Title: Experimentally Probing Topological Order and Its Breakdown via Modular Matrices

Date: Mar 06, 2018 03:30 PM

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Abstract: <p>The modern conception of phases of matter has undergone tremendous developments since the first observation of topologically ordered states in fractional quantum Hall systems in the 1980s. In this paper, we explore the question: In principle, how much detail of the physics of topological orders can be observed using state of the art technologies? We find that using surprisingly little data, namely the toric code Hamiltonian in the presence of generic disorders and detuning from its exactly solvable point, the modular matrices -- characterizing anyonic statistics that are some of the most fundamental fingerprints of topological orders -- can be reconstructed with very good accuracy solely by experimental means. This is a first experimental realization of these fundamental signatures of a topological order, a test of their robustness against perturbations, and a proof of principle -- that current technologies have attained the precision to identify phases of matter and, as such, probe an extended region of phase space around the soluble point before its breakdown. Given the special role of anyonic statistics in quantum computation, our work promises myriad applications in both probing and realistically harnessing these exotic phases of matter.</p>

EXPERIMENTALLY PROBING TOPOLOGICAL ORDER AND ITS BREAKDOWN VIA A QUANTUM SIMULATOR

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6 Mar 2018 PI condensed matter seminar

based on Nature Physics 14 (2), 160 (2017)

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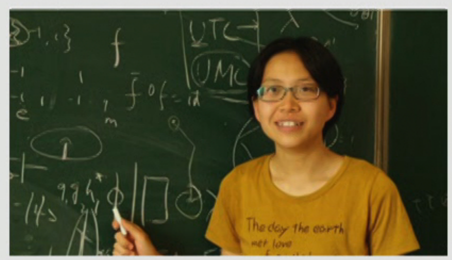
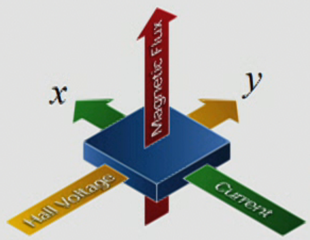


TOPOLOGICALLY ORDERED MATTER PHASES

Fractional quantum Hall states



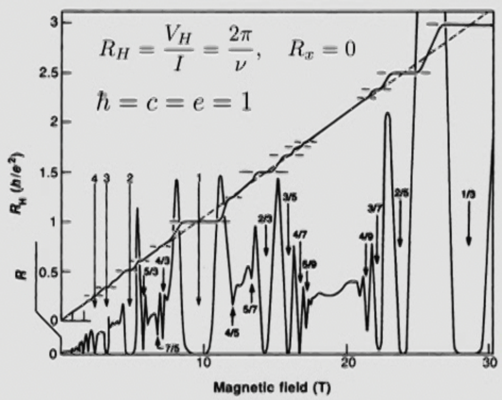
Distinct FQHS share the same symmetry

↓

Landau-Ginzburg symmetry breaking fails

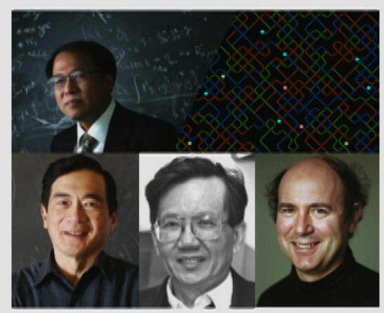
↓

Topological orders

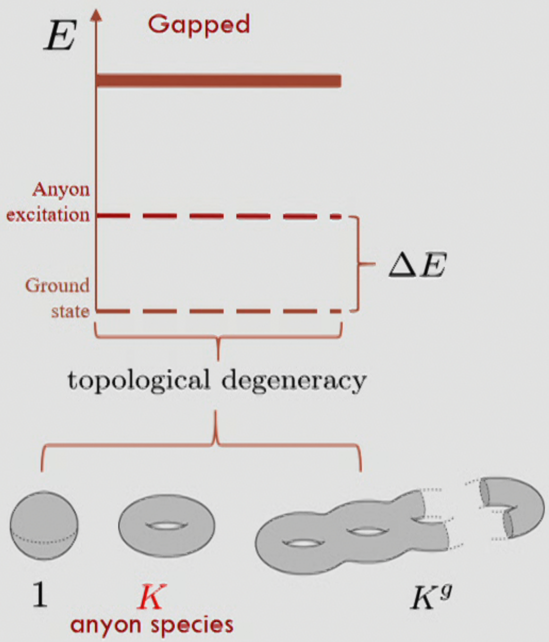


$$\nu = \frac{1}{3}, \frac{2}{3}, \frac{2}{5}, \frac{2}{7}, \frac{3}{7}, \cdots$$

Very particular fractions

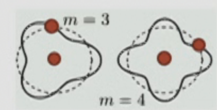


TOPOLOGICAL ORDER IN 2D



$\nu = \frac{1}{m}$ FQHS Abelian topological order

- Landau levels.
- $K = m$.



- Anyons species: $\{1, a_1, \ldots, a_{m-1}\}$
- Self-Statistics:

Topological spin

$$a_j \ a_j|\Psi\rangle = e^{i\pi \frac{j^2}{m}}|\Psi\rangle$$

- Braiding (Aharonov-Bohm):

$$a_j \ a_k|\Psi\rangle = e^{i\pi \frac{jk}{m}}|\Psi\rangle$$

- Fusion

$$a_j \times a_k = a_{j+k} \mod m$$

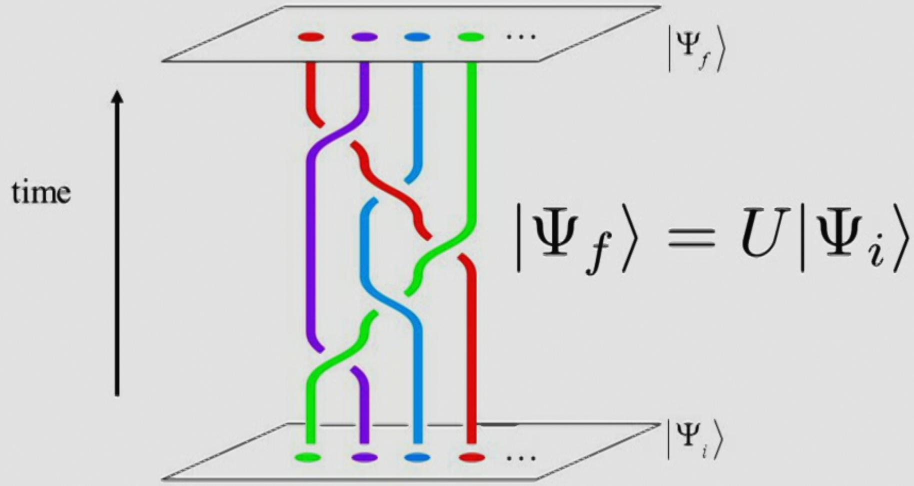
APPLICATIONS OF TOPOLOGICAL ORDERS

Alexei Kitaev

Topologically protected ground state space

Robust quantum memory

Non-Abelian anyons



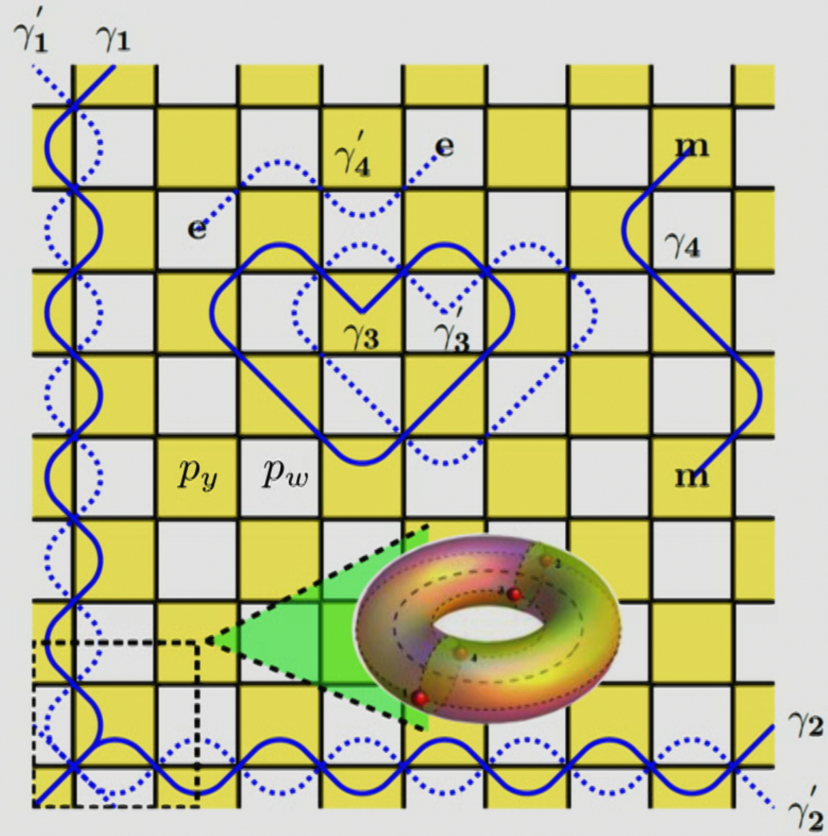
Topological quantum computing

Z2 TORIC CODE ON A TORUS

$$\hat{H}_{\mathbb{Z}_2} = \sum_{\text{white plaquettes}} X_{p_w} + \sum_{\text{yellow plaquettes}} Z_{p_y}$$

$$X_{p_w} = \prod_{j \in \partial p_w} \hat{\sigma}_j^x, \quad Z_{p_y} = \prod_{j \in \partial p_y} \hat{\sigma}_j^z$$

Ground states: all configurations of closed loops
Ground-state basis: 4 non-contractible loops
Excitations (anyons): 1, e, m, em

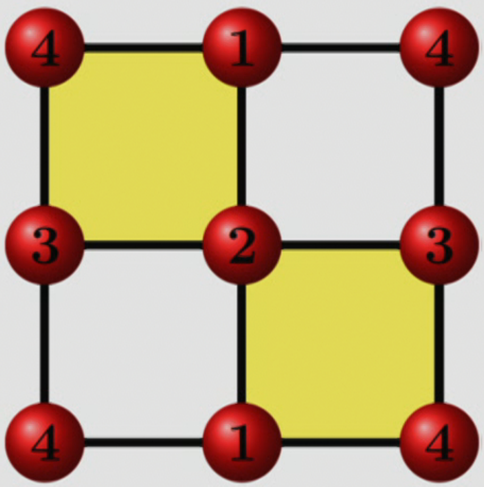


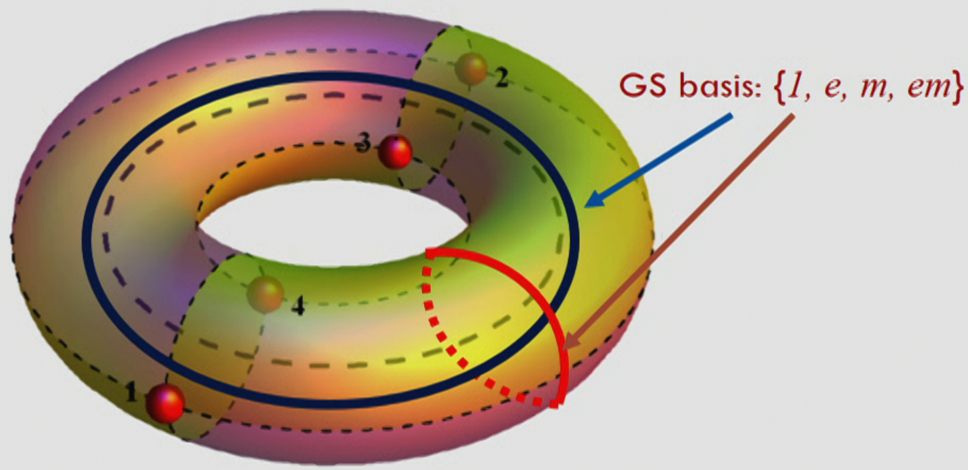
A. Kitaev, Ann. Phys. 303, 2 (2003); X. G. Wen, PRL. 90, 016803 (2003)

Z2 TORIC CODE ON A TORUS

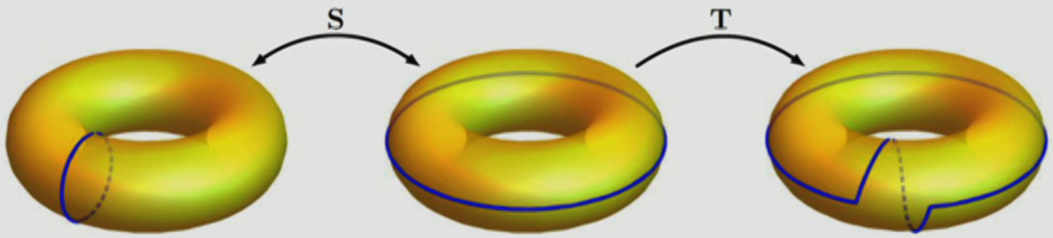
4-QUBIT SUFFICES

Topological invariance of the ground states



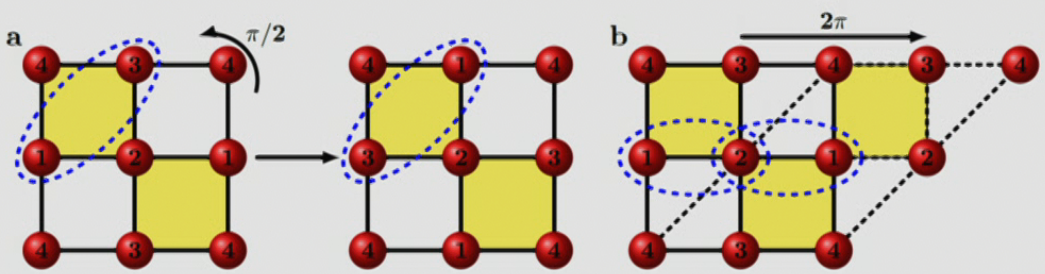


MODULAR MATRICES



$$S = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

Braiding



$$T = \mathrm{diag}\{1, 1, 1, -1\}$$

Self statistics

WHY NEED TO SIMULATE TOPOLOGICAL ORDERS

Difficult to realize topological orders directly in real systems

Real systems deviate from exact soluble models because of disorders

To identify topological phases of matter using current techniques

A lot of numerical investigations:

[1] H.-C. Jiang et al., Identifying topological order by entanglement entropy. Nat Phys, 8, 902–905 (2012).
[2] Y. Zhang et al., Quasiparticle statistics and braiding from ground-state entanglement. Phys. Rev. B, 85:235151 (2012).
[3] M. P. Zaletel et al., Topological characterization of fractional quantum hall ground states from microscopic hamiltonians. Phys. Rev. Lett., 110:236801 (2013).
[4] P. Bonderson et al. Probing non-abelian statistics with quasiparticle interferometry. Phys. Rev. Lett., 97:016401 (2006).
[5] L. Cincio & G. Vidal. Characterizing topological order by studying the ground states on an infinite cylinder. Phys. Rev. Lett., 110:06720, (2013).
[6]H. He et al. Modular matrices as topological order parameter by a gauge-symmetry-preserved tensor renormalization approach. Phys. Rev. B, 90:205114 (2014).
[7] Fangzhou Liu et al., Modular transformations and topological orders in two dimensions. arXiv: 1303.0829v2 (2013)
[8] Jacob C. Bridgeman et al., Detecting Topological Order with Ribbon Operators. arXiv:1603.02275v3 (2016)
……

OUTLINE

- What are topological orders?
  - Z2 Toric Code
- **Quantum Simulator – NMR Simulator**
- NMR simulation of Z2 toric code
- What lies ahead

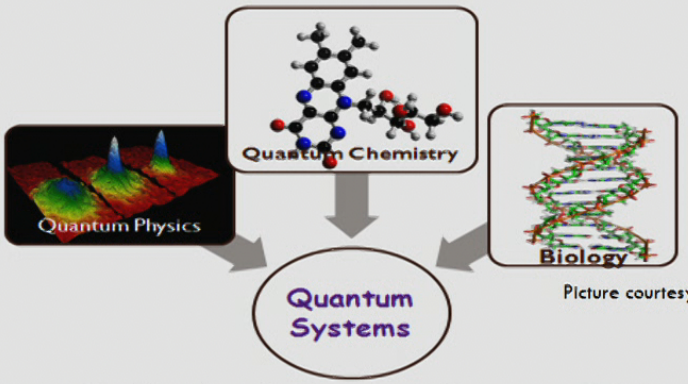
QUANTUM SIMULATIONS

Simulating Physics with Computers

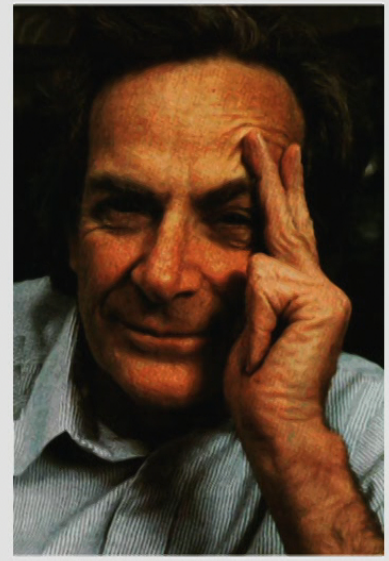
Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981



Picture courtesy Prof. Xinhua Peng



For example, the spin waves in a spin lattice imitating Bose-particles in the field theory. I therefore believe it's true that with a suitable class of quantum machines you could imitate any quantum system, including the physical world. But I don't know

QUANTUM SIMULATOR

1) mapping

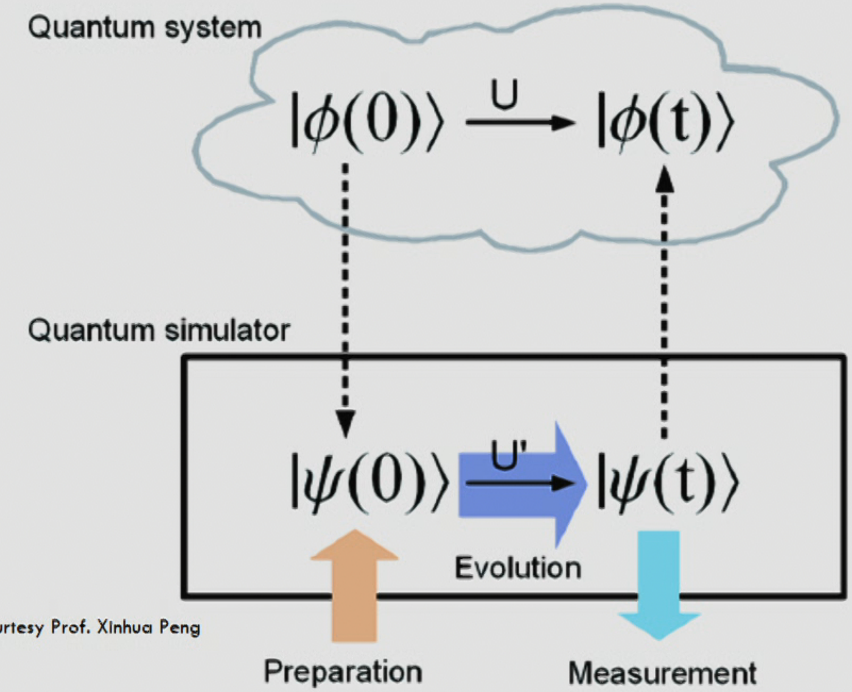
2) Hamiltonian engineering

Lloyd's method :

Quantum gates implemented by sequence of Hamiltonian

(Average Hamiltonian theory)

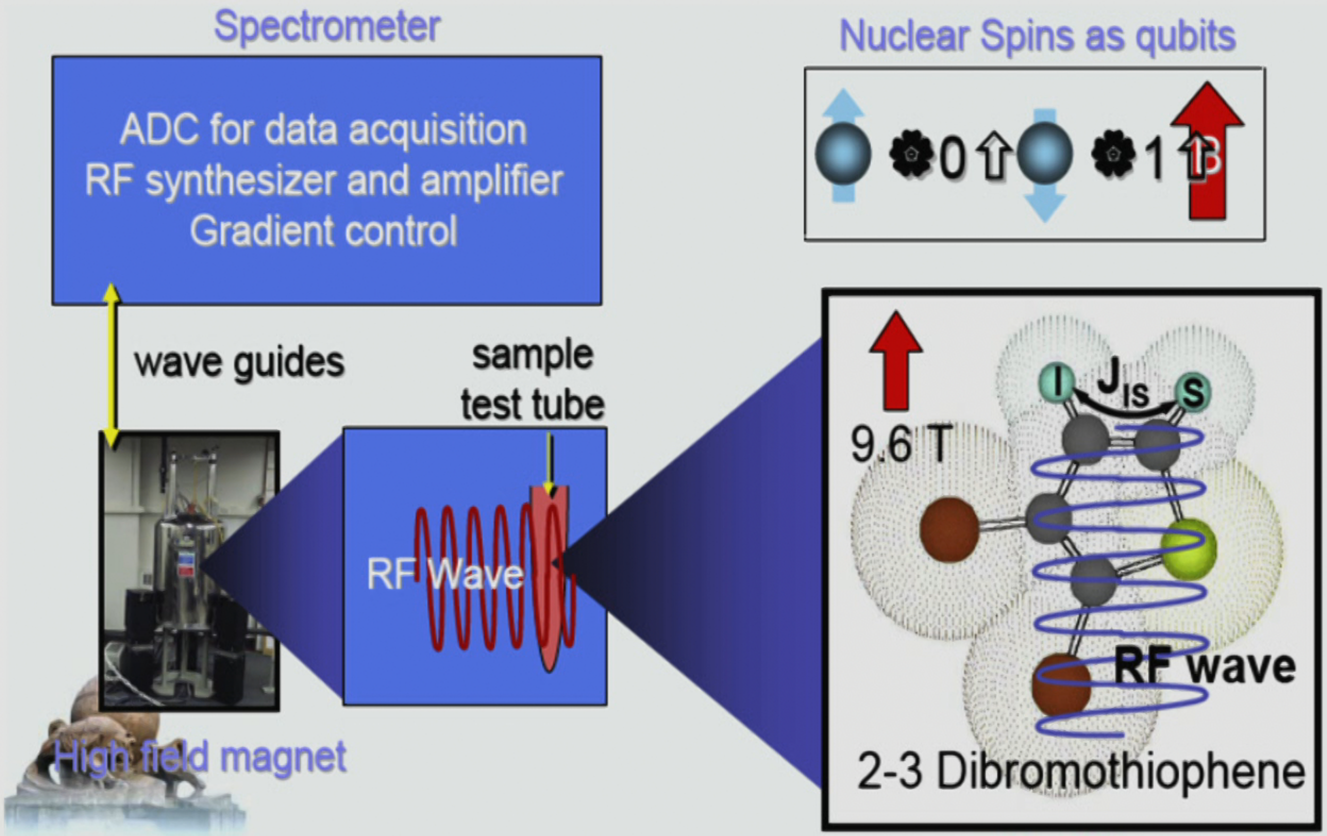
Measurement



Picture courtesy Prof. Xinhua Peng

I. M. Georgescu et al., Rev. Mod. Phys., Vol. 86, No. 1, January–March 2014

NMR QUANTUM SIMULATOR



Picture courtesy Prof. Xinhua Peng

HOW (LIQUID) NMR QUANTUM COMPUTATION WORKS

Operates at room temperature and pressure. Long coherence time ~ seconds

(upto ~ 1000 pulses in an experiment).

Different molecules have slightly different energy levels and so allow for suitable choice of pulses to control them individually

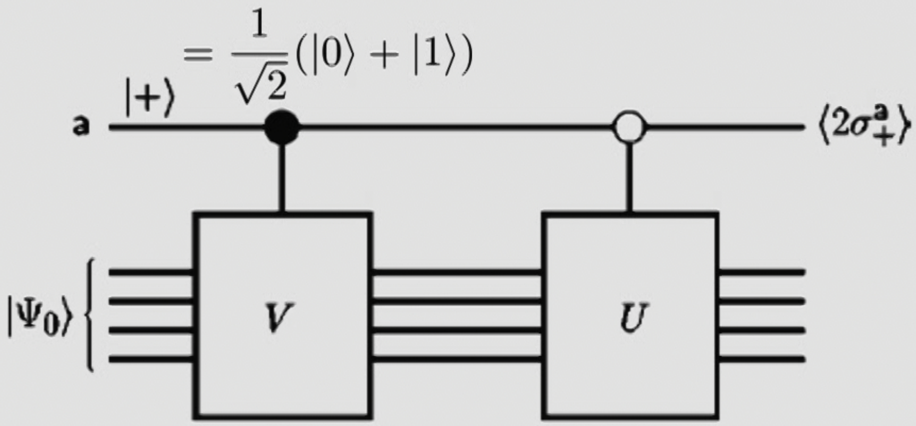
The system is in Pseudo pure state $\Psi = \frac{(1-\alpha)\mathbf{1} + 2\alpha|\psi\rangle\langle\psi|}{(1-\alpha)2^n + 2\alpha}$ $(-1 \leq \alpha \leq 1)$

Ensemble computing: measure small magnetization can detect occupation and allows one to measure

$$\mathrm{Tr}(K\Psi) = (1-\alpha)\mathrm{Tr}(K) + 2\alpha\langle\psi|K|\psi\rangle$$

Parallel computation without wavefunction collapse.

MEASUREMENT



$$2\sigma_+^{a} = \sigma_x^{a} + i\sigma_y^{a}$$

$$\tilde{V} = |0\rangle\langle 0| \otimes \mathbb{1} + |1\rangle\langle 1| \otimes V$$

$$\tilde{U} = |0\rangle\langle 0| \otimes U + |1\rangle\langle 1| \otimes \mathbb{1}.$$

$$\langle 2\sigma_+^{a} \rangle = \langle \Psi_0 | U^\dagger V | \Psi_0 \rangle$$

R. Somma et al., Phys. Rev. A, 65, 042323, (2002)

Z2 TORIC CODE ON A NMR SIMULATOR

Taget:

$$H = H_{\mathbb{Z}_2} + H_z + H_{\text{disorder}}$$
$$= 2(\hat{\sigma}_1^x\hat{\sigma}_2^x\hat{\sigma}_3^x\hat{\sigma}_4^x + \hat{\sigma}_1^z\hat{\sigma}_2^z\hat{\sigma}_3^z\hat{\sigma}_4^z) + h\sum_i \sigma_i^z + \sum_i \epsilon_i\sigma_i^z$$

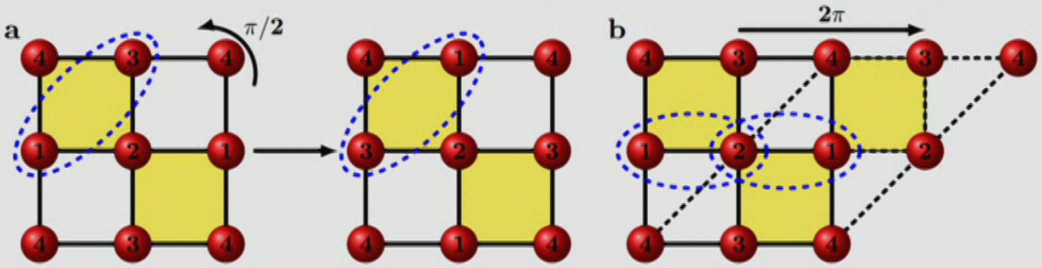
Certain larger $h=h_c$ should trigger a phase transition

$$S = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

Braiding

$$T = \text{diag}\{1, 1, 1, -1\}$$

Self statistics



Goal: identify the Z2 order and phase transition with minimal theoretical input by measuring the S and T matrices.

1. STATE PREPARATION

RANDOM ADIABATIC EVOLUTION

Obtaining the 4- fold degenerate ground state without prior knowledge:

$$H_{adiabatic}(s) = sH + (1-s)H_{random}$$

$$H_{random} = \sum_{j=1}^{4}\sum_{i=x}^{z}\alpha_j^i\sigma_j^i$$

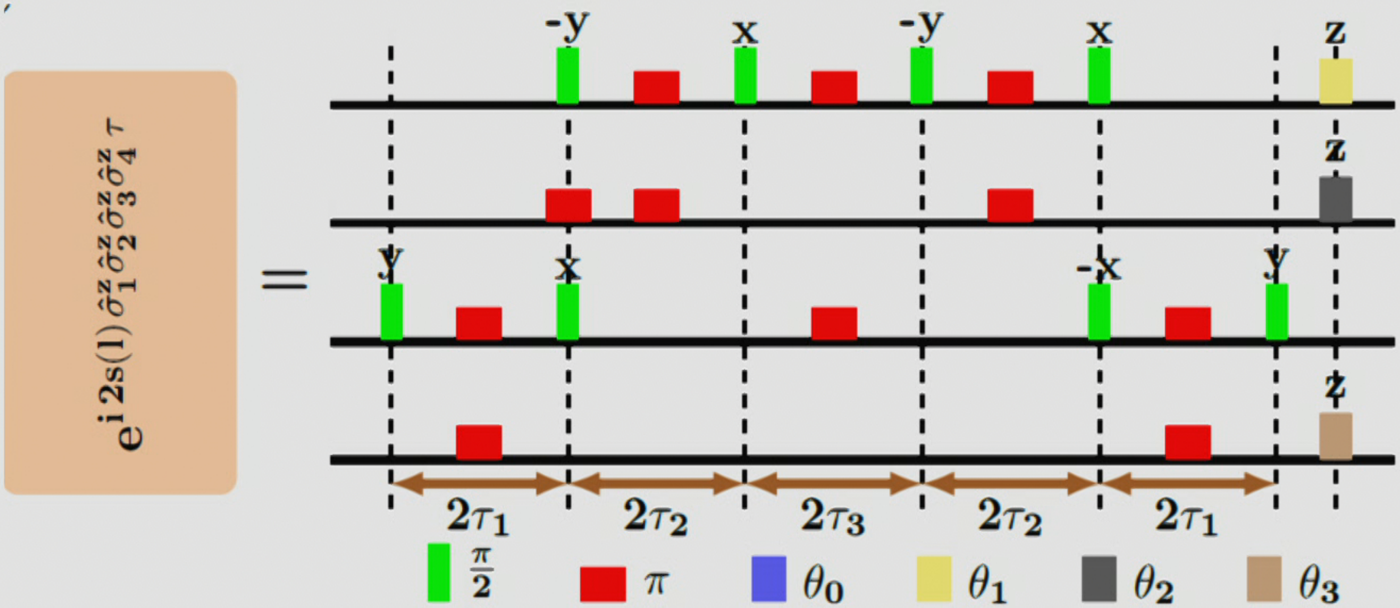
↓ Adiabatically obtain

Ground states $|\psi_i^{rd}\rangle$ $\quad i = 1, \ldots, 4$

↓

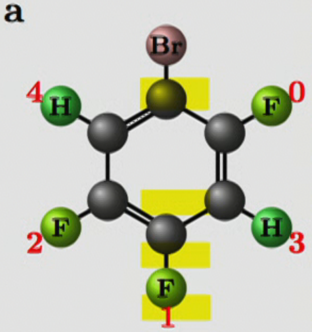
Orthonormal basis $|\phi_i\rangle$ $\quad i = 1, \ldots, 4$

2. SIMULATE THE 4-BODY INTERACTION



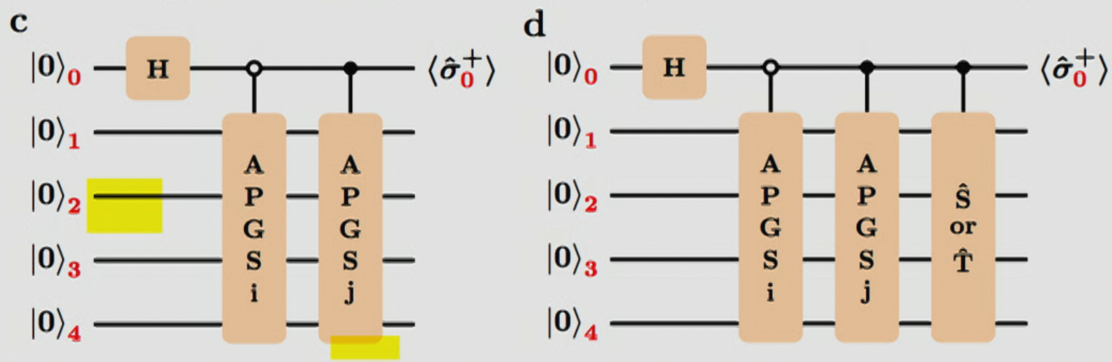
3. IMPLEMENT SWAP OPERATION

a



b

	0	1	2	3	4	T_2^* (ms)	T_1 (s)
0	-37342					40	0.8
1	334.8	-44793				40	0.8
2	114.0	-51.4	-47243			40	0.8
3	1875.9	1523.4	64.0	2701		100	1.5
4	62.8	127.4	1849.6	-11.8	2691	100	1.5



3. RECOVERY OF THE MODULAR MATRICES

Ground states $|\psi_i^{rd}\rangle$ $\quad i = 1, \ldots, 4$

$$\langle\psi_i^{rd}|O|\psi_j^{rd}\rangle = \sum_{kl} A_{ij}^{kl}\langle\phi_k|O|\phi_l\rangle$$

