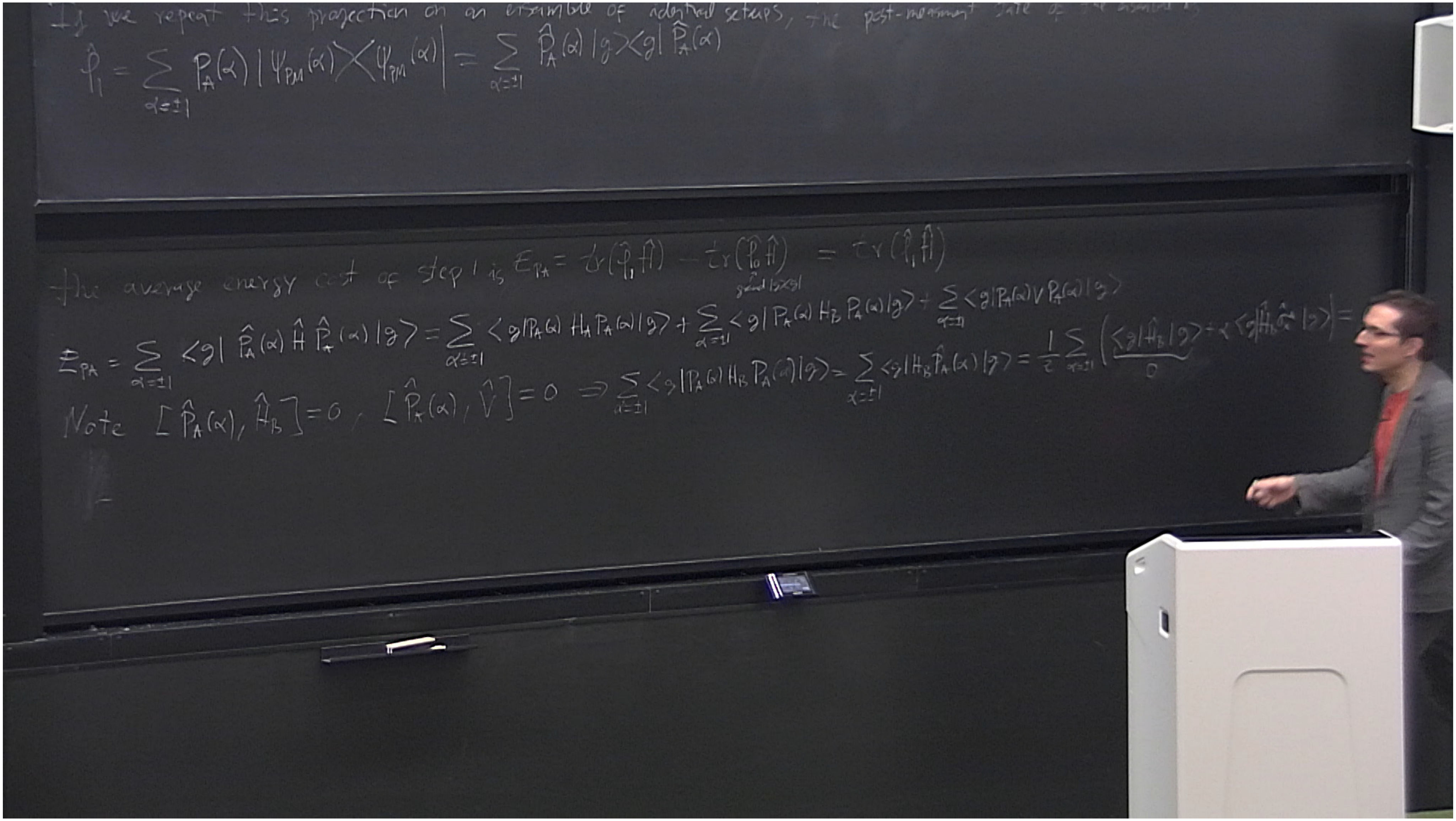
the average energy cost of step 1 is $E_{PA} = tr(\hat{\rho}_1 \hat{H}) - tr(\hat{\rho}_0 \hat{H}) = tr(\hat{\rho}_1 \hat{H})$
$E_{PA} = \sum_{\alpha=\pm1} \langle g| \hat{P}_A(\alpha) \hat{H} \hat{P}_A(\alpha) |g\rangle = \sum_{\alpha=\pm1} \langle g| P_A(\alpha) H_A P_A(\alpha) |g\rangle + \sum_{\alpha=\pm1} \langle g| P_A(\alpha) H_B P_A(\alpha) |g\rangle + \sum_{\alpha=\pm1} \langle g| P_A(\alpha) V P_A(\alpha) |g\rangle$
Note $[\hat{P}_A(\alpha), \hat{H}_B] = 0$, $[\hat{P}_A(\alpha), \hat{V}] = 0 \Rightarrow \sum_{\alpha=\pm1} \langle g| P_A(\alpha) H_B P_A(\alpha) |g\rangle = \sum_{\alpha=\pm1} \langle g| H_B \hat{P}_A(\alpha) |g\rangle = \frac{1}{2} \sum_{\alpha=\pm1} \left(\langle g|\hat{H}_B|g\rangle + \alpha \langle g|\hat{H}_B \hat{\sigma}_A |g\rangle \right)$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the average energy cost of step 1 is $E_{P_A} = tr(\hat{\rho}_1\hat{H}) - tr(\hat{\rho}_0\hat{H}) = tr(\hat{\rho}_1\hat{H})$
$\hat{\rho}_0 = |g\rangle\langle g|$
$E_{P_A} = \sum_{\alpha=\pm1}\langle g|\hat{P}_A(\alpha)\hat{H}\hat{P}_A(\alpha)|g\rangle = \sum_{\alpha=\pm1}\langle g|P_A(\alpha)H_A P_A(\alpha)|g\rangle + \sum_{\alpha=\pm1}\langle g|P_A(\alpha)H_B P_A(\alpha)|g\rangle + \sum_{\alpha=\pm1}\langle g|P_A(\alpha)VP_A(\alpha)|g\rangle$
Note $[\hat{P}_A(\alpha),\hat{H}_B]=0$, $[\hat{P}_A(\alpha),\hat{V}]=0 \Rightarrow \sum_{\alpha=\pm1}\langle g|P_A(\alpha)H_B P_A(\alpha)|g\rangle = \sum_{\alpha=\pm1}\langle g|H_B\hat{P}_A(\alpha)|g\rangle = \frac{1}{2}\sum_{\alpha=\pm1}\left(\langle g|\hat{H}_B|g\rangle + \alpha\langle g|\hat{H}_B\hat{\sigma}|g\rangle\right) =$
0

