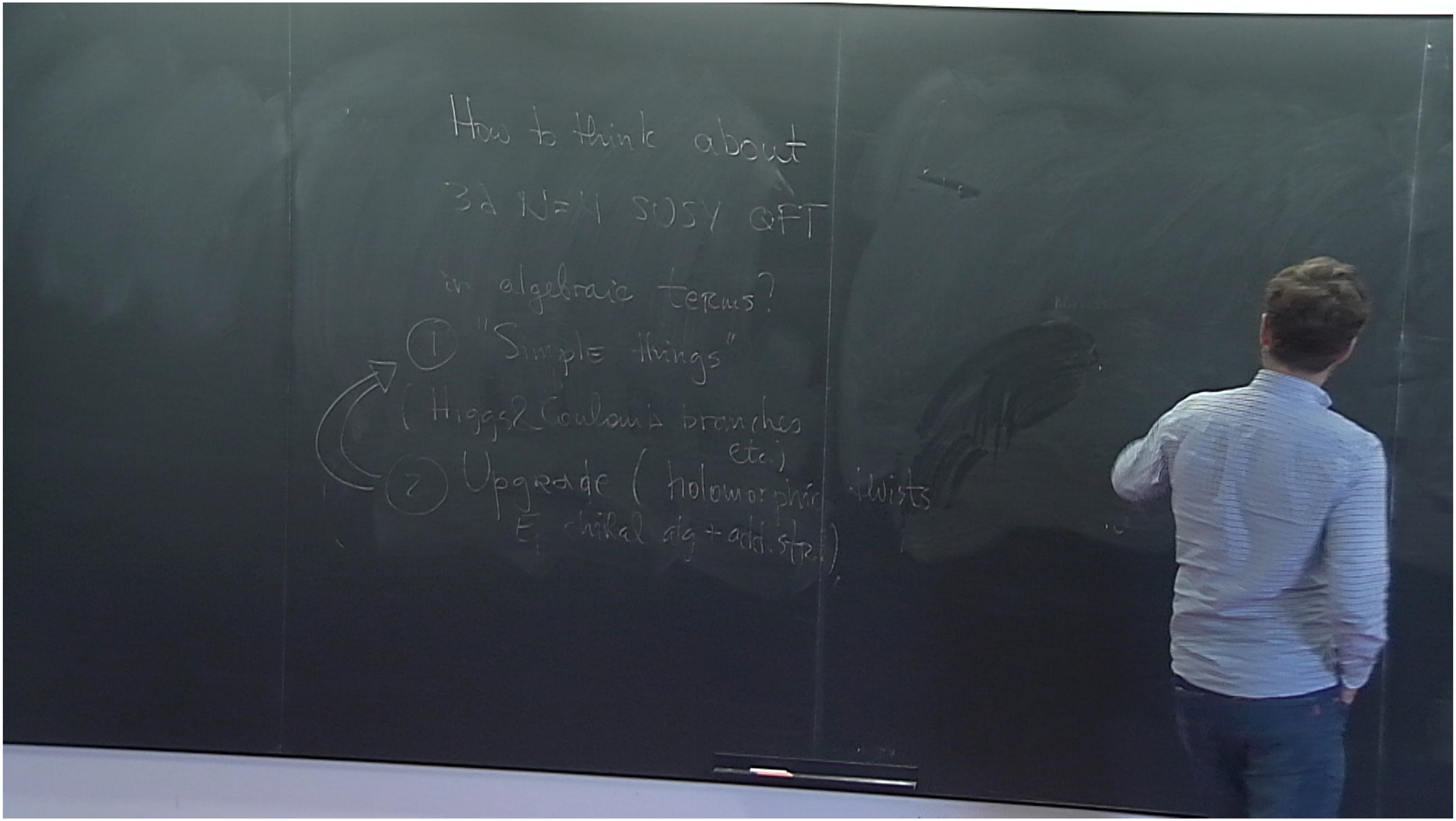
Title: Vertex algebras from holomorphically twisted 3d theories and quasi-classical limit of geometric Langlands duality

Date: Mar 24, 2018 03:30 PM

URL: http://pirsa.org/18030094

Abstract:


How to think about
3d N=4 SUSY QFT
in algebraic terms?
1 "Simple things"
(Higgs & Coulomb branches
etc.)
2 Upgrade (holomorphic twists
E. chiral alg + add. str.)

