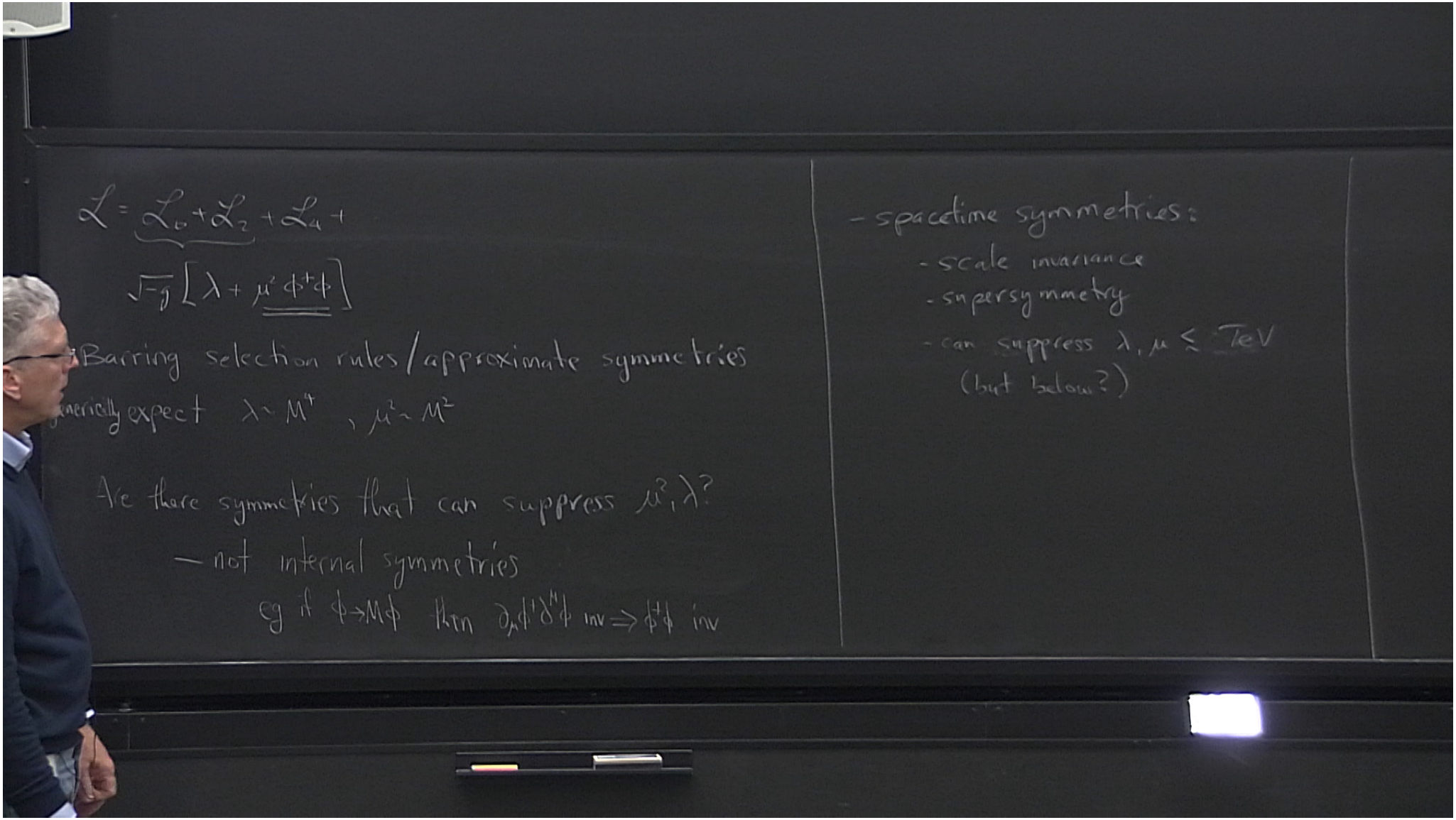
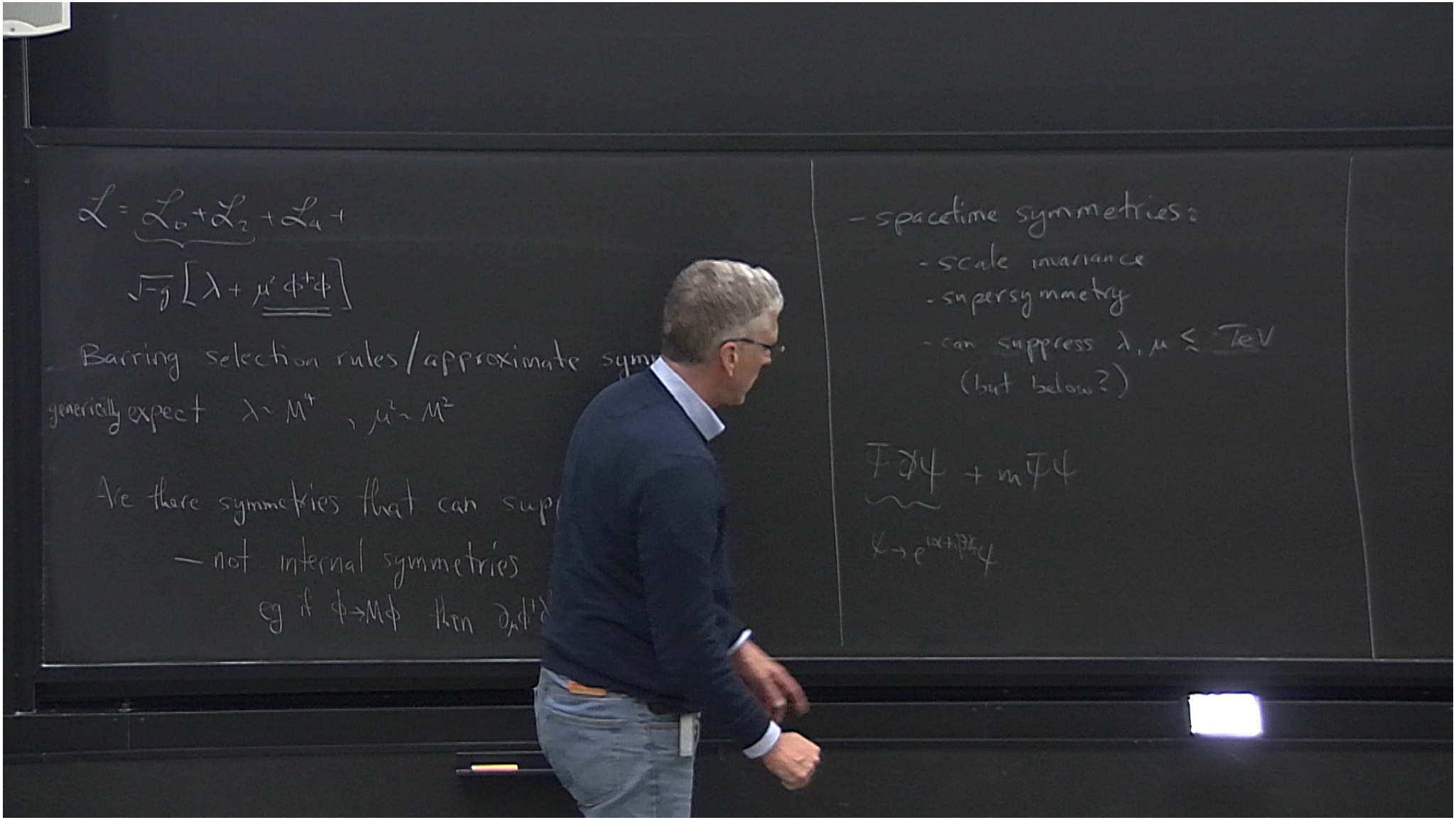
Title: PSI 2017/2018 - Beyond Standard Model - Lecture 5

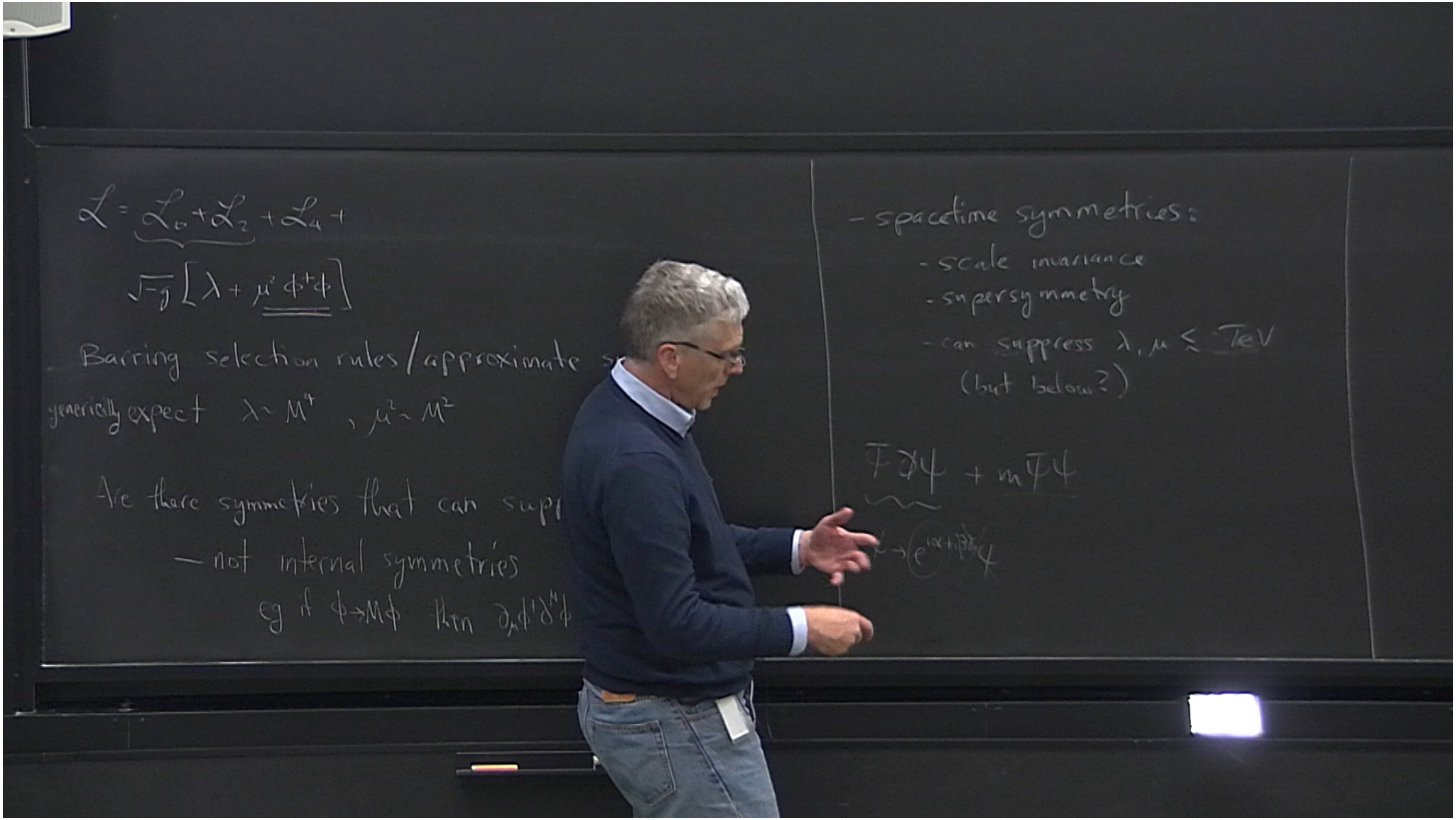
Date: Mar 02, 2018 02:00 PM

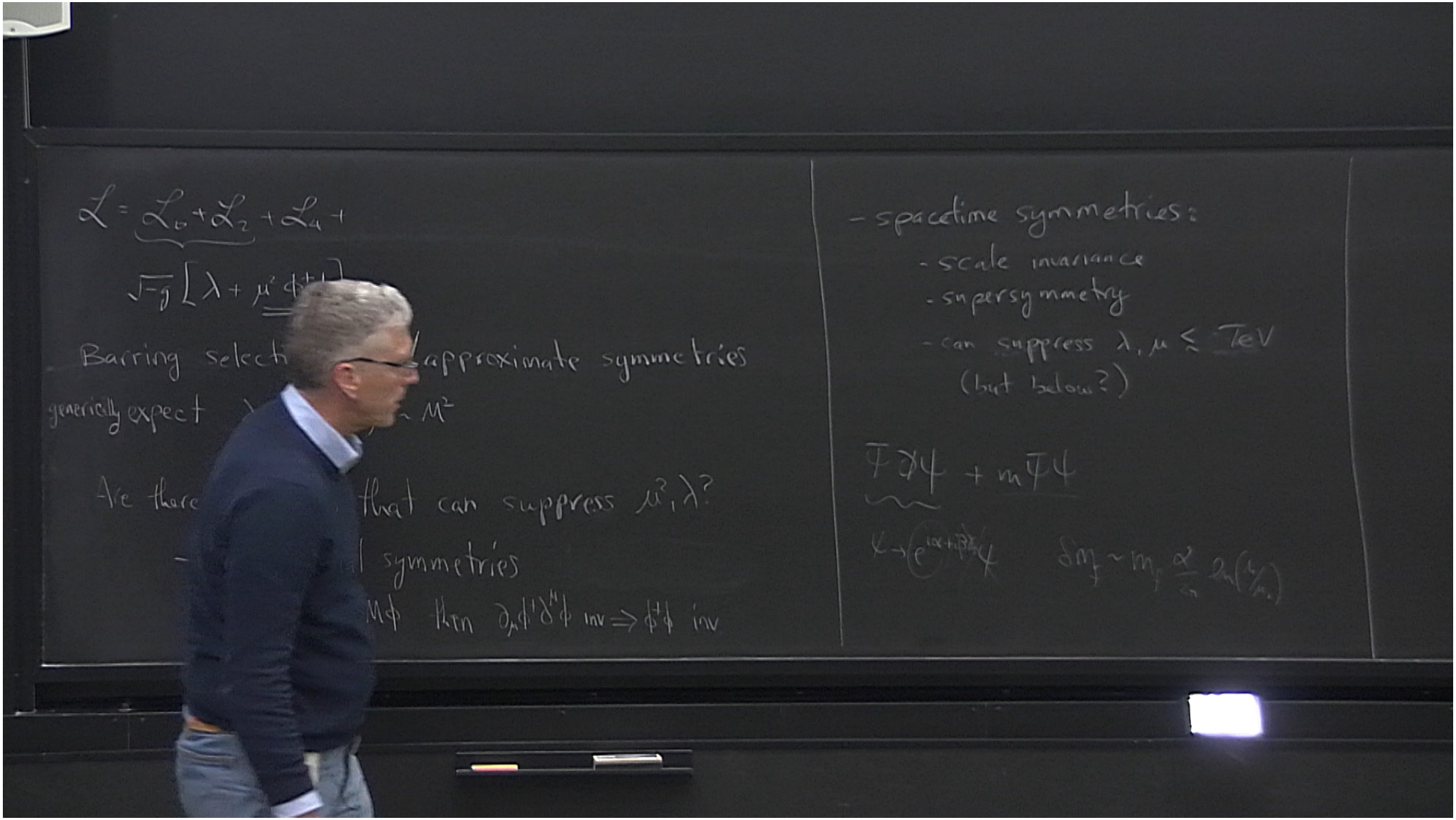
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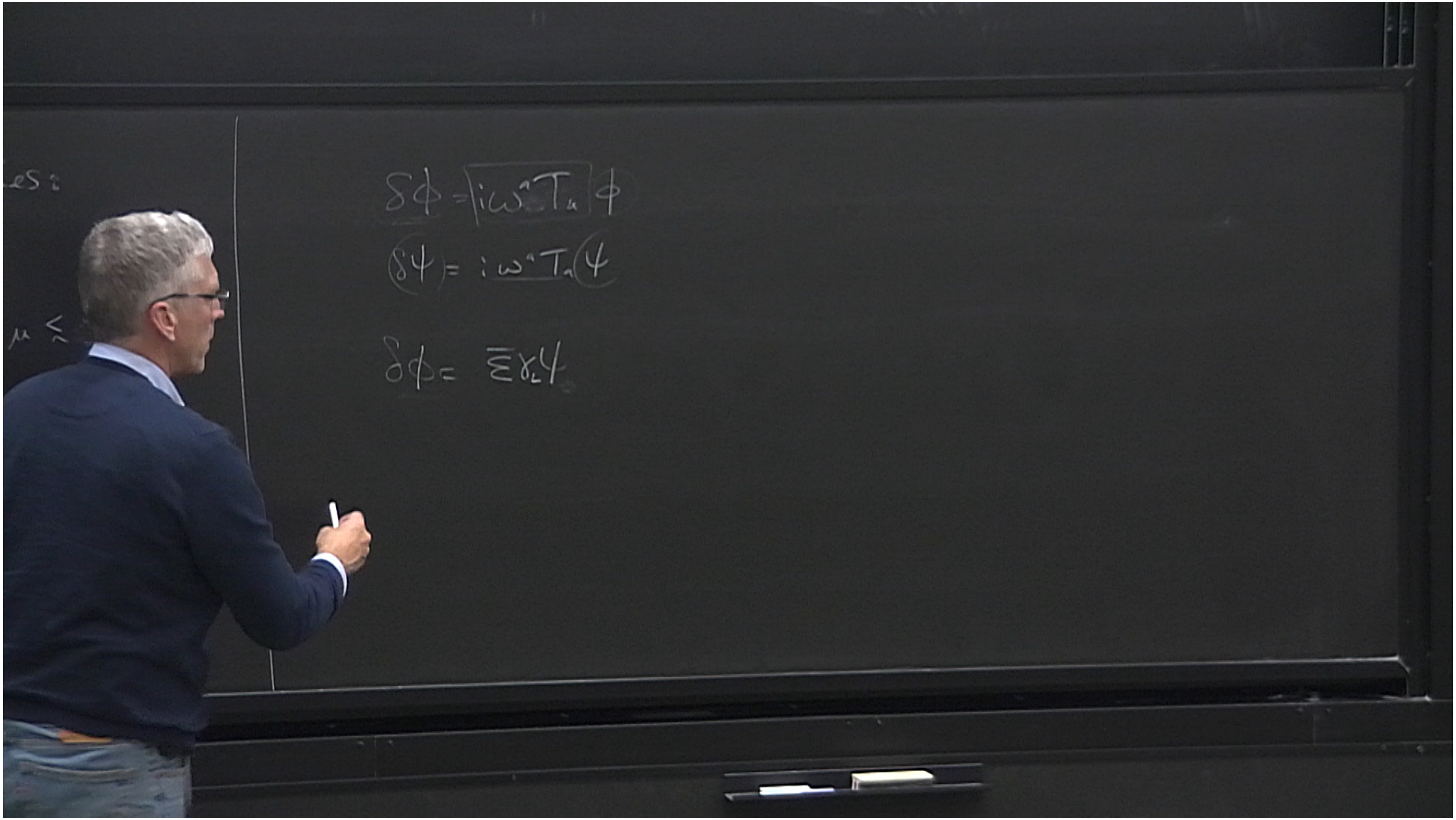
Abstract:

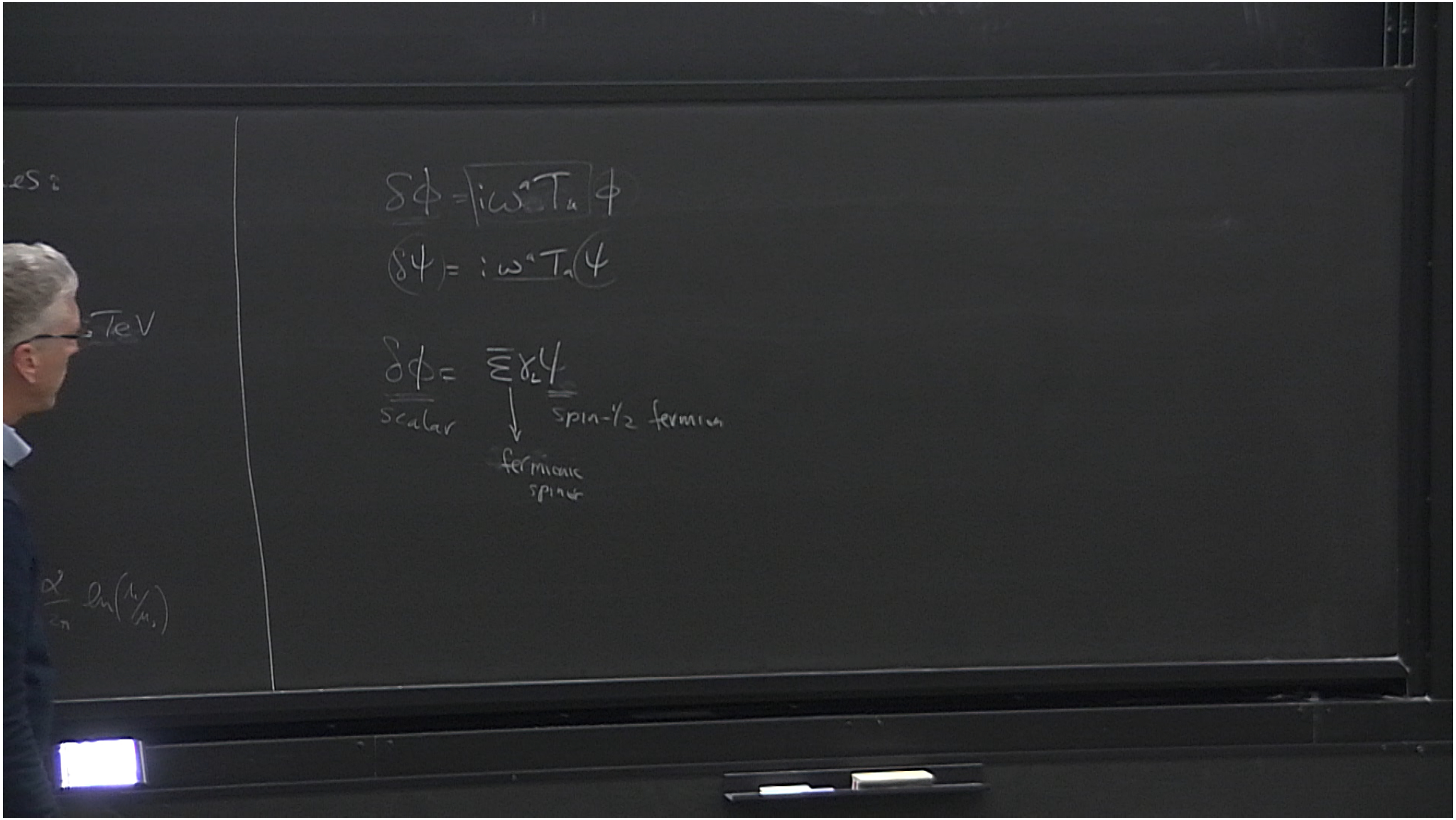

$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_2 + \mathcal{L}_4 +$
$\sqrt{-g}\left[\lambda + \mu^2 \phi^\dagger \phi\right]$
Barring selection rules / approximate symmetries
generically expect $\lambda \sim M^4$, $\mu^2 \sim M^2$
Are there symmetries that can suppress μ^2, λ?
— not internal symmetries
eg if $\phi \to M\phi$ then $\partial_\mu \phi^\dagger \partial^\mu \phi$ inv $\Rightarrow \phi^\dagger \phi$ inv
- spacetime symmetries:
- scale invariance
- supersymmetry
- can suppress $\lambda, \mu \lesssim$ TeV
(but below?)