If we repeat this projection on an ensemble of identical setups, the post-measurement state of the ensemble is

$$\hat{\rho}_1 = \sum_{\alpha=\pm1} P_A(\alpha) \, |\psi_{PM}(\alpha)\rangle\langle\psi_{PM}(\alpha)| = \sum_{\alpha=\pm1} \hat{P}_A(\alpha) \, |g\rangle\langle g| \, \hat{P}_A(\alpha)$$

the average energy cost of step 1 is $E_{P_A} = tr(\hat{\rho}_1 \hat{H}) - tr(\hat{\rho}_0 \hat{H}) = tr(\hat{\rho}_1 \hat{H})$, with $\hat{\rho}_0 = |g\rangle\langle g|$

$$E_{P_A} = \sum_{\alpha=\pm1} \langle g| \hat{P}_A(\alpha) \hat{H} \hat{P}_A(\alpha) |g\rangle = \sum_{\alpha=\pm1} \langle g| P_A(\alpha) H_A P_A(\alpha) |g\rangle + \sum_{\alpha=\pm1} \langle g| P_A(\alpha) H_B P_A(\alpha) |g\rangle + \sum_{\alpha=\pm1} \langle g| P_A(\alpha) V P_A(\alpha) |g\rangle$$

$$\Rightarrow \sum_{\alpha=\pm1} \langle g| P_A(\alpha) H_B P_A(\alpha) |g\rangle = \sum_{\alpha=\pm1} \langle g| H_B \hat{P}_A(\alpha) |g\rangle = \frac{1}{2} \sum_{\alpha=\pm1} \Big(\underbrace{\langle g| \hat{H}_B |g\rangle}_{0} + \alpha \underbrace{\langle g| \hat{H}_B \hat{\sigma}_x^A |g\rangle}_{\langle g| \sigma_x^A |g\rangle} \Big) = 0$$