Orthonormal basis $|\phi_i\rangle$ $\quad i = 1, \ldots, 4$

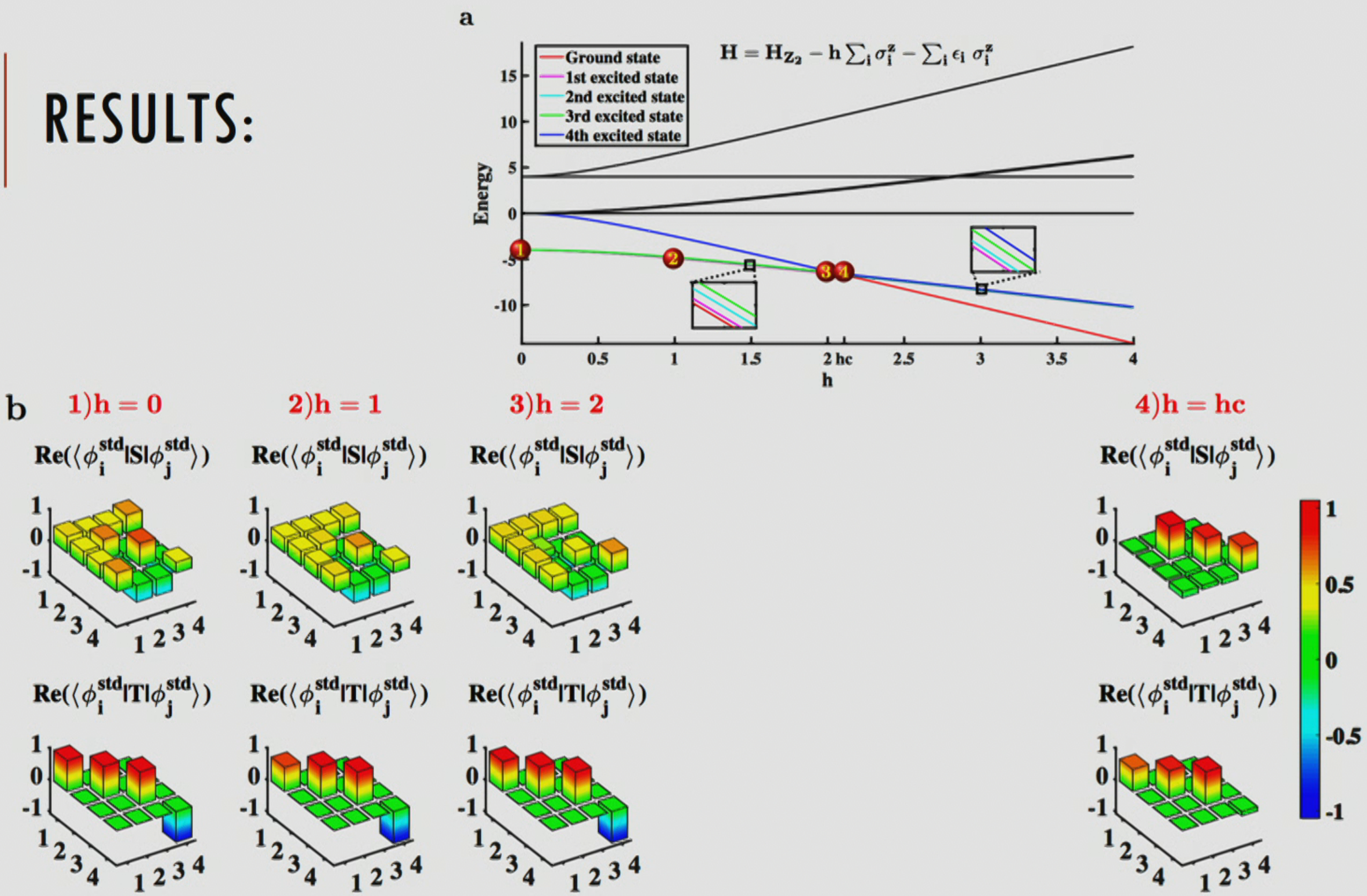
Optimize, diagonalize, and transform

$$T_{ab} = \exp(\mathrm{i}\theta_a)\delta_{ab}$$
$$S_{1a} = 1$$

Standard basis $\underbrace{|a\rangle}_{1,e,m,em} = \sum_i \beta_i^a|\phi_i\rangle$ $\qquad \langle a|b\rangle = \delta_{ab}$

$$T = \mathrm{diag}\{1,1,1,-1\} \qquad S = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

RESULTS:



a
Ground state
1st excited state
2nd excited state
3rd excited state
4th excited state
$H = H_{Z_2} - h\sum_i \sigma_i^z - \sum_i \epsilon_i \sigma_i^z$
Energy
h
hc
b
1)h = 0
2)h = 1
3)h = 2
4)h = hc
$Re(\langle\phi_i^{std}|S|\phi_j^{std}\rangle)$
$Re(\langle\phi_i^{std}|T|\phi_j^{std}\rangle)$

SUMMARY

We use minimal theoretical input to extract phase structure of the Z2 topological order via measurements of the modular matrices. It has mild requirements on symmetries.

Our experiment is the first to measure modular matrices without using the knowledge of string operators, and the first to move away from the exactly solvable point.

Bigger systems? (Complexity of our method only grows polynomially)

More general topological orders? Braiding operations?

THANK YOU FOR YOUR ATTENTION!