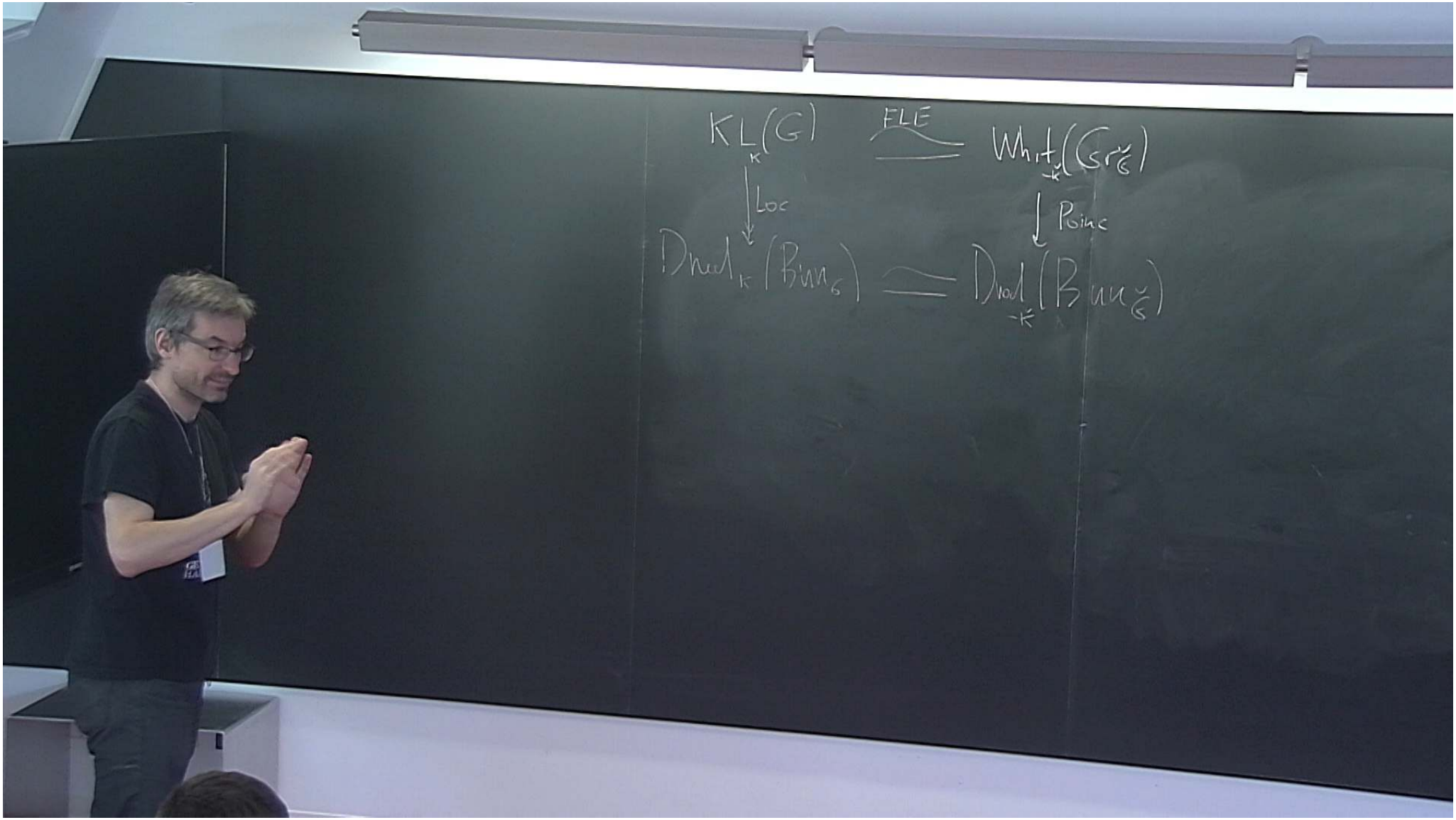
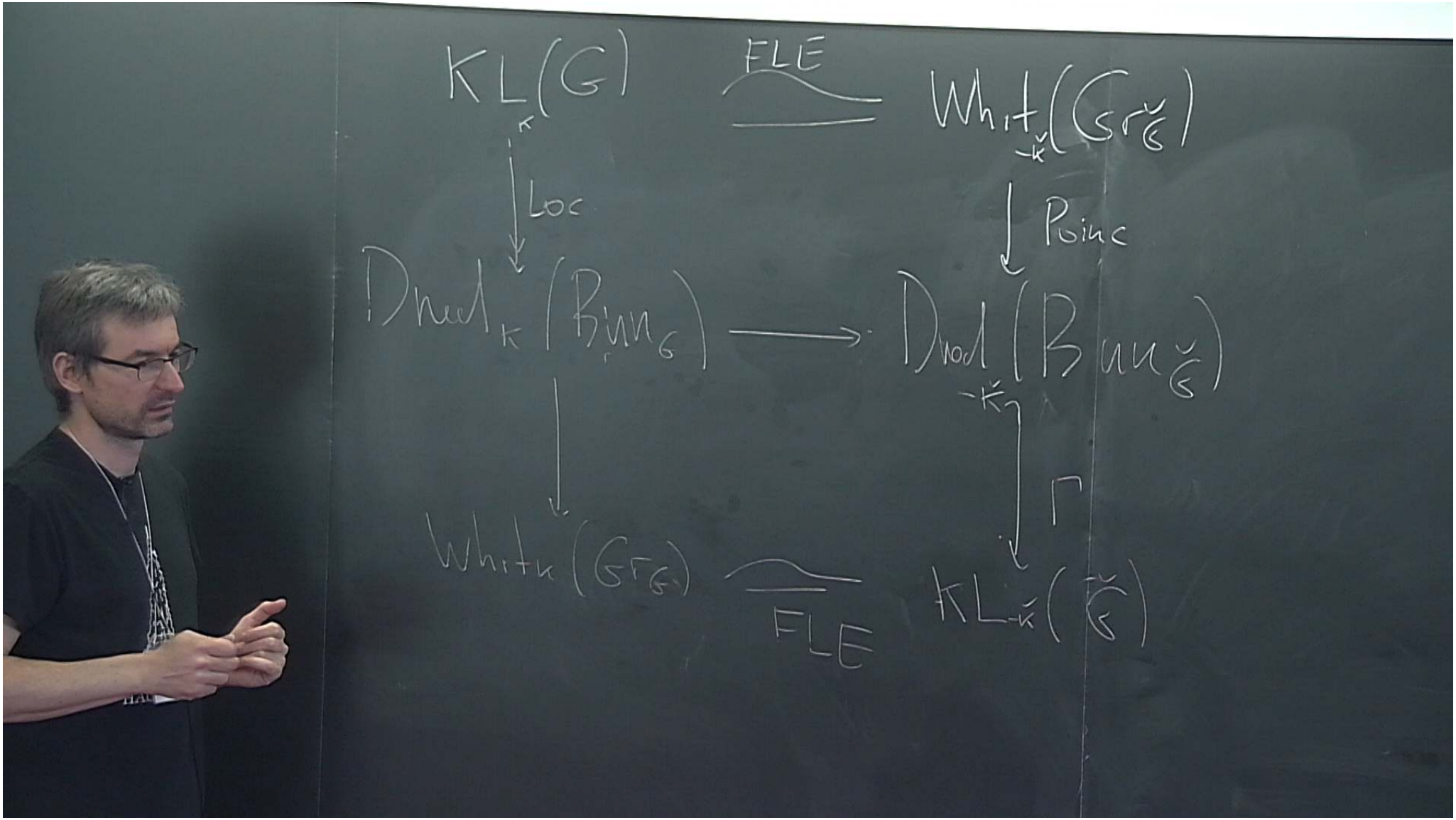
Title: The Master Chiral Algebra