Operations

① Product $T_1, T_2 \longrightarrow T_1 \times T_2$

$\longrightarrow$ product of $\mathcal{M}_H$ & $\mathcal{M}_C$.

② Mirror symmetry

$T \longrightarrow T^*$ interchanges $\mathcal{M}_C, \mathcal{M}_H$.

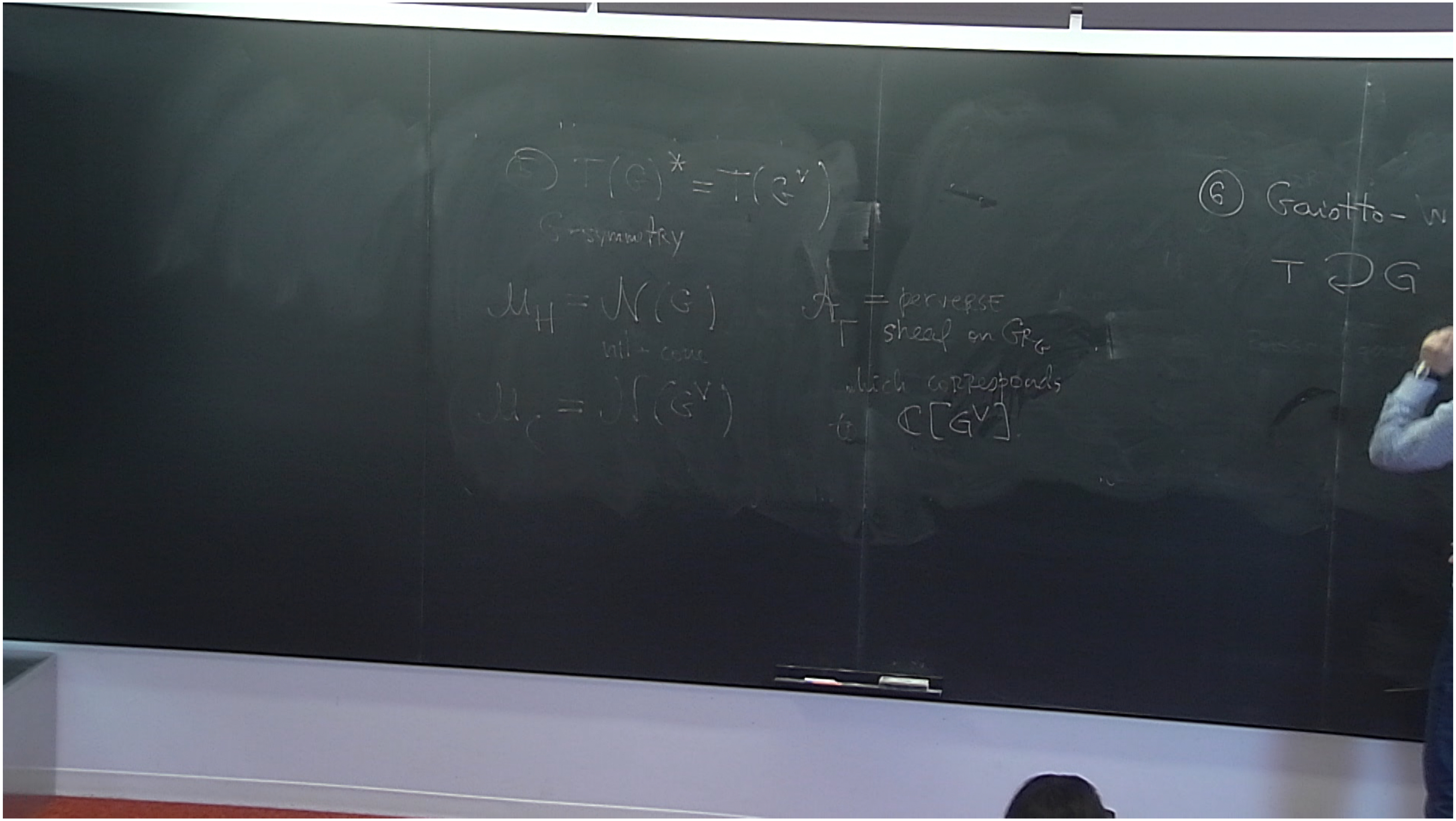
③ $\exists$ notion

G acting on a theory T

$\Longrightarrow$ ⓐ G-action on $\mathcal{M}_H$.

ⓑ Ring object $\mathcal{A}_T$

in $D_{G(\mathcal{O})}(Gr_G)$ s.t. !-stalk at 1 is $\mathbb{C}[\mathcal{M}_C]$.


5 T(G)* = T(G∨)
S-symmetry
M_H = N(G)
nil-cone
M_C = N(G∨)
A_Γ = perverse sheaf on Gr_G
which corresponds to C[G∨]
6 Gaiotto-W
T ⊃ G

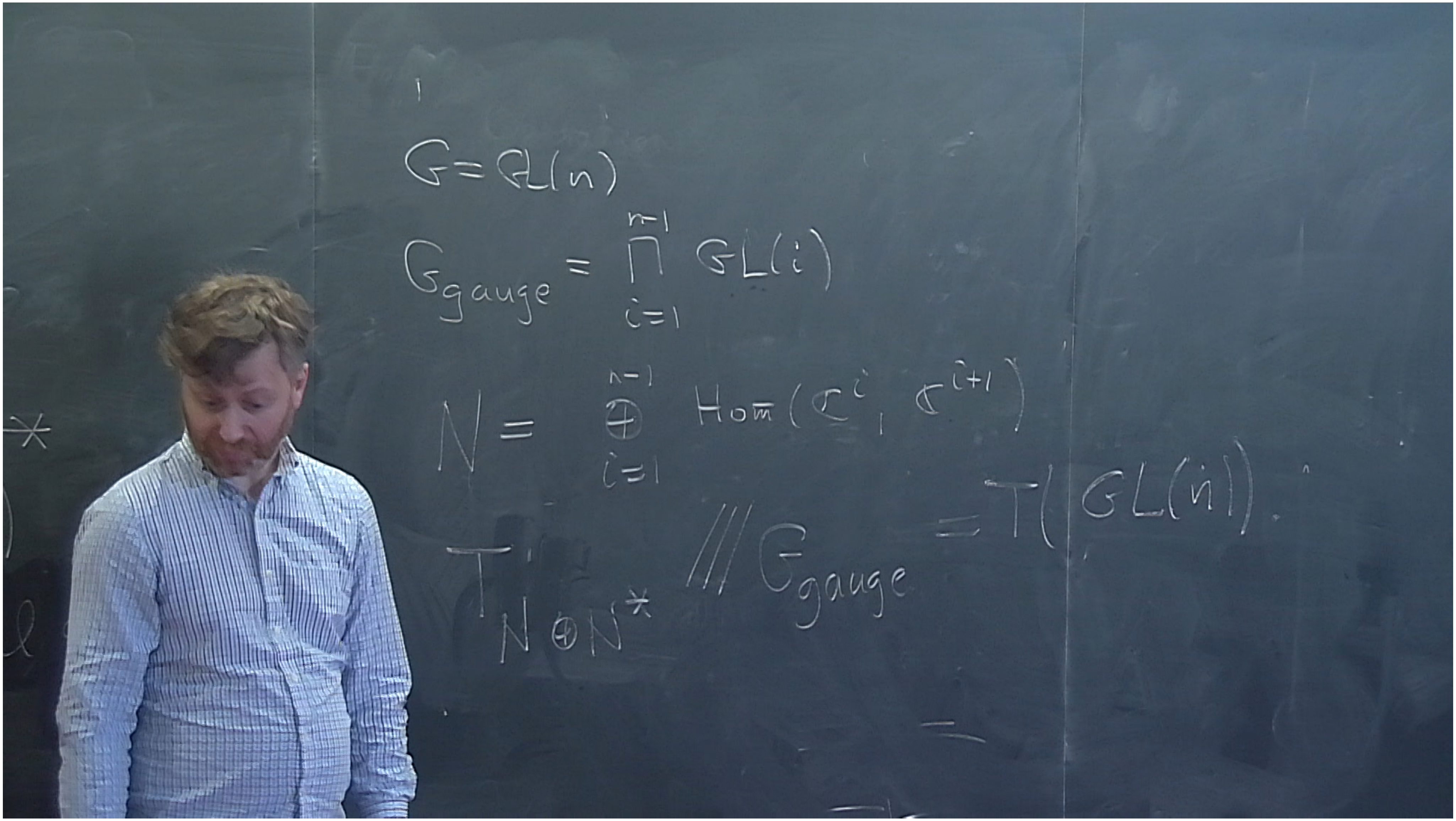
SE
on GR_G
esponds
$V]$

6 Gaiotto-Witten

$$T \circlearrowleft G$$

$$T^\vee \circlearrowleft G^\vee$$

$$T^\vee = \Big(T \times T(G) /\!/\!/ \underset{\substack{\uparrow \\ \text{diagonal copy}}}{G} \Big)^*$$


$G = GL(n)$
$G_{gauge} = \prod_{i=1}^{n-1} GL(i)$
$N = \bigoplus_{i=1}^{n-1} \mathrm{Hom}(\mathbb{C}^i, \mathbb{C}^{i+1})$
$T^*_{N \oplus N^*} /\!/\!/ G_{gauge} = T^*(GL(n))$.

$L(n))$.

(3) $\exists$ notion

G acting on a theory T

$\Longrightarrow$ (a) G-action on $\mathcal{M}_H$

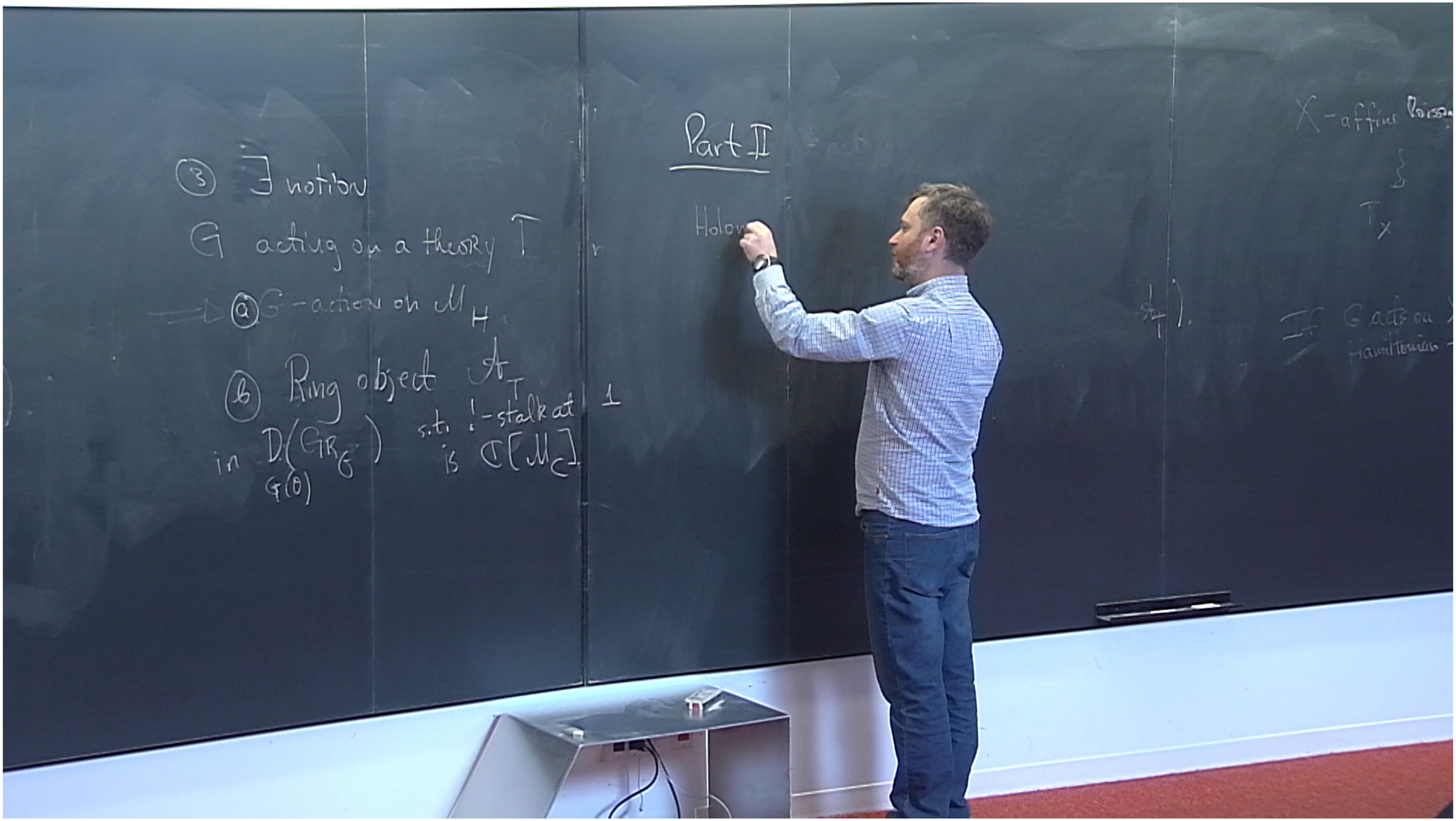
(b) Ring object $\mathcal{A}_T$ in $D_{G(\mathcal{O})}(\mathrm{Gr}_G)$ s.t. !-stalk at 1 is $\mathbb{C}[\mathcal{M}_C]$.

(4) G

$T_1 G$ —

$\mathcal{M}_H(T$

$\mathcal{M}_C(T$


Part II
③ ∃ notion
G acting on a theory T
ⓐ G-action on M_H
ⓑ Ring object A_T
in D_G(O)(Gr_G)
s.t. !-stalk at 1
is C[M_C]
Holom
X - affine Poisson
T_X
If G acts on X
Hamiltonian

notion

...ting on a theory T

-action on $\mathcal{M}_H$

Ring object $\mathcal{A}_T$

(Gr_G) s.t. !-stalk at 1

is $\mathbb{C}[\mathcal{M}_C]$.

(0)

Part II

Holomorphic twist

(some super-symmetry is lost).

$\uparrow$

E_1 - vertex algebra

= associative alg in the category of VOA

$\mathbb{V} \supset \mathrm{Vir}_+$ = vector fields on $\mathrm{Spec}\, \mathbb{C}[[z]]$

Super-symmetry means that some

$S\mathrm{Vir}_+$ acts.

$S\mathrm{Vir}_+$ - vector fields on $\mathrm{Spec}\, \mathbb{C}[[z, \varepsilon]]$

z -- even $\quad \varepsilon^2 = 0$

ε - odd.

$$Q_H = \varepsilon \frac{\partial}{\partial z}$$

$$Q_C = \frac{\partial}{\partial \varepsilon}$$

Expectation

$$H^*(\mathbb{V}_{T^1}, Q_H) = \mathbb{C}[\mathcal{M}_H]$$

$$H^*(\mathbb{V}_{T^1}, Q_C) = \mathbb{C}[\mathcal{M}_C]$$

Lemma

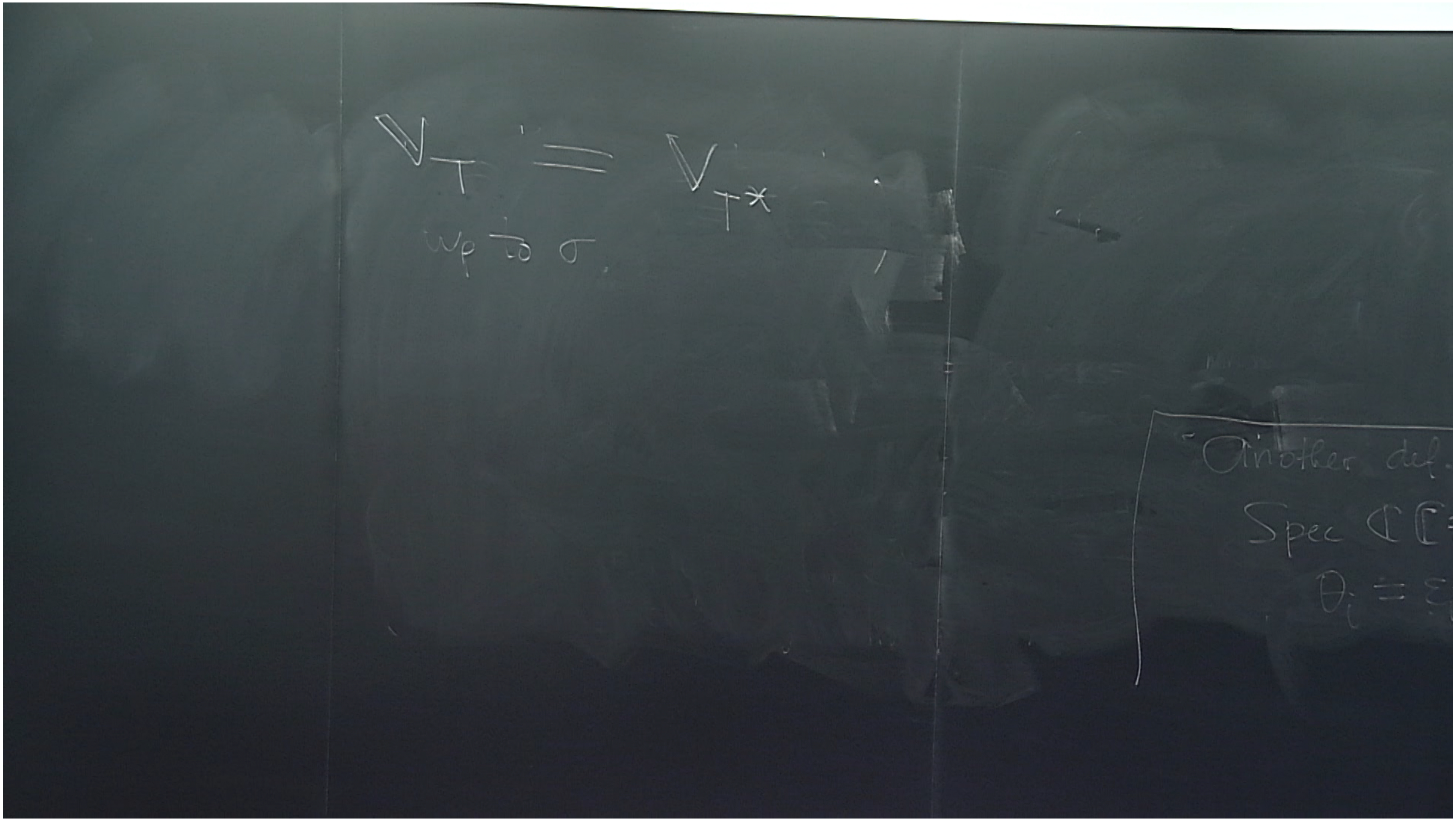
$SVir_+$ has an involution σ

$\sigma(Q_c) = Q_H$

"Another def. on $SVir_+$"

$\operatorname{Spec} \mathbb{C}[\![z, \varepsilon_1, \varepsilon_2]\!]$ $\quad$ z – even, $\varepsilon_1, \varepsilon_2$ – odd.

$$\theta_i \doteq \varepsilon_i \frac{\partial}{\partial z} + \frac{\partial}{\partial \varepsilon_i}$$


$\mathbb{V}_T = \mathbb{V}_{T^*}$
w_p to σ.
"another def.
Spec $\mathbb{C}[[$
$\theta_i = $

an involution σ

Q_H

z - even
$\varepsilon_1, \varepsilon_2$ - odd.

Let L be the dist spanned by θ_1, θ_2

Lemma

$SViR_+$ = vector fields on $\mathrm{Spec}\, \mathbb{C}[[z, \varepsilon_1, \varepsilon_2]]$ preserving L.

③ $\exists$

G act

⟹ ⓐ G

ⓑ R

in $D_{\cdot}(\ $
$G(0)$

Let L be the dist spanned by θ_1, θ_2

Lemma

$SViR_+$ = vector fields on $\mathrm{Spec}\ \mathbb{C}[[z, \varepsilon_1, \varepsilon_2]]$ preserving L.

Let X be a symplectic dg-stack

<u>Ex.</u> G-reductive group

N - rep. of G.

$$\mu : T^*N \longrightarrow \mathfrak{g}^*$$
$$T^*N = N \oplus N^*$$

$$\mu^{-1}(0)/G = X$$

conical : $\mathbb{C}^*$ acts on X.

acts on ω (symp. form) by standard character.

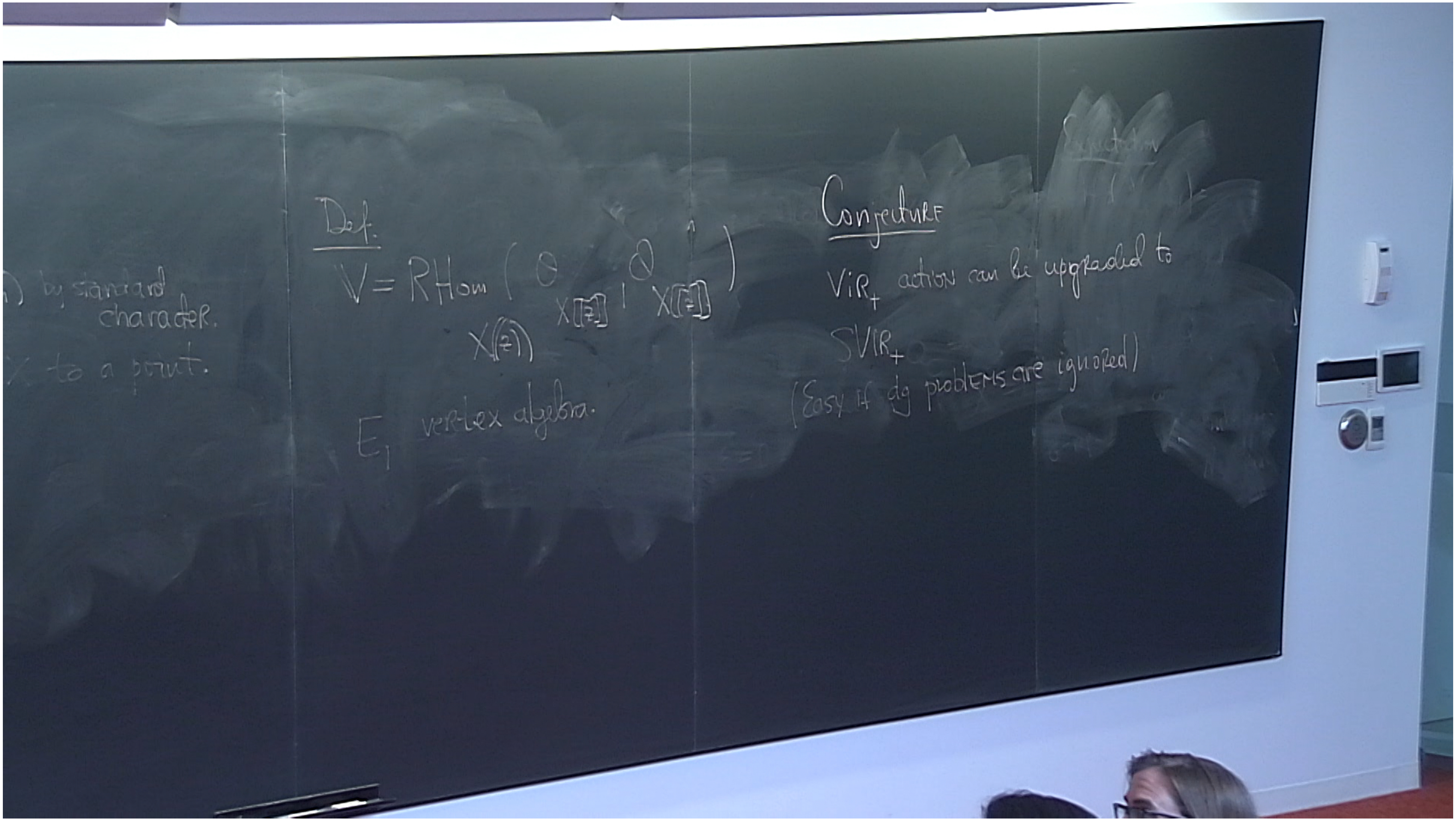
Contracts all of X to a point.

Rm) by standard character.

X to a point.

Def.

$$\mathbb{V} = R\mathrm{Hom}_{X((z))}\left(\mathcal{O}_{X[[z]]}, \hat{\mathcal{O}}_{X[[z]]}\right)$$


) by standard
character.
X to a point.
Def.
V = RHom(O_{X[[z]]}, O_{X[[z]]})
X((z))
E_1 vertex algebra.
Conjecture
Vir_+ action can be upgraded to
SVir_+
(Easy if dg problems are ignored)

$T^{*}_{G,N}$ is sometimes of the form

$T^{*}_{G',N'}$

Ex $X = \mathbb{C}^2$.

$T^{*}_{\mathbb{C}^2} = T^{*}_{G,\ \text{standard char.}}$

$G = \mathbb{C}^*$

$\mathbb{C}^2 \xrightarrow{\mu} \mathbb{C}$

$(x,y) \to xy$

$\mu^{-1}(0)/\mathbb{C}^*$

"another

Spec

$\theta_i =$

e dist spanned

$, \theta_2$

vector fields on

$\mathbb{C}[[z, \varepsilon_1, \varepsilon_2]]$ preserving L.

Assume that G acts on T

$G[[z]]$ must act on V_T.

$G[[z]]$-equiv. quasi-coherent sheaf on $Gr_G = G((z))/G[[z]]$

(can be upgraded to a sheaf on $T^* Gr$).

conical

ts on T ... stack

on V_T.

si-coherent

$G((z))/G[[z]]$

to a sheaf on $T^*(Gr)$

algebra.

conical: $\mathbb{C}^*$ acts on X

acts on ω (symp. form) by standard character.

Contracts all of X to a point.

$T^* Gr_G \supset St_G$ $\qquad St_{G^\vee} \subset T^* Gr_{G^\vee}$

$QCoh_{G[[z]]}(St_G) \simeq QCoh_{G^\vee[[z]]}(St_{G^\vee})$

Def.

$\mathbb{V} = RHom$

X

E_1 verte