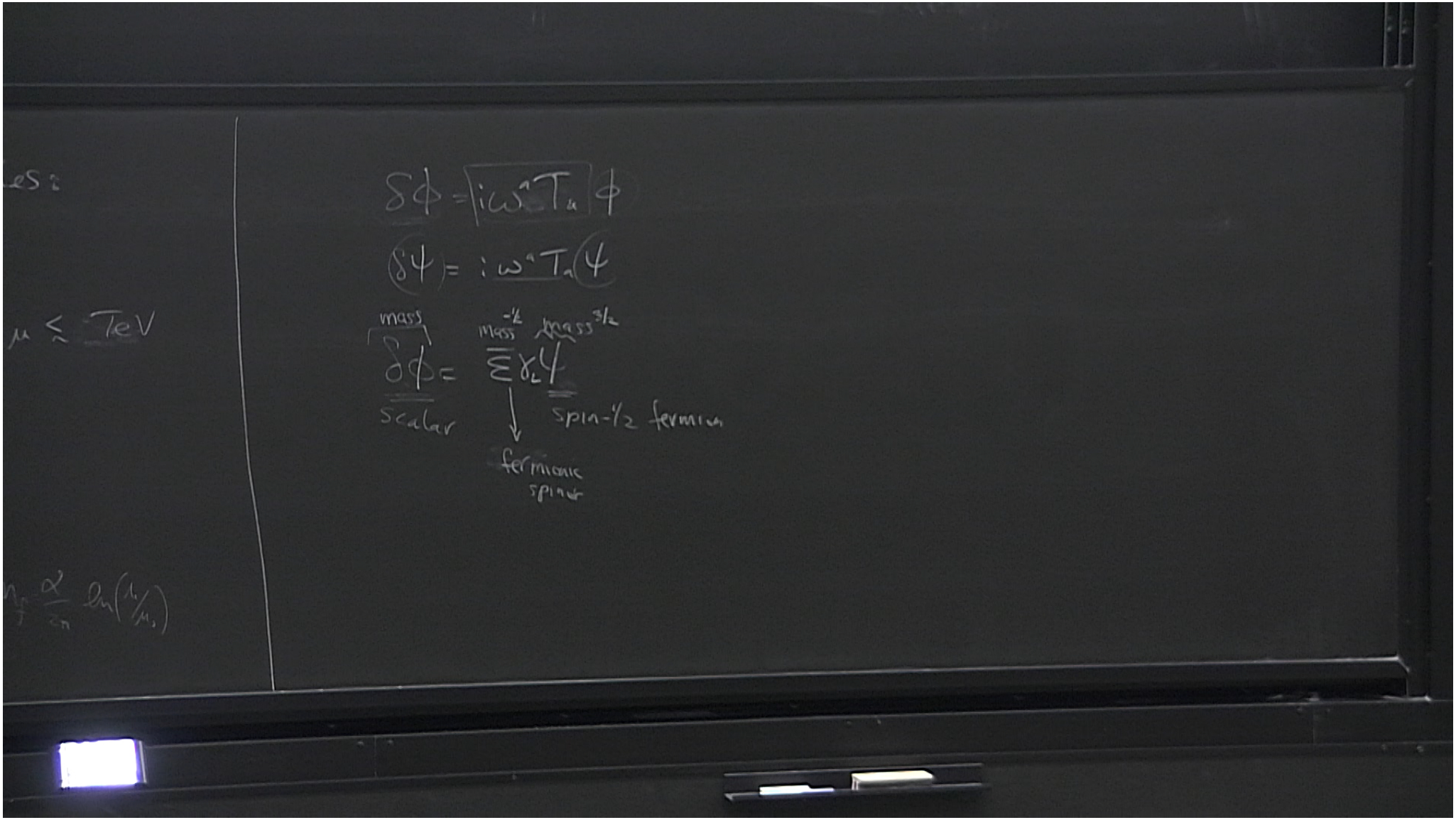

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- spacetime symmetries:
- scale invariance
- supersymmetry
- can suppress $\lambda, \mu \lesssim$ TeV
(but below?)
$\bar{\psi} \partial\!\!\!/ \psi + m \bar{\psi} \psi$
$\psi \to e^{i\alpha + i\beta\gamma_5}\psi$

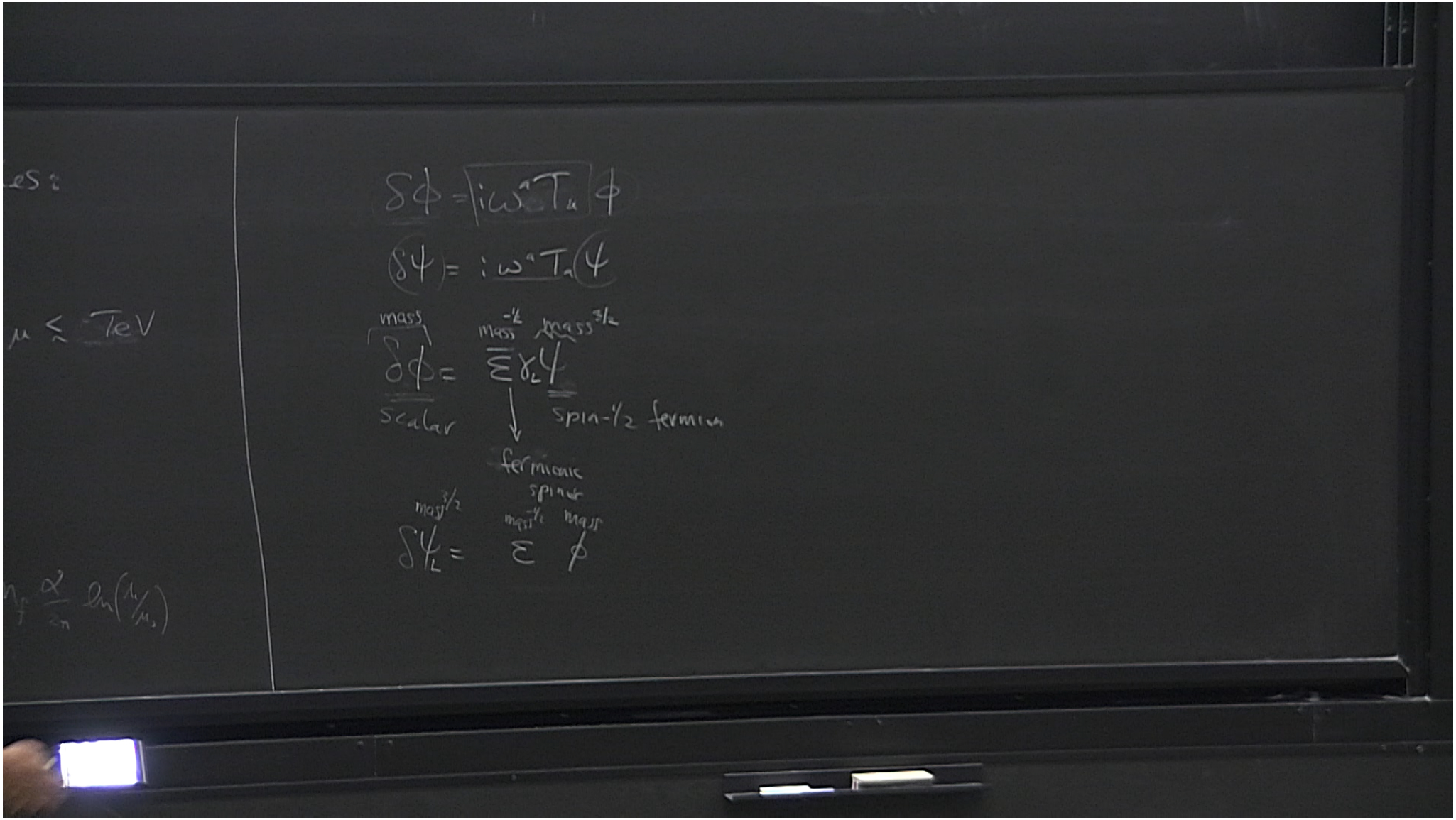

$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_2 + \mathcal{L}_4 +$
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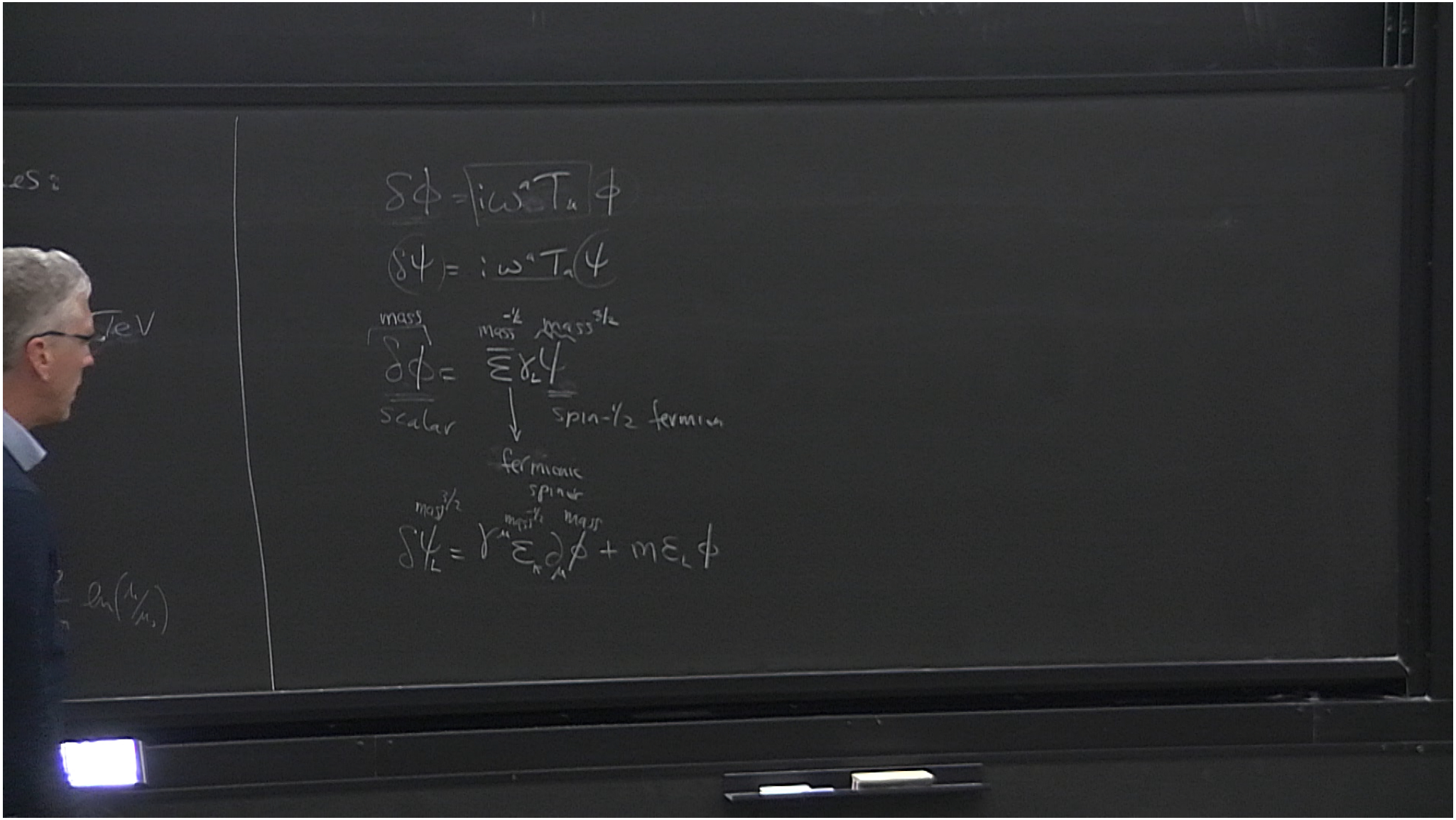

$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_2 + \mathcal{L}_4 +$
$\sqrt{-g}\left[\lambda + \mu^2 \phi^\dagger\phi + \right]$
Barring selecti approximate symmetries
generically expect $\sim M^2$
Are there that can suppress μ^2, λ?
- symmetries
$\phi \to M\phi$ then $\partial_\mu\phi^\dagger\partial^\mu\phi$ inv $\Rightarrow \phi^\dagger\phi$ inv
- spacetime symmetries:
- scale invariance
- supersymmetry
- can suppress $\lambda, \mu \lesssim$ TeV
(but below?)
$\bar{\psi}\partial\!\!\!/\psi + m\bar{\psi}\psi$
$\psi \to e^{i\alpha + i\beta\gamma^5}\psi$
$\dim_\gamma \sim m_\gamma \frac{\alpha}{4\pi} \ln(\frac{\Lambda}{m_\gamma})$

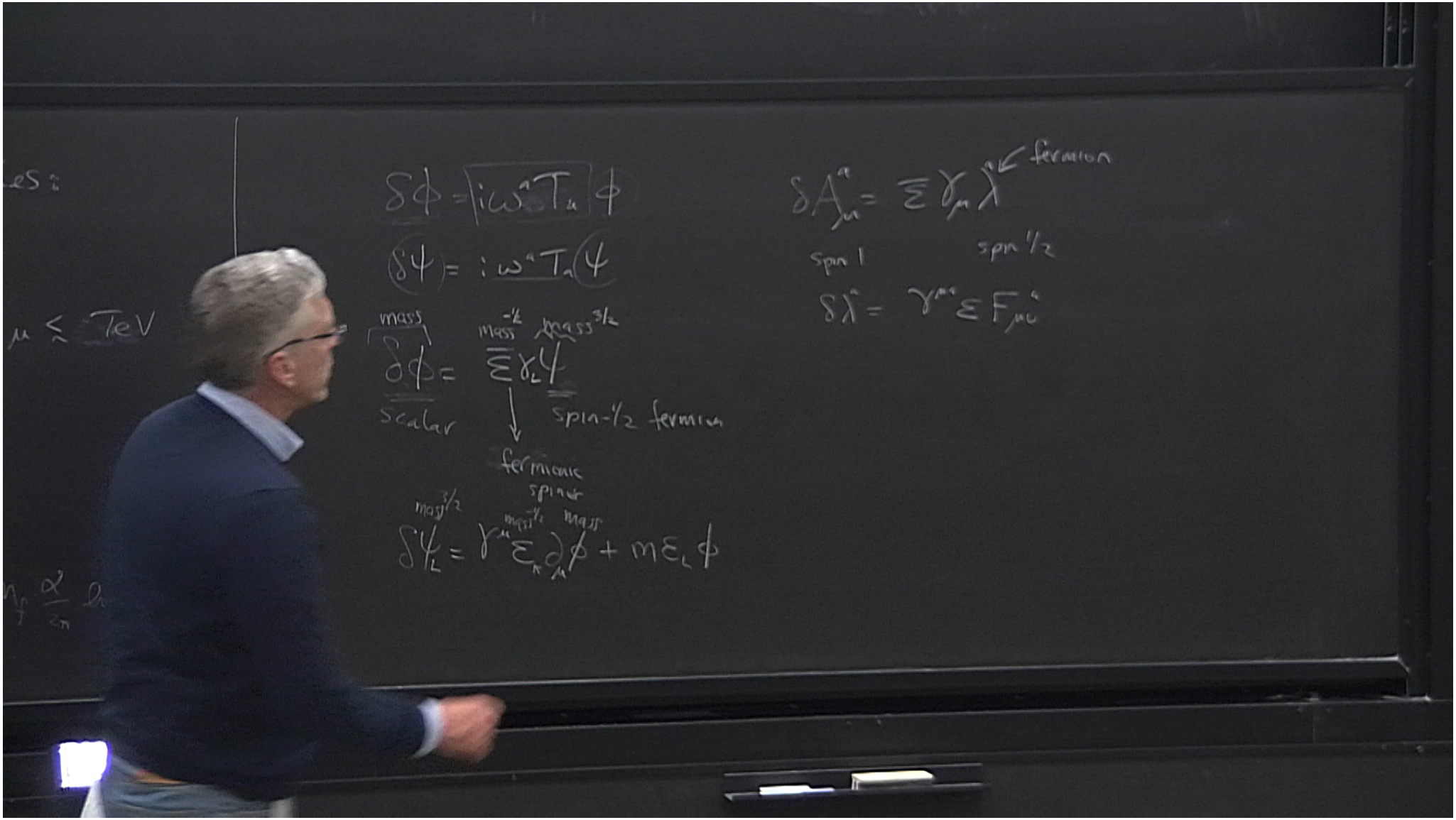

$\delta\phi = i\omega^a T_a \phi$
$(\delta\psi) = i\omega^a T_a (\psi$
$\delta\phi = \bar{\varepsilon}\gamma_L\psi$

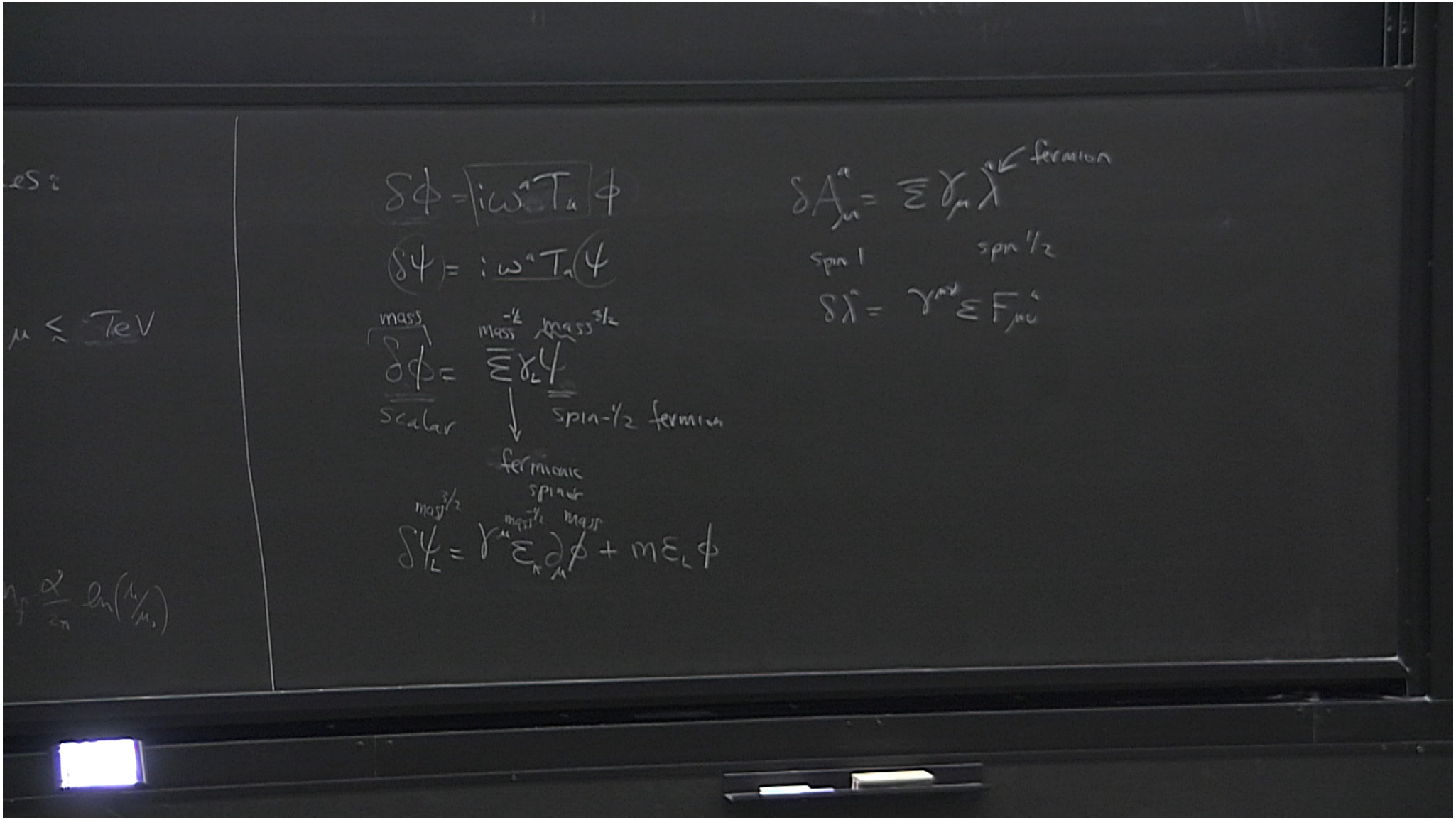

$\delta\phi = i\omega^a T_a \phi$
$(\delta\psi) = i\omega^a T_a (\psi)$
$\delta\phi = \bar{\varepsilon}\gamma_L\psi$
scalar
spin-½ fermion
fermionic spinor
TeV

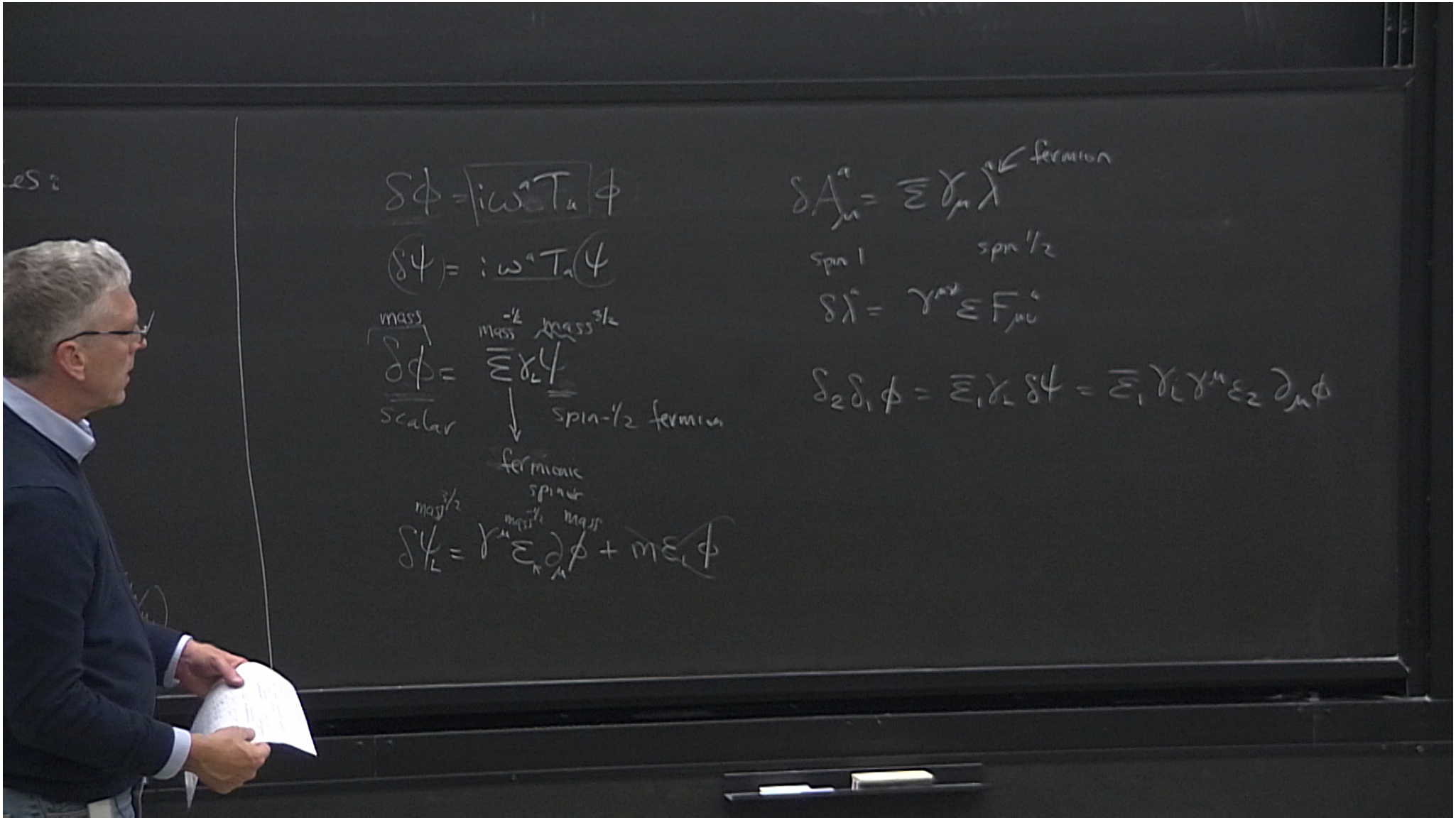

$\delta\phi = i\omega^a T_a \phi$
$(\delta\psi) = i\omega^a T_a (\psi)$
mass
mass$^{-1/2}$
mass$^{3/2}$
$\delta\phi = \bar{\varepsilon}\gamma_\mu\psi$
scalar
spin-1/2 fermion
fermionic spinor
$\mu \lesssim$ TeV

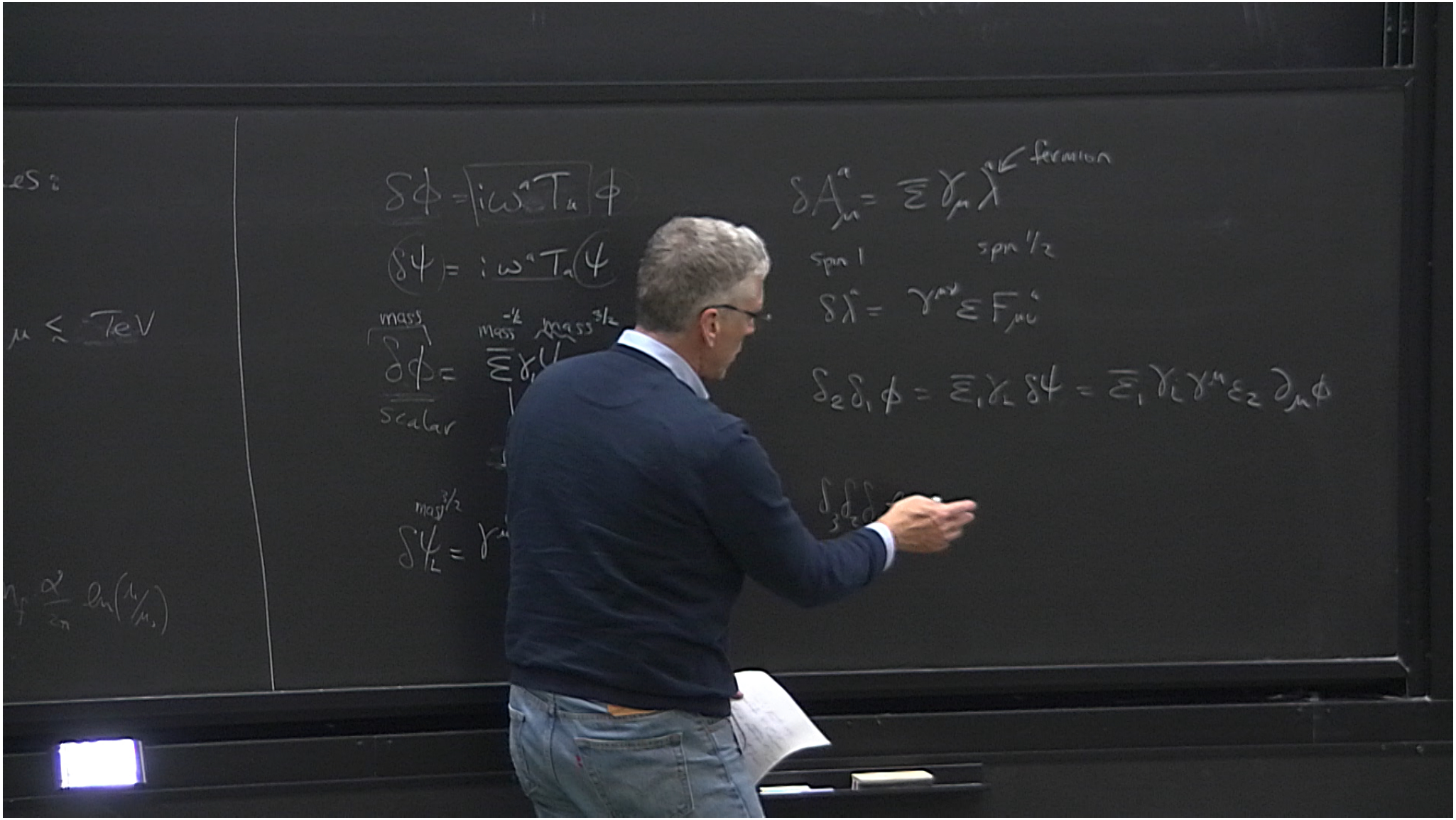

$\delta\phi = i\omega^a T_a \phi$
$\delta\psi = i\omega^a T_a \psi$
mass
mass$^{-1/2}$
mass$^{3/2}$
$\delta\phi = \bar{\varepsilon}\gamma_L\psi$
scalar
spin-1/2 fermion
fermionic spinor
mass$^{3/2}$
mass$^{-1/2}$
mass
$\delta\psi_L = \varepsilon\,\phi$

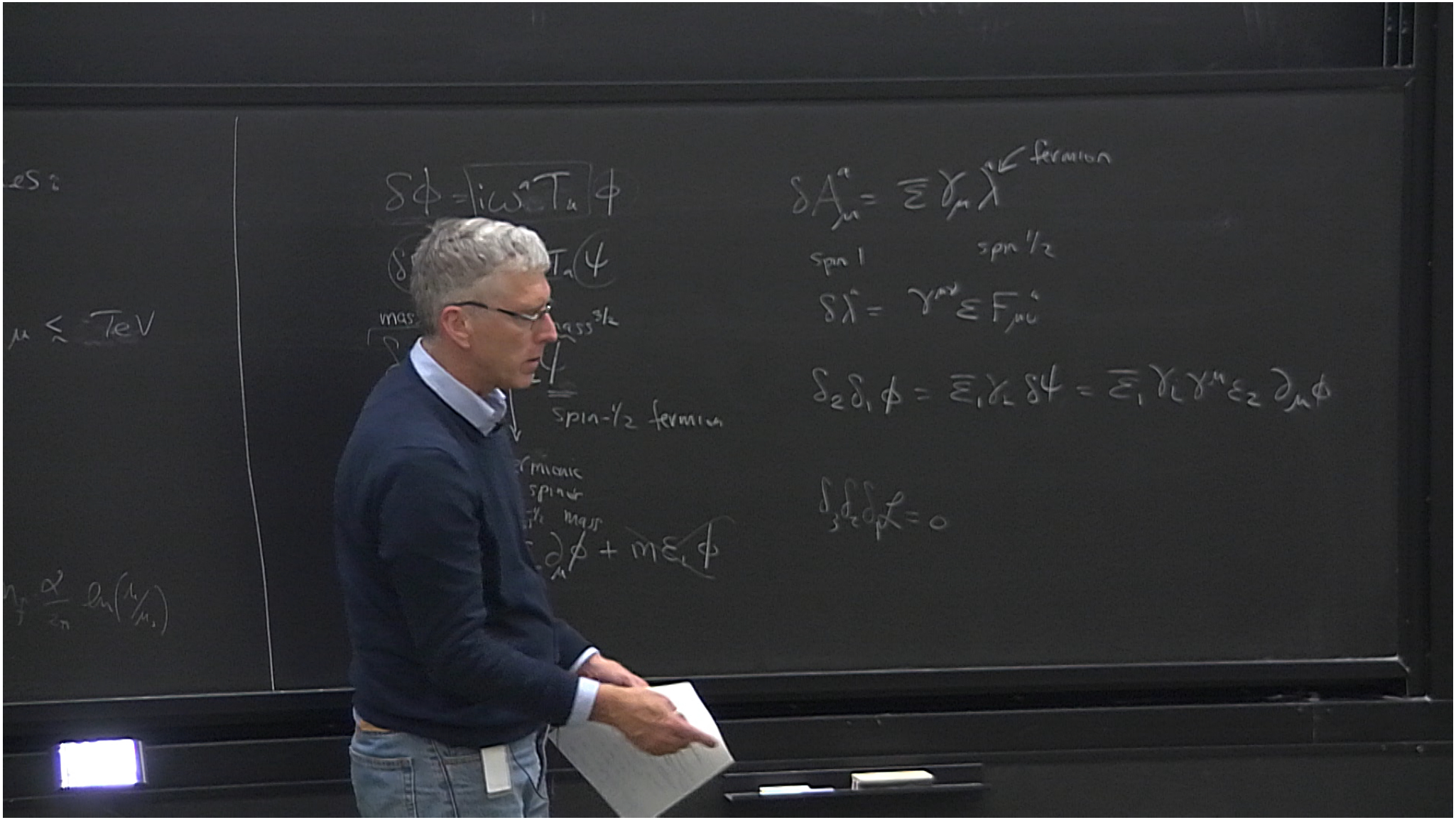

$\delta\phi = i\omega^a T_a \phi$
$(\delta\psi) = i\omega^a T_a (\psi)$
mass
mass$^{-1/2}$
mass$^{3/2}$
$\delta\phi = \bar{\varepsilon}\gamma_L\psi$
scalar
spin-1/2 fermion
fermionic spinor
mass$^{3/2}$
mass$^{1/2}$
mass
$\delta\psi_L = \gamma^\mu \varepsilon_R \partial_\mu \phi + m\varepsilon_L \phi$

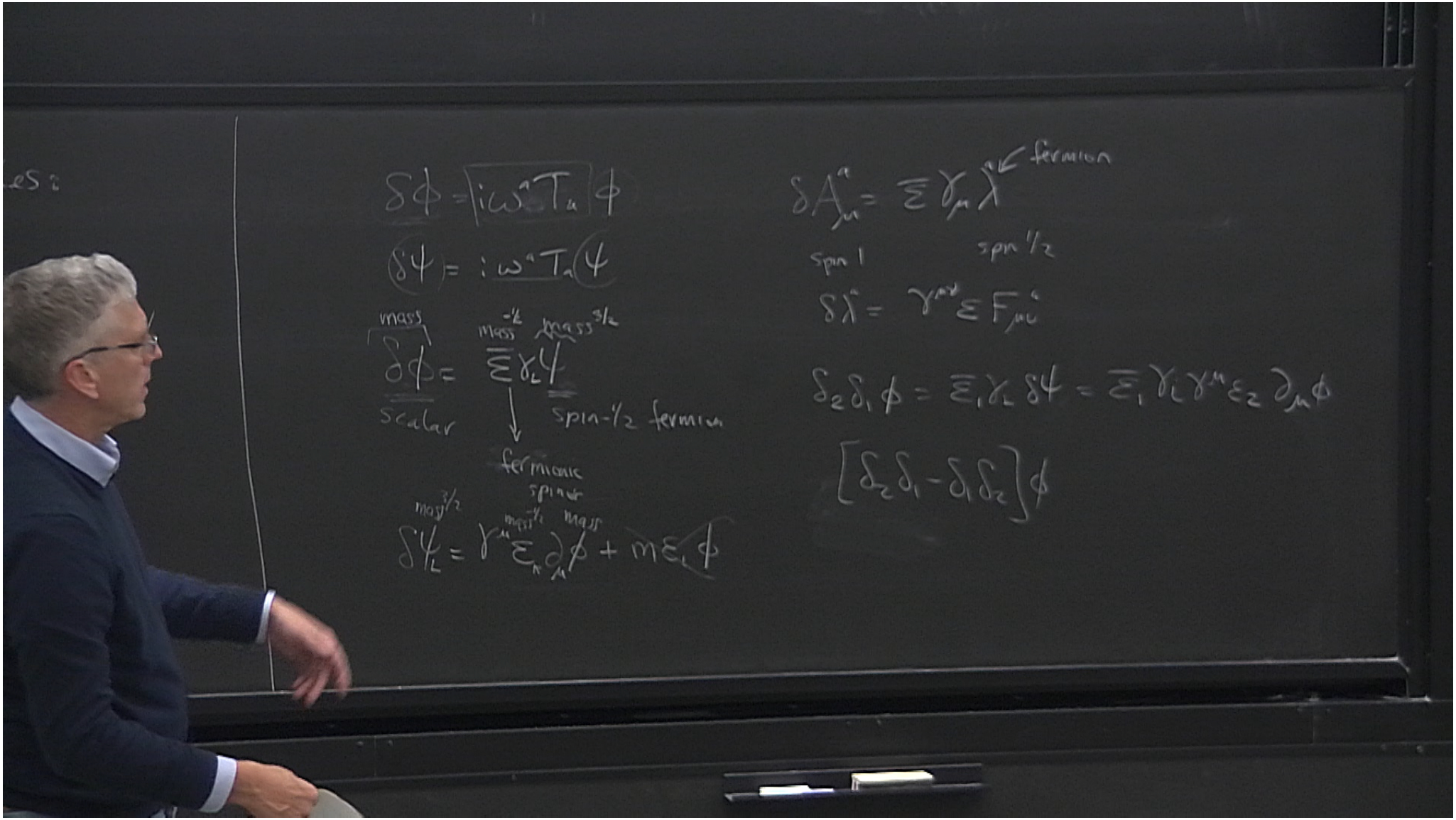

$\delta\phi = i\omega^a T_a \phi$
$(\delta\psi) = i\omega^a T_a (\psi)$
mass
$\delta\phi = \bar{\varepsilon}\gamma_L\psi$
scalar
spin-½ fermion
fermionic spinor
$\delta\psi_L = \gamma^\mu \varepsilon_R \partial_\mu \phi + m\varepsilon_L \phi$
$\delta A^a_\mu = \bar{\varepsilon}\gamma_\mu \lambda^a$
fermion
spin 1
spin ½
$\delta\lambda^a = \gamma^{\mu\nu}\varepsilon F^a_{\mu\nu}$
$\mu \lesssim$ TeV

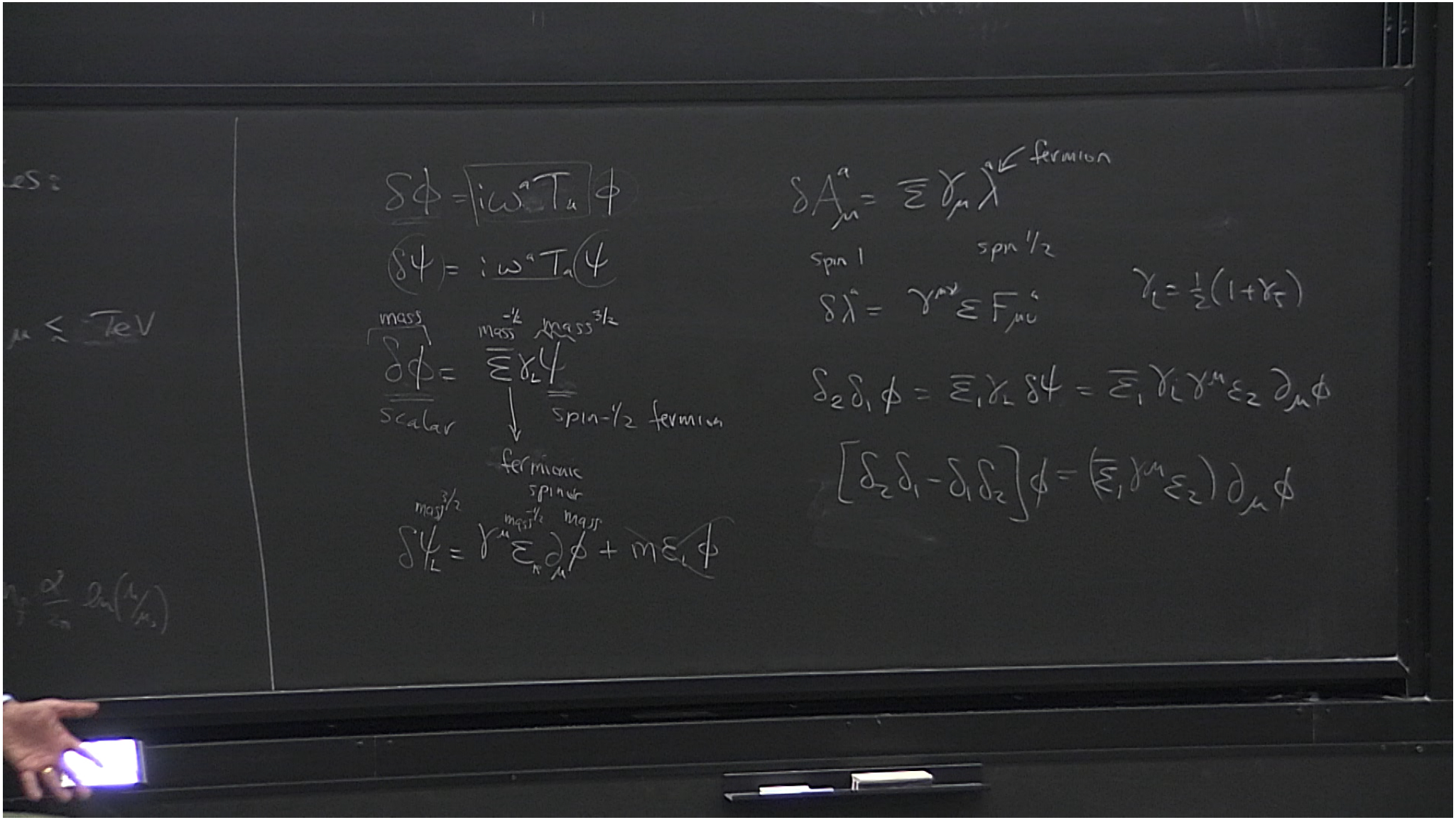

$\delta\phi = i\omega^a T_a \phi$
$\delta\psi = i\omega^a T_a \psi$
$\delta A^a_\mu = \bar{\varepsilon}\gamma_\mu \lambda^a$ ← fermion
spin 1
spin 1/2
$\delta\lambda = \gamma^{\mu\nu}\varepsilon F^a_{\mu\nu}$
mass
mass$^{-1/2}$
mass$^{3/2}$
$\delta\phi = \bar{\varepsilon}\gamma_L\psi$
scalar
spin-1/2 fermion
fermionic spinor
mass$^{3/2}$
mass$^{-1/2}$
mass
$\delta\psi_L = \gamma^\mu \varepsilon_R \partial_\mu \phi + m\varepsilon_L \phi$
$\mu \lesssim$ TeV

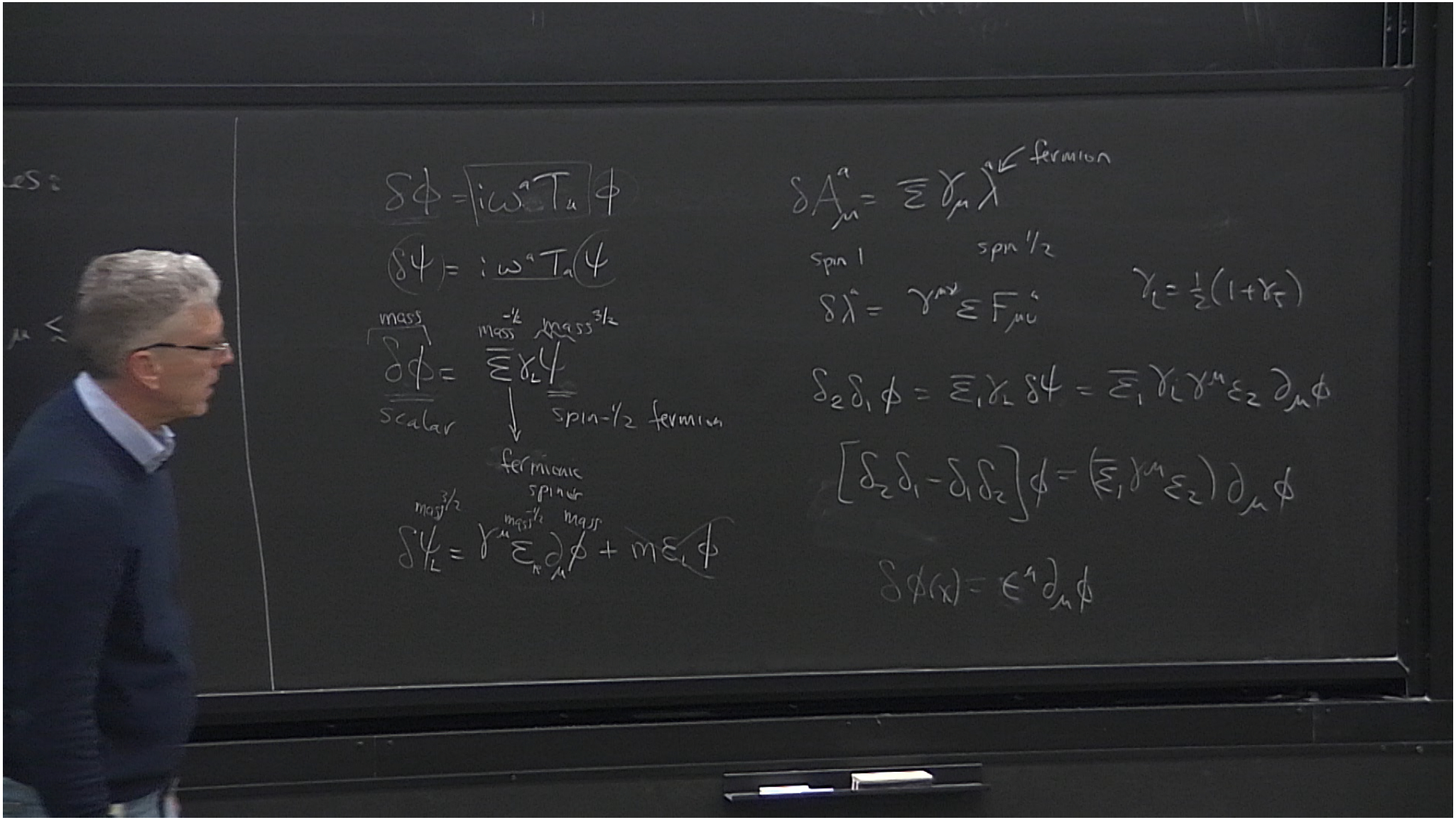

$\delta\phi = i\omega^a T_a \phi$
$(\delta\psi) = i\omega^a T_a (\psi)$
mass
mass$^{-1/2}$ mass$^{3/2}$
$\delta\phi = \bar{\varepsilon}\gamma_L\psi$
scalar
spin-1/2 fermion
fermionic spinor
mass$^{3/2}$ mass$^{-1/2}$ mass
$\delta\psi_L = \gamma^\mu \varepsilon_R \partial_\mu \phi + m\varepsilon_L \phi$
$\delta A^a_\mu = \bar{\varepsilon}\gamma_\mu \lambda^a$
fermion
spin 1
spin 1/2
$\delta\lambda^a = \gamma^{\mu\nu}\varepsilon F^a_{\mu\nu}$
$\delta_2\delta_1\phi = \bar{\varepsilon}_1\gamma_L\delta\psi = \bar{\varepsilon}_1\gamma_L\gamma^\mu\varepsilon_2\partial_\mu\phi$

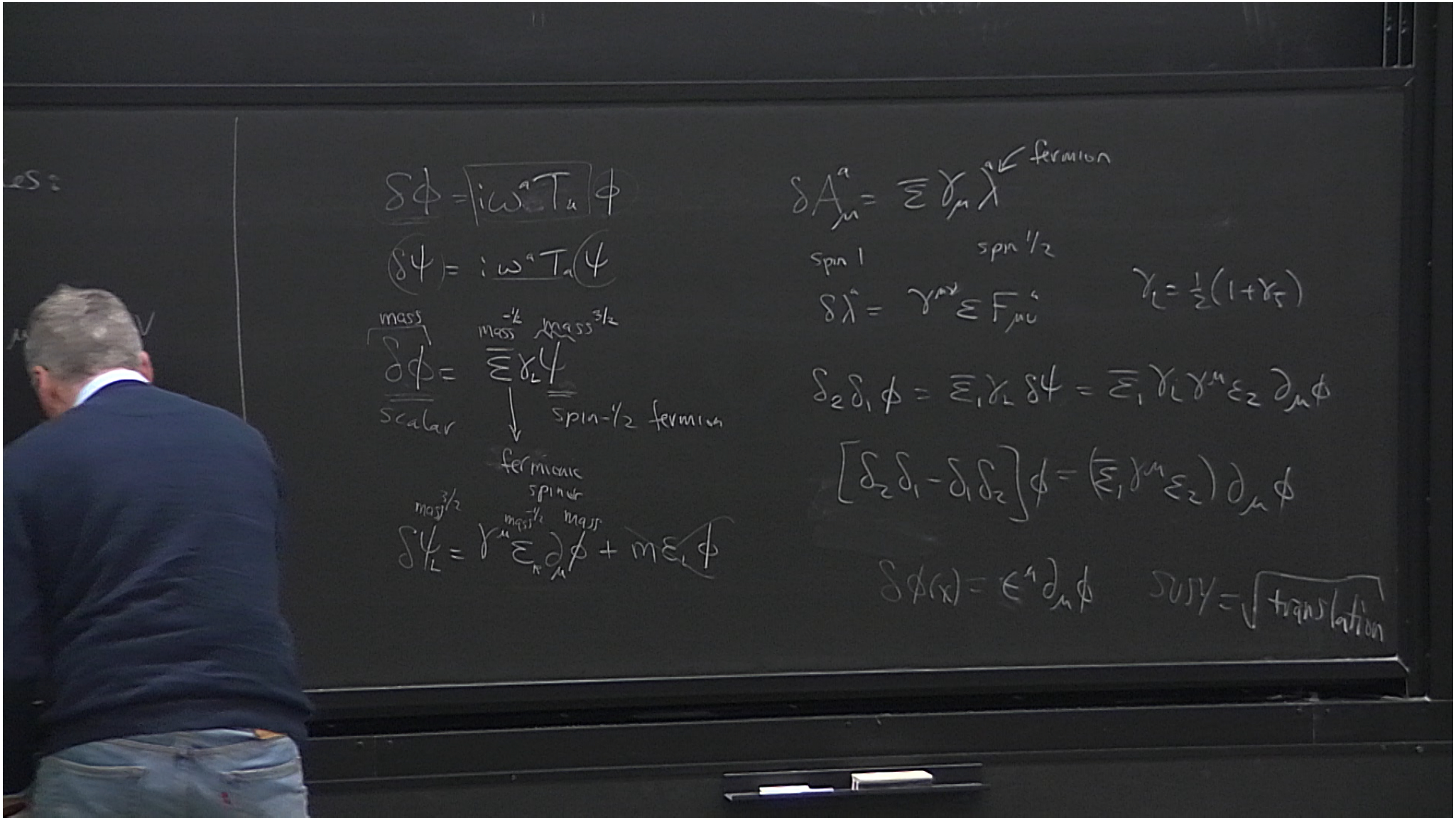

fermion
spin 1
spin 1/2
scalar

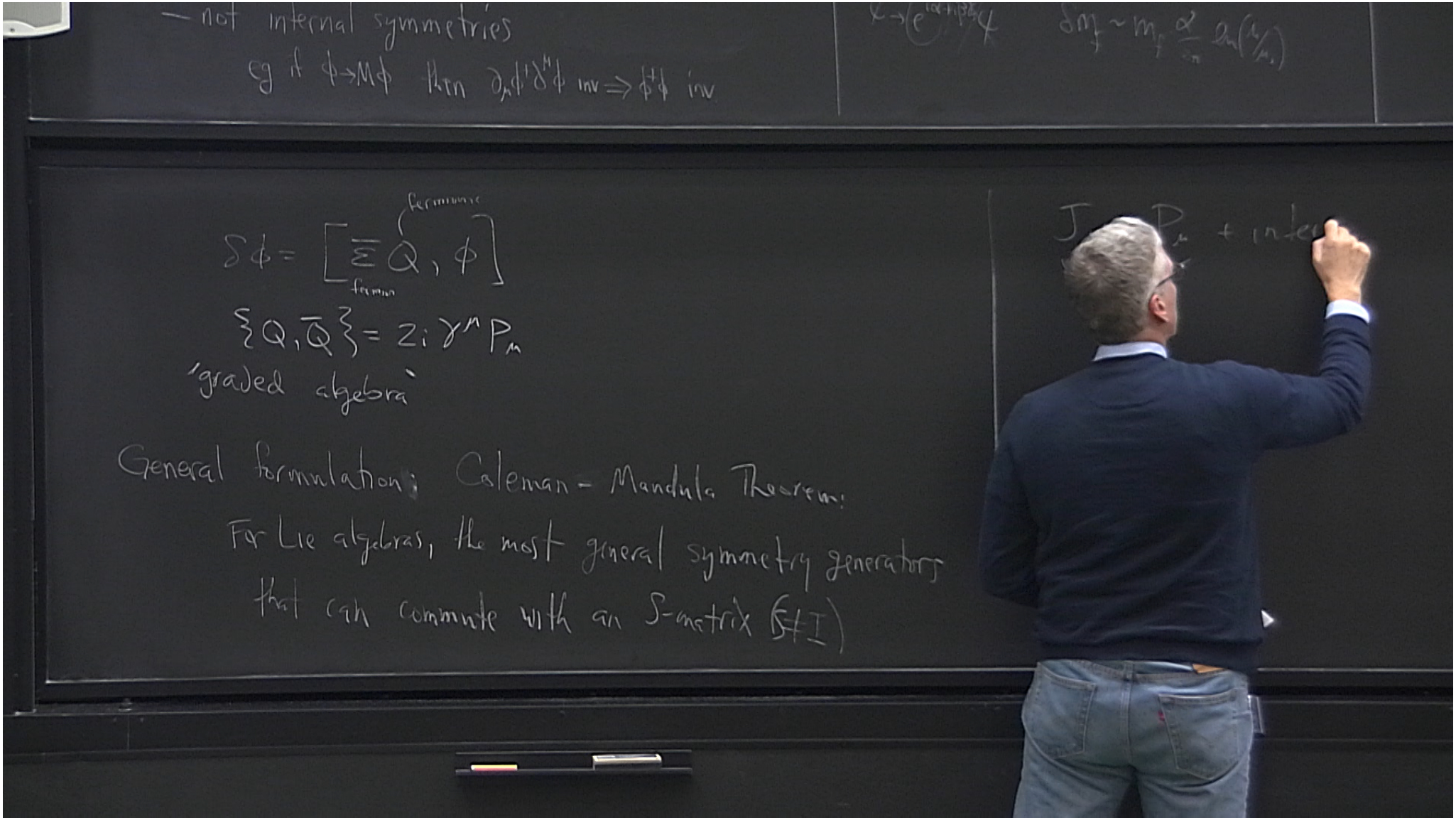

fermion
spin 1
spin 1/2
spin-1/2 fermion
mass
mass

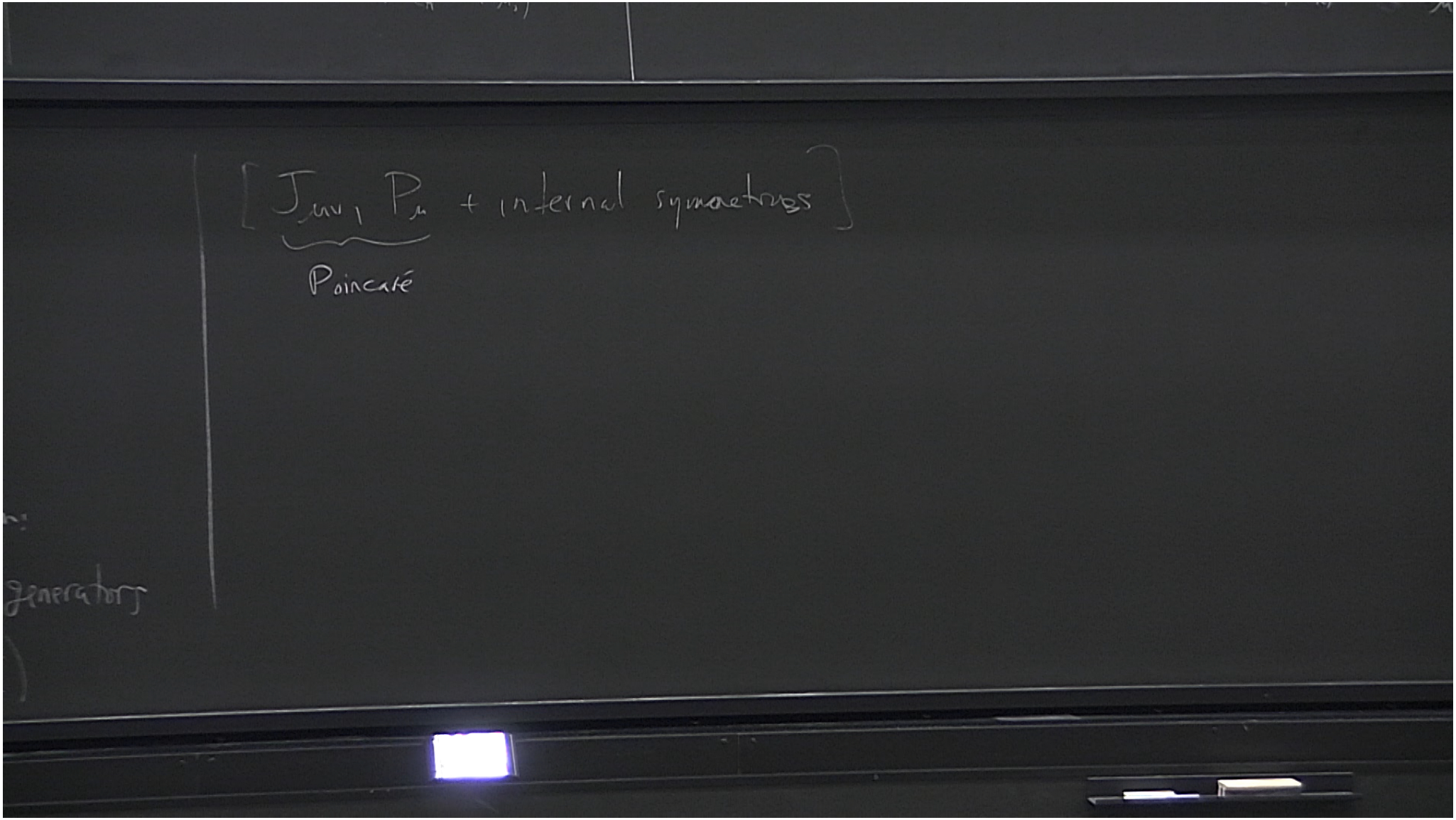

$\delta\phi = i\omega^a T_a \phi$
$\delta\psi = i\omega^a T_a \psi$
mass
mass $-\frac{1}{2}$
mass $3/2$
$\delta\phi = \bar{\varepsilon}\gamma_L\psi$
scalar
spin-$\frac{1}{2}$ fermion
fermionic spinor
mass $3/2$
mass $-\frac{1}{2}$
mass
$\delta\psi_L = \gamma^\mu\varepsilon_R\partial_\mu\phi + m\varepsilon_L\phi$
$\delta A^a_\mu = \bar{\varepsilon}\gamma_\mu\lambda^a$
fermion
spin 1
spin $1/2$
$\delta\lambda^a = \gamma^{\mu\nu}\varepsilon F^a_{\mu\nu}$
$\delta_2\delta_1\phi = \bar{\varepsilon}_1\gamma_L\delta\psi = \bar{\varepsilon}_1\gamma_L\gamma^\mu\varepsilon_2\partial_\mu\phi$
$[\delta_2\delta_1 - \delta_1\delta_2]\phi$

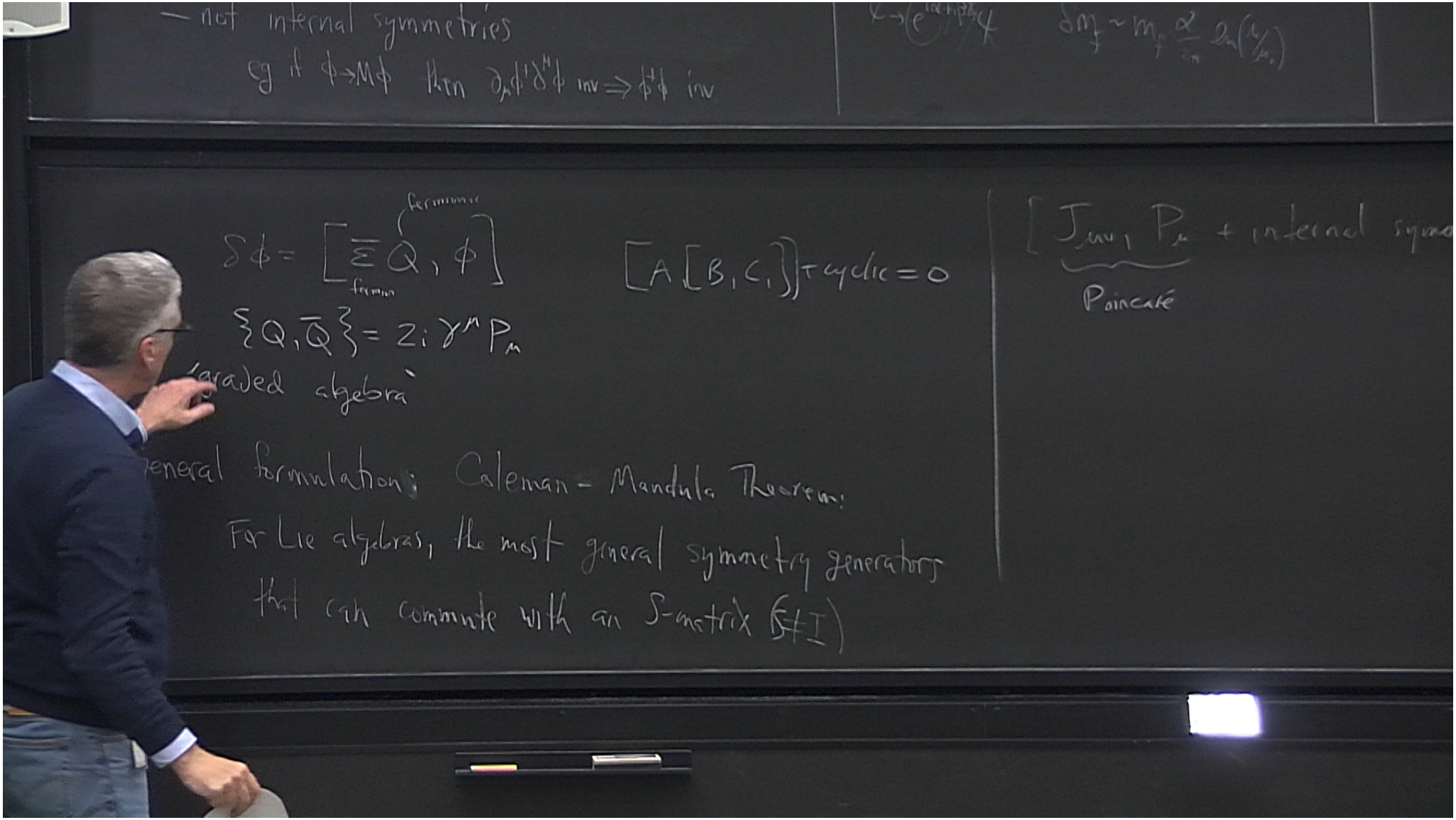

$\delta\phi = i\omega^a T_a \phi$
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mass
mass$^{-1/2}$ mass$^{3/2}$
$\delta\phi = \bar{\varepsilon}\gamma_L\psi$
scalar
spin-1/2 fermion
fermionic spinor
mass$^{3/2}$ mass$^{-1/2}$ mass
$\delta\psi_L = \gamma^\mu \varepsilon \partial_\mu \phi + m\varepsilon_L \phi$
$\delta A^a_\mu = \bar{\varepsilon}\gamma_\mu \lambda^a$
fermion
spin 1
spin 1/2
$\delta\lambda^a = \gamma^{\mu\nu}\varepsilon F^a_{\mu\nu}$
$\gamma_L = \frac{1}{2}(1+\gamma_5)$
$\delta_2\delta_1\phi = \bar{\varepsilon}_1\gamma_L\delta\psi = \bar{\varepsilon}_1\gamma_L\gamma^\mu\varepsilon_2\partial_\mu\phi$
$[\delta_2\delta_1 - \delta_1\delta_2]\phi = (\bar{\varepsilon}_1\gamma^\mu\varepsilon_2)\partial_\mu\phi$

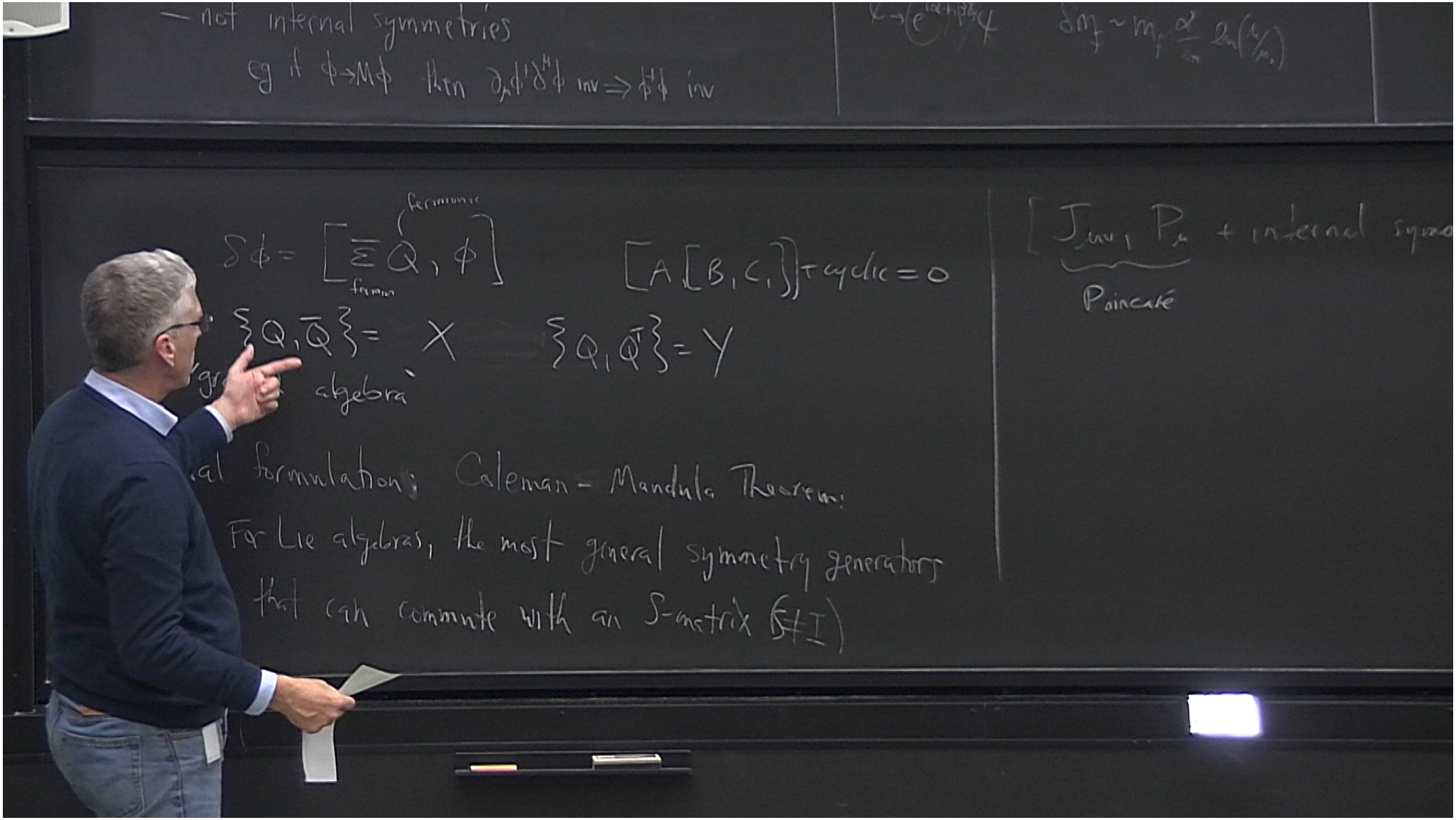

$\delta\phi = i\omega^a T_a \phi$
$(\delta\psi) = i\omega^a T_a (\psi)$
mass
$\delta\phi = \bar{\varepsilon}\gamma_L\psi$
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fermion
spin 1
spin ½
$\delta\lambda^a = \gamma^{\mu\nu}\varepsilon F^a_{\mu\nu}$
$\gamma_L = \frac{1}{2}(1+\gamma_5)$
$\delta_2\delta_1\phi = \bar{\varepsilon}_1\gamma_L\delta\psi = \bar{\varepsilon}_1\gamma_L\gamma^\mu\varepsilon_2\partial_\mu\phi$
$[\delta_2\delta_1 - \delta_1\delta_2]\phi = (\bar{\varepsilon}_1\gamma^\mu\varepsilon_2)\partial_\mu\phi$
$\delta\phi(x) = \epsilon^\mu\partial_\mu\phi$

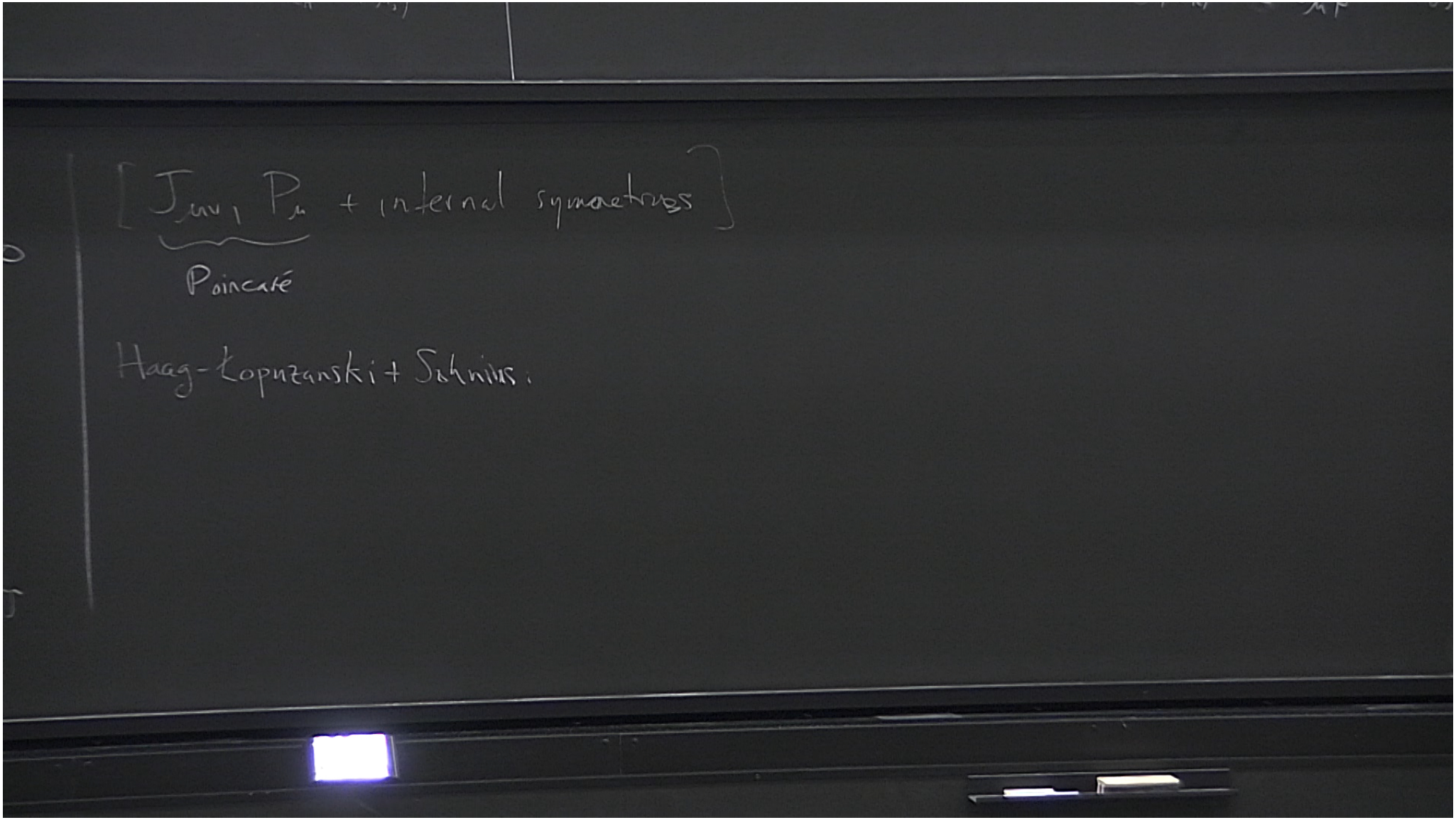

$\delta\phi = i\omega^a T_a \phi$
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mass
$\delta\phi = \bar{\varepsilon}\gamma_L\psi$
scalar
spin-½ fermion
fermionic spinor
$\delta\psi_L = \gamma^\mu \varepsilon \partial_\mu \phi + m\varepsilon_L \phi$
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fermion
spin 1
spin ½
$\delta\lambda^a = \gamma^{\mu\nu}\varepsilon F^a_{\mu\nu}$
$\gamma_L = \frac{1}{2}(1+\gamma_5)$
$\delta_2\delta_1\phi = \bar{\varepsilon}_1\gamma_L\delta\psi = \bar{\varepsilon}_1\gamma_L\gamma^\mu\varepsilon_2\partial_\mu\phi$
$[\delta_2\delta_1 - \delta_1\delta_2]\phi = (\bar{\varepsilon}_1\gamma^\mu\varepsilon_2)\partial_\mu\phi$
$\delta\phi(x) = \epsilon^\mu\partial_\mu\phi$
SUSY = "translation"

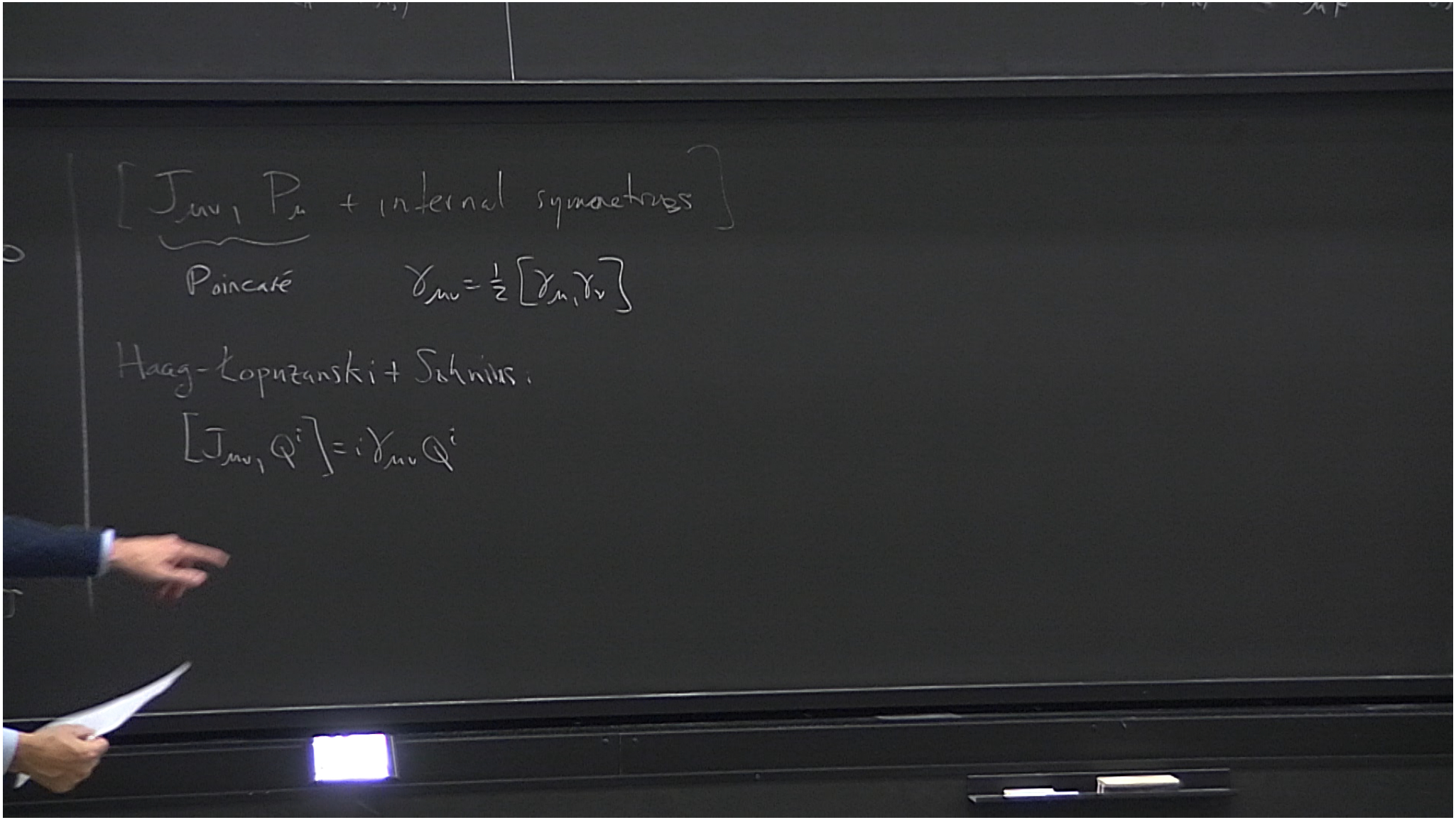

— not internal symmetries
eg if $\phi \to M\phi$ then $\partial_\mu \phi^\dagger \gamma^\mu \phi$ inv $\Rightarrow \phi^\dagger \phi$ inv
$\delta\phi = [\bar{\epsilon} Q, \phi]$
fermionic
fermion
$\{Q, \bar{Q}\} = 2i \gamma^\mu P_\mu$
'graded algebra'
General formulation: Coleman - Mandula Theorem:
For Lie algebras, the most general symmetry generators
that can commute with an S-matrix ($S \neq I$)


$[J_{\mu\nu}, P_\mu$ + internal symmetries $]$
Poincaré
generators


— not internal symmetries
eg if $\phi \to M\phi$ then $\partial_\mu \phi^\dagger \gamma^\mu \phi$ inv $\Rightarrow \phi^\dagger \phi$ inv
$\delta\phi = [\bar{\varepsilon} Q, \phi]$
fermionic
fermion
$\{Q, \bar{Q}\} = 2i \gamma^\mu P_\mu$
graded algebra
$[A,[B,C]] + \text{cyclic} = 0$
$[J_{\mu\nu}, P_\mu]$ + internal symm
Poincaré
eneral formulation: Coleman - Mandula Theorem:
For Lie algebras, the most general symmetry generators
that can commute with an S-matrix (S ≠ I)


— not internal symmetries
eg if $\phi \to M\phi$ then $\partial_\mu \phi^\dagger \gamma^\mu \phi$ inv $\Rightarrow \phi^\dagger \phi$ inv
$\delta\phi = [\bar{\varepsilon} Q, \phi]$
fermionic
fermion
$[A,[B,C]] + \text{cyclic} = 0$
$\{Q, \bar{Q}\} = X$
$\{Q, Q^\dagger\} = Y$
algebra
[$J_{\mu\nu}$, P_μ] + internal symm
Poincaré
al formulation; Coleman - Mandula Theorem:
For Lie algebras, the most general symmetry generators
that can commute with an S-matrix (S ≠ I)


[$J_{\mu\nu}$, P_μ + internal symmetries]
Poincaré
Haag-Łopuszański + Sohnius.


$[J_{\mu\nu}, P_\mu$ + internal symmetries$]$
Poincaré
$\gamma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$
Haag-Łopuszański + Sohnius:
$[J_{\mu\nu}, Q^i] = i\gamma_{\mu\nu} Q^i$

$[J_{\mu\nu}, P_\mu$ + internal symmetries $]$

Poincaré $\qquad \gamma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$

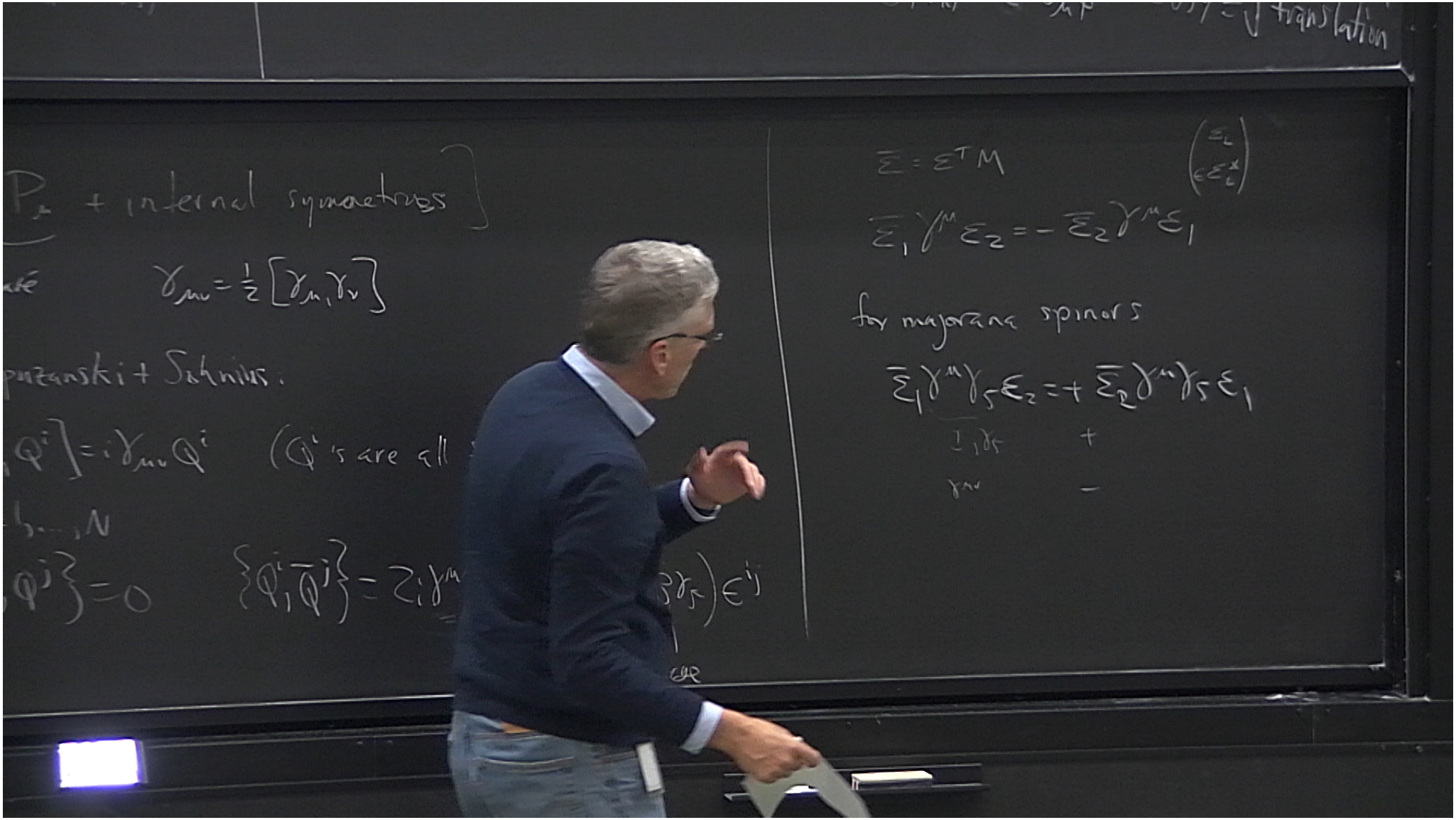
Haag-Lopuszanski + Sohnius.

$[J_{\mu\nu}, Q^i] = i\gamma_{\mu\nu} Q^i$ (Q^i's are all spinors)

$i = 1, \ldots, N$

$\{Q^i, Q^j\} = 0 \qquad \{Q^i, \bar{Q}^j\} = 2i\gamma^\mu P_\mu \delta^{ij} + (A + B\gamma_5)\epsilon^{ij}$

Central Charge


+ internal symmetries
$\gamma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$
$\bar{\varepsilon} = \varepsilon^T M$
$\bar{\varepsilon}_1 \gamma^\mu \varepsilon_2 = -\bar{\varepsilon}_2 \gamma^\mu \varepsilon_1$
for majorana spinors
$\bar{\varepsilon}_1 \gamma^\mu \gamma_5 \varepsilon_2 = +\bar{\varepsilon}_2 \gamma^\mu \gamma_5 \varepsilon_1$
translation

$[J_{\mu\nu}, P_\mu$ + internal symmetries$]$

Poincaré $\qquad \gamma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$

Haag-Lopuszanski + Sohnius.

$[J_{\mu\nu}, Q^i] = i\gamma_{\mu\nu} Q^i$ (Q^i's are all spinors)

$i = 1, \ldots, N$ ← "extended susy"

$\{Q^i, Q^j\} = 0 \qquad \{Q^i, \bar{Q}^j\} = 2i\gamma^\mu P_\mu \delta^{ij} + (A + B\gamma_5)\epsilon^{ij}$

Central Charge

$0 \leq N \leq 8$

$\bar{\varepsilon} = \varepsilon^T M$

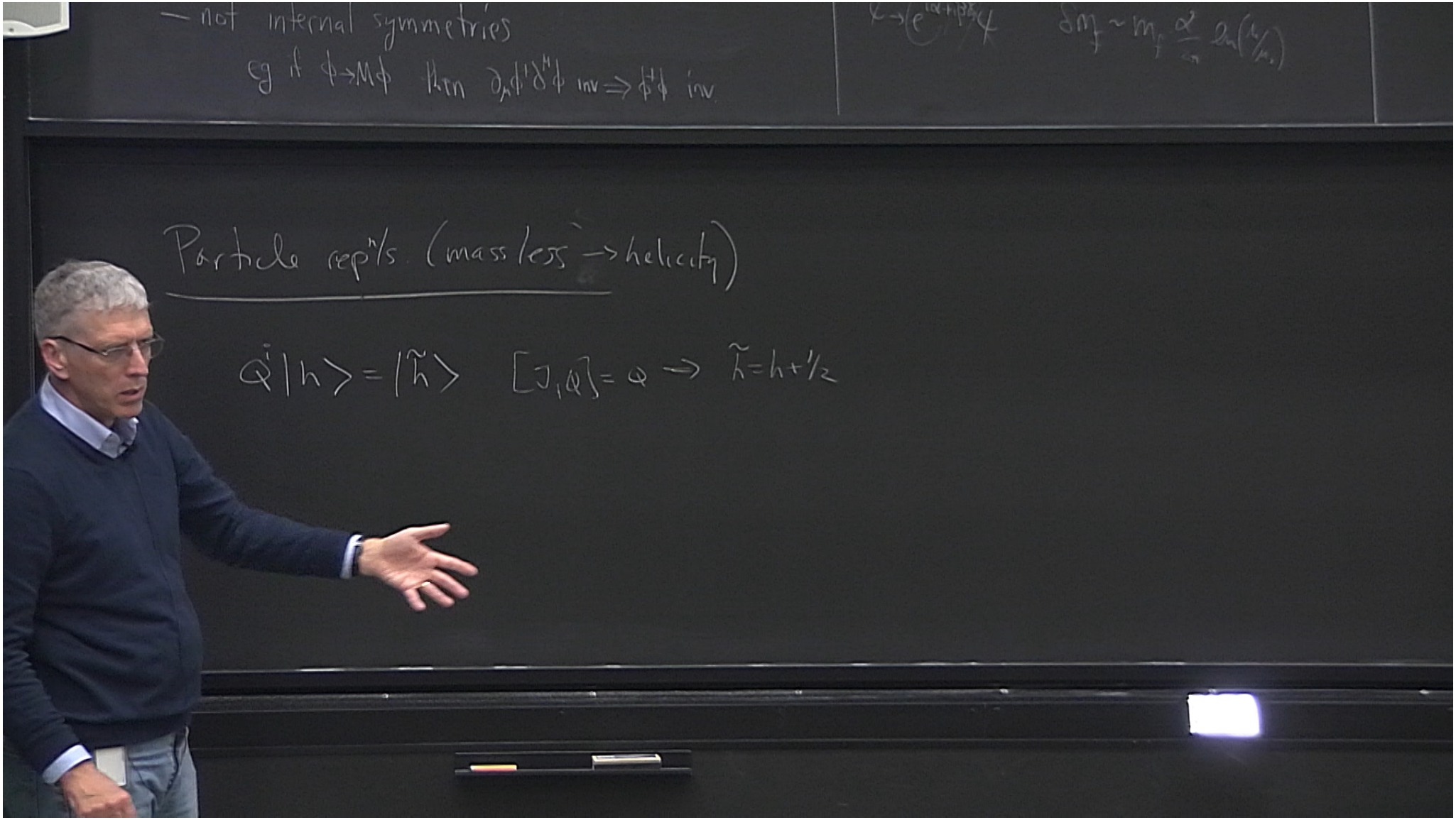
$\bar{\varepsilon}_1 \gamma^\mu \varepsilon_2 =$

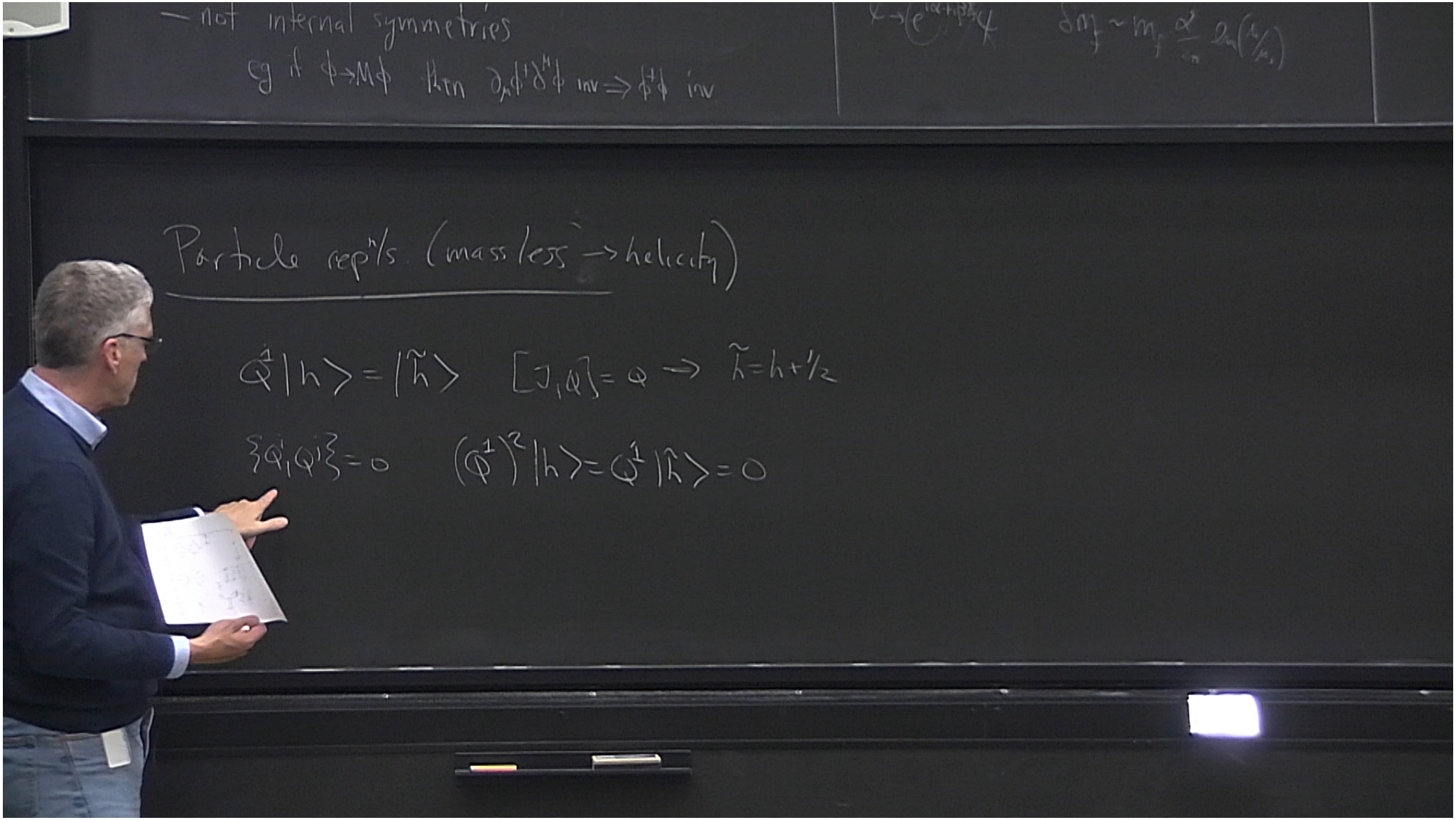
for majorana

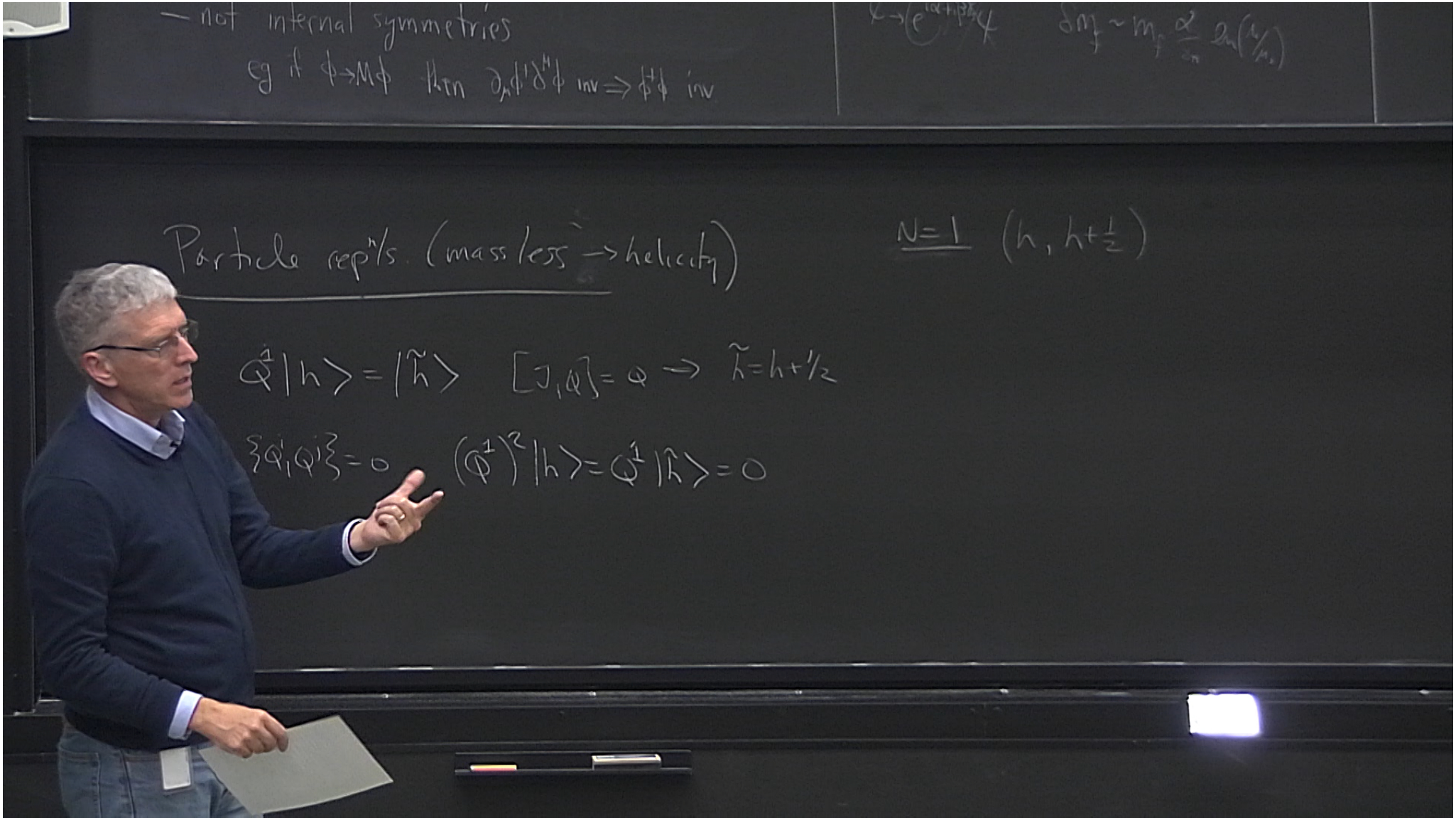
$\bar{\varepsilon}_1 \gamma^\mu \gamma_5 \varepsilon_2$

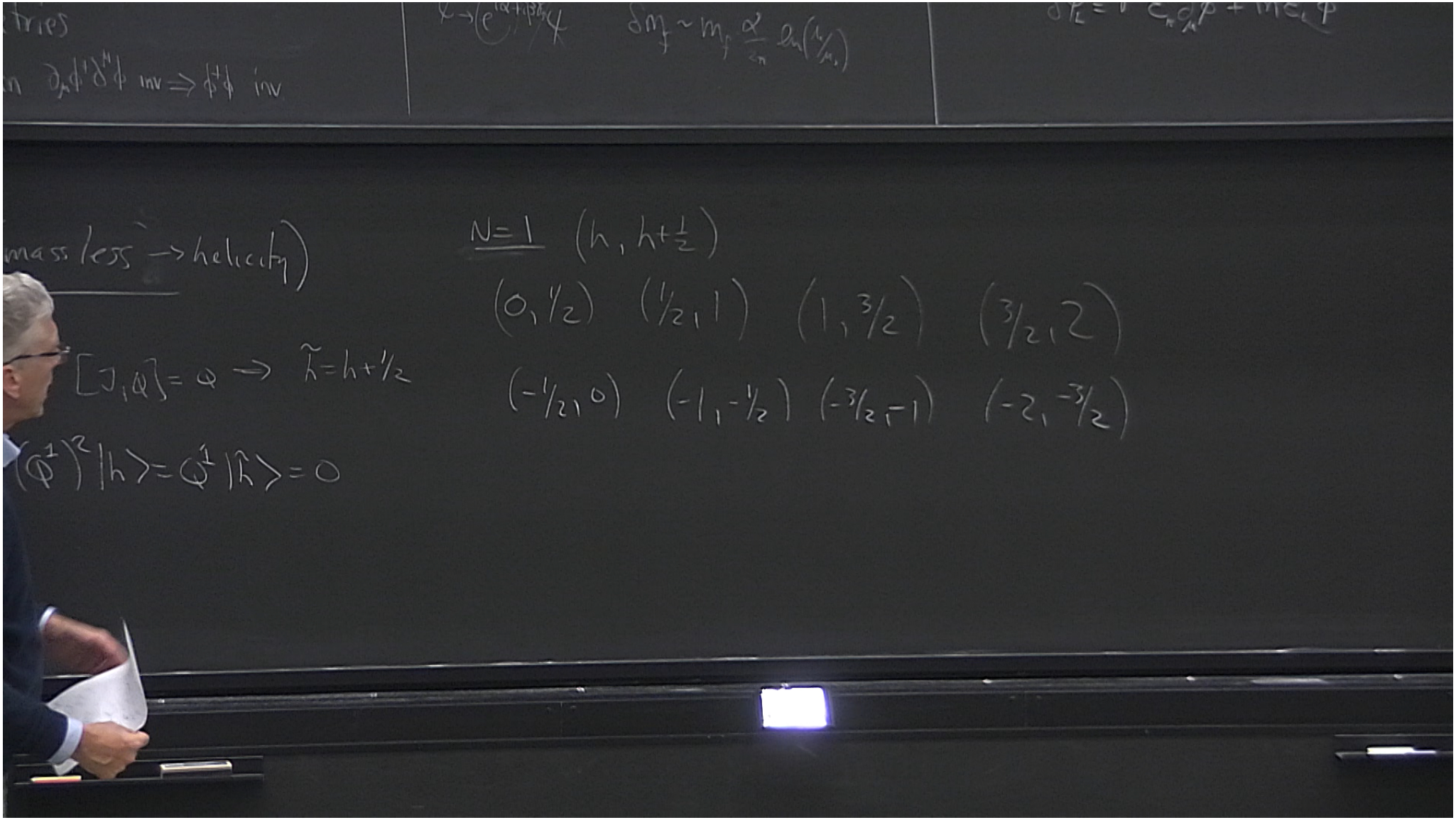
γ_5

$\gamma_{\mu\nu}$


– not internal symmetries
eg if $\phi \to M\phi$ then $\partial_\mu \phi^\dagger \gamma^\mu \phi$ inv $\Rightarrow \phi^\dagger \phi$ inv
Particle rep'ns (massless → helicity)
$Q^i|h\rangle = |\tilde{h}\rangle \quad [J, Q] = Q \to \tilde{h} = h + 1/2$


– not internal symmetries
Particle rep'ns (massless → helicity)


– not internal symmetries
eg if $\phi \to M\phi$ then $\partial_\mu \phi^\dagger \bar{\sigma}^\mu \phi$ inv $\Rightarrow \phi^\dagger \phi$ inv
Particle rep'ns (massless → helicity)
N=1 $(h, h+\frac{1}{2})$
$Q^1|h\rangle = |\tilde{h}\rangle$ $[J, Q] = Q \to \tilde{h} = h + 1/2$
$\{Q^i, Q^j\} = 0$ $(Q^1)^2|h\rangle = Q^1|\tilde{h}\rangle = 0$


tries
$\partial_\mu\phi^\dagger\partial^\mu\phi$ inv $\Rightarrow \phi^\dagger\phi$ inv
$\delta m_f \sim m_f \frac{\alpha}{\pi} \ln(\Lambda/\mu)$
mass less → helicity
$[J, Q] = Q \rightarrow \tilde{h} = h + 1/2$
$(Q^1)^2|h\rangle = Q^1|\tilde{h}\rangle = 0$
N=1 $(h, h+\frac{1}{2})$
$(0, 1/2)$ $(1/2, 1)$ $(1, 3/2)$ $(3/2, 2)$
$(-1/2, 0)$ $(-1, -1/2)$ $(-3/2, -1)$ $(-2, -3/2)$

tries

$\partial_\mu \phi^\dagger \partial^\mu \phi$ inv $\Rightarrow \phi^\dagger \phi$ inv

$\phi \to e^{iq\alpha + ip\beta x}\phi$ $\quad \delta m_f \sim m_f \frac{\alpha}{\pi} \ln\left(\frac{1}{\mu_0}\right)$

$\delta \mathcal{L} = \bar{\psi} \gamma^\mu \partial_\mu \psi + m \bar{\psi} \psi$

massless $\to$ helicity)

$[J, Q] = q \to \tilde{h} = h + 1/2$

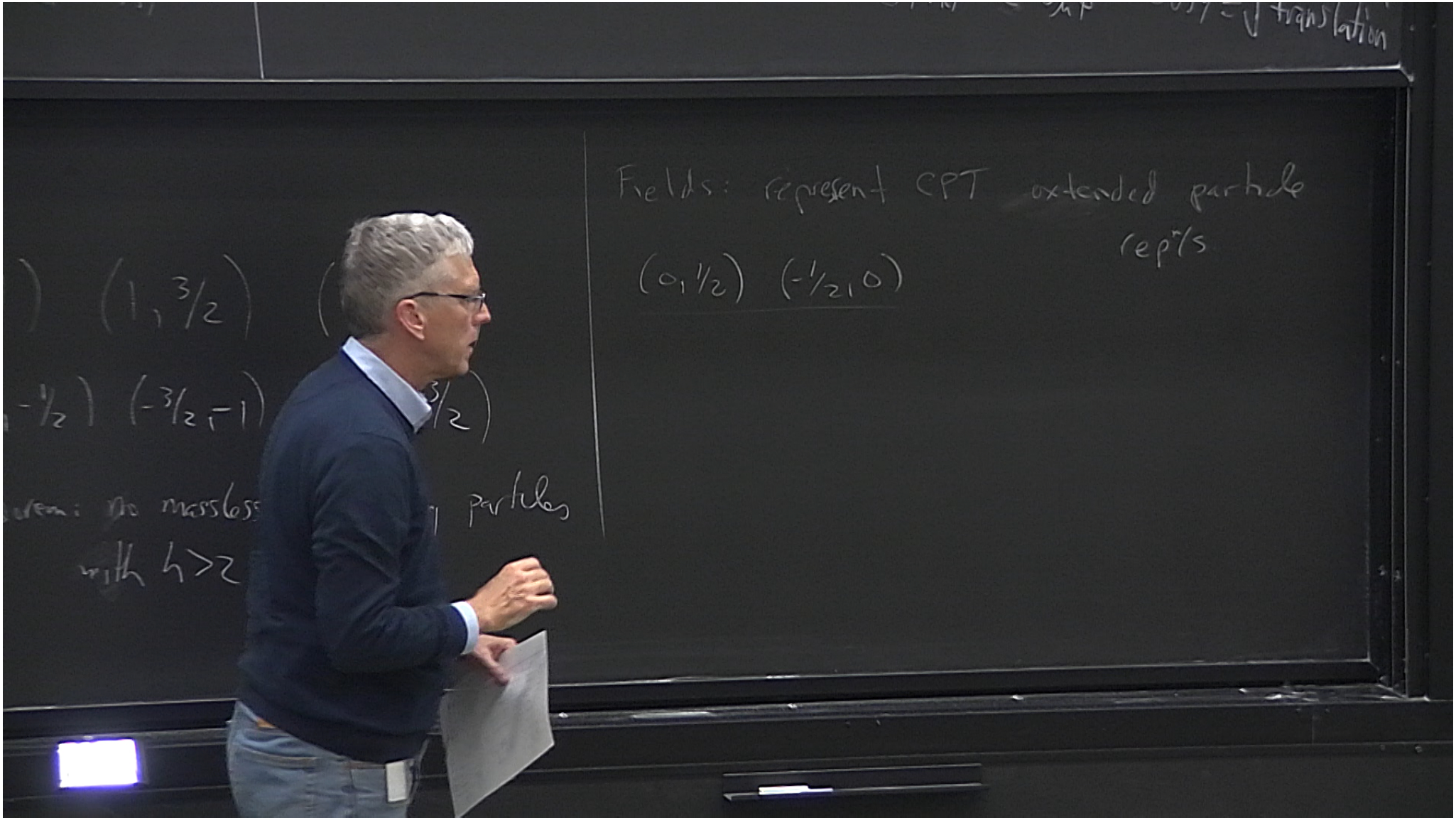
$(Q^1)^2 |h\rangle = Q^1 |\tilde{h}\rangle = 0$

N=1 $(h, h+\frac{1}{2})$

$(0, 1/2)$ $(1/2, 1)$ $(1, 3/2)$ $(3/2, 2)$

$(-1/2, 0)$ $(-1, -1/2)$ $(-3/2, -1)$ $(-2, -3/2)$

Weinberg-Witten theorem: no massless interacting particles with $h > 2$


translation
Fields: represent CPT extended particle
reps
(0, 1/2) (-1/2, 0)
(1, 3/2)
-1/2)
(-3/2, -1)
3/2)
particles
with h>2

translation

$(1, 3/2)$ $(3/2, 2)$

$(-1/2)$ $(-3/2, -1)$ $(-2, -3/2)$

...rem: no massless interacting particles with $h > 2$

Fields: represent CPT extended particle rep"s

$(0, 1/2)$ $(-1/2, 0)$

2 real scalars (or one complex scalar)

$1/2$ spinor

translation

$(1, 3/2)$ $(3/2, 2)$

$(-1/2)$ $(-3/2, -1)$ $(-2, -3/2)$

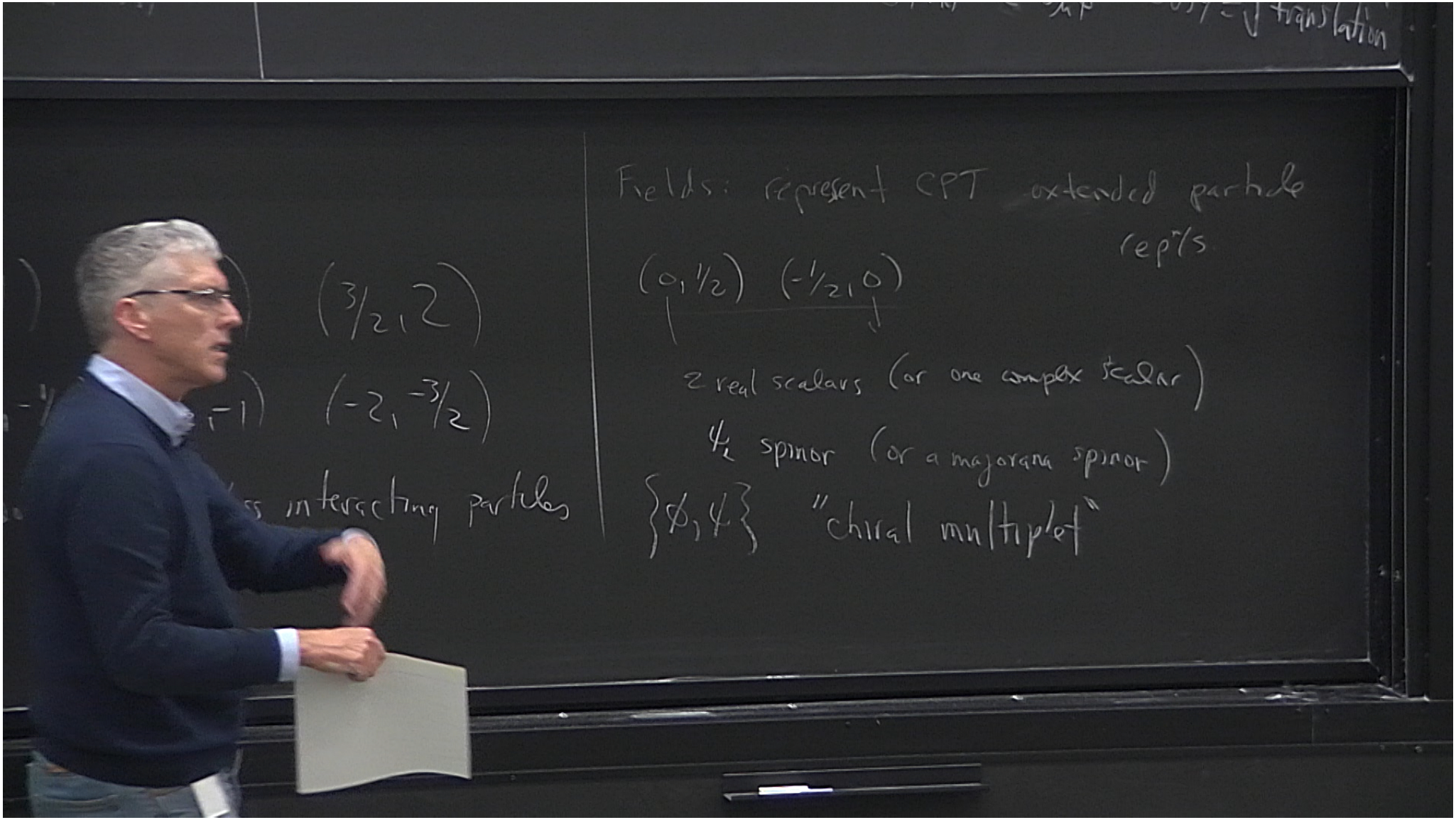
...rem: No massless interacting particles with $4 > 2$

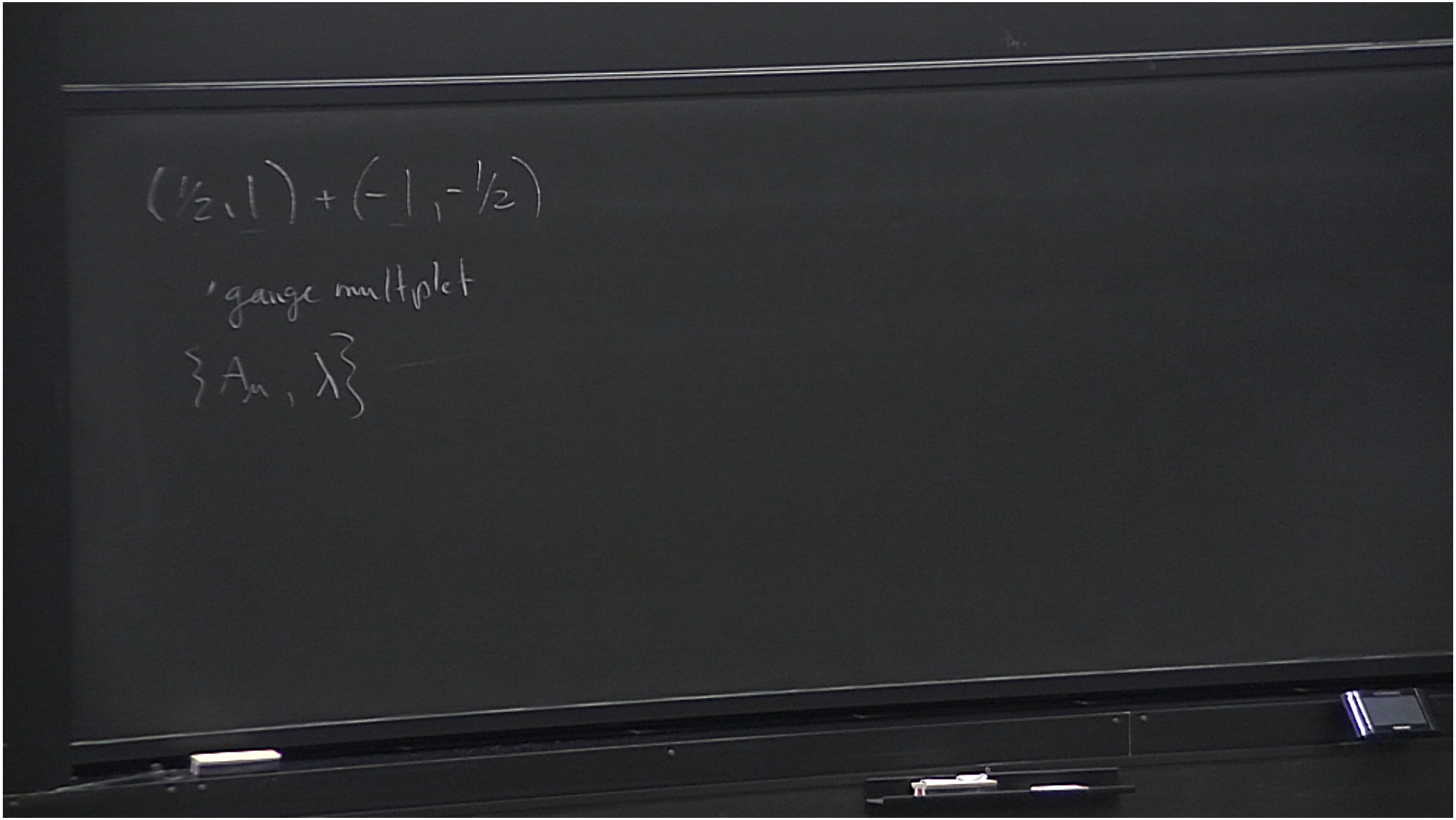
Fields: represent CPT extended particle reps

$(0, 1/2)$ $(-1/2, 0)$

2 real scalars (or one complex scalar)

$1/2$ spinor (or a majorana spinor)


translation
$(3/2, 2)$
$(-2, -3/2)$
interacting particles
Fields: represent CPT extended particle reps
$(0, 1/2)$ $(-1/2, 0)$
2 real scalars (or one complex scalar)
$1/2$ spinor (or a majorana spinor)
$\{\phi, \psi\}$ "chiral multiplet"


(½, 1) + (−1, −½)
'gauge multiplet
{A_μ, λ}

$(\frac{1}{2}, 1) + (-1, -\frac{1}{2})$

'gauge multiplet

$\{A^a_\mu, \lambda^a\}$ both A^a_μ, λ^a will be in the adjoint if $[G, Q] = 0$

$(1/2, \underline{1}) + (\underline{-1}, -1/2)$

'gauge multiplet

$\{A^a_\mu, \lambda^a\}$ both A^a_μ, λ^a will be in the adjoint if $[G, Q] = 0$

$(3/2, 2) + (-2, -3/2)$: $g_{\mu\nu}, \psi_\mu$

$(1/2, 1) + (-1, -1/2)$

'gauge multiplet

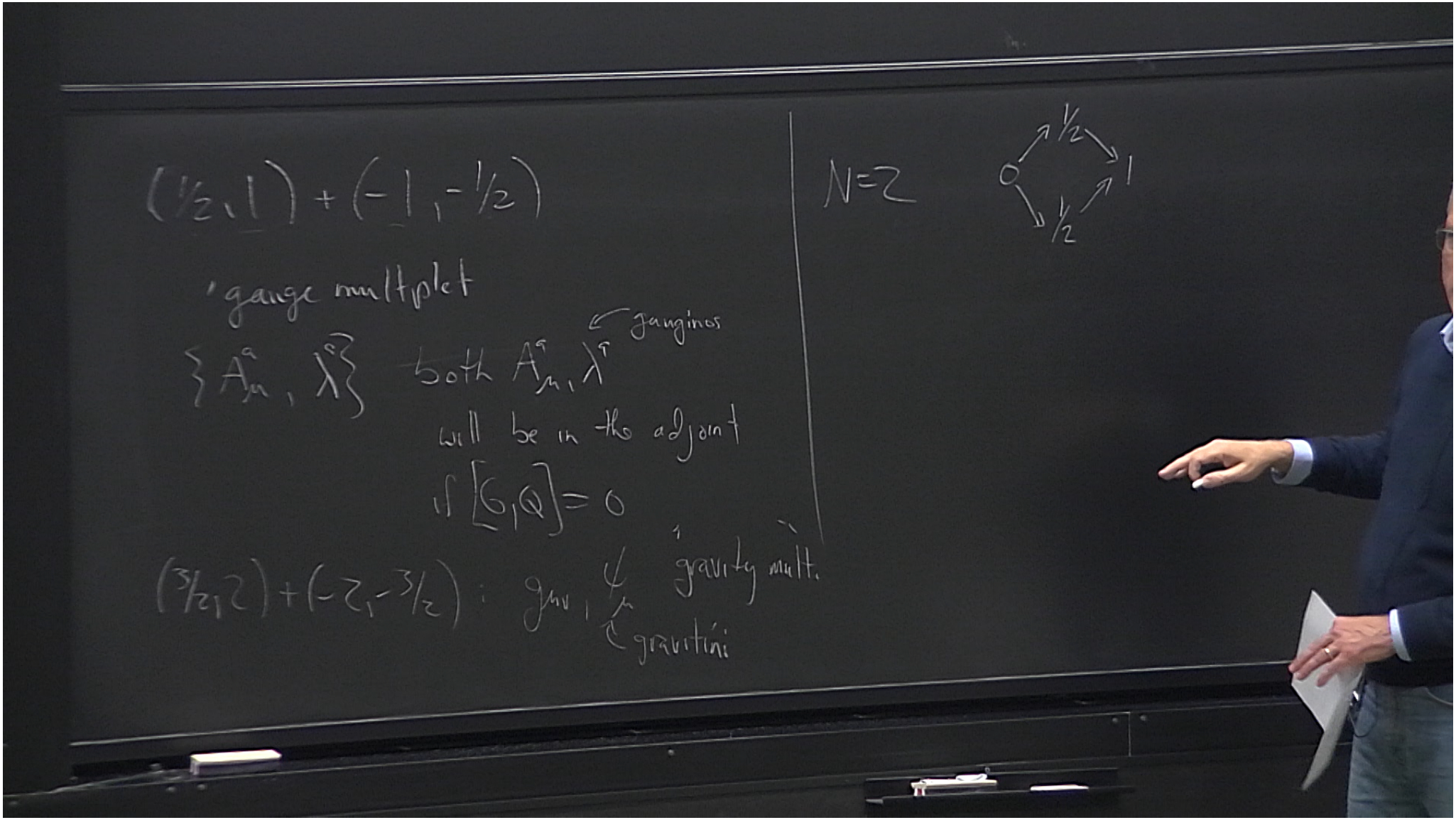
$\{A^a_\mu, \lambda^a\}$ both A^a_μ, λ^a ← gauginos

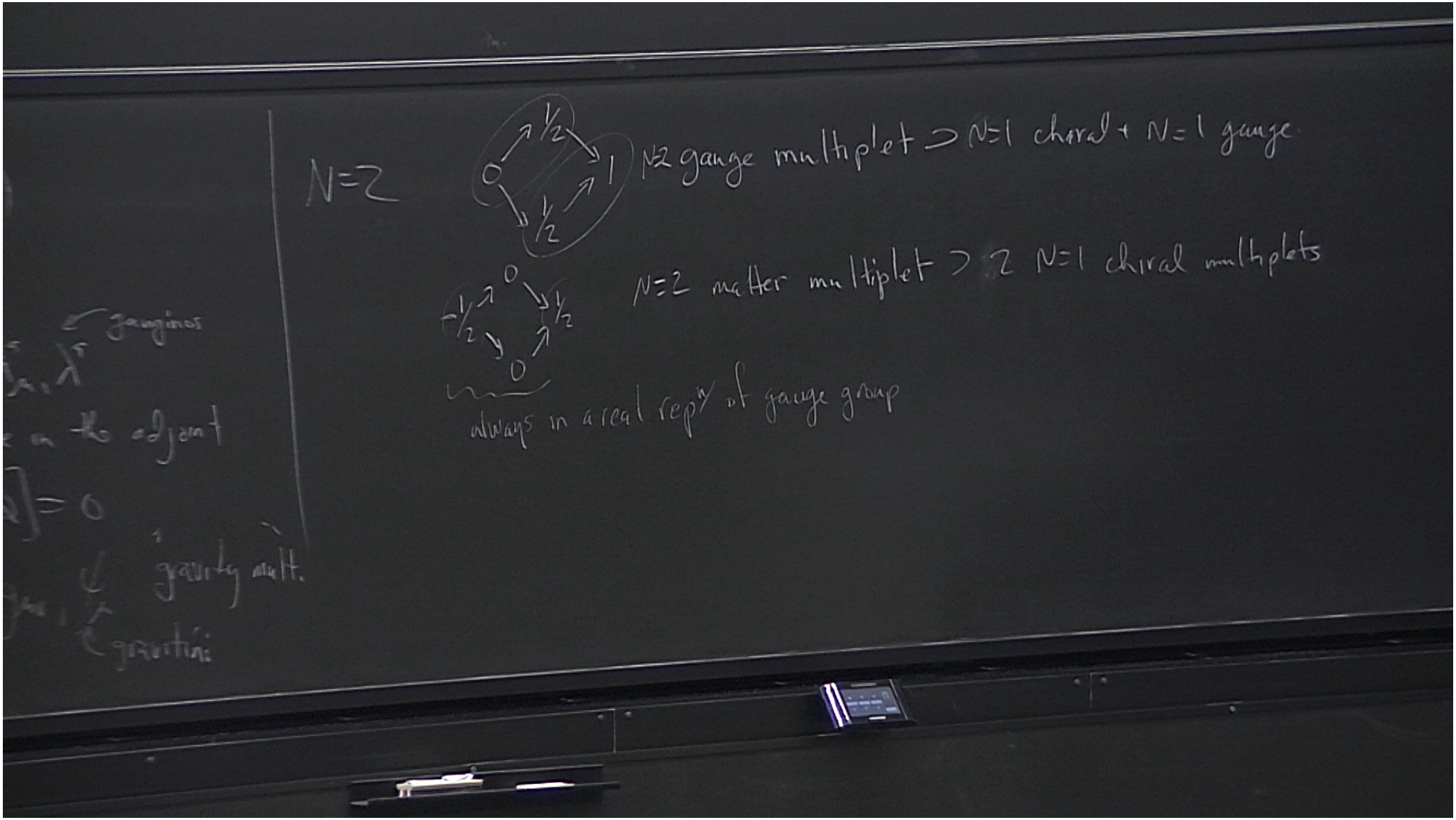
will be in the adjoint

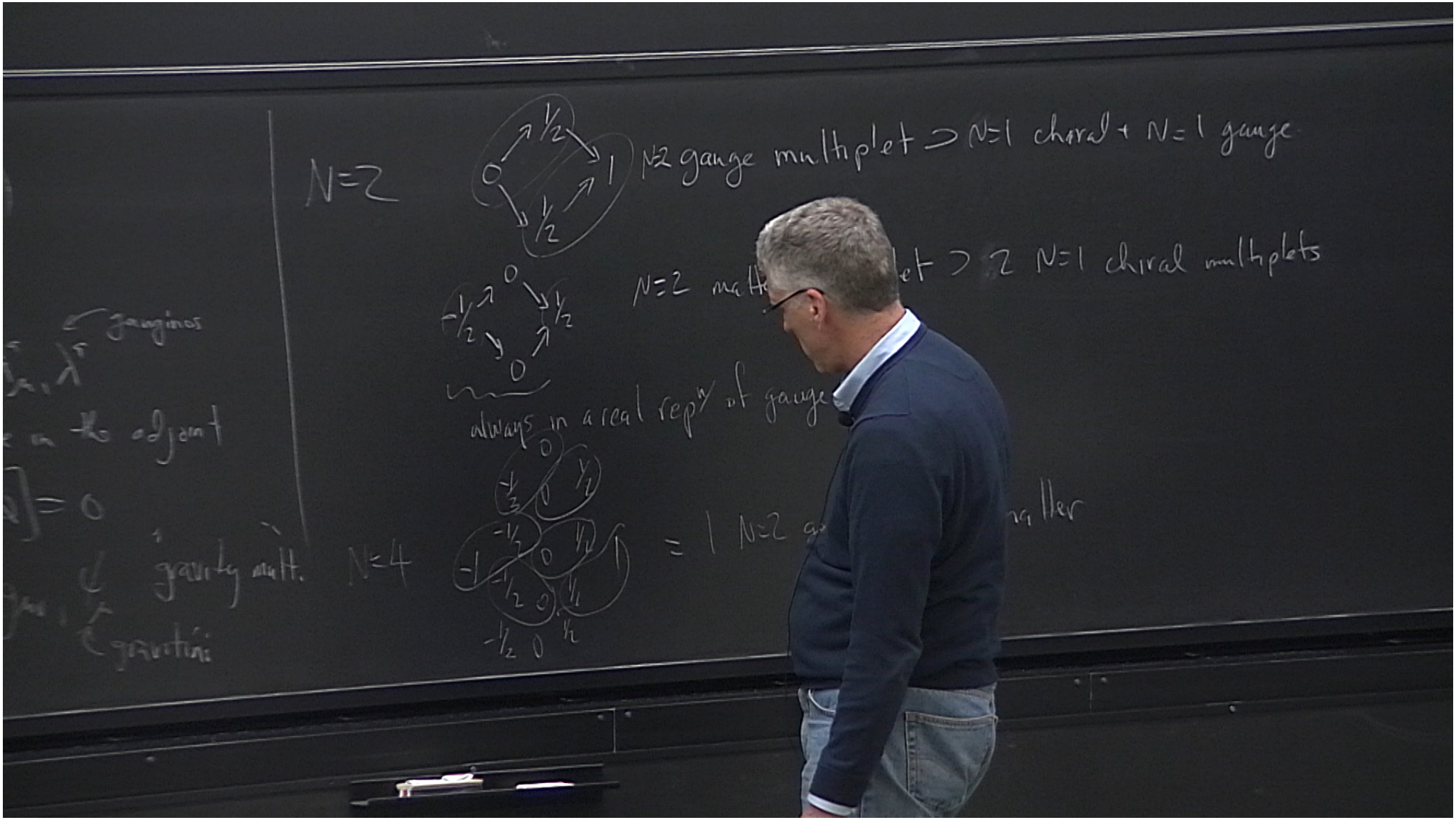
if $[G, Q] = 0$

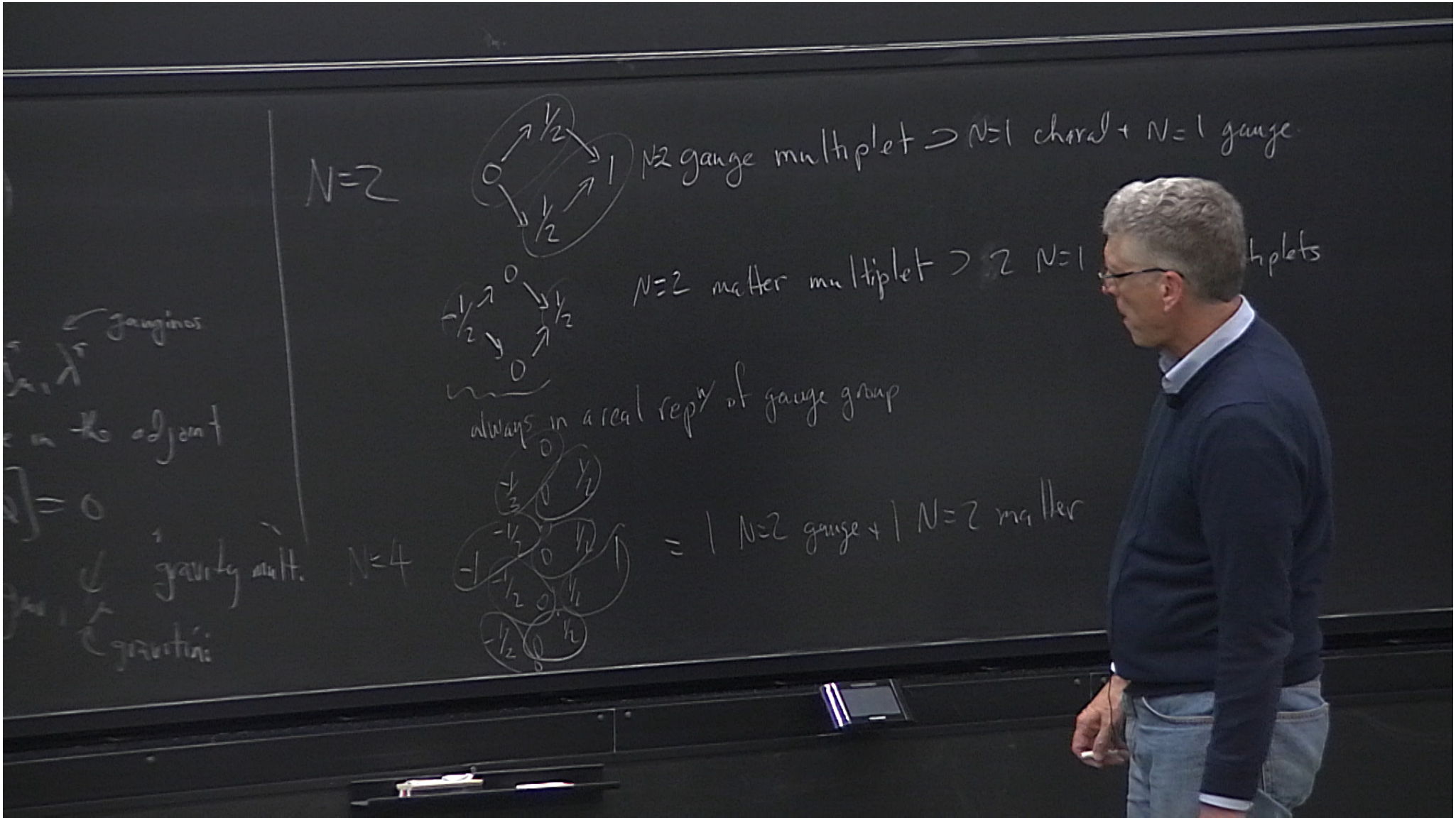
$(3/2, 2) + (-2, -3/2)$: $g_{\mu\nu}$, ψ_μ ← gravitini

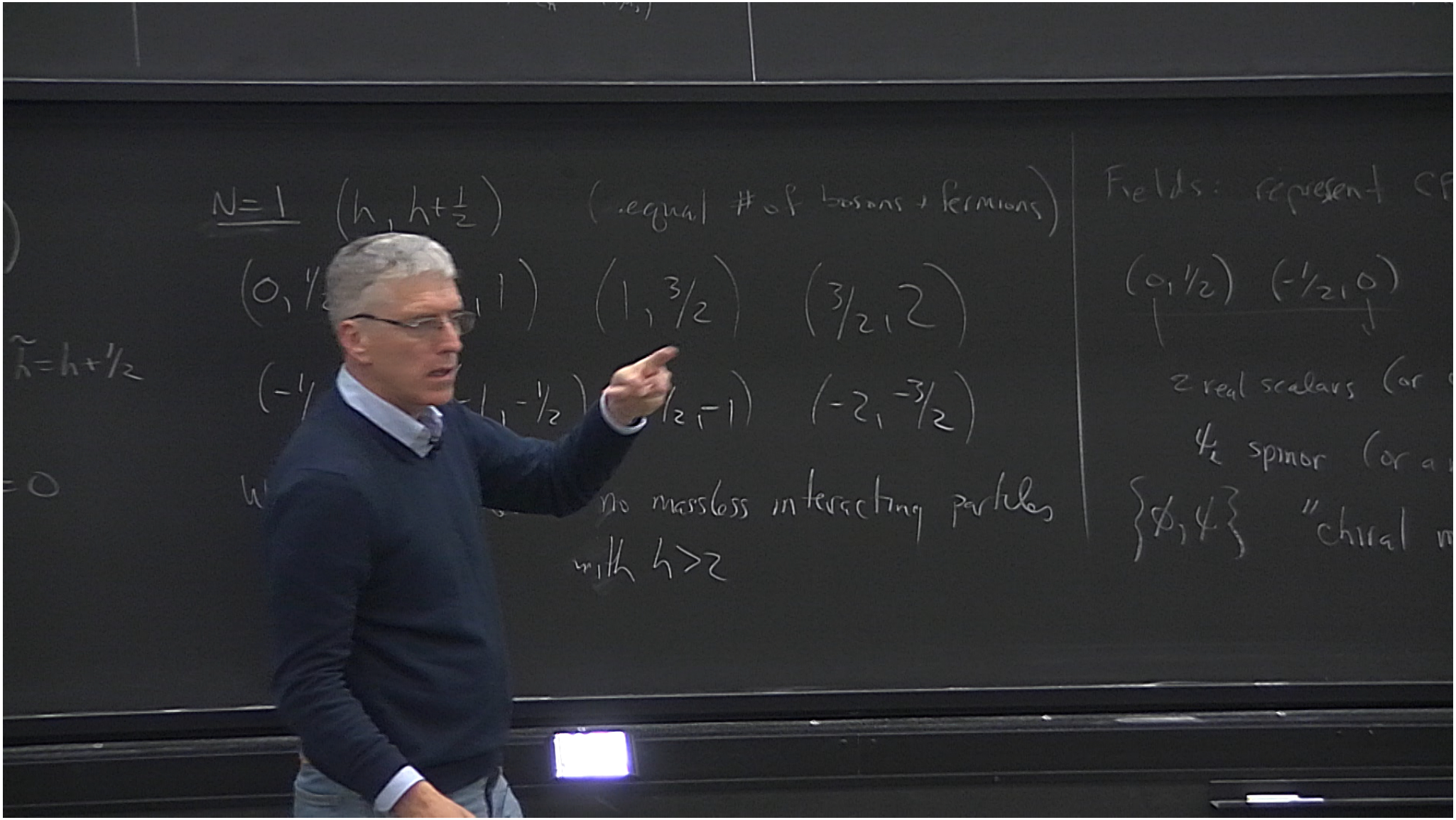
gravity mult.

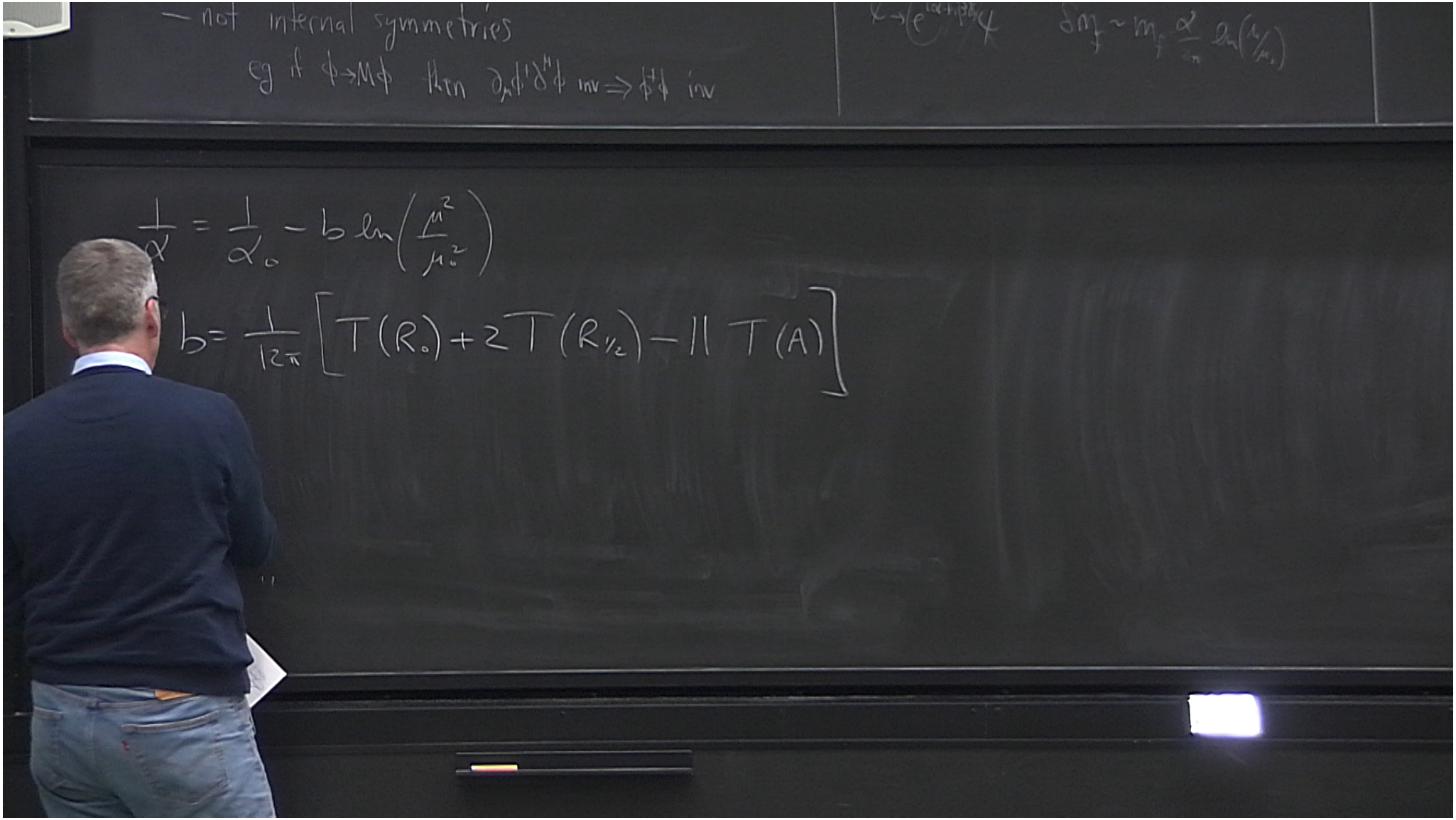

(½, 1) + (−1, −½)
'gauge multiplet
{A^a_μ, λ^a}
gauginos
both A^a_μ, λ^a
will be in the adjoint
if [G, Q] = 0
(3/2, 2) + (−2, −3/2) : g_μν, ψ_μ
gravitini
gravity mult.
N=2
½
1
½

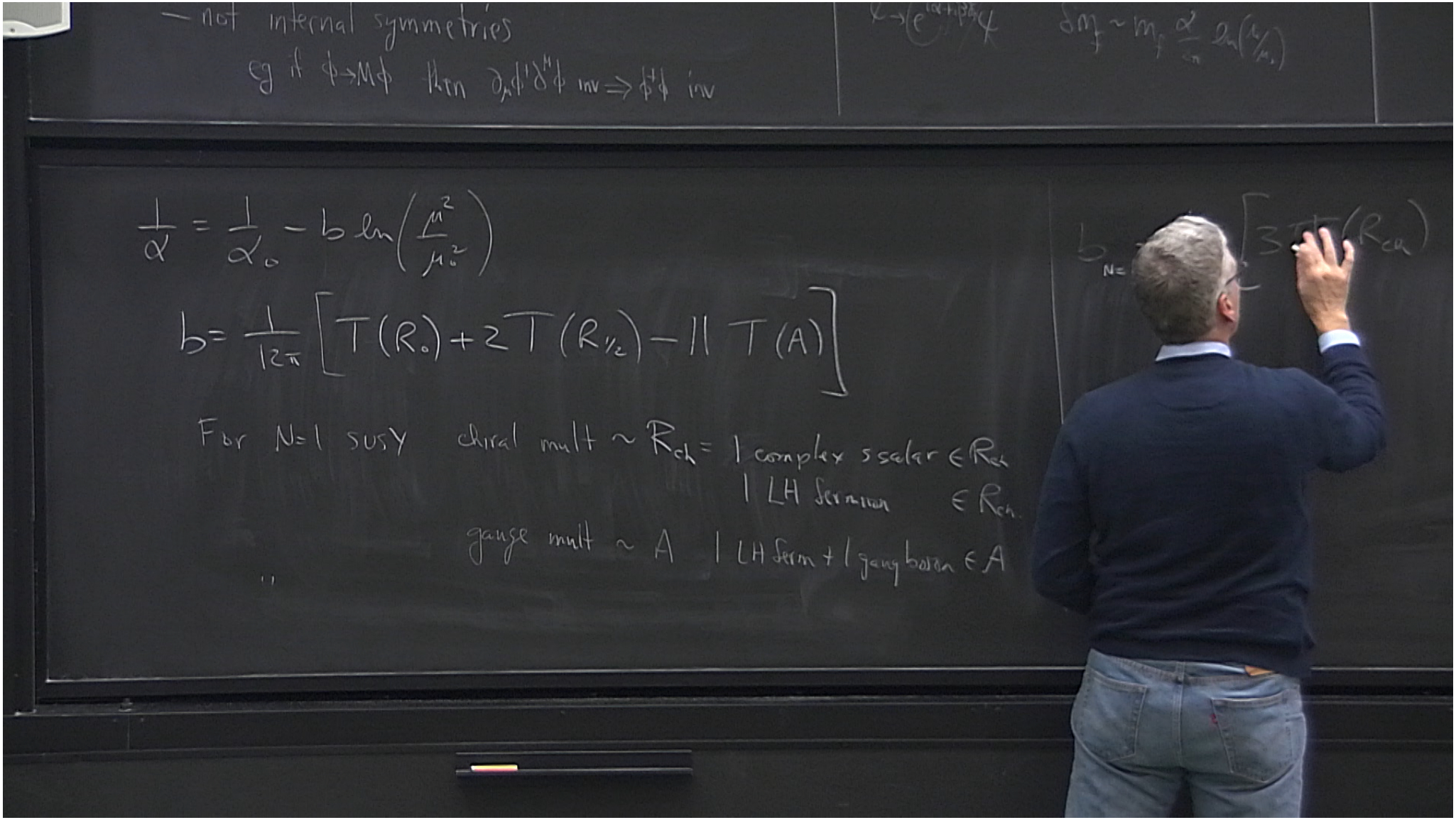

N=2
1/2
0
1
1/2
N=2 gauge multiplet ⊃ N=1 chiral + N=1 gauge
0
-1/2
1/2
0
N=2 matter multiplet ⊃ 2 N=1 chiral multiplets
always in a real rep'n of gauge group
gauginos
gravity mult.

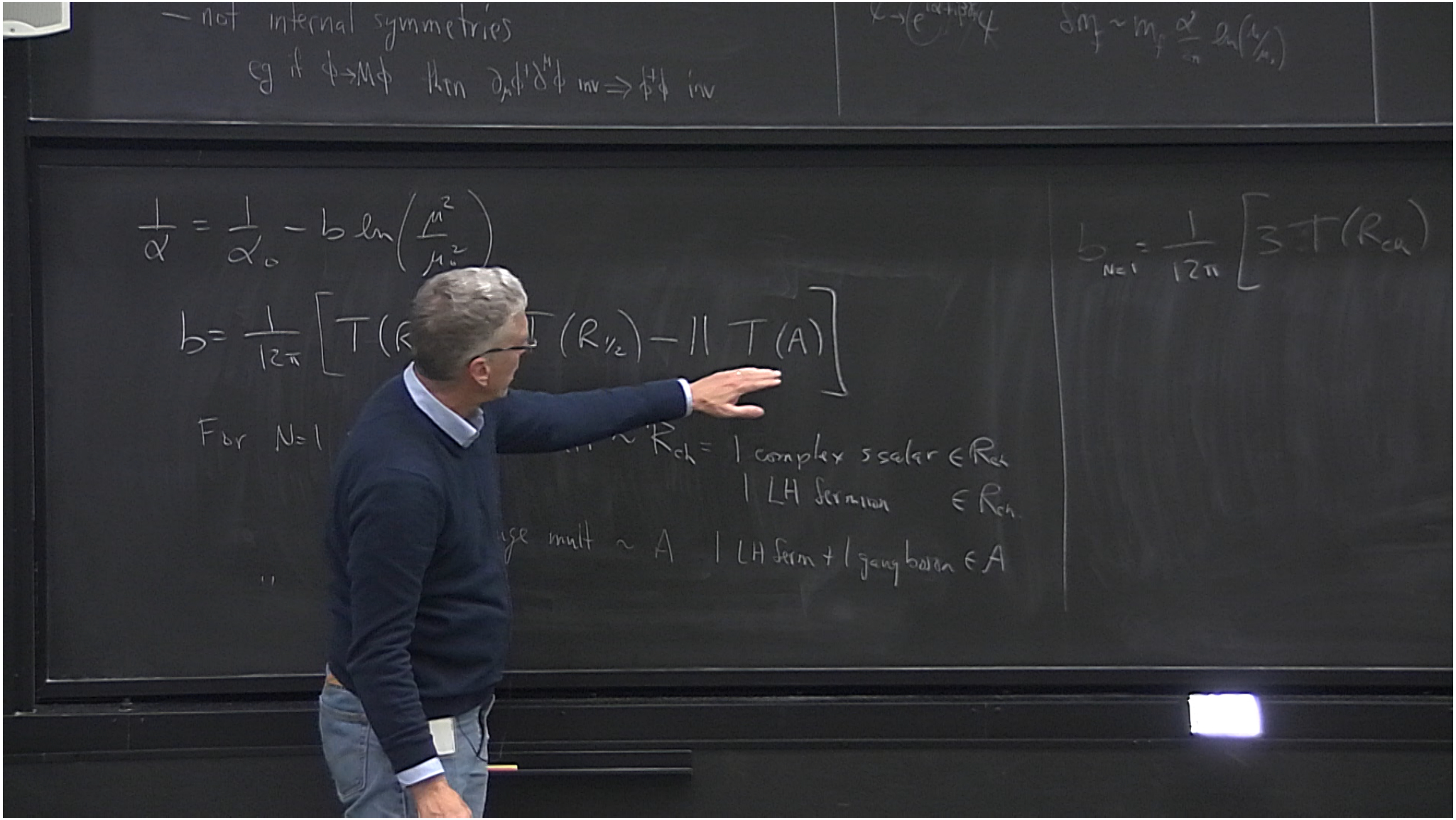

N=2
N=2 gauge multiplet ⊃ N=1 chiral + N=1 gauge
N=2 matter multiplet ⊃ 2 N=1 chiral multiplets
always in a real rep'n of gauge
N=4
gravitons
gravitini
gauginos
adjoint
gravity mult.

