Title: PI Day - Mar. 14th, 2018 - Part 1

Date: Mar 14, 2018 10:30 AM

URL: http://pirsa.org/18030095

Abstract:

The energy-entropy diagram as a fundamental tool of thermodynamics

arXiv:1607.01302, arXiv:1504.03661

March 2018

We consider a system with finite-dimensional Hilbert space $\mathbb{C}^d$ and Hamiltonian H. A state ρ has energy $E(\rho) = \mathrm{tr}(\rho H)$ and entropy $S(\rho) = -\mathrm{tr}(\rho \log \rho)$.

Theorem

For states ρ and σ, the following are equivalent:

1. There exists an ancilla system of size $O(\sqrt{n \log n})$ with state η and Hamiltonian H_{anc} satisfying $\|H_{\mathrm{anc}}\| \leq O(n^{2/3})$ as well as an energy-preserving unitary U such that

$$\left\|\mathrm{Tr}_{\mathrm{anc}}\left[U(\rho^{\otimes n} \otimes \eta)U^{\dagger}\right] - \sigma^{\otimes n}\right\|_1 \overset{n\to\infty}{\longrightarrow} 0.$$

2. There exists an ancilla system of size $o(n)$ with states η and ν and Hamiltonian H_{anc} satisfying $\|H_{\mathrm{anc}}\| \leq o(n)$ as well as energy-preserving unitaries U and V such that

$$\left\|\mathrm{Tr}_{\mathrm{anc}}\left[U(\rho^{\otimes n} \otimes \eta)U^{\dagger}\right] - \mathrm{Tr}_{\mathrm{anc}}\left[V(\sigma^{\otimes n} \otimes \eta)V^{\dagger}\right]\right\|_1 \overset{n\to\infty}{\longrightarrow} 0.$$

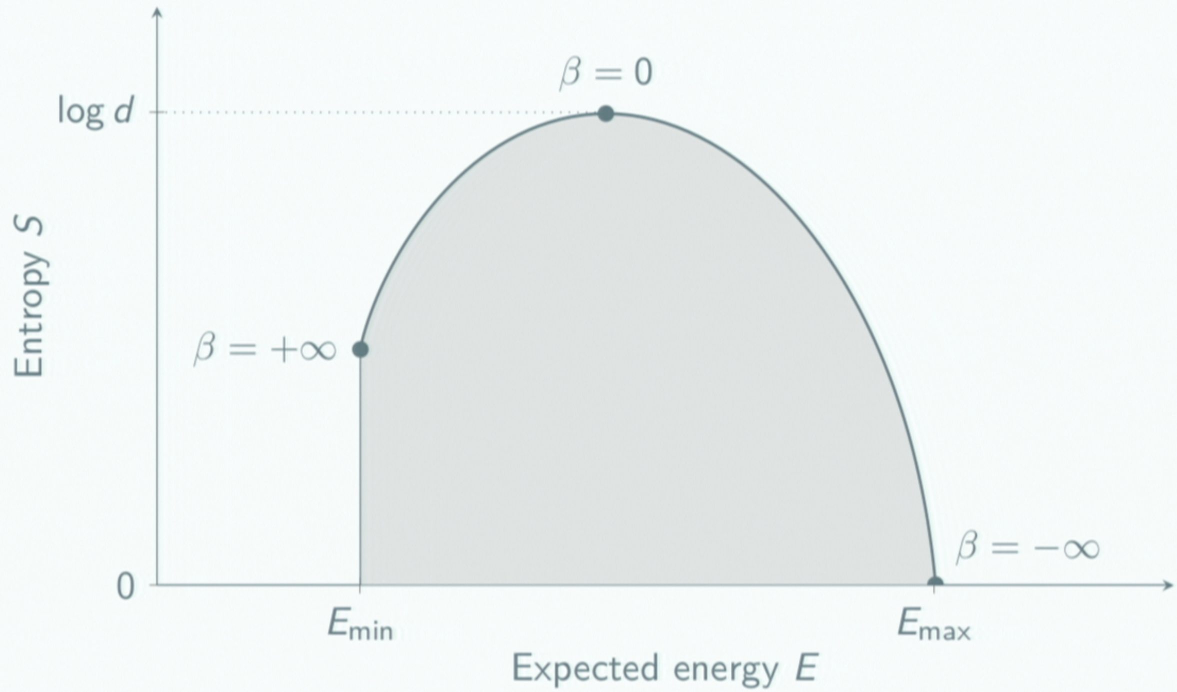
3. The states have equal energy and entropy,

$$E(\rho) = E(\sigma), \qquad S(\rho) = S(\sigma).$$

Definition

A **macrostate** is an equivalence class of states with respect to asymptotic interconvertibility as in the theorem.

$\Rightarrow$ Macrostates correspond to pairs (E, S) that can be jointly achieved. The set of macrostates makes up the **energy-entropy diagram**:



So *what is it all good for*? For example, let's determine how much work can be extracted out of many copies $\rho^{\otimes n}$ of a given state ρ.

Definition

Extraction of work is coupling the system to an **empty battery**,

$$\rho^{\otimes n} \otimes |E_1\rangle\langle E_1|^{\otimes \ell}, \tag{1}$$

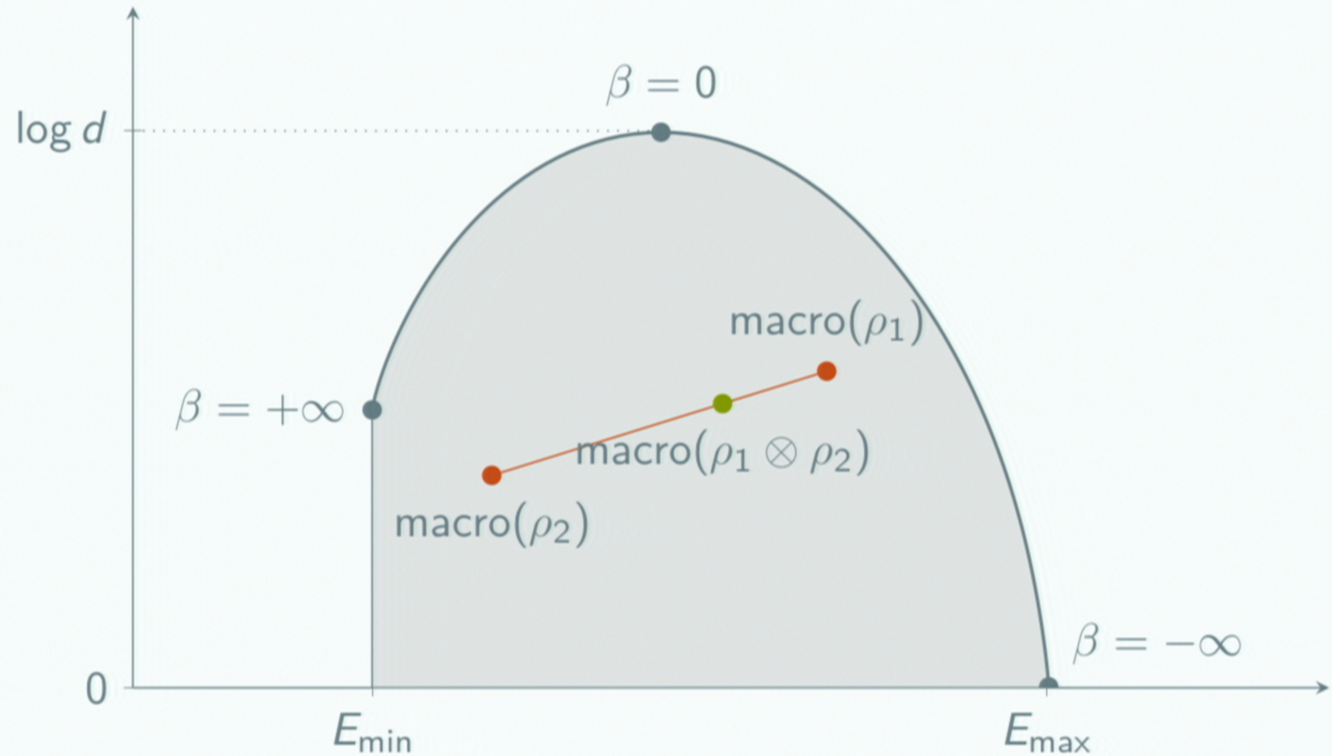
performing a thermodynamic transformation, and obtaining a final state of the form

$$\sigma^{\otimes n} \otimes |E_2\rangle\langle E_2|^{\otimes \ell} \tag{2}$$

with $E_2 > E_1$. The amount of work extracted is then

$$\ell \cdot (E_2 - E_1).$$

The maximal amount of work that can be extracted can now be easily read off geometrically from the energy-entropy diagram!


$\beta = 0$
$\log d$
$\text{macro}(\rho_1)$
$\beta = +\infty$
$\text{macro}(\rho_1 \otimes \rho_2)$
$\text{macro}(\rho_2)$
$\beta = -\infty$
0
E_{min}
E_{max}

We consider a system with finite-dimensional Hilbert space $\mathbb{C}^d$ and Hamiltonian H. A state ρ has energy $E(\rho) = \mathrm{tr}(\rho H)$ and entropy $S(\rho) = -\mathrm{tr}(\rho \log \rho)$.

Theorem

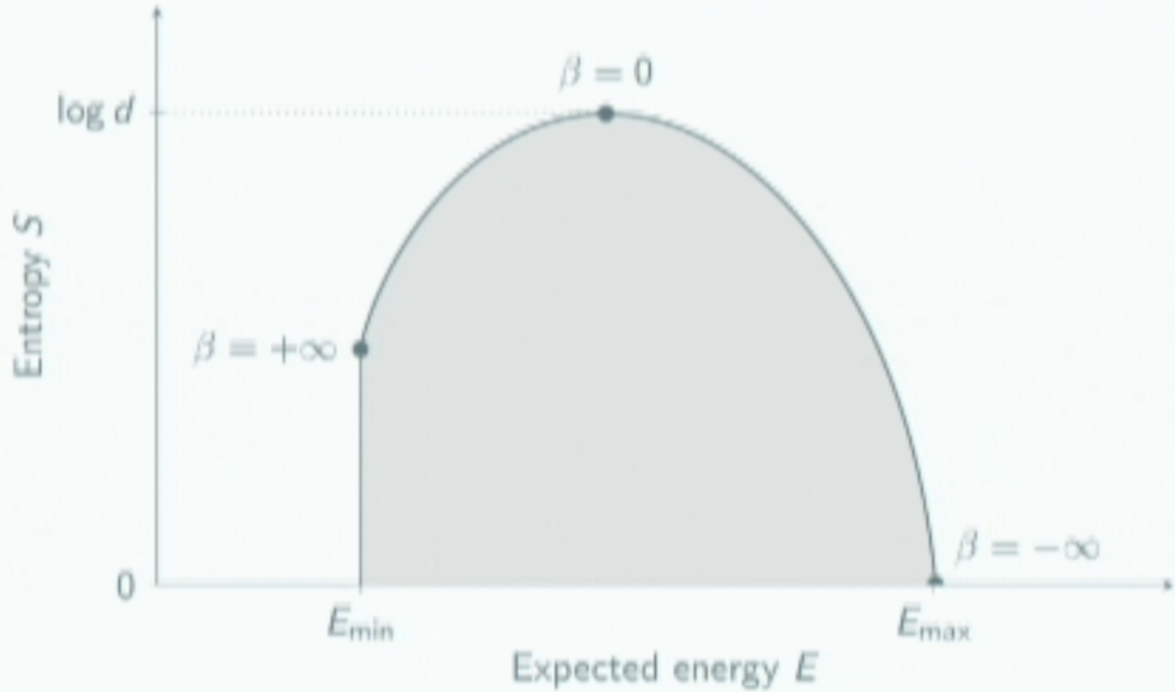
For states ρ and σ, the following are equivalent:

1. There exists an ancilla system of size $O(\sqrt{n \log n})$ with state η and Hamiltonian H_{anc} satisfying $\|H_{\mathrm{anc}}\| \leq O(n^{2/3})$ as well as an energy-preserving unitary U such that
$$\left\|\mathrm{Tr}_{\mathrm{anc}}\left[U(\rho^{\otimes n} \otimes \eta)U^\dagger\right] - \sigma^{\otimes n}\right\|_1 \overset{n\to\infty}{\longrightarrow} 0.$$
2. There exists an ancilla system of size $o(n)$ with states η and ν and Hamiltonian H_{anc} satisfying $\|H_{\mathrm{anc}}\| \leq o(n)$ as well as energy-preserving unitaries U and V such that
$$\left\|\mathrm{Tr}_{\mathrm{anc}}\left[U(\rho^{\otimes n} \otimes \eta)U^\dagger\right] - \mathrm{Tr}_{\mathrm{anc}}\left[V(\sigma^{\otimes n} \otimes \eta)V^\dagger\right]\right\|_1 \overset{n\to\infty}{\longrightarrow} 0.$$
3. The states have equal energy and entropy,
$$E(\rho) = E(\sigma), \qquad S(\rho) = S(\sigma).$$

Definition

A **macrostate** is an equivalence class of states with respect to asymptotic interconvertibility as in the theorem.

$\Rightarrow$ Macrostates correspond to pairs (E, S) that can be jointly achieved. The set of macrostates makes up the **energy-entropy diagram**:



Given a state ρ on N copies of the system $(\mathbb{C}^d)^{\otimes N}$, we renormalize energy and entropy for convenience,

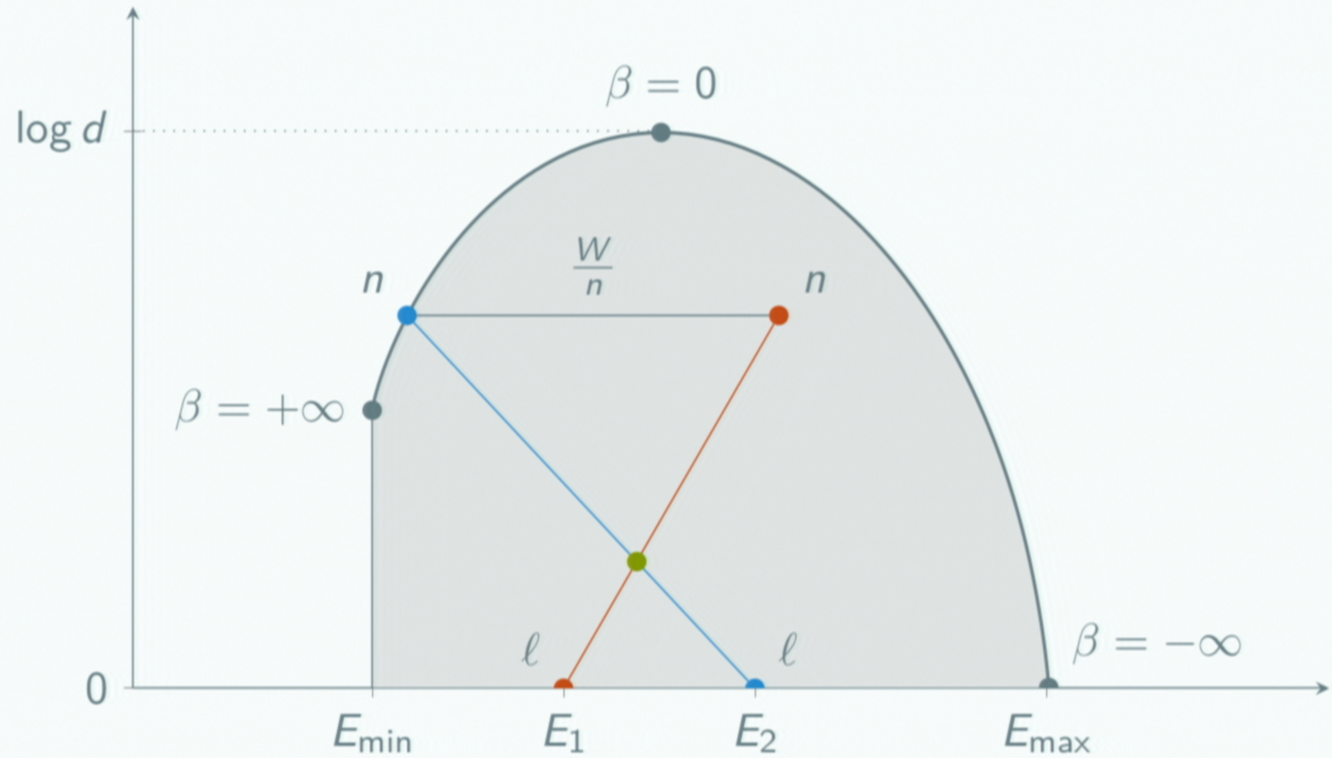
$$\text{macro}(\rho) := \left(\frac{E(\rho)}{N}, \frac{S(\rho)}{N}\right).$$

Like this, we can represent systems of any amount of substance in the energy-entropy diagram.

Now forming a total system out of ρ_1 on $(\mathbb{C}^d)^{\otimes N_1}$ and ρ_2 on $(\mathbb{C}^d)^{\otimes N_2}$ results in a **convex combination** of normalized macrostates,

$$\text{macro}(\rho_1 \otimes \rho_2) = \frac{N_1}{N_1 + N_2}\text{macro}(\rho_1) + \frac{N_2}{N_1 + N_2}\text{macro}(\rho_2).$$

The maximal extracted work per copy is $\frac{W}{n}$, given by the horizontal distance to the boundary:



A similar analysis applies to the case of a heat engine. Let's take the initial state to be

$$\tau_{\beta_{\mathrm{cold}}}^{\otimes n} \otimes \tau_{\beta_{\mathrm{hot}}}^{\otimes m} \otimes |E_1\rangle\langle E_1|^{\otimes \ell},$$

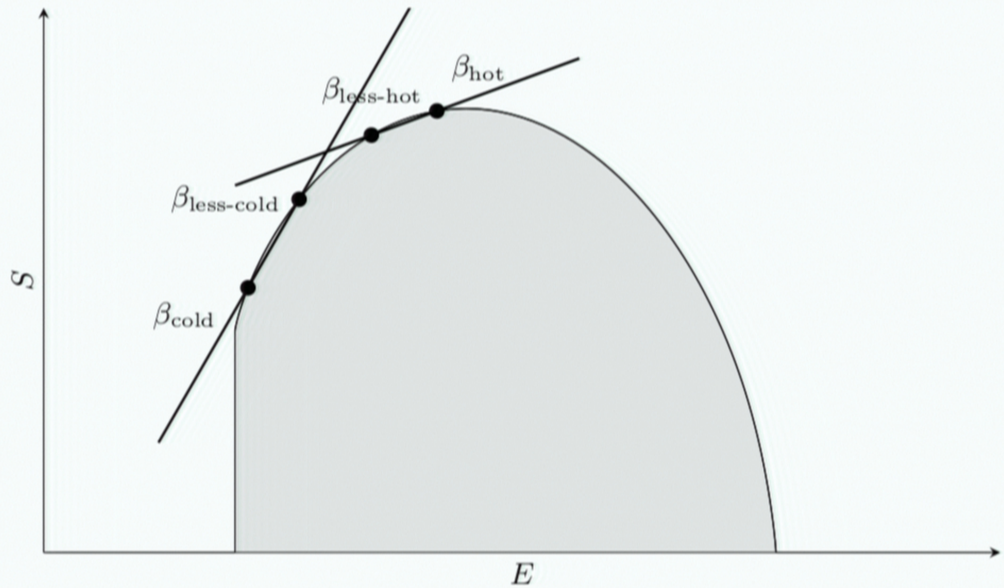
and by symmetry the final state

$$\tau_{\beta_{\mathrm{less\text{-}cold}}}^{\otimes n} \otimes \tau_{\beta_{\mathrm{less\text{-}hot}}}^{\otimes m} \otimes |E_2\rangle\langle E_2|^{\otimes \ell},$$

$\Rightarrow$ Determine $\beta_{\mathrm{less\text{-}cold}}$ and $\beta_{\mathrm{less\text{-}hot}}$ such that the final macrostate coincides with the initial macrostate.

- This model of a heat engine *abstracts away* from concepts of "working body" or "cycle".
- Instead, we only need to consider the initial and final states!
- There exists **some** protocol transforming one into the other if and only if these states define the same macrostate.

Let $\beta_{\text{eff-cold}}$ and $\beta_{\text{eff-hot}}$ correspond to the slopes in



Then a straightforward computation determines the efficiency to be

$$\eta = 1 - \frac{\beta_{\text{eff-hot}}}{\beta_{\text{eff-cold}}}.$$

For very small battery $\ell \ll n, m$, this approaches the Carnot efficiency!

Interpreting the Geometry of a Quantum State of Spacetime

Barak Shoshany
(with Laurent Freidel and Florian Girelli)

Perimeter Institute, Waterloo, Ontario, Canada

PI Day, March 14, 2018

About Me...

- Grew up in Israel, BSc Tel Aviv University, MSc PSI program @ PI

About Me...

- Grew up in Israel, BSc Tel Aviv University, MSc PSI program @ PI
- Currently PhD student at PI, supervisor: Laurent Freidel.
- Answered more than 1000 math and physics questions on Quora (Top Writer 2015 and 2016).
- Mentored high school students in PI's ISSYP program for the last 3 years.
- Many other outreach activities at PI.
- Hobbies: Board games and role-playing games, playing and composing music.

About Me...

- Grew up in Israel, BSc Tel Aviv University, MSc PSI program @ PI
- Currently PhD student at PI, supervisor: Laurent Freidel.
- Answered more than 1000 math and physics questions on Quora (Top Writer 2015 and 2016).
- Mentored high school students in PI's ISSYP program for the last 3 years.
- Many other outreach activities at PI.
- Hobbies: Board games and role-playing games, playing and composing music.

About Me...

- Grew up in Israel, BSc Tel Aviv University, MSc PSI program @ PI
- Currently PhD student at PI, supervisor: Laurent Freidel.
- Answered more than 1000 math and physics questions on Quora (Top Writer 2015 and 2016).
- Mentored high school students in PI's ISSYP program for the last 3 years.
- Many other outreach activities at PI.
- Hobbies: Board games and role-playing games, playing and composing music.
- I've been running a game of Dungeons and Dragons at PI for the last 2 years (let me know if you want to join!)

Yang-Mills Gauge Theory

- Yang-Mills action, choose gauge group = SU (2):

$$S = \int \mathrm{Tr}\left(\mathbf{F} \wedge \star\mathbf{F}\right) = \frac{1}{2}\int F_i^{\mu\nu} F^i_{\mu\nu}\, \mathrm{d}^4 x,$$

Yang-Mills Gauge Theory

- Yang-Mills action, choose gauge group = SU (2):

$$S = \int \mathrm{Tr}\,(\mathbf{F} \wedge \star\mathbf{F}) = \frac{1}{2}\int F_i^{\mu\nu} F^i_{\mu\nu}\,\mathrm{d}^4 x,$$

  - $\mathbf{A} = \mathfrak{su}\,(2)$-valued connection 1-form.

Yang-Mills Gauge Theory

- Yang-Mills action, choose gauge group = SU (2):

$$S = \int \mathrm{Tr}\left(\mathbf{F} \wedge \star\mathbf{F}\right) = \frac{1}{2}\int F_i^{\mu\nu} F_{\mu\nu}^i \, \mathrm{d}^4 x,$$

  - $\mathbf{A} = \mathfrak{su}\,(2)$-valued connection 1-form.
  - $\mathbf{F} \equiv \mathrm{d}\mathbf{A} + \frac{1}{2}\left[\mathbf{A}, \mathbf{A}\right] = \mathfrak{su}\,(2)$-valued curvature 2-form.

Yang-Mills Gauge Theory

- Yang-Mills action, choose gauge group = SU (2):

$$S = \int \mathrm{Tr}\,(\mathbf{F} \wedge \star\mathbf{F}) = \frac{1}{2}\int F_i^{\mu\nu} F^i_{\mu\nu}\, \mathrm{d}^4 x,$$

  - $\mathbf{A}$ = $\mathfrak{su}\,(2)$-valued connection 1-form.
  - $\mathbf{F} \equiv \mathrm{d}\mathbf{A} + \frac{1}{2}[\mathbf{A}, \mathbf{A}]$ = $\mathfrak{su}\,(2)$-valued curvature 2-form.
  - **Bold** = $\mathfrak{su}\,(2)$-valued quantities throughout!

Yang-Mills Gauge Theory

- Yang-Mills action, choose gauge group = SU (2):

$$S = \int \mathrm{Tr}\left(\mathbf{F} \wedge \star\mathbf{F}\right) = \frac{1}{2}\int F_i^{\mu\nu} F_{\mu\nu}^i \, \mathrm{d}^4 x,$$

  - $\mathbf{A} = \mathfrak{su}(2)$-valued connection 1-form.
  - $\mathbf{F} \equiv \mathrm{d}\mathbf{A} + \frac{1}{2}[\mathbf{A}, \mathbf{A}] = \mathfrak{su}(2)$-valued curvature 2-form.
  - **Bold** = $\mathfrak{su}(2)$-valued quantities throughout!
- Invariant under SU (2) gauge transformation:

$$g \in \mathrm{SU}(2) \quad \Longrightarrow \quad \mathbf{A} \mapsto g^{-1}\mathbf{A}g + g^{-1}\mathrm{d}g \quad \Longrightarrow \quad \mathbf{F} \mapsto g^{-1}\mathbf{F}g.$$

Yang-Mills Gauge Theory

- Hamiltonian formulation (3+1 split, Σ = spatial slice):

$$S = \int \mathrm{d}t \int_\Sigma \left(\mathbf{E} \cdot \partial_t \mathbf{A} + \boldsymbol{\lambda} \cdot \mathbf{G} - \frac{1}{2} \left(\mathbf{E}^2 + (\star \mathbf{F})^2 \right) \right) .$$

Yang-Mills Gauge Theory

- Hamiltonian formulation (3+1 split, Σ = spatial slice):

$$S = \int \mathrm{d}t \int_{\Sigma} \left(\mathbf{E} \cdot \partial_t \mathbf{A} + \boldsymbol{\lambda} \cdot \mathbf{G} - \frac{1}{2} \left(\mathbf{E}^2 + (\star \mathbf{F})^2 \right) \right) .$$

  - First term: $\mathbf{A}$ = configuration variable, $\mathbf{E}$ = conjugate momentum ($E_i^a \equiv F_i^{0a}$).

Yang-Mills Gauge Theory

- Hamiltonian formulation (3+1 split, Σ = spatial slice):

$$S=\int \mathrm{d}t\int_{\Sigma}\left(\mathbf{E}\cdot\partial_t\mathbf{A}+\boldsymbol{\lambda}\cdot\mathbf{G}-\frac{1}{2}\left(\mathbf{E}^2+(\star\mathbf{F})^2\right)\right).$$

  - First term: $\mathbf{A}$ = configuration variable, $\mathbf{E}$ = conjugate momentum ($E_i^a\equiv F_i^{0a}$).
  - Second term: $\boldsymbol{\lambda}$ is a Lagrange multiplier imposing the Gauss constraint $\mathbf{G}\equiv \mathrm{d}_A\mathbf{E}=0$, which generates infinitesimal $\mathrm{SU}(2)$ gauge transformations:

$$\{\mathbf{A},\mathbf{G}(\boldsymbol{\alpha})\}=\mathrm{d}_A\boldsymbol{\alpha},\qquad g\equiv \mathrm{e}^{\boldsymbol{\alpha}},\qquad \boldsymbol{\alpha}\in\mathfrak{su}(2).$$

Yang-Mills Gauge Theory

- Hamiltonian formulation (3+1 split, Σ = spatial slice):

$$S=\int \mathrm{d}t\int_{\Sigma}\left(\mathbf{E}\cdot\partial_t\mathbf{A}+\boldsymbol{\lambda}\cdot\mathbf{G}-\frac{1}{2}\left(\mathbf{E}^2+(\star\mathbf{F})^2\right)\right).$$

  - First term: $\mathbf{A}$ = configuration variable, $\mathbf{E}$ = conjugate momentum ($E_i^a\equiv F_i^{0a}$).
  - Second term: $\boldsymbol{\lambda}$ is a Lagrange multiplier imposing the Gauss constraint $\mathbf{G}\equiv\mathrm{d}_A\mathbf{E}=0$, which generates infinitesimal $\mathrm{SU}(2)$ gauge transformations:

$$\{\mathbf{A},\mathbf{G}(\boldsymbol{\alpha})\}=\mathrm{d}_A\boldsymbol{\alpha},\qquad g\equiv\mathrm{e}^{\boldsymbol{\alpha}},\qquad \boldsymbol{\alpha}\in\mathfrak{su}(2).$$

  - Third term: energy density.

Holonomies and Wilson Loops

- Holonomy = parallel transport along a curve γ:

$$h_\gamma \equiv \overrightarrow{\exp}\left(\int_\gamma \mathbf{A}\right).$$

$\overrightarrow{\exp}$ = path-ordered exponential.

Holonomies and Wilson Loops

- Holonomy = parallel transport along a curve γ:

$$h_\gamma \equiv \overrightarrow{\exp}\left(\int_\gamma \mathbf{A}\right).$$

  $\overrightarrow{\exp}$ = path-ordered exponential.
- Holonomy around an infinitesimal loop measures the curvature at a point.

$$h_\gamma = \overrightarrow{\exp}\left(\oint_\gamma \mathbf{A}\right) \approx 1 + \varepsilon^2 \mathbf{F}.$$

Loop Gravity

- Hamiltonian formulation of GR (3+1 split), change variables to Ashtekar variables:

$$S = \int \mathrm{d}t \int_{\Sigma} \left(\mathbf{E} \cdot \partial_t \mathbf{A} + \boldsymbol{\lambda} \cdot \mathbf{G} + \vec{N} \cdot \vec{V} + NC \right).$$

Loop Gravity

- Hamiltonian formulation of GR (3+1 split), change variables to Ashtekar variables:

$$S = \int \mathrm{d}t \int_{\Sigma} \left(\mathbf{E} \cdot \partial_t \mathbf{A} + \boldsymbol{\lambda} \cdot \mathbf{G} + \vec{N} \cdot \vec{V} + NC \right).$$

  - First term: $\mathbf{A}$ = configuration variable, $\mathbf{E}$ = conjugate momentum. ($\mathbf{E} \equiv [\mathbf{e}, \mathbf{e}]$ where $\mathbf{e}$ is the frame field, $g_{ab} \equiv \mathbf{e}_a \cdot \mathbf{e}_b$.)

Loop Gravity

- Hamiltonian formulation of GR (3+1 split), change variables to Ashtekar variables:

$$S = \int \mathrm{d}t \int_{\Sigma} \left(\mathbf{E} \cdot \partial_t \mathbf{A} + \boldsymbol{\lambda} \cdot \mathbf{G} + \vec{N} \cdot \vec{V} + NC \right).$$

  - First term: **A** = configuration variable, **E** = conjugate momentum. ($\mathbf{E} \equiv [\mathbf{e}, \mathbf{e}]$ where **e** is the frame field, $g_{ab} \equiv \mathbf{e}_a \cdot \mathbf{e}_b$.)
  - **A** and **E** are valued in $\mathfrak{su}(2)$ since we started with $\mathfrak{so}(3,1)$ (Lorentz) in 3+1D, so on the 3D spatial slice Σ it reduces to $\mathfrak{so}(3) \cong \mathfrak{su}(2)$.

Loop Gravity

- Hamiltonian formulation of GR (3+1 split), change variables to Ashtekar variables:

$$S = \int \mathrm{d}t \int_{\Sigma} \left(\mathbf{E} \cdot \partial_t \mathbf{A} + \boldsymbol{\lambda} \cdot \mathbf{G} + \vec{N} \cdot \vec{V} + NC \right).$$

  - First term: $\mathbf{A}$ = configuration variable, $\mathbf{E}$ = conjugate momentum. ($\mathbf{E} \equiv [\mathbf{e}, \mathbf{e}]$ where $\mathbf{e}$ is the frame field, $g_{ab} \equiv \mathbf{e}_a \cdot \mathbf{e}_b$.)
  - $\mathbf{A}$ and $\mathbf{E}$ are valued in $\mathfrak{su}(2)$ since we started with $\mathfrak{so}(3,1)$ (Lorentz) in 3+1D, so on the 3D spatial slice Σ it reduces to $\mathfrak{so}(3) \cong \mathfrak{su}(2)$.
  - Second term: λ is a Lagrange multiplier imposing the Gauss constraint $\mathbf{G} \equiv \mathrm{d}_A \mathbf{E} = 0$.

Loop Gravity

- Hamiltonian formulation of GR (3+1 split), change variables to Ashtekar variables:

$$S = \int \mathrm{d}t \int_{\Sigma} \left(\mathbf{E} \cdot \partial_t \mathbf{A} + \boldsymbol{\lambda} \cdot \mathbf{G} + \vec{N} \cdot \vec{V} + NC \right).$$

  - First term: $\mathbf{A}$ = configuration variable, $\mathbf{E}$ = conjugate momentum. ($\mathbf{E} \equiv [\mathbf{e}, \mathbf{e}]$ where $\mathbf{e}$ is the frame field, $g_{ab} \equiv \mathbf{e}_a \cdot \mathbf{e}_b$.)
  - $\mathbf{A}$ and $\mathbf{E}$ are valued in $\mathfrak{su}(2)$ since we started with $\mathfrak{so}(3,1)$ (Lorentz) in 3+1D, so on the 3D spatial slice Σ it reduces to $\mathfrak{so}(3) \cong \mathfrak{su}(2)$.
  - Second term: λ is a Lagrange multiplier imposing the Gauss constraint $\mathbf{G} \equiv \mathrm{d}_A \mathbf{E} = 0$.
  - Third term: the shift $\vec{N}$ imposes the vector constraint $\vec{V} \equiv \mathbf{E} \lrcorner \mathbf{F}$, which generates infinitesimal spatial diffeomorphisms.

Loop Gravity

- Hamiltonian formulation of GR (3+1 split), change variables to Ashtekar variables:

$$S = \int \mathrm{d}t \int_{\Sigma} \left(\mathbf{E} \cdot \partial_t \mathbf{A} + \boldsymbol{\lambda} \cdot \mathbf{G} + \vec{N} \cdot \vec{V} + NC \right).$$

  - First term: $\mathbf{A}$ = configuration variable, $\mathbf{E}$ = conjugate momentum. ($\mathbf{E} \equiv [\mathbf{e}, \mathbf{e}]$ where $\mathbf{e}$ is the frame field, $g_{ab} \equiv \mathbf{e}_a \cdot \mathbf{e}_b$.)
  - $\mathbf{A}$ and $\mathbf{E}$ are valued in $\mathfrak{su}(2)$ since we started with $\mathfrak{so}(3,1)$ (Lorentz) in 3+1D, so on the 3D spatial slice Σ it reduces to $\mathfrak{so}(3) \cong \mathfrak{su}(2)$.
  - Second term: λ is a Lagrange multiplier imposing the Gauss constraint $\mathbf{G} \equiv \mathrm{d}_A \mathbf{E} = 0$.
  - Third term: the shift $\vec{N}$ imposes the vector constraint $\vec{V} \equiv \mathbf{E} \lrcorner \mathbf{F}$, which generates infinitesimal spatial diffeomorphisms.
  - Fourth term: the lapse N imposes the scalar constraint C, which generates time evolution.

Loop Gravity

$$S = \int \mathrm{d}t \int \left(\mathbf{E} \cdot \partial_t \mathbf{A} + \boldsymbol{\lambda} \cdot \mathbf{G} + \vec{N} \cdot \vec{V} + NC \right).$$

- Gravity now looks like Yang-Mills theory, with two additional constraints and no energy density (totally constrained system).

Loop Gravity

$$S = \int \mathrm{d}t \int \left(\mathbf{E} \cdot \partial_t \mathbf{A} + \boldsymbol{\lambda} \cdot \mathbf{G} + \vec{N} \cdot \vec{V} + NC \right).$$

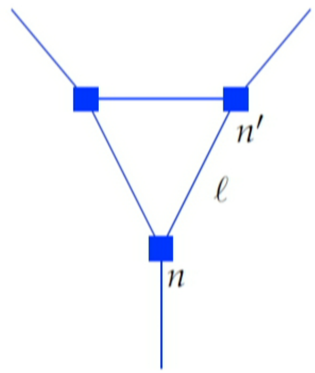
- Gravity now looks like Yang-Mills theory, with two additional constraints and no energy density (totally constrained system).

Loop Gravity

$$S = \int \mathrm{d}t \int \left(\mathbf{E} \cdot \partial_t \mathbf{A} + \boldsymbol{\lambda} \cdot \mathbf{G} + \vec{N} \cdot \vec{V} + NC \right).$$

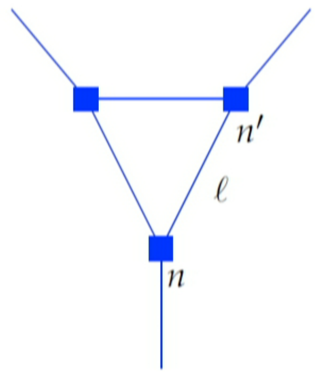
- Gravity now looks like Yang-Mills theory, with two additional constraints and no energy density (totally constrained system).
- Now we can quantize gravity similarly to how we quantize Yang-Mills theory. In particular, the theory is background independent: we quantize the full geometry, not just perturbations over a flat background.

Spin Networks



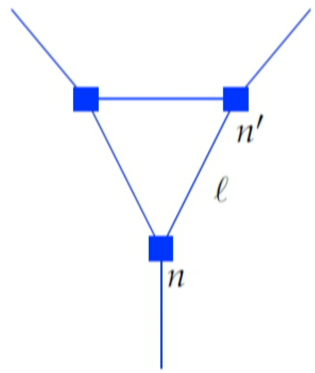
- Classical spin networks are graphs composed of nodes n connected by links $\ell = (nn')$.

Spin Networks



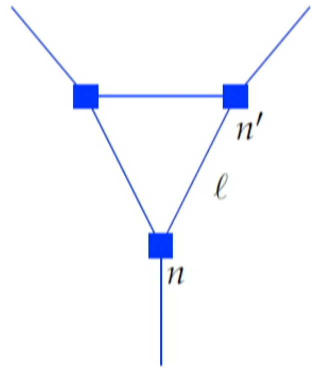
- Classical spin networks are graphs composed of nodes n connected by links $\ell = (nn')$.
- There is a holonomy $h_\ell \in \mathrm{SU}(2)$ and a flux $\mathbf{X}_\ell \in \mathfrak{su}(2)$ associated to each link.

Spin Networks



- Classical spin networks are graphs composed of nodes n connected by links $\ell = (nn')$.
- There is a holonomy $h_\ell \in \mathrm{SU}(2)$ and a flux $\mathbf{X}_\ell \in \mathfrak{su}(2)$ associated to each link.
- The Gauss constraint implies, for each node, $\sum \mathbf{X}_\ell = 0$ where the sum is over the links meeting at the node.

Spin Networks

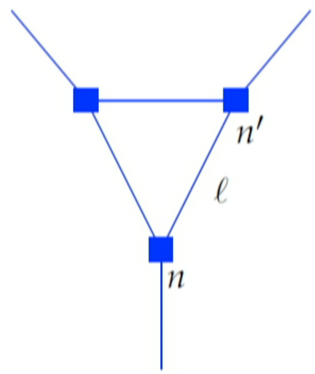


- The phase space structure is given by the following symplectic potential, describing a product of $T^*\mathrm{SU}(2)$ cotangent bundles per link:

$$\Theta = \sum_{\ell \in \Gamma} \mathbf{\Delta}(h_\ell) \cdot \mathbf{X}_\ell, \qquad \mathbf{\Delta}(h) \equiv \delta h h^{-1} \in \mathfrak{su}(2).$$

The symplectic form is given by $\Omega \equiv \delta\Theta$.

Spin Networks



- When quantizing loop gravity, we find that the quantum states of space are quantum spin networks. We would like to understand the geometry described by such a spin network.
- In the usual approach, loop gravity is both discretized and quantized in one step. Our approach is to disentangle the two. First we deal with the discretization classically, and see what we can learn from it. Later, we can proceed to quantization.

A Toy Model: 2+1 Gravity

- *BF* action:

$$S = \int \mathbf{e} \cdot \mathbf{F}(\mathbf{A}) .$$

A Toy Model: 2+1 Gravity

- *BF* action:

$$S = \int \mathbf{e} \cdot \mathbf{F}(\mathbf{A}) \, .$$

- Symplectic potential:

$$\Theta = \int_{\Sigma} \mathbf{e} \cdot \delta \mathbf{A}.$$

A and **e** are the phase space variables.

A Toy Model: 2+1 Gravity

- *BF* action:

$$S = \int \mathbf{e} \cdot \mathbf{F}(\mathbf{A}) \, .$$

- Symplectic potential:

$$\Theta = \int_{\Sigma} \mathbf{e} \cdot \delta \mathbf{A}.$$

A and **e** are the phase space variables.

- Equations of motions: no curvature or torsion.

$$\mathbf{F} \equiv \mathrm{d}_A \mathbf{A} = \mathrm{d}\mathbf{A} + \frac{1}{2}[\mathbf{A}, \mathbf{A}] = 0,$$

$$\mathbf{T} \equiv \mathrm{d}_A \mathbf{e} = \mathrm{d}\mathbf{e} + [\mathbf{A}, \mathbf{e}] = 0.$$

Curvature and Torsion Defects

- Since $\mathbf{F} = \mathbf{T} = 0$ everywhere, we introduce curvature and torsion as topological defects. At a point v, we take:

$$\mathbf{F} \equiv \mathbf{M}_v \delta(v), \qquad \mathbf{T} \equiv \mathbf{S}_v \delta(v).$$

$\mathbf{M}_v$ is the mass and $\mathbf{S}_v$ is the spin of a point particle at v.

Curvature and Torsion Defects

- Since $\mathbf{F} = \mathbf{T} = 0$ everywhere, we introduce curvature and torsion as topological defects. At a point v, we take:

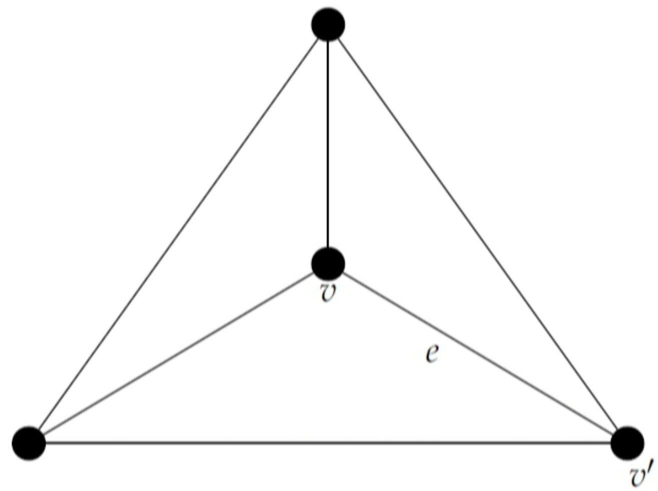
$$\mathbf{F} \equiv \mathbf{M}_v \delta(v), \qquad \mathbf{T} \equiv \mathbf{S}_v \delta(v).$$

  $\mathbf{M}_v$ is the mass and $\mathbf{S}_v$ is the spin of a point particle at v.

- Let v^* be a disk around v and let ∂v^* be its boundary, then by Stokes' theorem

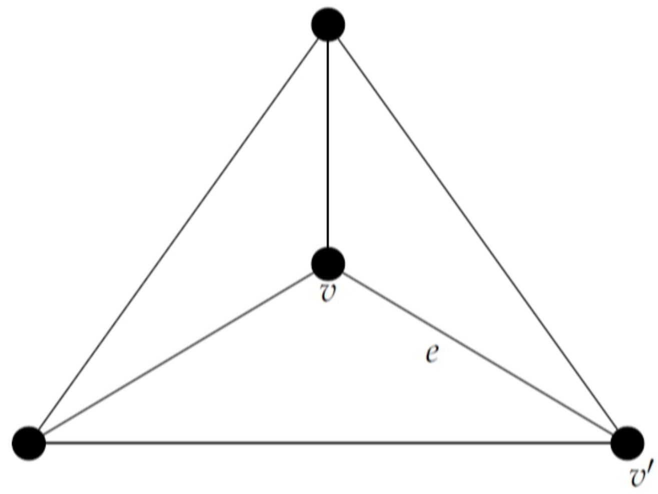
$$h_{\partial v^*} = \overrightarrow{\exp}\left(\oint_{\partial v^*} \mathbf{A}\right) = \overrightarrow{\exp}\left(\int_{v^*} \mathbf{F}\right) = \exp(\mathbf{M}_v).$$

The Cellular Decomposition



- We sprinkle some point-like defects at various points v, v', etc. and then connect them by edges $e \equiv (vv')$ as shown.

The Cellular Decomposition



- We sprinkle some point-like defects at various points v, v', etc. and then connect them by edges $e \equiv (vv')$ as shown.
- This splits the spatial manifold into cells.
- By construction, $\mathbf{F} = \mathbf{T} = 0$ inside the cells and the curvature and torsion are located only at the vertices v. We call this a piecewise-flat geometry.

Winding Holonomies

- At each vertex, we define a connection and a frame field:

$$\mathbf{A} \equiv \frac{\mathbf{M}_v}{2\pi} \mathrm{d}\phi, \qquad \mathbf{e} \equiv \frac{\mathbf{S}_v}{2\pi} \mathrm{d}\phi$$

  Since $\mathrm{d}^2\phi = 2\pi\delta(v)$, we get the desired distributional curvature and torsion: $\mathbf{F} = \mathbf{M}_v \delta(v)$ and $\mathbf{T} = \mathbf{S}_v \delta(v)$.

Winding Holonomies

- At each vertex, we define a connection and a frame field:

$$\mathbf{A} \equiv \frac{\mathbf{M}_v}{2\pi} \mathrm{d}\phi, \qquad \mathbf{e} \equiv \frac{\mathbf{S}_v}{2\pi} \mathrm{d}\phi$$

Since $\mathrm{d}^2\phi = 2\pi\delta(v)$, we get the desired distributional curvature and torsion: $\mathbf{F} = \mathbf{M}_v \delta(v)$ and $\mathbf{T} = \mathbf{S}_v \delta(v)$.

Winding Holonomies

- At each vertex, we define a connection and a frame field:

$$\mathbf{A} \equiv \frac{\mathbf{M}_v}{2\pi}\mathrm{d}\phi, \qquad \mathbf{e} \equiv \frac{\mathbf{S}_v}{2\pi}\mathrm{d}\phi$$

  Since $\mathrm{d}^2\phi = 2\pi\delta(v)$, we get the desired distributional curvature and torsion: $\mathbf{F} = \mathbf{M}_v\delta(v)$ and $\mathbf{T} = \mathbf{S}_v\delta(v)$.

- Using this connection, we define a winding holonomy:

$$\psi_v^{\phi\phi'} \equiv \exp\left(\int_\phi^{\phi'} \mathbf{A}\right) = \exp\left(\frac{\mathbf{M}_v}{2\pi}(\phi' - \phi)\right) \in \mathrm{SU}(2),$$

  where ϕ and ϕ' are any two angles around v.

Winding Holonomies

- At each vertex, we define a connection and a frame field:

$$\mathbf{A} \equiv \frac{\mathbf{M}_v}{2\pi} \mathrm{d}\phi, \qquad \mathbf{e} \equiv \frac{\mathbf{S}_v}{2\pi} \mathrm{d}\phi$$

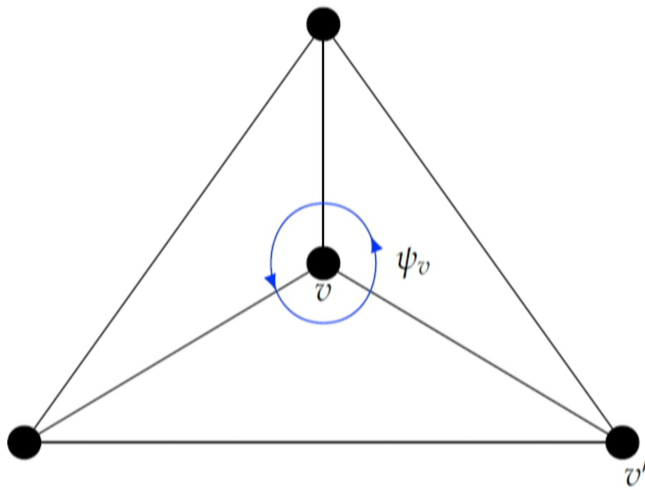
  Since $\mathrm{d}^2\phi = 2\pi\delta(v)$, we get the desired distributional curvature and torsion: $\mathbf{F} = \mathbf{M}_v \delta(v)$ and $\mathbf{T} = \mathbf{S}_v \delta(v)$.

- Using this connection, we define a winding holonomy:

$$\psi_v^{\phi\phi'} \equiv \exp\left(\int_\phi^{\phi'} \mathbf{A}\right) = \exp\left(\frac{\mathbf{M}_v}{2\pi}(\phi' - \phi)\right) \in \mathrm{SU}(2),$$

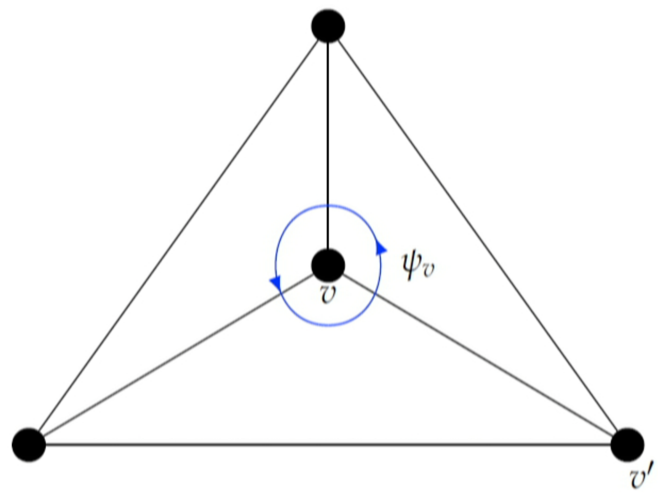
  where ϕ and ϕ' are any two angles around v.

Winding Holonomies



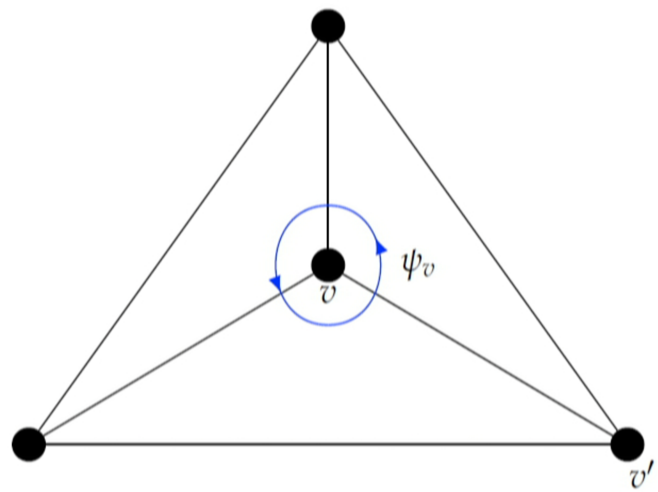
- Note that the holonomy is calculated at v itself, with the angles ϕ on an infinitesimal circle (enlarged in the figure for clarity).

Winding Holonomies



- Note that the holonomy is calculated at v itself, with the angles ϕ on an infinitesimal circle (enlarged in the figure for clarity).

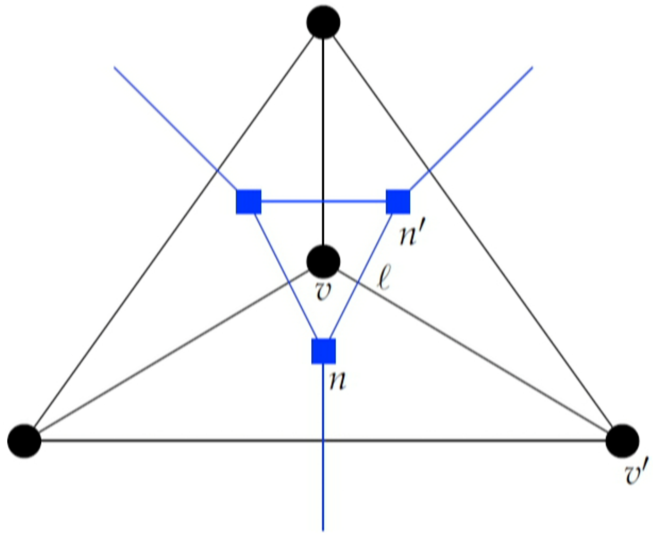
Winding Holonomies



- Note that the holonomy is calculated at v itself, with the angles ϕ on an infinitesimal circle (enlarged in the figure for clarity).
- The holonomy in a loop from some angle ϕ to itself depends on the number, s, of windings around v:

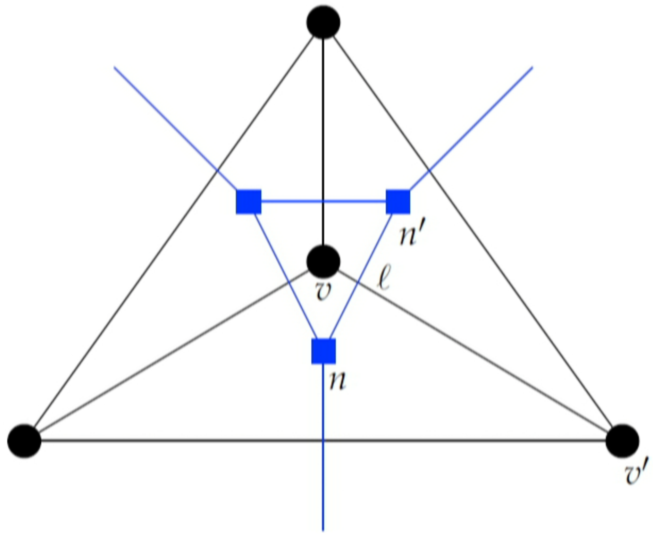
$$\phi' = \phi + 2\pi s \quad \Longrightarrow \quad \psi_v^{\phi\phi'} = \exp\left(\frac{\mathbf{M}_v}{2\pi}\left(\phi' - \phi\right)\right) = \exp\left(s\mathbf{M}_v\right).$$

The Dual Graph



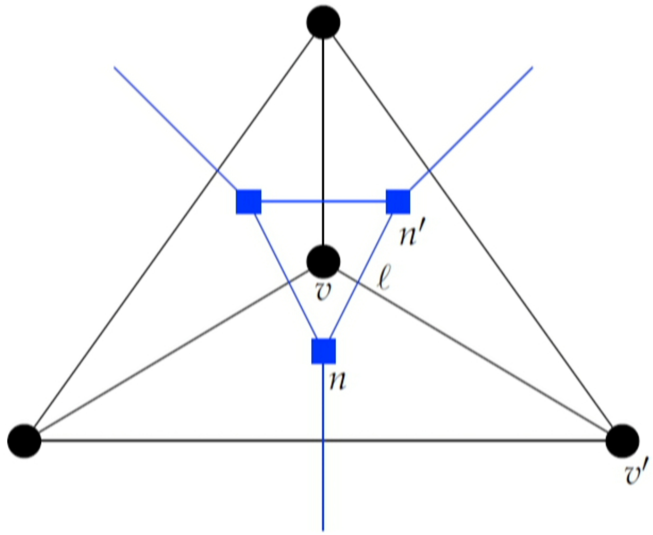
- We place a blue node n inside each cell and connect those nodes with blue links $\ell = (nn')$.

The Dual Graph



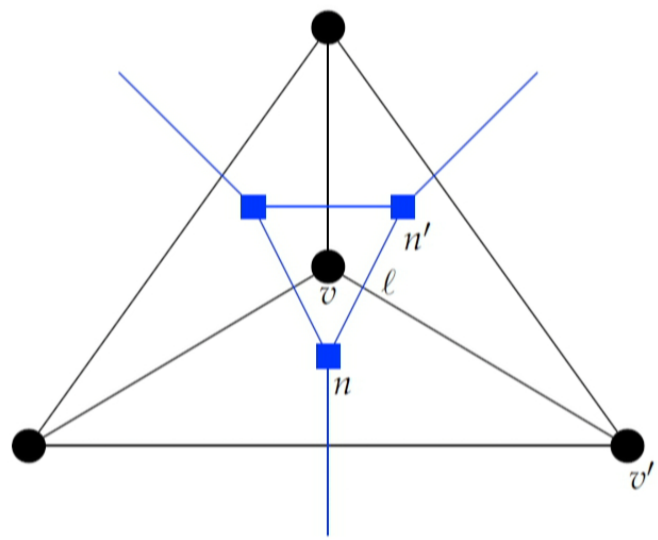
- We place a blue node n inside each cell and connect those nodes with blue links $\ell = (nn')$.
- There is a 1-to-1 correspondence: nodes⇔cells and links⇔edges.

The Dual Graph



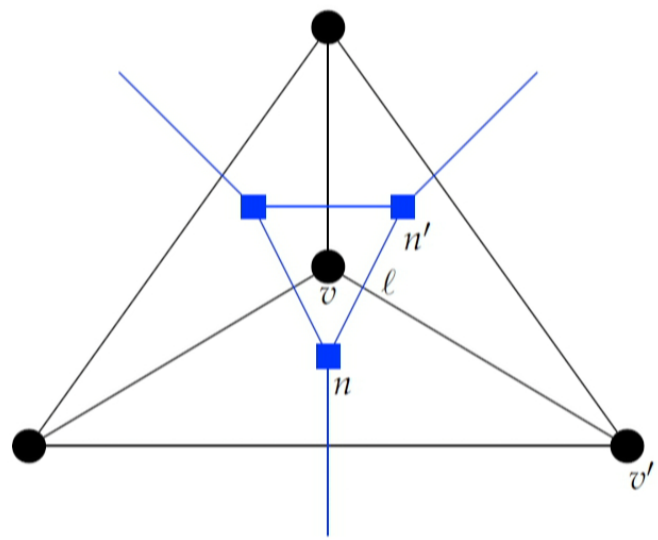
- We place a blue node n inside each cell and connect those nodes with blue links $\ell = (nn')$.
- There is a 1-to-1 correspondence: nodes⇔cells and links⇔edges.
- We define a holonomy $h_{nn'}$ on each link $\ell = (nn')$. The holonomy simply allows us to parallel-transport from one cell to the other.

The Dual Graph



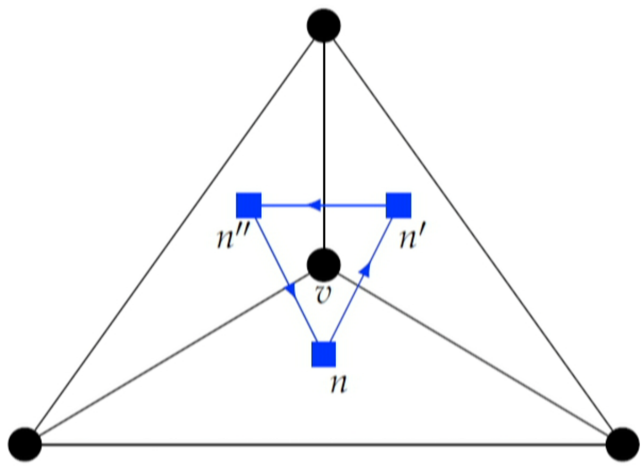
- The blue graph look suspiciously like a spin network. But does it have the same phase space structure?

The Dual Graph



- The blue graph look suspiciously like a spin network. But does it have the same phase space structure?
- Let us find the phase space structure of the piecewise-flat geometry, and see how it can be related to the phase space of spin networks.

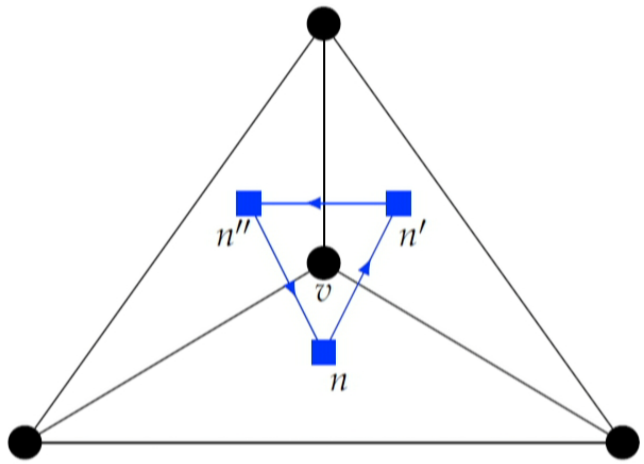
Probing Curvature Defects with Holonomies



- We would like to probe the curvature at v by taking a loop of holonomies along the blue links:

$$\mathcal{O}_v = h_{nn'} h_{n'n''} h_{n''n}.$$

Probing Curvature Defects with Holonomies

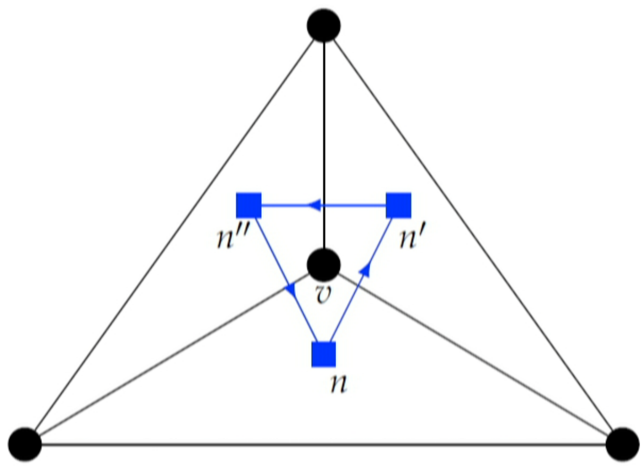


- We would like to probe the curvature at v by taking a loop of holonomies along the blue links:

$$\mathcal{O}_v = h_{nn'} h_{n'n''} h_{n''n}.$$

- How will these holonomies know about the curvature at v?

Probing Curvature Defects with Holonomies



- We would like to probe the curvature at v by taking a loop of holonomies along the blue links:

$$\mathcal{O}_v = h_{nn'} h_{n'n''} h_{n''n}.$$

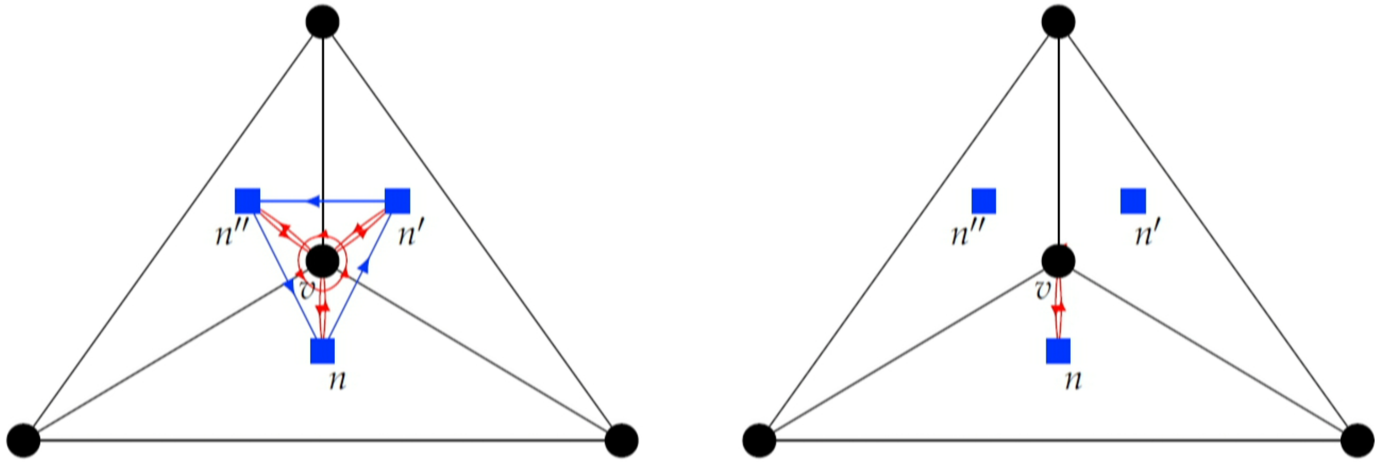
- How will these holonomies know about the curvature at v?
- We deform them such that they take into account the winding holonomies around v.

Deforming the Links

$$h_{nn'} = h_{nv}\psi_v^{nn'}h_{vn'}$$

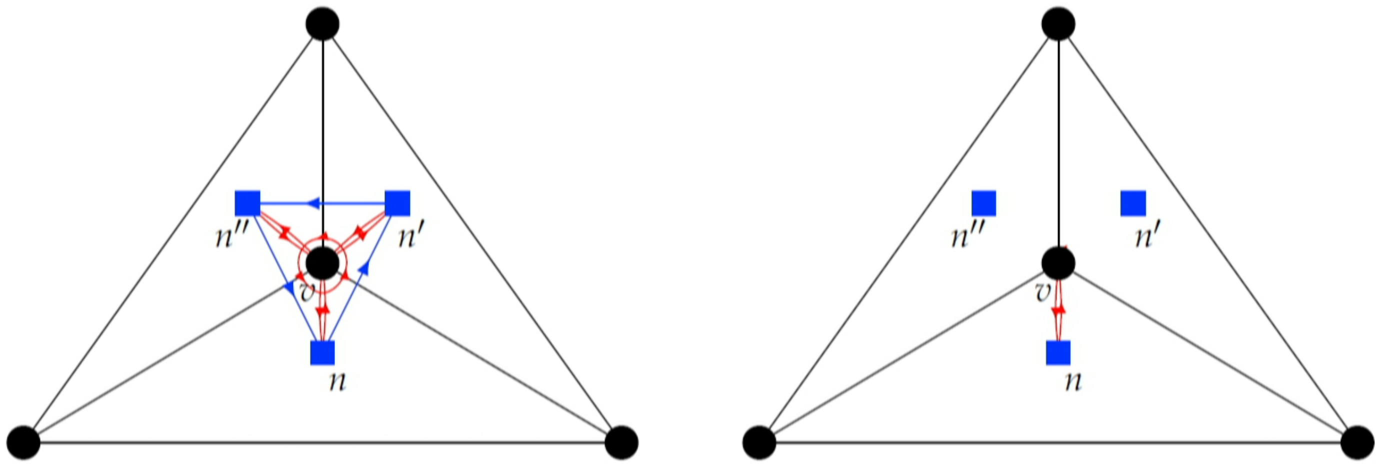
- h_{nv} is the holonomy from n to v,

Deforming the Links



- We deform every one of the links in the loop in this way.

Deforming the Loops



- If there is curvature at v, then we get:

$$
\begin{aligned}
\mathcal{O}_v &= h_{nn'} h_{n'n''} h_{n''n} \\
&= \left(h_{nv} \psi_v^{nn'} h_{vn'}\right) \left(h_{n'v} \psi_v^{n'n''} h_{vn''}\right) \left(h_{n''v} \psi_v^{n''n} h_{vn}\right) \\
&= h_{nv} \left(\psi_v^{nn'} \psi_v^{n'n''} \psi_v^{n''n}\right) h_{vn} \\
&= h_{nv}\, \mathrm{e}^{\mathbf{M}_v}\, h_{vn}.
\end{aligned}
$$

Translational Holonomies

- So far, we have only discussed holonomies $h \in \mathrm{SU}(2)$. These holonomies rotate group and algebra elements, and thus we call them rotational holonomies.

Translational Holonomies

- So far, we have only discussed holonomies $h \in \mathrm{SU}(2)$. These holonomies rotate group and algebra elements, and thus we call them rotational holonomies.
- We also define translational holonomies $\mathbf{w} \in \mathfrak{su}(2)$, which translate algebra elements. Thus, for example, for a group element g_n and algebra element $\mathbf{z}_n$, we have the relations:

$$g_n = h_{nv} g_v, \qquad \mathbf{z}_n = h_{nv} \left(\mathbf{z}_v - \mathbf{w}_v^n \right) h_{vn}.$$

Translational Holonomies

- So far, we have only discussed holonomies $h \in \mathrm{SU}(2)$. These holonomies rotate group and algebra elements, and thus we call them rotational holonomies.
- We also define translational holonomies $\mathbf{w} \in \mathfrak{su}(2)$, which translate algebra elements. Thus, for example, for a group element g_n and algebra element $\mathbf{z}_n$, we have the relations:

$$g_n = h_{nv} g_v, \qquad \mathbf{z}_n = h_{nv} \left(\mathbf{z}_v - \mathbf{w}_v^n \right) h_{vn}.$$

- The group element g_v (located at v) is rotated by h_{nv} into g_n (located at n). The algebra element $\mathbf{z}_v$ (located at v) is first translated by $\mathbf{w}_v^n$, and the result is then rotated by h_{nv} into $\mathbf{z}_n$.

Translational Holonomies

- These relations follow directly from considering the action of the ISU (2) group. Indeed, ISU (2) = SU (2) $\ltimes \mathbb{R}^3$ and $\mathbb{R}^3 \cong \mathfrak{su}(2)$, so we can write
$$(h_{nv}, \mathbf{w}_n^v), (g_n, \mathbf{z}_n) \in \mathrm{ISU}(2),$$
and then use the usual ISU (2) group product to get
$$(g_n, \mathbf{z}_n) = (h_{nv}, \mathbf{w}_n^v) \triangleright (g_v, \mathbf{z}_v) = (h_{nv} g_v, h_{nv}(\mathbf{z}_v - \mathbf{w}_v^n) h_{vn}).$$

Translational Holonomies

- These relations follow directly from considering the action of the ISU (2) group. Indeed, ISU $(2) = \mathrm{SU}(2) \ltimes \mathbb{R}^3$ and $\mathbb{R}^3 \cong \mathfrak{su}(2)$, so we can write

$$\left(h_{nv}, \mathbf{w}_n^v\right), \left(g_n, \mathbf{z}_n\right) \in \mathrm{ISU}(2),$$

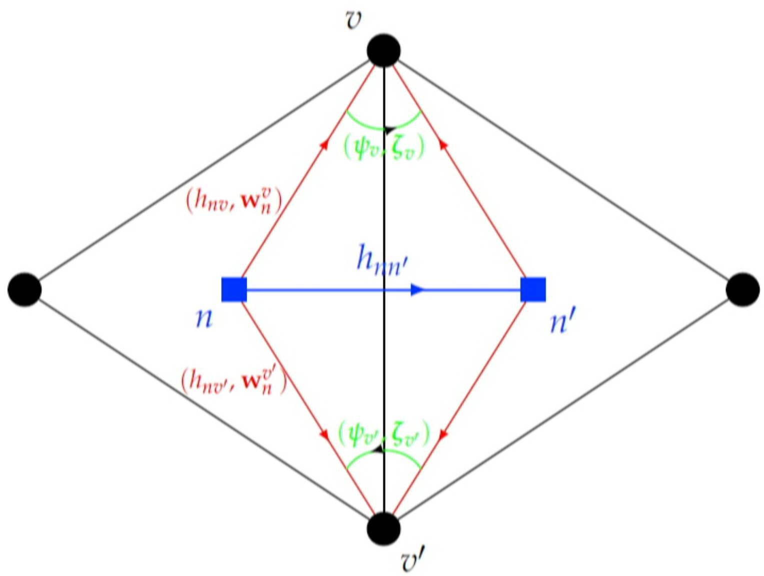
and then use the usual ISU (2) group product to get

$$\left(g_n, \mathbf{z}_n\right) = \left(h_{nv}, \mathbf{w}_n^v\right) \triangleright \left(g_v, \mathbf{z}_v\right) = \left(h_{nv} g_v, h_{nv}\left(\mathbf{z}_v - \mathbf{w}_v^n\right) h_{vn}\right).$$

- We also define the translational winding holonomy

$$\zeta_v^{\phi\phi'} \equiv \frac{\mathbf{S}_v}{2\pi}\left(\phi' - \phi\right) \in \mathfrak{su}(2).$$

The Phase Space of Piecewise-Flat Geometries

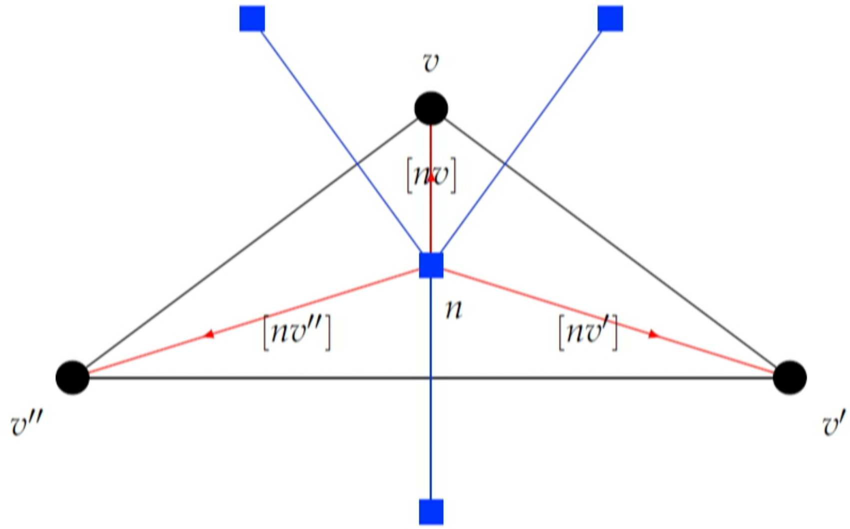


From this construction, we can work out the symplectic potential:

$$\Theta = \sum_{\ell=(nn')} \boldsymbol{\Delta}(h_{nn'}) \cdot \left(\mathbf{w}_n^{v'} - \mathbf{w}_n^v\right) + \sum_{[vn]} \boldsymbol{\Delta}(\psi_v^n h_{vn}) \cdot \zeta_v^n - \frac{1}{2}\sum_v \mathbf{S}_v \cdot \delta \mathbf{M}_v,$$

$$\left(h_{nn'}, \mathbf{w}_n^{n'}\right), (h_{nv}, \mathbf{w}_n^v), (\psi_v, \zeta_v) \in \mathrm{ISU}(2), \qquad \boldsymbol{\Delta}(h) \equiv \delta h h^{-1}.$$

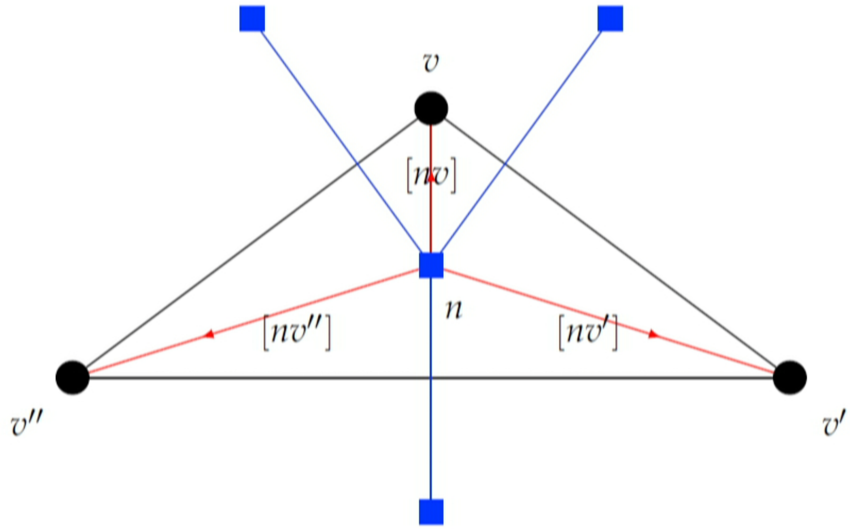
The Phase Space of Piecewise-Flat Geometries



$$\Theta = \sum_{\ell=(nn')} \mathbf{\Delta}\left(h_{nn'}\right) \cdot \left(\mathbf{w}_n^{v'} - \mathbf{w}_n^{v}\right) + \sum_{[vn]} \mathbf{\Delta}\left(\psi_v^n h_{vn}\right) \cdot \zeta_v^n - \frac{1}{2}\sum_v \mathbf{S}_v \cdot \delta\mathbf{M}_v,$$

- First term = links connecting the node n to adjacent nodes,

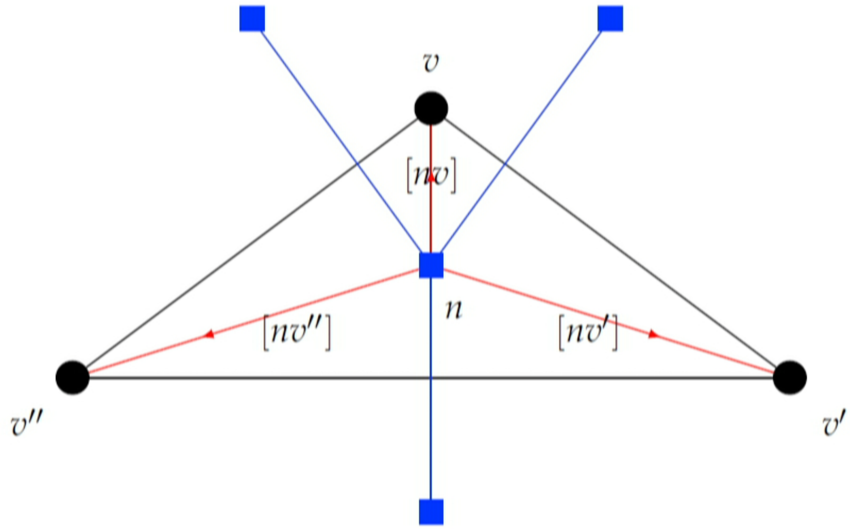
The Phase Space of Piecewise-Flat Geometries



$$\Theta = \sum_{\ell=(nn')} \mathbf{\Delta}\left(h_{nn'}\right) \cdot \left(\mathbf{w}_n^{v'} - \mathbf{w}_n^{v}\right) + \sum_{[vn]} \mathbf{\Delta}\left(\psi_v^n h_{vn}\right) \cdot \zeta_v^n - \frac{1}{2}\sum_v \mathbf{S}_v \cdot \delta \mathbf{M}_v,$$

- First term = links connecting the node n to adjacent nodes,
- Second term = extra links connecting the node n to each vertex on its boundary,

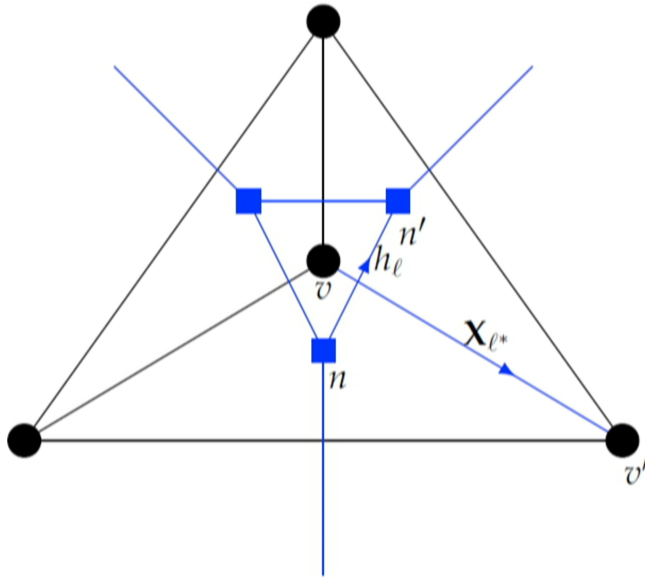
The Phase Space of Piecewise-Flat Geometries



$$\Theta = \sum_{\ell=(nn')} \boldsymbol{\Delta}\left(h_{nn'}\right) \cdot \left(\mathbf{w}_n^{v'} - \mathbf{w}_n^{v}\right) + \sum_{[vn]} \boldsymbol{\Delta}\left(\psi_v^n h_{vn}\right) \cdot \zeta_v^n - \frac{1}{2} \sum_v \mathbf{S}_v \cdot \delta \mathbf{M}_v,$$

- First term = links connecting the node n to adjacent nodes,
- Second term = extra links connecting the node n to each vertex on its boundary,
- Third term = point particle.

Relation to the Spin Network Phase Space



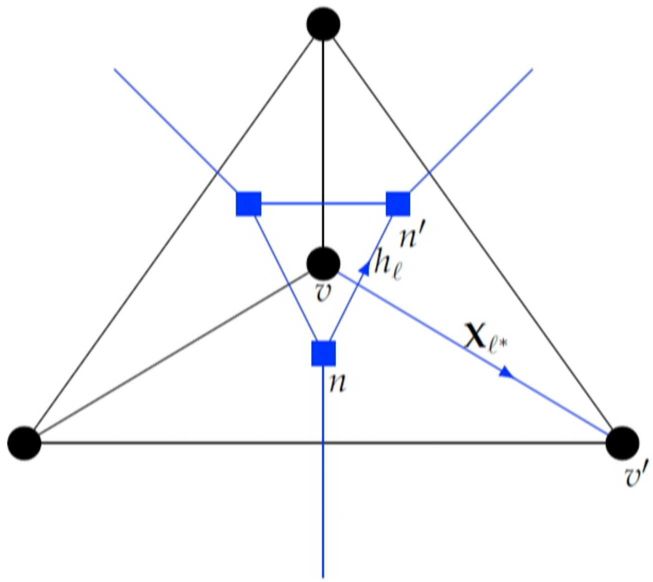
If there is no torsion, $\mathbf{S}_v = \boldsymbol{\zeta}_v = 0$ for all v:

$$\Theta = \sum_{\ell \in \Gamma} \boldsymbol{\Delta}\left(h_\ell\right) \cdot \mathbf{X}_{\ell^*},$$

where the holonomies and fluxes per link are given by

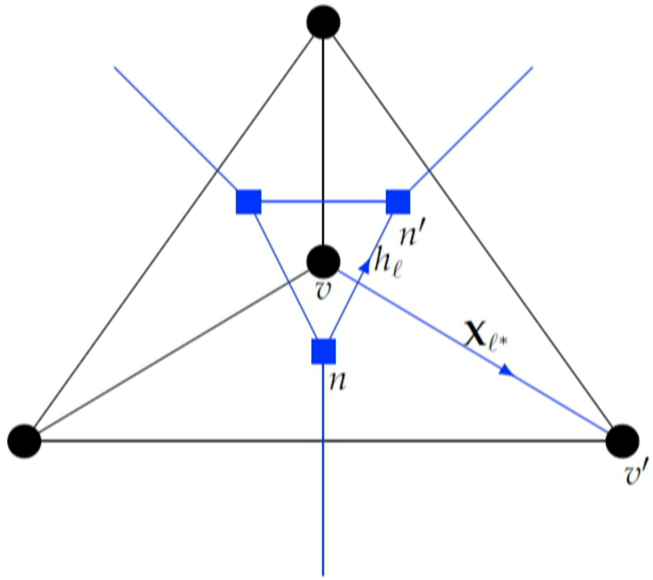
$$h_\ell \equiv h_{nn'}, \qquad \mathbf{X}_{\ell^*} \equiv \mathbf{w}_n^{v'} - \mathbf{w}_n^{v}.$$

Relation to the Spin Network Phase Space



- This is the spin network symplectic potential that we introduced before, describing a product of $T^*\mathrm{SU}(2)$ cotangent bundles per link.

Relation to the Spin Network Phase Space



- This is the spin network symplectic potential that we introduced before, describing a product of $T^*\mathrm{SU}(2)$ cotangent bundles per link.
- Thus, we have embedded the spin network phase space into the larger piecewise-flat geometry phase space!

Outlook (Work in Progress)

- Generalization to 3+1 gravity with Ashtekar variables. Vanishing curvature and torsion are imposed by hand to mimic the 2+1 case. We get a piecewise-flat geometry, with curvature and torsion defects on the edges (instead of vertices).
- This again gives rise to a discretized piecewise-flat geometry phase space, which we expect to reduce to the spin network phase space if torsion is assumed to vanish.

Outlook (Work in Progress)

- Generalization to 3+1 gravity with Ashtekar variables. Vanishing curvature and torsion are imposed by hand to mimic the 2+1 case. We get a piecewise-flat geometry, with curvature and torsion defects on the edges (instead of vertices).
- This again gives rise to a discretized piecewise-flat geometry phase space, which we expect to reduce to the spin network phase space if torsion is assumed to vanish.
- We expect that this will provide a rigorous interpretation of the classical geometry corresponding to spin network states in 3+1 dimensions.
- Our work is heavily related to recent work by Bianca Dittrich, Clement Delcamp, Jonathan Ziprick, Maite Dupuis, Marc Geiller, and others.

WILFRID LAURIER UNIVERSITY Waterloo | Brantford | Kitchener | Toronto

LAURIER

Inspiring Lives.

Equal portions of pi(e):

Balancing the gender equation in science

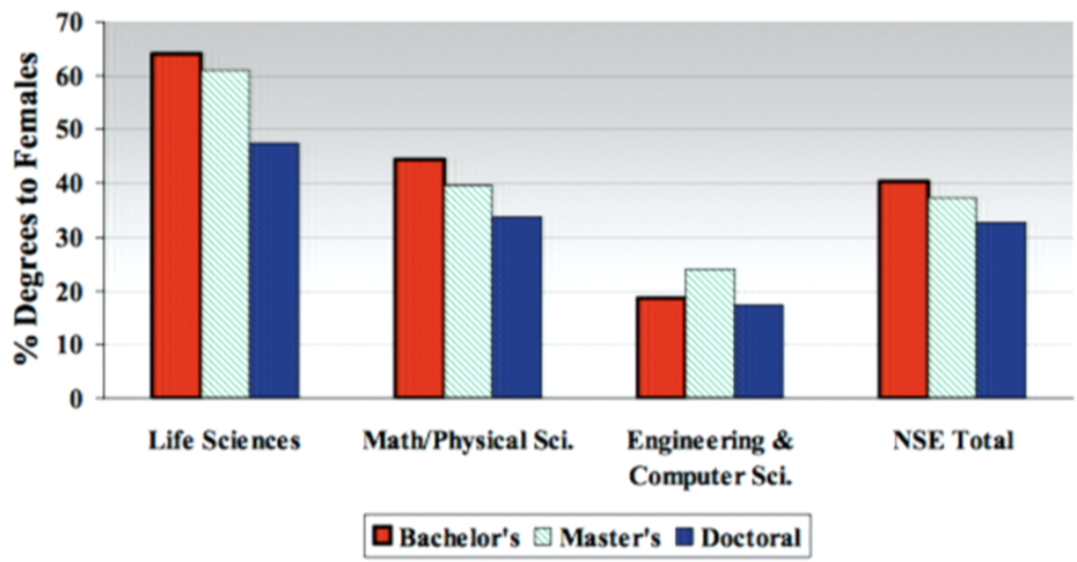
Shohini Ghose

Female Nobel Laureates in physics…

Female Nobel Laureates in physics

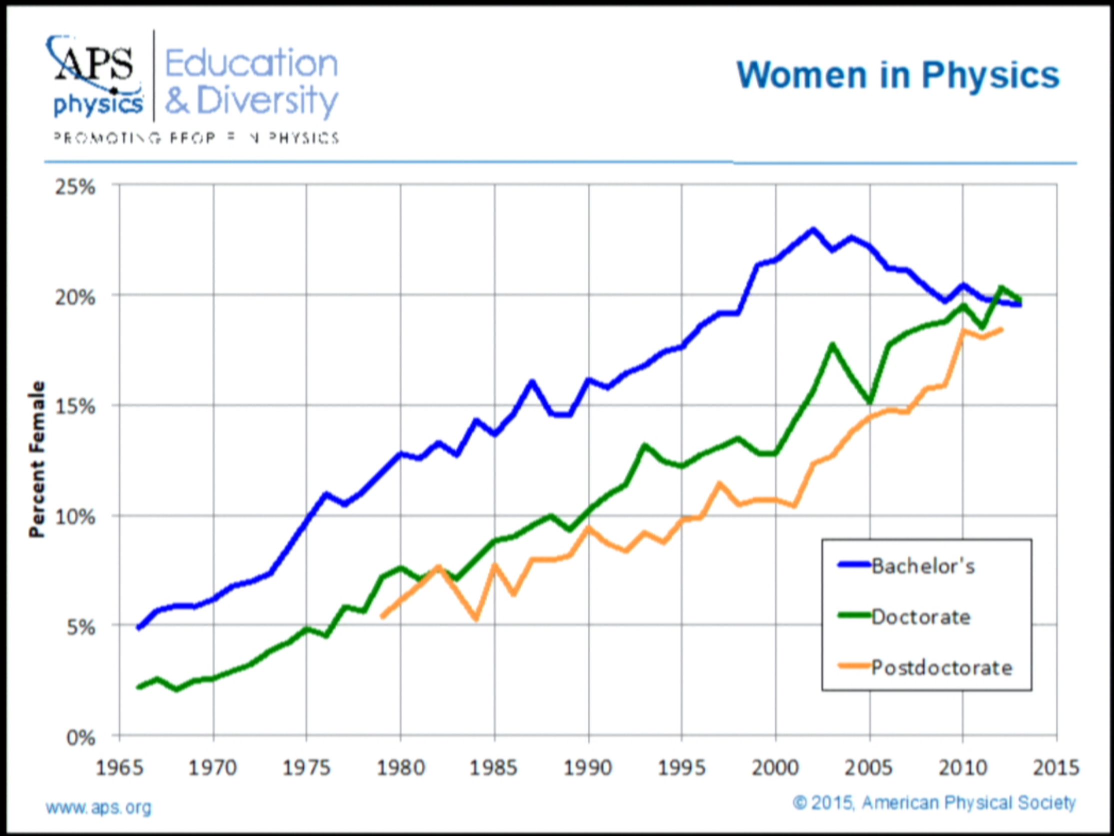
Marie Curie (1903)

Maria Goeppert Mayer (1963)

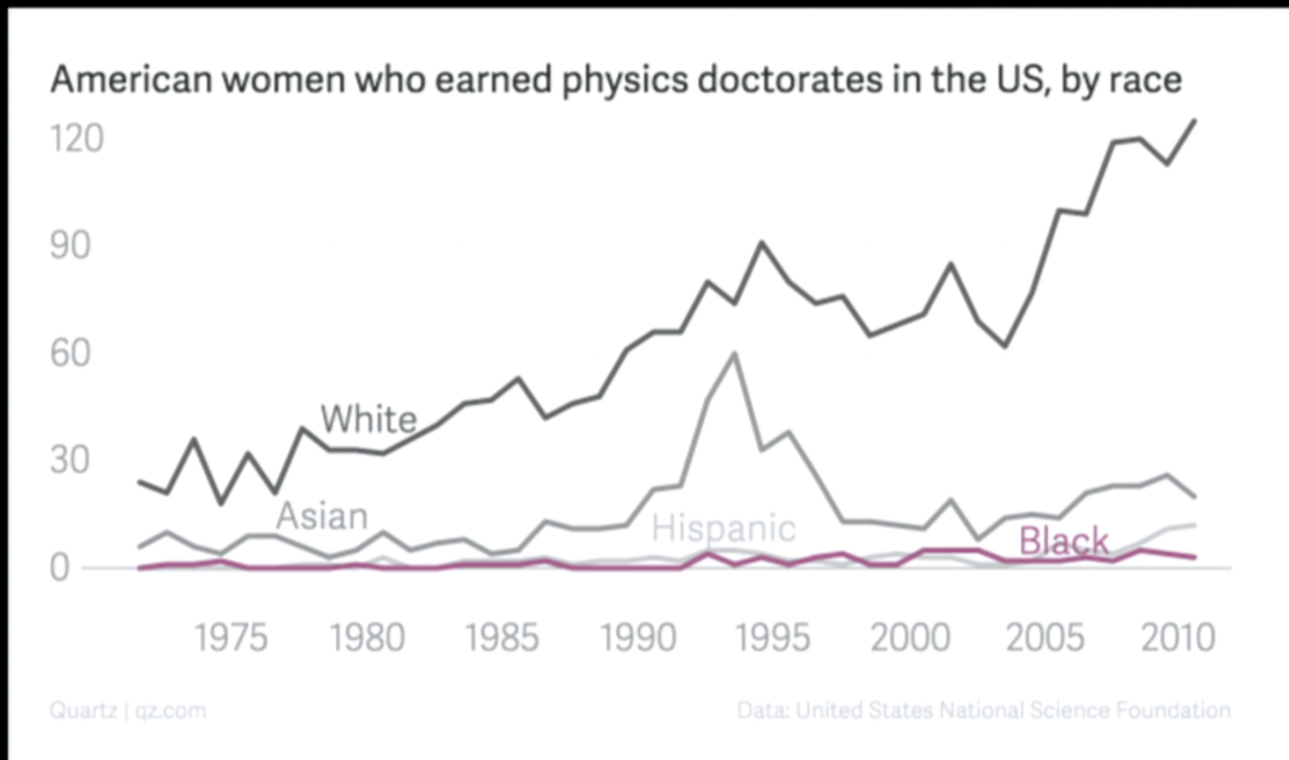


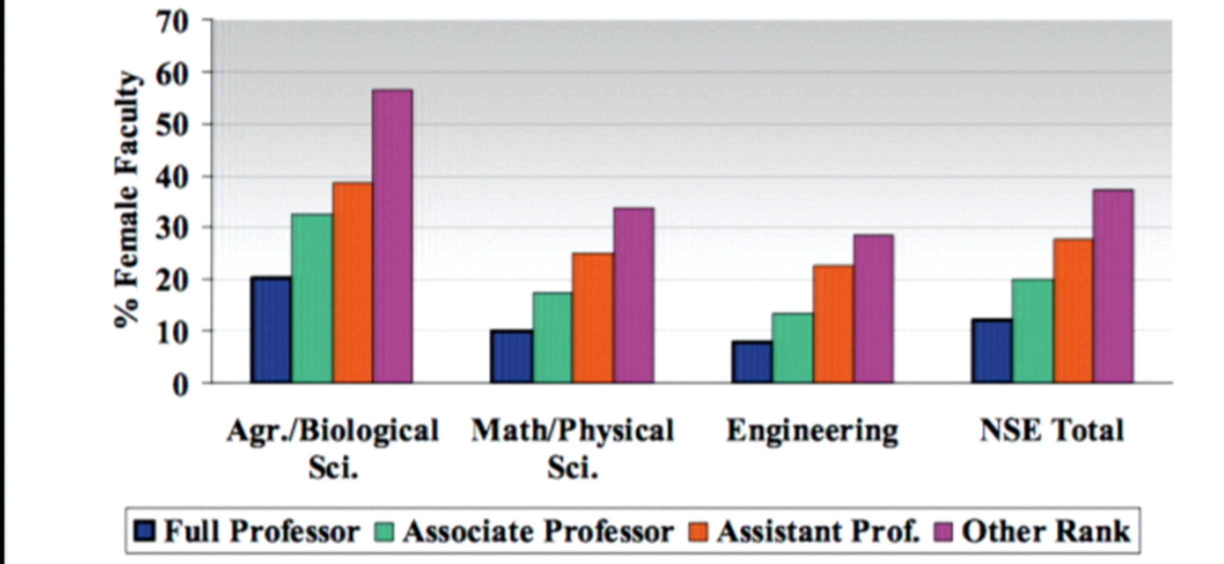
Source: Statistics Canada.

NSERC Report, 2010



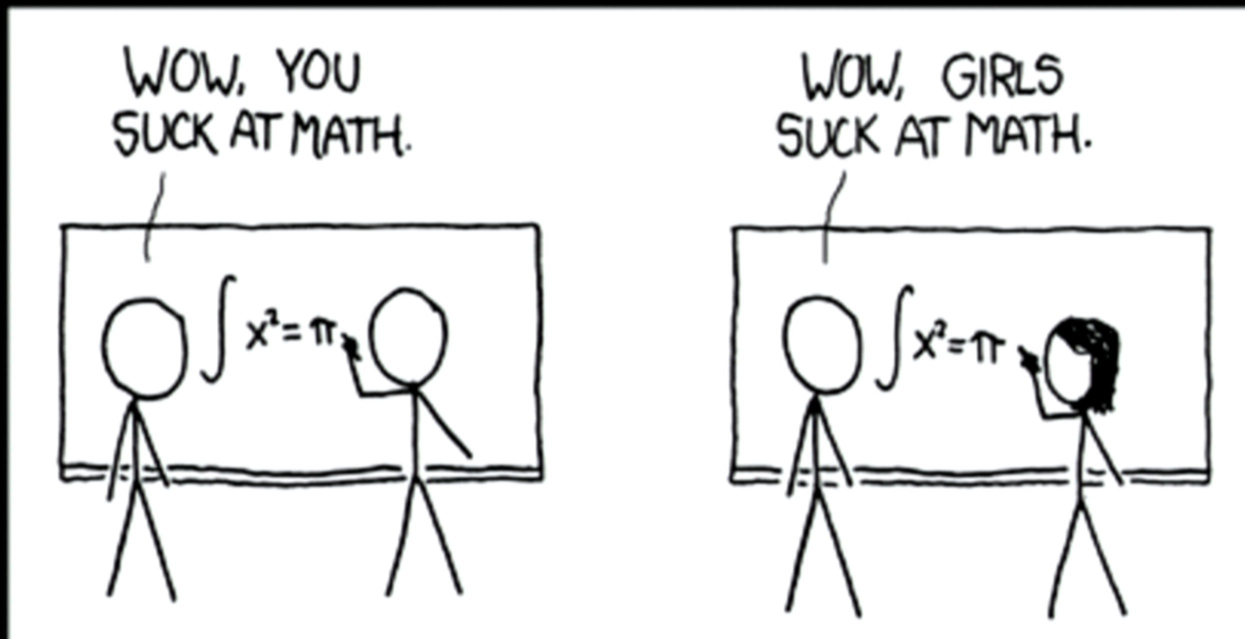
APS physics
Education & Diversity
Women in Physics
25%
20%
15%
10%
5%
0%
Percent Female
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015
Bachelor's
Doctorate
Postdoctorate
www.aps.org
© 2015, American Physical Society


American women who earned physics doctorates in the US, by race
120
90
60
30
0
White
Asian
Hispanic
Black
1975
1980
1985
1990
1995
2000
2005
2010
Quartz | qz.com
Data: United States National Science Foundation



Source: Statistics Canada.

Physics: Assistant: 25%, Associate: 15.7%, Full: 5.6% (CAUT 2013-2014)

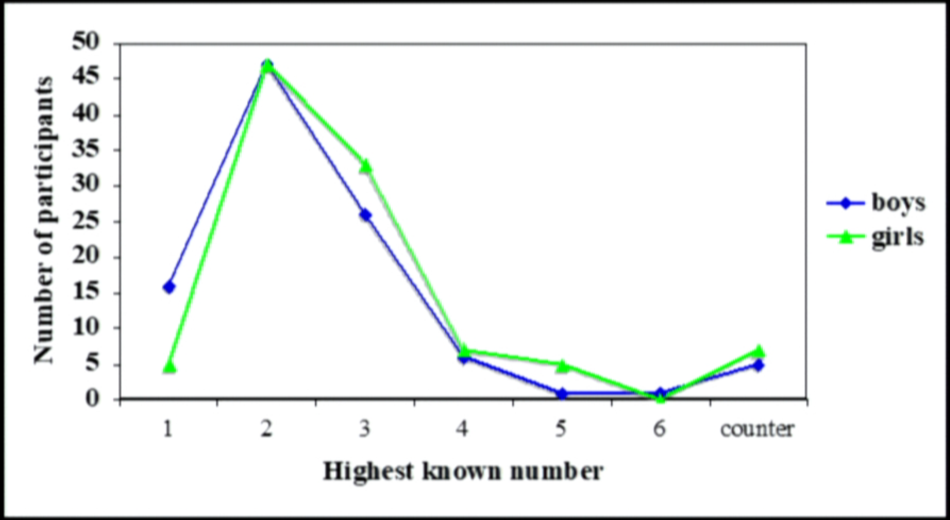


xkcd.com

Math performance

Learning to count

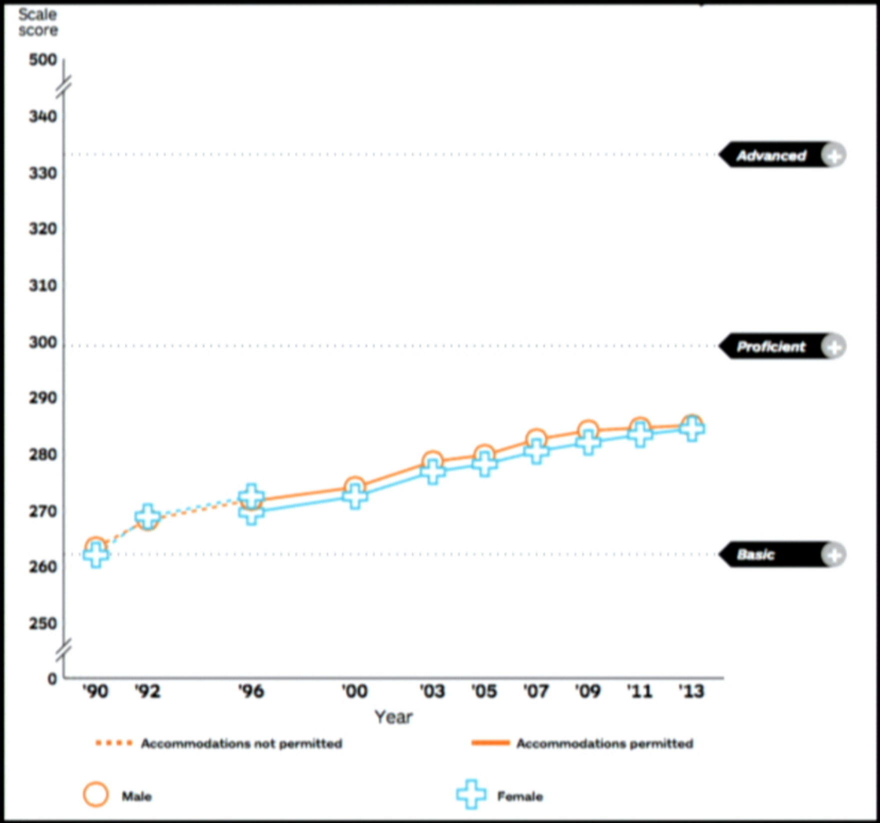
3 year old children



"not a shred of evidence" Spelke, American Psychologist 2005

Math performance

NAEP report cards: 8th grade



Math performance

Grade 8 science

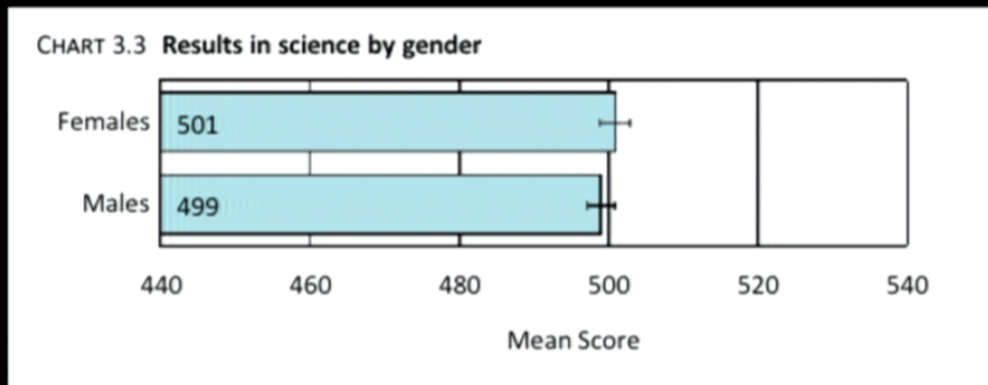
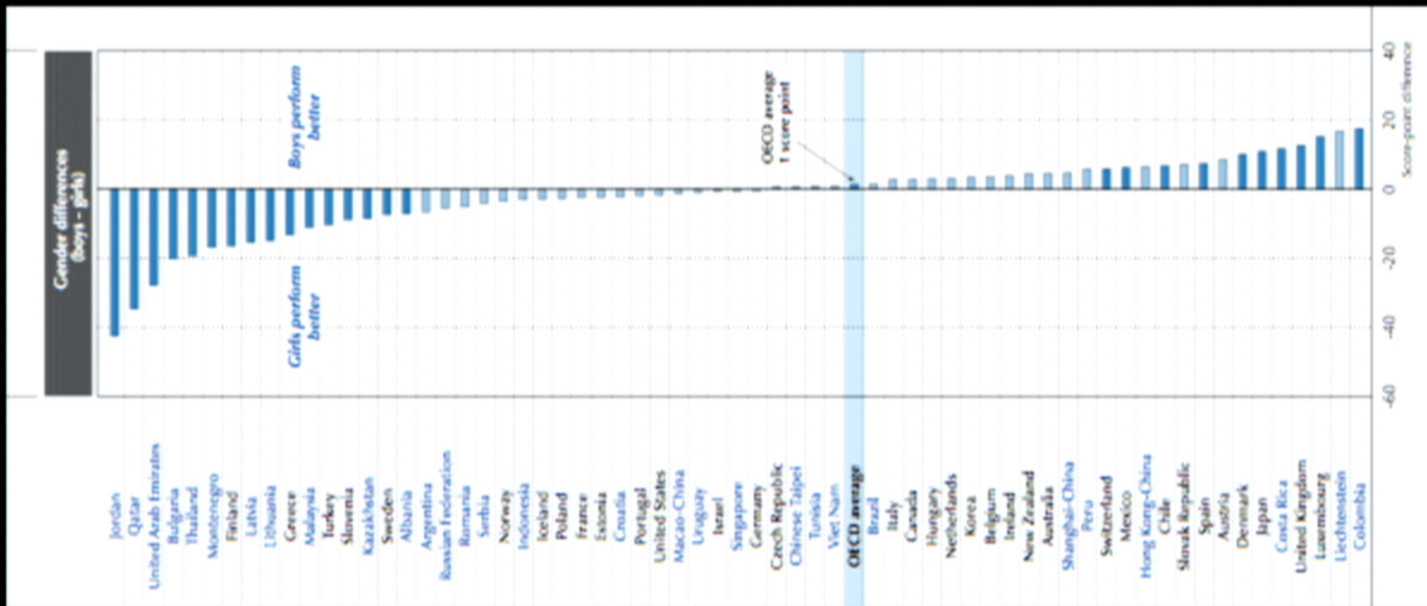


TABLE 3.5 **Results by sub-domain in science by gender**

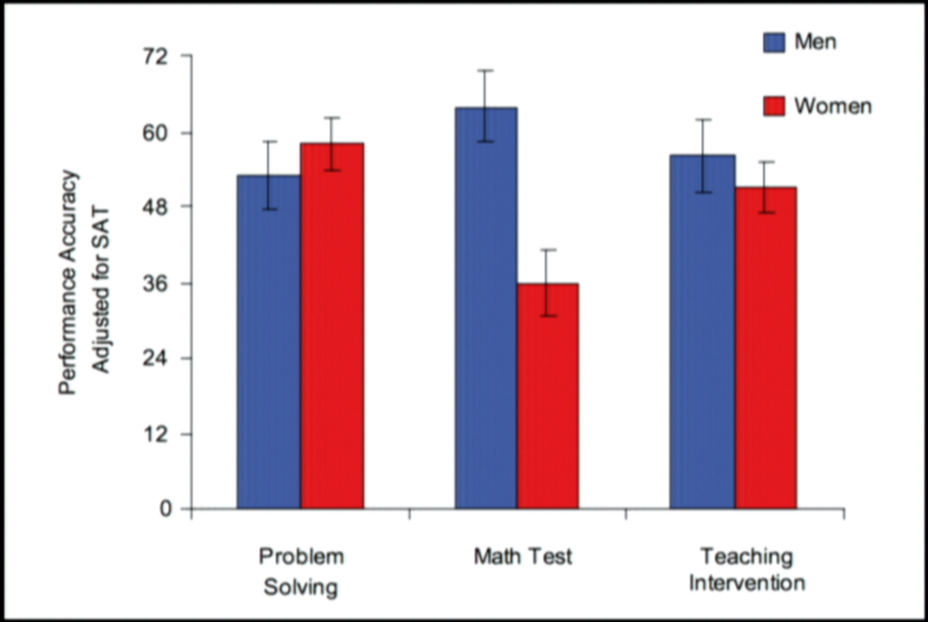
	Nature of science		Life science		Physical science		Earth science	
	Mean	**CI**	**Mean**	**CI**	**Mean**	**CI**	**Mean**	**CI**
Females	501	2.7	501	2.5	499	2.5	501	3.3
Males	499	2.8	499	2.1	501	2.4	500	2.9
Difference	2		2		2		1	

Pan Canadian-Academic Program Report 2013

PISA 2012: Science results across countries



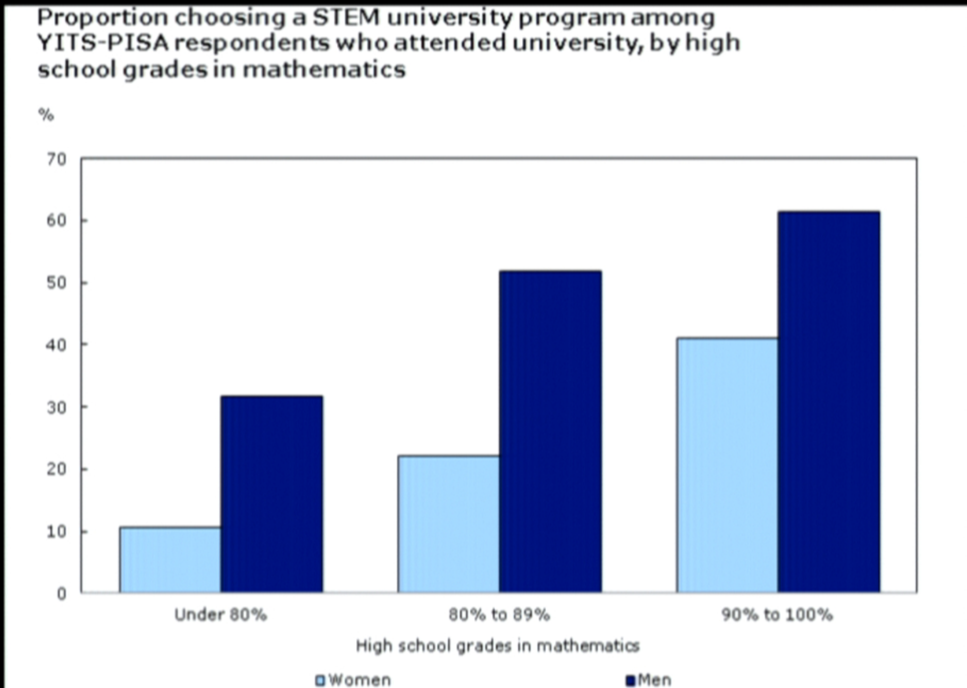
Perceptions and environment



Johns, M., Schmader, T., & Martens, A. (2005). “Knowing is half the battle: Teaching stereotype threat as a means of improving women’s math performance.” *Psychological Science*, 16, 175–179.

Spencer, S. J., Steele, C. M., & Quinn, D. M., (1999), "Stereotype threat and women's math performance," *Journal of Experimental Social Psychology*, 35(1), p. 13.

Math performance and university choices



Note: STEM includes science, technology, engineering, mathematics and computer science.

Sources: Statistics Canada and Human Resources and Skills Development Canada, Youth in Transition Survey (YITS); Organisation for Economic Co-operation and Development, Programme for International Student Assessment (PISA), 2000 to 2010.

Perceptions and environment

What does a scientist look like?

Perceptions and environment

What does a scientist look like?

www.open.ac.uk/invisible-witnesses

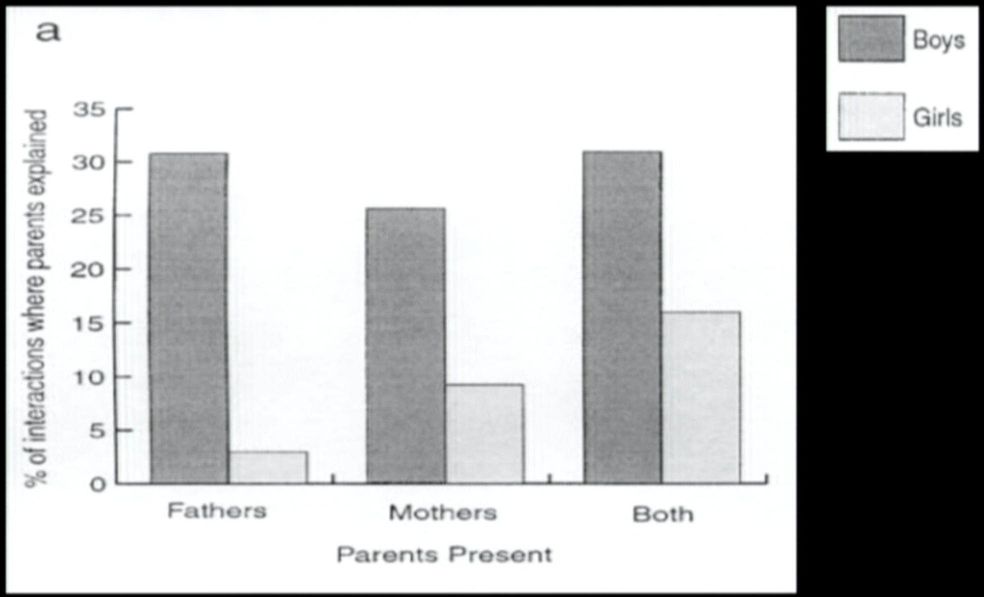
Perceptions and environment

What does a scientist look like?

A. Bodzin, M. Gehringer, Breaking science stereotypes, Science and Children (2001)

Perceptions and environment

Parents/mentors



Crowley, K., Callanan, M. A., Tenenbaum, H. R., & Allen, E. (2001). Parents explain more often to boys than to girls during shared scientific thinking. *Psychological Science, 12*(3).

Perceptions and environment

Implicit bias

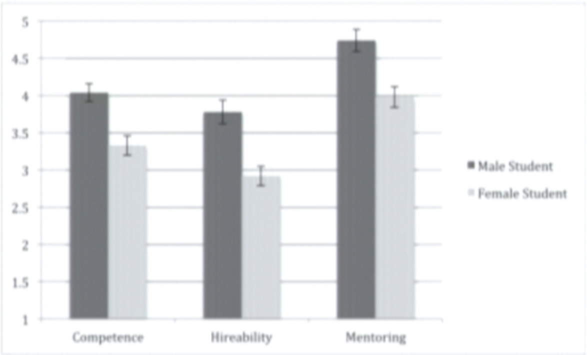


Fig. 1. Competence, hireability, and mentoring by student gender condition (collapsed across faculty gender). All student gender differences are significant ($P < 0.001$). Scales range from 1 to 7, with higher numbers reflecting a greater extent of each variable. Error bars represent SEs. $n_{\text{male student condition}} = 63$, $n_{\text{female student condition}} = 64$.

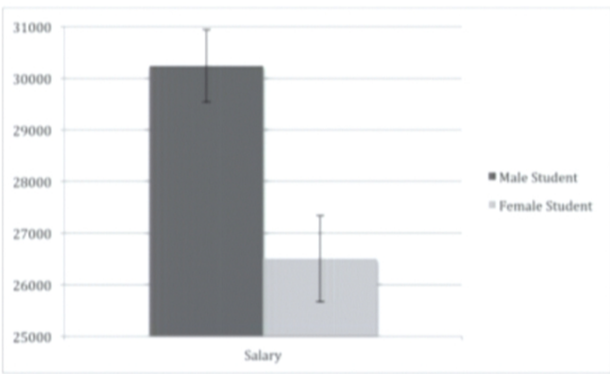


Fig. 2. Salary conferral by student gender condition (collapsed across faculty gender). The student gender difference is significant ($P < 0.01$). The scale ranges from $15,000 to $50,000. Error bars represent SEs. $n_{\text{male student condition}} =$ 63, $n_{\text{female student condition}} = 64$.

Moss-Racusin, C. A., Dovidio, J. F., Brescoll, V. L., Graham, M., & Handelsman, J., *Proceedings of the National Academy of Sciences 109, 16474* (2012)

Perceptions and environment

IUPAP Global Survey of physicists

- Women were:
  - Less likely to have adequate resources
  - More likely to do majority of housework/childrearing
  - More likely to experience slower career advancement

https://www.aip.org/statistics/reports/global-survey-physicists

Strategies for change

- Destroy invisibility
- Un-normalize
- Lay down the law
- Measure
- Connect

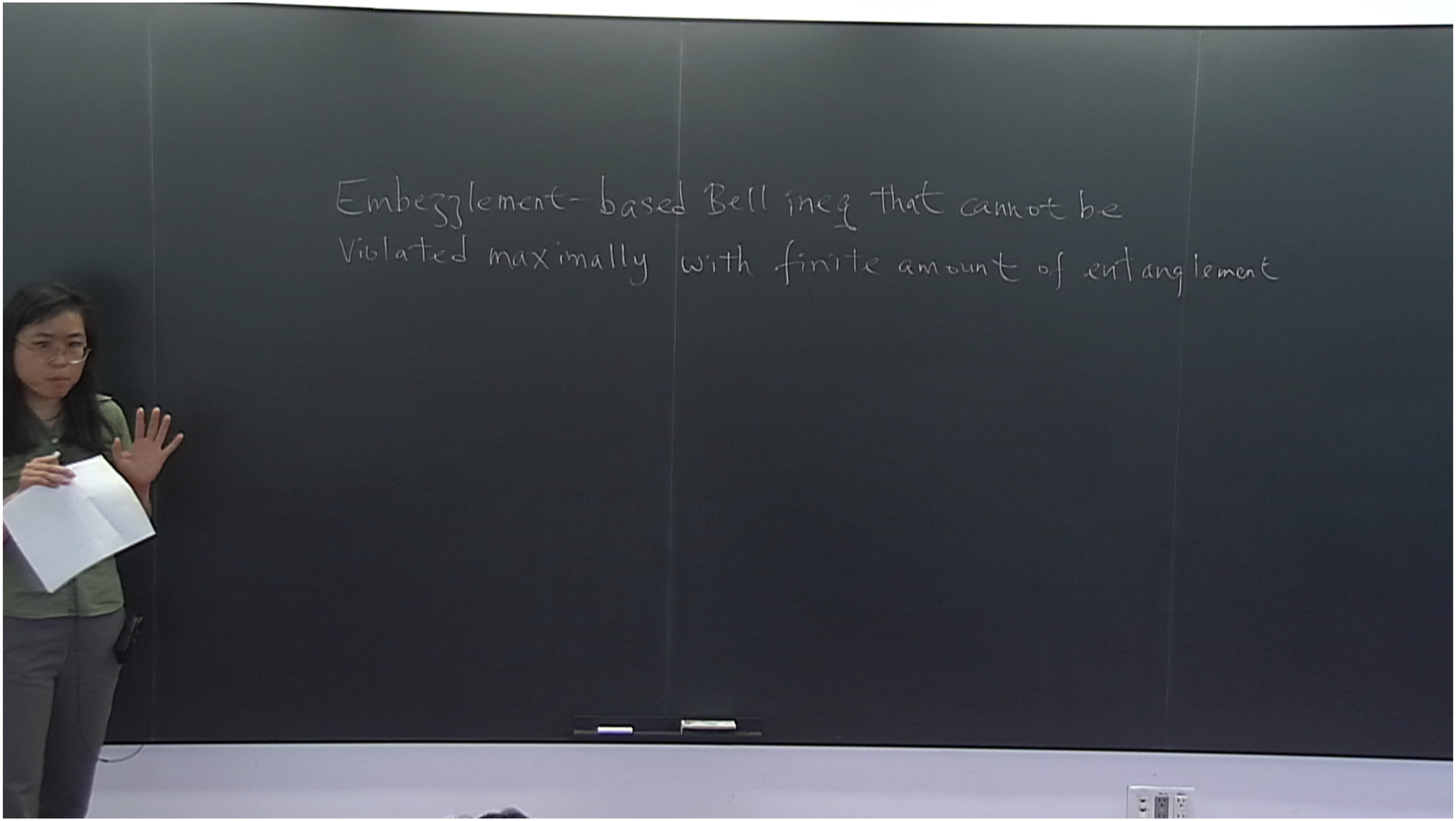
WOMEN IN SCIENCE

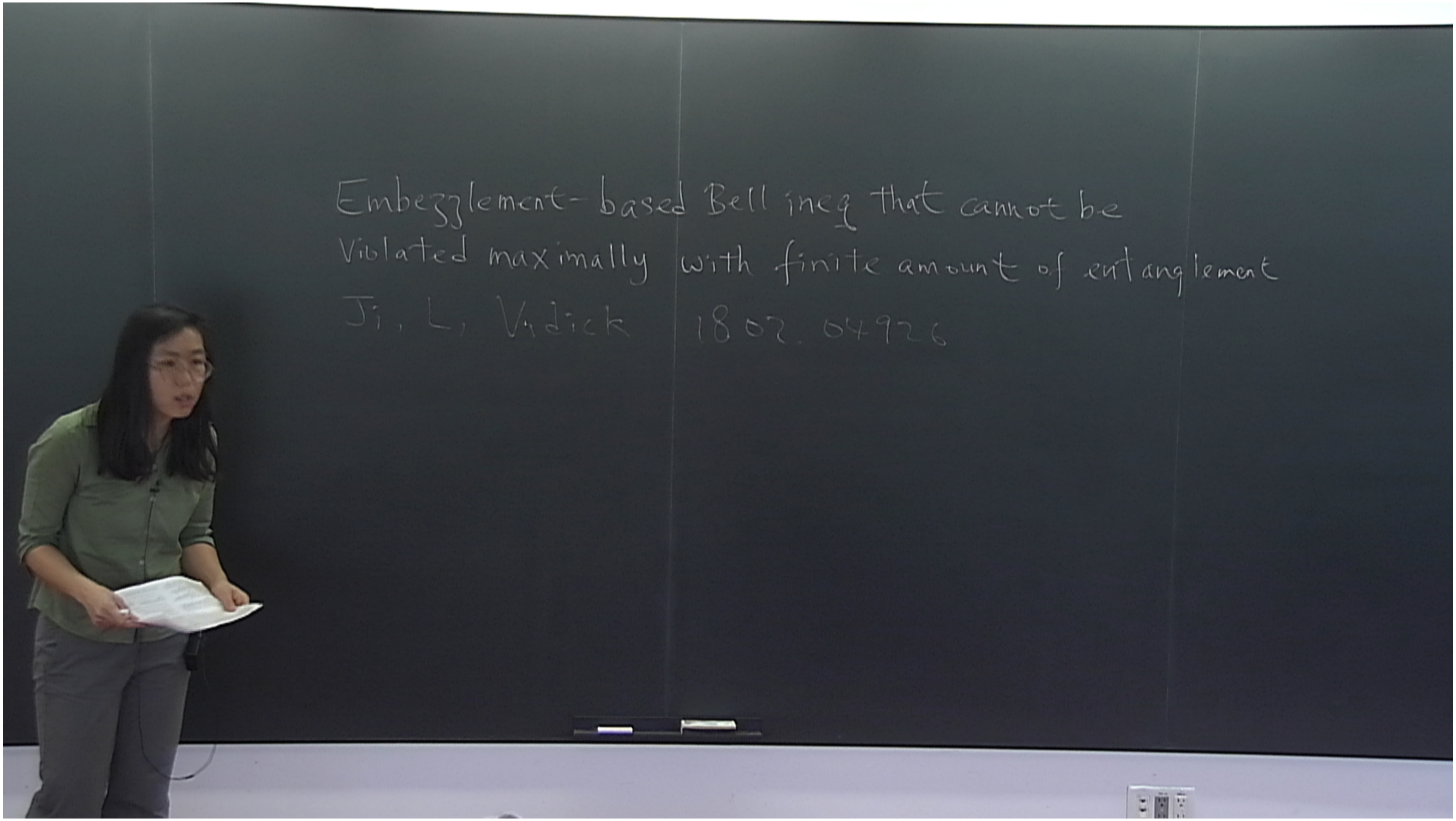
wlu.ca/wins

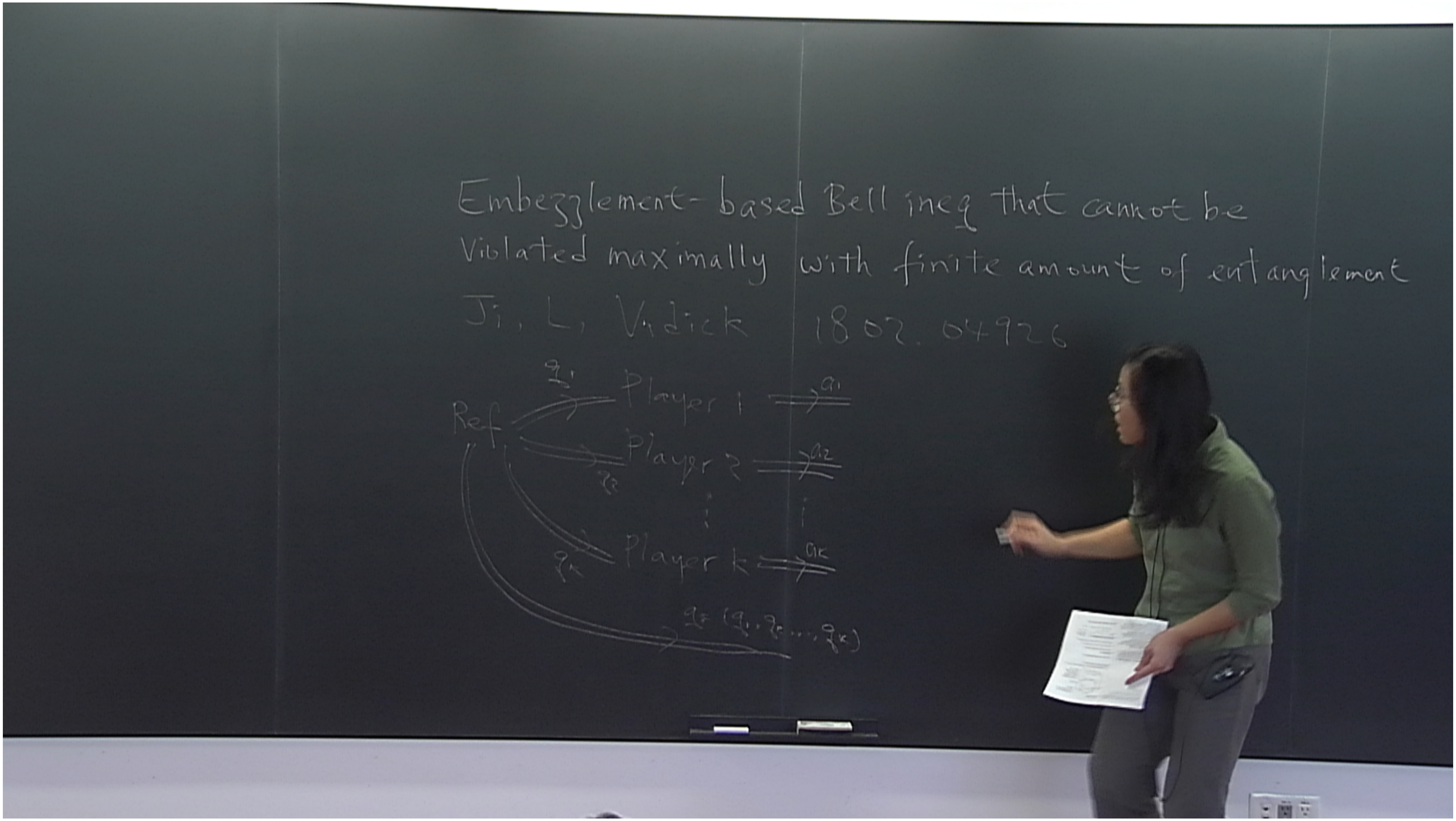
International Conference on Women in Physics
Waterloo, Canada 2014

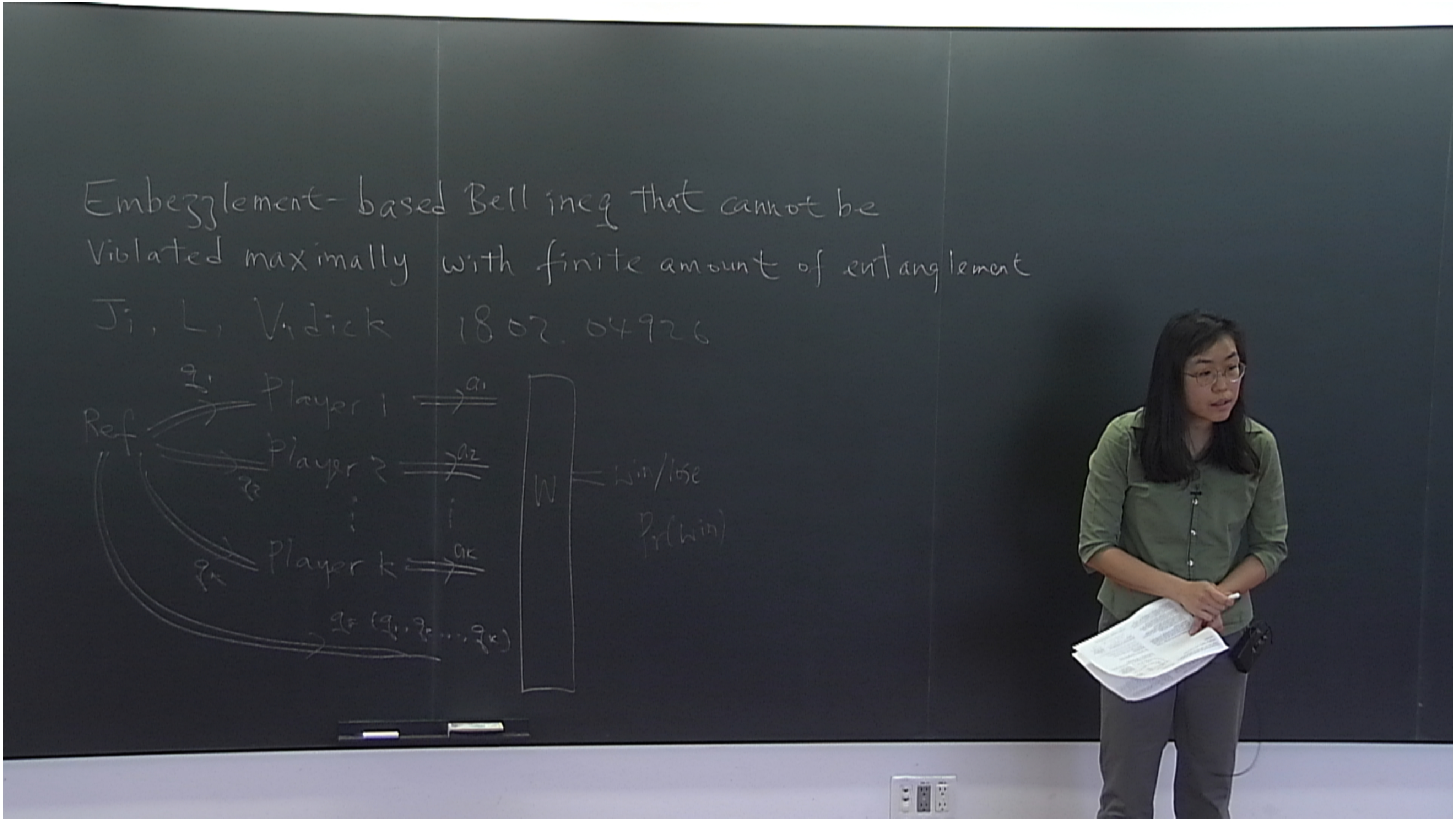
“…black holes ain’t as black as they are painted. They are not the eternal prisons they were once thought…things can get out of a black hole both on the outside and possibly to another universe. So if you feel you are in a black hole, don’t give up – there’s a way out.”

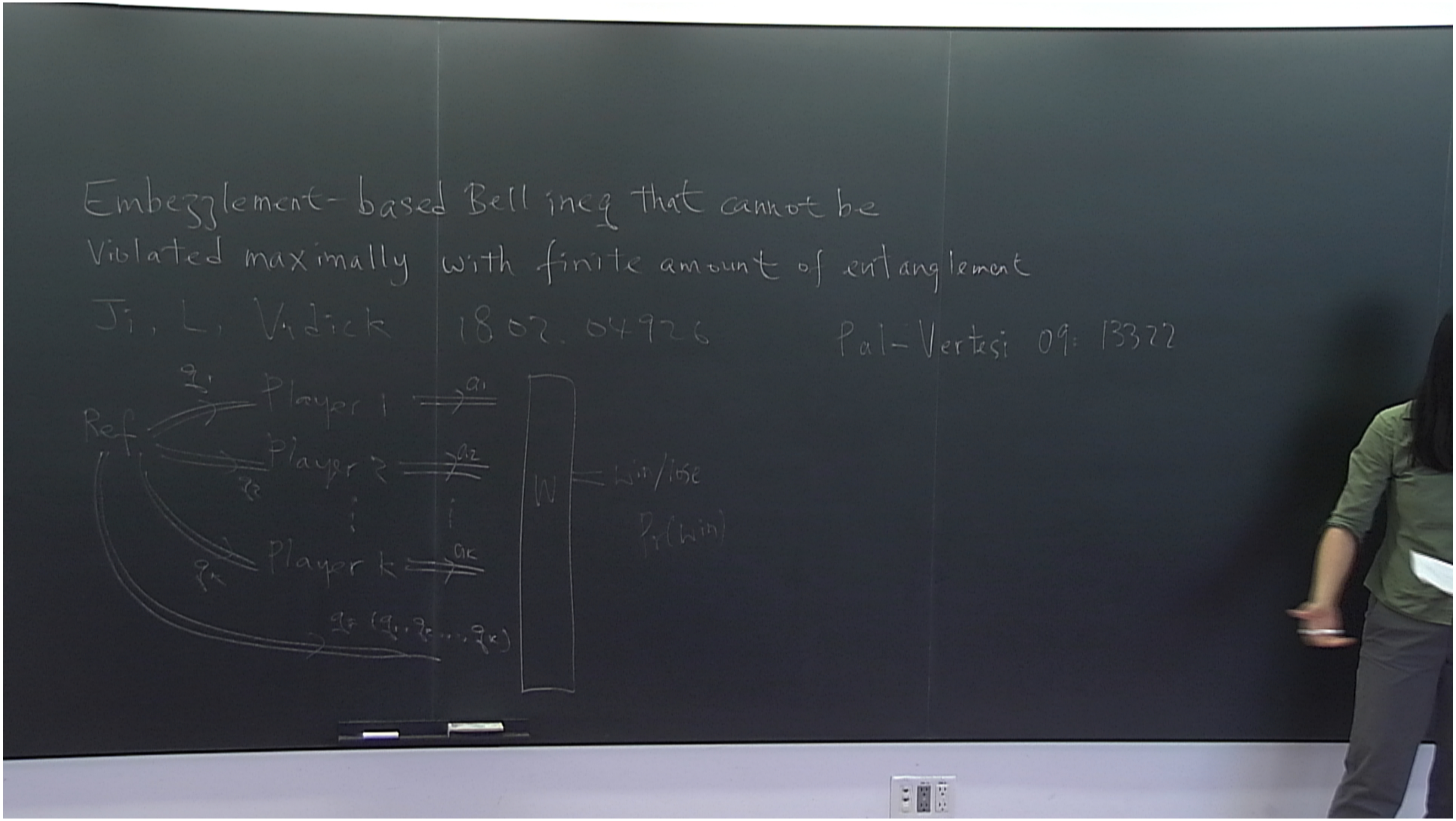
-Stephen Hawking
1942-2018

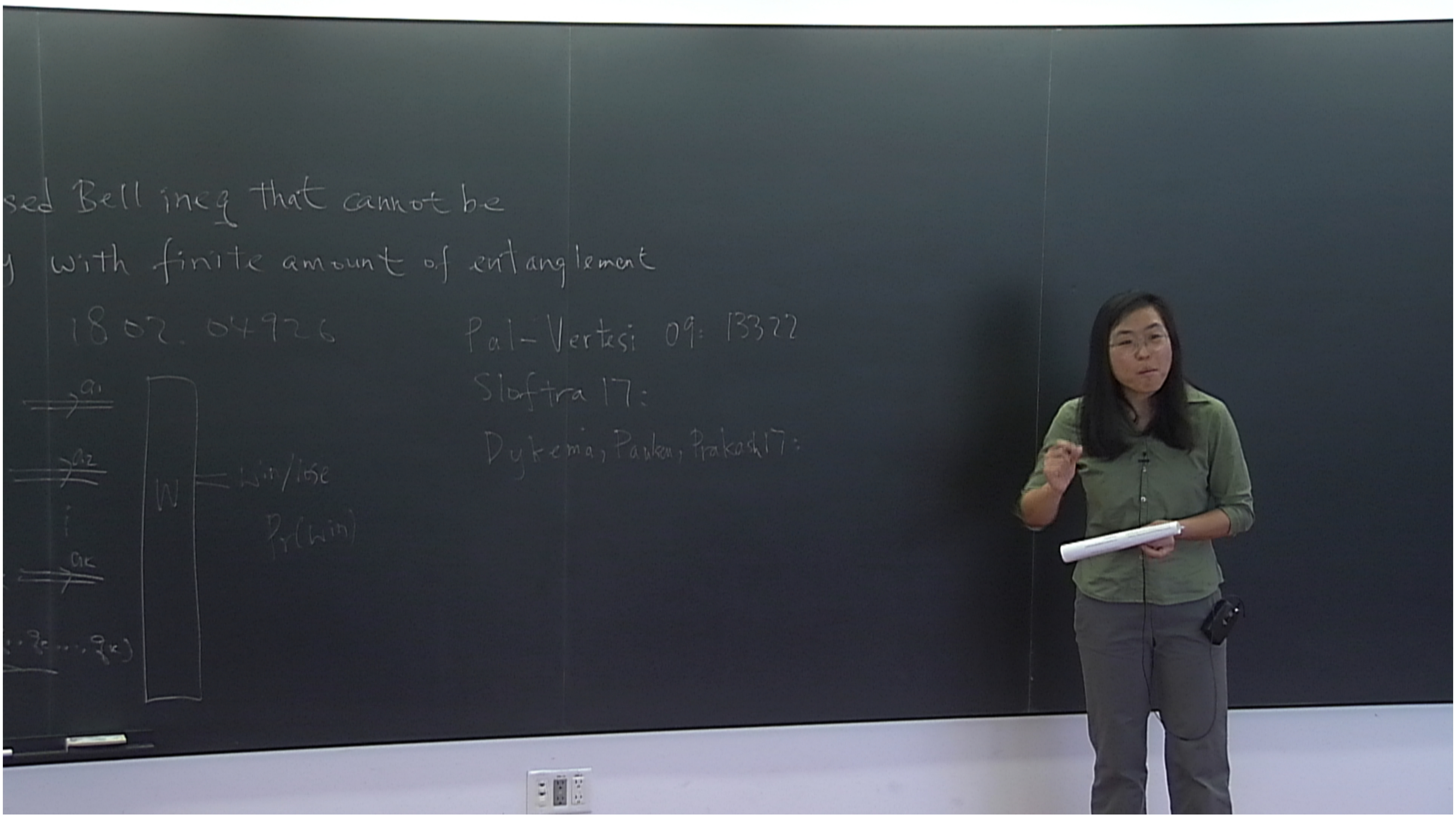

Embezzlement-based Bell ineq that cannot be
Violated maximally with finite amount of entanglement

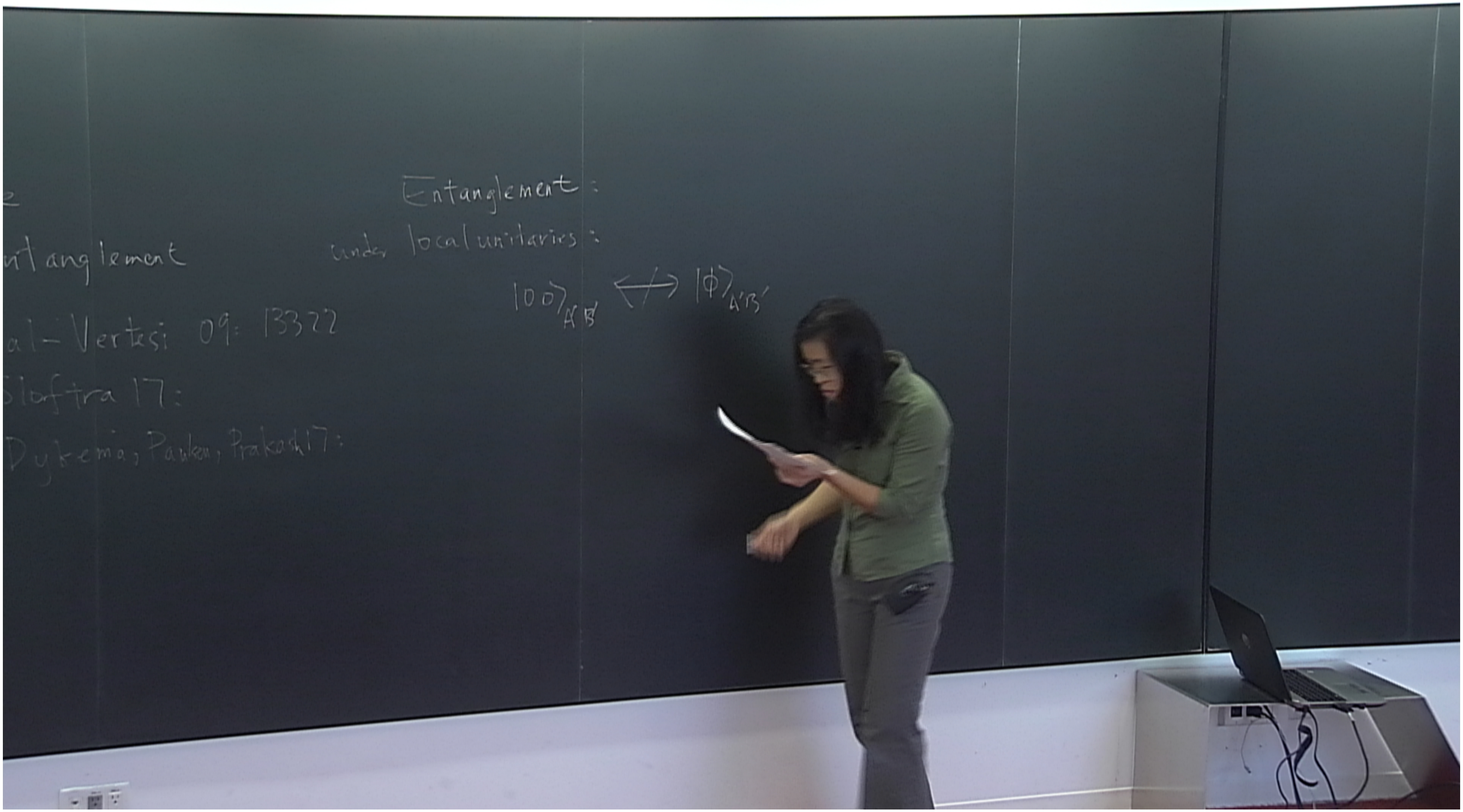

Embezzlement-based Bell ineq that cannot be
violated maximally with finite amount of entanglement
Ji, L, Vidick 1802.04926

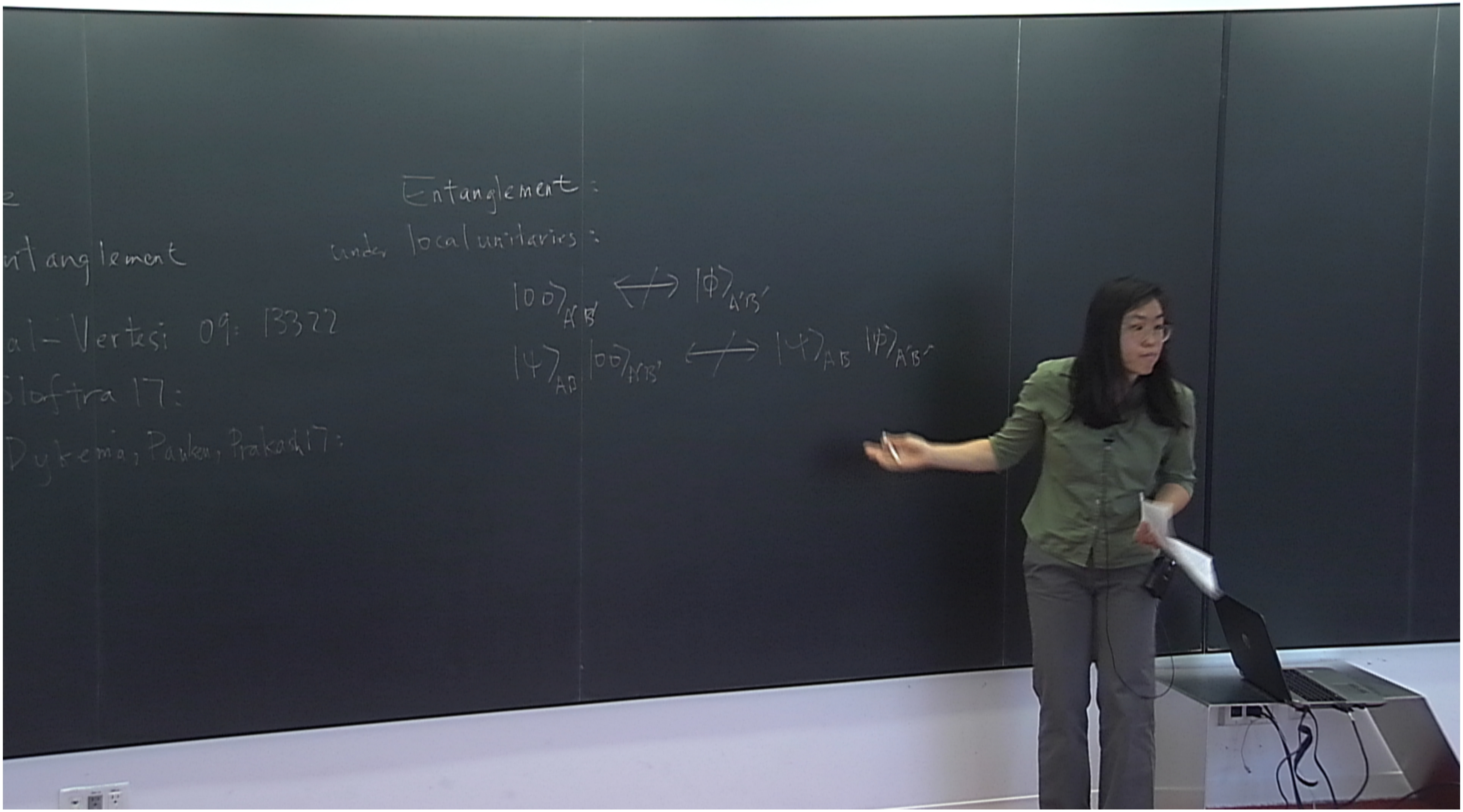

Embezzlement-based Bell ineq that cannot be
violated maximally with finite amount of entanglement
Ji, L, Vidick 1802.04926
Ref
q1
Player 1
a1
Player 2
q2
a2
Player k
qk
ak
q = (q1, q2, ..., qk)


Embezzlement-based Bell ineq that cannot be
violated maximally with finite amount of entanglement
Ji, L, Vidick 1802.04926
Ref
q1
Player 1
a1
Player 2
q2
a2
Player k
qk
ak
W
win/lose
Pr(win)
q=(q1,q2,...,qk)

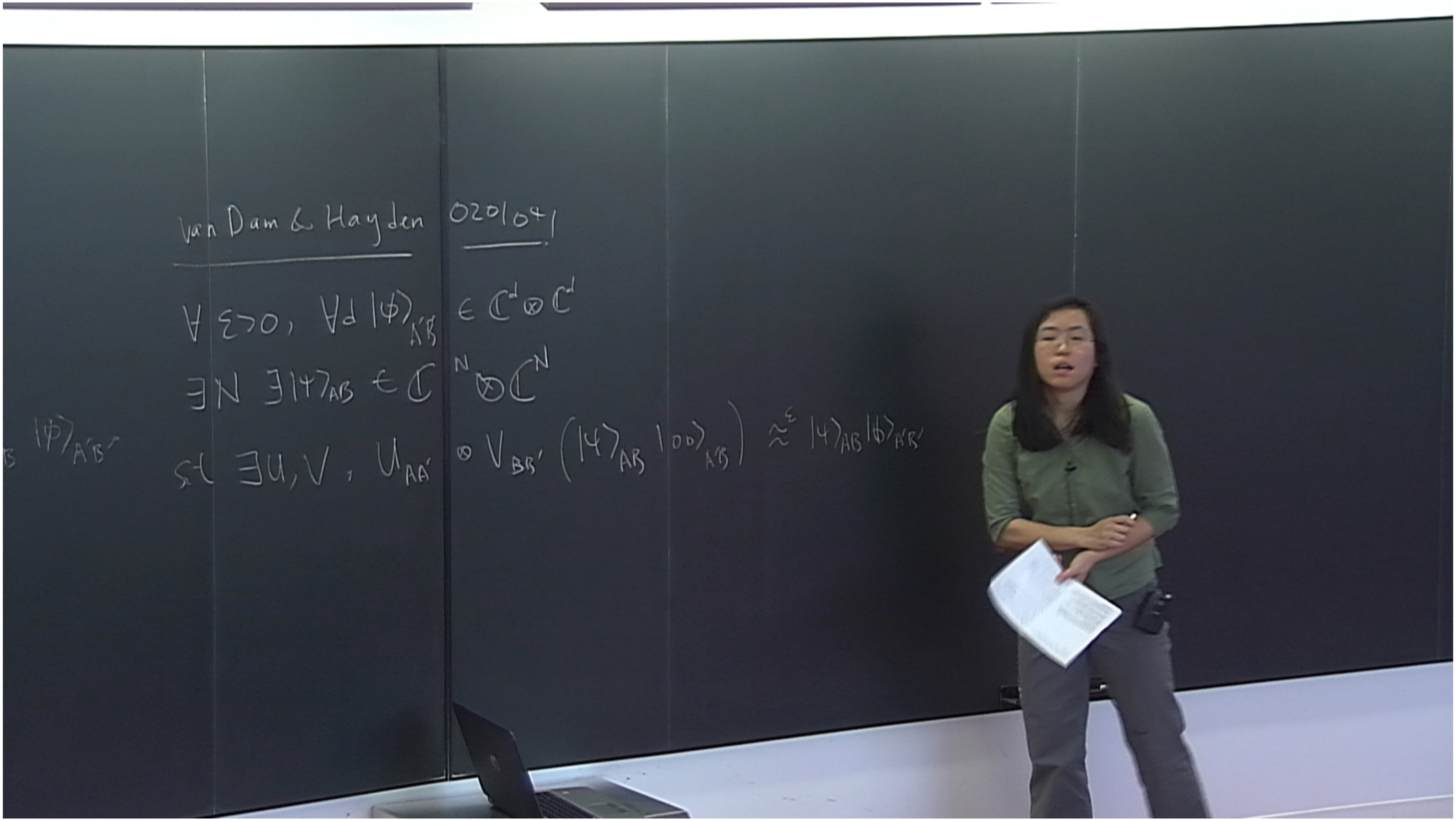

Embezzlement-based Bell ineq that cannot be
violated maximally with finite amount of entanglement
Ji, L, Vidick 1802.04926
Pal-Vertesi 09:13322
Ref
Player 1
Player 2
Player k
W
win/lose
Pr(win)

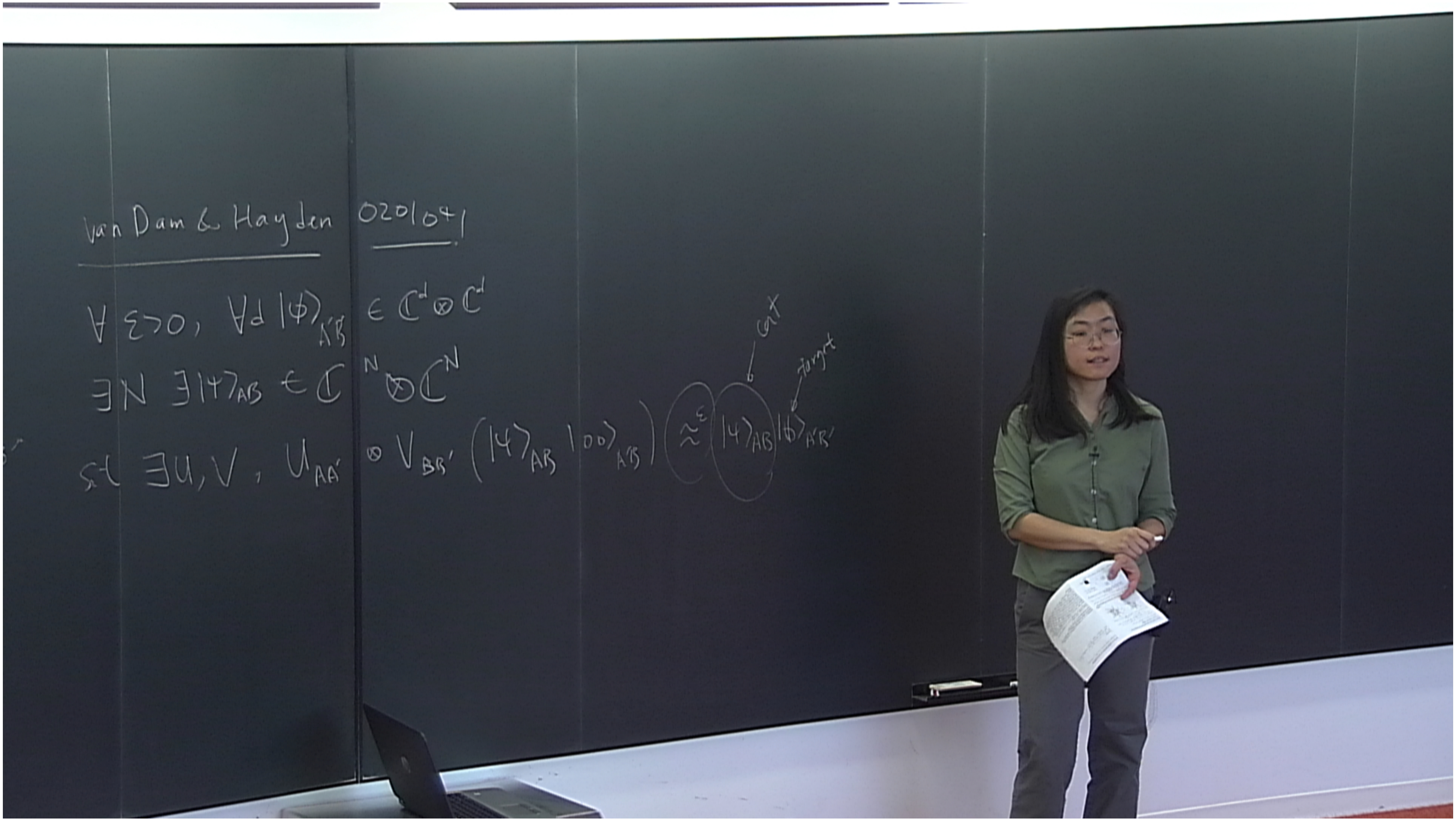

sed Bell ineq that cannot be
y with finite amount of entanglement
1802.04926
Pal-Vertesi 09: 13322
Slofstra 17:
Dykema, Paulsen, Prakash 17:
W
win/lose
Pr(win)

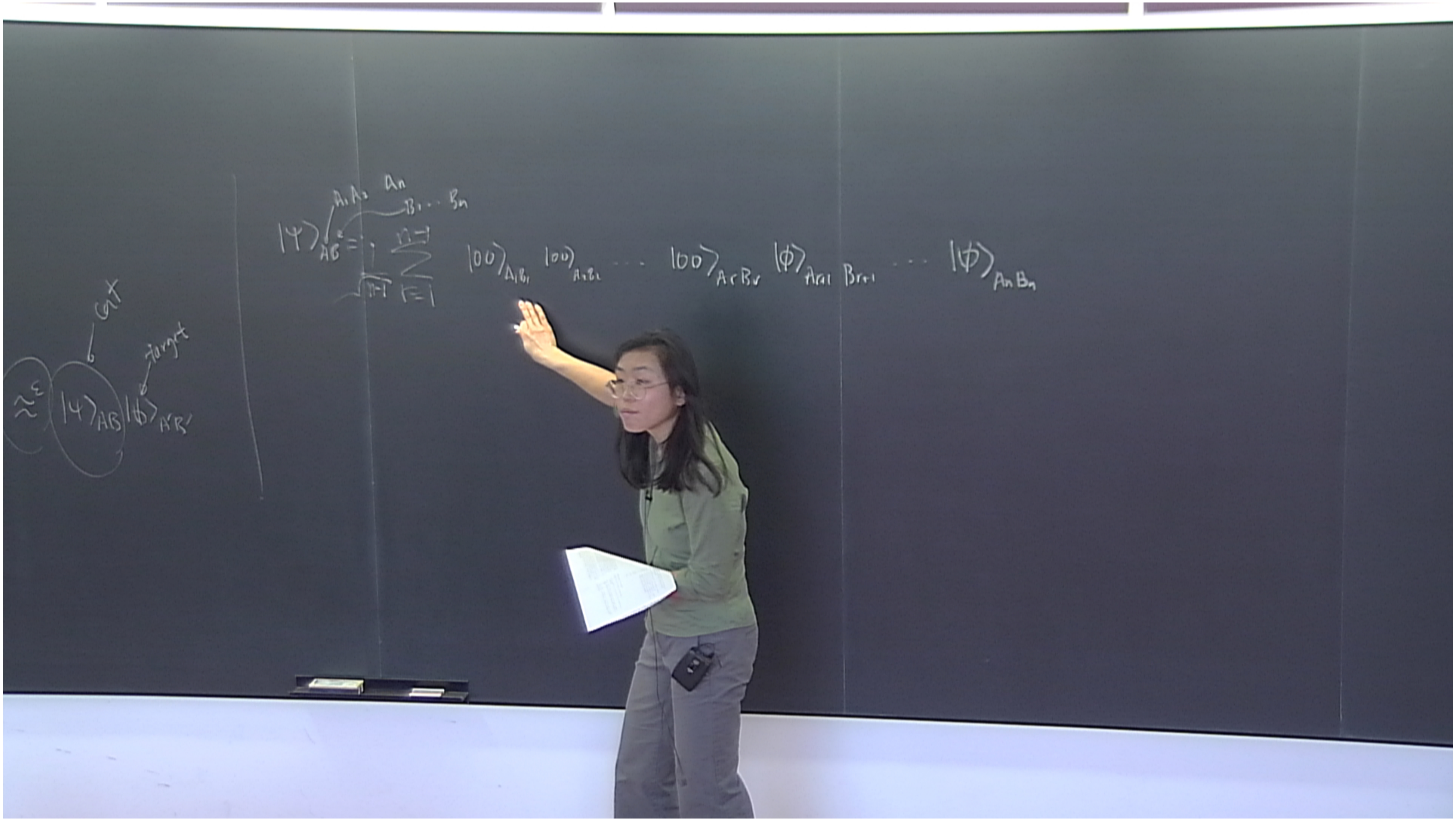

Entanglement:
under local unitaries:
|00⟩_AB ⟷ |φ⟩_A'B'
ntanglement
al-Vertesi 09: 13322
Sloftra 17:
Dykema, Paulsen, Prakash 17:

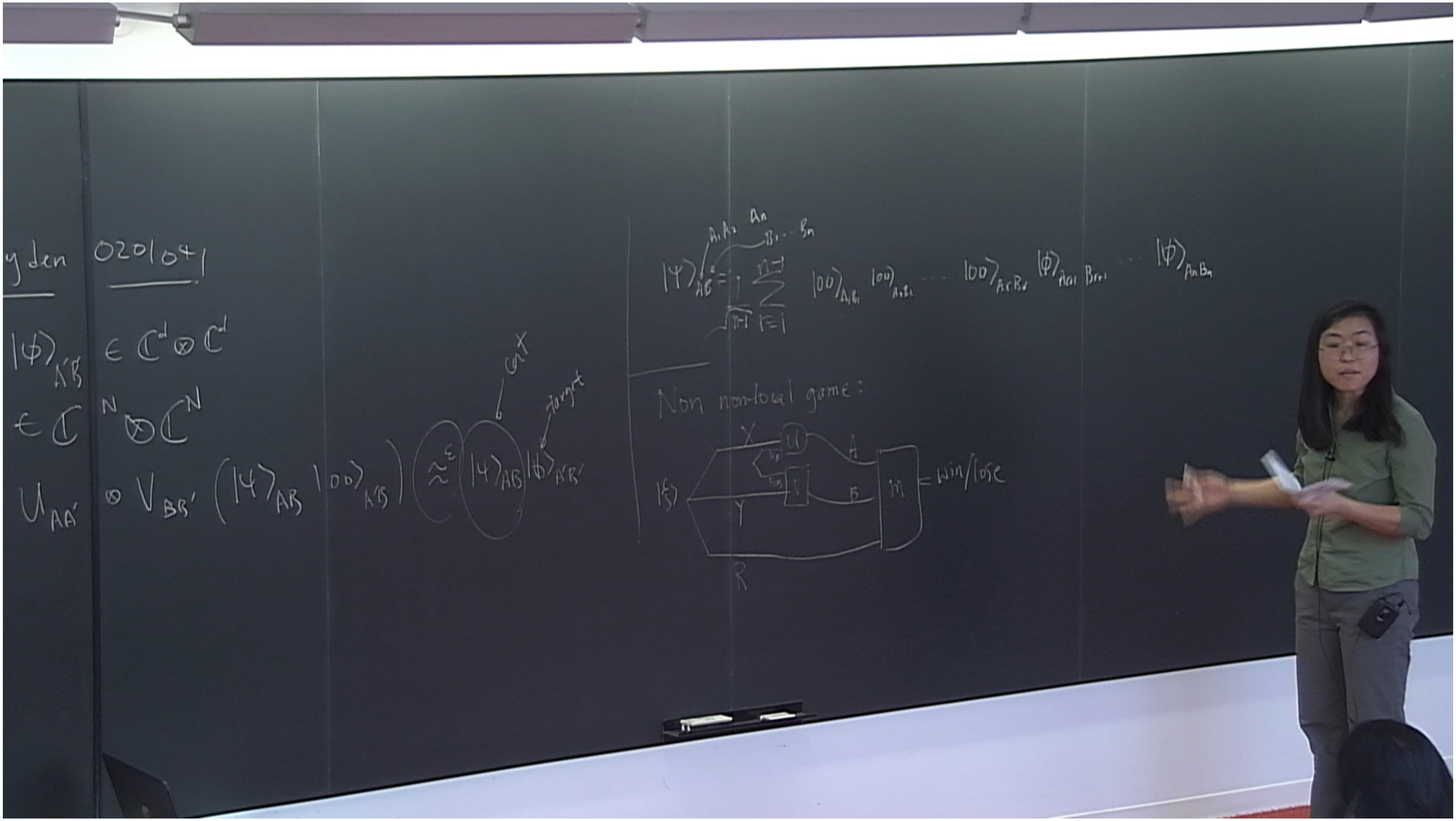

Entanglement:
under local unitaries:
|00⟩A'B' ↮ |Φ⟩A'B'
|ψ⟩AB |00⟩A'B' ↮ |ψ⟩AB |φ⟩A'B'
ntanglement
al-Vertesi 09: 13322
Slofstra 17:
Dykema, Paulsen, Prakash17:

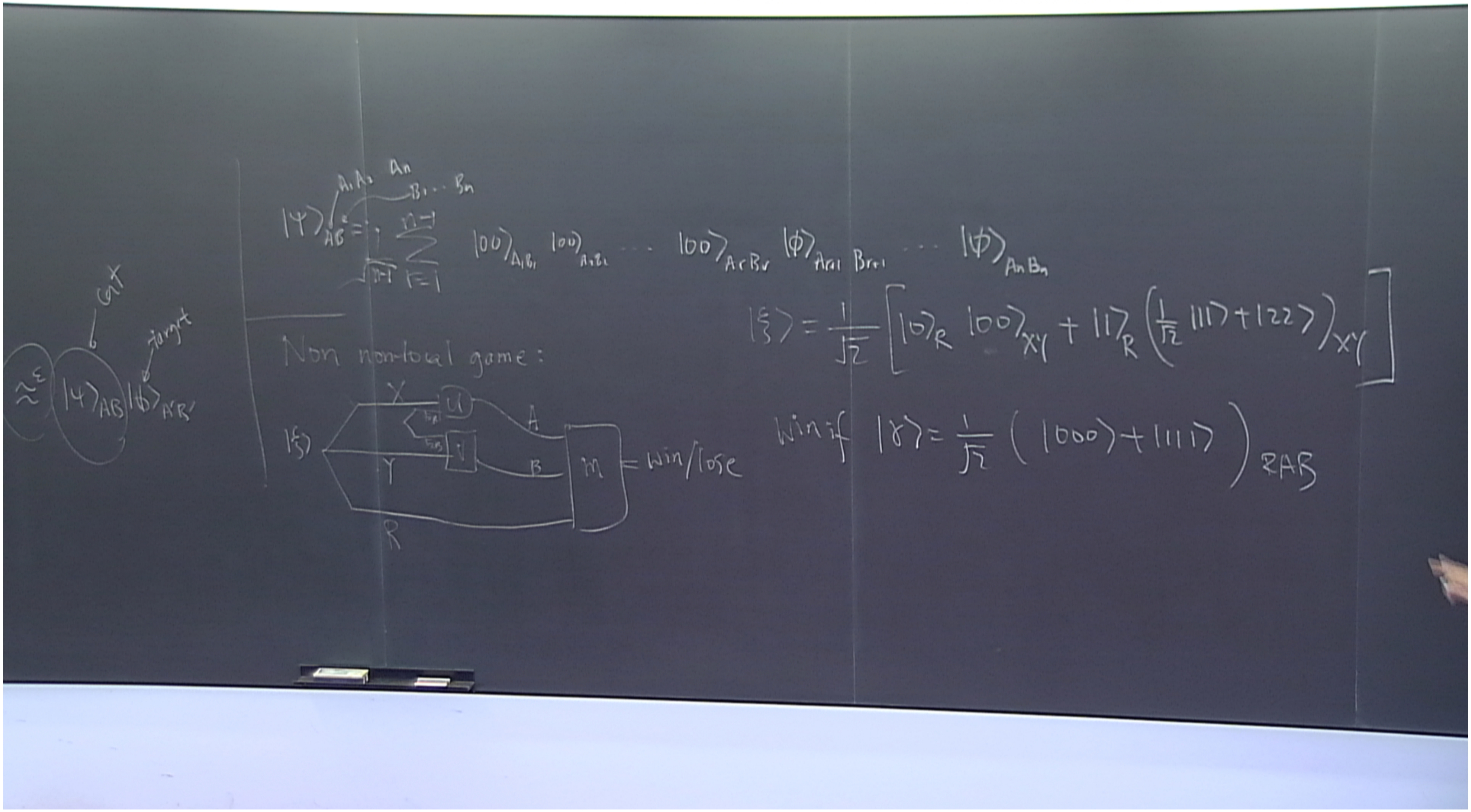

van Dam & Hayden 0201041

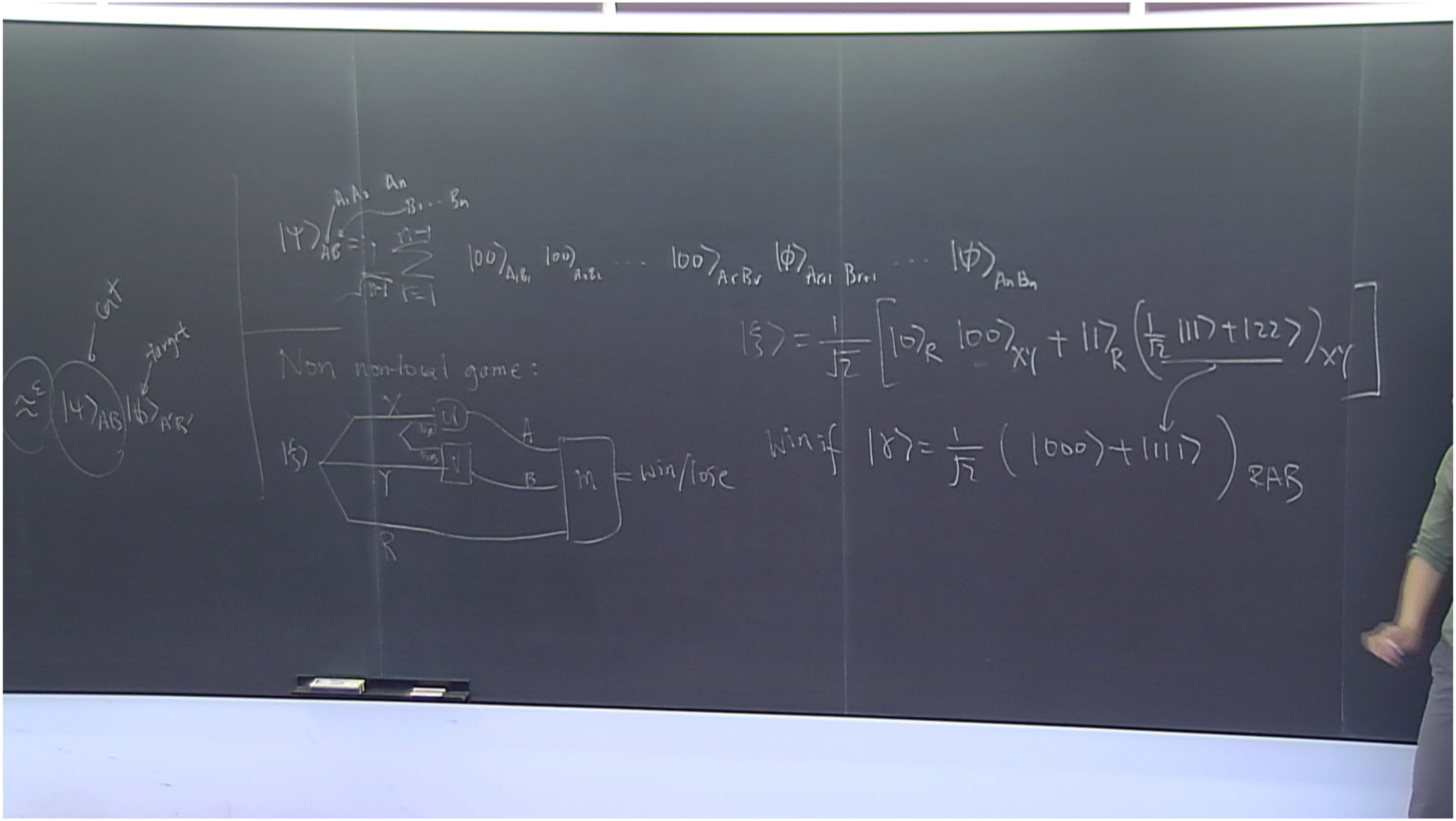

van Dam & Hayden 0201041
∀ ε>0, ∀d |φ⟩A'B' ∈ ℂ^d ⊗ ℂ^d
∃N ∃|ψ⟩AB ∈ ℂ^N ⊗ ℂ^N
s.t. ∃U,V, UAA' ⊗ VBB' (|ψ⟩AB |00⟩A'B') ≈ε |ψ⟩AB |φ⟩A'B'

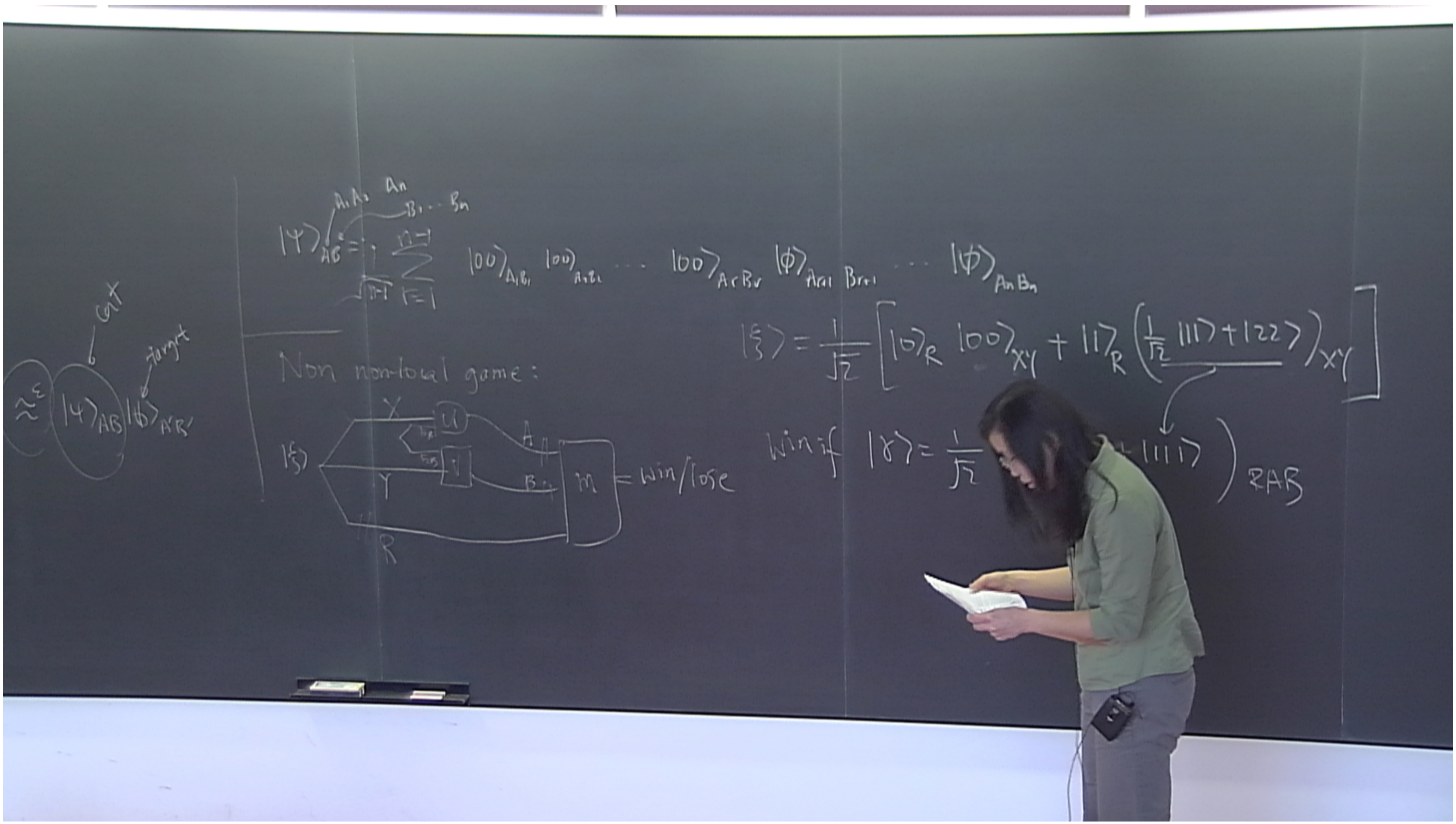

van Dam & Hayden 0201041
∀ ε>0, ∀d |φ⟩A'B' ∈ C^d ⊗ C^d
∃N ∃|ψ⟩AB ∈ C^N ⊗ C^N
s.t. ∃U,V, UAA' ⊗ VBB' (|ψ⟩AB |00⟩A'B') ≈ε |ψ⟩AB |φ⟩A'B'
cat
target

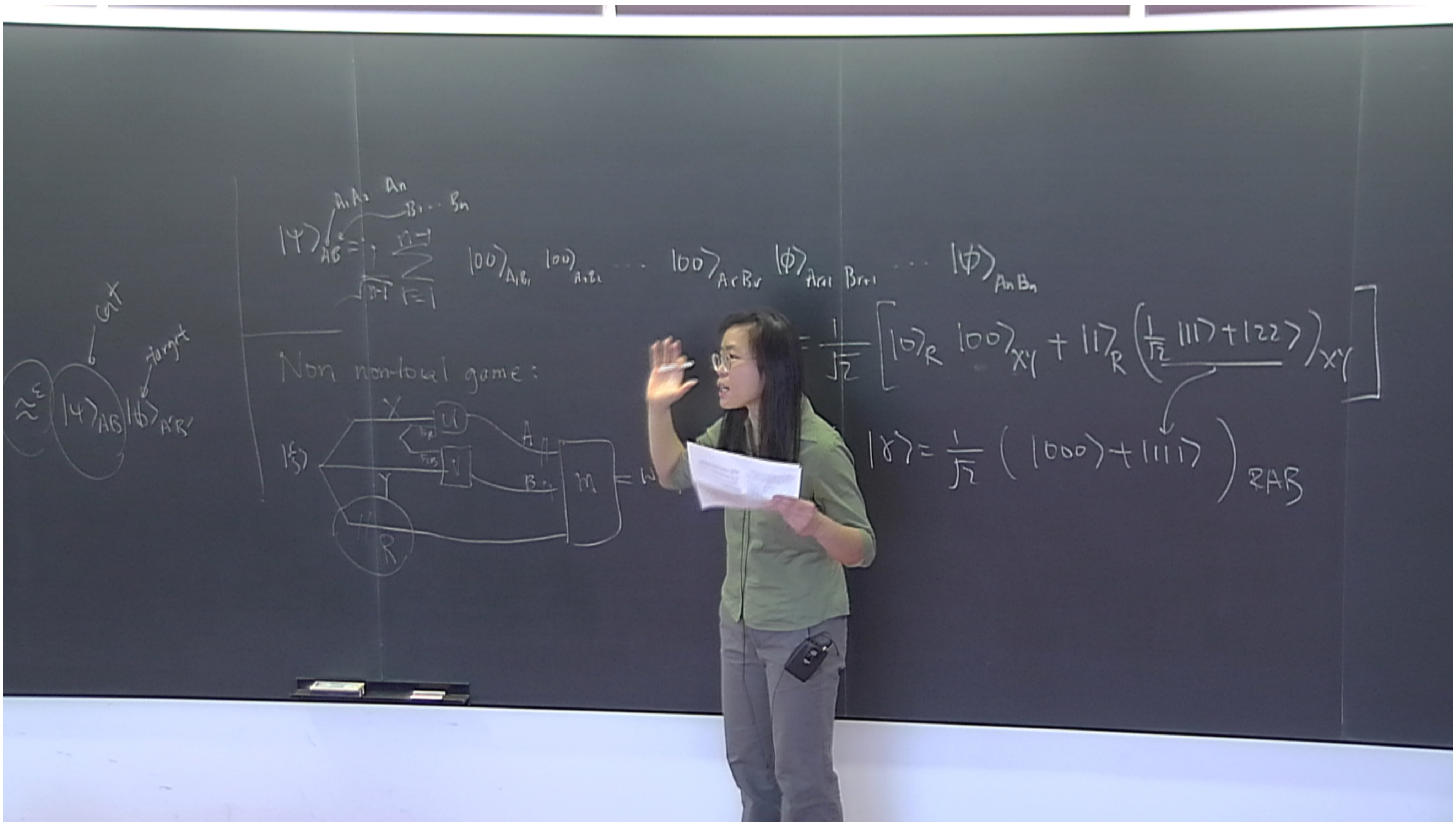


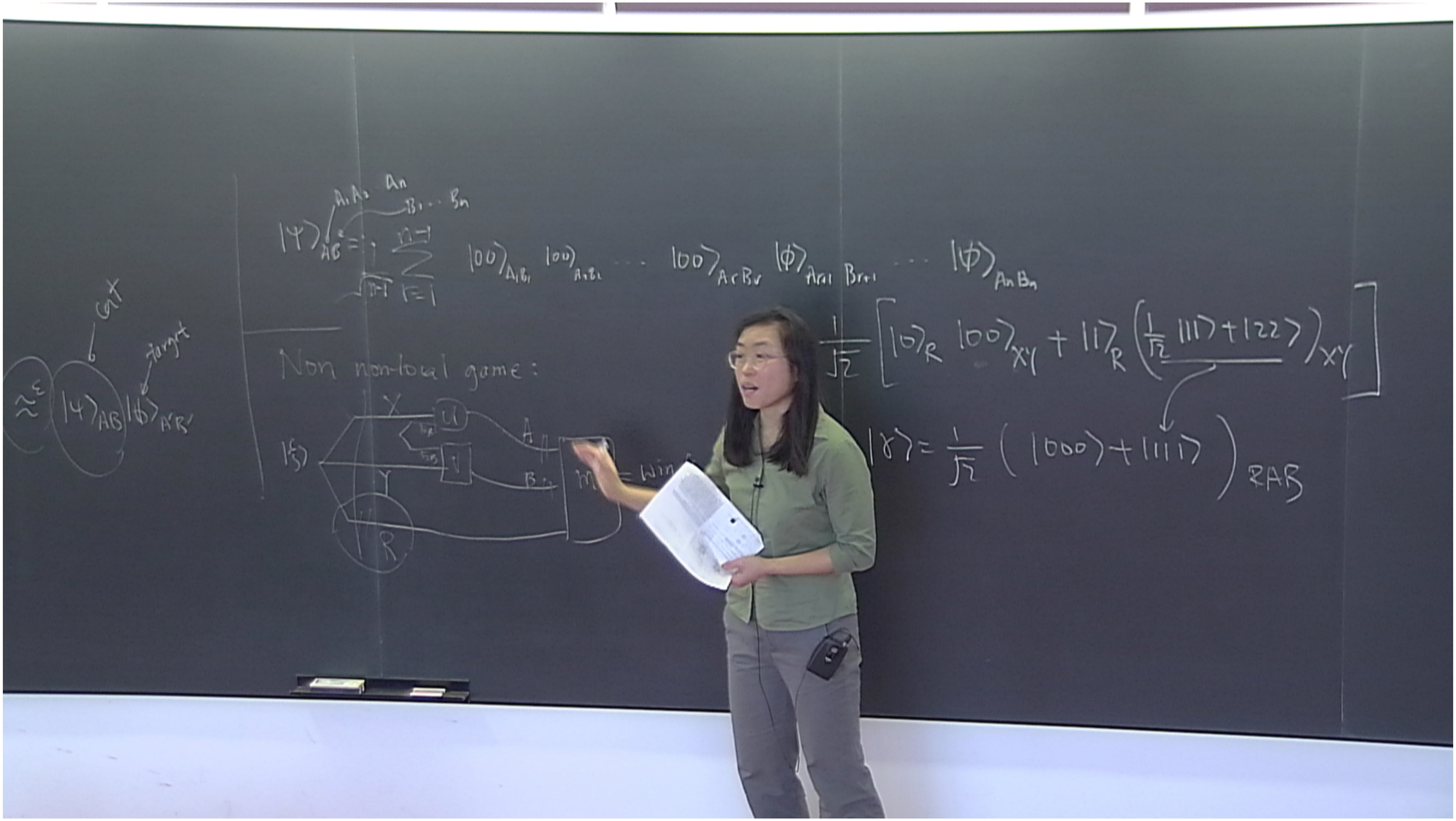

Non non-local game:
win/lose

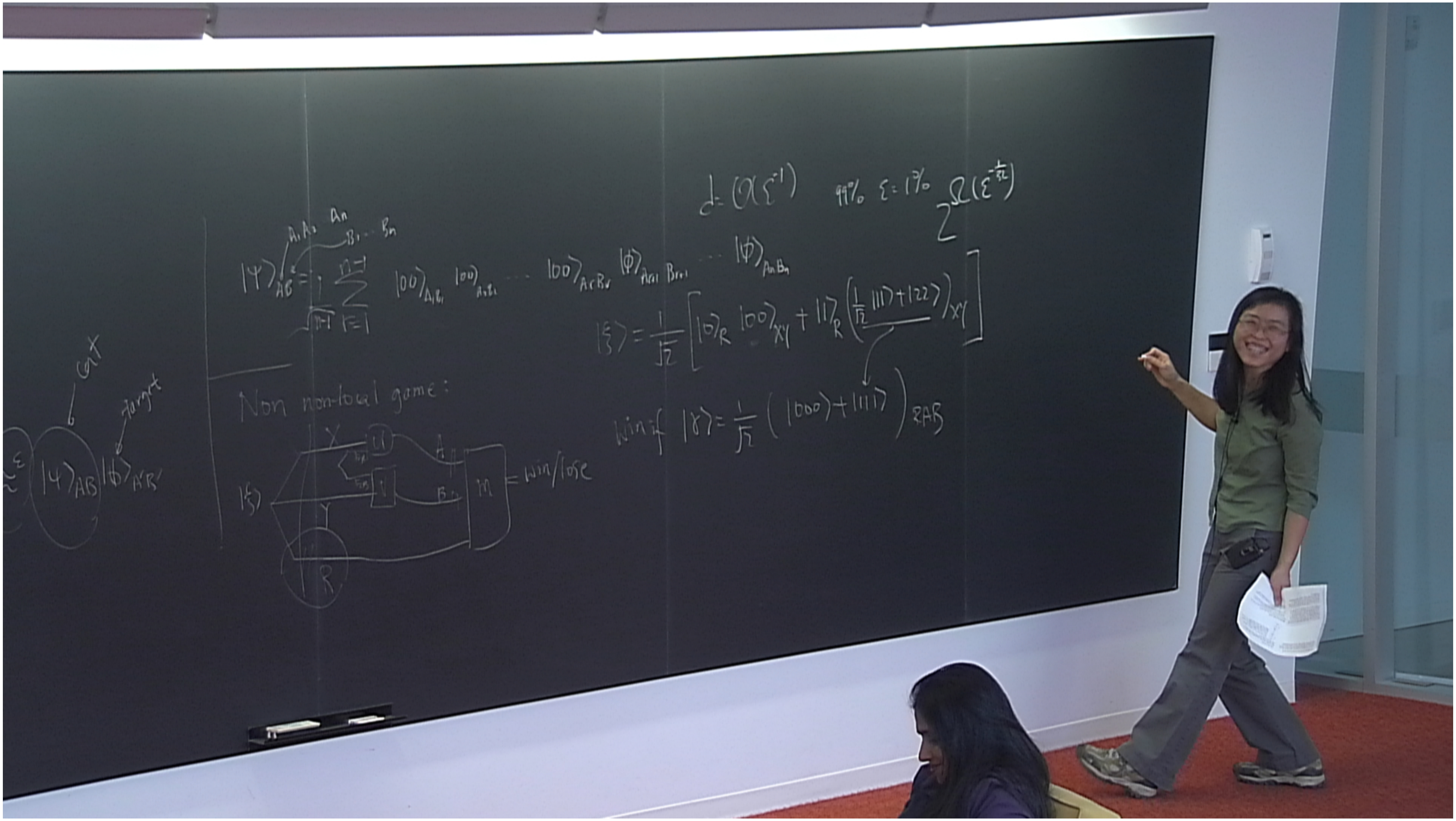

Non non-local game:
win/lose
Win if
cat
target

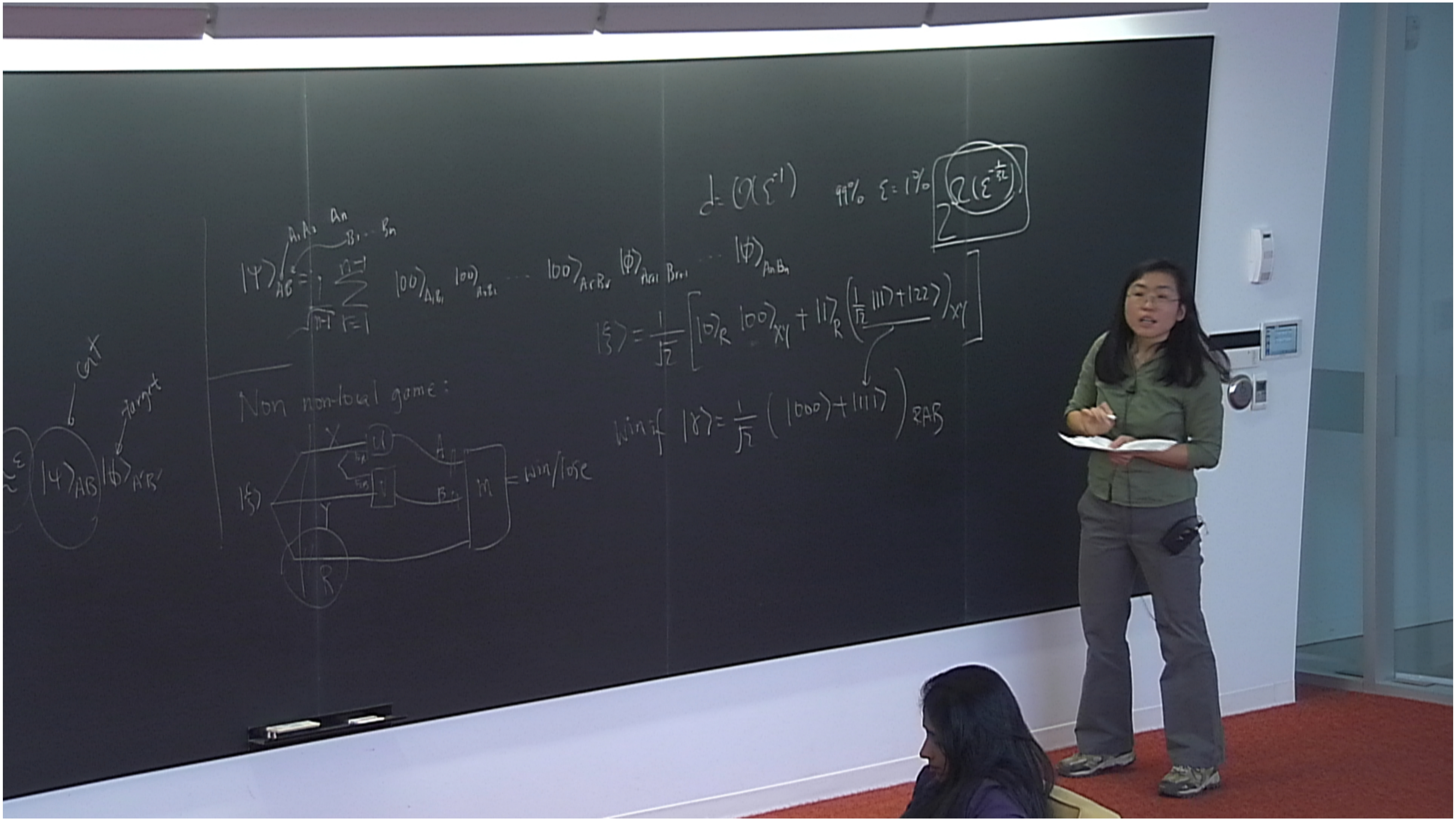

Non non-local game:
win/lose
win if
ctrl
target


Non non-local game:
win/lose
win if
target


Non non-local game:
ctrl
target
R


Non non-local game:
target


Non non-local game:
win/lose
win if
ctrl
target


Non non-local game:
win/lose
target