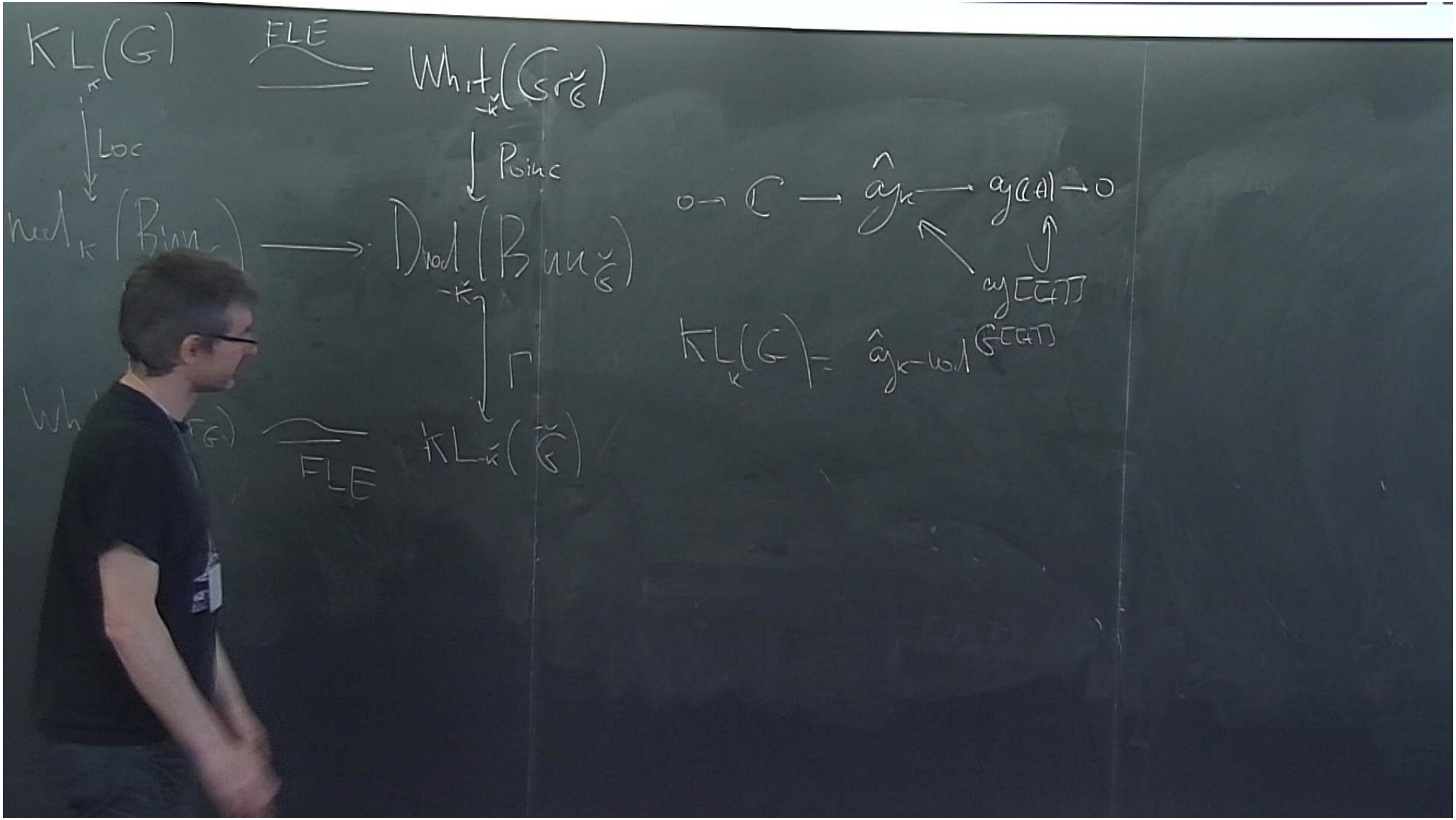
Date: Mar 23, 2018 02:00 PM

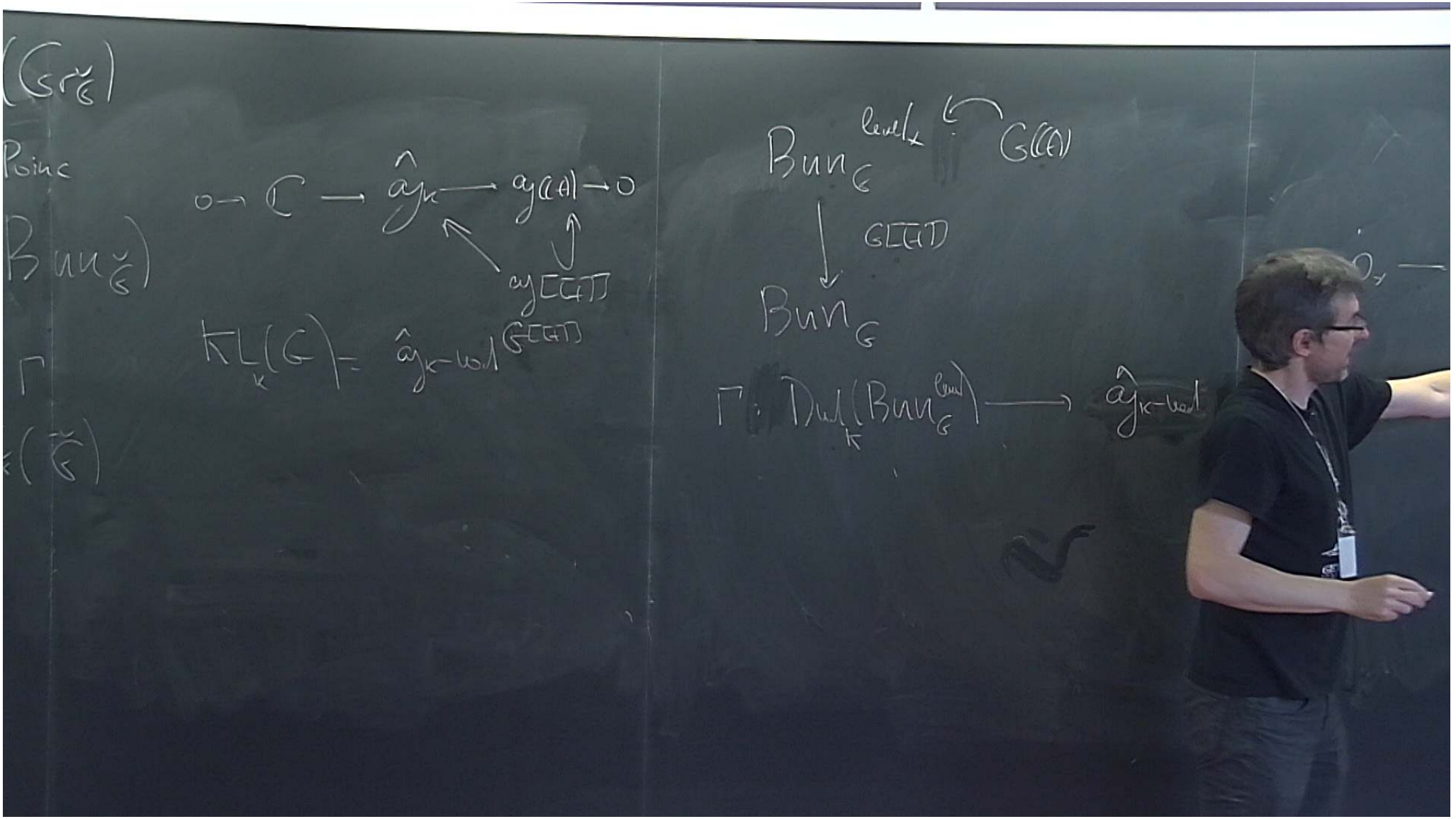
URL: http://pirsa.org/18030089

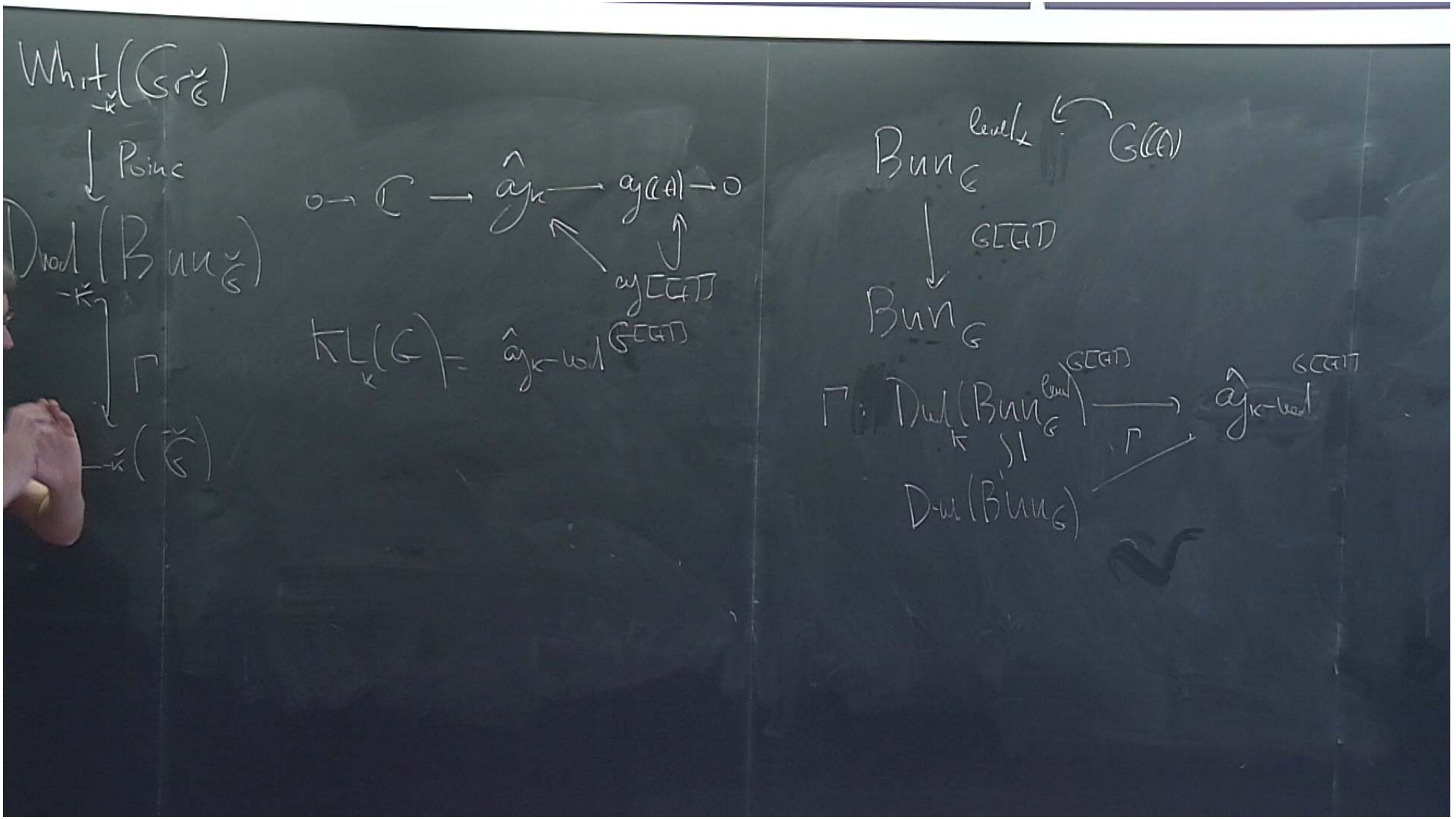
Abstract:


KL(G)
FLE
Loc
Poinc


KL_κ(G)
FLE
Whit_{-κ̌}(Gr_{Ǧ})
Loc
Poinc
Dmod_κ(Bun_G)
Dmod_{-κ̌}(Bun_{Ǧ})
Γ
Whit_κ(Gr_G)
FLE
KL_{-κ̌}(Ǧ)


KL_κ(G)
FLE
Whit_{-κ̌}(Gr_{Ǧ})
Loc
Poinc
Dmod_{-κ̌}(Bun_{Ǧ})
Γ
KL_{-κ̌}(Ǧ)
FLE
0 → ℂ → ĝ_κ → g((t)) → 0
g[[t]]
KL_κ(G) = ĝ_κ-mod^{G[[t]]}


Poinc
0 → ℂ → ĝ_κ → g((t)) → 0
g[[t]]
KL_κ(G) = ĝ_κ-mod^{G[[t]]}
Bun_G^{level_x} ↺ G((t))
G[[t]]
Bun_G
Γ : Dmod_κ(Bun_G^{level}) → ĝ_κ-mod



$$\mathrm{Whit}_{-\check{\kappa}}(\mathrm{Gr}_{\check{G}})$$

$$\downarrow \text{Poinc}$$

$$\mathrm{Dmod}_{-\check{\kappa}}(\mathrm{Bun}_{\check{G}})$$

$$\downarrow \Gamma$$

$$\mathrm{Dmod}_{-\check{\kappa}}(\check{G})$$

$$0 \to \mathbb{C} \to \hat{\mathfrak{g}}_\kappa \to \mathfrak{g}((t)) \to 0$$

$$\mathfrak{g}[[t]] \hookrightarrow \mathfrak{g}((t))$$

$$KL_\kappa(G) = \hat{\mathfrak{g}}_\kappa\text{-mod}^{G[[t]]}$$

$$\mathrm{Bun}_G^{\mathrm{level}_\kappa} \circlearrowleft G((t))$$

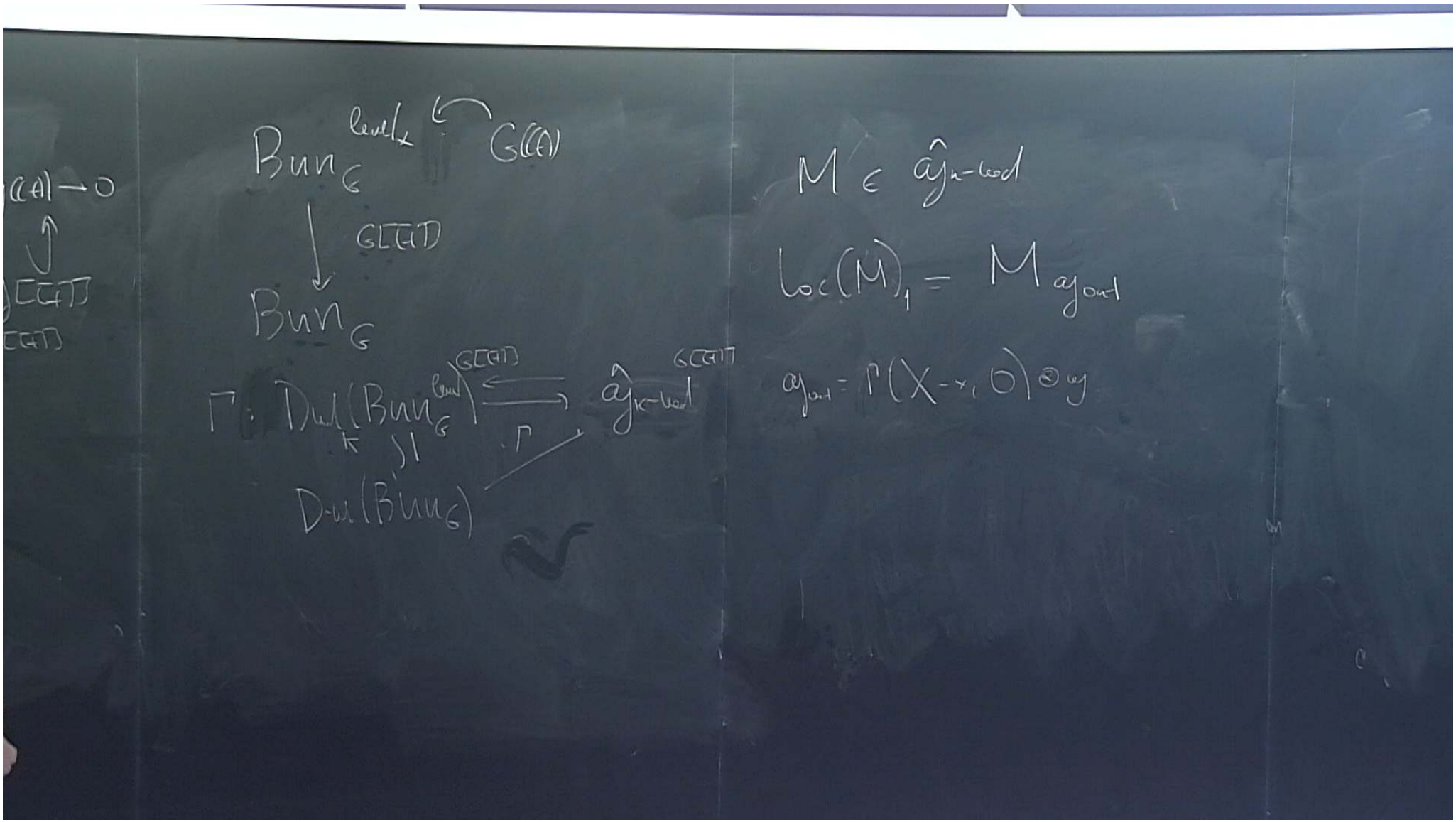
$$\downarrow G[[t]]$$

$$\mathrm{Bun}_G$$

$$\Gamma : \mathrm{Dmod}_\kappa(\mathrm{Bun}_G^{\mathrm{level}})^{G[[t]]} \longrightarrow \hat{\mathfrak{g}}_\kappa\text{-mod}^{G[[t]]}$$

$$\wr\!\!\mid$$

$$\mathrm{Dmod}(\mathrm{Bun}_G) \xrightarrow{\Gamma} \hat{\mathfrak{g}}_\kappa\text{-mod}^{G[[t]]}$$



$$\mathrm{Bun}_G^{\mathrm{level}_\kappa} \circlearrowleft G((t))$$

$$\downarrow G[[t]]$$

$$\mathrm{Bun}_G$$

$$\Gamma : \mathrm{Dmod}_\kappa(\mathrm{Bun}_G^{\mathrm{level}})^{G[[t]]} \rightleftarrows \hat{\mathfrak{g}}_\kappa\text{-mod}^{G[[t]]}$$

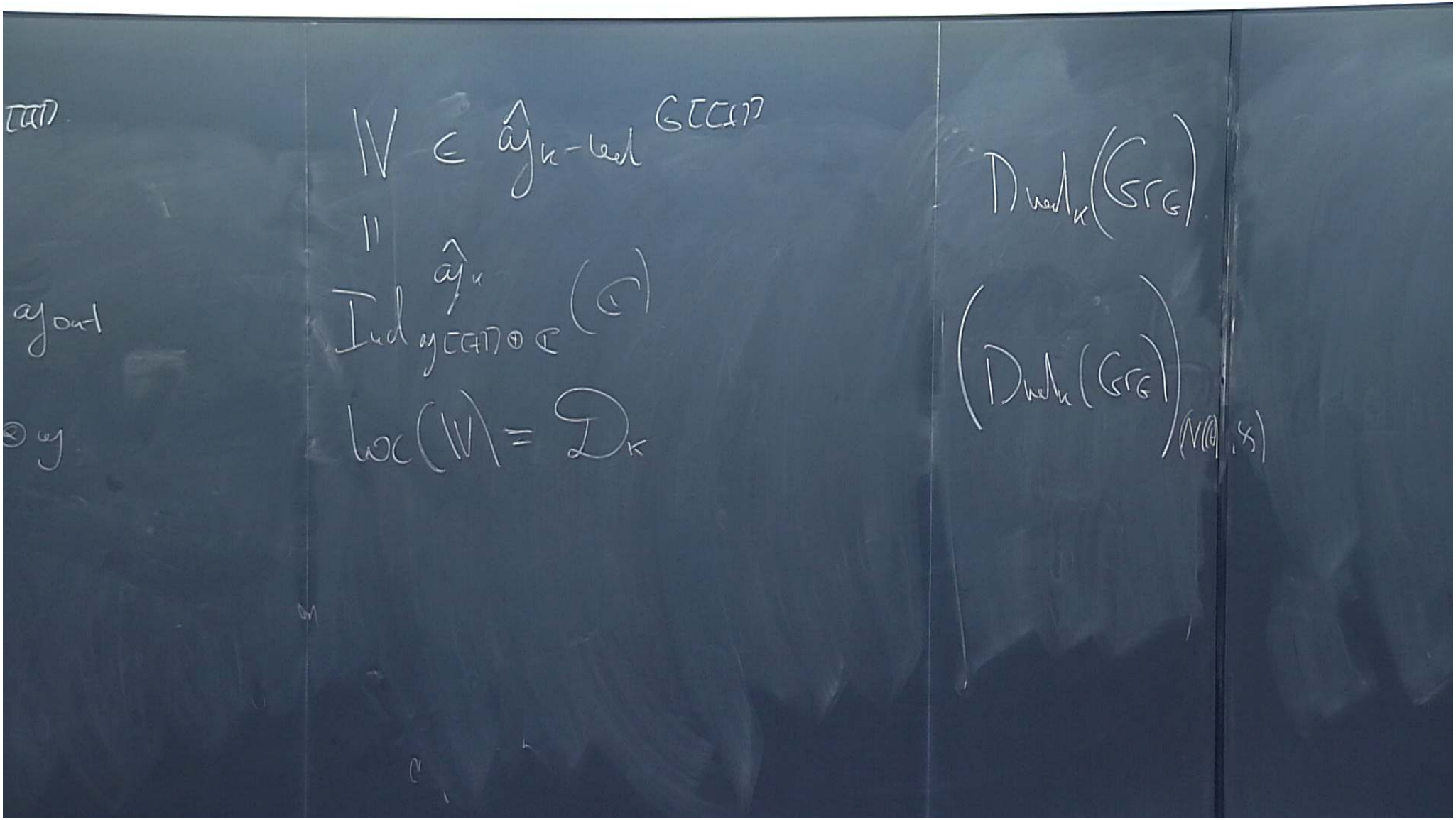
$$\wr\!\!\mid \qquad \Gamma$$

$$\mathrm{D\text{-}mod}(\mathrm{Bun}_G)$$

$$M \in \hat{\mathfrak{g}}_\kappa\text{-mod}$$

$$\mathrm{Loc}(M)_1 = M_{\mathfrak{g}_{out}}$$

$$\mathfrak{g}_{out} = \Gamma(X - x, \mathcal{O}) \otimes \mathfrak{g}$$



$[[t]]$

$\mathfrak{g}_{out}$

$\otimes \mathfrak{g}$

$$\mathbb{V} \in \hat{\mathfrak{g}}_\kappa\text{-mod}^{G[[t]]}$$

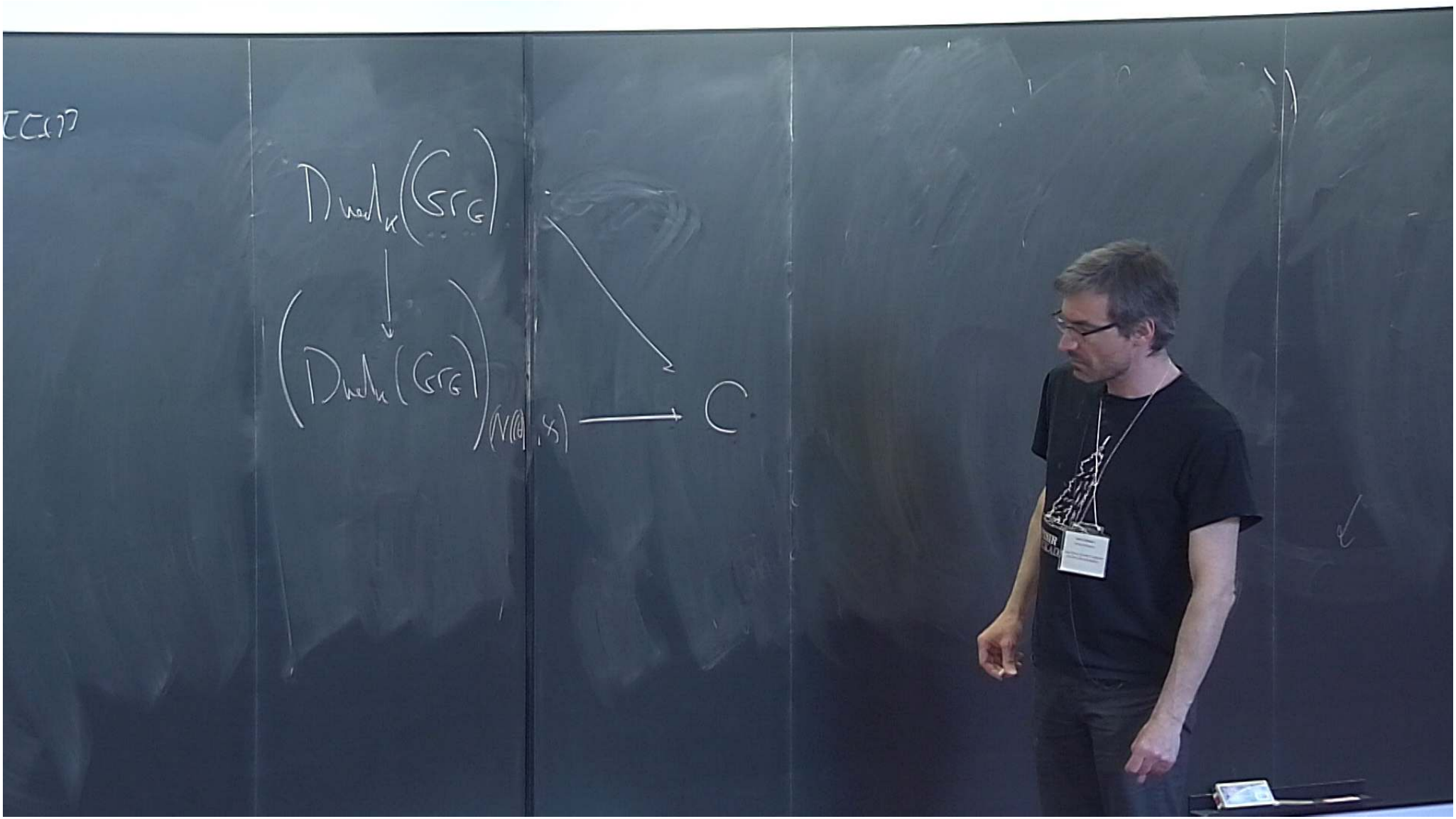
$$\| $$

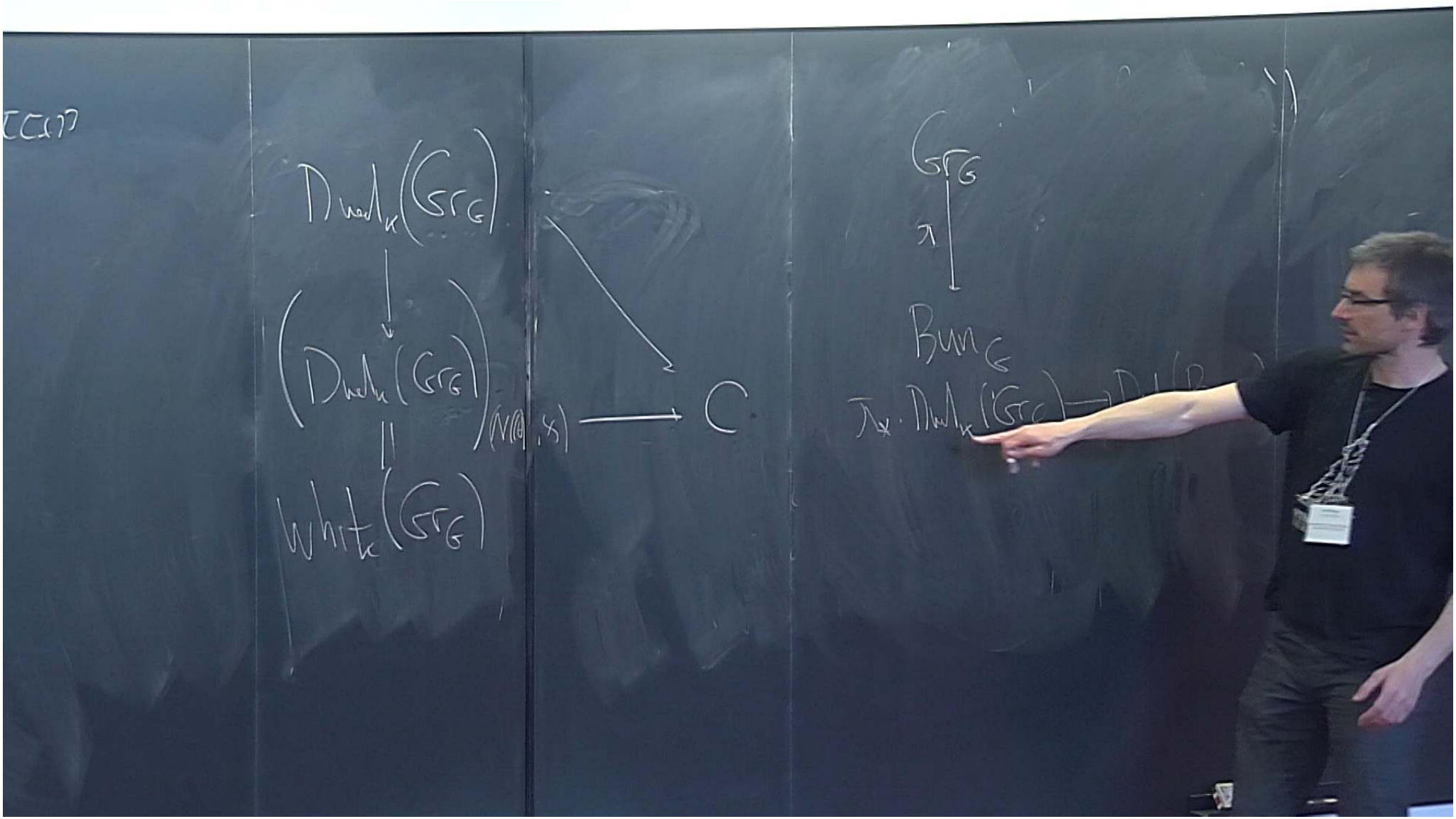
$$\mathrm{Ind}_{\mathfrak{g}[[t]]\oplus\mathbb{C}}^{\hat{\mathfrak{g}}_\kappa}(\mathbb{C})$$

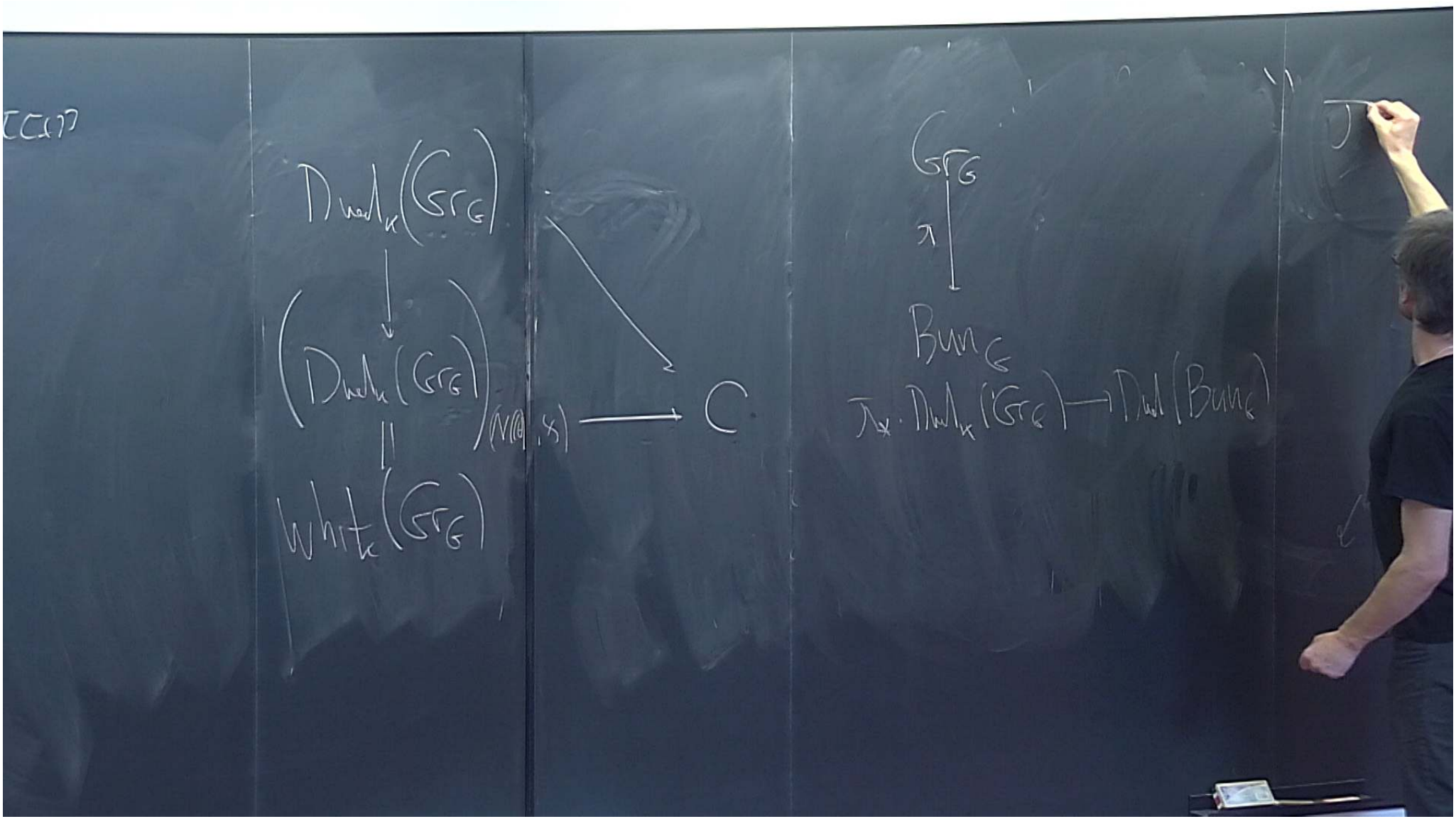
$$\mathrm{loc}(\mathbb{V}) = \mathcal{D}_\kappa$$

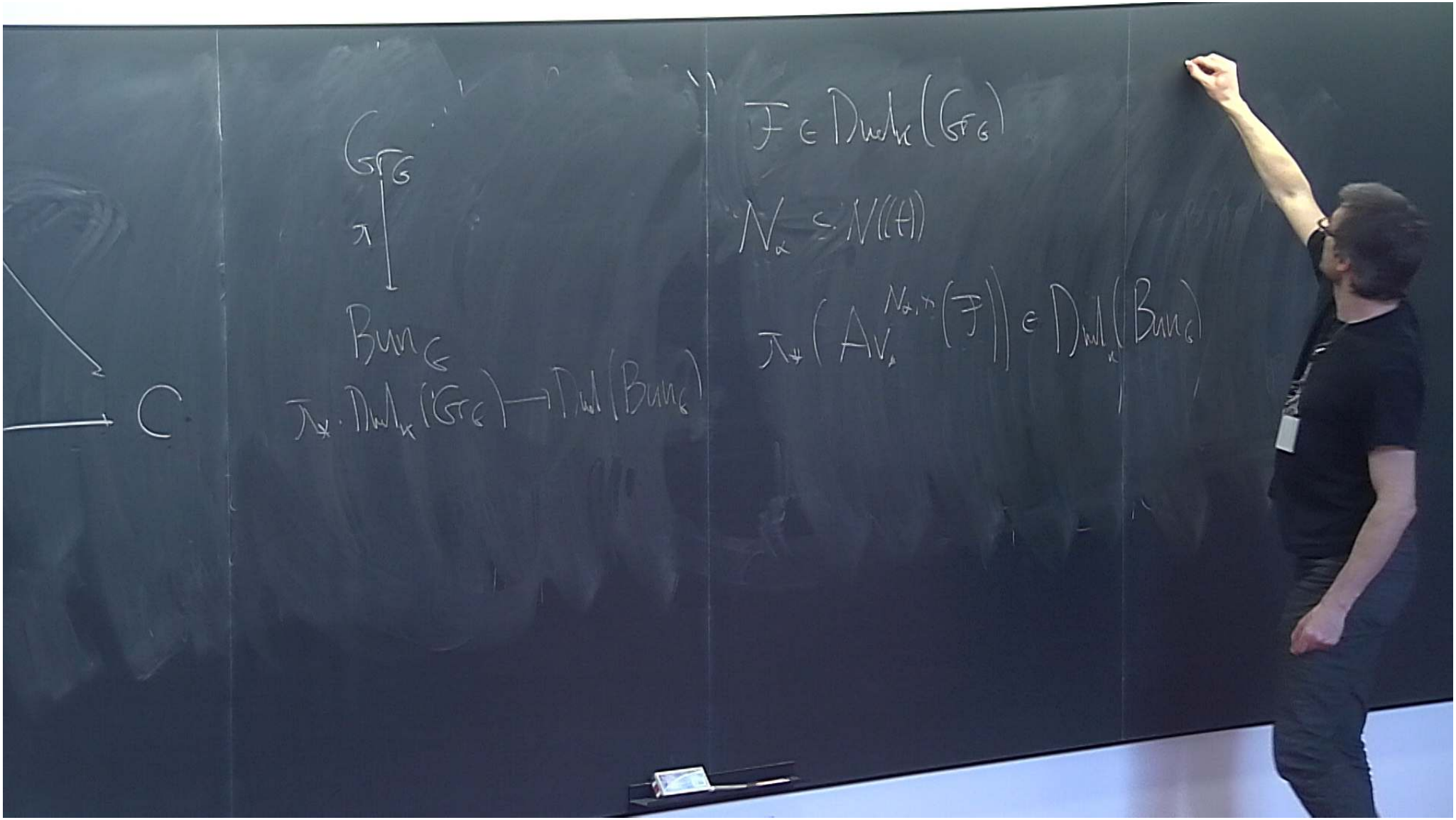
$$\mathrm{Dmod}_\kappa(\mathrm{Gr}_G)$$

$$\left(\mathrm{Dmod}_\kappa(\mathrm{Gr}_G)\right)_{(N(G),\chi)}$$


C


Dmod_k(Gr_G)
(Dmod_k(Gr_G))
(N(G), x)
C
Whit_k(Gr_G)
Gr_G
π
Bun_G
π_* · Dmod_k(Gr_G)


$\mathrm{Dmod}_\kappa(\mathrm{Gr}_G)$
$\mathrm{Whit}_\kappa(\mathrm{Gr}_G)$
Gr_G
Bun_G
$\pi_* \cdot \mathrm{Dmod}_\kappa(\mathrm{Gr}_G) \to \mathrm{Dmod}(\mathrm{Bun}_G)$


C
$G_F G$
Bun_G
$\pi_* \cdot D_{mod_K}(G_F G) \to D_{mod}(\mathrm{Bun}_G)$
$\mathcal{F} \in D_{mod_K}(G_F G)$
$N_\alpha \subseteq N((t))$
$\pi_* \left(Av_!^{N_\alpha, \psi}(\mathcal{F}) \right) \in D_{mod_\alpha}(\mathrm{Bun}_G)$

$\mathcal{F} \in \mathrm{Dmod}_k(\mathrm{Gr}_G)$

$N_\alpha \subseteq N((t))$

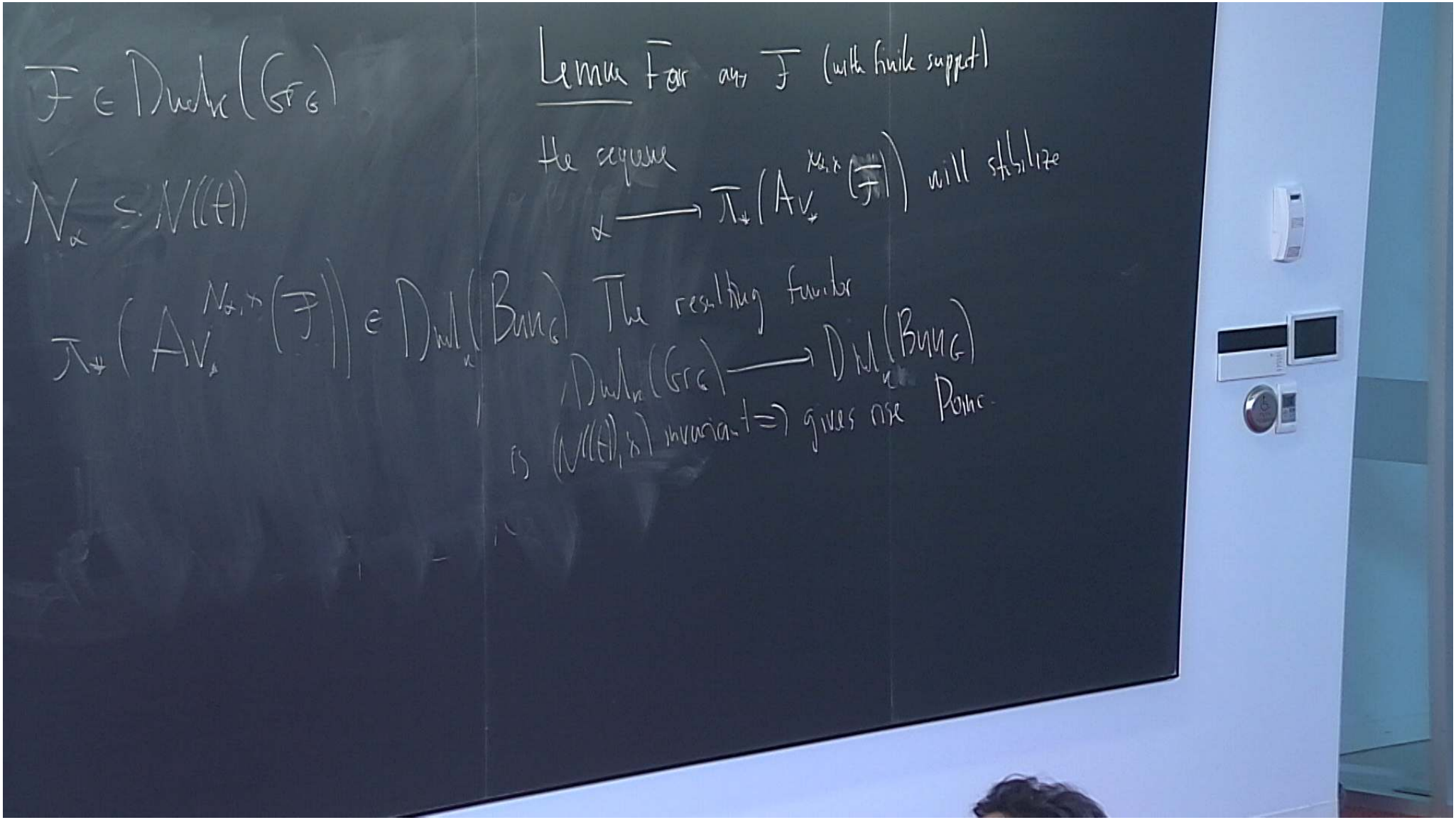
$\pi_*\left(\mathrm{Av}_*^{N_\alpha, x}(\mathcal{F})\right) \in \mathrm{Dmod}_k(\mathrm{Bun}_G)$

$\mathrm{Dmod}(\mathrm{Bun}_G)$

<u>Lemma</u> For any $\mathcal{F}$ (with finite support)

the sequence

$\alpha \longrightarrow \pi_*\left(\mathrm{Av}_*^{N_\alpha, x}(\mathcal{F})\right)$ will stabilize



$\mathcal{F} \in \mathrm{Dmod}_\kappa(\mathrm{Gr}_G)$

$N_\alpha \subseteq N((t))$

$\pi_*\left(\mathrm{Av}_*^{N_\alpha, \chi}(\mathcal{F})\right) \in \mathrm{Dmod}_\kappa(\mathrm{Bun}_G)$

Lemma For any $\mathcal{F}$ (with finite support)

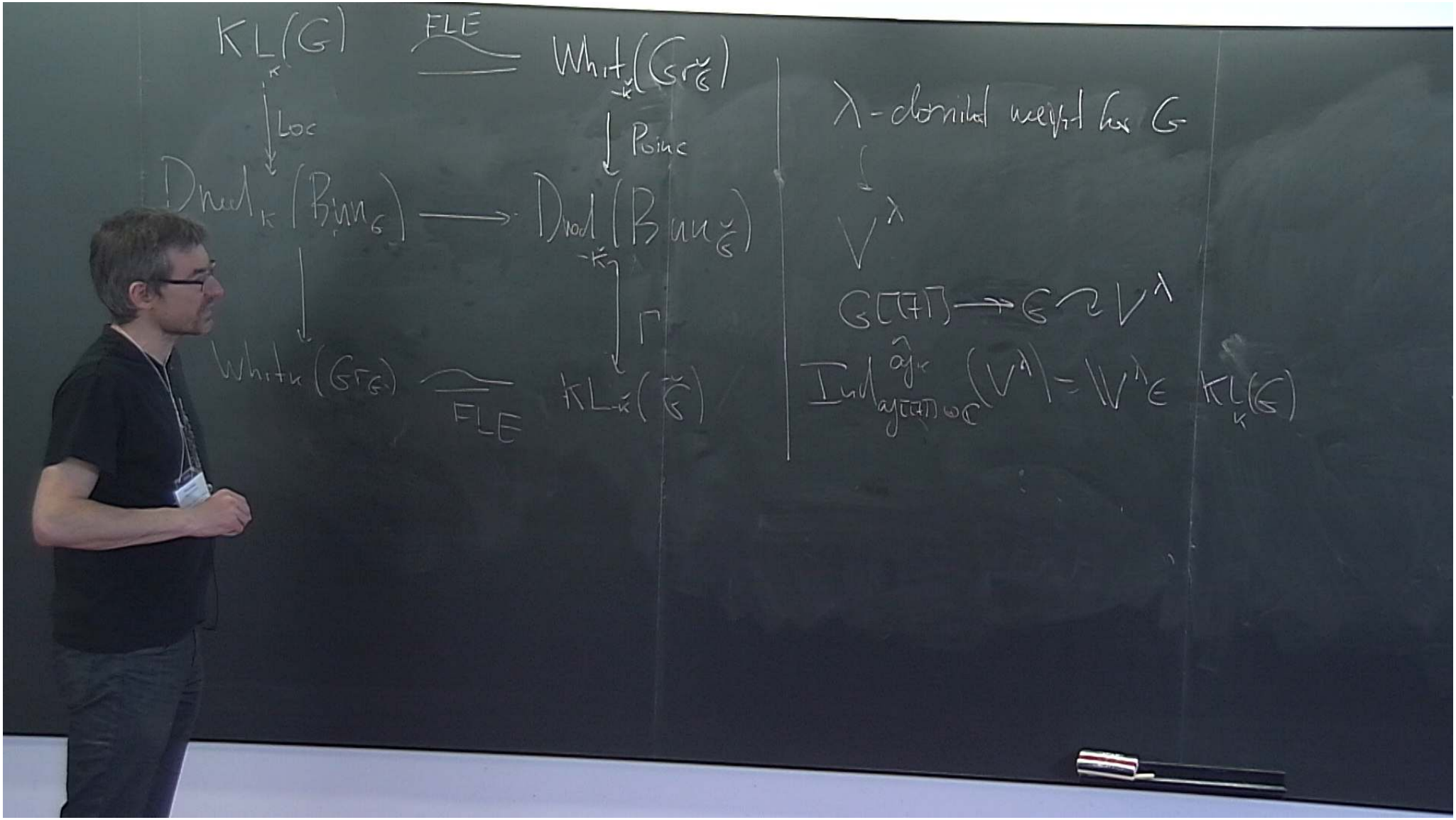
the sequence

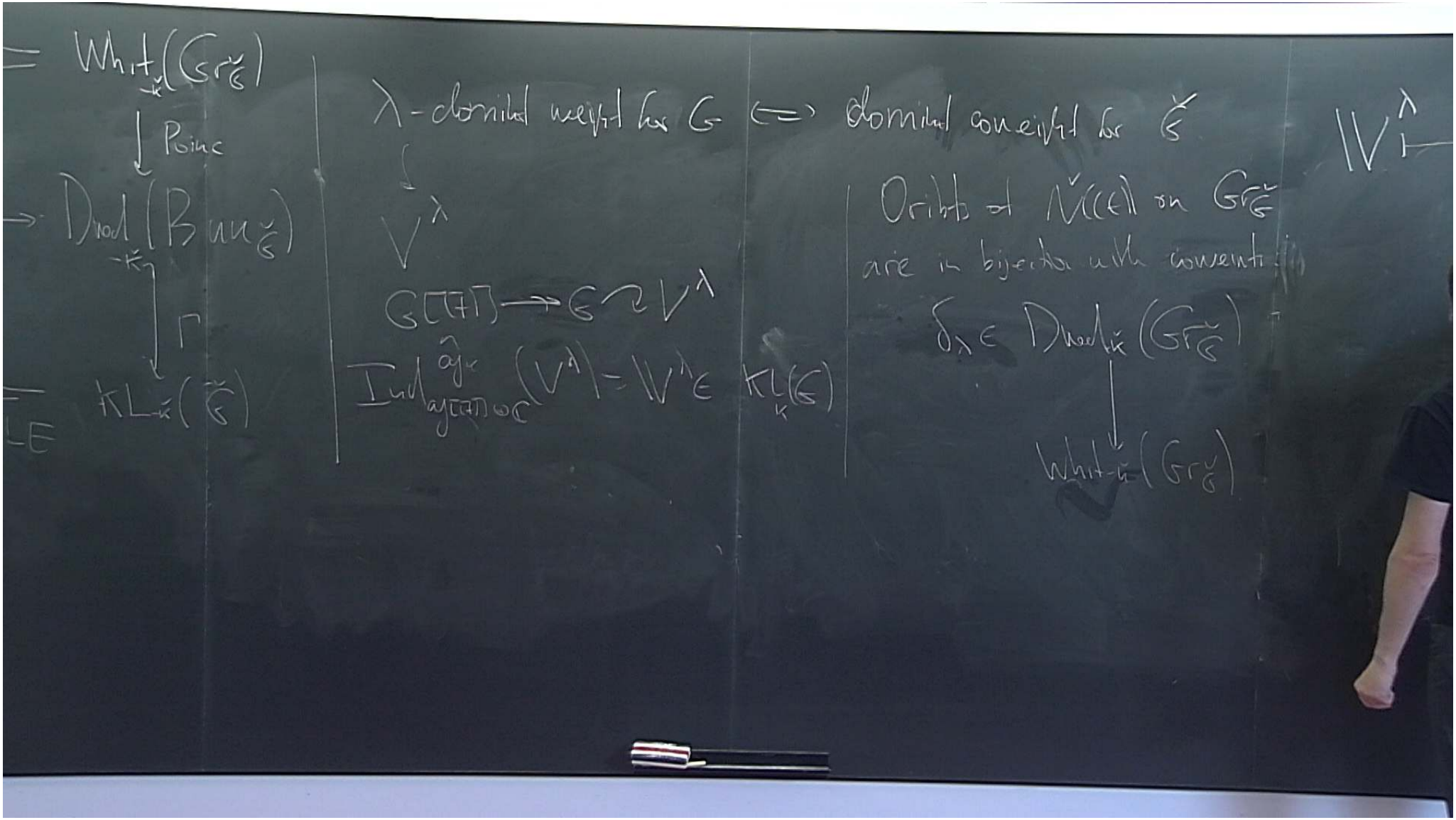
$$\alpha \longrightarrow \pi_*\left(\mathrm{Av}_*^{N_\alpha, \chi}(\mathcal{F})\right) \text{ will stabilize}$$

The resulting functor

$$\mathrm{Dmod}_\kappa(\mathrm{Gr}_G) \longrightarrow \mathrm{Dmod}_\kappa(\mathrm{Bun}_G)$$

is $(N((t)), \chi)$ invariant $\Rightarrow$ gives rise Poinc.


KL(G)
FLE
Whit(Gr_G^∨)
Loc
Poinc
Whit(GrG)
FLE
λ-dominant weight for G
V^λ
G[[t]] ⟶ G ↷ V^λ



$= \mathrm{Whit}_{-\check{\kappa}}(\mathrm{Gr}_{\check{G}})$

$\downarrow$ Poinc

$\to \mathrm{Dmod}_{-\check{\kappa}}(\mathrm{Bun}_{\check{G}})$

$\downarrow \Gamma$

$= \mathrm{KL}_{-\check{\kappa}}(\check{G})$

LE

λ-dominant weight for G $\Longleftrightarrow$ dominant coweight for $\check{G}$

$\downarrow$

V^λ

$G[[t]] \twoheadrightarrow G \curvearrowright V^\lambda$

$\mathrm{Ind}_{\mathfrak{g}[[t]]\otimes\mathbb{C}}^{\hat{\mathfrak{g}}_\kappa}(V^\lambda) = \mathbb{V}^\lambda \in \mathrm{KL}_\kappa(G)$

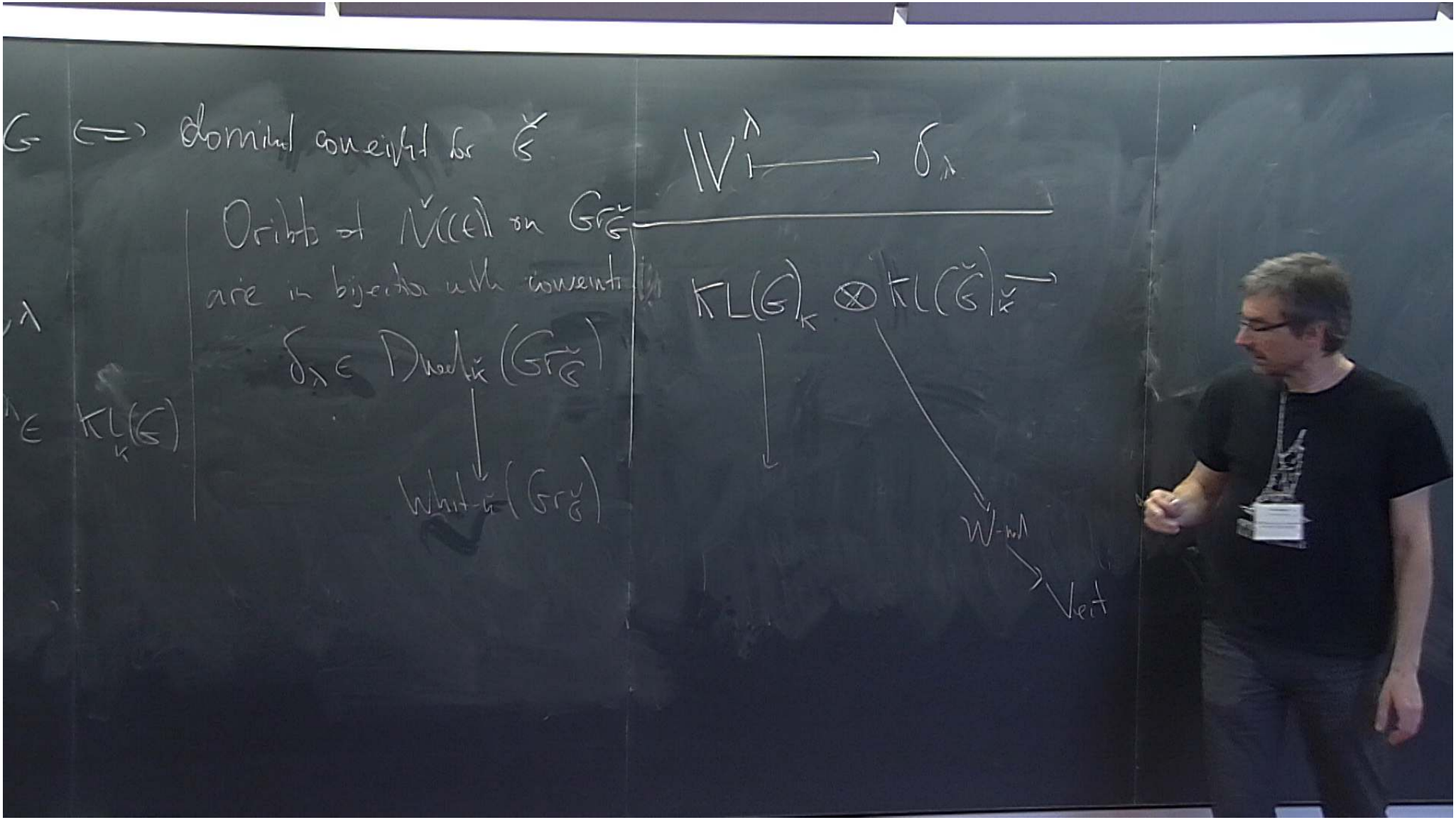
Orbits of $\check{N}((t))$ on $\mathrm{Gr}_{\check{G}}$ are in bijection with coweights

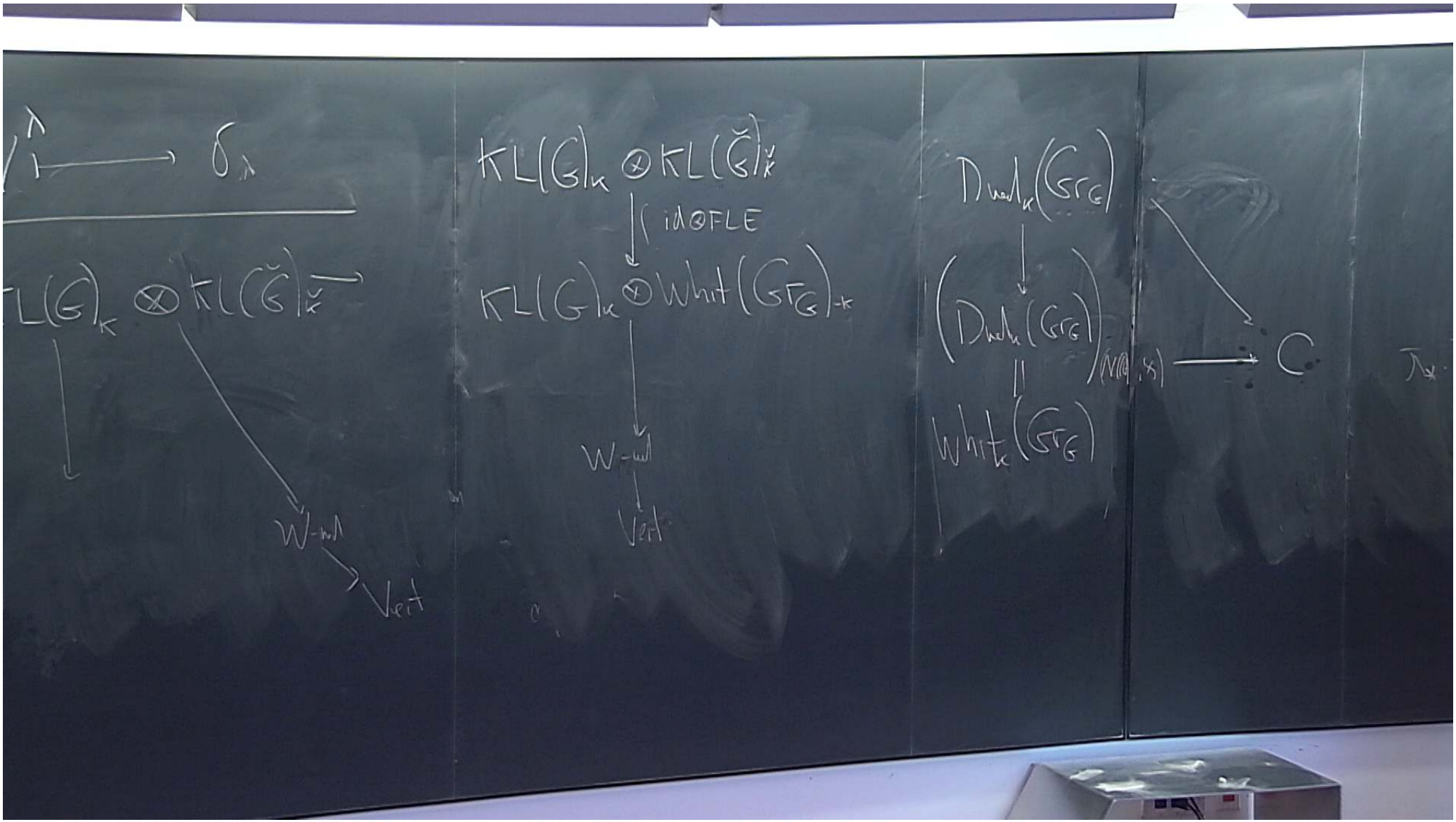
$\delta_\lambda \in \mathrm{Dmod}_{-\check{\kappa}}(\mathrm{Gr}_{\check{G}})$

$\downarrow$

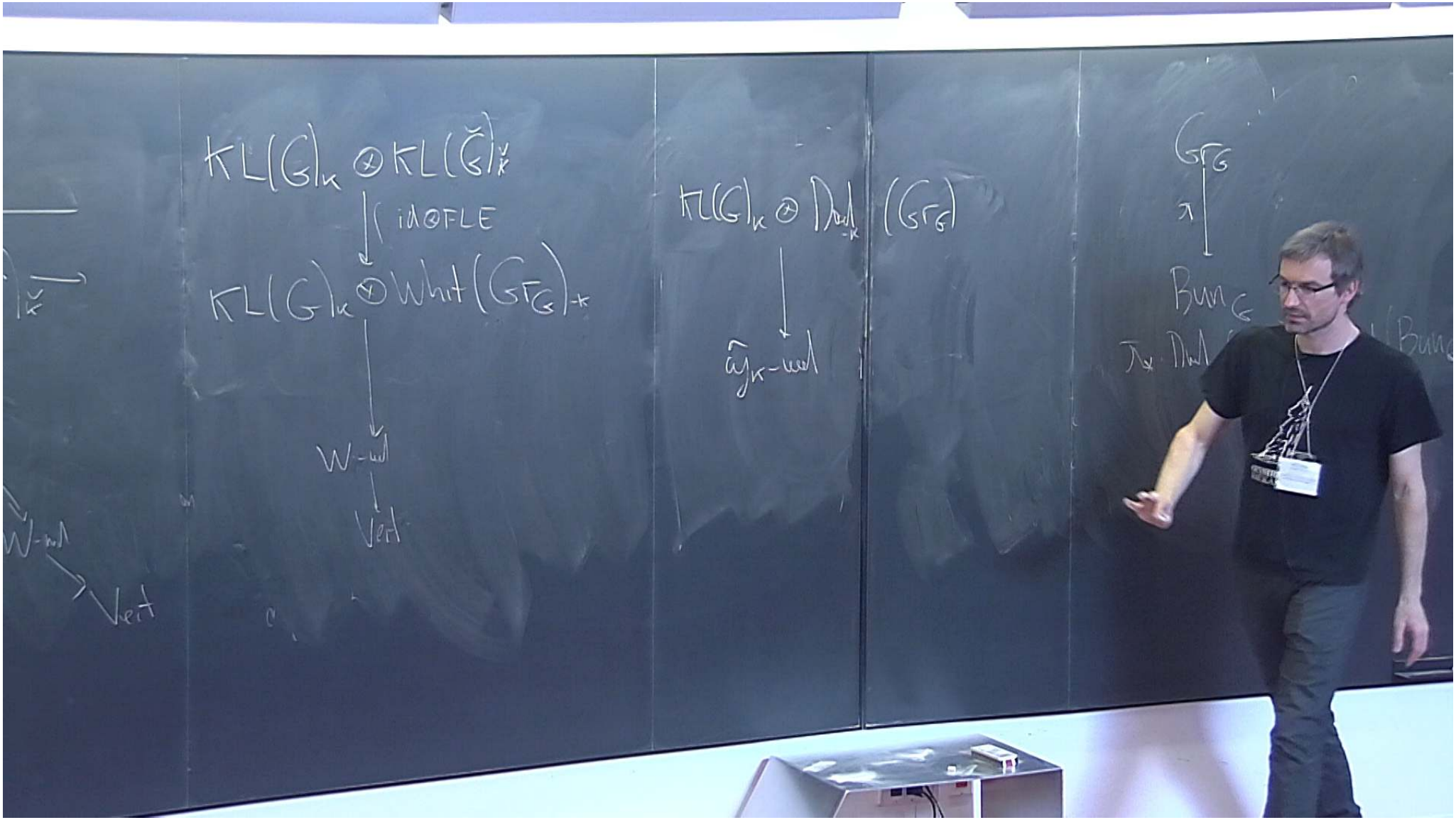
$\mathrm{Whit}_{-\check{\kappa}}(\mathrm{Gr}_{\check{G}})$

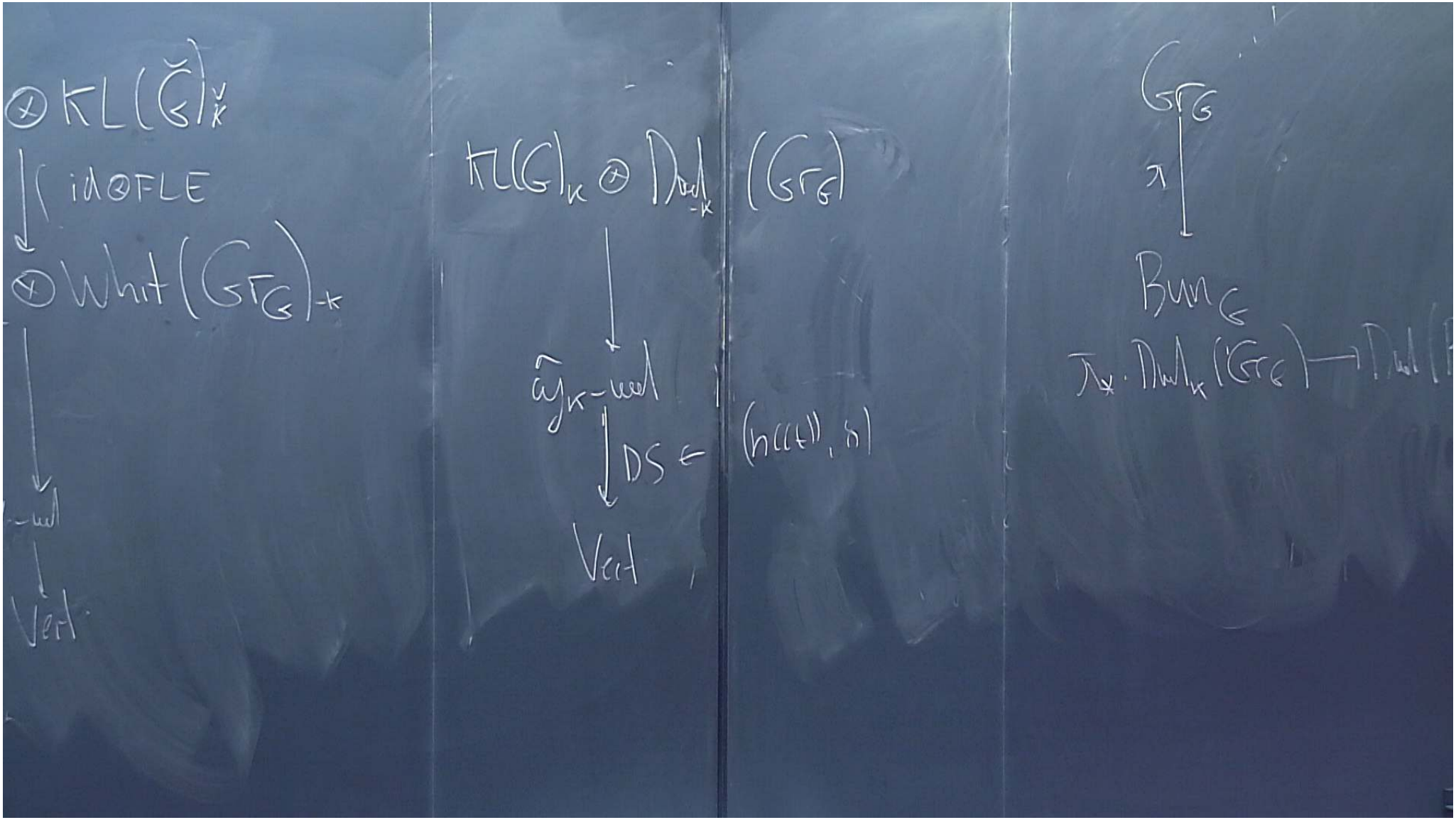
$\mathbb{V}^\lambda \mapsto$

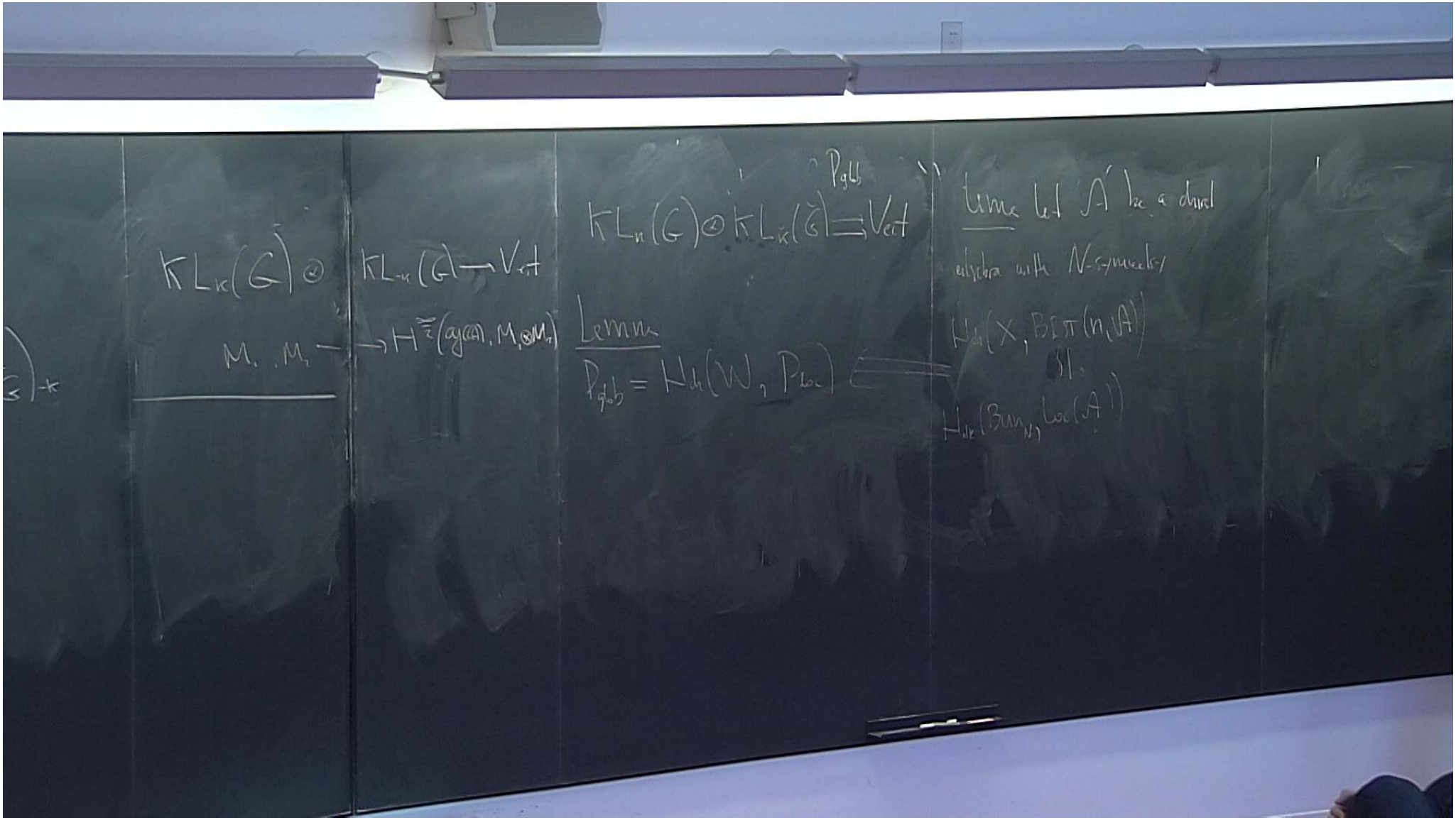

G ⟺ dominant coweight for Ǧ
Orbits of Ň((t)) on Gr_Ǧ
are in bijection with coweights
δ_λ ∈ Dmod_κ̌(Gr_Ǧ)
Whit-κ̌(Gr_Ǧ)
KL(G)_κ ⊗ KL(Ǧ)_κ̌
W-mod
Vect

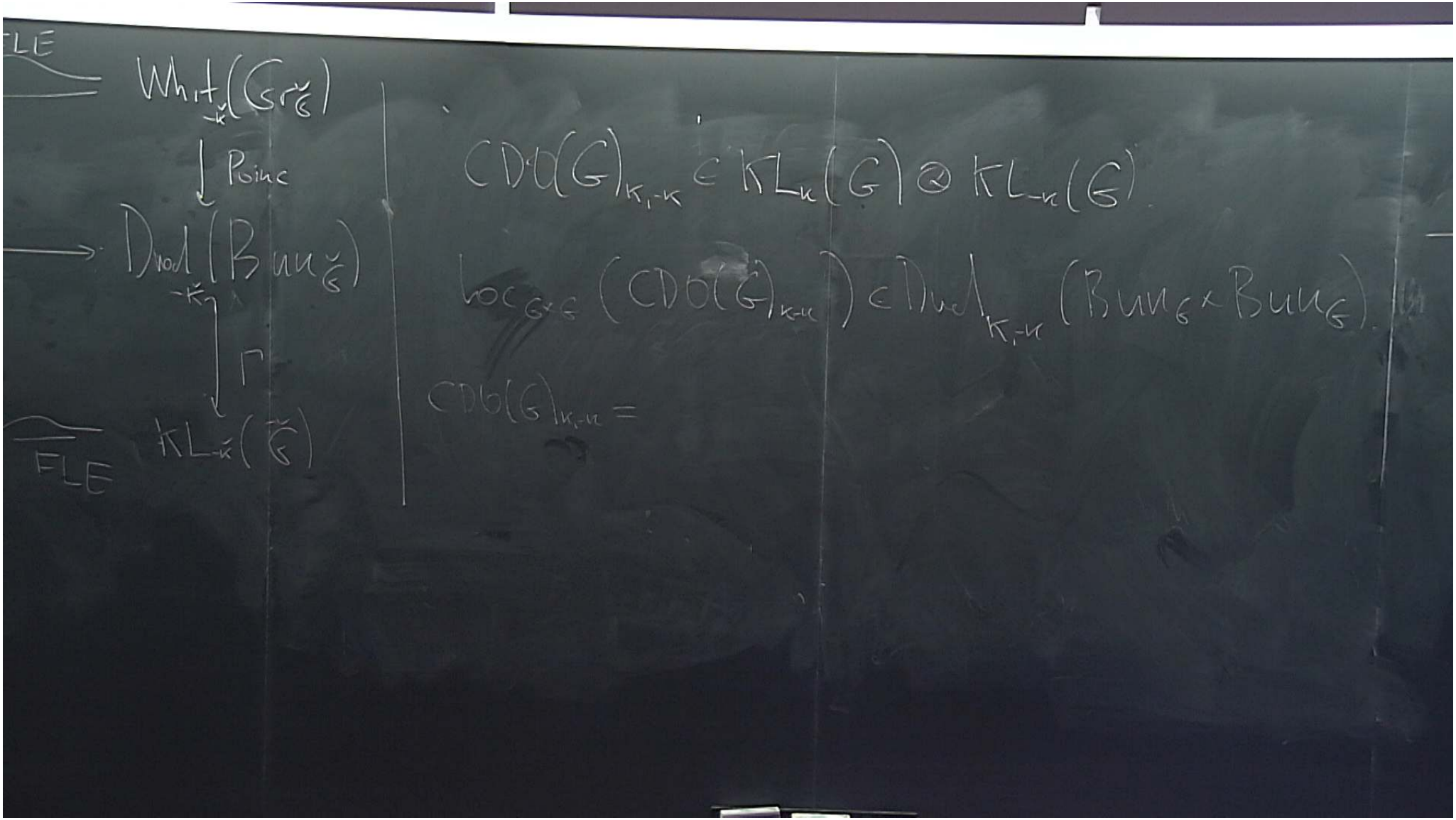


$\lambda \longmapsto \delta_\lambda$
$KL(G)_\kappa \otimes KL(\check{G})_{\check{\kappa}}$
W-mod
Vect
$KL(G)_\kappa \otimes KL(\check{G})_{\check{\kappa}}$
id⊗FLE
$KL(G)_\kappa \otimes Whit(Gr_G)_{-\kappa}$
W-mod
Vect
$Dmod_\kappa(Gr_G)$
$(Dmod_\kappa(Gr_G))_{(N(\mathbb{Q}),\chi)}$
$Whit_\kappa(Gr_G)$
C
π_*


KL(G)_κ ⊗ KL(Ǧ)_κ
id ⊗ FLE
KL(G)_κ ⊗ Whit(Gr_G)_κ
W-mod
Vect
KL(G)_κ ⊗ D-mod_{-κ}(Gr_G)
ω̃_{κ-mod}
Gr_G
Bun_G
Vect


KL(Ǧ)
id⊗FLE
Whit(GrG)
Vect
KL(G) ⊗ Dmod
(GrG)
DS
(h(t), n)
Vect
GrG
Bun
Vect


KL_κ(G) ⊗ KL_-κ(G) → Vect
M_1, M_2 → H^{≥2}(g(A), M_1 ⊗ M_2)
KL_κ(G) ⊗ KL_-κ(G) ⇉ Vect
P_glob
Lemma
P_glob = H_dR(W, P_loc)
time let A be a chiral algebra with N-symmetry
H_dR(X, BiSt(n, A))
≅
H_dR(Bun_N, Loc(A))



$$\begin{tikzcd}
\mathrm{Whit}_{-\check{\kappa}}(\mathrm{Gr}_{\check{G}}) \arrow[d, "\mathrm{Poinc}"] \\
\mathrm{Dual}_{-\check{\kappa}}(\mathrm{Bun}_{\check{G}}) \arrow[d, "\Gamma"] \\
\mathrm{KL}_{-\check{\kappa}}(\check{G})
\end{tikzcd}$$

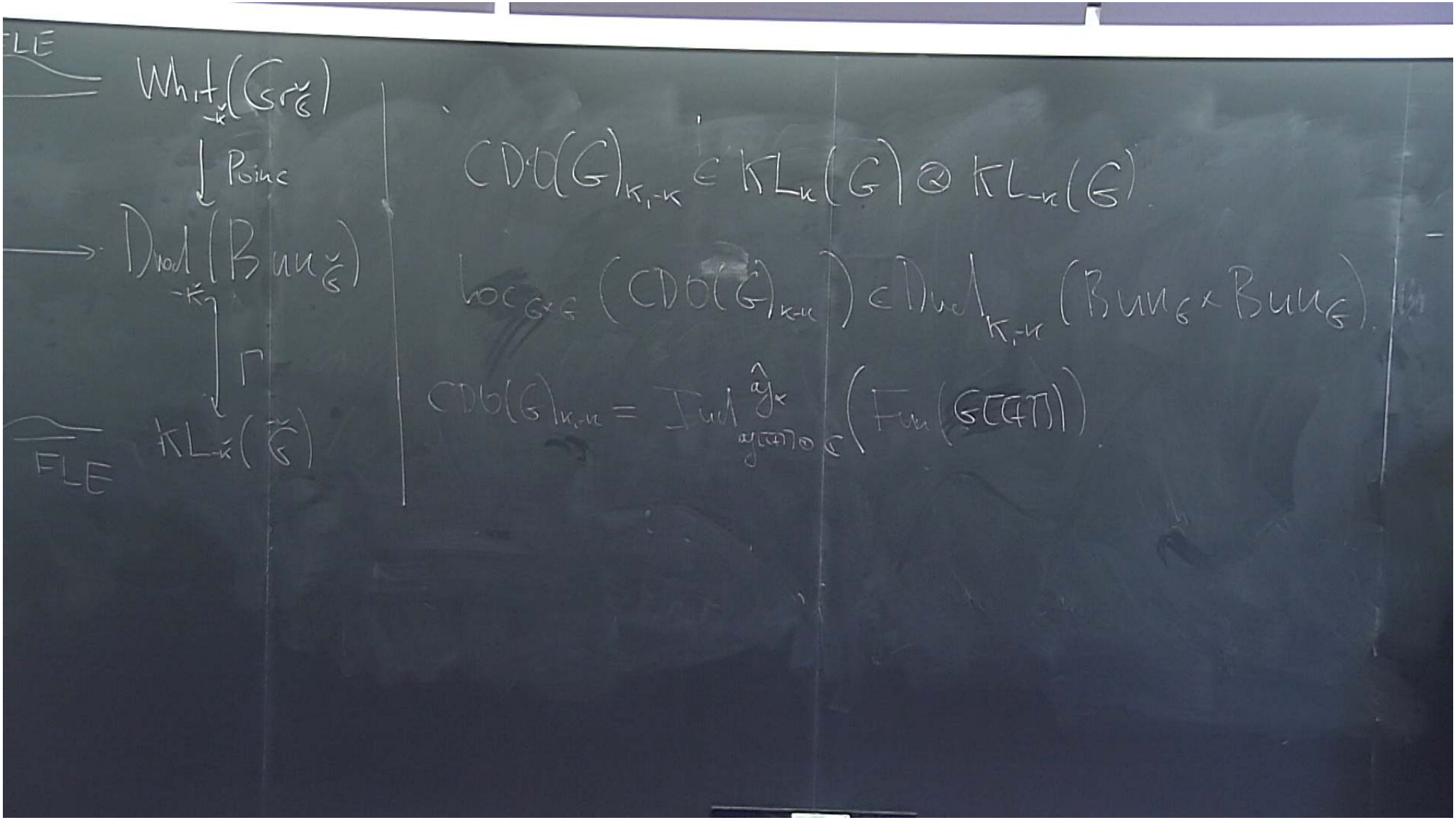
FLE

FLE

$$CDO(G)_{\kappa,-\kappa} \in KL_{\kappa}(G) \otimes KL_{-\kappa}(G)$$

$$\mathrm{loc}_{G\times G}(CDO(G)_{\kappa,-\kappa}) \in \mathrm{Dual}_{\kappa,-\kappa}(\mathrm{Bun}_G \times \mathrm{Bun}_G)$$

$$CDO(G)_{\kappa,-\kappa} =$$



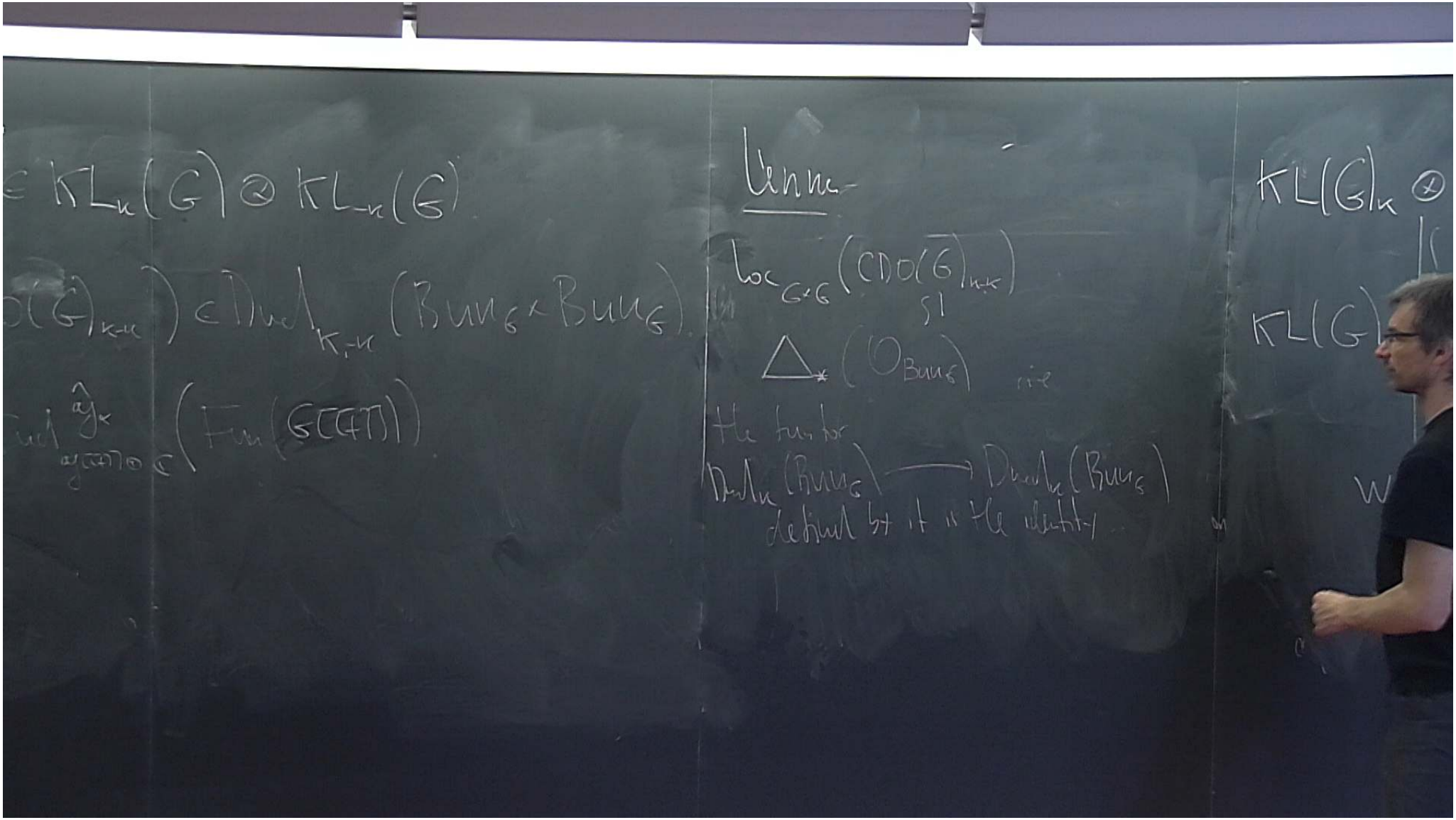
$$\begin{tikzcd}
\mathrm{Whit}_{-\check{\kappa}}(\mathrm{Gr}_{\check{G}}) \arrow[d, "\mathrm{Poinc}"] \\
\mathrm{Dual}_{-\check{\kappa}}(\mathrm{Bun}_{\check{G}}) \arrow[d, "\Gamma"] \\
\mathrm{KL}_{-\check{\kappa}}(\check{G})
\end{tikzcd}$$

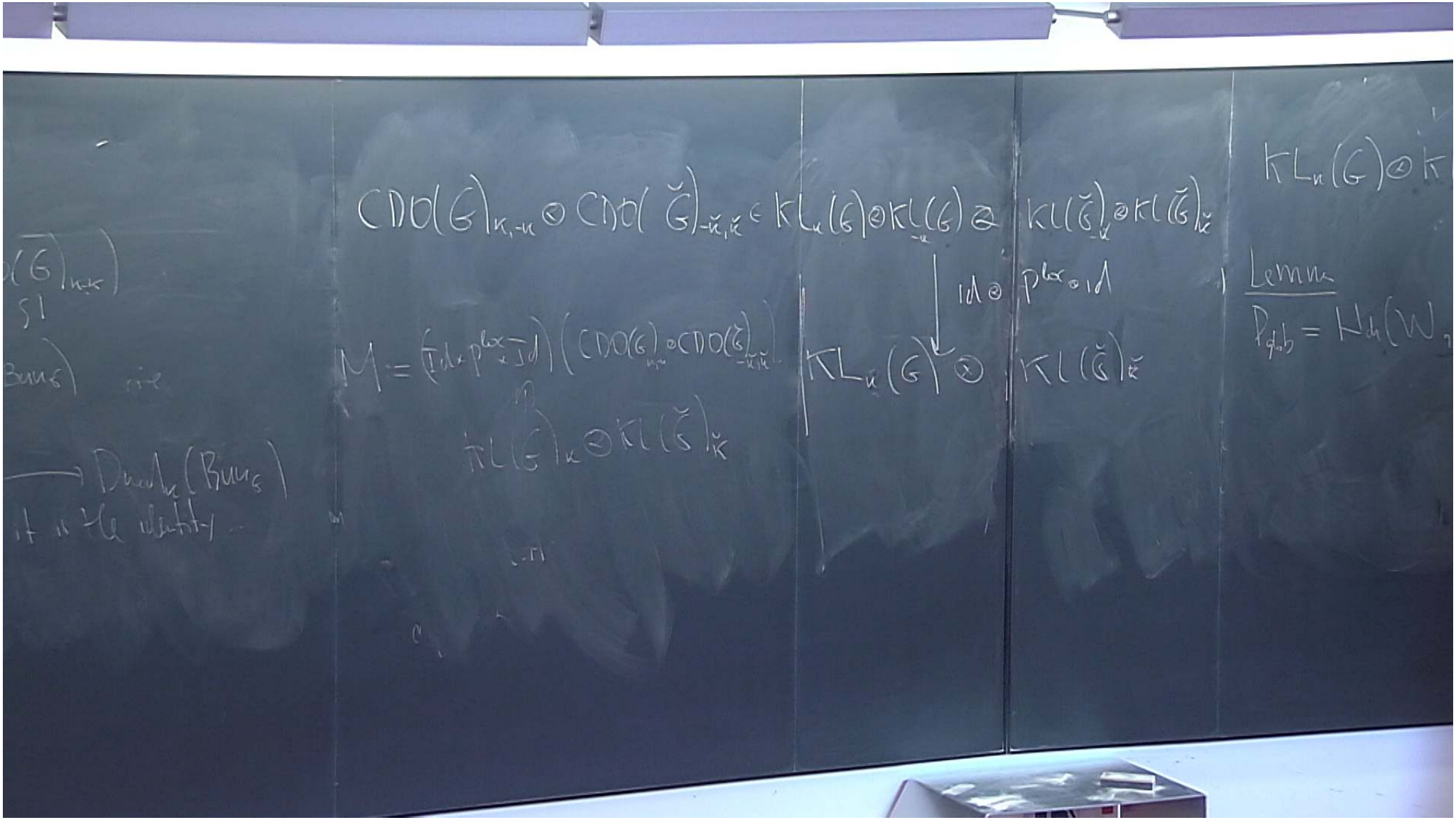
FLE

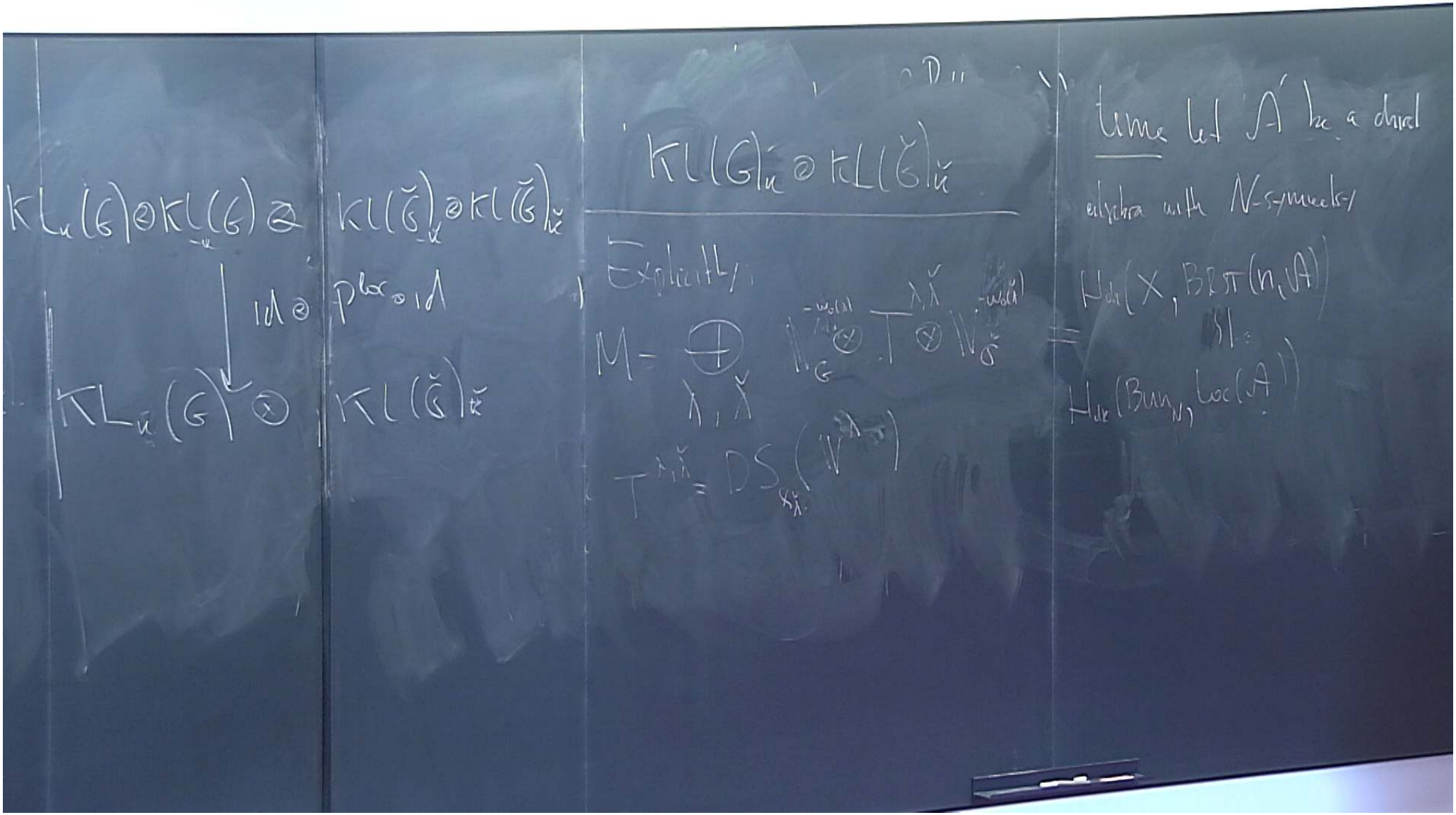
$$\mathrm{CDO}(G)_{\kappa,-\kappa} \in \mathrm{KL}_{\kappa}(G) \otimes \mathrm{KL}_{-\kappa}(G)$$

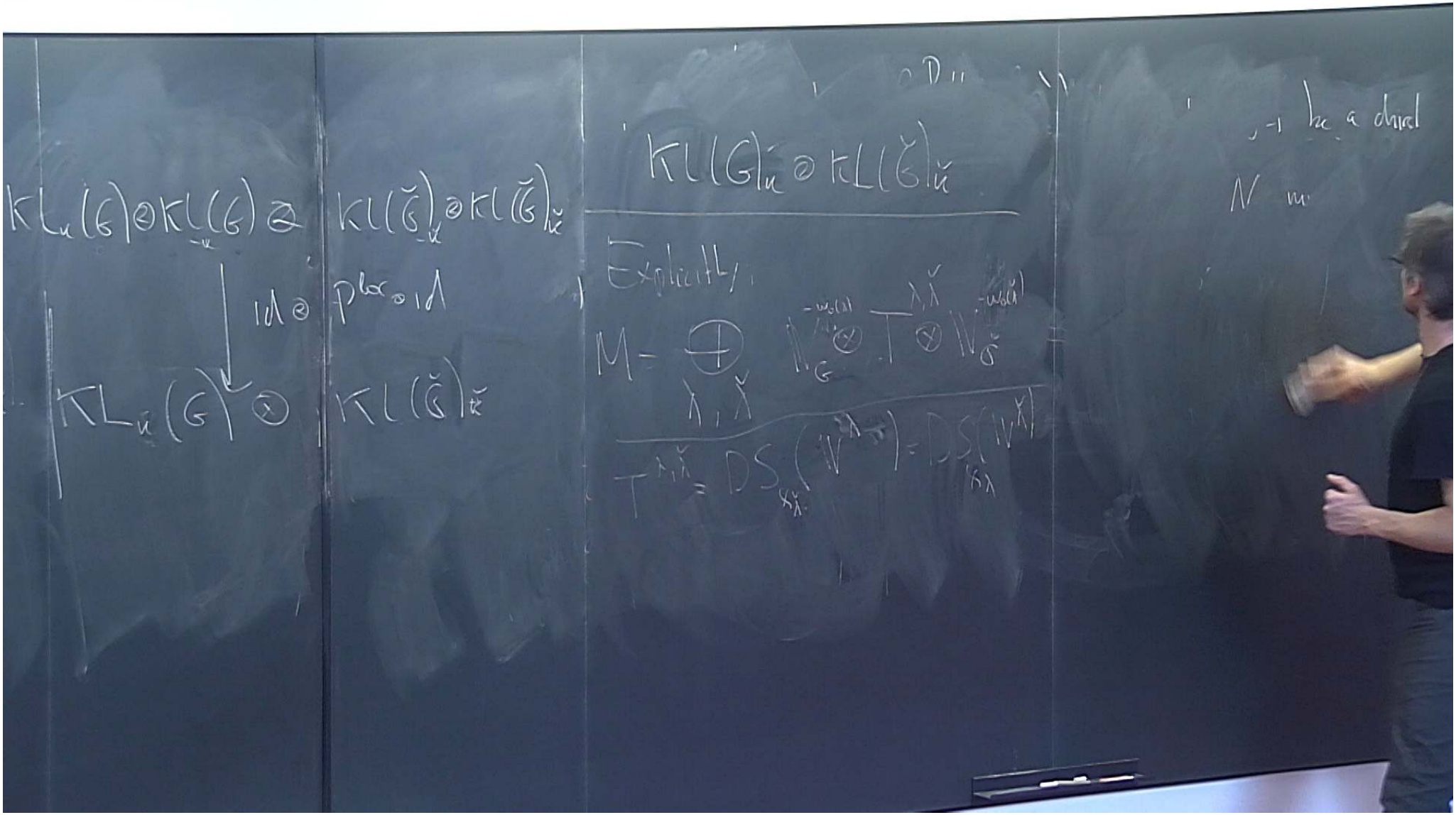
$$\mathrm{loc}_{G\times G}\left(\mathrm{CDO}(G)_{\kappa,-\kappa}\right) \in \mathrm{Dual}_{\kappa,-\kappa}(\mathrm{Bun}_G \times \mathrm{Bun}_G)$$

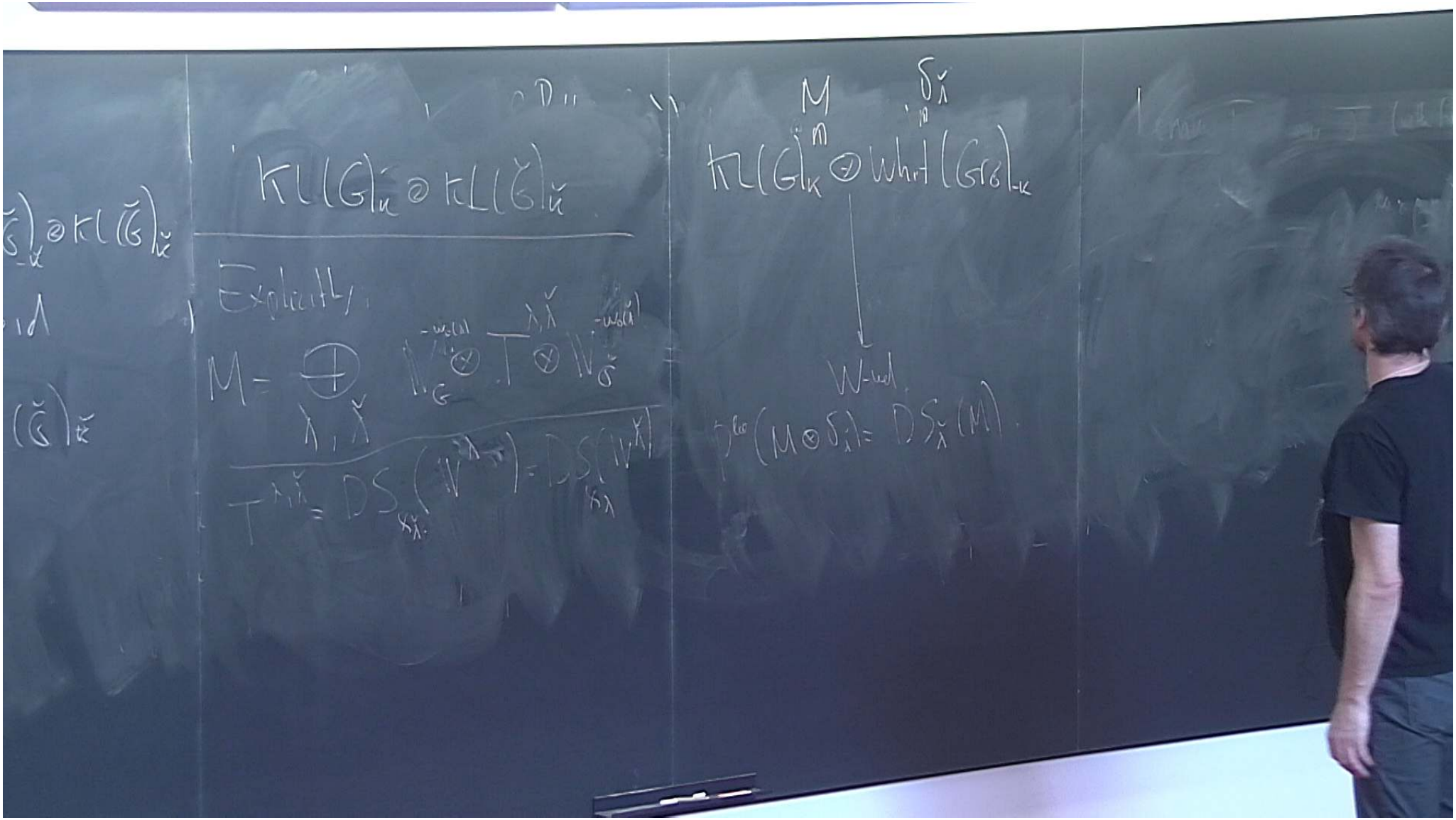
$$\mathrm{CDO}(G)_{\kappa,-\kappa} = \mathrm{Ind}^{\hat{\mathfrak{g}}_\kappa}_{\mathfrak{g}[[t]]\otimes\mathbb{C}}\left(\mathrm{Fun}(G[[t]])\right)$$

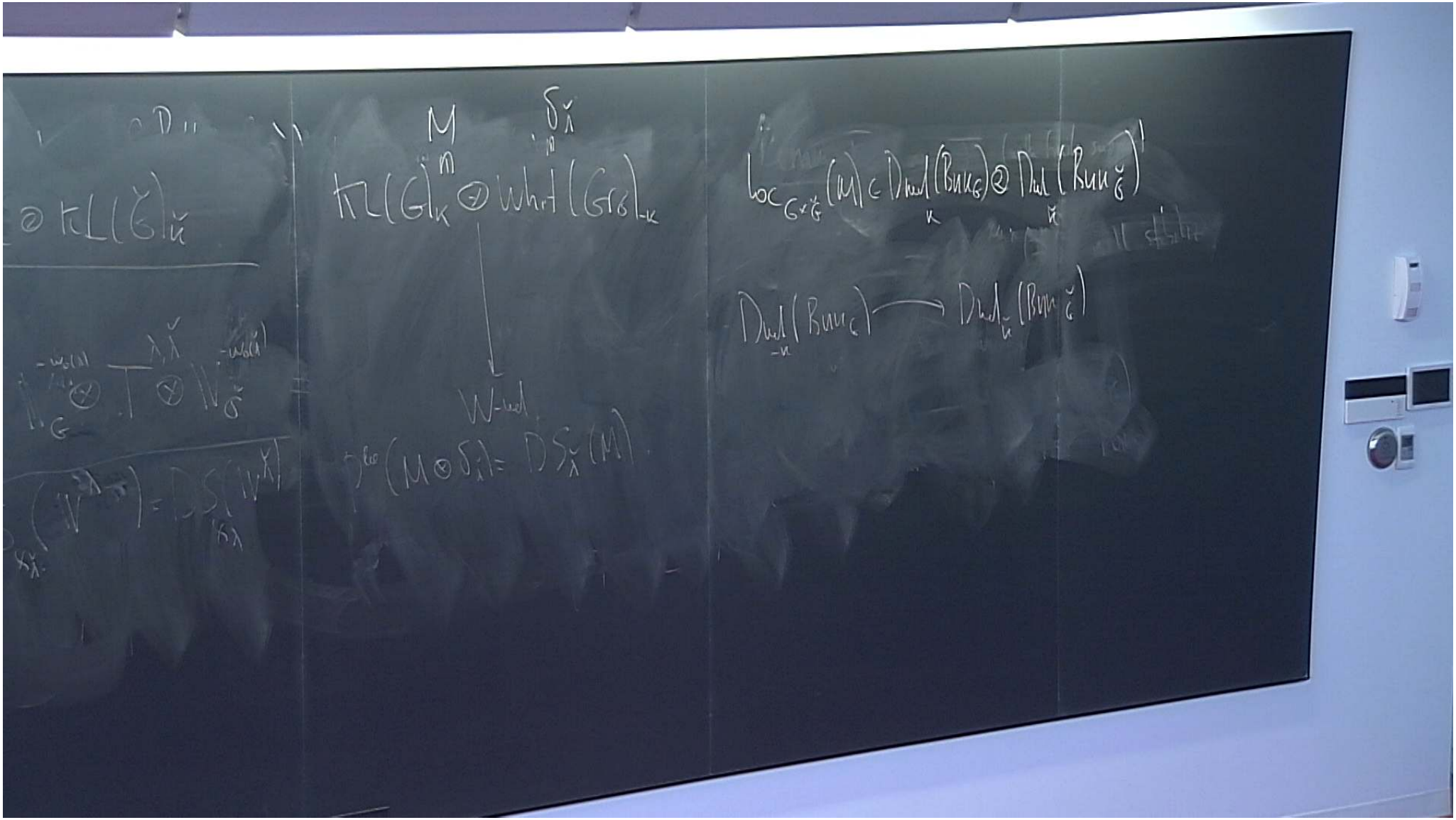

$\in KL_\kappa(G) \otimes KL_{-\kappa}(G)$
$\in \mathrm{Dmod}_{\kappa,-\kappa}(\mathrm{Bun}_G \times \mathrm{Bun}_G)$
$\mathrm{Fun}(G[[t]])$
Lemma
$\mathrm{loc}_{G\times G}(\mathrm{CDO}(\overline{G})_{\kappa,-\kappa})$
$\Delta_*(\mathcal{O}_{\mathrm{Bun}_G})$
the functor
$\mathrm{Dmod}_\kappa(\mathrm{Bun}_G) \longrightarrow \mathrm{Dmod}_\kappa(\mathrm{Bun}_G)$
defined by it is the identity
$KL(G)_\kappa \otimes$
$KL(G)$

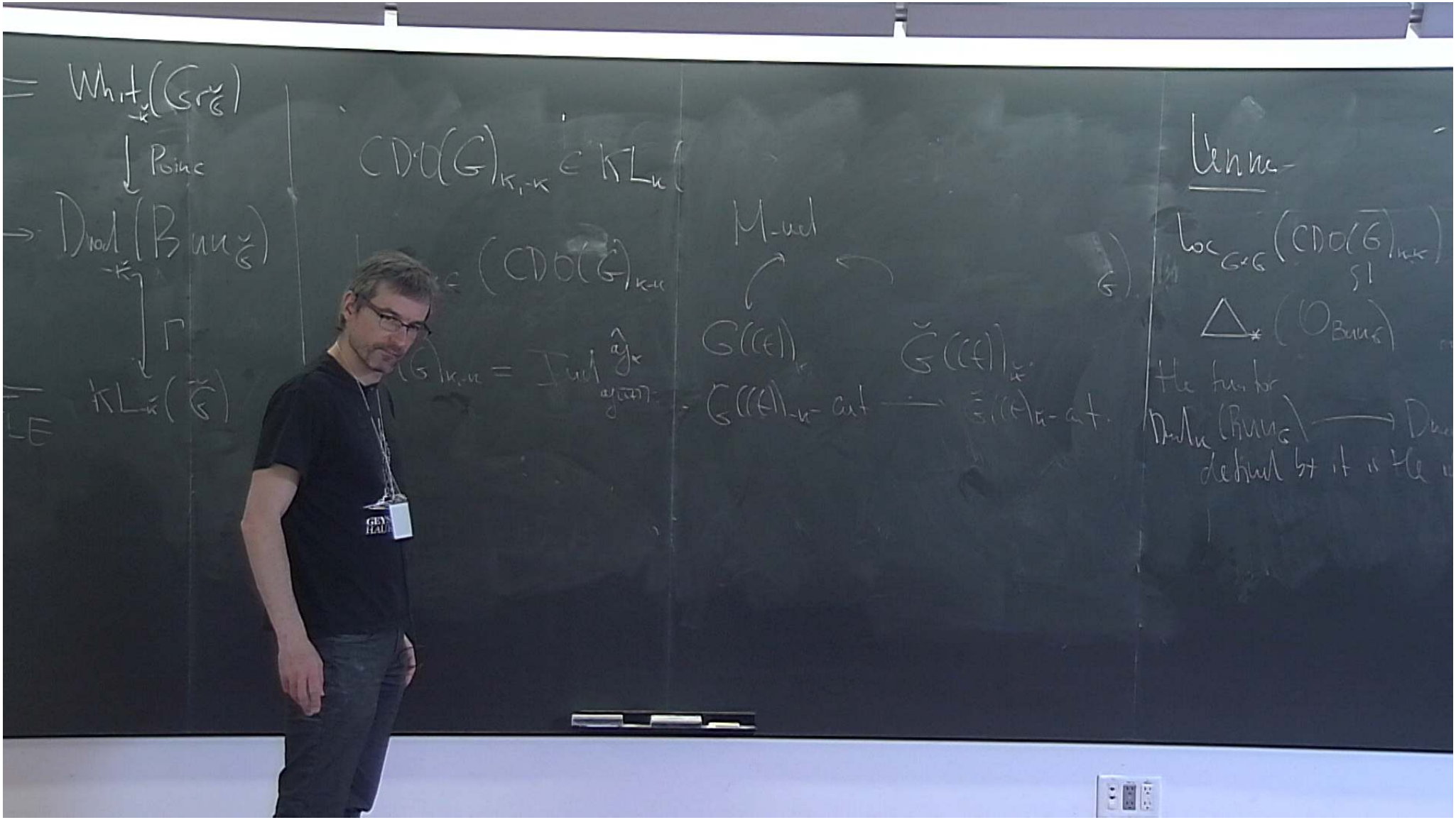

Lemma

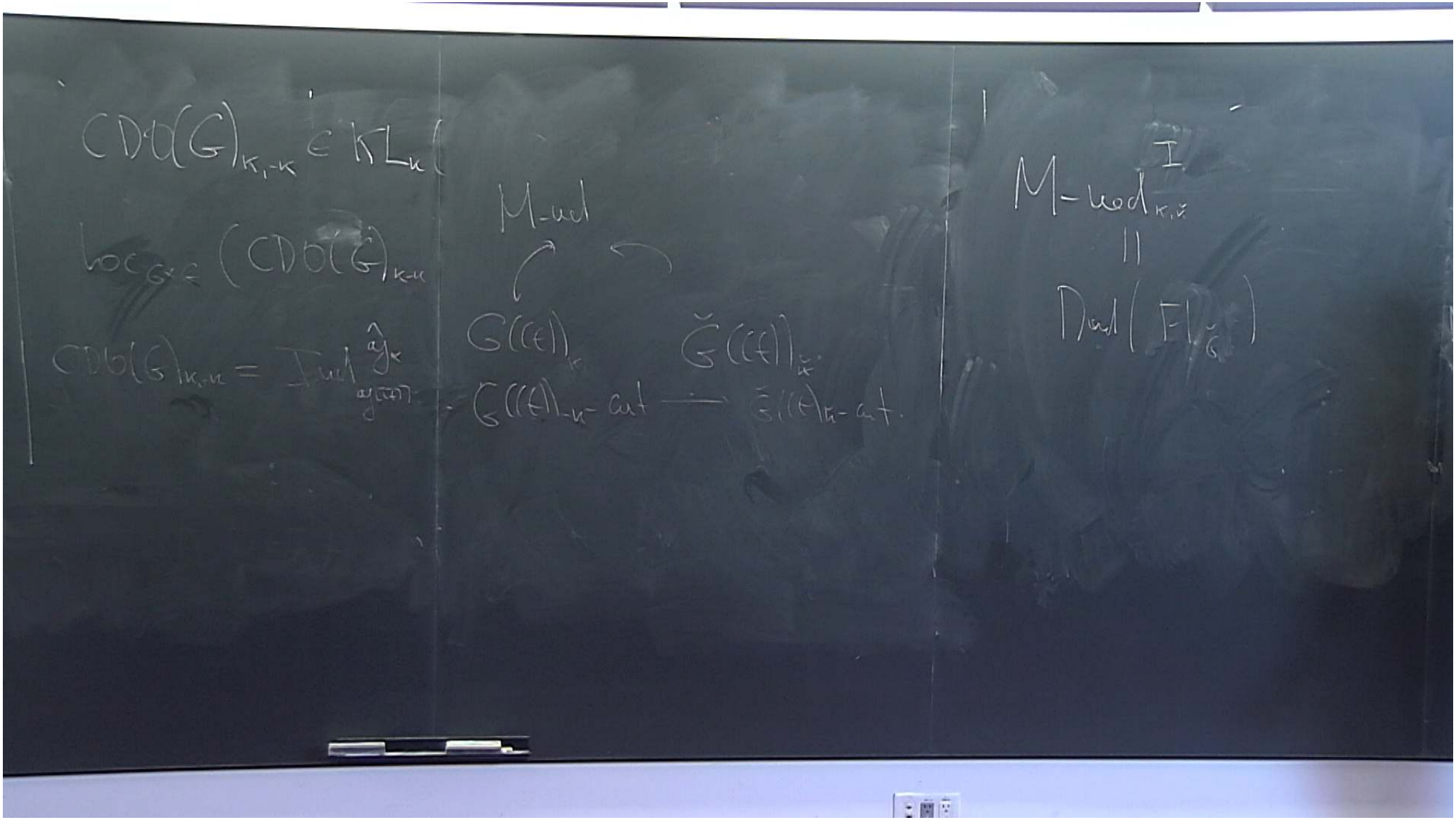

$KL_u(G) \otimes KL_{-u}(G) \otimes$
$KL(\check{G})_u \otimes KL(\check{G})_{\check{u}}$
$Id \otimes$
$p^{loc} \circ Id$
$KL_u(G)$
$KL(\check{G})_{\check{u}}$
$KL(G)_u \otimes KL(\check{G})_{\check{u}}$
Explicitly:
$M = \bigoplus_{\lambda, \check{\lambda}}$
$T^{\lambda, \check{\lambda}} = DS_{\chi^{\check{\lambda}}}(V^{\lambda})$
time let Λ' be a chiral
algebra with N-symmetry
$= H_{dR}(Bun_N, Loc(A))$

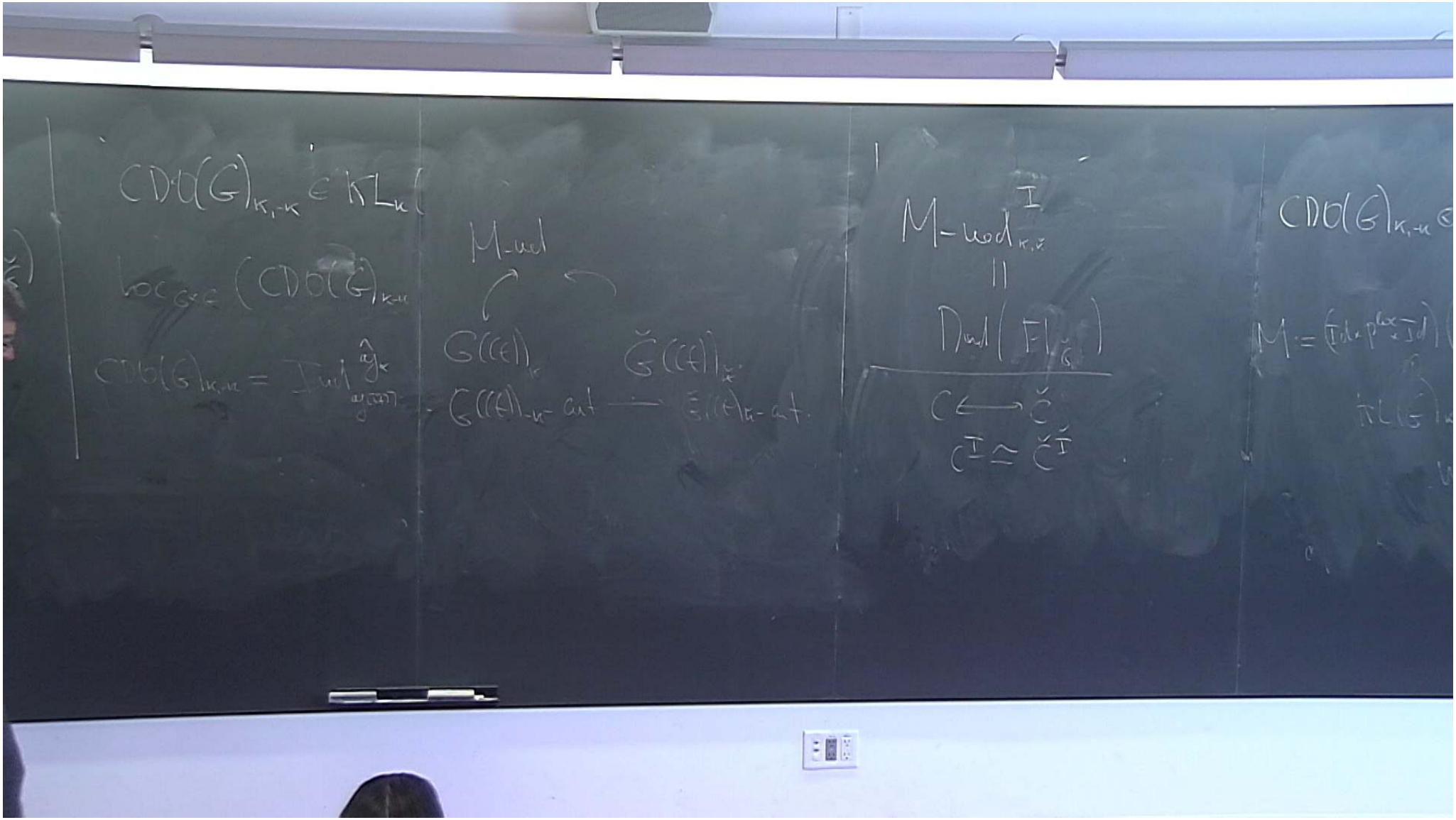

Explicitly:


Explicitly:




Poinc
M-mod
Lemma
the functor
defined by it is the


M-mod
M-mod
Dmod( )


M-mod
M-mod^I
Dmod(Fl)
C ⟷ Č
C^I ≃ Č^Ĭ