Title: The approach of a black hole to equilibrium

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Abstract: <p>In this talk I will consider the black hole formed as a result of a binary black hole coalescence. As expected, the final black hole eventually approaches a Kerr solution. We show numerically, in full general relativity and in the highly non-linear merger regime, how the final black hole approaches equilibrium. In particular we show how the infalling radiation wipes out the deviations from Kerr and the rate at which the multipole moments decay to their asymptotic values.</p>

The approach of a black hole horizon to equilibrium

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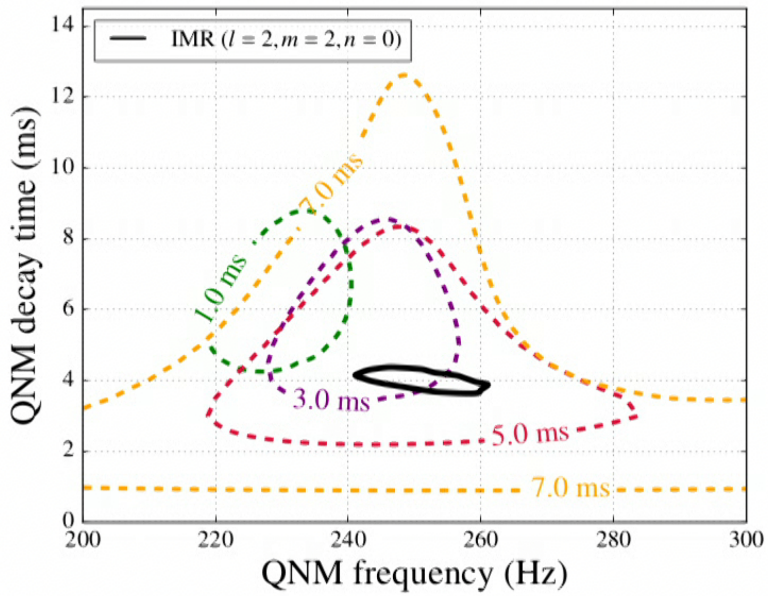
March 22, 2018

Post merger analysis for GW150914

- BBH detections allow us to observe dynamical black holes
- Simple analysis of postmerger signal with damped sinusoids carried out in one of the LIGO papers (PRL 116, 221101 (2016)) (R. Prix, AEI)



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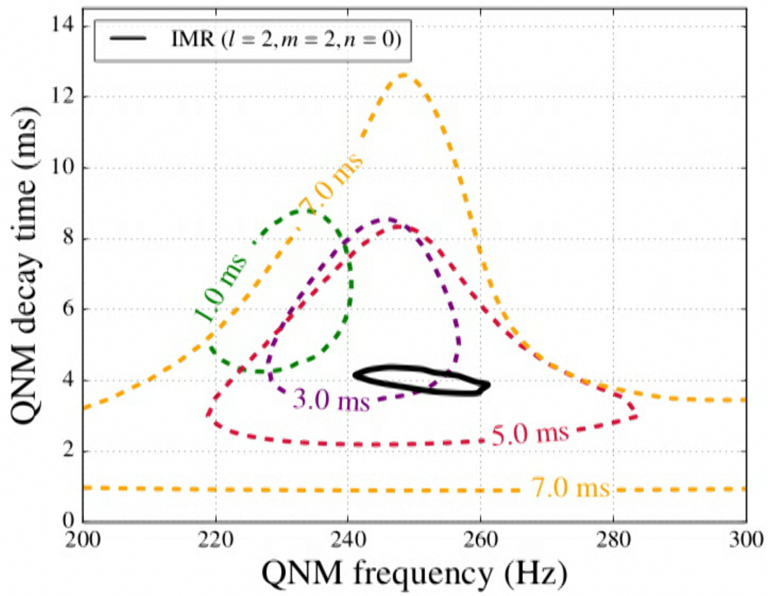
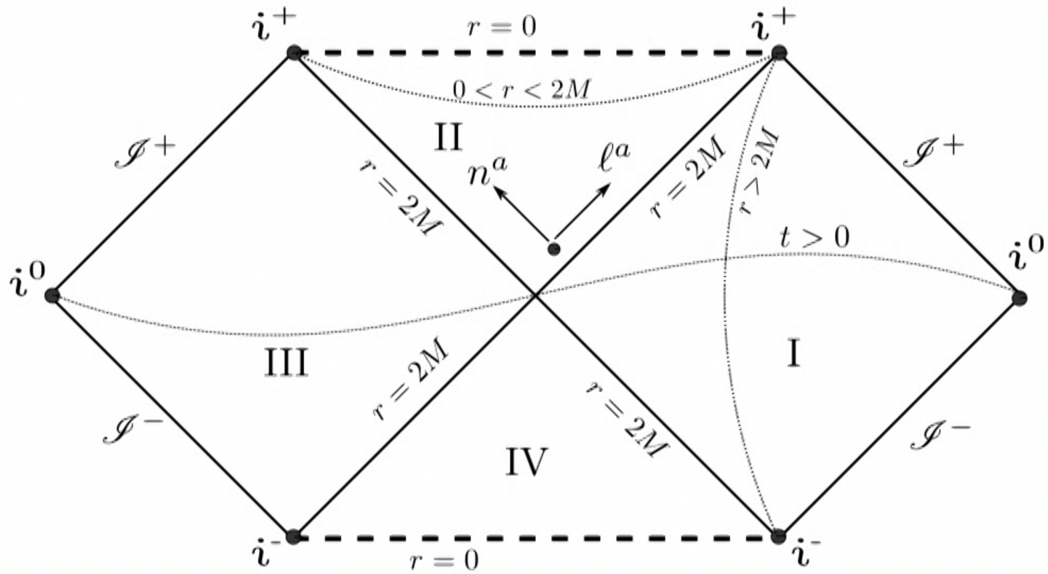


Post merger analysis for GW150914

- This suggests that the final black hole has reached equilibrium 5 ms after the peak of the signal
- This is $\sim 15M$ in geometric units in terms of the total mass of the system ($\sim 70M_\odot$)
- Can ask a similar question with gravitational wave signals obtained from numerical relativity
- Kamaretsos et al (PRD, 2012) looked at the frequency evolution of the GW signal
- They found that the frequencies become consistent with the QNM frequencies $\sim 10M$ after the peak of the $\ell = m = 2$ luminosity
- Can we see similar effects at the black hole horizon?

Event Horizons

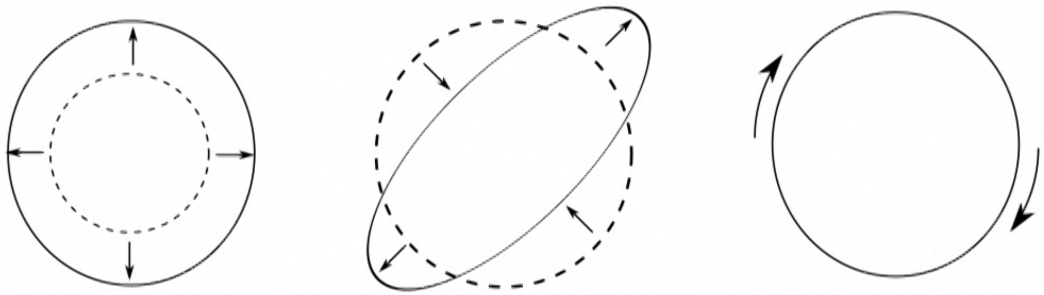
- Black hole surfaces traditionally defined via event horizons



The singularity theorems

- There is one classic result which does not use event horizons directly
- The singularity theorem: the presence of a *closed trapped surface* (+energy condition) implies geodesic incompleteness in the future
- First singularity theorem proved by Penrose in 1965
- Subsequently generalized by Penrose & Hawking in 1970
- Penrose's trapped surfaces are the key to understanding black holes quasi-locally

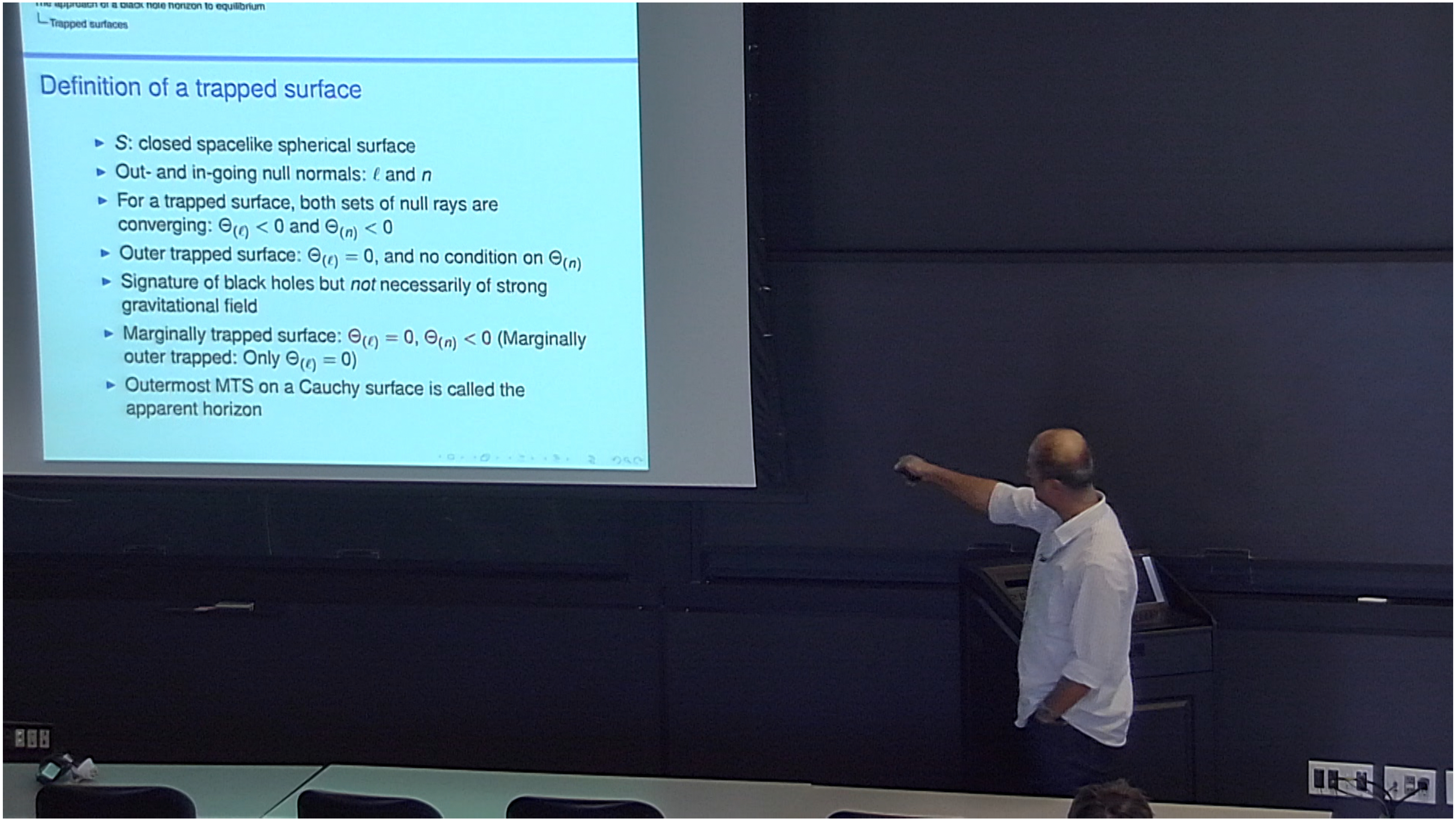
The expansion of a null geodesic congruence