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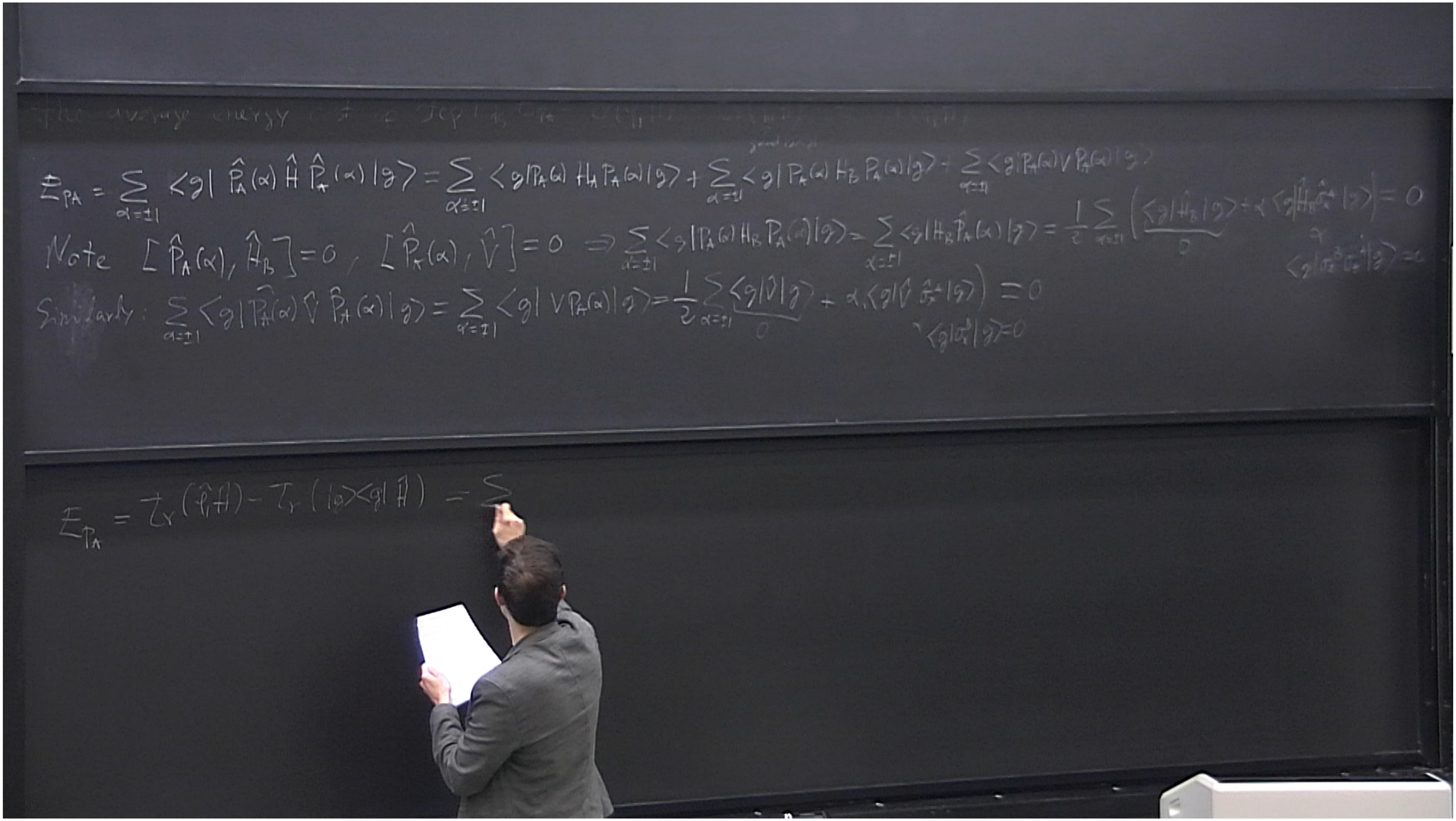
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Similarly: