Title: Gauge Theory, Geometric Langlands, and All That

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Abstract:

# GAUGE THEORY & GEOMETRIC LANGLANDS

CONFORMAL

$SO(2,6)$

STORY STARTS IN $D=6$

FOR A-D-E

there's a $D=6$ SUPERCONFORMAL FIELD THEORY

$SOSp(2,6|4)$

STORY STARTS IN D=6

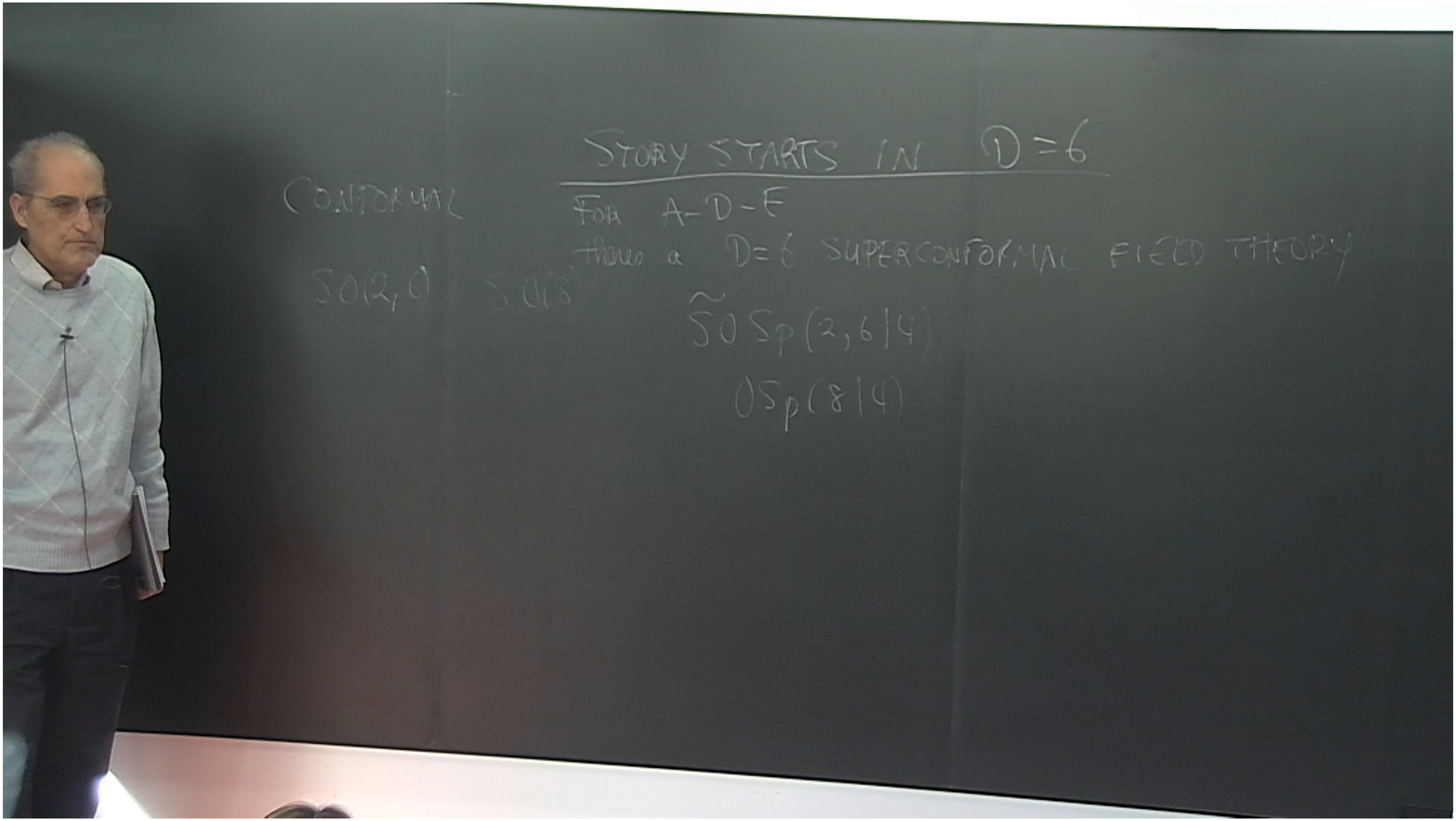
CONFORMAL

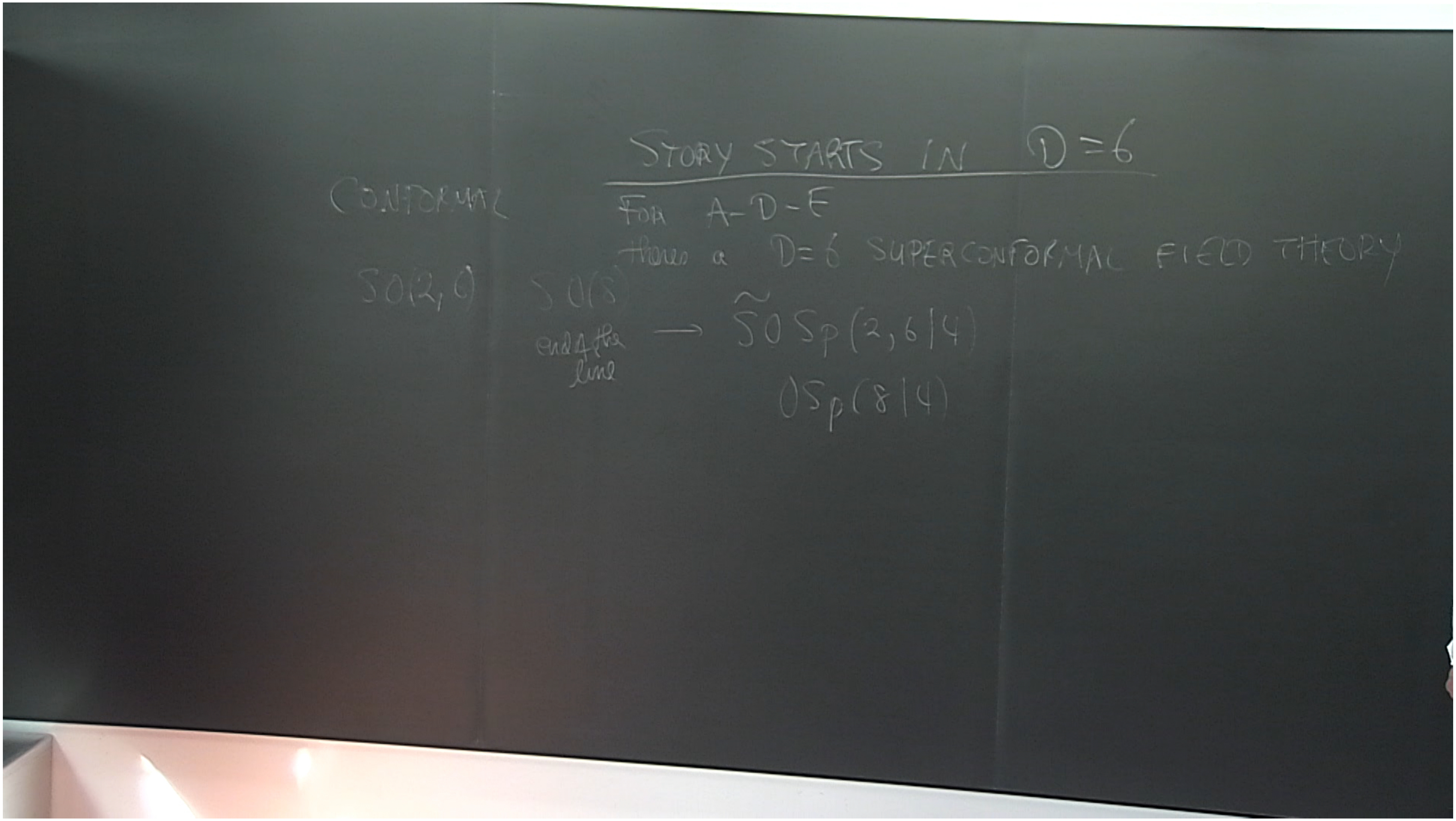
FOR A-D-E

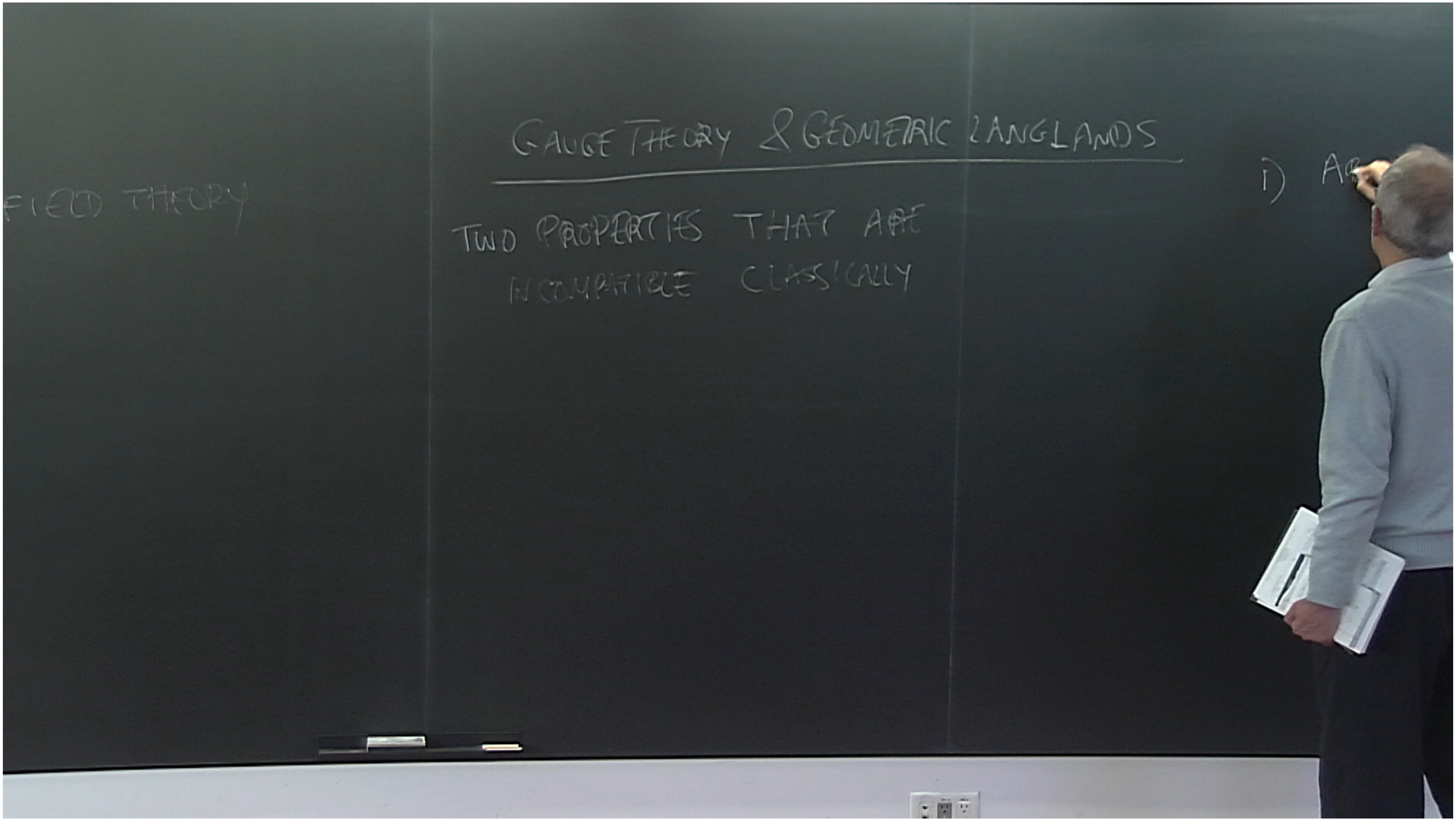
there's a D=6 SUPERCONFORMAL FIELD THEORY

$SO(2,6)$ $SO(8)$

$\widetilde{SO}Sp(2,6|4)$


STORY STARTS IN D=6
CONFORMAL
FOR A-D-E
theres a D=6 SUPERCONFORMAL FIELD THEORY
SO(2,6
SO(8
$\widetilde{SOSp}(2,6|4)$
$OSp(8|4)$


STORY STARTS IN D=6
CONFORMAL
FOR A-D-E
there's a D=6 SUPERCONFORMAL FIELD THEORY
SO(2,6)
SO(8)
end of the line
→ $\widetilde{SO}$ Sp(2,6|4)
OSp(8|4)

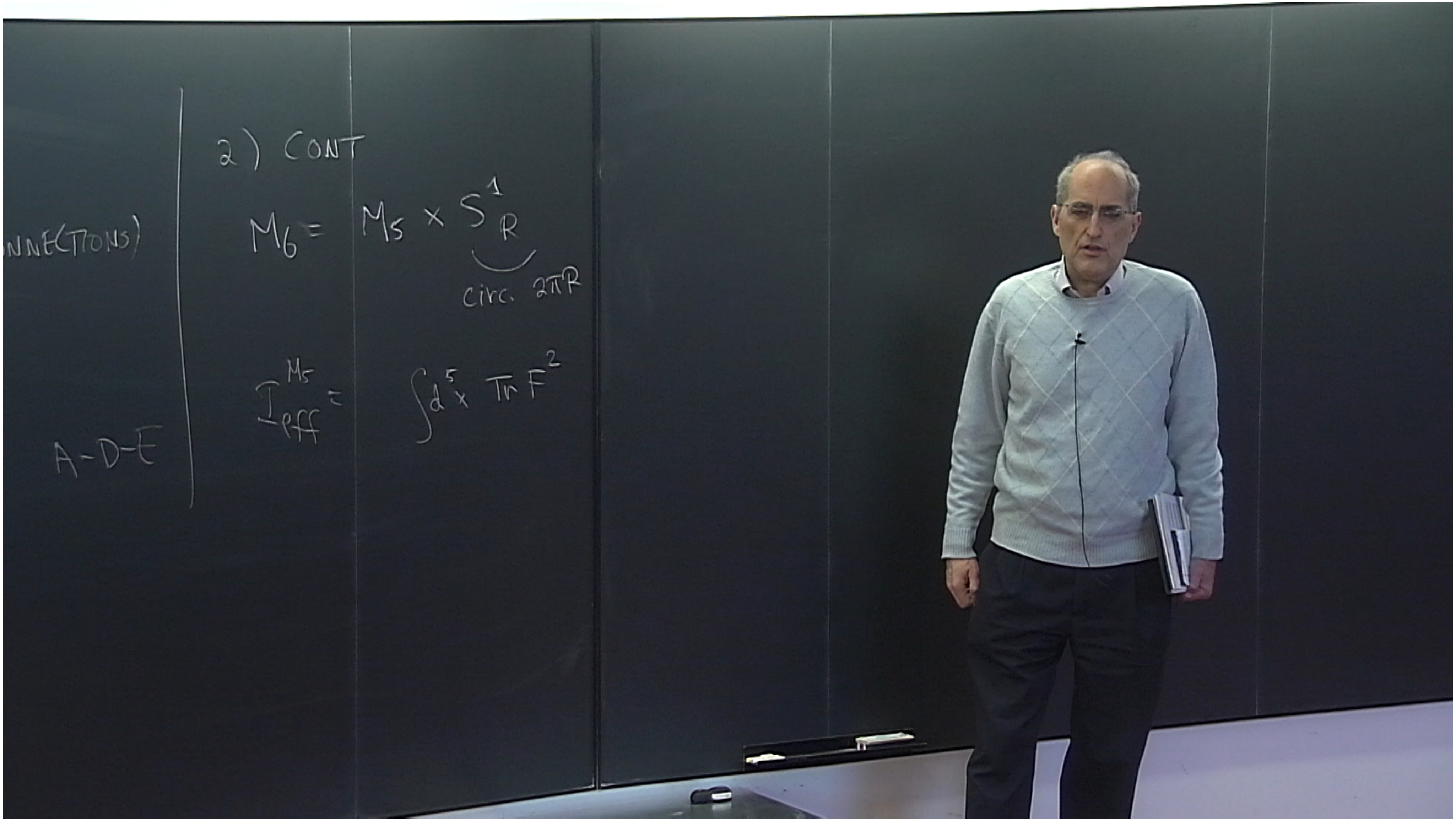

GAUGE THEORY & GEOMETRIC LANGLANDS
FIELD THEORY
TWO PROPERTIES THAT ARE
INCOMPATIBLE CLASSICALLY
1) A

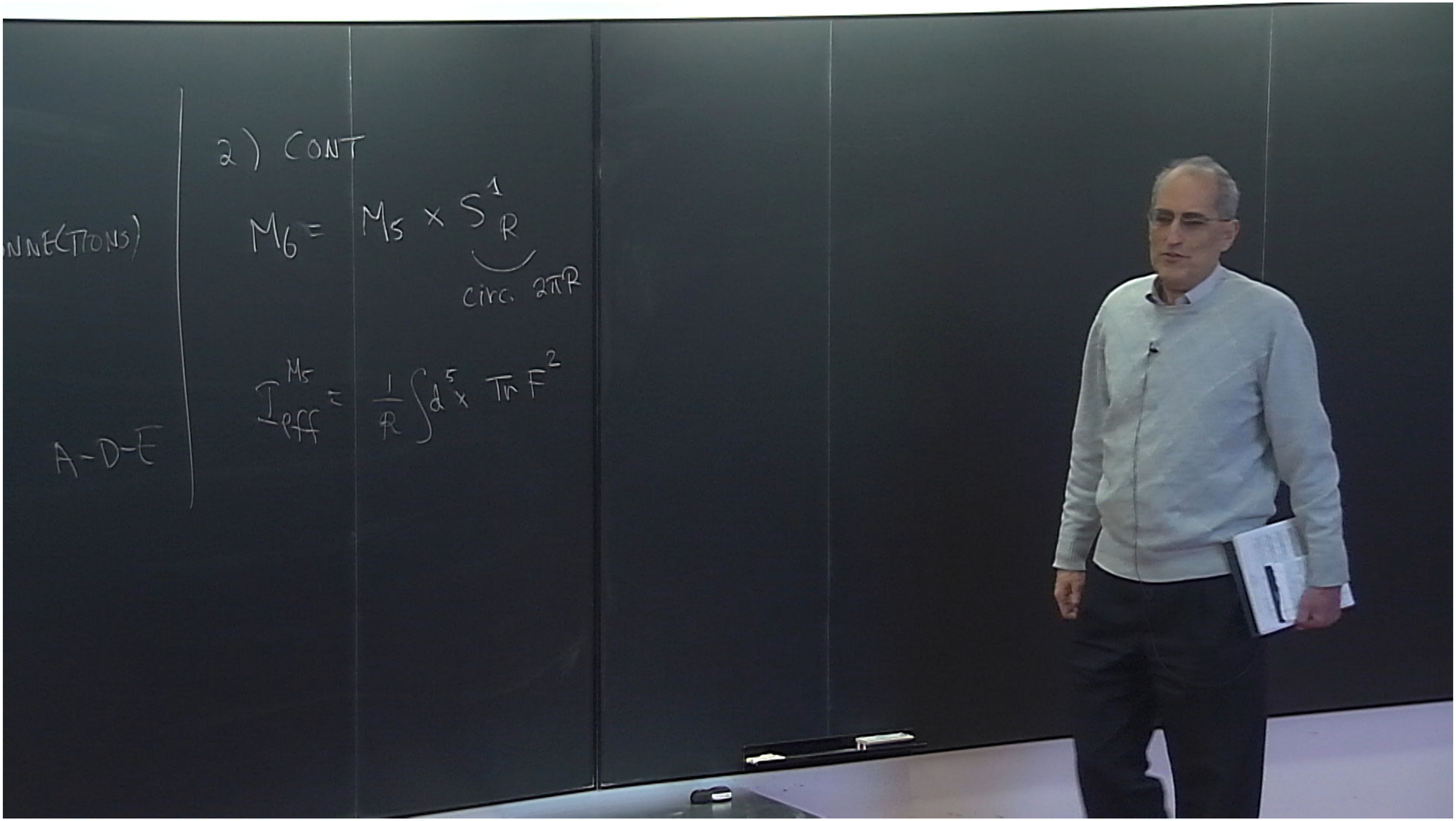
& GEOMETRIC LANGLANDS

THAT ARE
CLASSICALLY

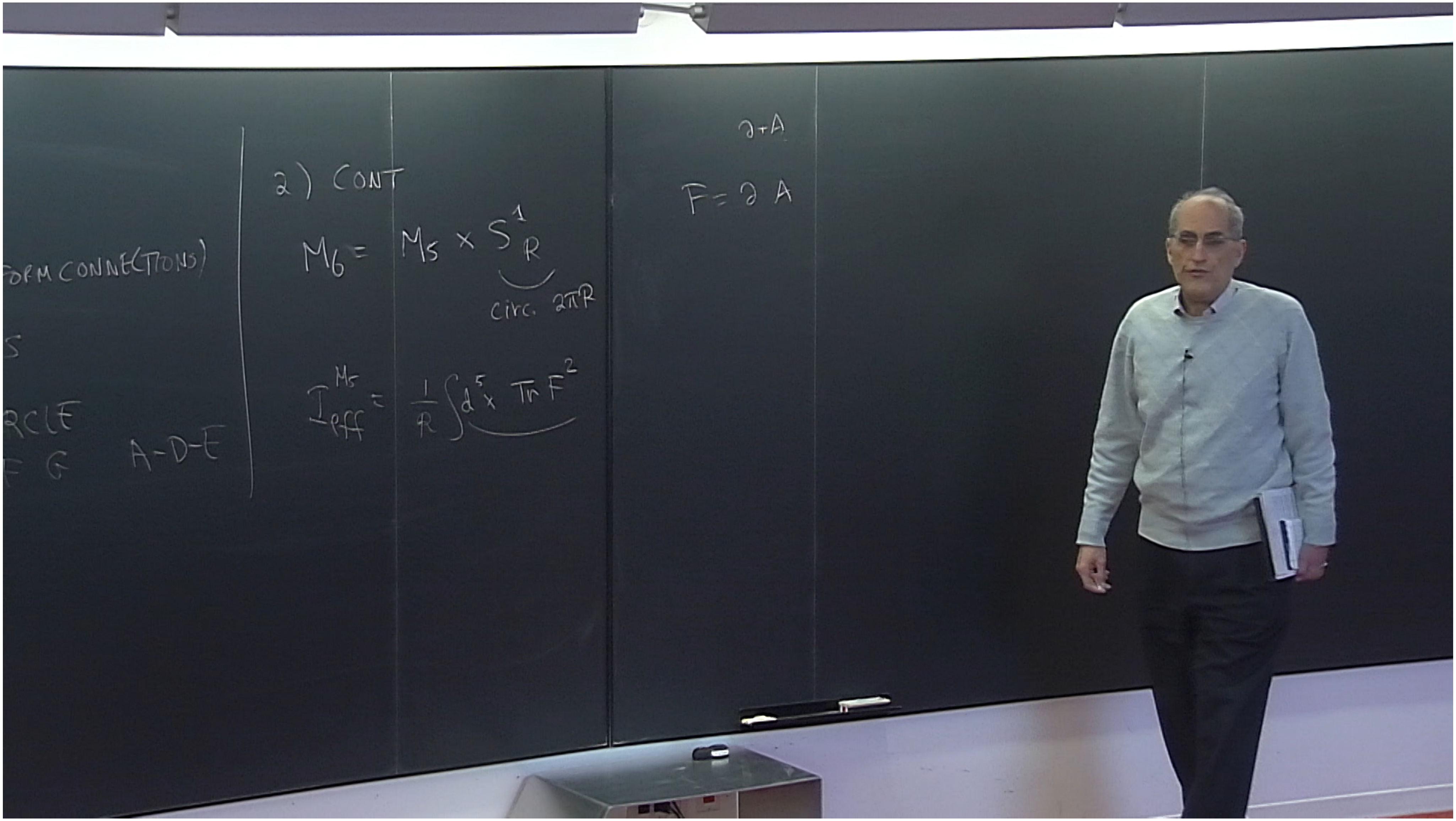
1) ABELIANIZE
GET GERBES (2-FORM CONNECTIONS)
OF MAXIMAL TORUS

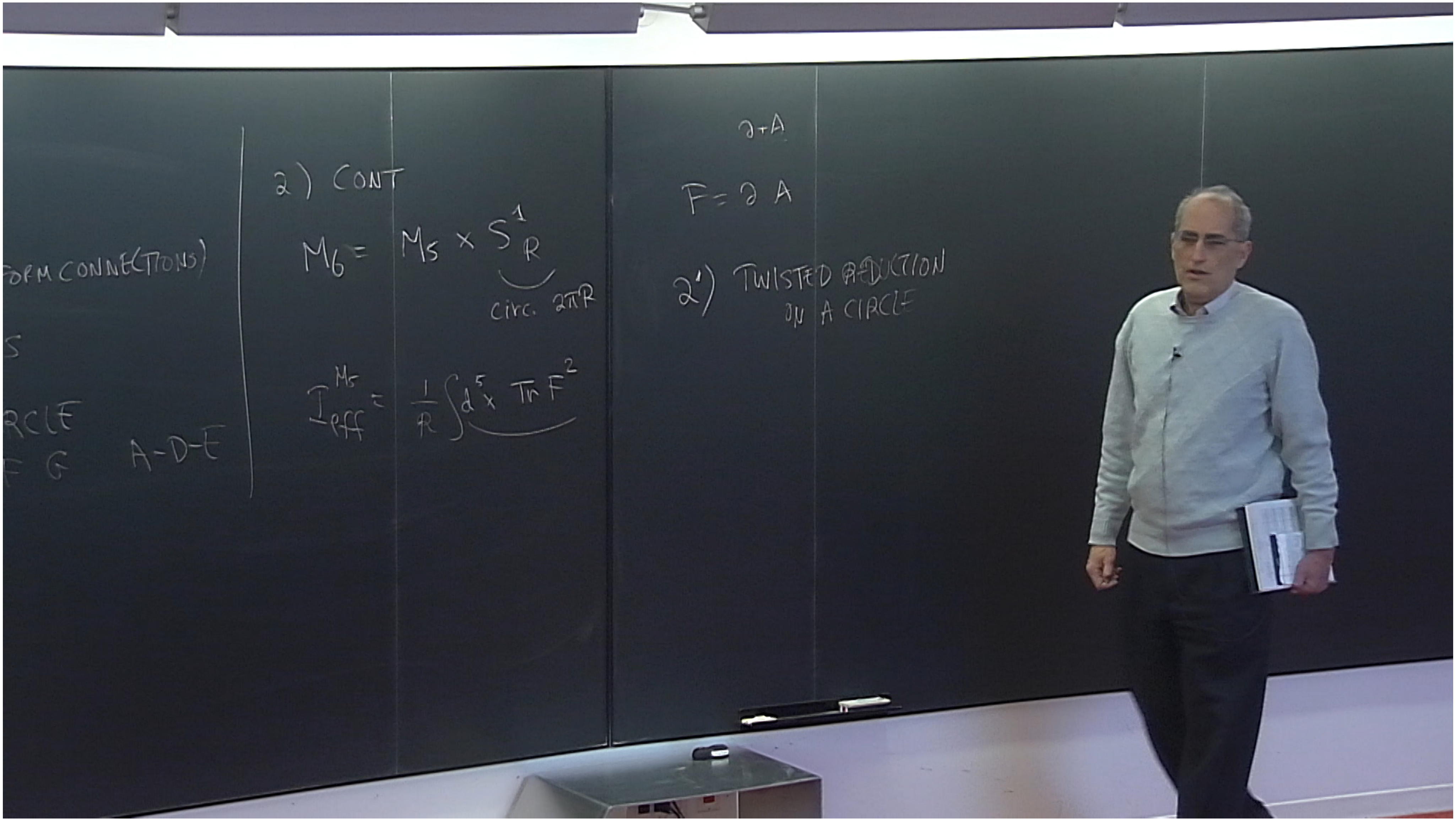
2) REDUCE ON A CIRCLE
GAUGE FIELDS OF G A-D-E


NNECTIONS)
A-D-E
2) CONT
$M_6 = M_5 \times S^1_R$
circ. $2\pi R$
$I^{M_5}_{eff} = \int d^5x \, \mathrm{Tr} F^2$



NNECTIONS)
A-D-E
2) CONT
M6 = M5 × S1R
circ. 2πR
IeffM5 = 1/R ∫d5x Tr F2


ORM CONNECTIONS)
S
RCLE
F G A-D-E
2) CONT
M6 = M5 × S1R
circ. 2πR
IeffM5 = 1/R ∫d5x Tr F2
∂+A
F = ∂ A


ORM CONNECTIONS)
RCLE
A-D-E
2) CONT
$M_6 = M_5 \times S^1_R$
circ. $2\pi R$
$I_{eff}^{M_5} = \frac{1}{R}\int d^5x \, \mathrm{Tr} F^2$
$\partial + A$
$F = \partial A$
2') TWISTED REDUCTION
ON A CIRCLE

$\partial + A$

$S^1_R$

circ. $2\pi R$

$d^5x \; \mathrm{Tr}\, F^2$

$F = \partial A$

$G_2$

SPIN(8) TRIALITY

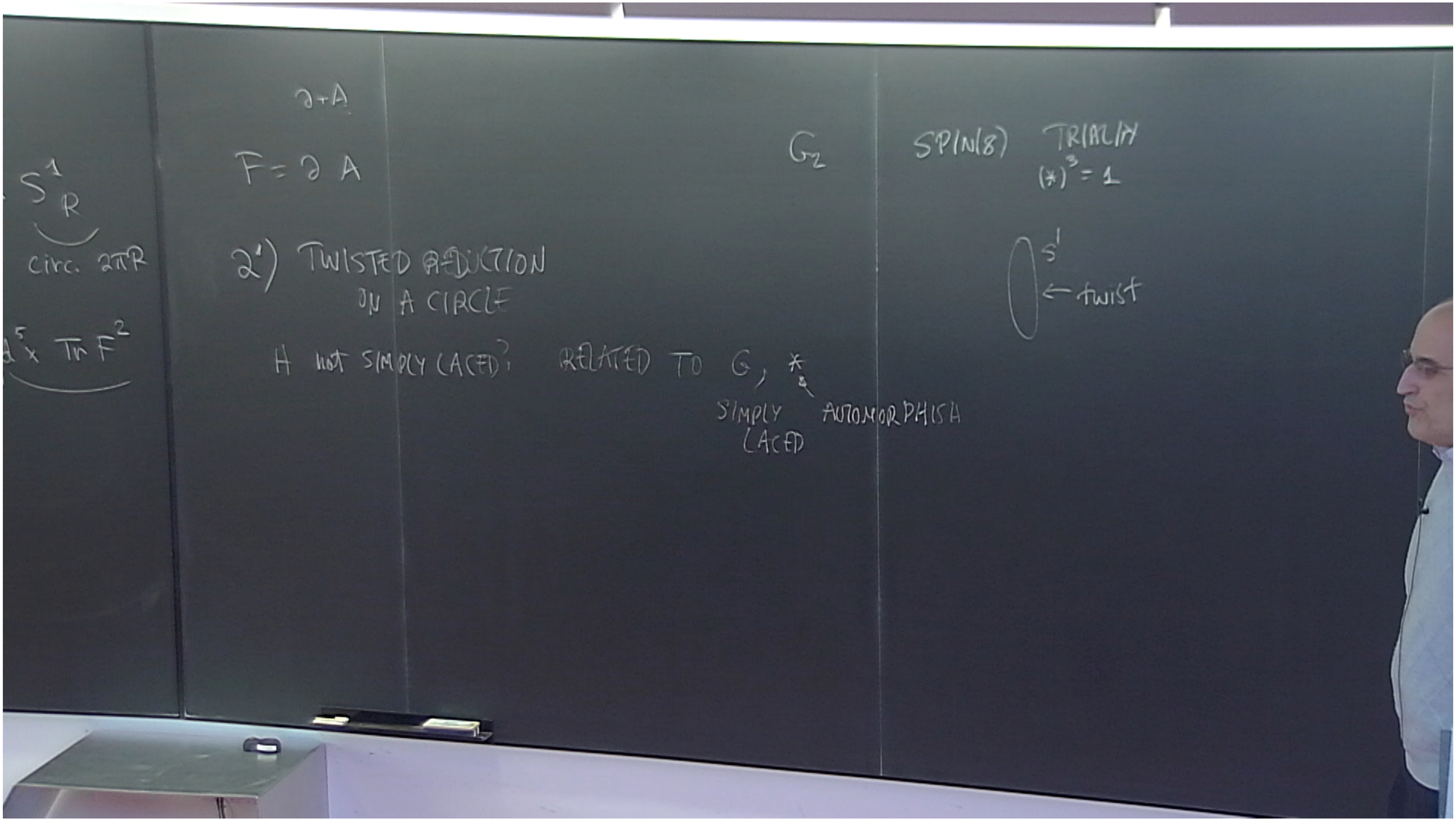
$(*)^3 = 1$

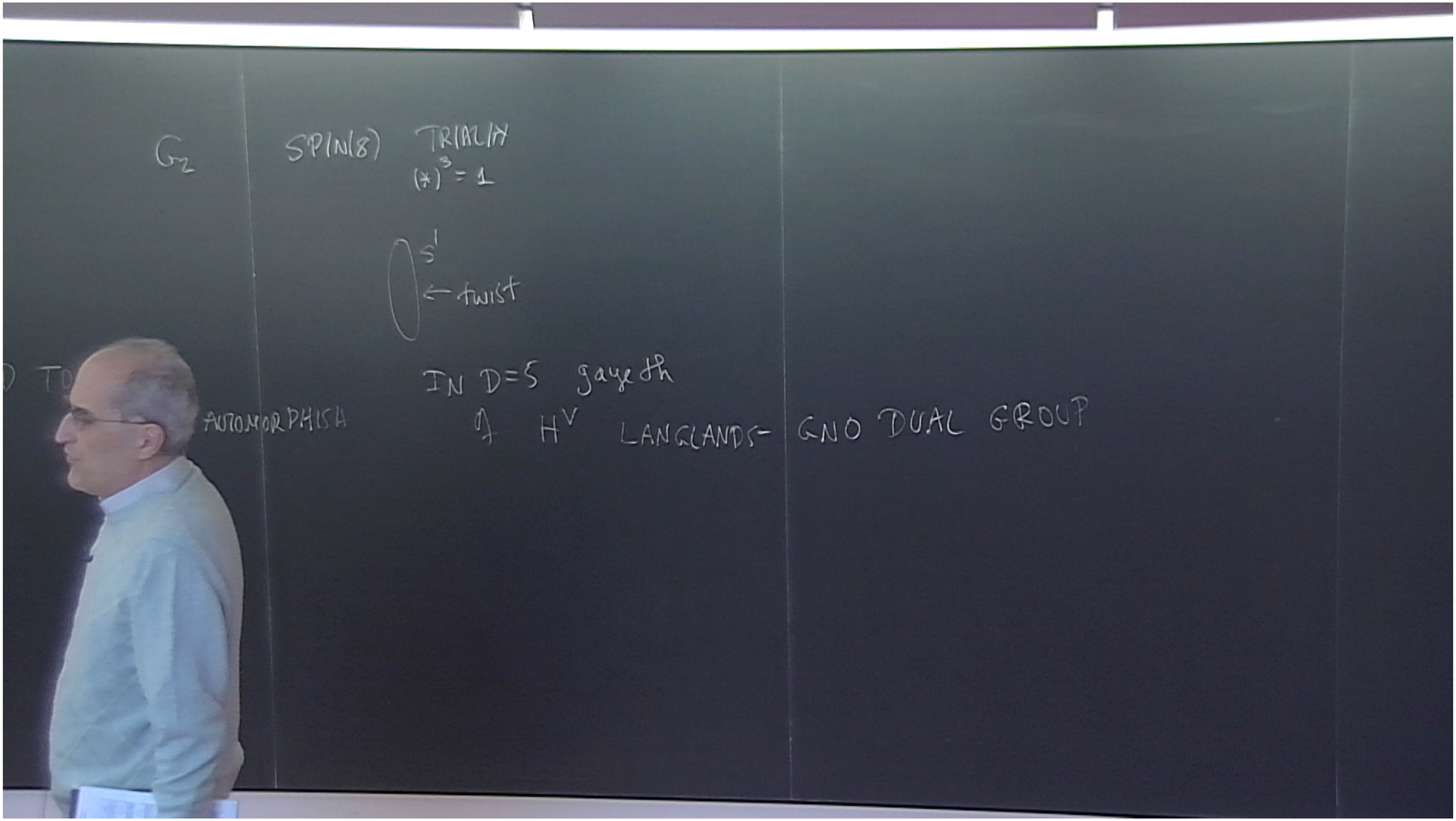
2') TWISTED REDUCTION ON A CIRCLE

H not SIMPLY LACED? RELATED TO G, *

SIMPLY LACED

AUTOMORPHISM

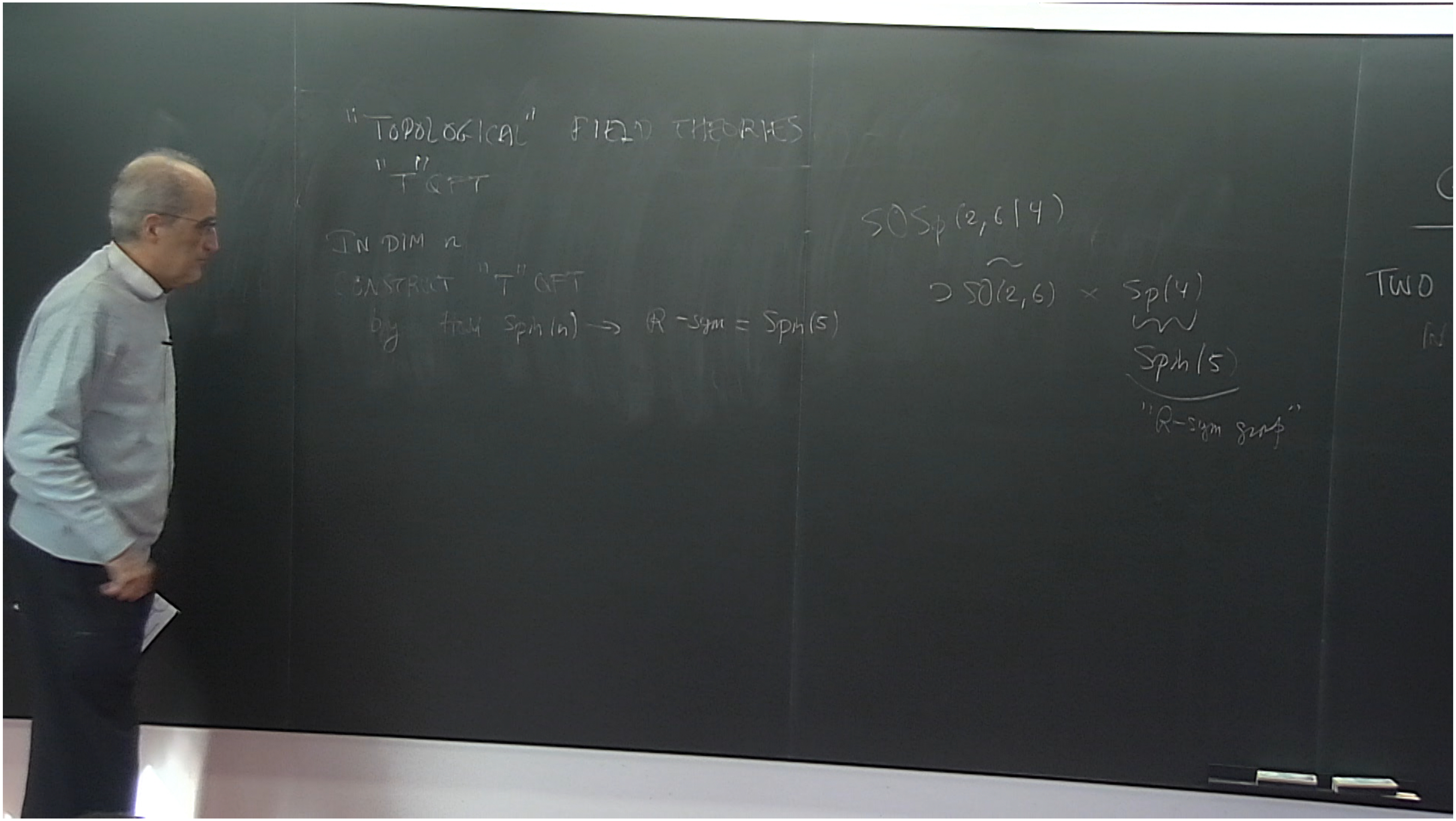

∂+A
F = ∂ A
S1 R
circ. 2πR
d5x Tr F2
2') TWISTED REDUCTION
ON A CIRCLE
H not SIMPLY LACED? RELATED TO G, *
SIMPLY LACED
AUTOMORPHISM
G2
SPIN(8) TRIALITY
(*)3 = 1
S1
← twist


G2
SPIN(8) TRIALITY
(*)^3 = 1
S^1
← twist
IN D=5 gauge th
of H^∨ LANGLANDS-GNO DUAL GROUP
AUTOMORPHISM

"TOPOLOGICAL" FIELD THEORIES

"T"QFT

TWO


"TOPOLOGICAL" FIELD THEORIES
"T" QFT
IN DIM n
CONSTRUCT "T" QFT
Spin(n) → R-sym = Spin(5)
SOSp(2,6|4)
⊃ SO(2,6) × Sp(4)
Spin(5)
"R-sym grp"
TWO

"TOPOLOGICAL" FIELD THEORIES

"T" QFT

IN DIM $n$

CONSTRUCT "T" QFT

by twist $Spin(n) \to R\text{-sym} = Spin(5)$

In dim 6 NONE

In dim 5

one $Spin(5) \to Spin(5)$

$SOSp(2,6|4)$

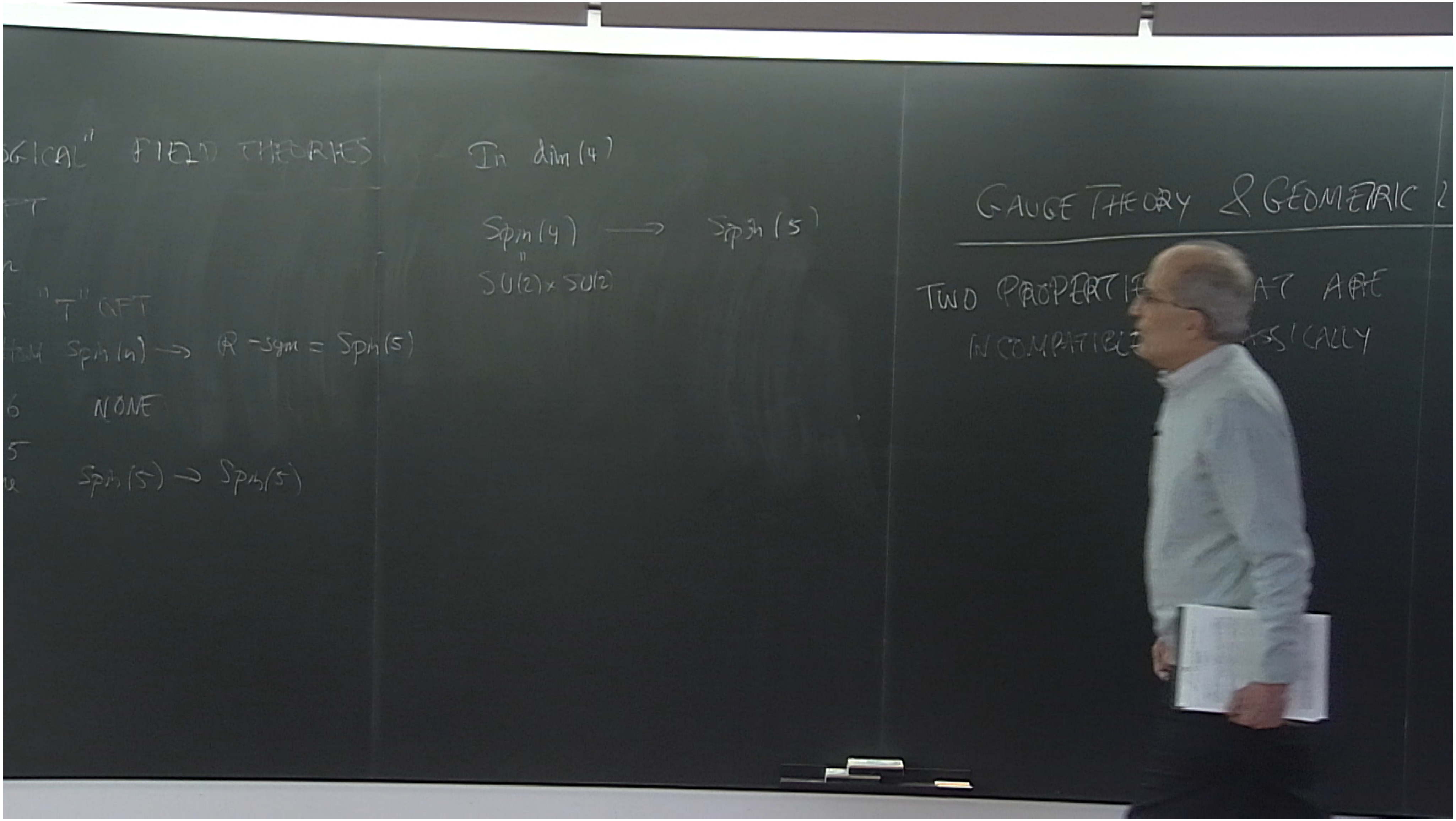
$\sim$

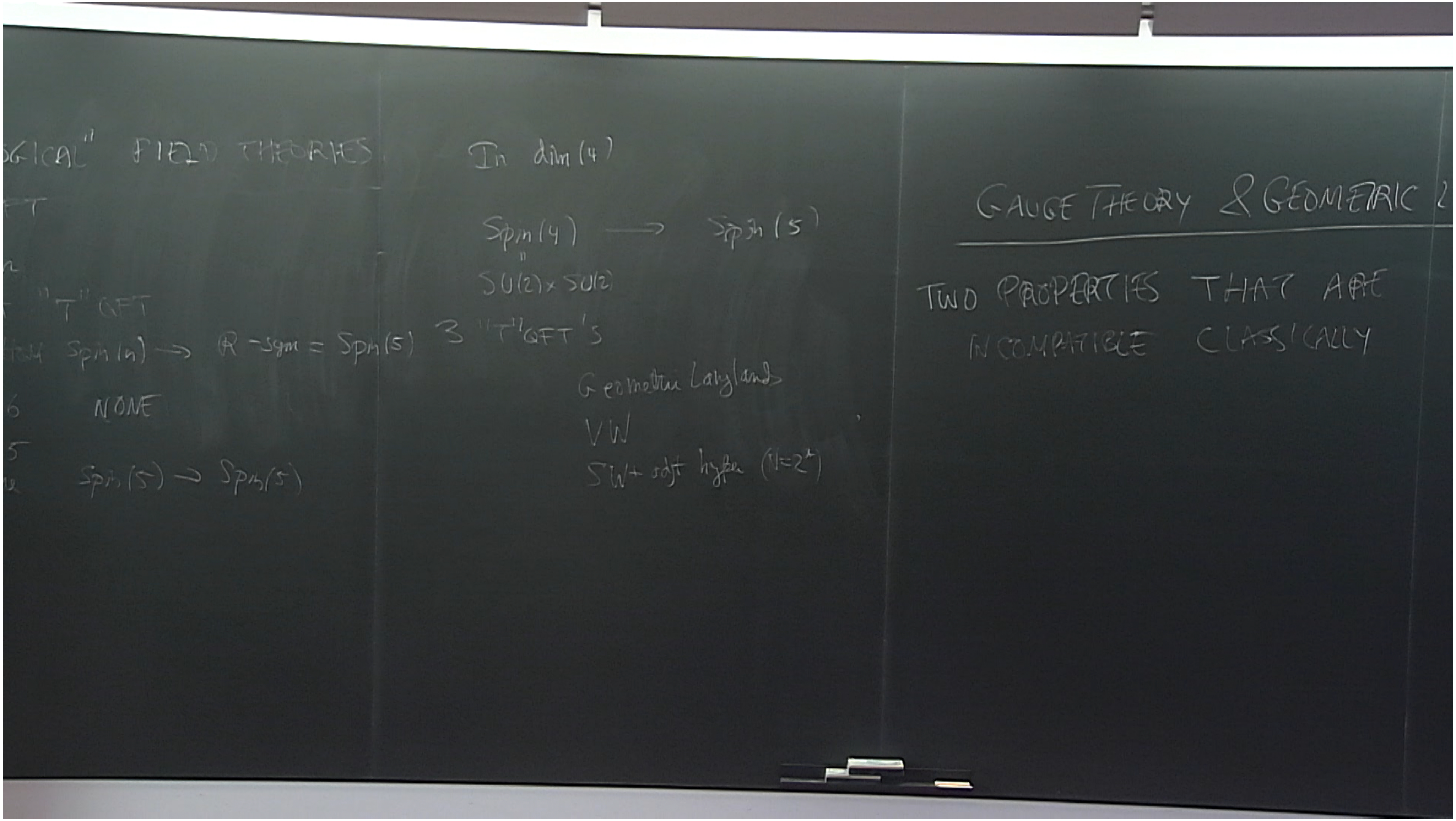
$\supset SO(2,6) \times \underbrace{Sp(4)}_{\substack{\sim \\ Spin(5)}}$

"R-sym group"

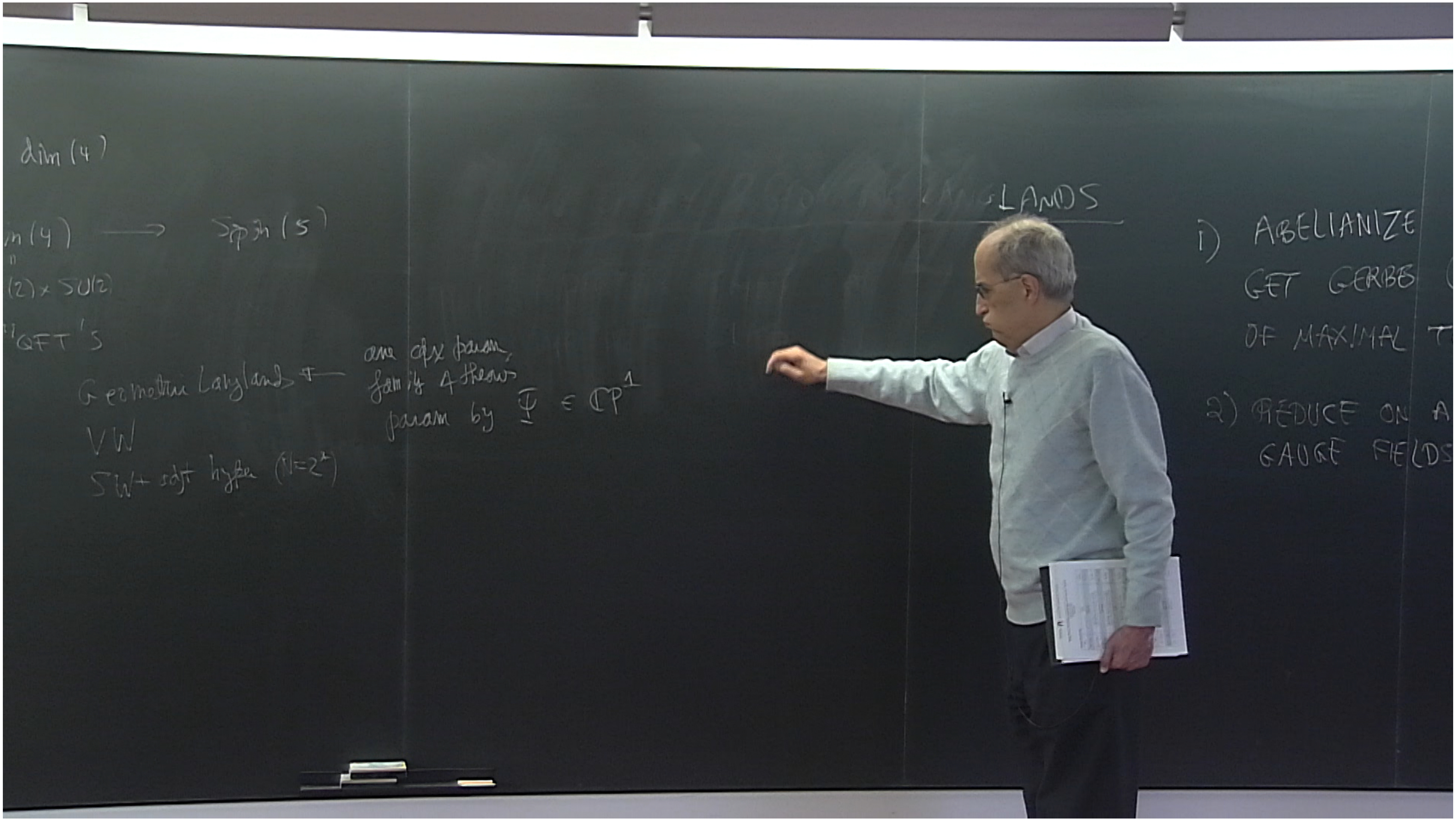
TWO

IN


FIELD THEORIES
In dim(4)
Spin(4) → Spin(5)
SU(2)×SU(2)
"T" QFT
Spin(n) → R-sym = Spin(5)
NONE
Spin(5) → Spin(5)
GAUGE THEORY & GEOMETRIC
TWO PROPERT
AT ARE
IN COMPATIBL
ICALLY



GICAL" FIELD THEORIES
"T" QFT
Spin(n) → R-sym = Spin(5)
NONE
Spin(5) → Spin(5)
In dim(4)
Spin(4) → Spin(5)
SU(2)×SU(2)
3 "T" QFT's
Geometric Langlands
VW
SW + adjt hyper (N=2*)
GAUGE THEORY & GEOMETRIC
TWO PROPERTIES THAT ARE
INCOMPATIBLE CLASSICALLY


dim (4)
Spin(5)
SU(2)
QFT's
Geometric Langlands
one cpx param,
param by $\Psi \in \mathbb{CP}^1$
VW
LANDS
1) ABELIANIZE
GET GERBES
OF MAXIMAL
2) REDUCE ON A
GAUGE FIELDS

dim (4)

$\text{…in}(4) \longrightarrow \text{Spin}(5)$

$\|$

$\text{…}(2) \times SU(2)$

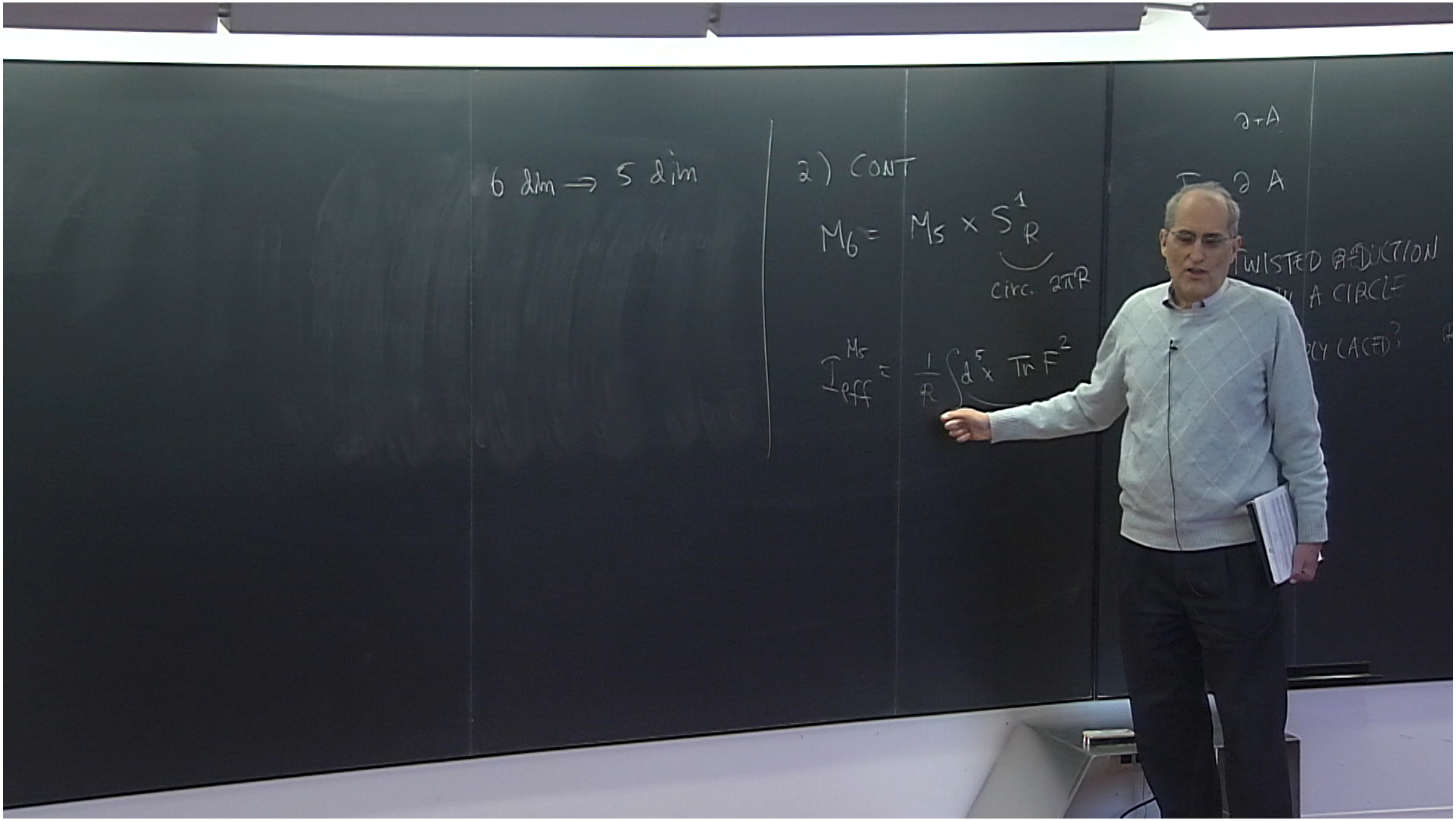
"QFT"'s

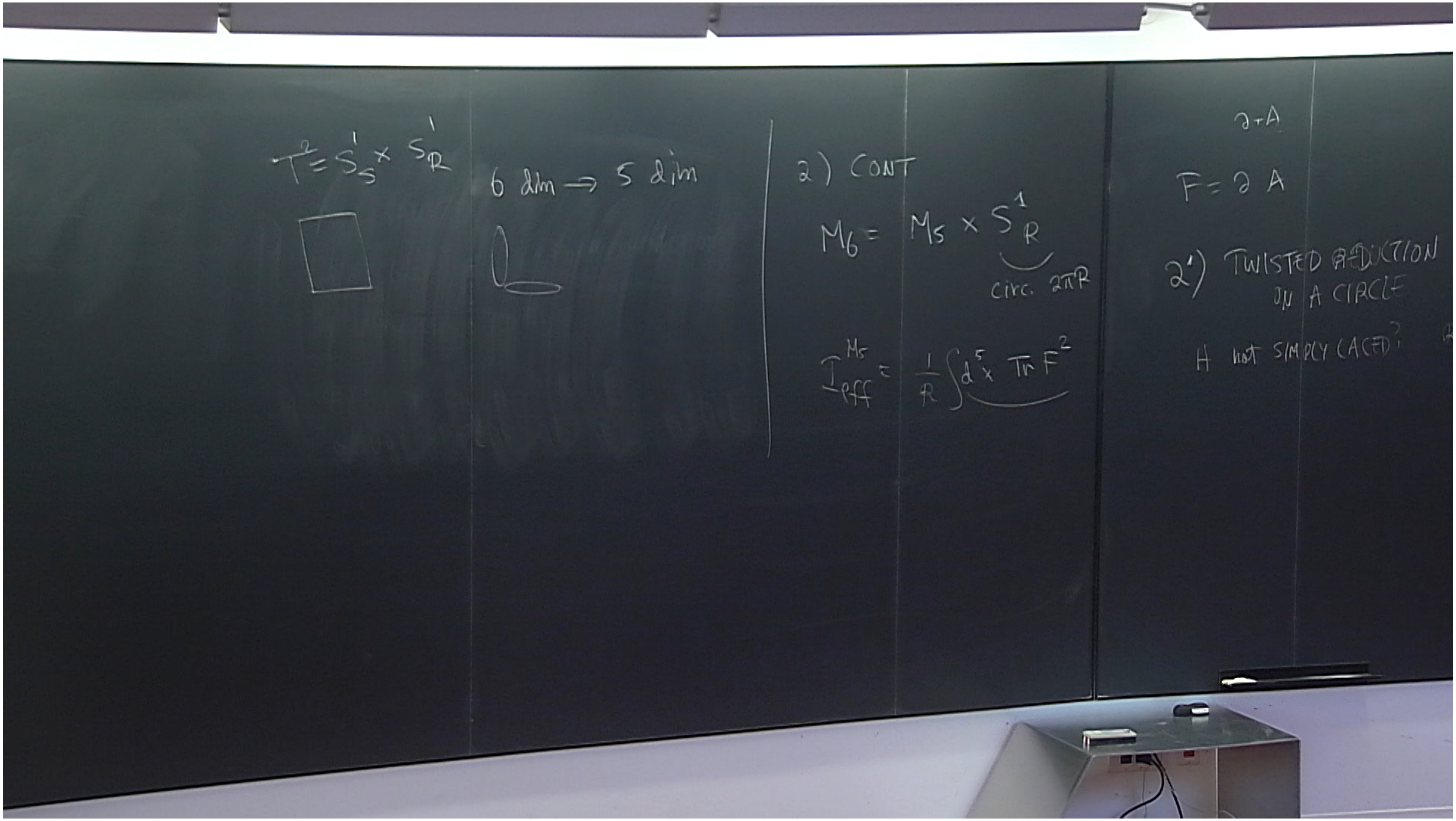
Geometric Langlands $\longleftarrow$ one cpx param, family 4 theories param by $\Psi \in \mathbb{CP}^1$

VW

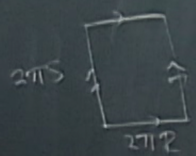
SW + adj hyper ($N=2^*$)

Geom Lan

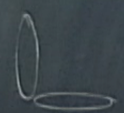

6 dim ⇒ 5 dim
2) CONT
M6 = M5 × S1R
circ. 2πR
Ieff M5 = 1/R ∫ d5x Tr F2
∂+A
∂ A
TWISTED REDUCTION
A CIRCLE


T² = S¹_S × S¹_R
6 dim ⇒ 5 dim
2) CONT
M₆ = M₅ × S¹_R
circ. 2πR
I_eff^M₅ = 1/R ∫ d⁵x Tr F²
∂+A
F = ∂ A
2') TWISTED REDUCTION
ON A CIRCLE
H not SIMPLY CONNECTED?

$T^2 = S^1_S \times S^1_R$




6 dim $\rightarrow$ 5 dim



2) CONT

$$M_6 = M_5 \times \underbrace{S^1_R}_{\text{circ. } 2\pi R}$$

$$I^{M_5}_{eff} = \frac{1}{R}\underbrace{\int d^5x \, \mathrm{Tr}\, F^2}$$

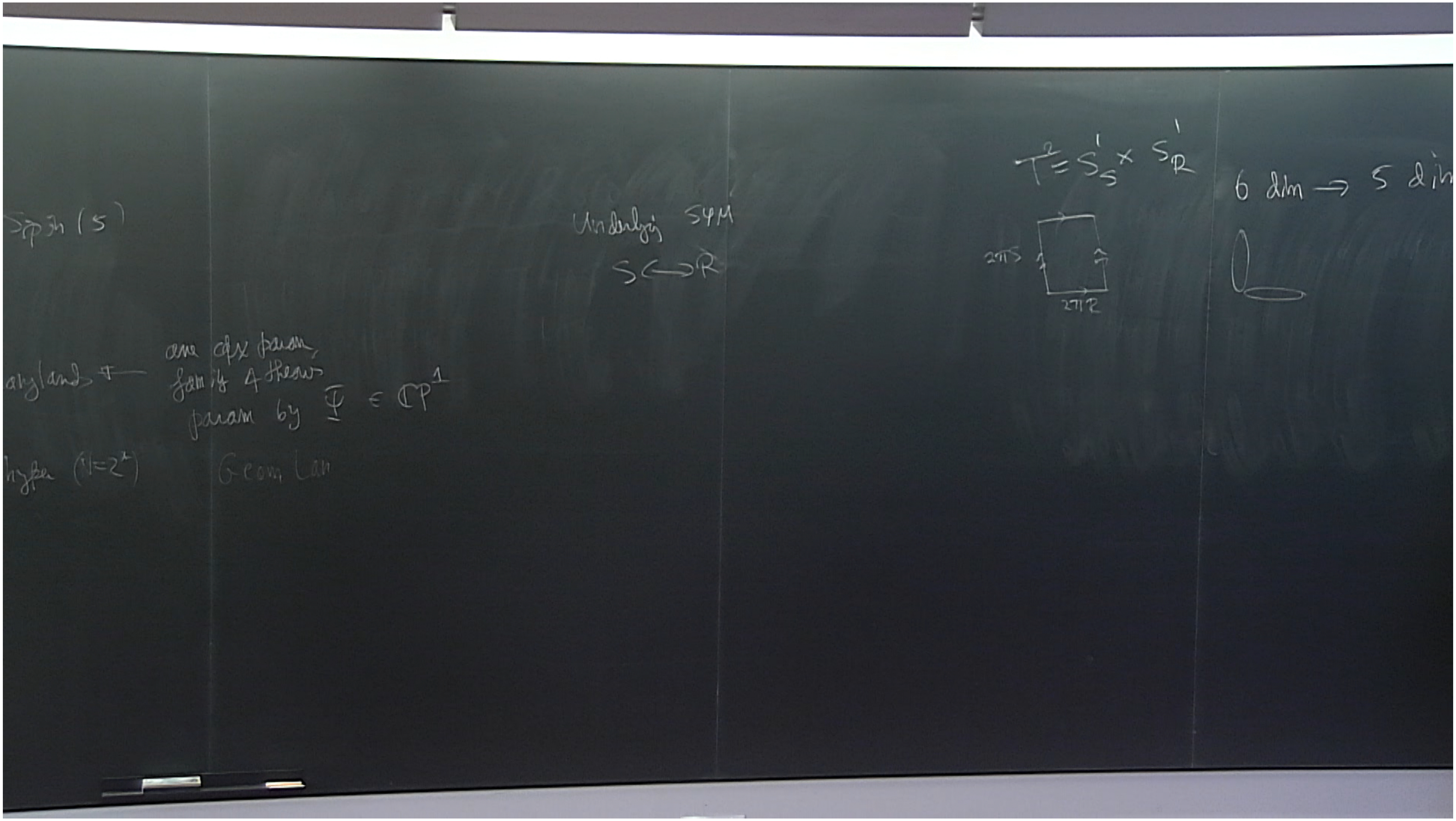
$$\underset{4\,\mathrm{dim}}{\Rightarrow} \frac{S}{R}\int d^4x \, \mathrm{Tr}\, F^2$$

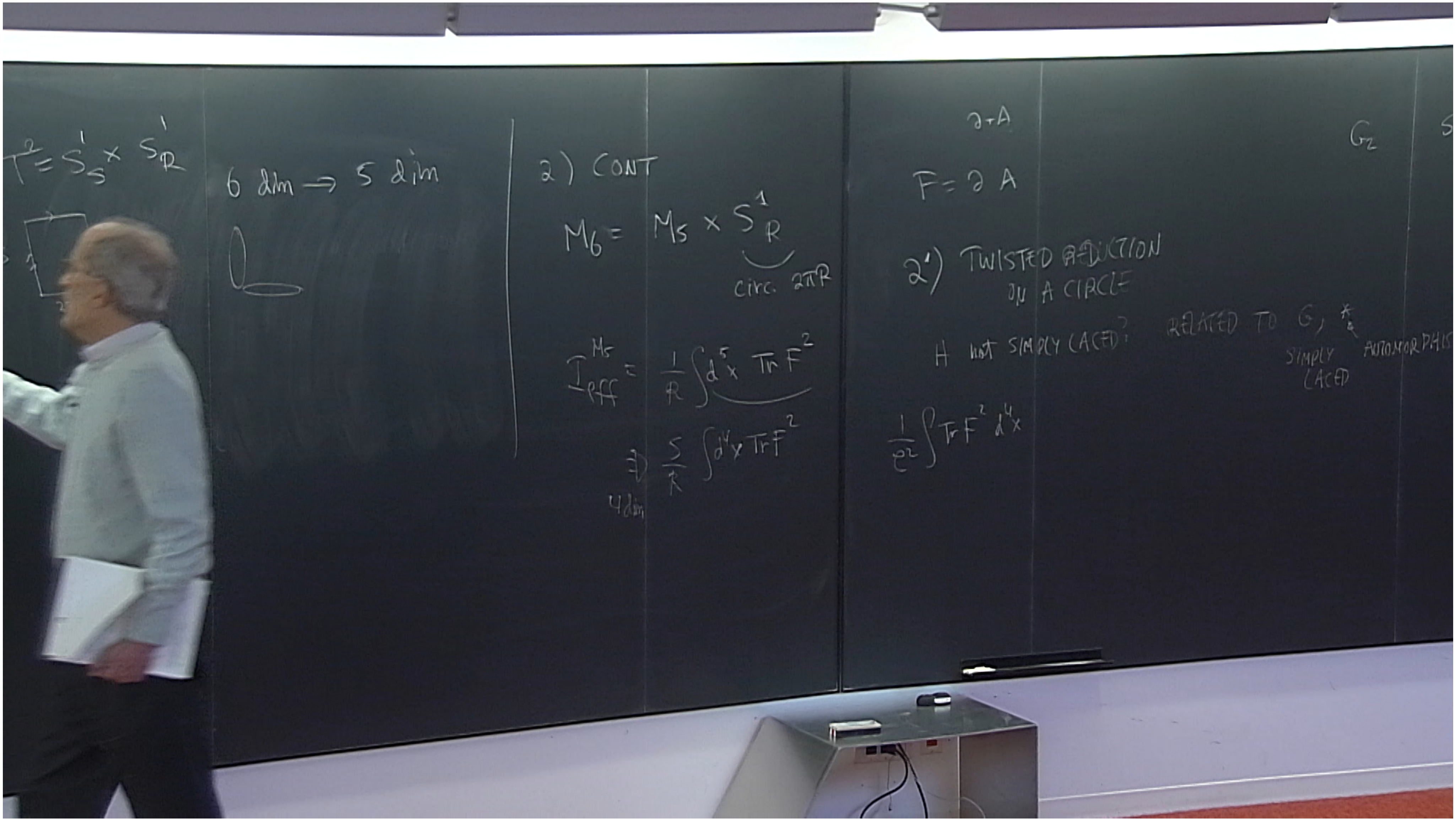
$\partial + A$

$F = \partial A$

2') TWISTED REDUCTION ON A CIRCLE

H not SIMPLY LACED?


Underlying SYM
S ⟷ R
T² = S¹_S × S¹_R
2πS
2πR
6 dim → 5 di
one cpx param
family 4 theories
param by Ψ ∈ CP¹
Geom Lan


$T^2 = S^1_S \times S^1_R$
6 dim → 5 dim
2) CONT
$M_6 = M_5 \times S^1_R$
circ. $2\pi R$
$I^{M_5}_{eff} = \frac{1}{R}\int d^5x \, \mathrm{Tr} F^2$
$\Rightarrow \frac{S}{R}\int d^4x \, \mathrm{Tr} F^2$
4 dim
$\partial_\tau A$
$F = \partial A$
2') TWISTED REDUCTION
ON A CIRCLE
H not SIMPLY LACED?
RELATED TO G, SIMPLY LACED
AUTOMORPHIS
$\frac{1}{e^2}\int \mathrm{Tr} F^2 d^4x$
$G_2$

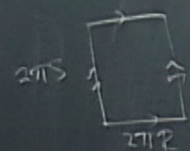
Underlying SYM

$S \longleftrightarrow R$

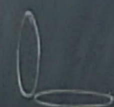
$e^2 = \frac{R}{S}$

$e^2 \longleftrightarrow \frac{1}{e^2}$

$T^2 = S^1_S \times S^1_R$




6 dim → 5 dim



2) CONT

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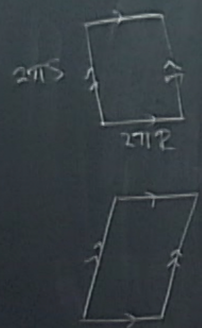
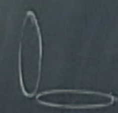



6 dim → 5 dim



2) CONT

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Underlying SYM

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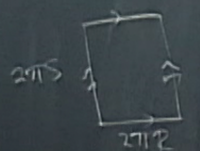
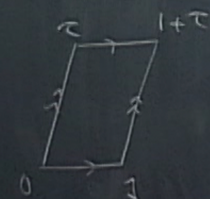
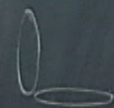



6 dim $\rightarrow$ 5 dim



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$I^{M_5}_{eff} = \frac{1}{R} \underbrace{\int d^5x \, \mathrm{Tr}\, F^2}$

$\underset{4\,\text{dim}}{\Rightarrow} \frac{S}{R} \int d^4x \, \mathrm{Tr}\, F^2$

cpx param,
... 4 theories
...ram by $\Psi \in \mathbb{CP}^1$
Geom Lan...

Underlying SYM

$S \longleftrightarrow R$

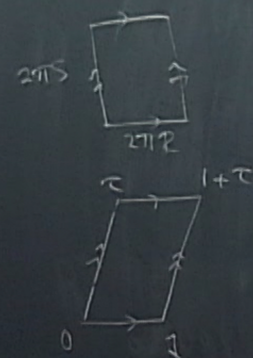
$\tau \to -\frac{1}{\tau}$

$\tau \to \frac{a\tau+b}{c\tau+d}$

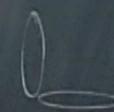
$e^2 = \frac{R}{S}$

$e^2 \longleftrightarrow \frac{1}{e^2}$

$T^2 = S^1_S \times S^1_R$




6 dim $\to$ 5 dim



2) Co...

$M_6 =$

$I_{eff}^{M_5} =$

4di...

cpx param,
ly 4 theory
param by $\Phi \in \mathbb{C}P^1$

Geom Lan

Underlying SYM
$S \longleftrightarrow R$

$\tau \to -\frac{1}{\tau}$

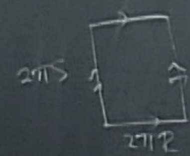
$\tau \to \frac{a\tau+b}{c\tau+d}$

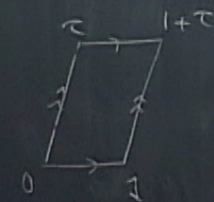
$e^2 = \frac{R}{S}$

$e^2 \longleftrightarrow \frac{1}{e^2}$

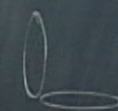
$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{e^2}$

$T^2 = S^1_S \times S^1_R$




6 dim → 5 dim



2) Co

$M_6 =$

$I^{M_5}_{eff} =$

4d

5 dim

2) CONT

$M_6 = M_5 \times \underbrace{S^1_R}_{\text{circ. } 2\pi R}$

$I^{M_5}_{eff} = \frac{1}{R} \int \underbrace{d^5x \ \mathrm{Tr}\, F^2}$

$\ni \frac{S}{R} \int d^4x \ \mathrm{Tr}\, F^2$

4 dim

$\partial_\tau A$

$F = \partial A$

2') TWISTED REDUCTION ON A CIRCLE

H not SIMPLY LACED? RELATED TO G, * SIMPLY LACED AUTOMORPHISM

$\frac{1}{e^2} \int \mathrm{Tr}\, F^2 \, d^4x \ + \frac{i\theta}{8\pi} \int \mathrm{Tr}\, F \wedge F$

$G_2$

SPIN(8) TRIALITY

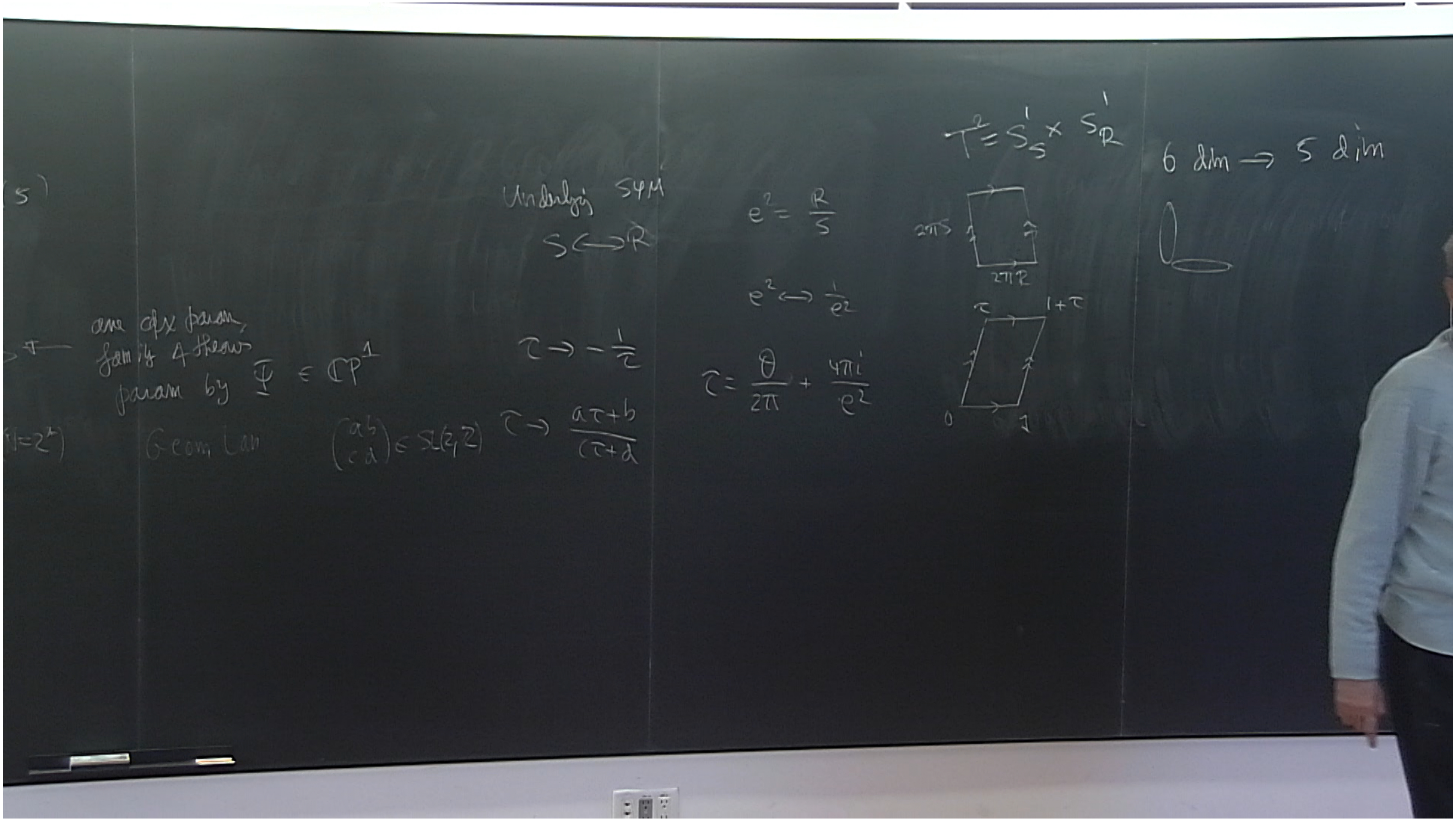
$(*)^3 = 1$

$S^1$

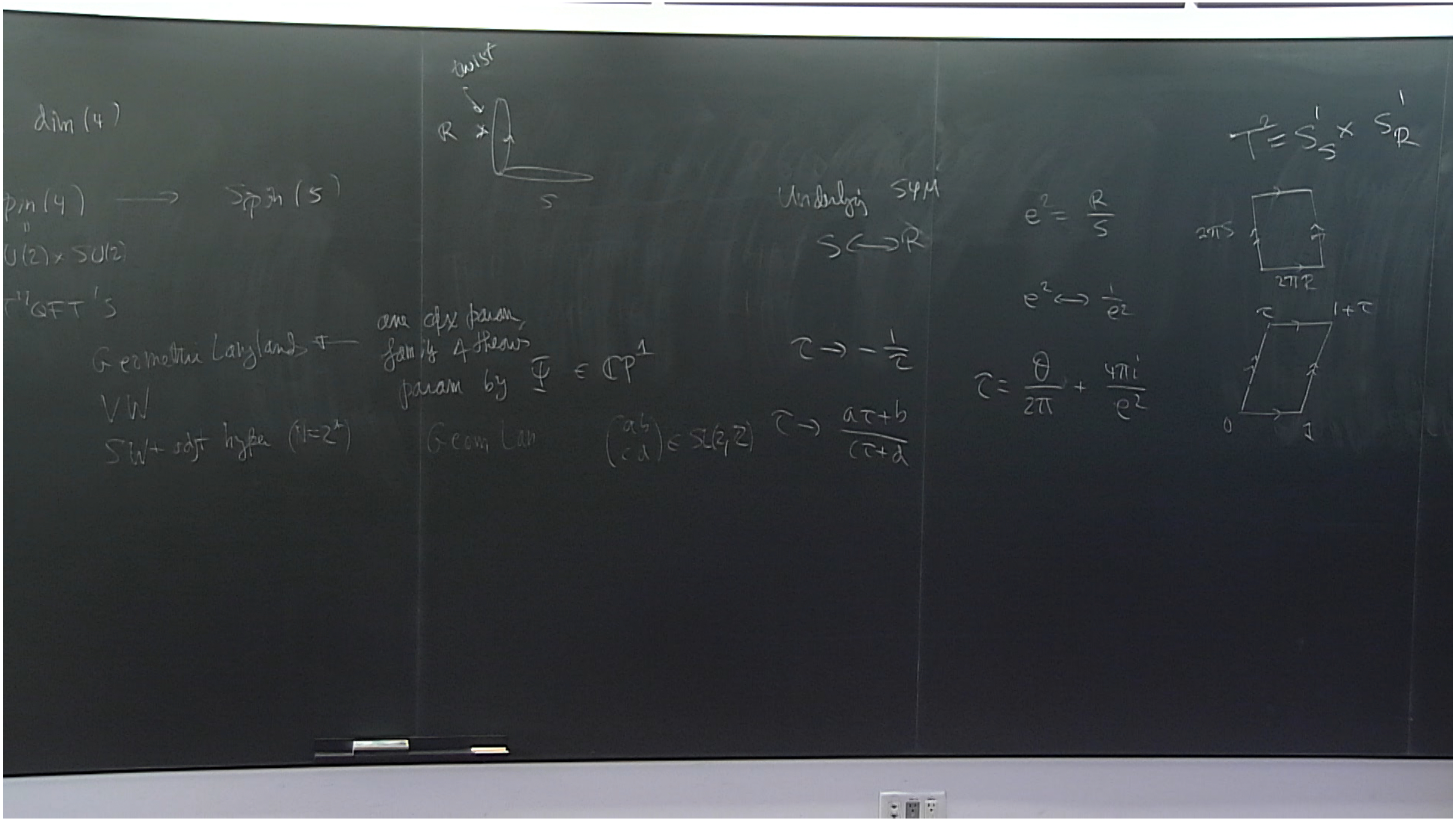
← twist

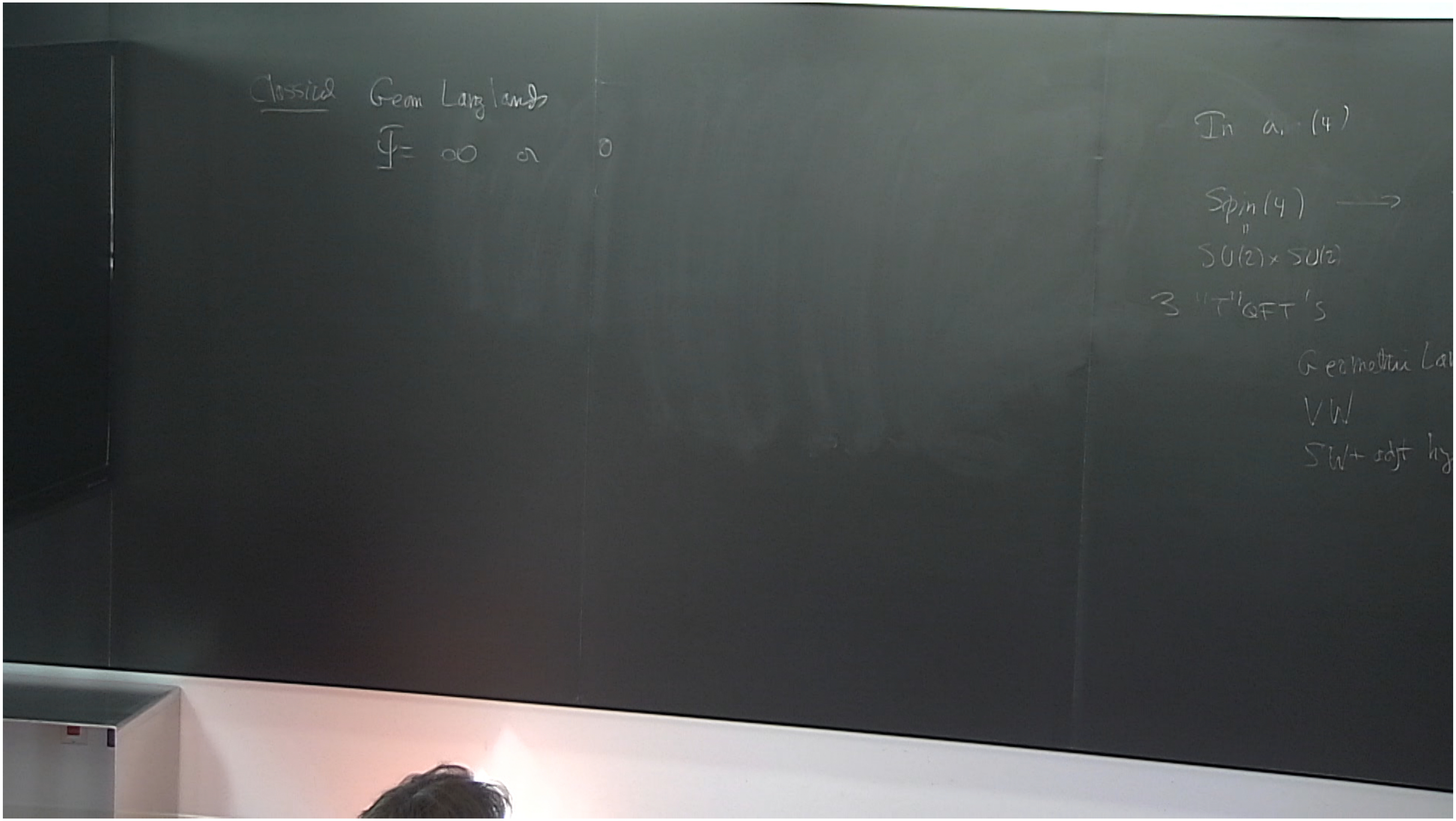
IN D=5 gauge th

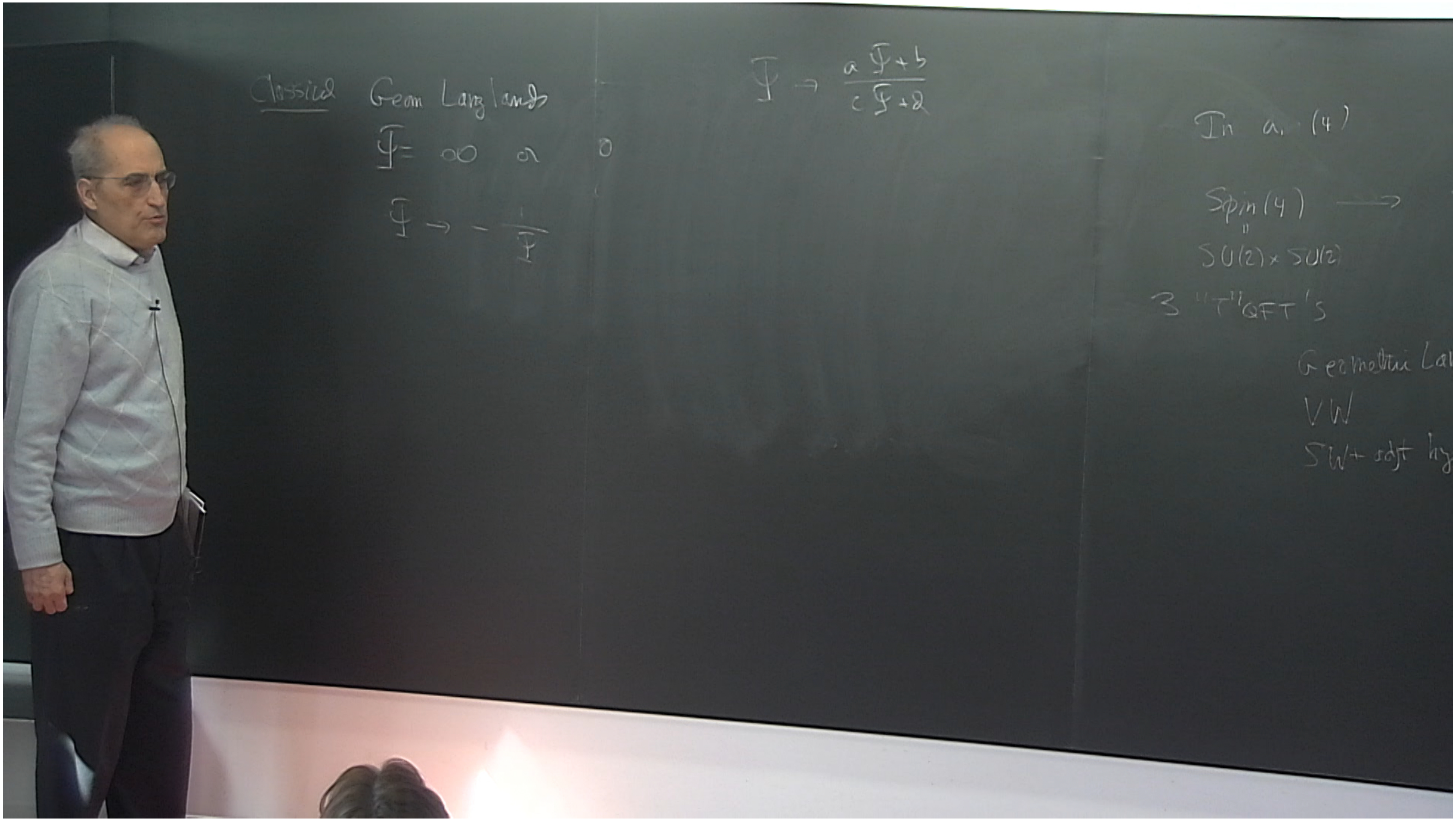
of $H^\vee$ LANGLANDS - GNO

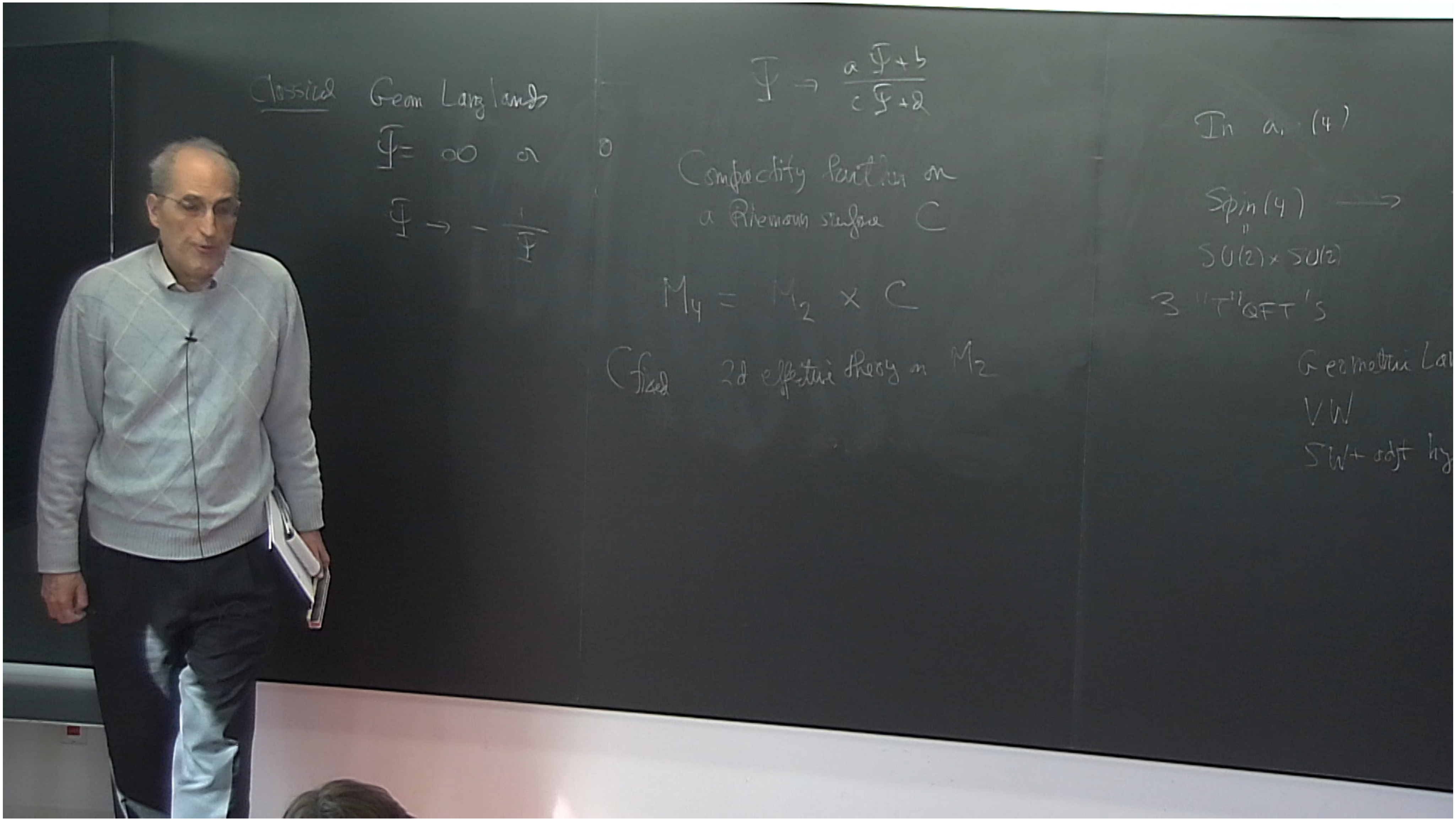


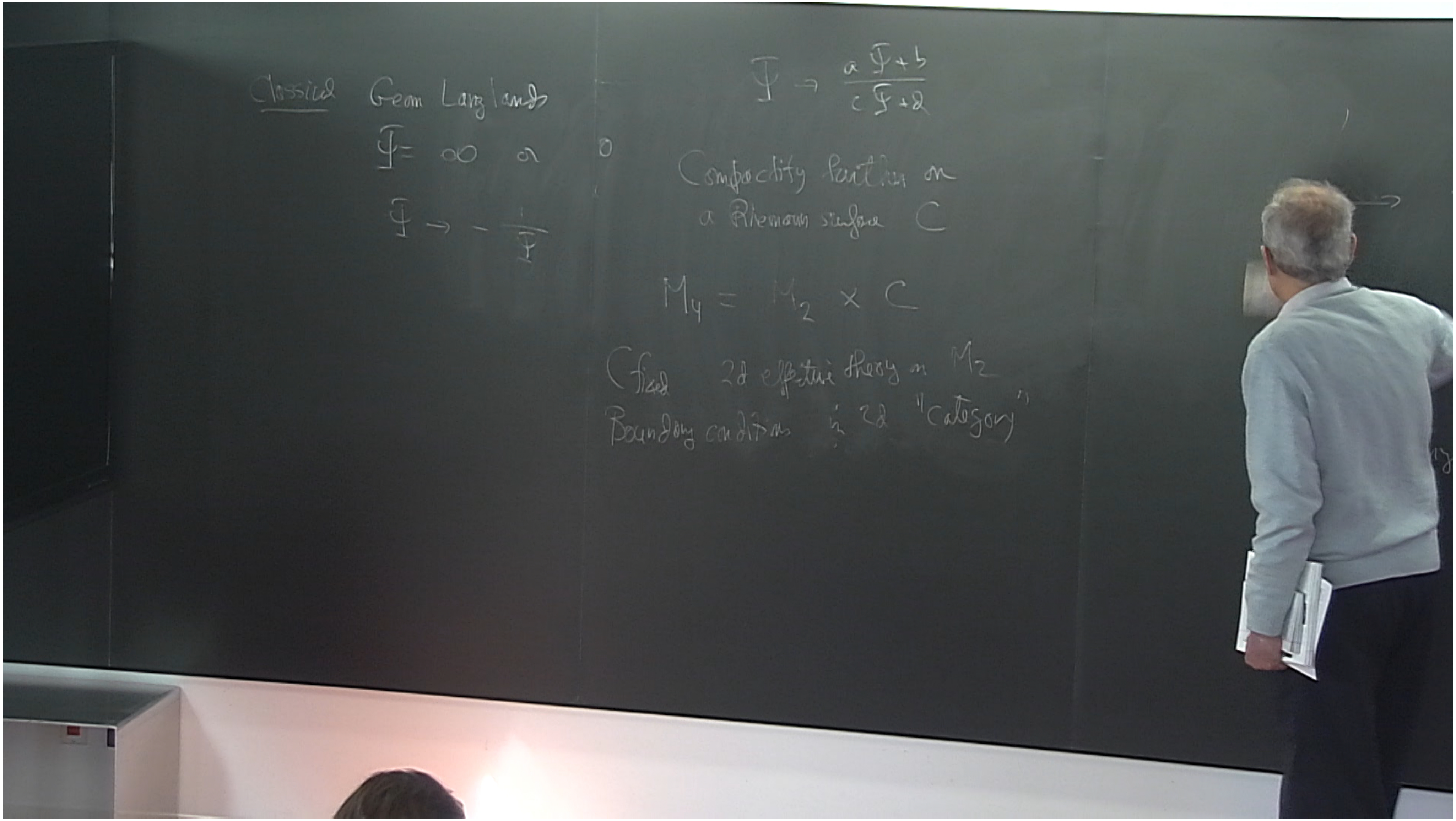
Underlying SYM
S ⟷ R
one cpx param,
family 4 theories
param by Ψ ∈ CP^1
Geom Lan
τ → −1/τ
(a b; c d) ∈ SL(2, Z)
τ → (aτ+b)/(cτ+d)
e^2 = R/S
e^2 ⟷ 1/e^2
τ = θ/2π + 4πi/e^2
T^2 = S^1_S × S^1_R
2πS
2πR
τ
1+τ
0
1
6 dim → 5 dim

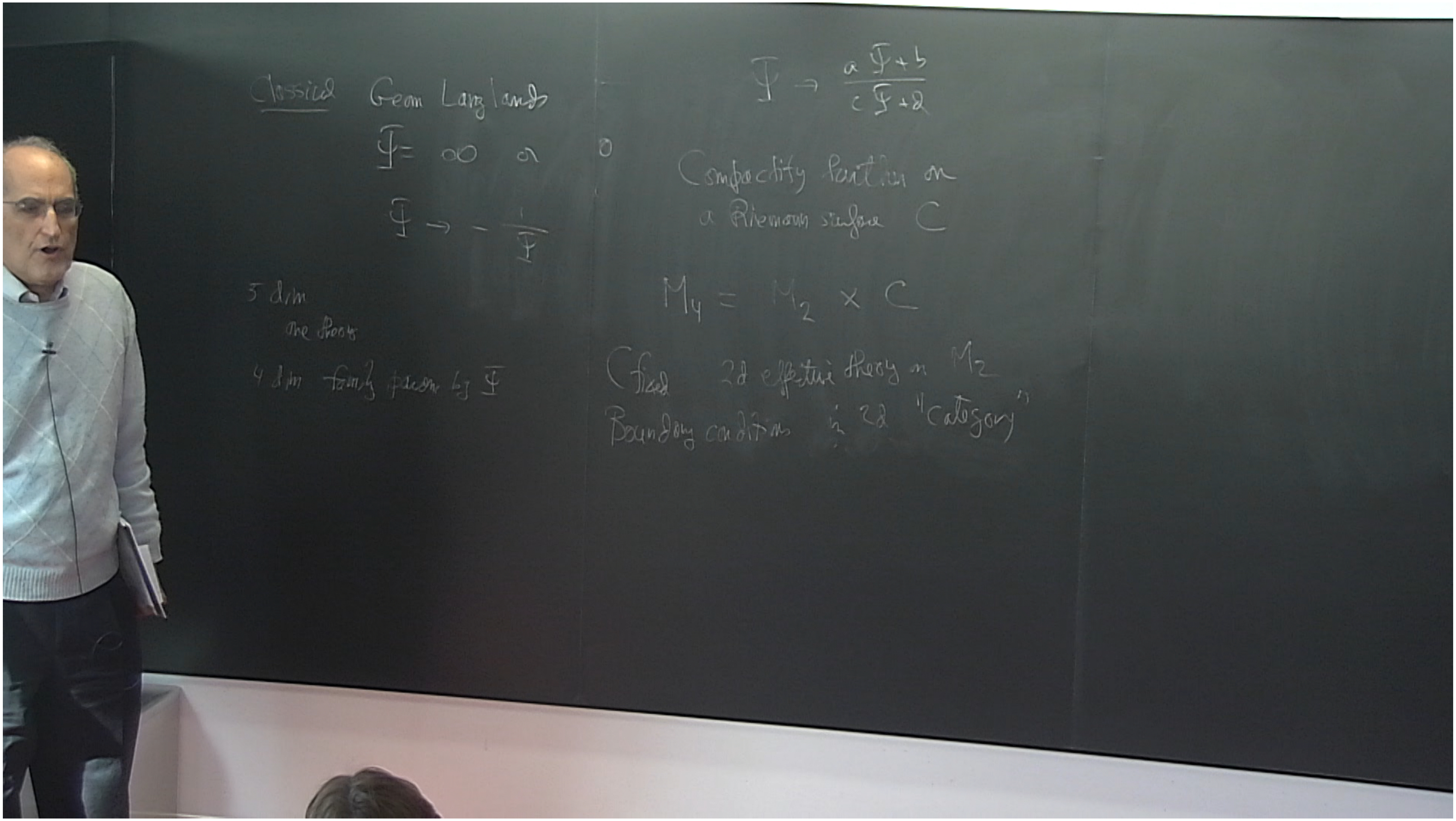

twist
R
S
dim (4)
pin(4) ⟶ Spin (5)
U(2) × SU(2)
QFT's
Geometric Langlands
VW
SW + adj hyper ($N=2^*$)
one cpx param,
family of theories
param by $\Psi \in \mathbb{CP}^1$
Geom Lan
$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$
Underlying SYM
$S \longleftrightarrow R$
$\tau \to -\frac{1}{\tau}$
$\tau \to \frac{a\tau+b}{c\tau+d}$
$e^2 = \frac{R}{S}$
$e^2 \longleftrightarrow \frac{1}{e^2}$
$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{e^2}$
$T^2 = S^1_S \times S^1_R$
$2\pi S$
$2\pi R$
$\tau$
$1+\tau$
0
1

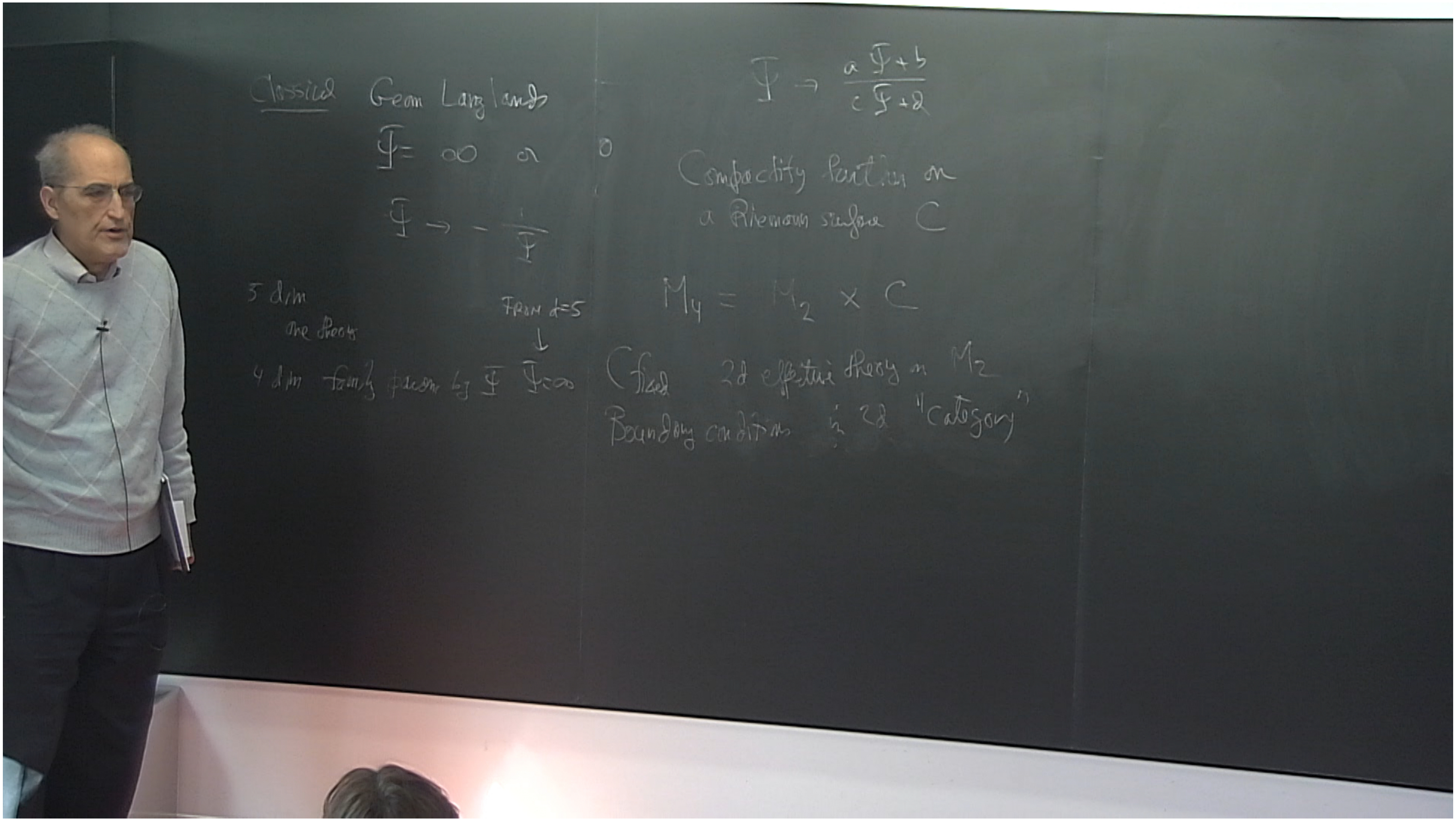

Classical Geom Langlands
Ψ = ∞ or 0
In
Spin(4)
SU(2) × SU(2)
3 "T" QFT's
VW


Classical Geom Langlands
Ψ = ∞ or 0
Ψ → − 1/Ψ
Ψ → (aΨ + b)/(cΨ + d)
Spin(4)
SU(2) × SU(2)
3 "T" QFT's
VW


Classical Geom Langlands
Ψ = ∞ or
Ψ → − 1/Ψ
Ψ → (aΨ + b)/(cΨ + d)
Compactify theory on
a Riemann surface C
M4 = M2 × C
C fixed 2d effective theory on M2
Spin(4)
SU(2) × SU(2)
3 "T" QFT's
VW


Classical Geom Langlands
$\Psi = \infty$ or $0$
$\Psi \to -\frac{1}{\Psi}$
$\Psi \to \frac{a\Psi + b}{c\Psi + d}$
Compactify further on a Riemann surface $C$
$M_4 = M_2 \times C$
$C$ fixed 2d effective theory on $M_2$
Boundary conditions in 2d "category"


Classical Geom Langlands
Ψ = ∞ or 0
Ψ → − 1/Ψ
5 dim
4 dim family param by Ψ
Ψ → (aΨ + b)/(cΨ + d)
Compactify Partition on
a Riemann surface C
M4 = M2 × C
C fixed 2d effective theory on M2
Boundary conditions is 2d "category"


Classical Geom Langlands
Ψ = ∞ or 0
Ψ → −1/Ψ
5 dim
the theory
From d=5
4 dim family param by Ψ Ψ=∞
Ψ → (aΨ+b)/(cΨ+d)
Compactify further on
a Riemann surface C
M4 = M2 × C
C fixed 2d effective theory on M2
Boundary conditions is 2d "category"

$$\Phi \to \frac{a\Phi + b}{c\Phi + d}$$

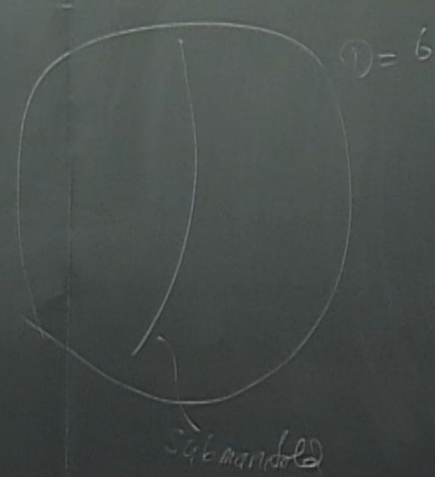
Compactify theory on a Riemann surface $C$

$M_4 = M_2 \times C$

2d effective theory on $M_2$

boundary conditions in 2d "category"

DEFECTS




Underlying

$S \longleftrightarrow$

$\tau \to -\frac{1}{\tau}$

$SL(2,\mathbb{Z}) \quad \tau \to \frac{a\tau + b}{c\tau + d}$

$$\Phi \to \frac{a\Phi+b}{c\Phi+d}$$

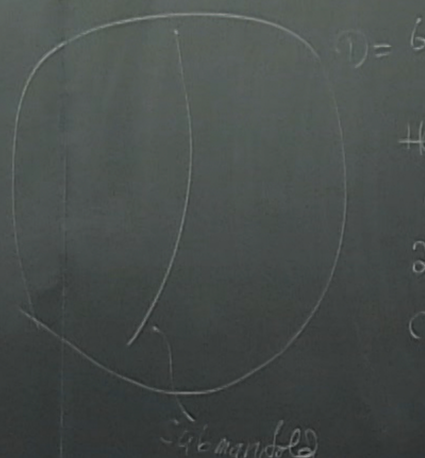
Compactify Partition on a Riemann surface $C$

$M_4 = M_2 \times C$

2d effective theory on $M_2$

...ry conditions in 2d "category"

DEFECTS




Here 2 important classes

2-dim reps of $G$

codim 2 defects nilpotent conjugacy classes

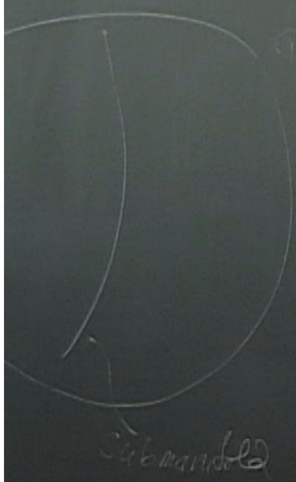
Underlying

$S \longleftrightarrow$

$\tau \to -\frac{1}{\tau}$

$SL(2,\mathbb{Z})$ $\tau \to \frac{a\tau+b}{c\tau+d}$

## DEFECTS




$D = 6$

Here 2 important classes

2-dim reps of G

codim 2 defects   nilpotent conjugacy classes

## 2 dim defects

At least 2 nobs

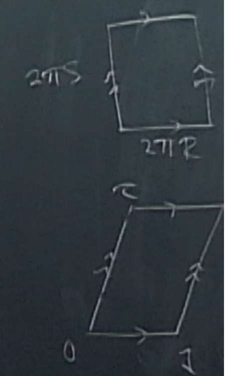
$$M_6 = T^2 \times M_4$$

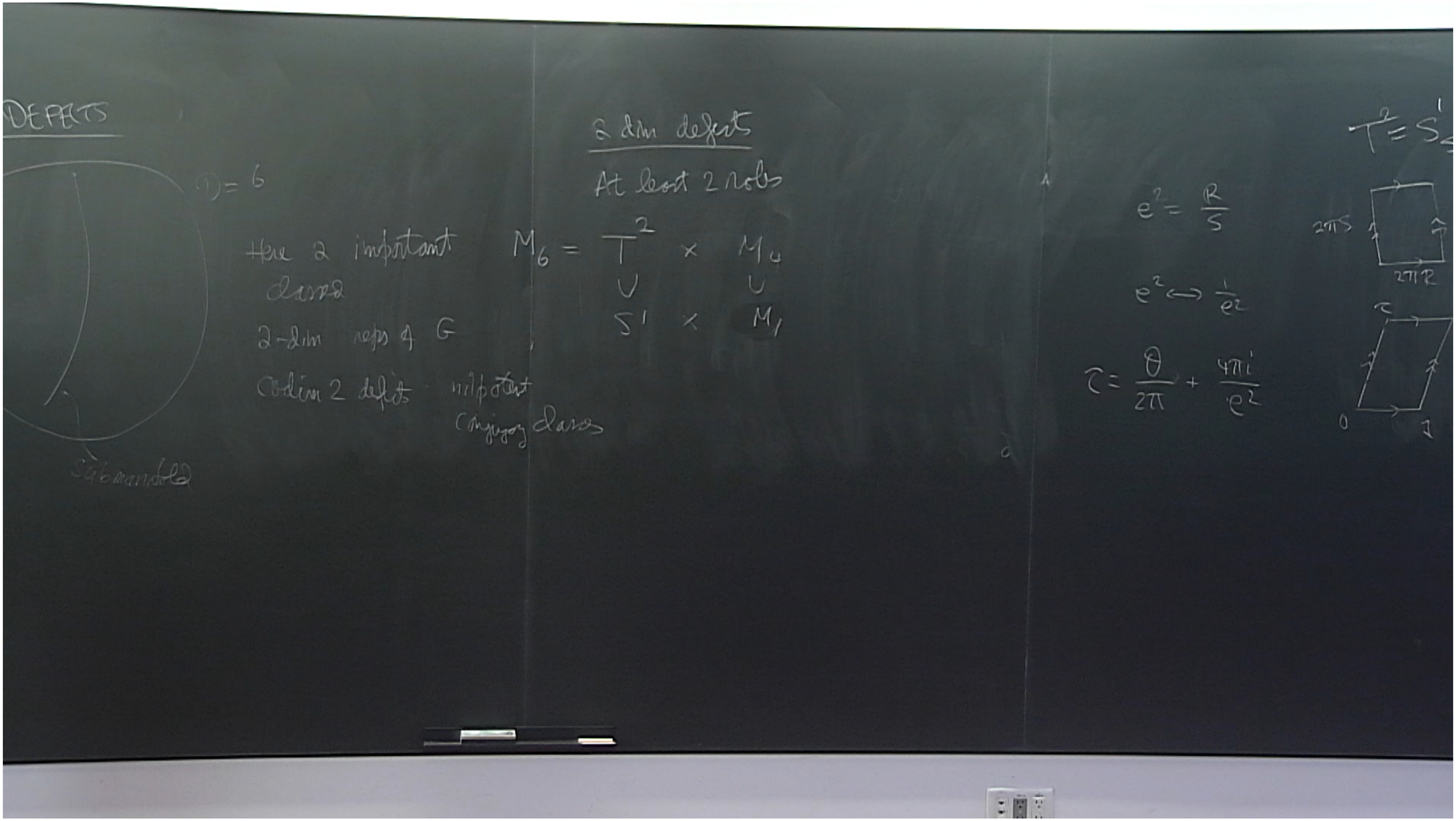
$$e^2 = \frac{R}{S}$$

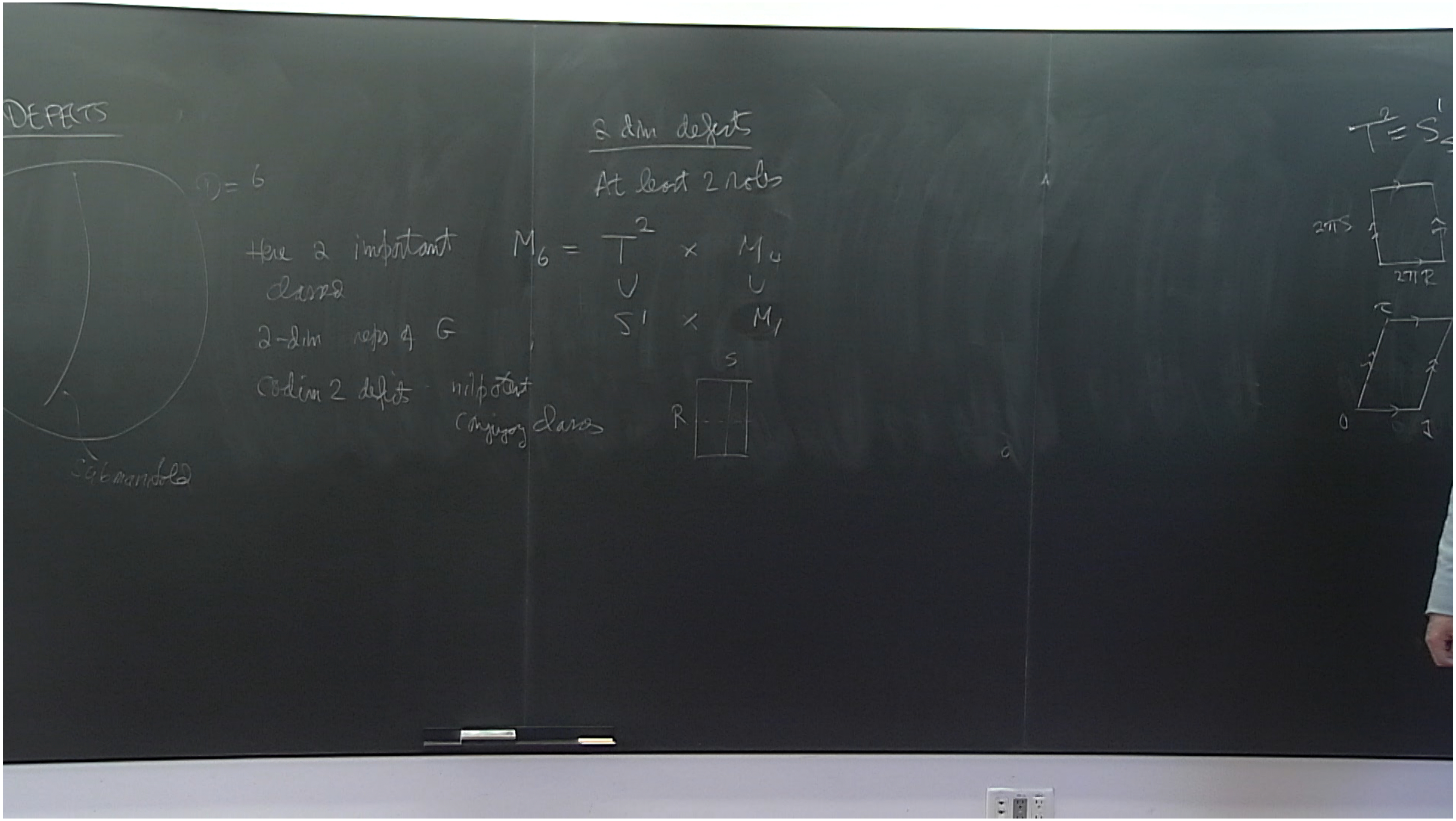
$$e^2 \longleftrightarrow \frac{1}{e^2}$$

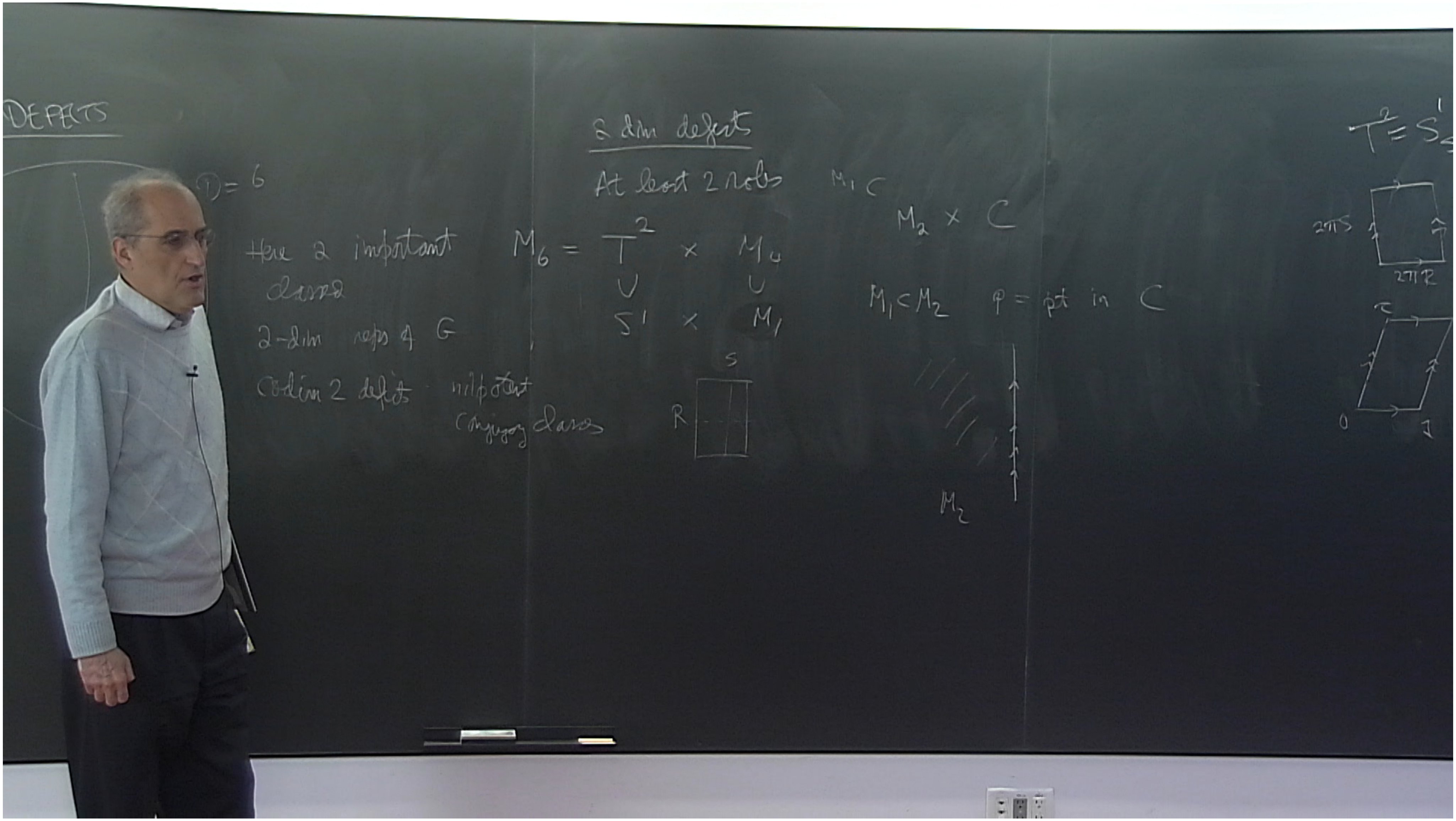
$$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{e^2}$$

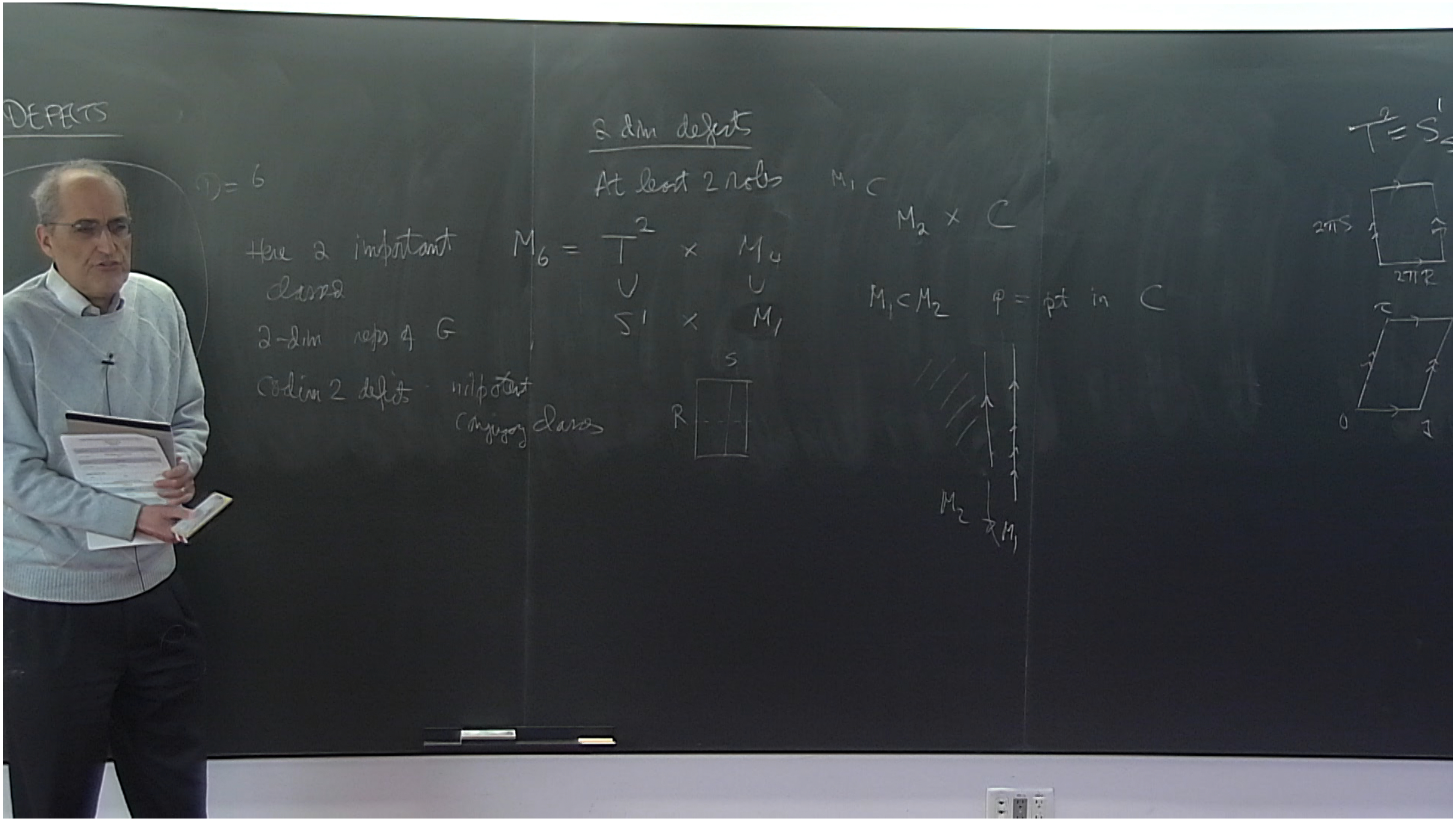
$T^2 = S^1$

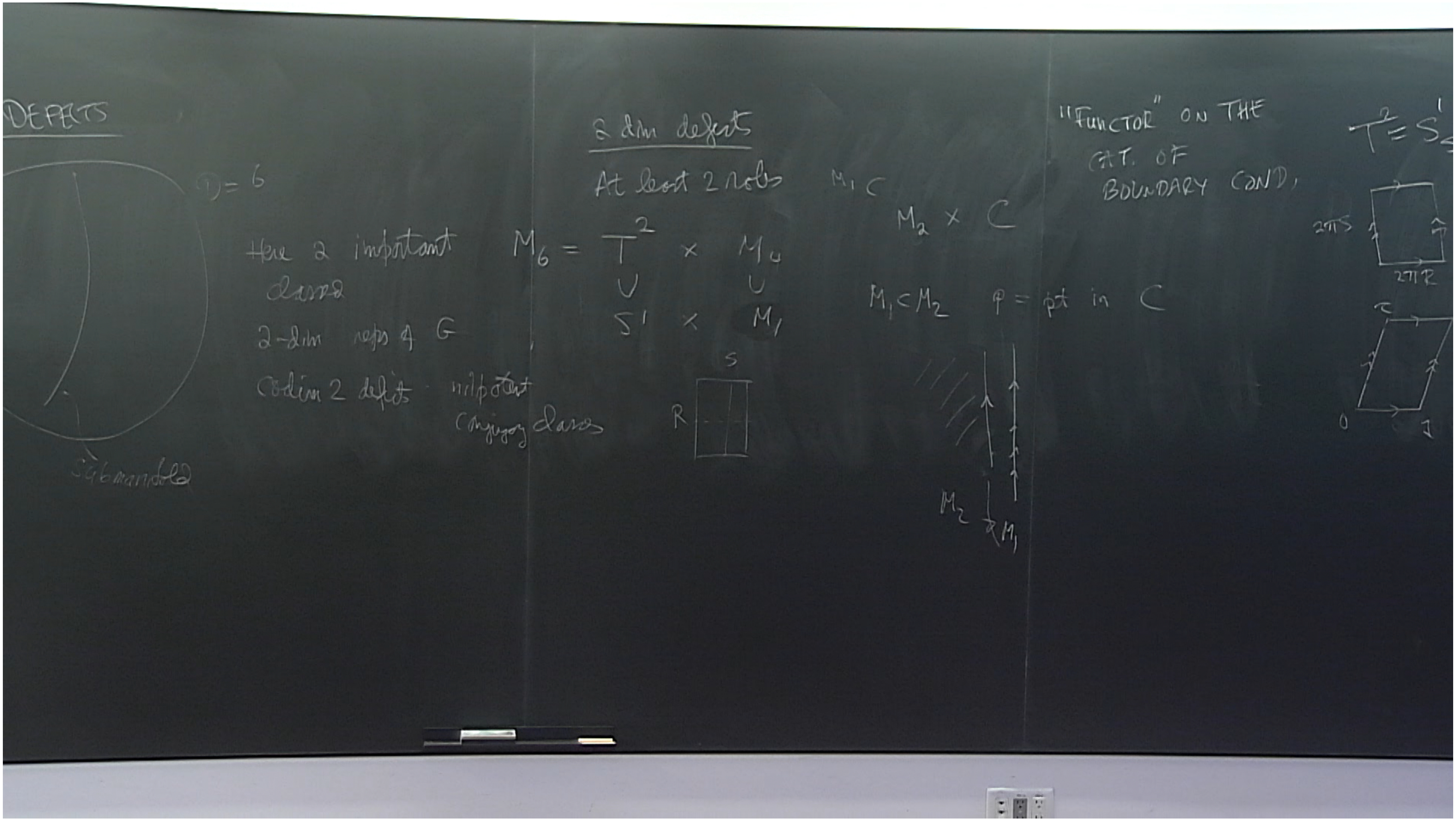


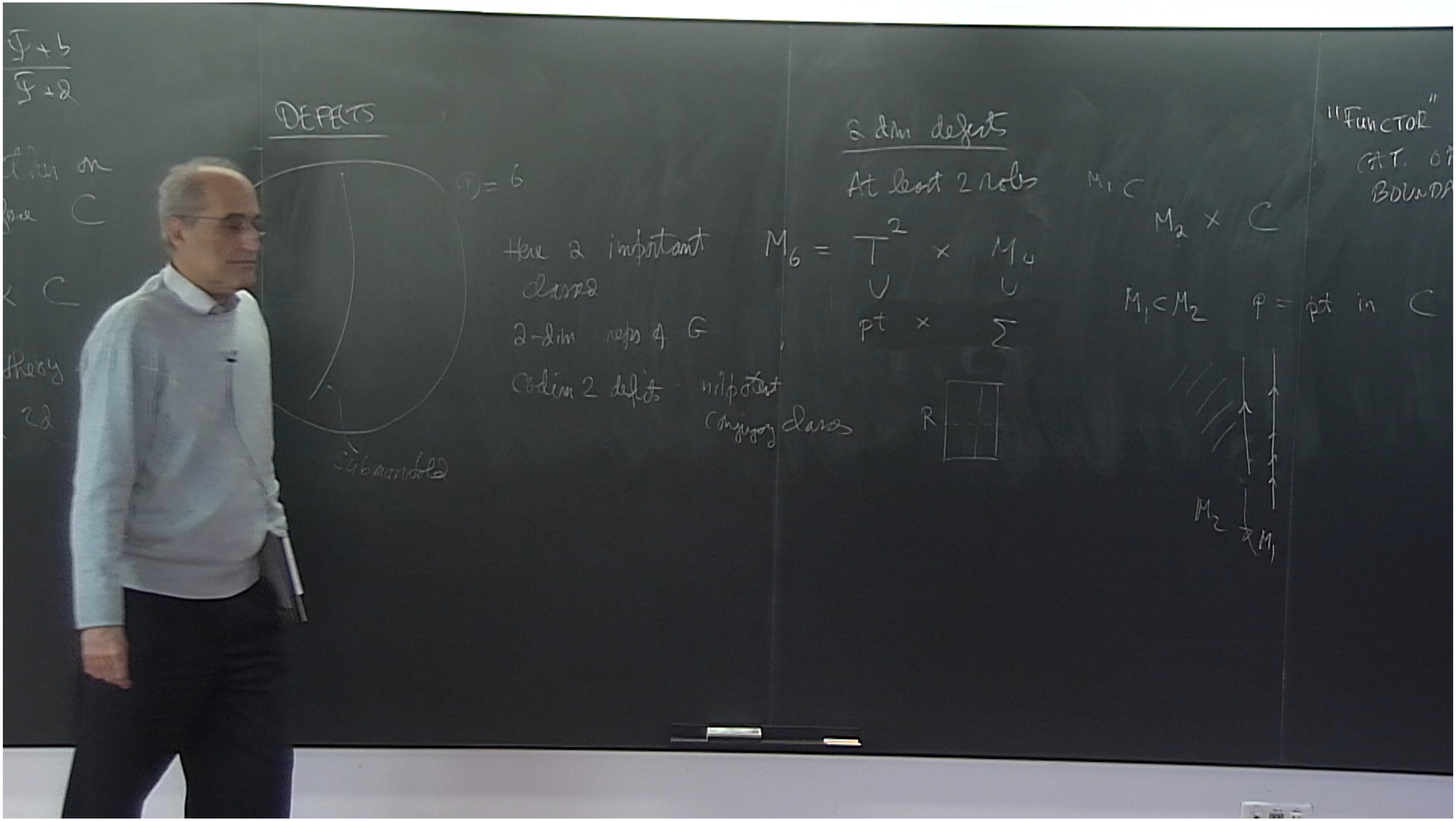

DEFECTS
= 6
Submanifold
Here 2 important classes
2-dim reps of G
Codim 2 defects nilpotent conjugacy classes
2 dim defects
At least 2
M6 = T2 × M4
S1 × M1
e2 = R/S
e2 ⟷ 1/e2
τ = θ/2π + 4πi/e2
2πS
2πR
0
1

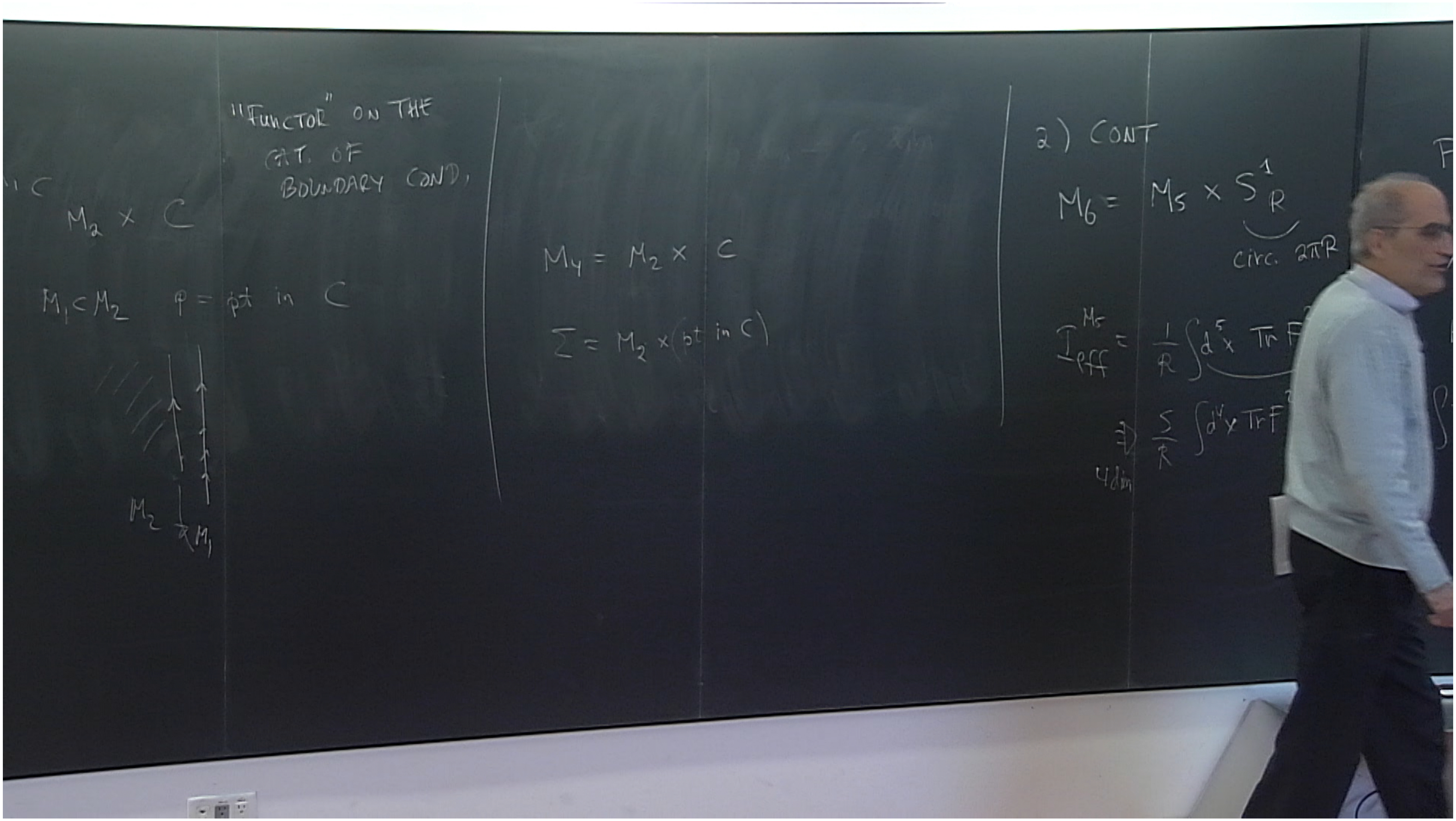

DEFECTS
Here 2 important classes
2-dim reps of G
Codim 2 defects nilpotent
conjugacy classes
submanifold
2 dim defects
At least 2 nobs
M6 = T2 × M4
S1 × M1
R
2πS
2πR
0
1


DEFECTS
D = 6
Here 2 important classes
2-dim reps of G
Codim 2 defects nilpotent conjugacy classes
2 dim defects
M_6 = T^2 × M_4
S^1 × M_1
M_2 × C
M_1 ⊂ M_2
p = pt in C
2πS
2πR
M_2


DEFECTS
D = 6
Here 2 important classes
2-dim reps of G
codim 2 defects nilpotent conjugacy classes
2 dim defects
At least 2 nobs
M6 = T2 × M4
S1 × M1
S
R
M1 C
M2 × C
M1 ⊂ M2
p = pt in C
M2
M1
2πS
2πR
τ
0
1


DEFECTS
D = 6
submanifold
2 dim defects
At least 2 roles
Here 2 important classes
2-dim reps of G
codim 2 defects · nilpotent conjugacy classes
M6 = T2 × M4
S1 × M1
S
R
M1 C
M2 × C
M1 < M2
p = pt in C
M2
M1
"FUNCTOR" ON THE CAT. OF BOUNDARY COND.
T2 = S1
2πS
2πR
τ
0
1


DEFECTS
submanifold
$D = 6$
Here 2 important classes
2-dim reps of G
Codim 2 defects
nilpotent conjugacy classes
2 dim defects
At least 2 roles
$M_6 = T^2 \times M_4$
$pt \times \Sigma$
R
$M_2 \times C$
$M_1 \subset M_2$
$p = pt$ in $C$
$M_2$
$M_1$
"FUNCTOR"


"FUNCTOR" ON THE
CAT. OF
BOUNDARY COND,
M2 × C
M1 < M2   φ = pt in C
M2
M1
M4 = M2 × C
Σ = M2 × (pt in C)
2) CONT
M6 = M5 × S1R
circ. 2πR
Ieff M5 = 1/R ∫d5x Tr F
⇒ 4dim S/R ∫d4x Tr F

"FUNCTOR" ON THE CAT. OF BOUNDARY COND.

in $C$

START IN $D=6$

$M_4 = M_2 \times C$

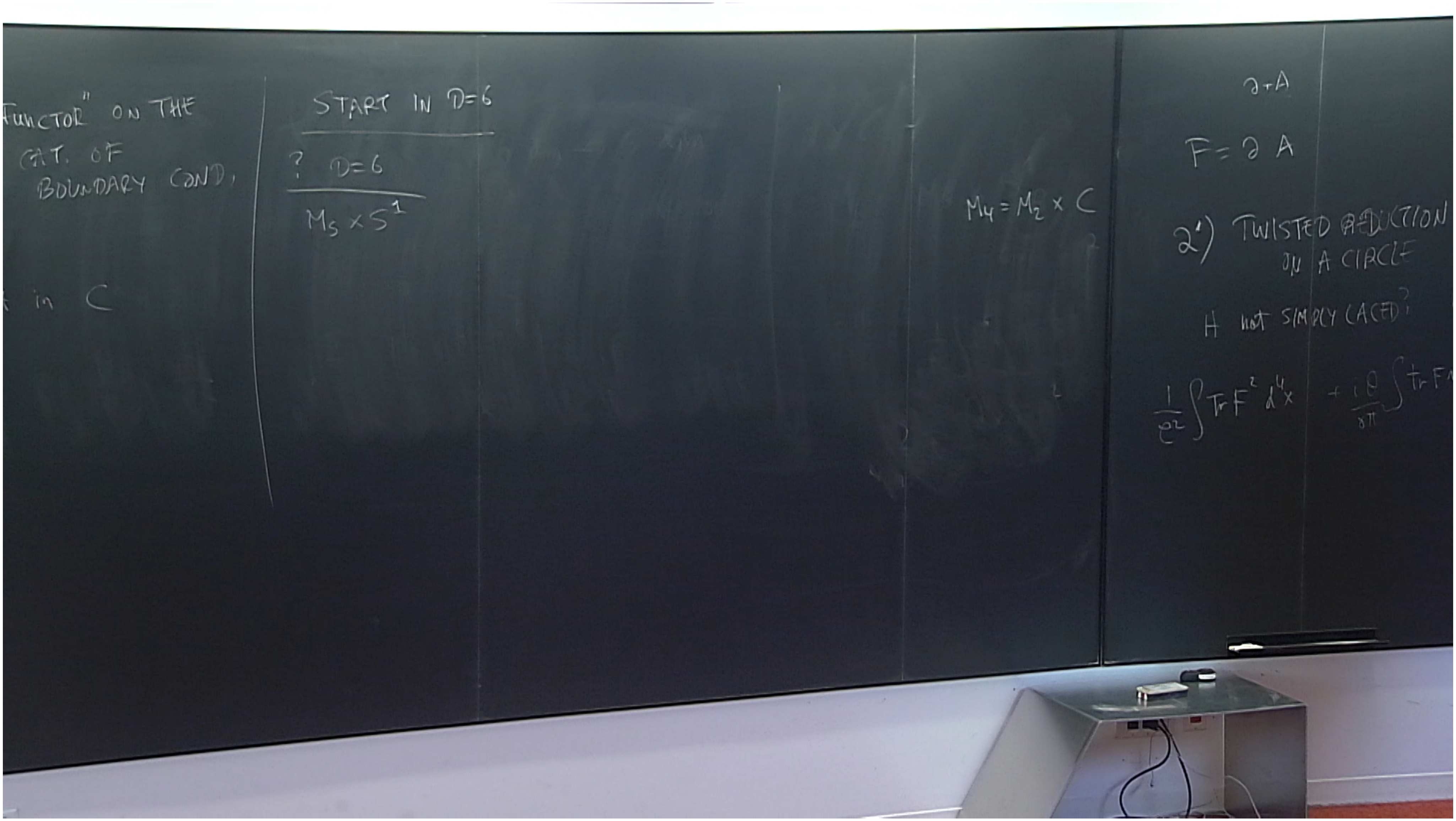
$\partial_\tau A$

$F = \partial A$

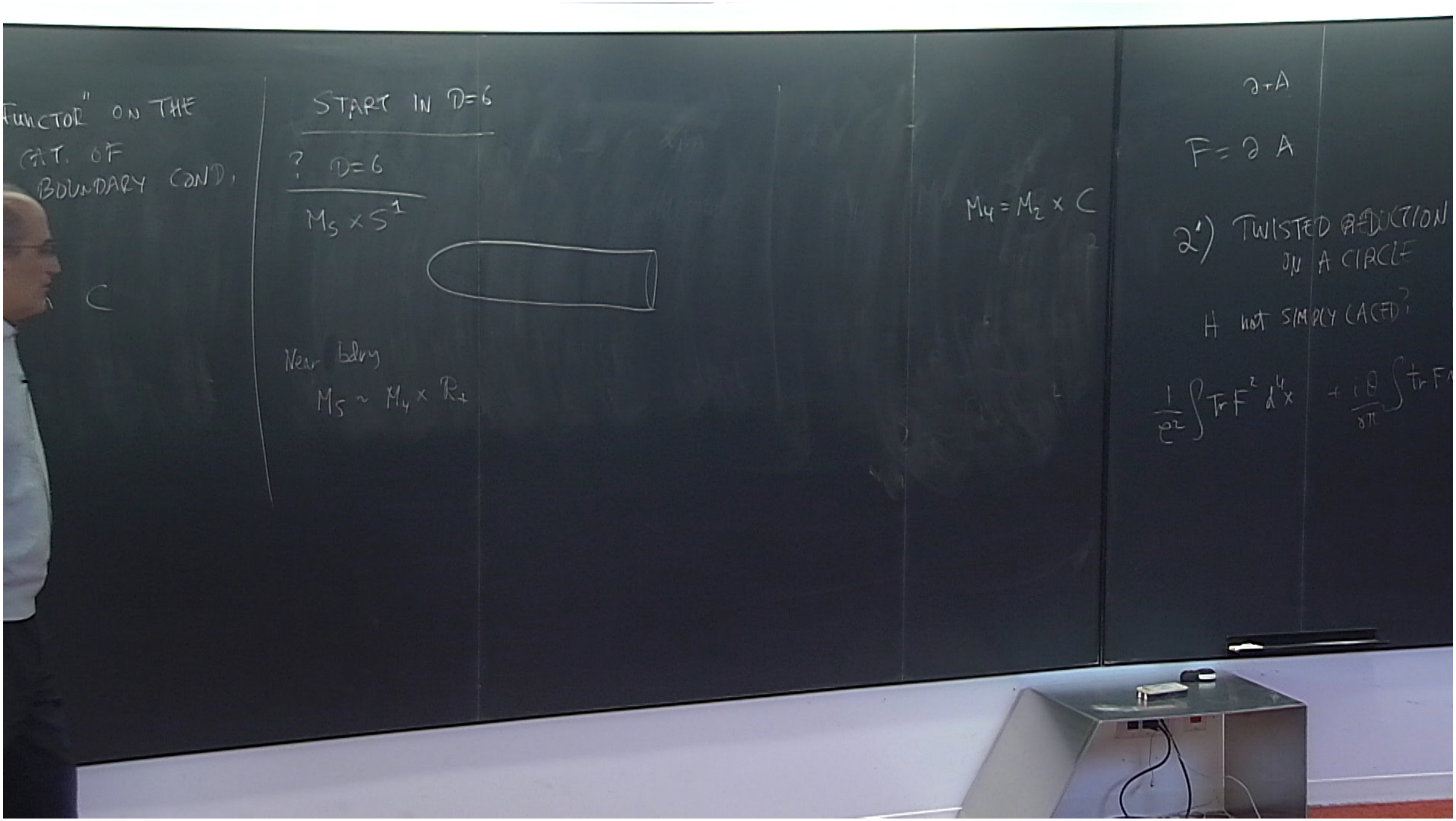
2') TWISTED REDUCTION ON A CIRCLE

H not SIMPLY LACED?

$\frac{1}{e^2}\int \mathrm{Tr}\, F^2\, d^4x + \frac{i\theta}{8\pi}\int \mathrm{Tr}\, F \wedge$



"FUNCTOR" ON THE
CAT. OF
BOUNDARY COND.
in C
START IN D=6
? D=6
M5 × S1
M4 = M2 × C
∂+A
F = ∂A
2') TWISTED REDUCTION
ON A CIRCLE
H not SIMPLY LACED?


"FUNCTOR" ON THE
CAT. OF
BOUNDARY COND,
C
START IN D=6
? D=6
M5 × S1
Near bdry
M5 ~ M4 × R+
M4 = M2 × C
∂τA
F = ∂ A
2') TWISTED REDUCTION
ON A CIRCLE
H not SIMPLY CONNECTED?
1/e2 ∫ Tr F2 d4x
+ iθ/8π ∫ Tr F

"FUNCTOR" ON THE
CAT. OF
BOUNDARY COND.

in $C$

START IN $D=6$

? $D=6$

$M_5 \times S^1$

Near bdry
$M_5 \sim M_4 \times \mathbb{R}_+$

$M_4 = M_2 \times C$

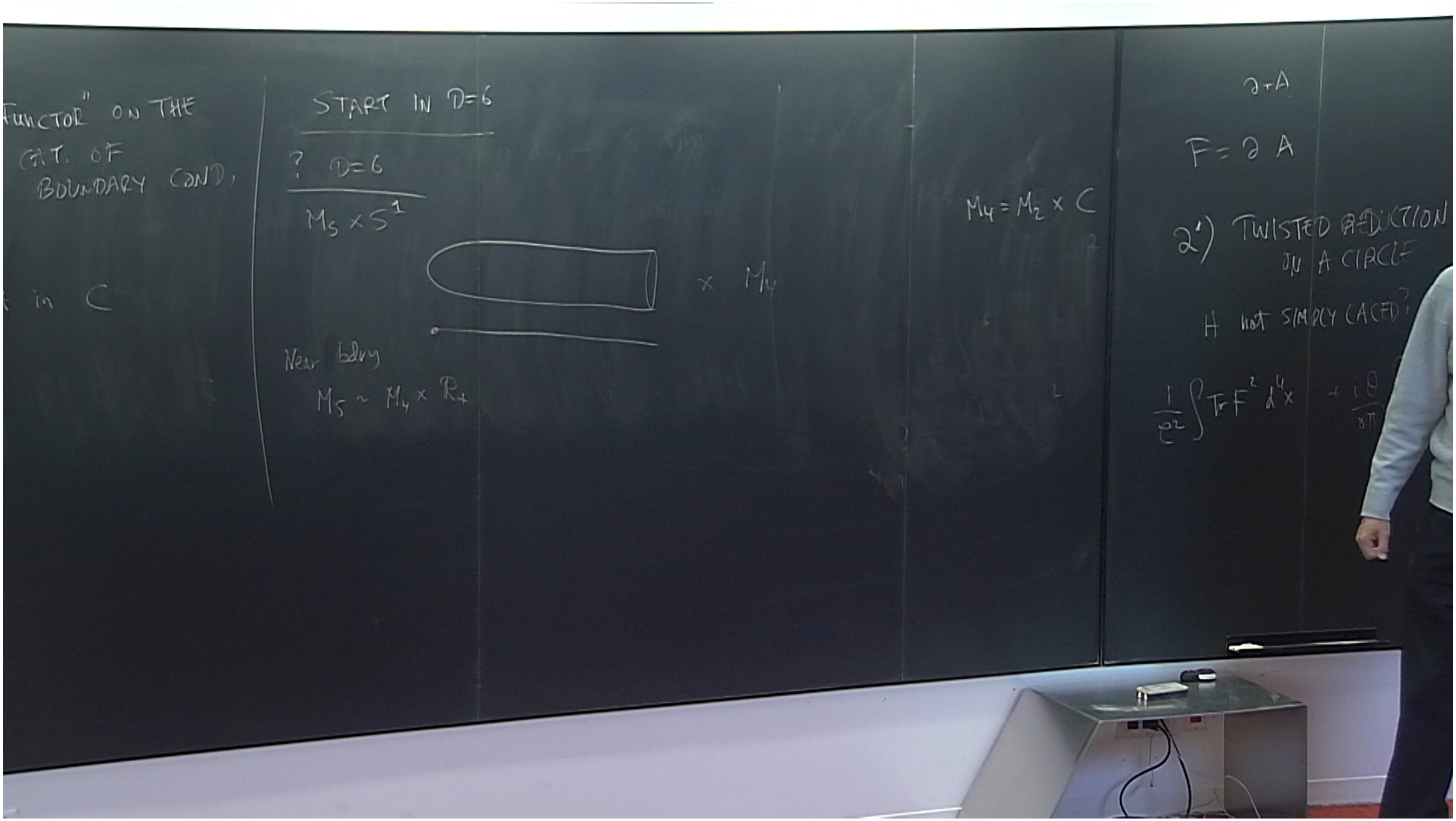
$\partial_+ A$

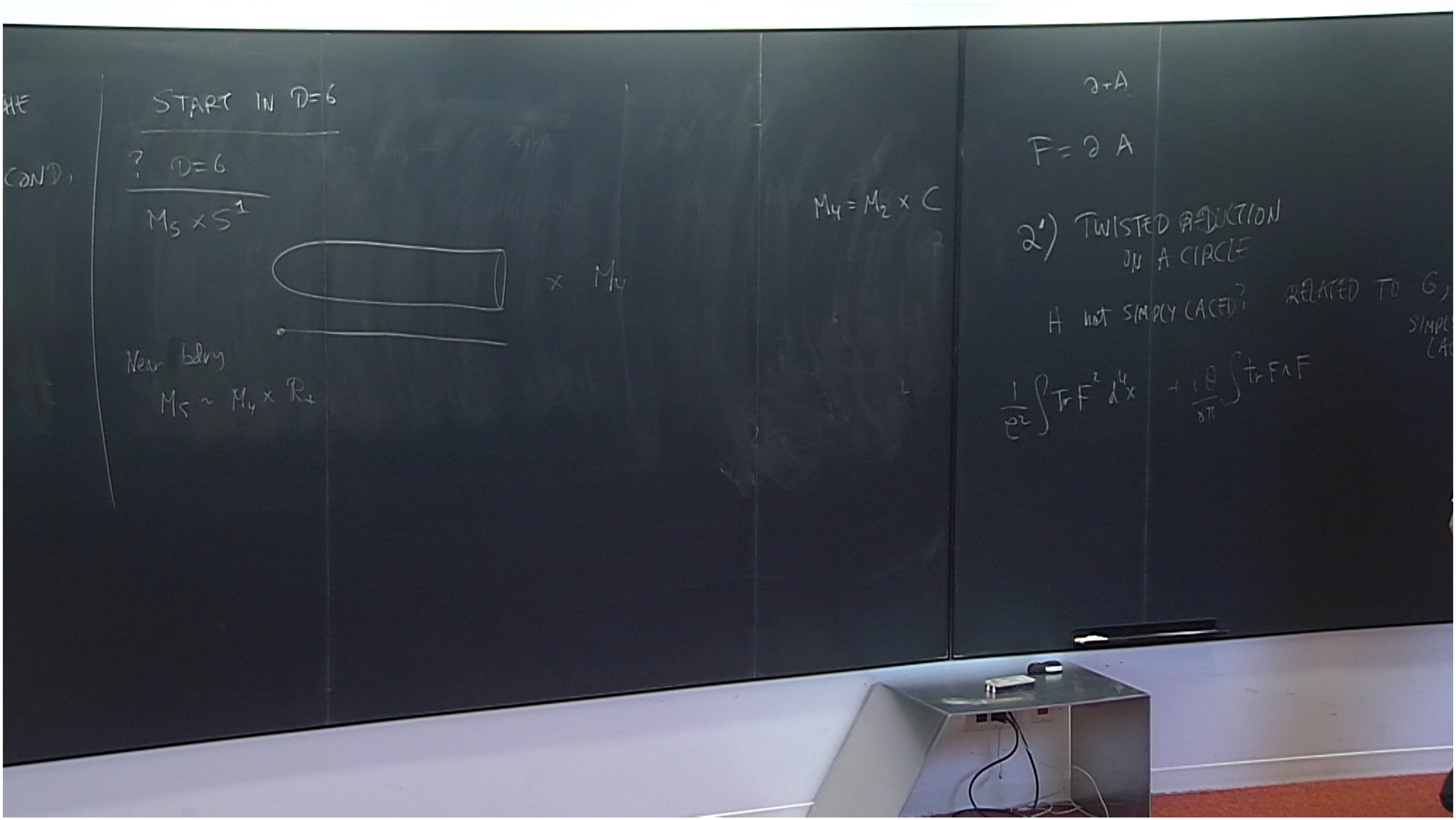
$F = \partial A$

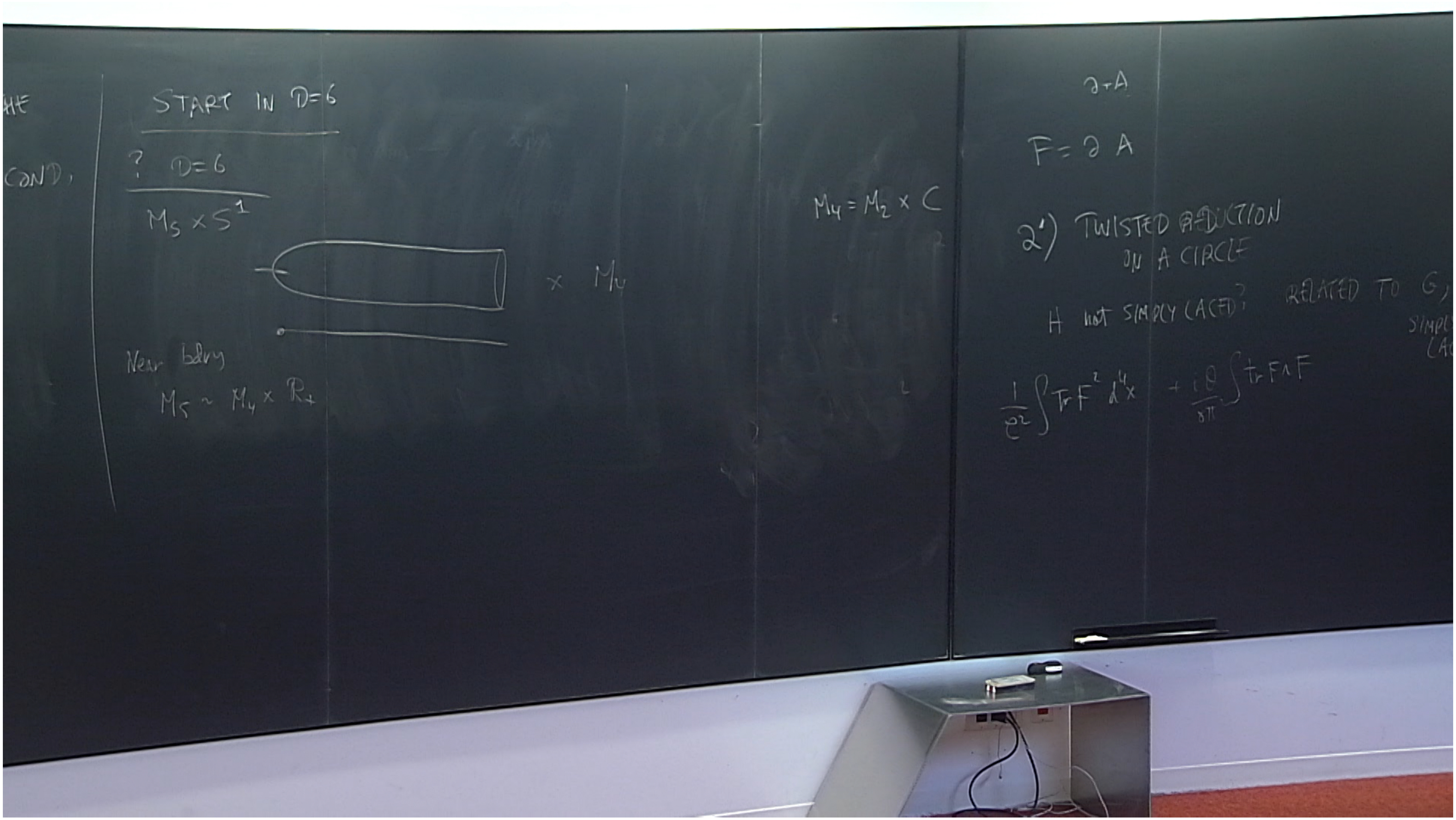
2') TWISTED REDUCTION
ON A CIRCLE

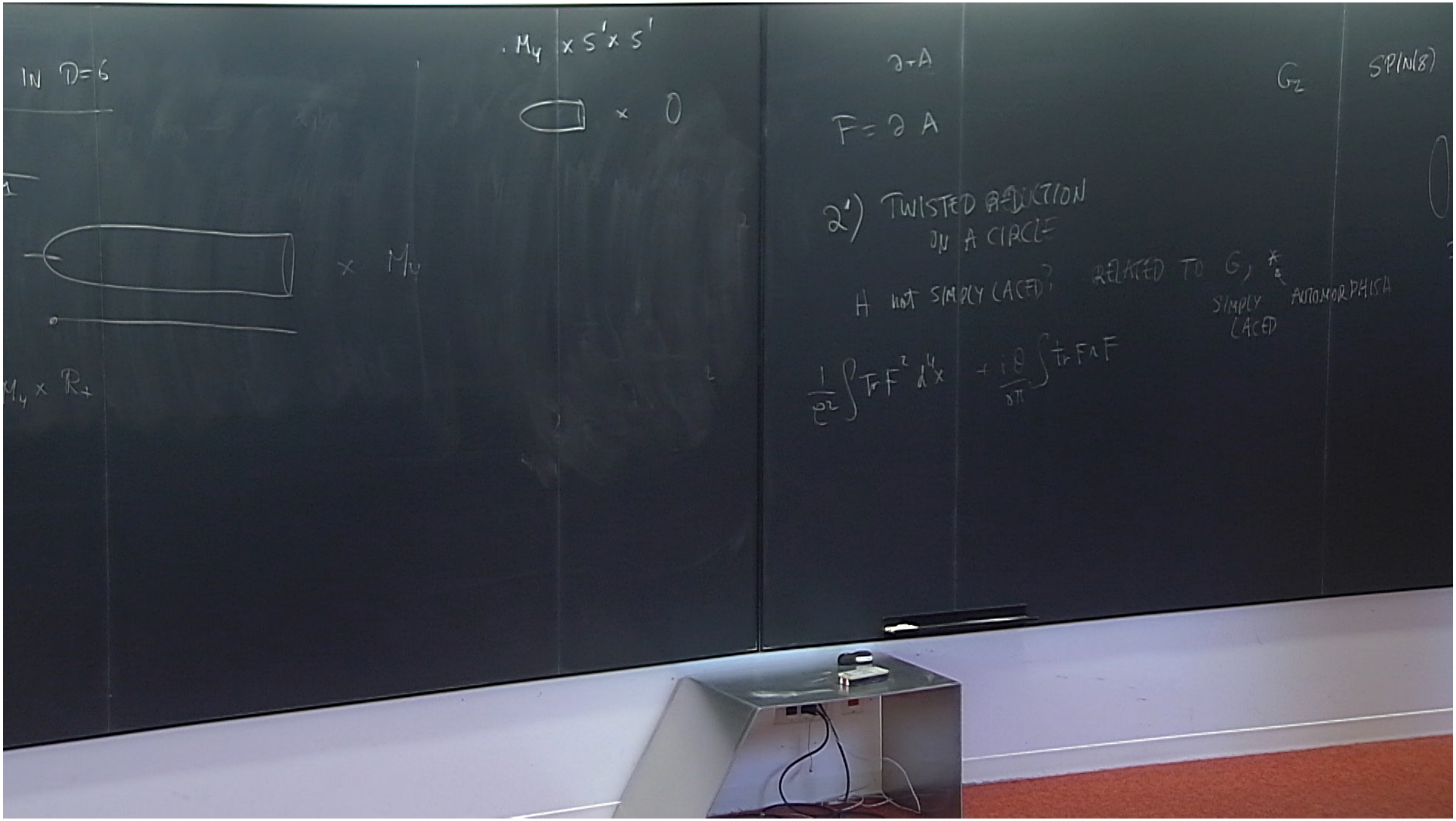
$H$ not SIMPLY LACED?

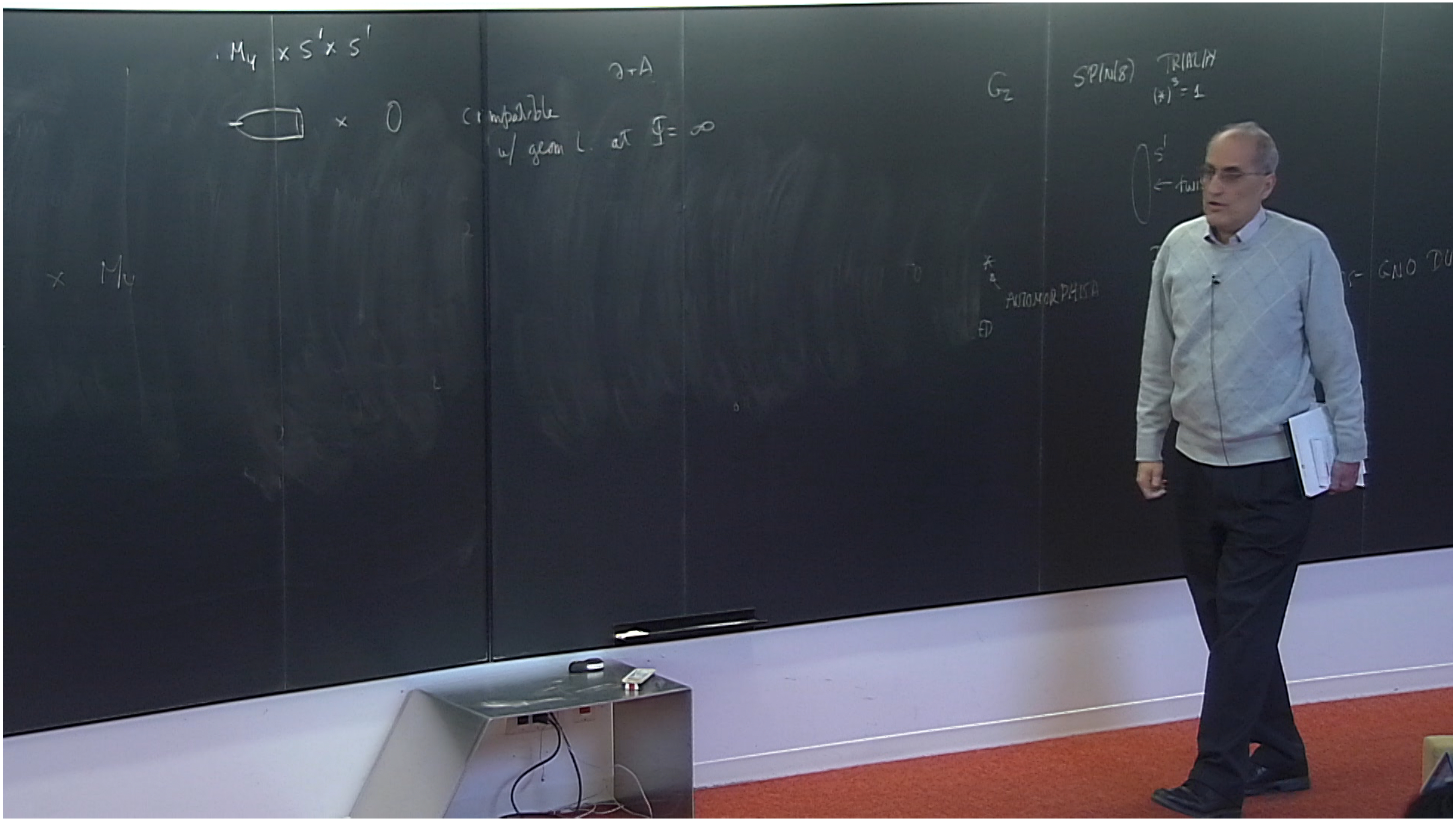
$$\frac{1}{e^2}\int \mathrm{Tr}\, F^2\, d^4x \quad + \frac{i\theta}{8\pi}\int \mathrm{Tr}\, F \wedge$$

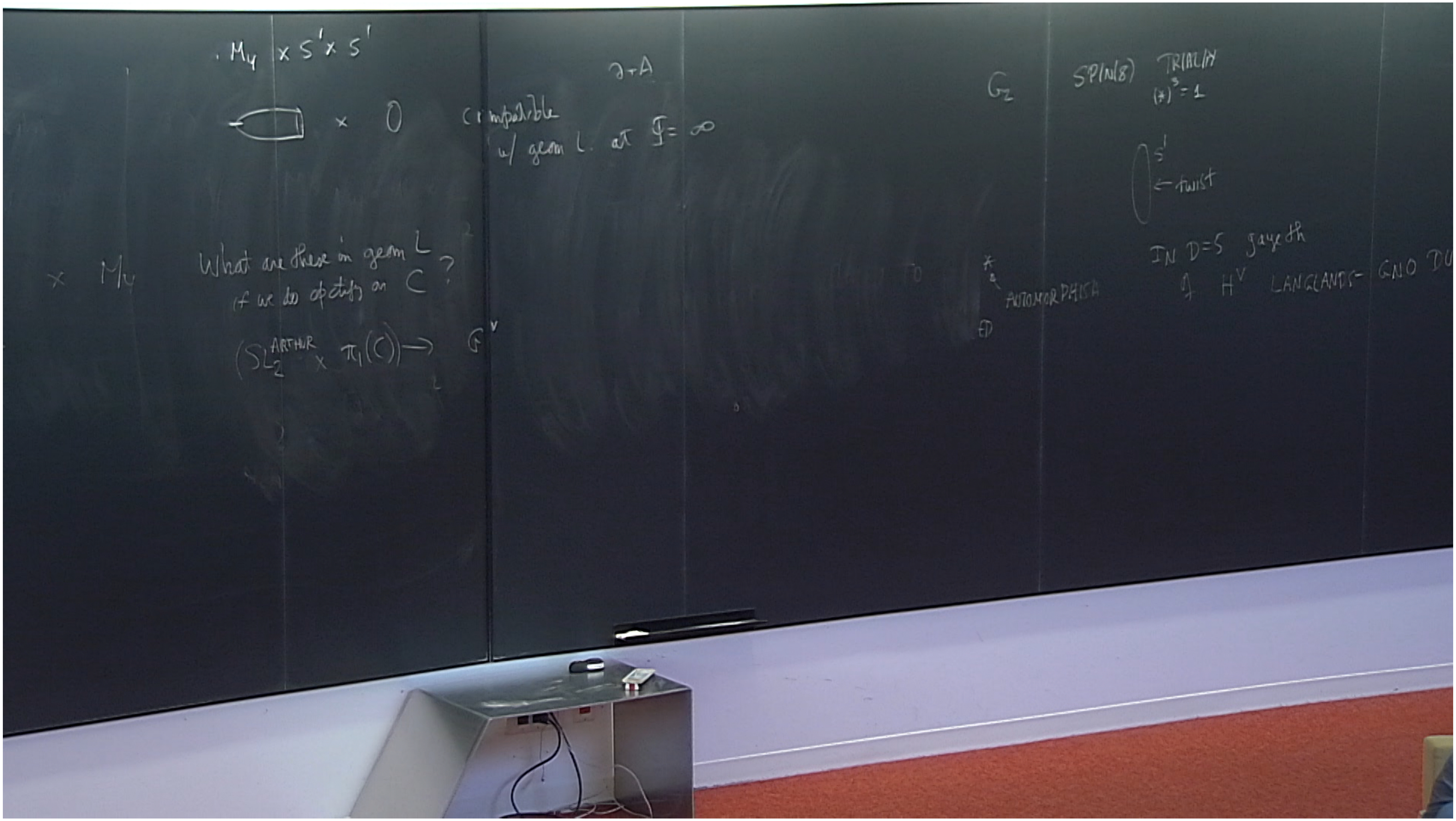

"FUNCTOR" ON THE
CAT. OF
BOUNDARY COND.
in C
START IN D=6
? D=6
$M_5 \times S^1$
$\times M_4$
Near bdry
$M_5 \sim M_4 \times \mathbb{R}_+$
$M_4 = M_2 \times C$
$F = \partial A$
2') TWISTED REDUCTION
ON A CIRCLE
H not SIMPLY CONNECTED?
$\frac{1}{e^2}\int \mathrm{Tr}\, F^2 \, d^4x$

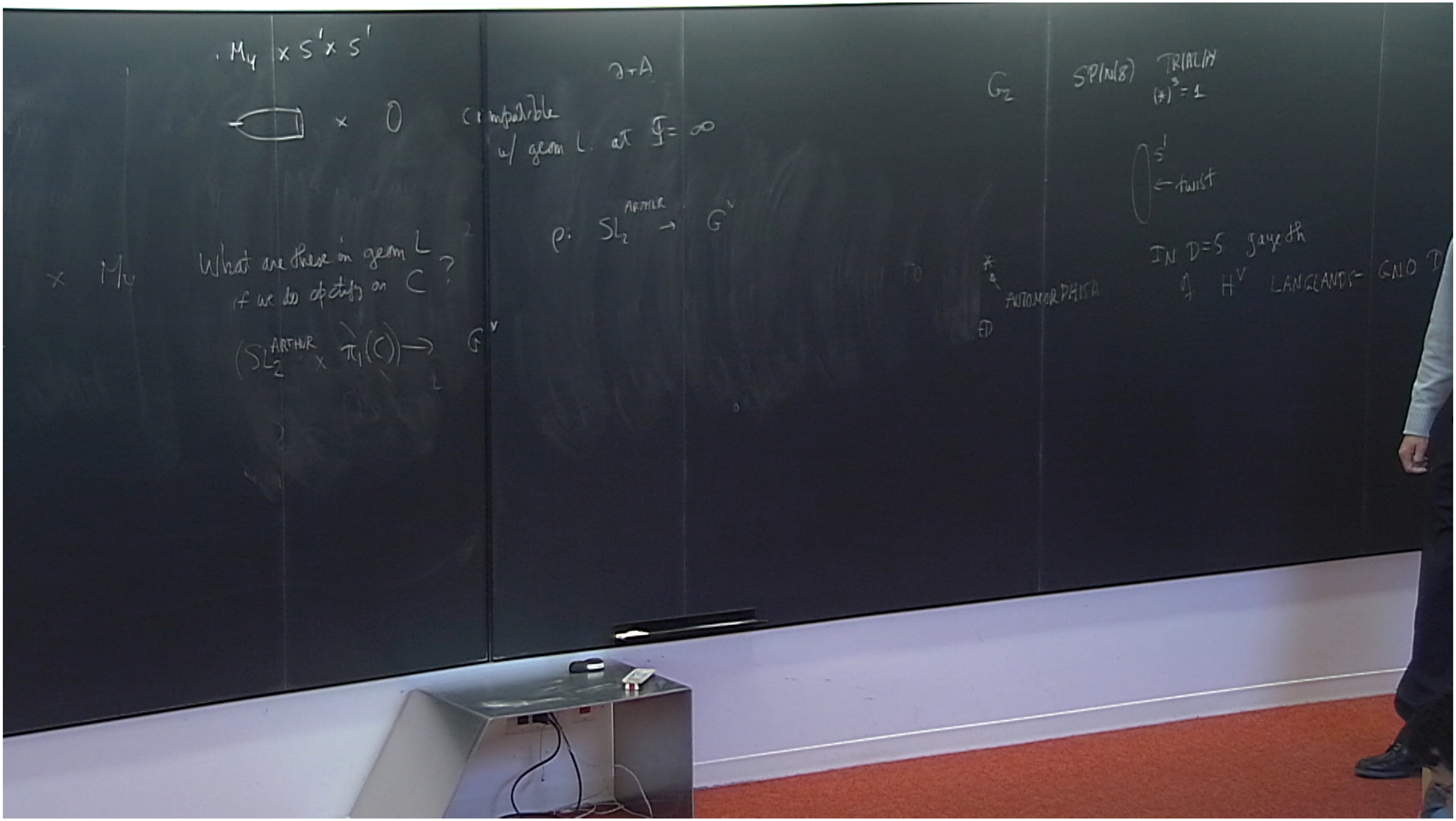

START IN D=6
? D=6
M5 × S1
× M4
Near bdry
M5 ~ M4 × R+
M4 = M2 × C
∂+A
F = ∂ A
2') TWISTED REDUCTION ON A CIRCLE
H not SIMPLY LACED? RELATED TO G,
1/e2 ∫ Tr F2 d4x + iθ/8π ∫ tr F∧F


COND,
START IN D=6
? D=6
M5 × S1
× M4
Near bdry
M5 ~ M4 × R+
M4 = M2 × C
F = ∂ A
2') TWISTED REDUCTION
ON A CIRCLE
H not SIMPLY LACED? RELATED TO G,
1/e2 ∫ Tr F2 d4x + iθ/8π ∫ tr F∧F

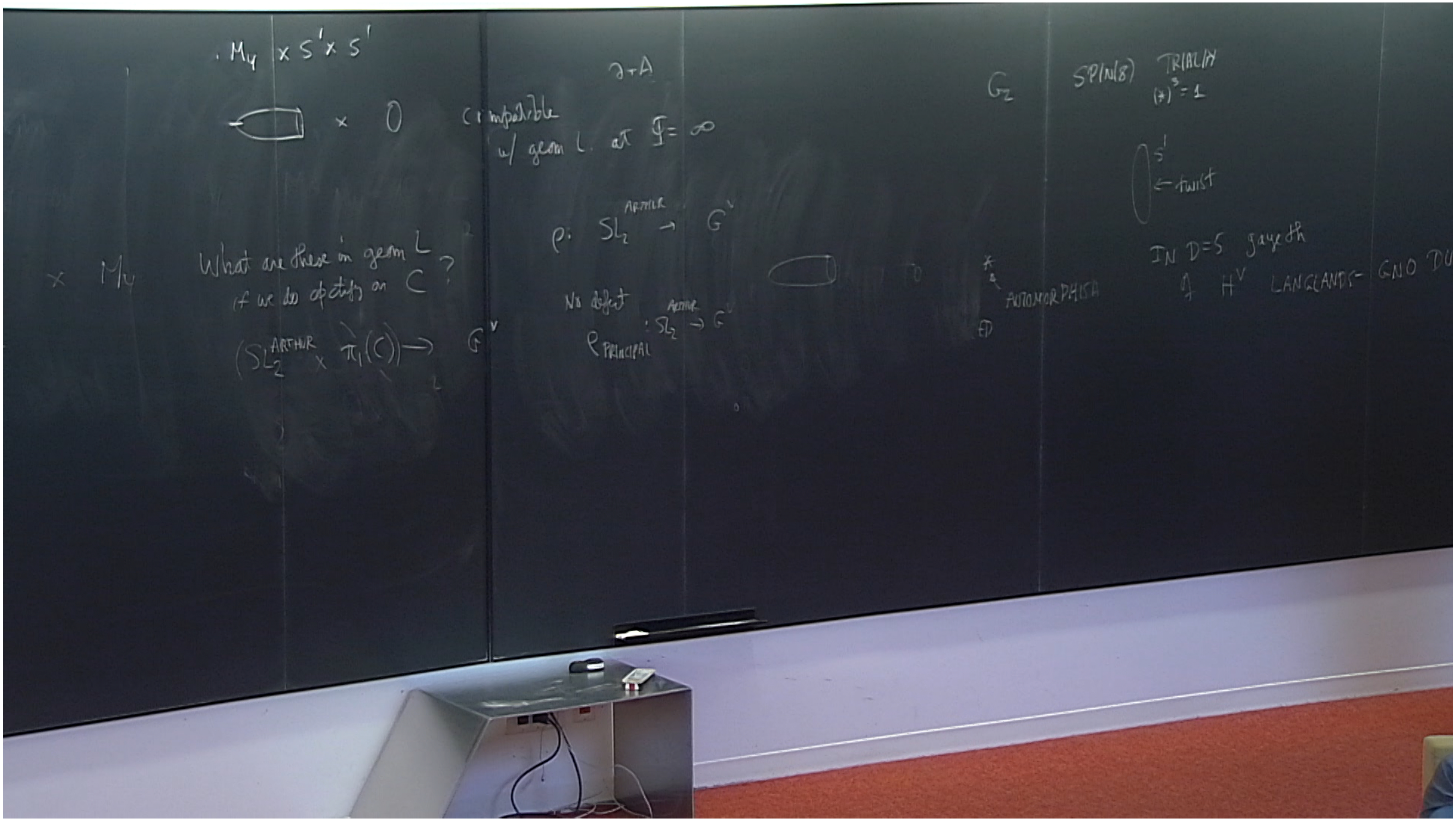

IN D=6
M4 x S1 x S1
x M4
M4 x R+
F = ∂A
2') TWISTED REDUCTION
ON A CIRCLE
H not SIMPLY LACED? RELATED TO G,
SIMPLY LACED
AUTOMORPHISM
G2
SPIN(8)


M4 x S'x S'
x O
compatible
w/ geom L. at Φ = ∞
∂+A
G2
SPIN(8) TRIALITY
(*)³ = 1
S'
× M4
AUTOMORPHISM

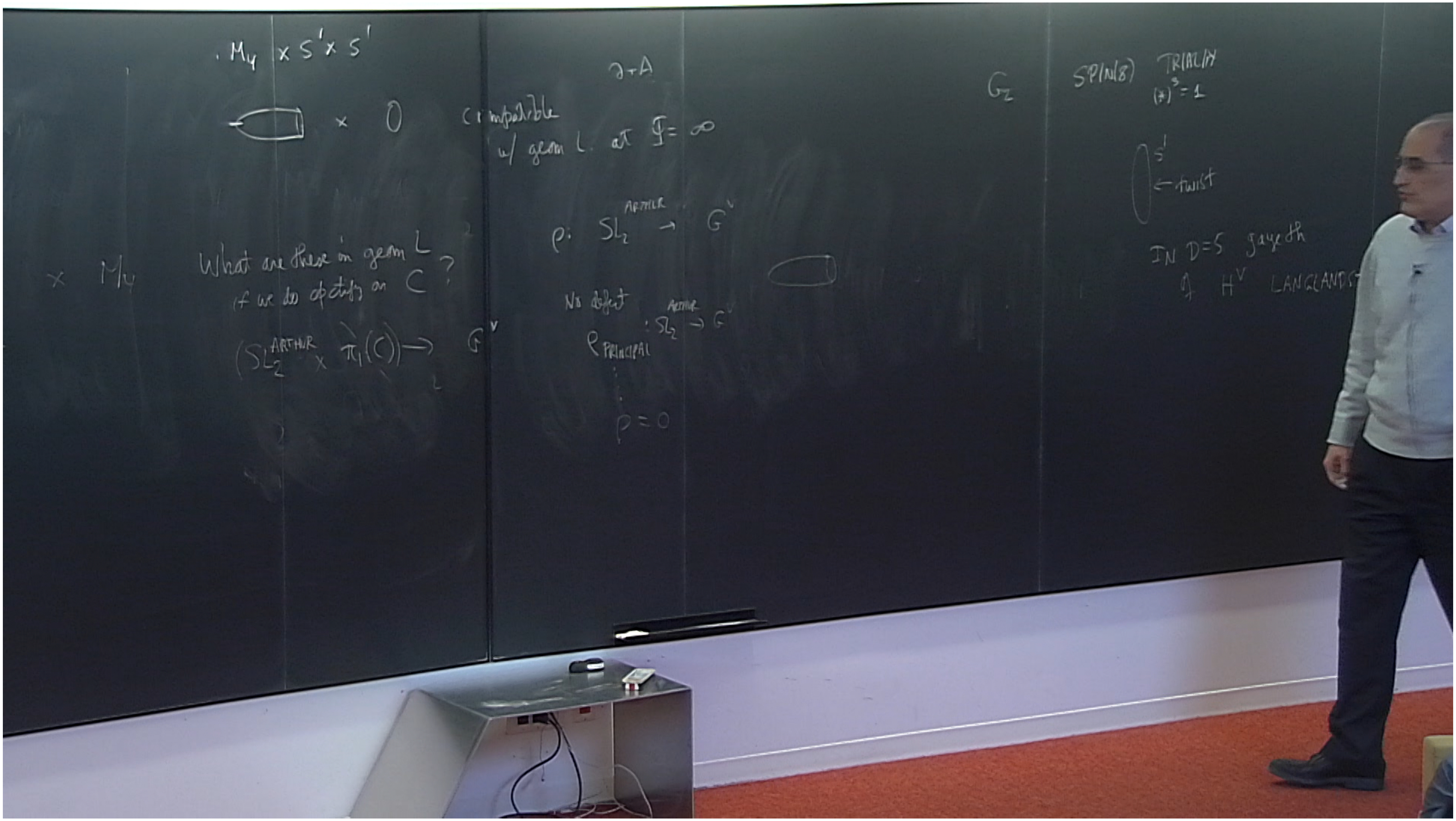

$M_4 \times S^1 \times S^1$
$\partial + A$
compatible
w/ geom L. at $\Psi = \infty$
$G_2$
SPIN(8) TRIALITY
$(*)^3 = 1$
$S^1$
← twist
$\times M_4$
What are these in geom L ?
if we do operations on C ?
$(SL_2^{ARTHUR} \times \pi_1(C)) \to G^\vee$
AUTOMORPHISM
IN D=5 gauge th
$H^\vee$ LANGLANDS-GNO DU



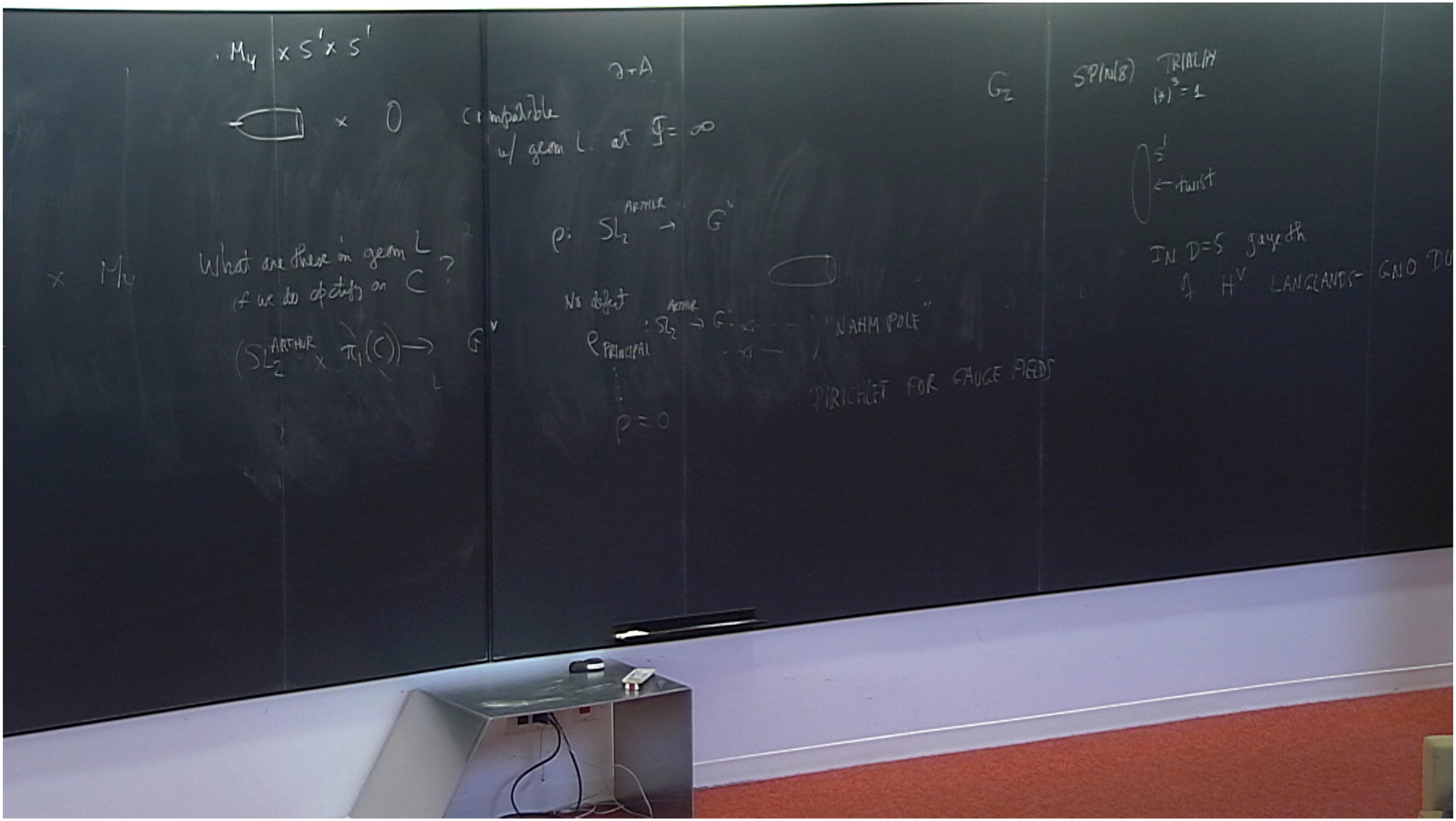
M4 x S1 x S1
compatible
w/ geom L. at Ψ = ∞
∂+A
G2
SPIN(8) TRIALITY
(*)3 = 1
S1
← twist
ρ: SL2 ARTHUR → G∨
What are these in geom L ?
(SL2 ARTHUR x π1(C)) → G∨
IN D=5 gauge th
H∨ LANGLANDS - GNO D
AUTOMORPHISM



. M4 x S1 x S1
compatible
w/ geom L. at Ψ = ∞
What are these in geom L
if we do ops on C ?
(SL2 ARTHUR x π1(C)) → G∨
∂+A
ρ: SL2 ARTHUR → G∨
No defect
ρ PRINCIPAL : SL2 ARTHUR → G∨
G2
AUTOMORPHISM
SPIN(8) TRIALITY
(*)3 = 1
S1
← twist
IN D=5 gauge th
H∨ LANGLANDS - GNO DU



$M_4 \times S^1 \times S^1$
compatible
w/ geom L. at $\Phi = \infty$
$\partial + A$
$G_2$
SPIN(8) TRIALITY
$(*)^3 = 1$
$S^1$
← twist
$\rho: SL_2^{ARTHUR} \to G^\vee$
What are these in geom L
if we do compactify on C ?
$(SL_2^{ARTHUR} \times \pi_1(C)) \to G^\vee$
No defect
$\rho_{PRINCIPAL}: SL_2^{ARTHUR} \to G^\vee$
$\rho = 0$
IN D=5 gauge th
$H^\vee$ LANGLANDS

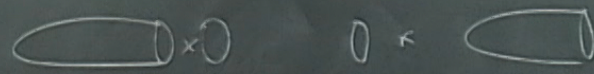

compatible
w/ geom L. at Φ = ∞
What are these in geom L ?
if we do defects on C ?
ARTHUR
No defect
ρ PRINCIPAL
"NAHM POLE"
DIRICHLET FOR GAUGE FIELDS
G2
SPIN(8) TRIALITY
twist
IN D=5 gauge th

= 6

Here 2 important classes

2-dim reps of G

Codim 2 defects nilpotent conjugacy classes



$\downarrow$

Nahm pole

$\vdots$

DIRICHLET
(Nahm = 0)

"FUNCTOR" ON THE
CAT. OF
BOUNDARY COND,

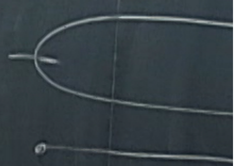
in C

$M_2 \to M_1$

START IN D=6

? D=6

$M_5 \times S^1$



Near bdry

$M_5 \sim M_4 \times \mathbb{R}_+$

$= 6$

Here 2 important classes

2-dim reps of $G$

Codim 2 defects · nilpotent conjugacy classes

$\times$ $\times$

Regular Nahm pole

DIRICHLET ($\mathrm{Nahm} = 0$)

"FUNCTOR" ON THE CAT. OF BOUNDARY COND.

in $\mathcal{C}$

START IN $D=6$

? $D=6$

$M_5 \times S^1$

Near bdry

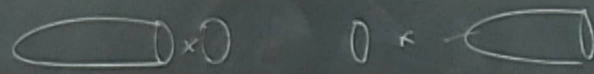
$M_5 \sim M_4 \times \mathbb{R}_+$

= 6

Here 2 important classes

2-dim reps of G

Codim 2 defects nilpotent conjugacy classes



↓

("Regular Nahm pole") ⟷ NEUMANN B. COND IN GAUGE THEORY

DIRICHLET (Nahm = 0)

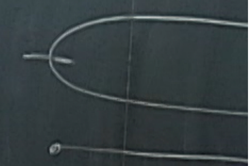
"FUNCTOR" ON THE CAT. OF BOUNDARY COND.

in $\mathbb{C}$

START IN D=6

? D=6

$M_5 \times S^1$



Near bdry

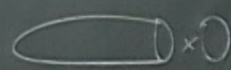
$M_5 \sim M_4 \times \mathbb{R}_+$

$= 6$

Here 2 important classes

2-dim reps of G

Codim 2 defects   nilpotent conjugacy class



$\downarrow$

("Regular Nahm pole") $\longleftrightarrow$ NEUMANN B. COND IN GAUGE THEORY

$\vdots$

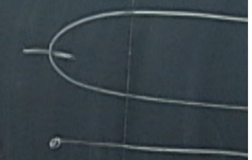
DIRICHLET (Nahm = 0)

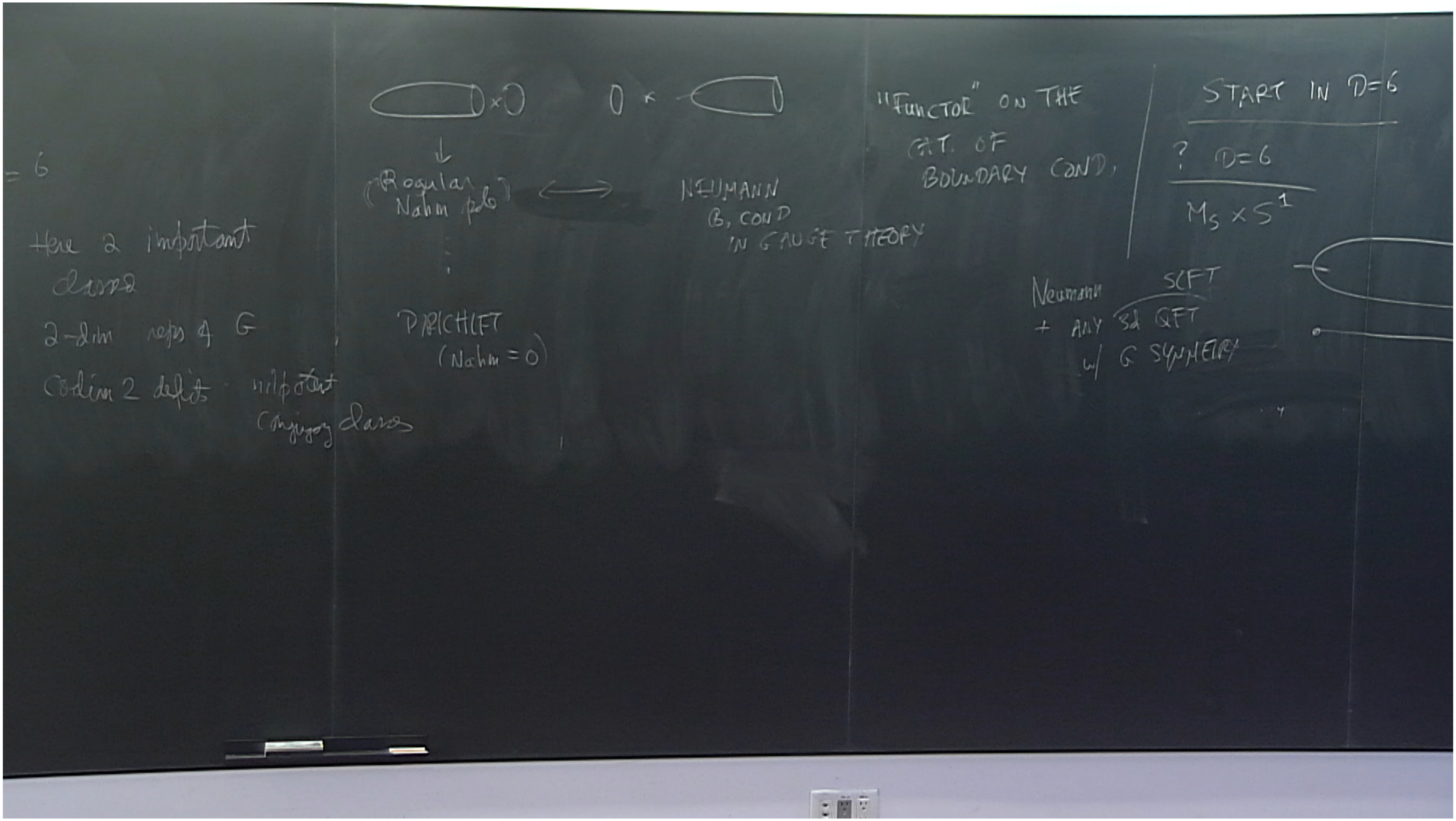
"FUNCTOR" ON THE CAT. OF BOUNDARY COND.

Neumann + ANY 3d QFT w/ G SYMMETRY

START IN $D=6$

$$\frac{?\ D=6}{M_5 \times S^1}$$




= 6
Here 2 important classes
2-dim reps of G
Codim 2 defects nilpotent conjugacy classes
(Regular Nahm pole) ⟷ NEUMANN B. COND IN GAUGE THEORY
DIRICHLET (Nahm = 0)
"FUNCTOR" ON THE CAT. OF BOUNDARY COND.
Neumann + ANY 3d QFT w/ G SYMMETRY
SCFT
START IN D=6
? D=6
M5 × S1

$= 6$

Here 2 important classes

2-dim reps of $G$

Codim 2 defects nilpotent conjugacy classes

("Regular Nahm pole") $\longleftrightarrow$ NEUMANN B. COND IN GAUGE THEORY

Neumann $\times$ SCFT w/ $G$ SYM

DIRICHLET (Nahm $= 0$)

"FUNCTOR" ON THE CAT. OF BOUNDARY COND.

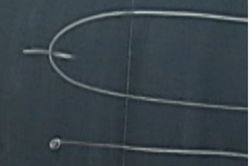
Neumann + ANY 3d QFT w/ $G$ SYMMETRY

SCFT

START IN $D=6$

? $D=6$

$M_5 \times S^1$



= 6

Here 2 important classes

2-dim reps of G

Codim 2 defects nilpotent conjugacy classes

$\times$ ○ ○ $\times$

↓

(Regular Nahm pole) ⟷ NEUMANN B. COND IN GAUGE THEORY

DIRICHLET (Nahm = 0)

Neumann $\times$ SCFT w/ G SYM

Neumann $\times$ $T[G]$

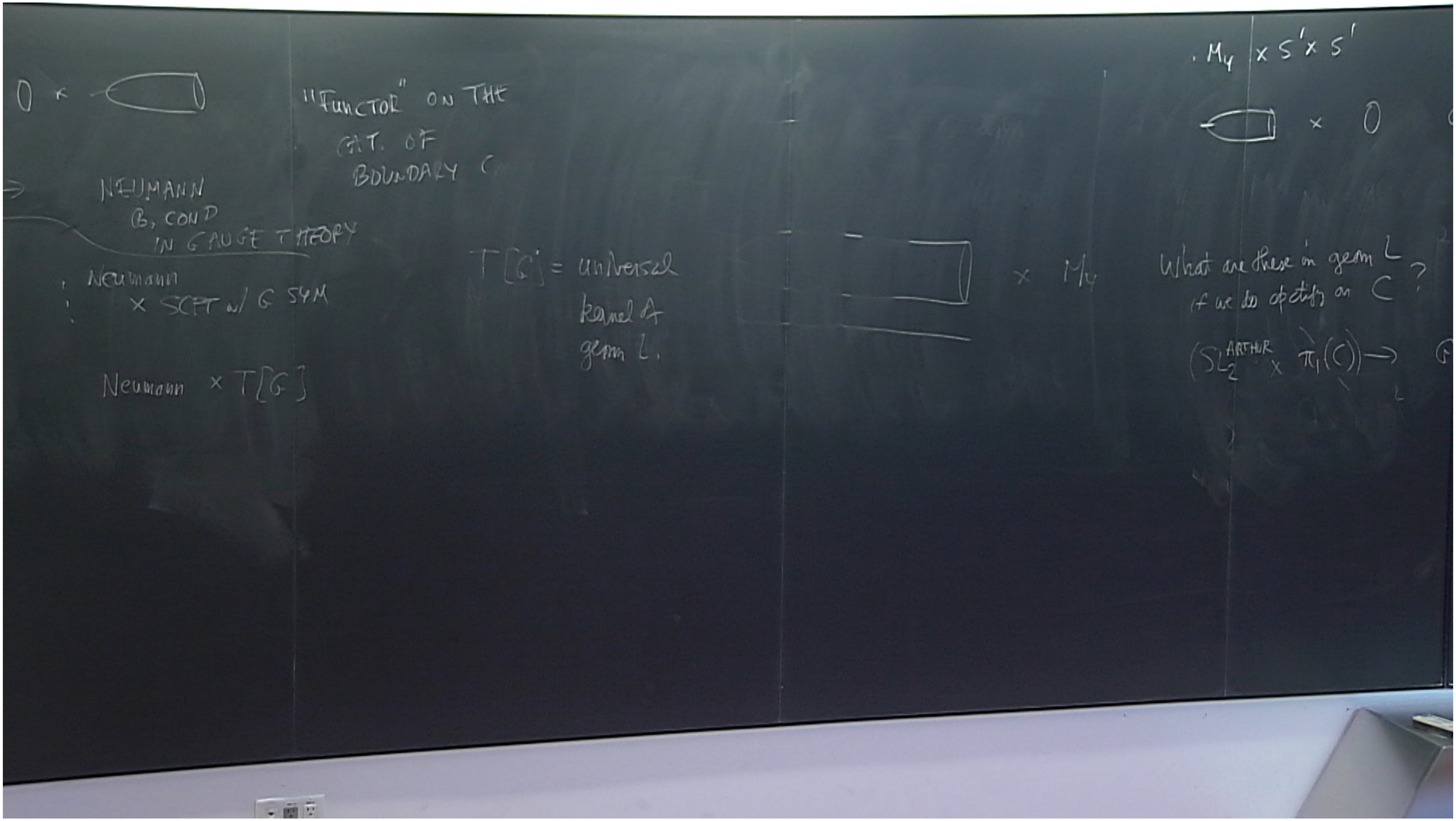
"FUNCTOR" ON THE CAT. OF BOUNDARY COND.