- The expansion of a congruence of null geodesics quantifies the rate of increase of the cross-section area

$$\Theta = \frac{1}{A}\frac{dA}{dt}$$

- Can also define the shear σ_{ab} and twist ω_{ab} using deformations of the infinitesimal cross-section


approach of a black hole horizon to equilibrium
Trapped surfaces
Definition of a trapped surface
S: closed spacelike spherical surface
Out- and in-going null normals: ℓ and n
For a trapped surface, both sets of null rays are converging: $\Theta_{(\ell)} < 0$ and $\Theta_{(n)} < 0$
Outer trapped surface: $\Theta_{(\ell)} = 0$, and no condition on $\Theta_{(n)}$
Signature of black holes but *not* necessarily of strong gravitational field
Marginally trapped surface: $\Theta_{(\ell)} = 0$, $\Theta_{(n)} < 0$ (Marginally outer trapped: Only $\Theta_{(\ell)} = 0$)
Outermost MTS on a Cauchy surface is called the apparent horizon

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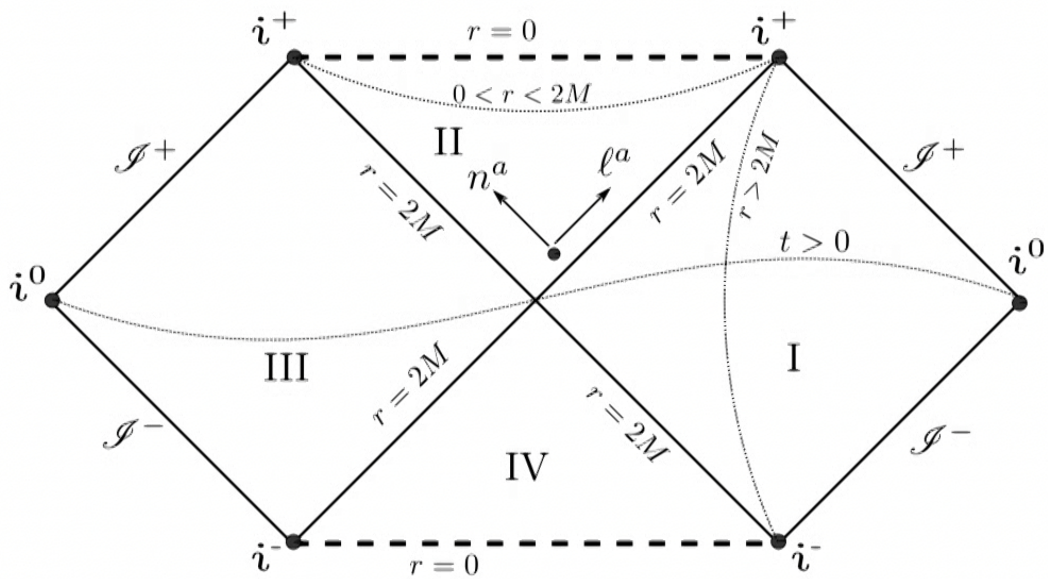

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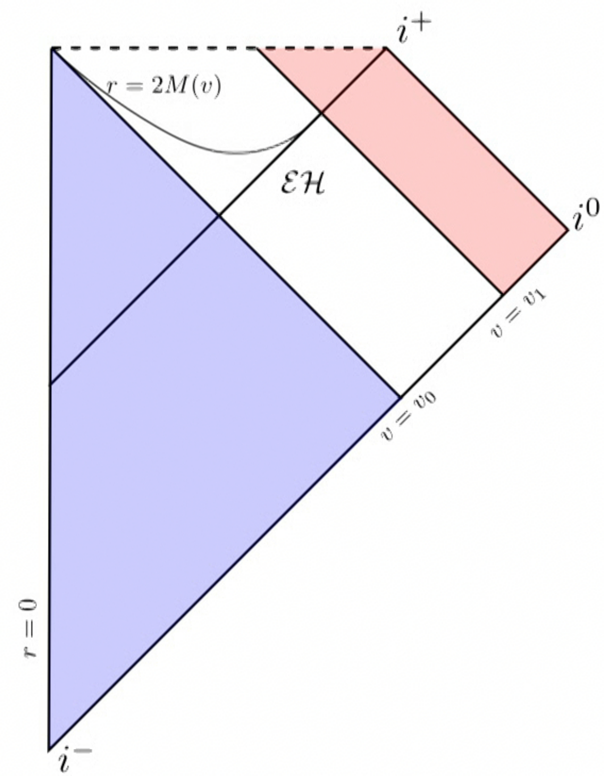
The Vaidya spacetime

- Simple generalization of Schwarzschild for a varying mass function $M(v)$ with $\dot{M}(v) \geq 0$ and $\lim_{v\to\infty} M(v)$ finite
- Vaidya spacetime represents collapse of spherically symmetric null-dust

$$ds^2 = -\left(1 - \frac{2M(v)}{r}\right) dv^2 - 2dv\,dr + r^2 d\Omega^2\,.$$

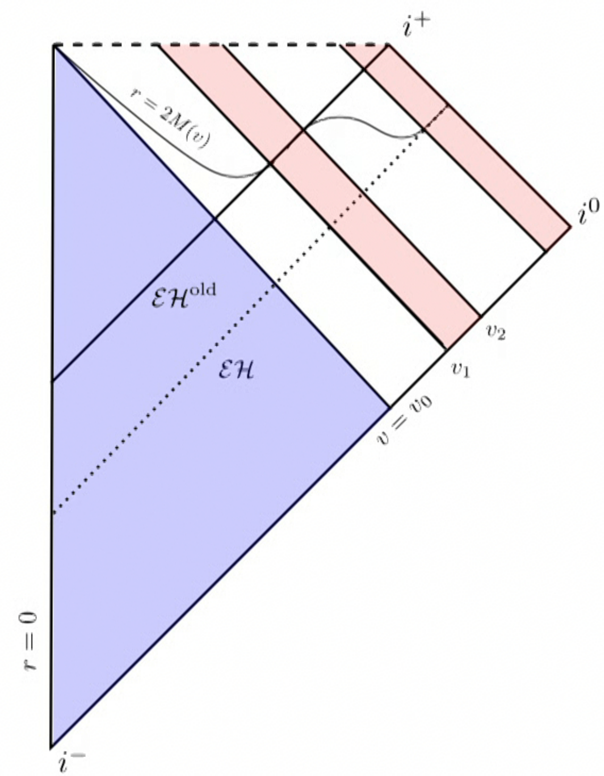
$$T_{ab} = \frac{\dot{M}(v)}{4\pi r^2} \nabla_a v \nabla_b v\,.$$

Trapped surfaces in Vaidya



- Unphysical but useful playground
- Spherically symmetric trapped surfaces do not extend up to event horizon

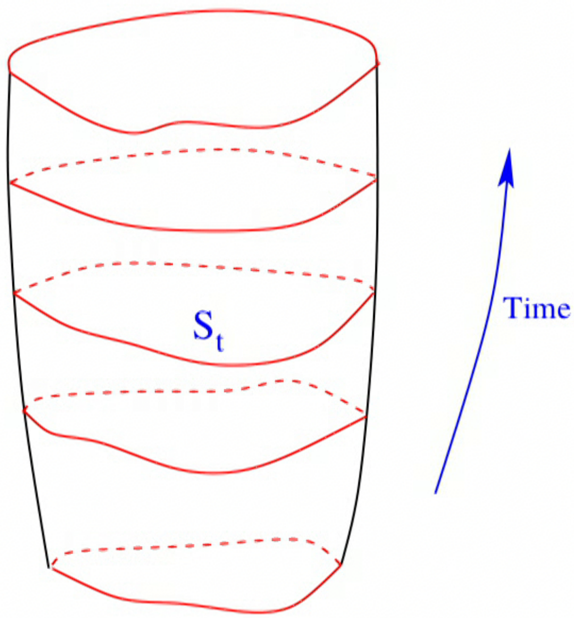
The Vaidya spacetime



- Event horizon extends to flat spacetime
- As before, location of EH depends on $M(v)$ for $v \to \infty$

Marginally trapped tubes (MTTs)

- Marginally trapped tubes obtained by stacking up Marginally outer trapped surfaces (MOTS)



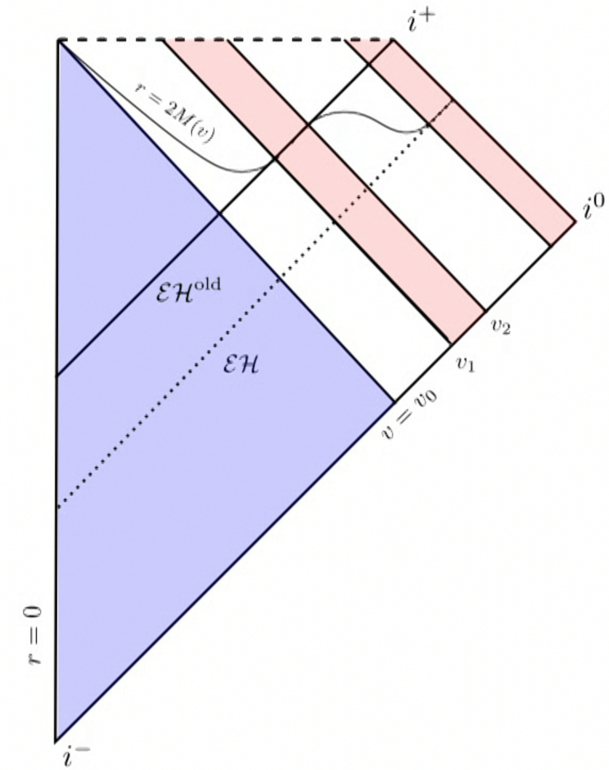
- Think of marginally trapped tube as the smooth time evolution of a MOTS
- Marginally trapped tube is defined as a 3-surface foliated by MOTSs
- This is a covariant spacetime definition even if we might use a 3+1 split in practice

Different kinds of horizons

All these are different kinds of MTTs ($\Theta_{(\ell)} = 0$)

- **Dynamical horizon**: spacelike MTT with $\Theta_{(n)} < 0$
- **Isolated horizon**: null MTT, no restriction on $\Theta_{(n)}$
- **Timelike membrane**: timelike MTT

The Vaidya spacetime



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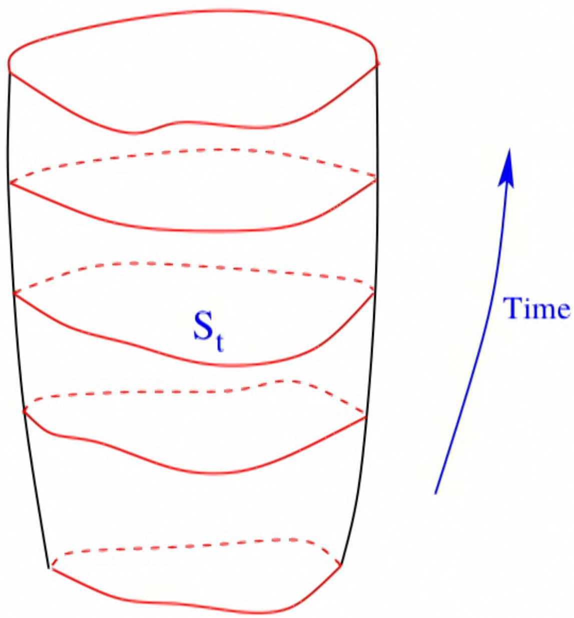

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Mass and angular momentum

- For the axisymmetric case we can perform a Hamiltonian analysis for spacetimes with inner boundaries
- These lead to definitions for angular momentum and energy

$$J_S = \frac{1}{8\pi} \oint_S K_{ab} R^a \varphi^b \, d^2 S$$

$$M_S = \frac{1}{2R_S} \sqrt{R_S^4 + 4J_S^2}$$

(used commonly in numerical simulations to calculate mass and spin)

- Black hole thermodynamics can be formulated quasilocally in this set up for isolated and dynamical black holes

Horizon multipole moments

- For a mass or current distribution, it is extremely useful to decompose in terms of $Y_{\ell m}(\theta, \varphi)$ (or vector/tensor spherical harmonics)
- At black hole horizons, which (θ, φ) should we use?
- Freedom to make coordinate transformations leads to ambiguity in multipole moments
- For axisymmetric isolated horizons, one can construct an invariant coordinate system (Ashtekar et al, 2004)
- If φ^a is the axial symmetry vector then choose ϕ as affine parameter with range $0 \leq \phi < 2\pi$
- Assume that φ^a has closed orbits and vanishes at exactly 2 points (the poles)

Horizon multipole moments

- The other coordinate ζ (analogous to $\cos\theta$) defined by

$$D_a\zeta = \frac{1}{R^2}\tilde{\epsilon}_{ab}\phi^b\,, \quad \int_S \zeta\tilde{\epsilon} = 0$$

- ζ^a is orthogonal to φ^a and the metric on S can be written as

$$R^2\left(f^{-1}(\zeta)d\zeta^2 + f(\zeta)d\phi^2\right)$$

  (Volume element is same as for a "round" sphere)
- Basic idea is to use $Y_{\ell m}$ in these invariant coordinates

(details in Ashtekar et al, 2004)

Horizon multipole moments

- Mass and current moments are obtained by decomposing the scalar curvature and $K_{ab}R^b$

$$M_n = \frac{M_\mathcal{S} R_\mathcal{S}^n}{8\pi} \oint_S \mathcal{R} P_n(\zeta) d^2 S$$

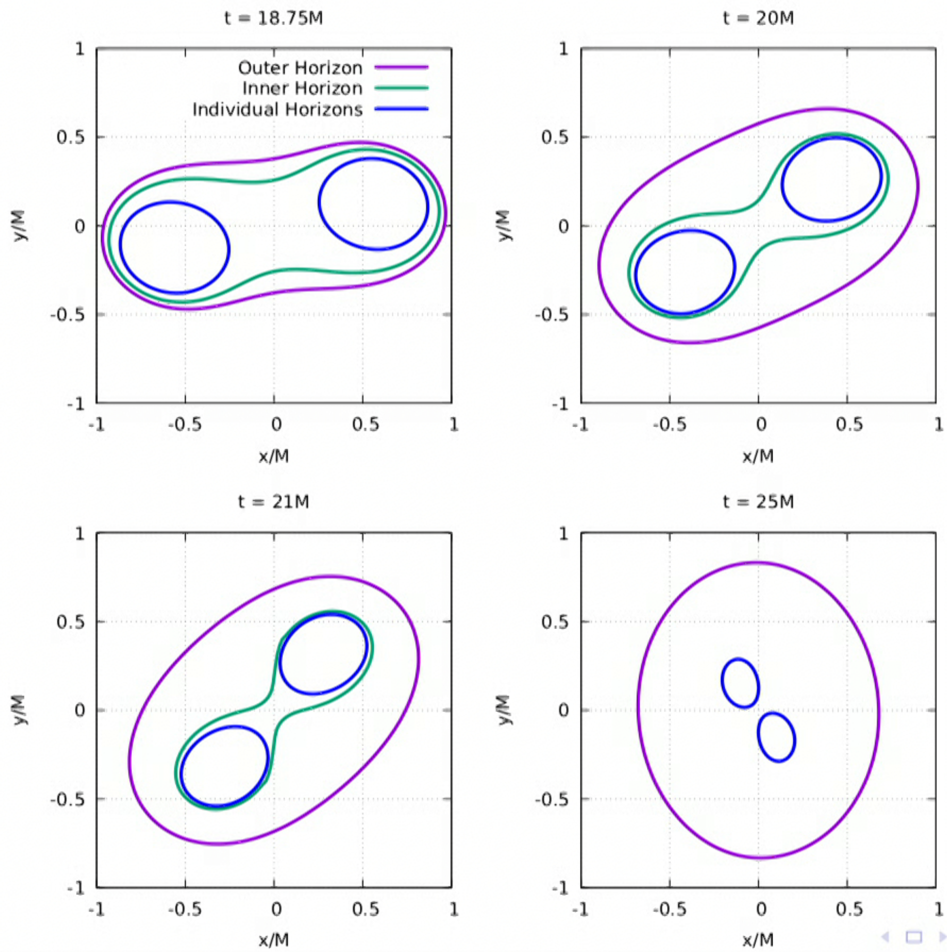
$$J_n = \frac{R_\mathcal{S}^{n-1}}{8\pi} \oint_S P_n'(\zeta) K_{ab} \varphi^a R^b d^2 S$$

- In axisymmetry only $m = 0$ multipole moments are non-zero
- These are different from the Geroch-Hansen moments at spatial infinity

The initial configuration

(arXiv:1801.07048)

- Our goal is not a GW waveform calculation with a long inspiral phase
- We start with a system that performs $\sim 3/4$ of an orbit before a common horizon forms
- We do not have control on the eccentricity of the inspiral
- The common horizon forms at $t \sim 18.66M$ and immediately bifurcates into inner and outer horizons
- The individual horizons continue to exist
- We lose track of the individual and inner horizon about $\sim 7M$ and $\sim 3M$ after the common horizon forms


t = 18.75M
Outer Horizon
Inner Horizon
Individual Horizons
t = 20M
t = 21M
t = 25M
y/M
x/M
1
0.5
0
-0.5
-1

Marginally trapped tubes (MTTs)

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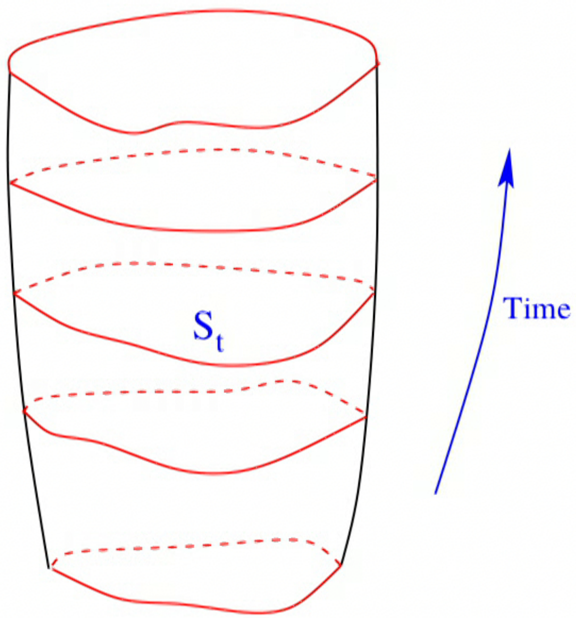

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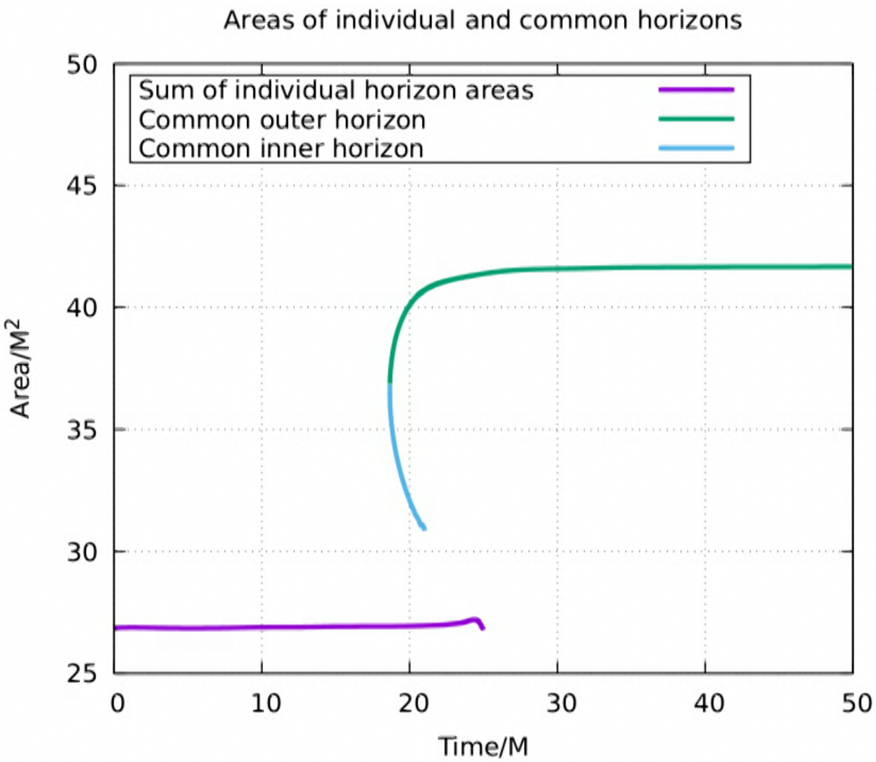
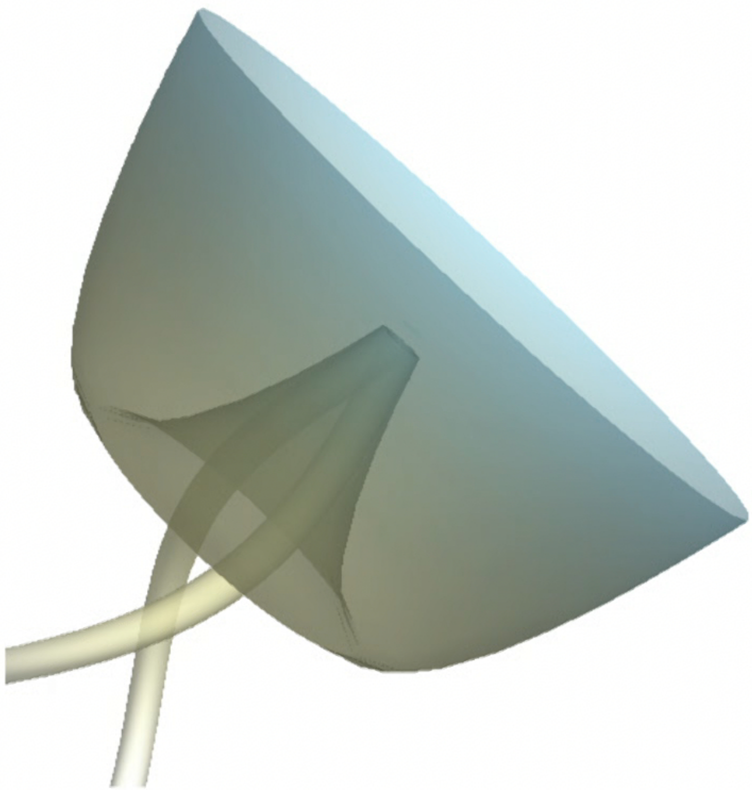
The horizon areas



A cartoon picture of the merger



Other properties

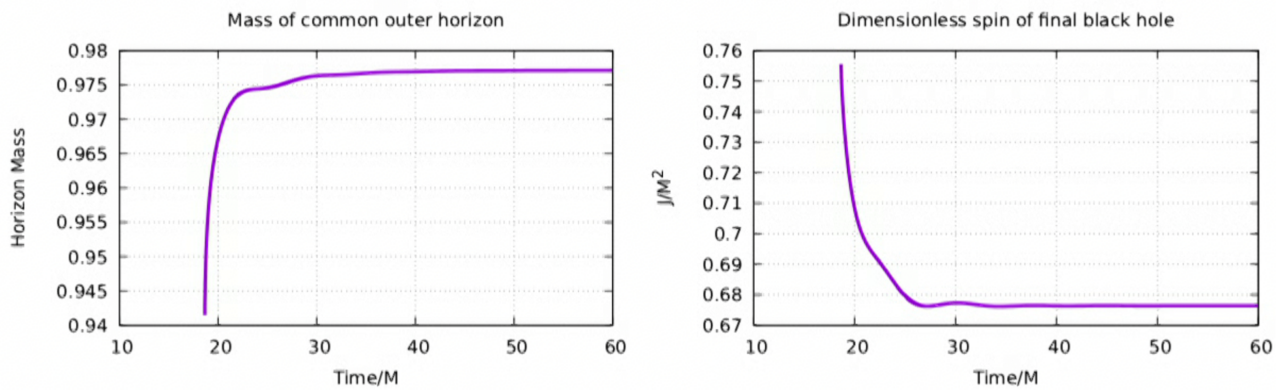
- The outer horizon does not have $\Theta_{(n)}$ negative everywhere
- It becomes negative everywhere only at $t \sim 31M$
- Its average on the horizon is always negative

$$\int_{\mathcal{S}} \Theta_{(n)} \widetilde{\epsilon} < 0$$

- The outer horizon is spacelike
- The inner horizon is initially spacelike (by continuity) but soon it becomes timelike
- The individual horizons remain null

Evolution of the mass and spin

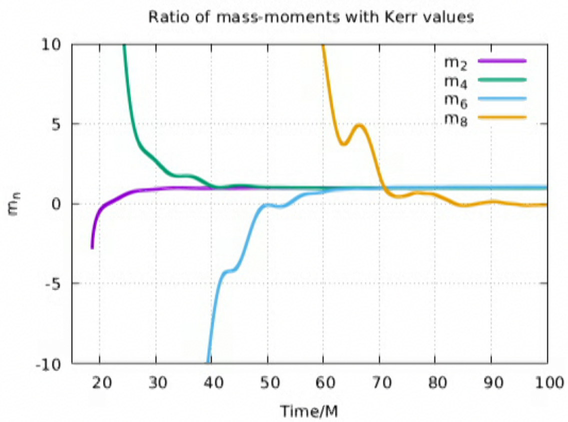
- Take $\partial/\partial\varphi$ to calculate spin – needs to be improved!
- The dimensionless spin J/M^2 reaches a final value of ~ 0.68 as expected
- The mass increases monotonically

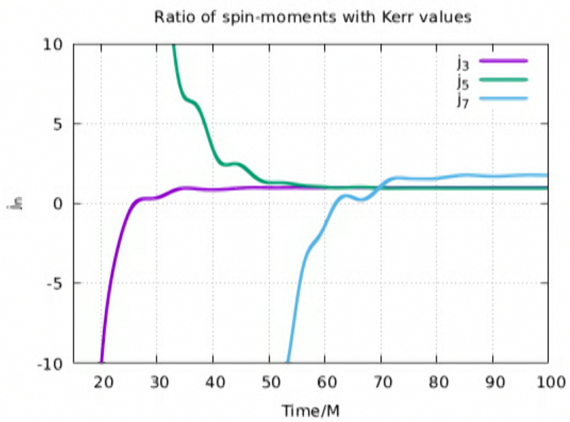


Approach to Kerr

- In Kerr, given M, J we can calculate all other moments
- Define then the ratios with the Kerr values:

$$m_n = \frac{M_n(t)}{M_n^{\mathrm{Kerr}}(t)}, \quad j_n = \frac{J_n(t)}{J_n^{\mathrm{Kerr}}(t)}$$



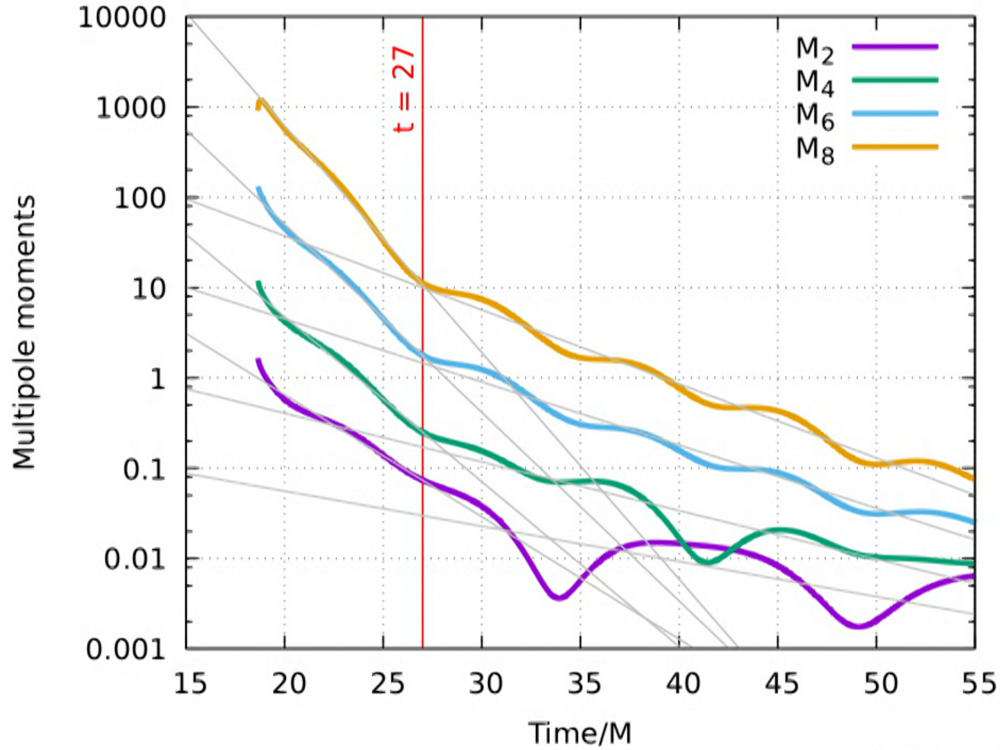


Approach to Kerr

- The moments generally approach their Kerr values
- J_7 and M_8 are less accurate
- Do the moments fall off exponentially?
- At late times we would expect power law decays (Price's law)
- In the immediate post-merger and quasi-normal ringing phase we see exponential decays with oscillations

The transition at $t = 27M$

But the fall off is not a single exponent!



The values of the exponents

- Take times before and after $27M$ and calculate best fit values of exponents ($e^{-\alpha t}$)

Multipole	$\alpha^{(t<27M)}$	$\alpha^{(t>27M)}$
M_2	0.31	0.09
M_4	0.42	0.12
M_6	0.48	0.16
M_8	0.58	0.19
J_3	0.43	0.16
J_5	0.51	0.17
J_7	0.64	0.18

The in-falling flux

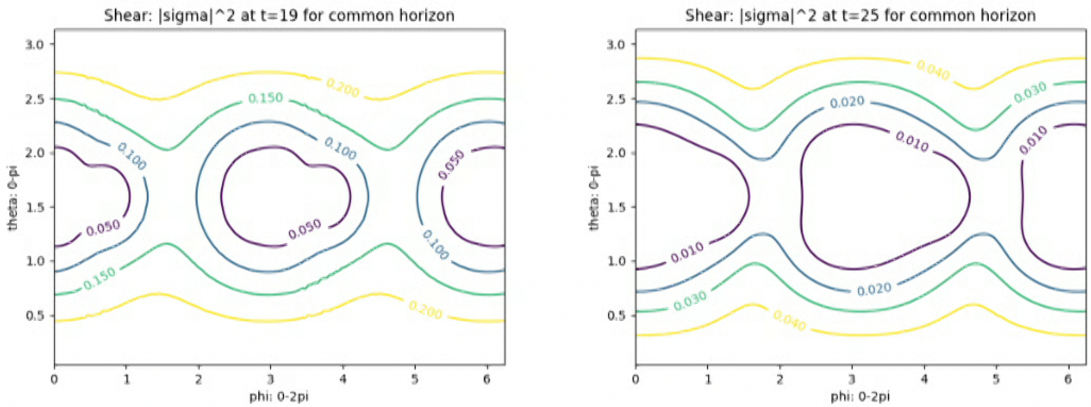
- Horizon grows due to in-falling flux of energy
- No general local flux law possible for event horizons
- For dynamical horizons we can show that the flux is

$$\mathfrak{f} = \frac{1}{2}\sigma_{ab}\sigma^{ab} + \zeta_a\zeta^a$$

- "time" on the horizon is radius of MOTS and $\zeta_a := h_a{}^b \hat{r}^c \nabla_c \ell_b$
- Exact result in GR with no approximations (analogous to Bondi flux at null infinity due to GWs)
- The shear dominates near equilibrium – we shall focus on shear

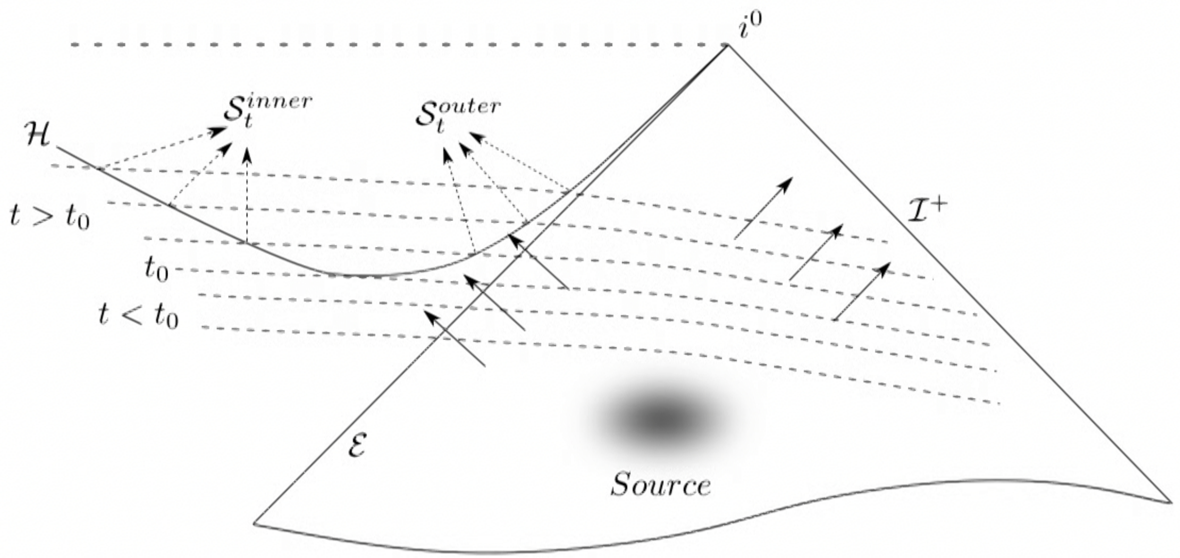
The shear at the horizon

- The shear decreases with time and has clear quadrupolar pattern



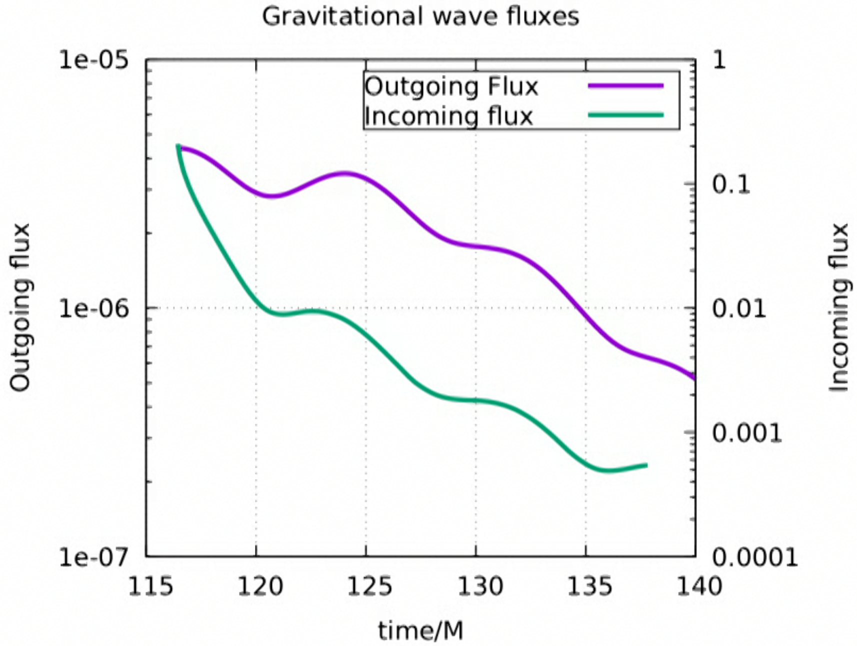
Correlations with the wavezone

- The in- and out-going fluxes have a common source and may be correlated (Jaramillo et al, 2010, 2011)



Correlations with the wavezone

- align the fluxes with their peaks – The shears are clearly correlated both in time and directions



Correlations with the wavezone

- The horizon can clearly not causally affect the GW signal
- But they have strong correlations
- Any effect seen at the horizon will likely have a counterpart in the GW signal
- We see again a qualitative change at $t \sim 27M$, i.e. $\sim 10M$ after the common horizon is formed
- Is this a generic feature of BBH mergers or perhaps only for astrophysical initial data?
- Very different gauge conditions in the wavezone and the horizon – no reason this should have worked!