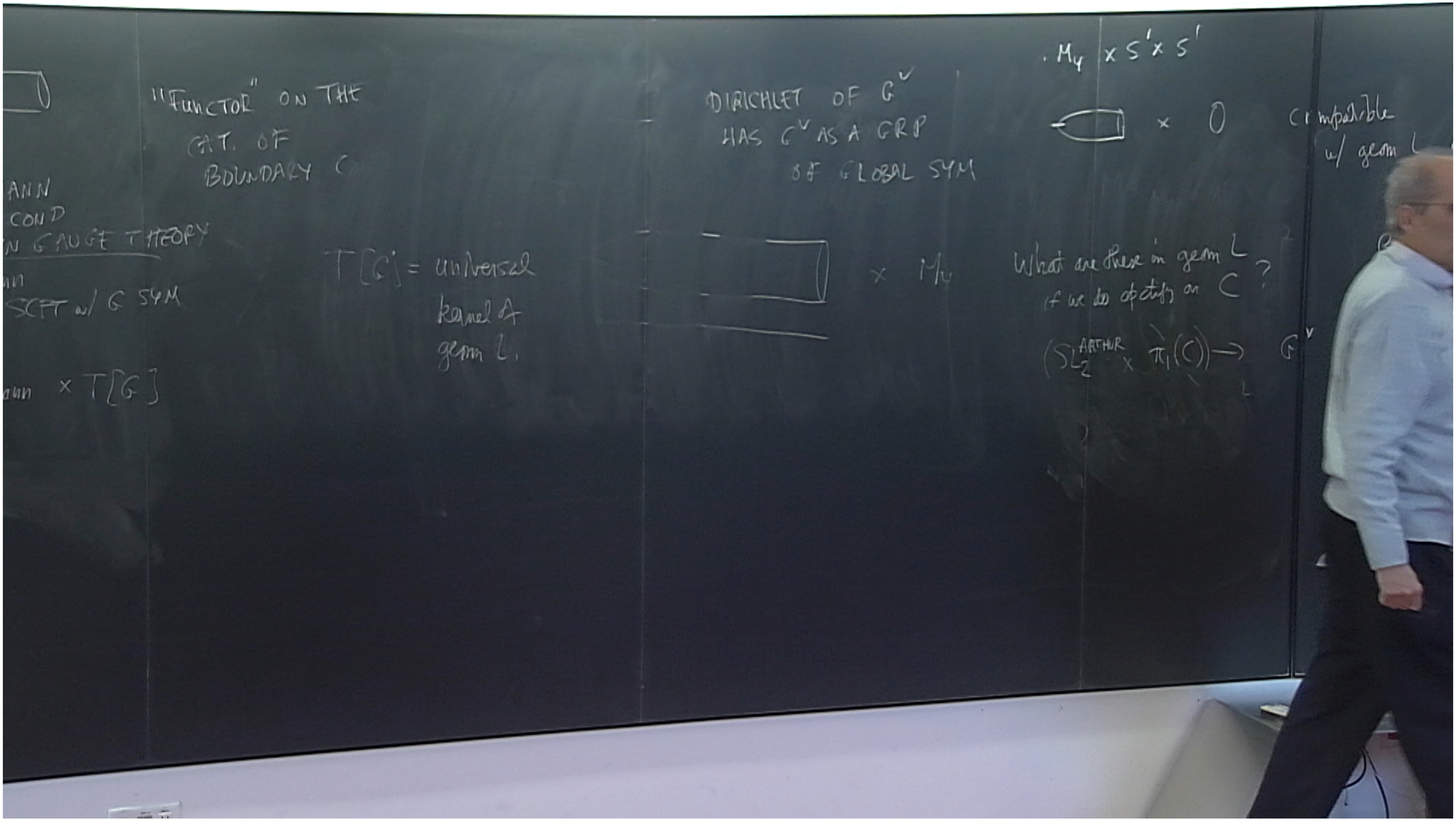
Neumann SCFT + ANY 3d QFT w/ G SYMMETRY

START IN D=6

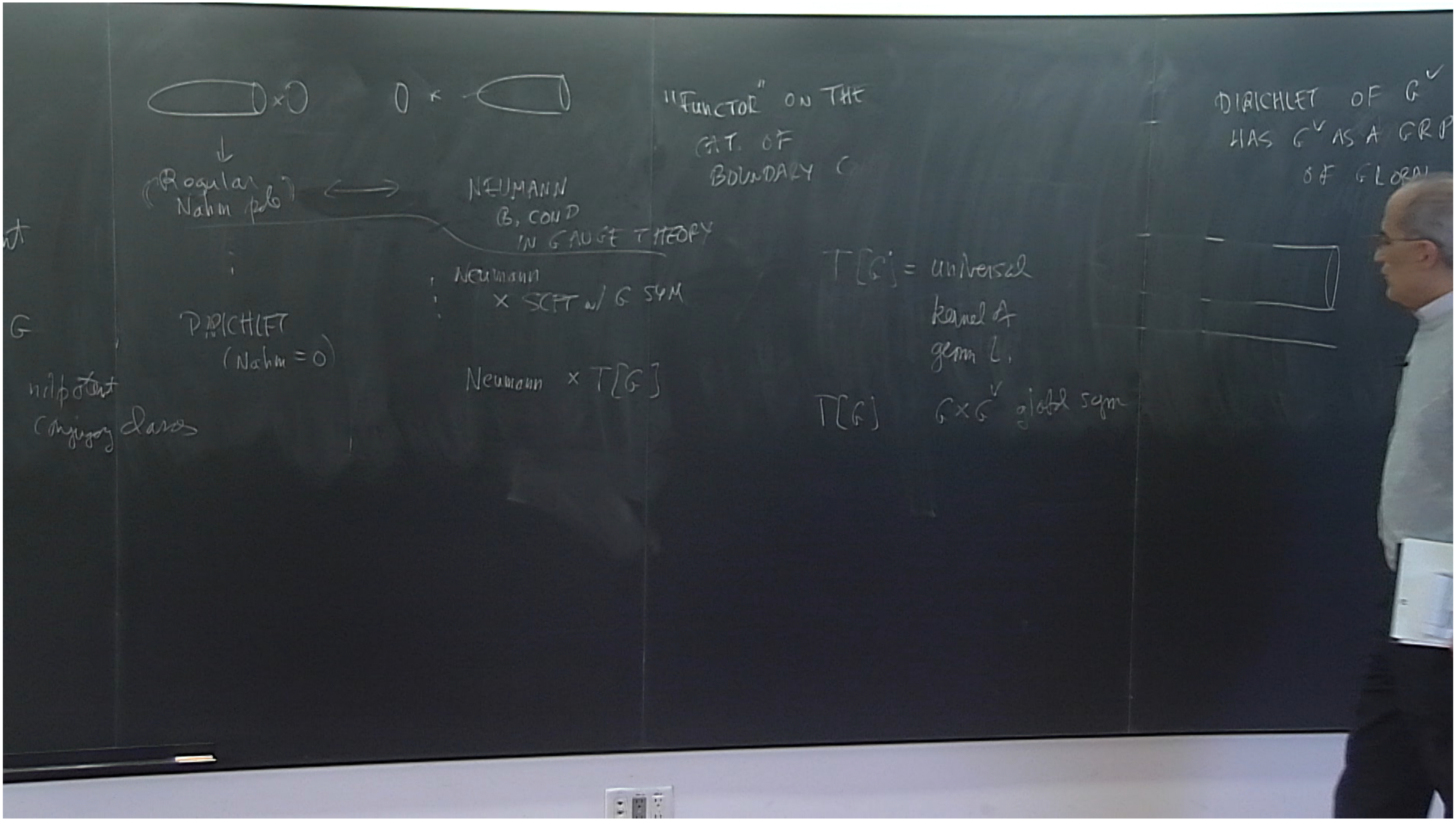
? D=6

$M_5 \times S^1$


"FUNCTOR" ON THE
CAT. OF
BOUNDARY C
NEUMANN
B. COND
IN GAUGE THEORY
Neumann
× SCFT w/ G SYM
Neumann × T[G]
T[G] = universal
kernel A
germ L.
× M4
M4 × S1 × S1
× 0
What are these in germ L ?
if we do opctify on C
(SL2ARTHUR × π1(C)) →



"FUNCTOR" ON THE
CAT. OF
BOUNDARY C
ANN
COND
N GAUGE THEORY
SCFT w/ G SYM
× T[G]
T[G] = universal
kernel of
geom L.
DIRICHLET OF G^∨
HAS G^∨ AS A GRP
OF GLOBAL SYM
× M4
. M4 × S1 × S1
× 0
compatible
w/ geom L
What are these in geom L ?
if we do opetify on C ?
(SL2^ARTHUR × π1(C)) →


"Regular Nahm pole"
NEUMANN B. COND IN GAUGE THEORY
DIRICHLET (Nahm = 0)
Neumann × SCFT w/ G SYM
Neumann × T[G]
"FUNCTOR" ON THE Q.T. OF BOUNDARY C
T[G] = universal kernel of geom L.
T[G] G×G∨ global sym
DIRICHLET OF G∨ HAS G∨ AS A GRP OF GLOBAL
G
nilpotent conjugacy classes

$\mathcal{O} \times$

Neumann B. COND IN GAUGE THEORY

Neumann $\times$ SCFT w/ $G$ SYM

Neumann $\times T[G]$

"FUNCTOR" ON THE CAT. OF BOUNDARY C

$T[G]$ = universal kernel 4 geom L.

$T[G]$ $G \times G^\vee$ global sym

DIRICHLET OF $G^\vee$ HAS $G^\vee$ AS A GRP OF GLOBAL SYM

$\times M_4$

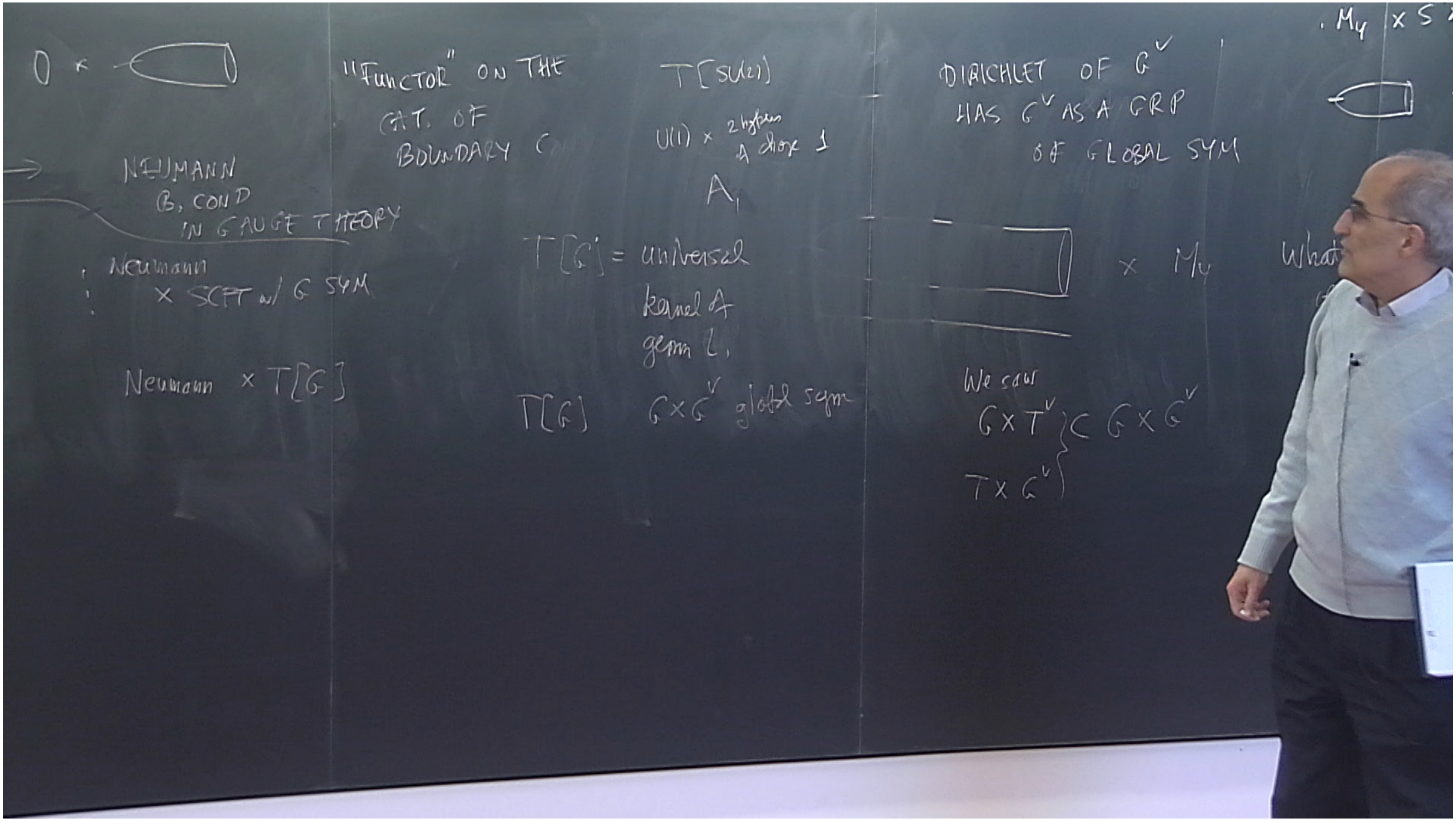
We saw $\left.\begin{array}{l} G \times T^\vee \\ T \times G^\vee \end{array}\right\} \subset G \times G^\vee$

$\cdot\, M_4 \times S^1 \times S^1$

$\times \mathcal{O}$

What are these in geom L if we do opetify on C?

$(SL_2^{ARTHUR} \times \pi_1(C)) \to$


"FUNCTOR" ON THE
CAT. OF
BOUNDARY C
NEUMANN
B. COND
IN GAUGE THEORY
Neumann
× SCFT w/ G SYM
Neumann × T[G]
T[SU(2)]
U(1) × 2 hypers
A charge 1
T[G] = universal
kernel A
geom L
T[G] G × G^∨ global sym
DIRICHLET OF G^∨
HAS G^∨ AS A GRP
OF GLOBAL SYM
× M4
We saw
G × T^∨ ⊂ G × G^∨
T × G^∨
M4 × S
What

$0 \times$ 

$\rightarrow$ NEUMANN
B. COND
IN GAUGE THEORY

: Neumann
$\times$ SCFT w/ G SYM

Neumann $\times T[G]$

"FUNCTOR" ON THE
CAT. OF
BOUNDARY C

$U(1)^\vee \times SU(2)$

$T[SU(2)]$

$U(1) \times$ 2 hypers
of charge 1

$A_1$

$T[G]$ = universal
kernel of
geom L.

$T[G]$ $\quad G \times G^\vee$ global sym

DIRICHLET OF $G^\vee$
HAS $G^\vee$ AS A GRP
OF GLOBAL SYM

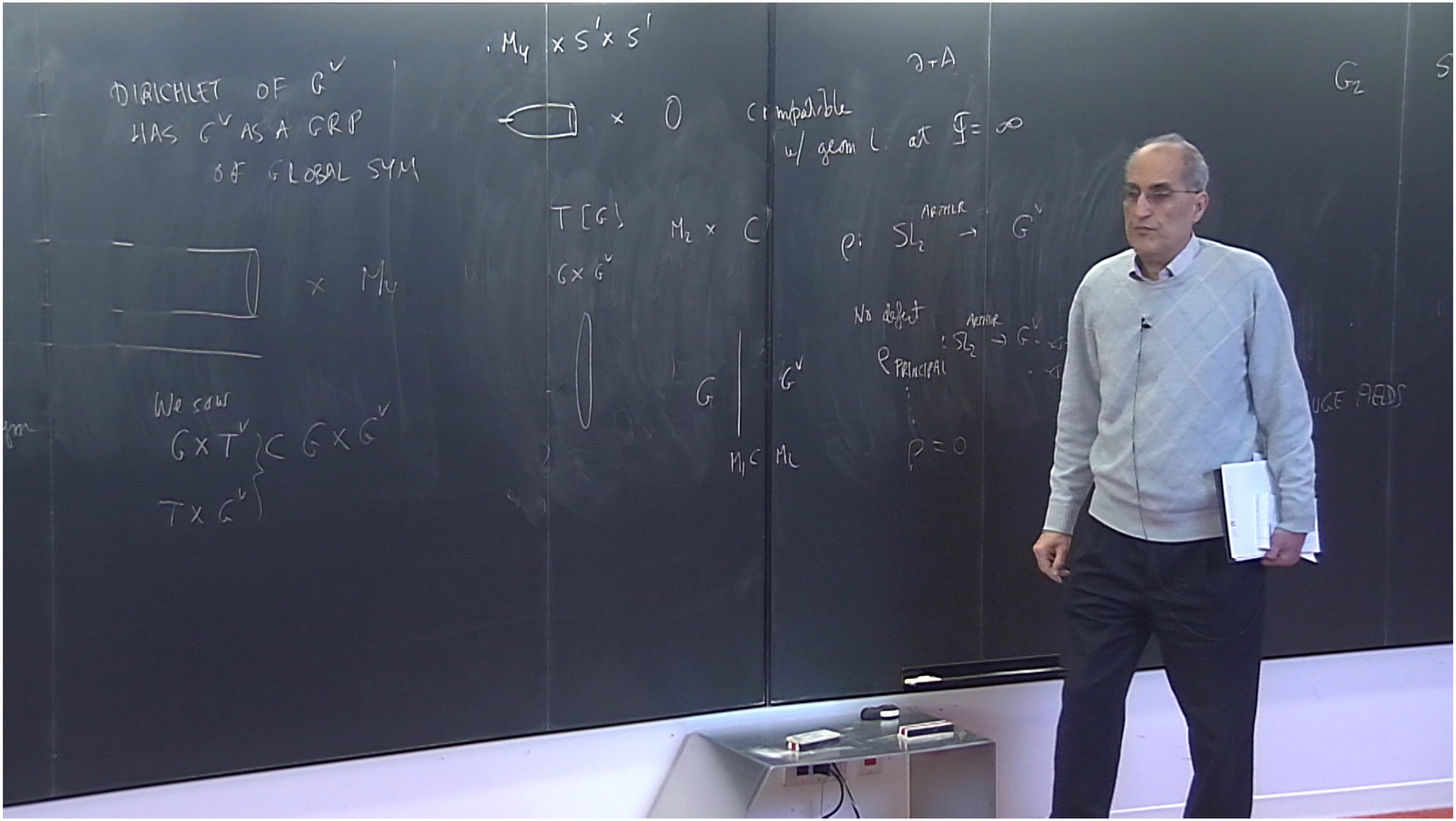
 $\times M_4$

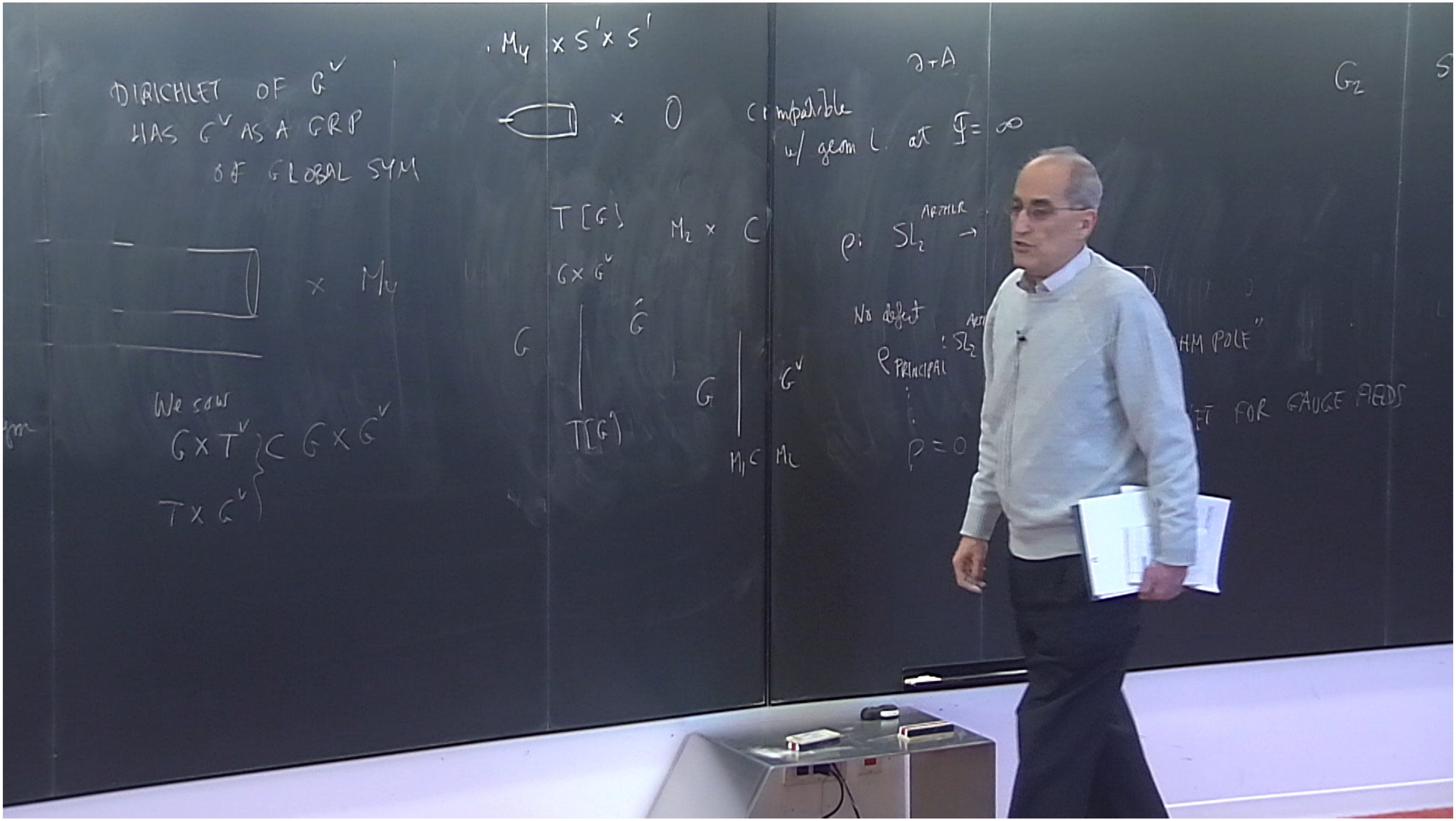
We saw
$\left.\begin{matrix} G \times T^\vee \\ T \times G^\vee \end{matrix}\right\} \subset G \times G^\vee$

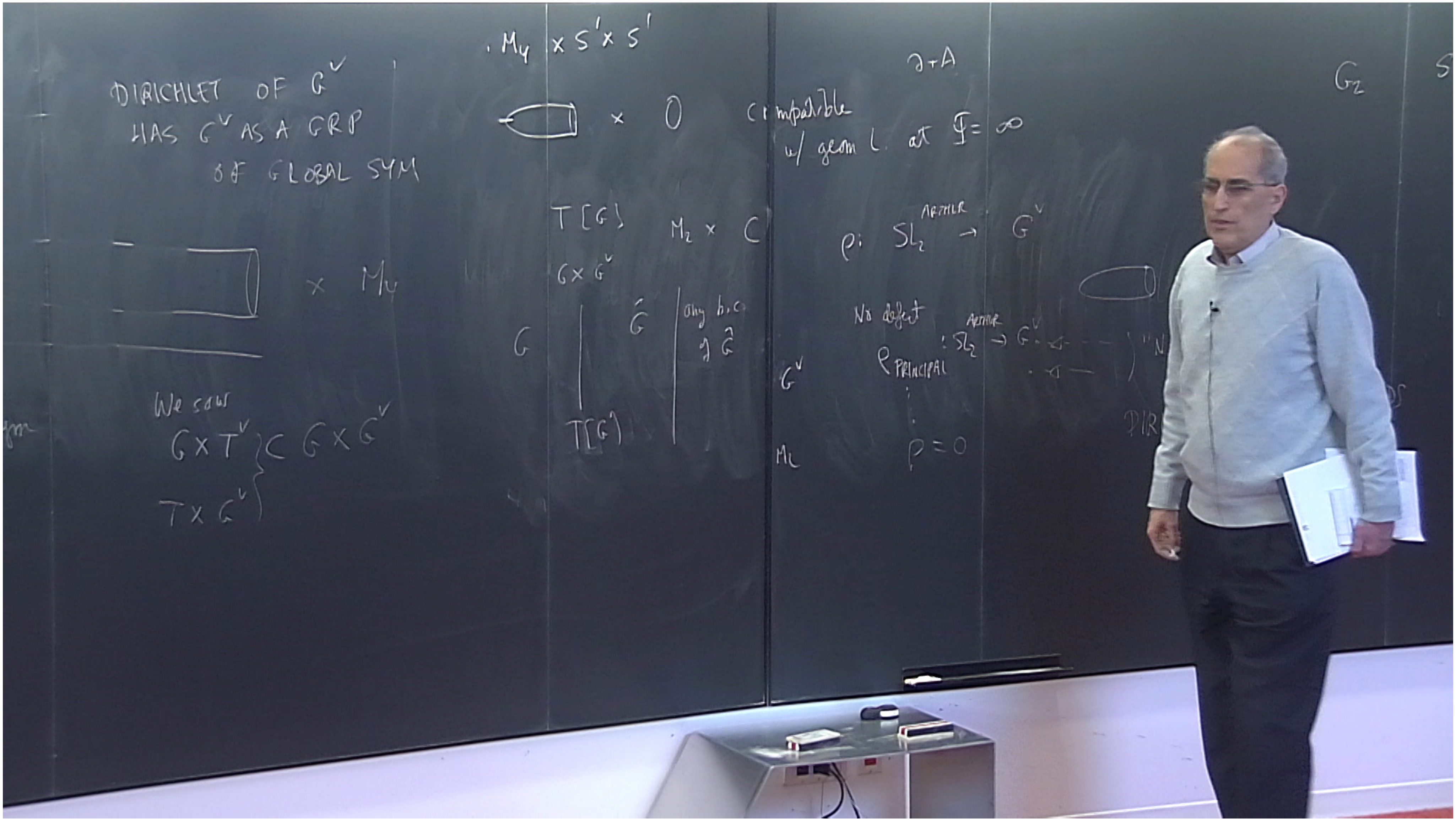
$M_4 \times S^1 \times$

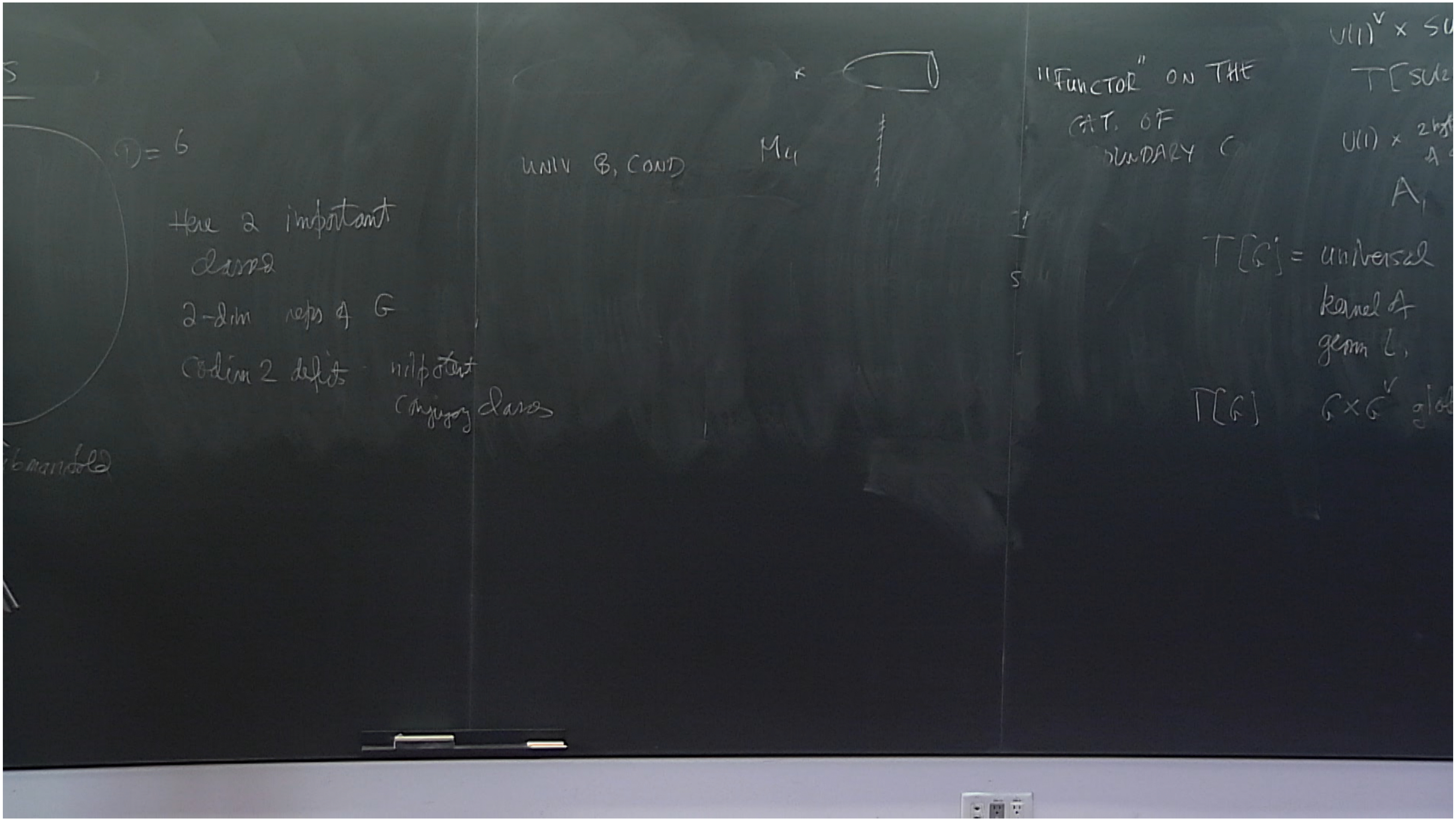
What are these
if we do optim

$(SL_2^{ARTHUR} \times \pi$


DIRICHLET OF G
HAS G AS A GRP
OF GLOBAL SYM
We saw
T[G]
compatible
w/ geom L at
No defect
ARTHUR
PRINCIPAL


DIRICHLET OF G^V
HAS G^V AS A GRP
OF GLOBAL SYM
We saw
G × T^V
T × G^V
C G × G^V
M4 × S'× S'
T[G]
G × G^V
T[G^V]
M2 × C
M1 C M2
compatible
w/ geom L at Φ = ∞
ρ: SL2 → 
No defect
ρ PRINCIPAL
ρ = 0
G2
FOR GAUGE FIELDS


DIRICHLET OF G
HAS G AS A GRP
OF GLOBAL SYM
We saw
compatible
w/ geom L at
ARTHUR
No defect
PRINCIPAL
any b.c.


= 6
Here 2 important
classes
2-dim reps of G
codim 2 defects - nilpotent
conjugacy classes
submanifold
UNIV B, COND
$M_4$
"FUNCTOR" ON THE
CAT. OF
UNDARY C
$U(1)^\vee \times SU$
$T[SU_2$
$U(1) \times$
$A_1$
$T[G] =$ universal
kernel of
geom L.
$T[G]$
$G \times G^\vee$ glob

$\textcircled{1} = G$

Here 2 important classes

2-dim reps of G

Codim 2 defects - nilpotent conjugacy classes

submanifold

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CORNERS

$M_4$

DIRICHLET

NEUMANN

"FUNCTOR" ON THE CAT. OF ...UNDARY C...

$U(1)^\vee \times S$...

$T[SU$...

$U(1) \times$ 2 hyp...

$A_1$

$T[G] =$ universal kernel of geom L.

$T[G]$ $\quad G \times G^\vee$ glob...