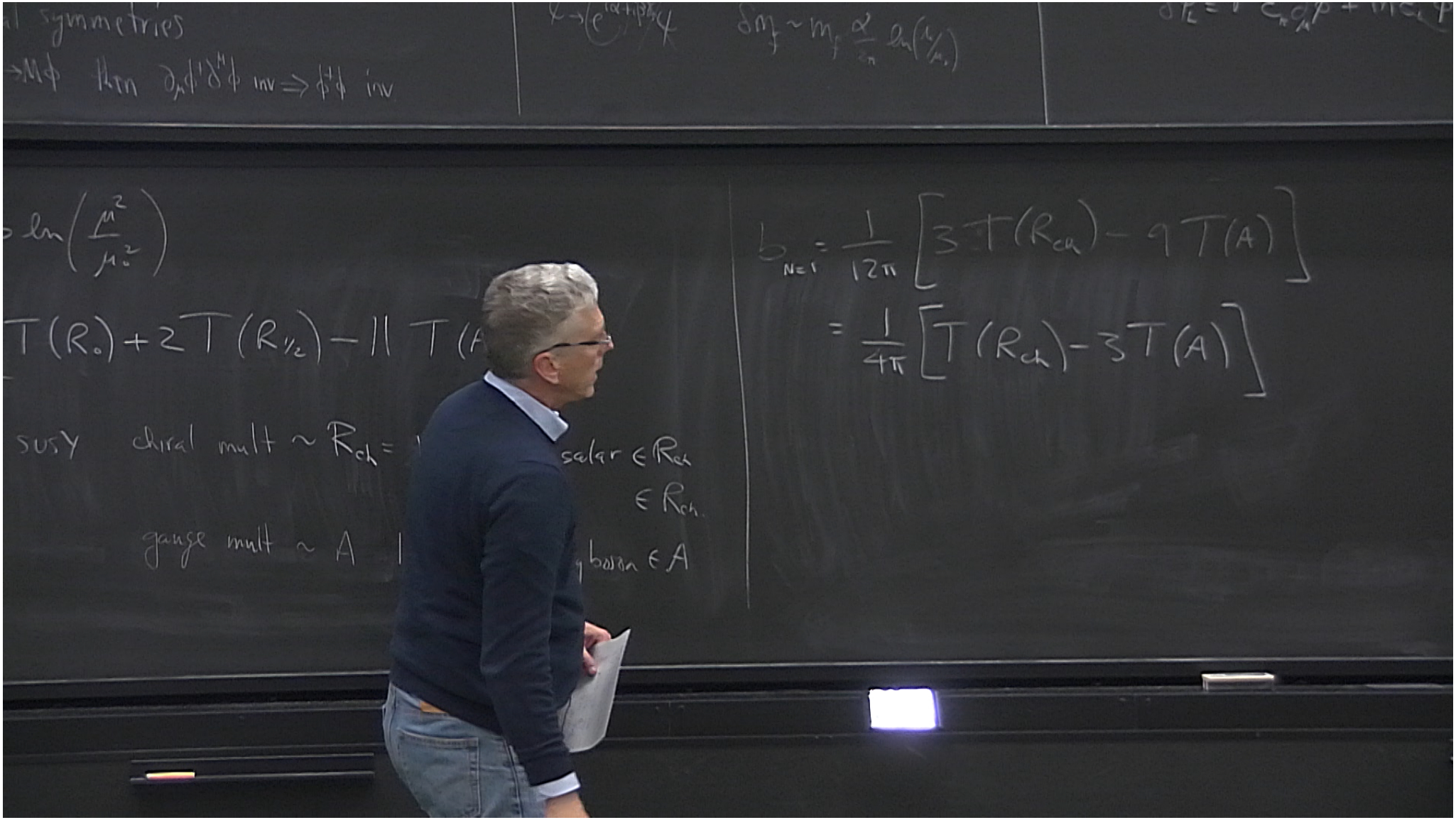

N=2
N=2 gauge multiplet ⊃ N=1 chiral + N=1 gauge
N=2 matter multiplet ⊃ 2 N=1
always in a real rep'n of gauge group
N=4
= 1 N=2 gauge + 1 N=2 matter
gravitini
gravity mult.
gauginos


N=1 (h, h+½) (equal # of bosons + fermions)
(0, ½) (1, 3/2) (3/2, 2)
(-½, -½) (-2, -3/2)
no massless interacting particles
with h>2
Fields: represent
(0, ½) (-½, 0)
2 real scalars
½ spinor
{φ, ψ}
"chiral


– not internal symmetries
eg if $\phi \to M\phi$ then $\partial_\mu \phi^\dagger \gamma^\mu \phi$ inv $\Rightarrow \phi^\dagger \phi$ inv
$\frac{1}{\alpha} = \frac{1}{\alpha_0} - b \ln\left(\frac{\Lambda^2}{\mu_0^2}\right)$
$b = \frac{1}{12\pi}\left[T(R_0) + 2T(R_{1/2}) - 11\,T(A)\right]$


– not internal symmetries
eg if $\phi \to M\phi$ then $\partial_\mu \phi^\dagger \gamma^\mu \phi$ inv $\Rightarrow \phi^\dagger \phi$ inv
$\frac{1}{\alpha} = \frac{1}{\alpha_0} - b \ln\left(\frac{\Lambda^2}{\mu_0^2}\right)$
$b = \frac{1}{12\pi}\left[T(R_0) + 2T(R_{1/2}) - 11\,T(A)\right]$
For N=1 SUSY chiral mult ~ R_{ch} = 1 complex scalar $\in R_{ch}$
1 LH fermion $\in R_{ch}$
gauge mult ~ A 1 LH ferm + 1 gauge boson $\in A$


– not internal symmetries
e.g. if $\phi \to M\phi$ then $\partial_\mu \phi^\dagger \bar{\sigma}^\mu \phi$ inv $\Rightarrow \phi^\dagger \phi$ inv
$\frac{1}{\alpha} = \frac{1}{\alpha_0} - b \ln\left(\frac{\Lambda^2}{\mu_0^2}\right)$
$b = \frac{1}{12\pi}\left[T(R) - T(R_{1/2}) - 11\, T(A)\right]$
$b_{N=1} = \frac{1}{12\pi}\left[3 \cdot T(R_{ch})\right.$
For N=1
R_{ch} = 1 complex scalar $\in R_{ch}$
1 LH fermion $\in R_{ch}$
gauge mult ~ A 1 LH ferm + 1 gauge boson $\in A$


symmetries
$\ln\left(\frac{\mu^2}{\mu_0^2}\right)$
$T(R_0)+2T(R_{1/2})-11\,T($
susy chiral mult ~ R_{ch} =
scalar $\in R_{ch}$
$\in R_{ch}$
gauge mult ~ A
boson $\in A$
$b_{N=1}=\frac{1}{12\pi}\left[3T(R_{ch})-9T(A)\right]$
$=\frac{1}{4\pi}\left[T(R_{ch})-3T(A)\right]$

$$+2T(R_{1/2}) - 11\,T(A)\Big]$$

chiral mult $\sim R_{ch}$ = 1 complex scalar $\in R_{ch}$
1 LH fermion $\in R_{ch}$

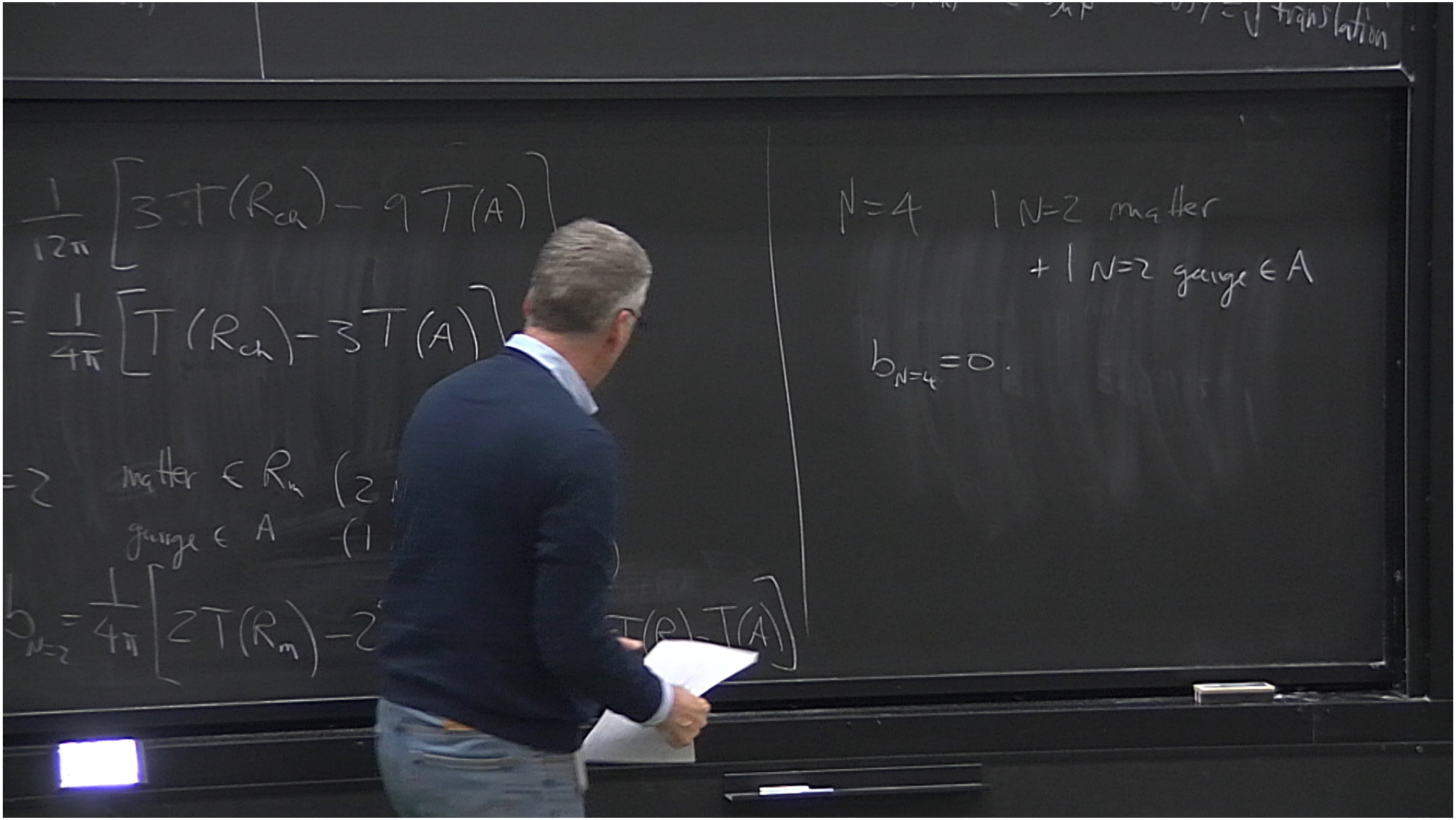
gauge mult $\sim A$ 1 LH ferm + 1 gauge boson $\in A$

$$b_{N=1} = \frac{1}{12\pi}\left[3T(R_{ch}) - 9T(A)\right]$$

$$= \frac{1}{4\pi}\left[T(R_{ch}) - 3T(A)\right]$$

$N=2$ matter $\in R_m$ (2 $N=1$ ch)
gauge $\in A$ (1 matter + 1 gauge $N=1$)

$$b_{N=2} = \frac{1}{4\pi}\left[2T(R_m) - 2T(A)\right] = \frac{1}{2\pi}\left[T(R_m) - T(A)\right]$$


$\frac{1}{12\pi}\left[3T(R_{ch}) - 9T(A)\right]$
$= \frac{1}{4\pi}\left[T(R_{ch}) - 3T(A)\right]$
$N=4$ | $N=2$ matter
$+ | N=2$ gauge $\in A$
$b_{N=4} = 0.$

translation

$$\frac{1}{12\pi}\left[3T(R_{cq}) - 9T(A)\right]$$

$$= \frac{1}{4\pi}\left[T(R_{ch}) - 3T(A)\right]$$

$=2$ matter $\in R_m$ (2 $N=1$ ch)

gauge $\in A$ (1 matter + 1 gauge $N=1$)

$$b_{N=2} = \frac{1}{4\pi}\left[2T(R_m) - 2T(A)\right] = \frac{1}{2\pi}\left[T(R_m) - T(A)\right]$$

$N=4$ 1 $N=2$ matter

+ 1 $N=2$ gauge $\in A$

$b_{N=4} = 0$.

Better UV properties because

in loops bosons & fermions cancel.

eg: 1-loop vacuum energy.

$$\delta\lambda = \frac{\#}{64\pi^2}\left[\mathrm{Tr}\left(M_0^2\right) - 2\,\mathrm{Tr}\left(M_{1/2}^2\right) + 3\,\mathrm{Tr}\left(M_1^2\right)\right]$$

$$= \frac{\#}{64\pi^2}\,\underbrace{S\mathrm{Tr}\left(M^2\right)}$$

$$\sum_S (-)^{2S}(2S+1)\,\mathrm{Tr}\left(M_S^2\right)$$

multiplet

matter multiplet

of gauge group

N=2 gauge + M

eg: 1-loop vacuum energy

$$\delta\lambda = \frac{\#}{64\pi^2}\left[\mathrm{Tr}\left(M_0^2\right) - 2\,\mathrm{Tr}\left(M_{1/2}^2\right) + 3\,\mathrm{Tr}\left(M_1^2\right)\right]$$

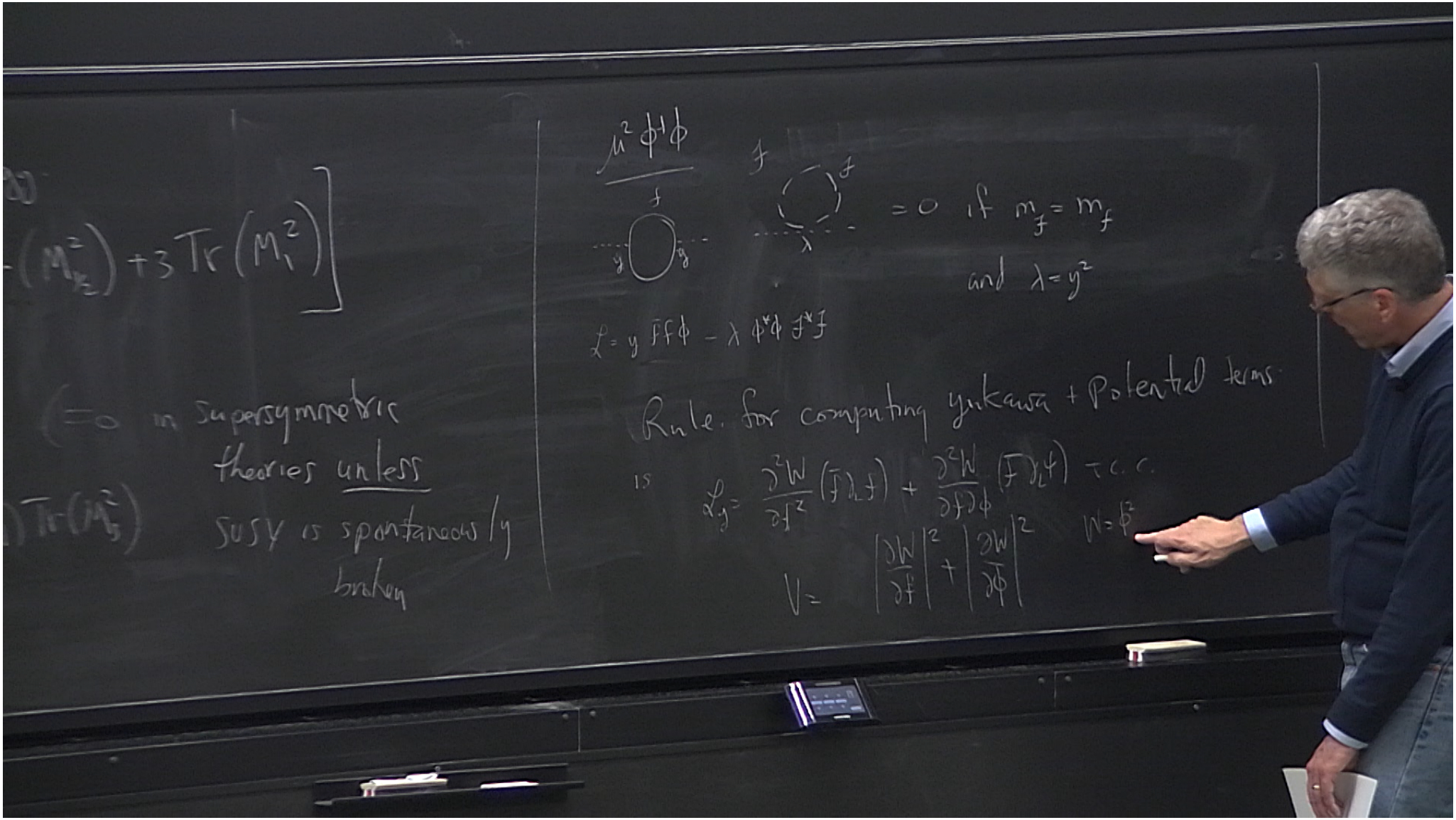
$$= \frac{\#}{64\pi^2}\,\underbrace{\mathrm{STr}\left(M^2\right)}_{\sum_s (-)^{2s}(2s+1)\,\mathrm{Tr}\left(M_s^2\right)} \qquad (=0 \text{ in supersymmetric theories } \underline{\text{unless}} \text{ SUSY is spontaneously broken}$$

multiplet

matter multiplet

or gauge group

N=2 gauge + 1

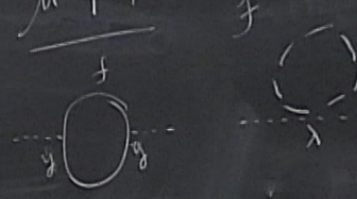

$\mu^2 \phi^\dagger \phi$
$= 0$ if $m_f = m_f$
and $\lambda = y^2$
$\mathcal{L} = y \bar{f} f \phi - \lambda \phi^\dagger \phi f^\dagger f$
Rule for computing yukawa + Potential terms
is
$\mathcal{L}_y = \frac{\partial^2 W}{\partial f^2} (\bar{f} \lambda_L f) + \frac{\partial^2 W}{\partial f \partial \phi} (\bar{f} \lambda_L f) + \text{c.c.}$
$V = \left|\frac{\partial W}{\partial f}\right|^2 + \left|\frac{\partial W}{\partial \phi}\right|^2$
$W = \phi^3$
$= 0$ in supersymmetric theories unless
SUSY is spontaneously broken

$\left(M_{\chi^2}^2\right) + 3\,\mathrm{Tr}\left(M_1^2\right)\Big]$

$= 0$ in Supersymmetric theories unless SUSY is spontaneously broken

$\mathrm{Tr}\left(M_3^2\right)$

$\mu^2 \phi^\dagger \phi$



$= 0$ if $m_{\mathcal{F}} = m_f$ and $\lambda = y^2$

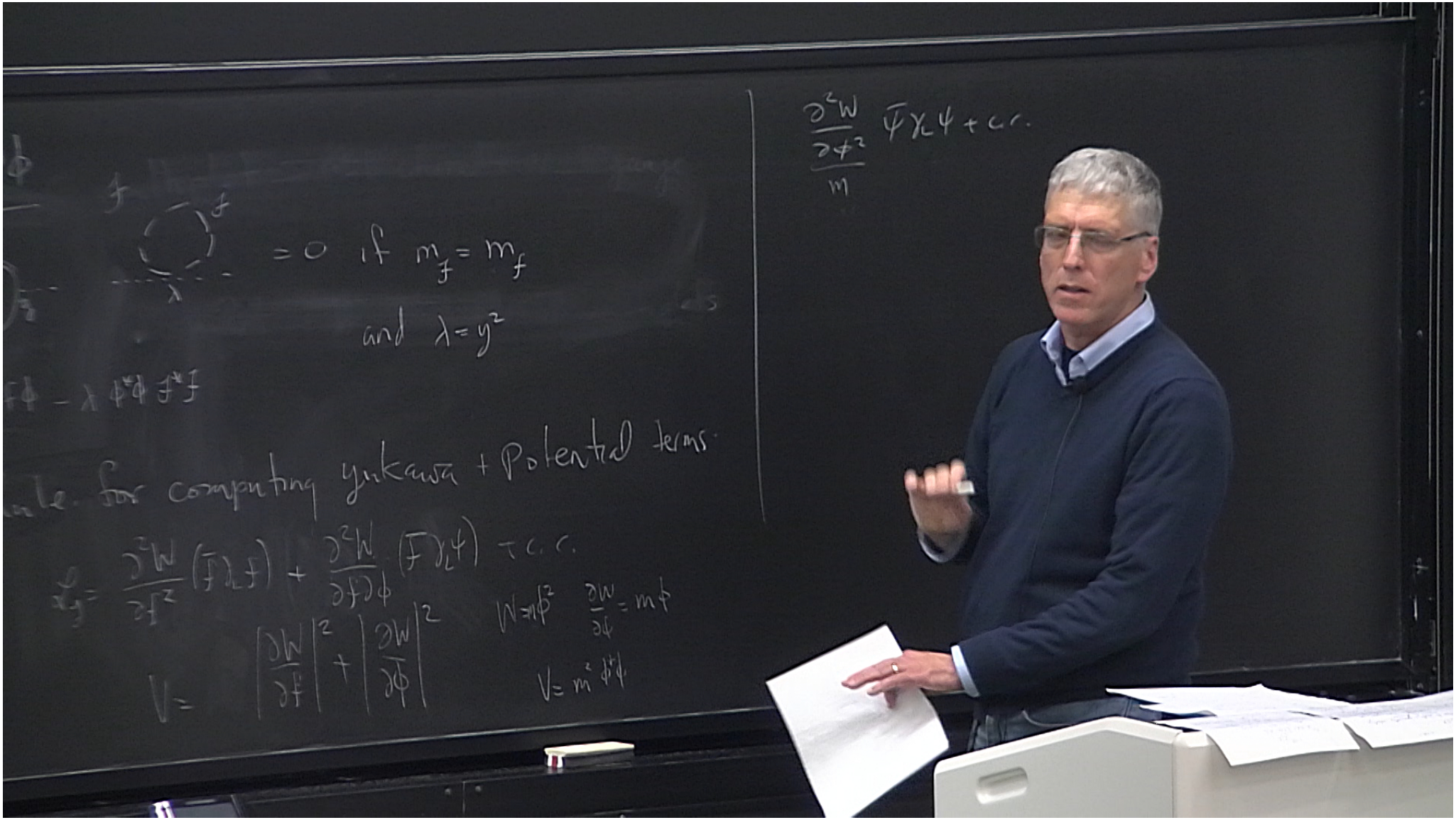
$\mathcal{L} = y\, \bar{f} f \phi - \lambda\, \phi^\dagger \phi\, \mathcal{F}^\dagger \mathcal{F}$

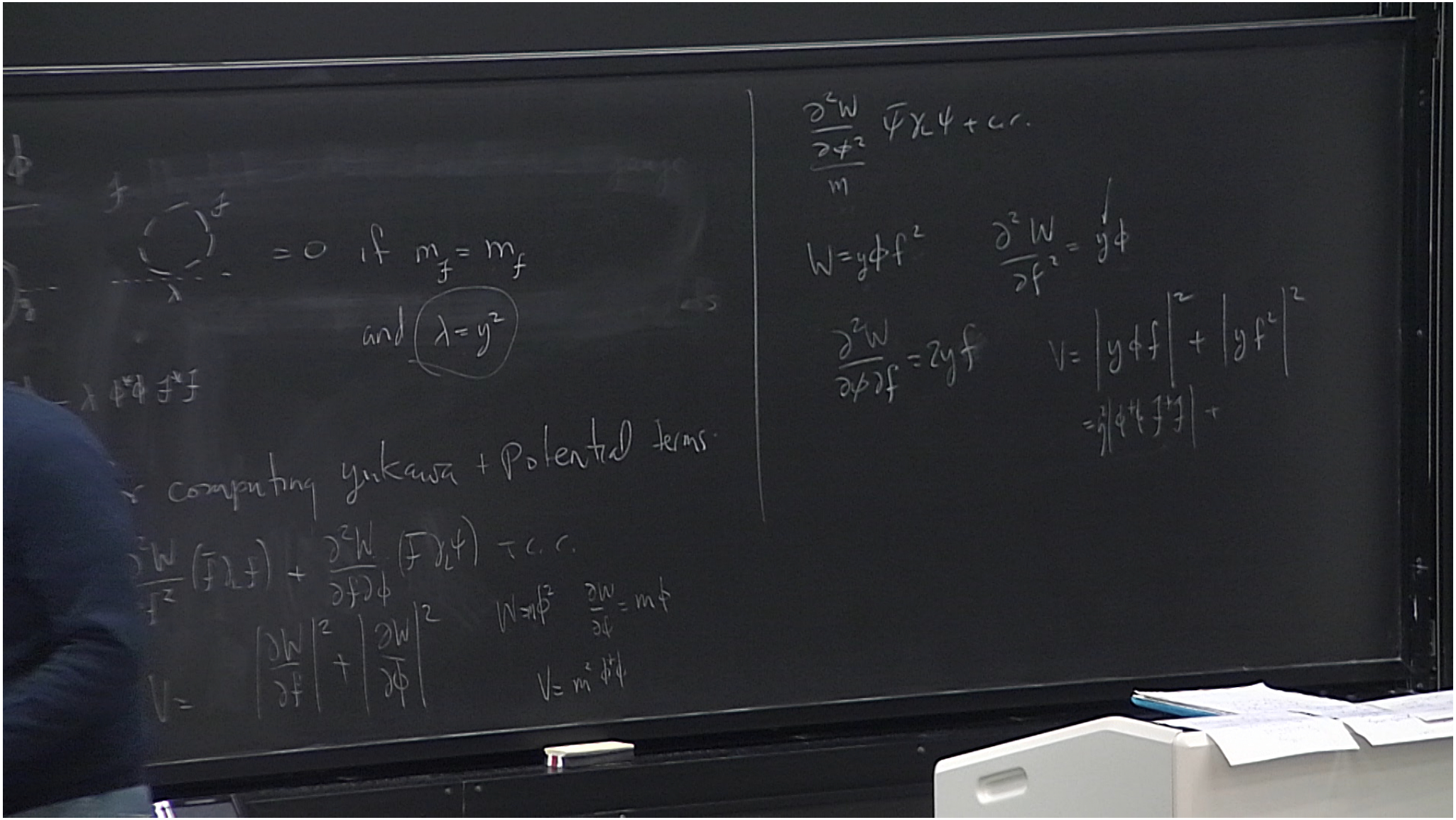
Rule for computing yukawa + Potential terms

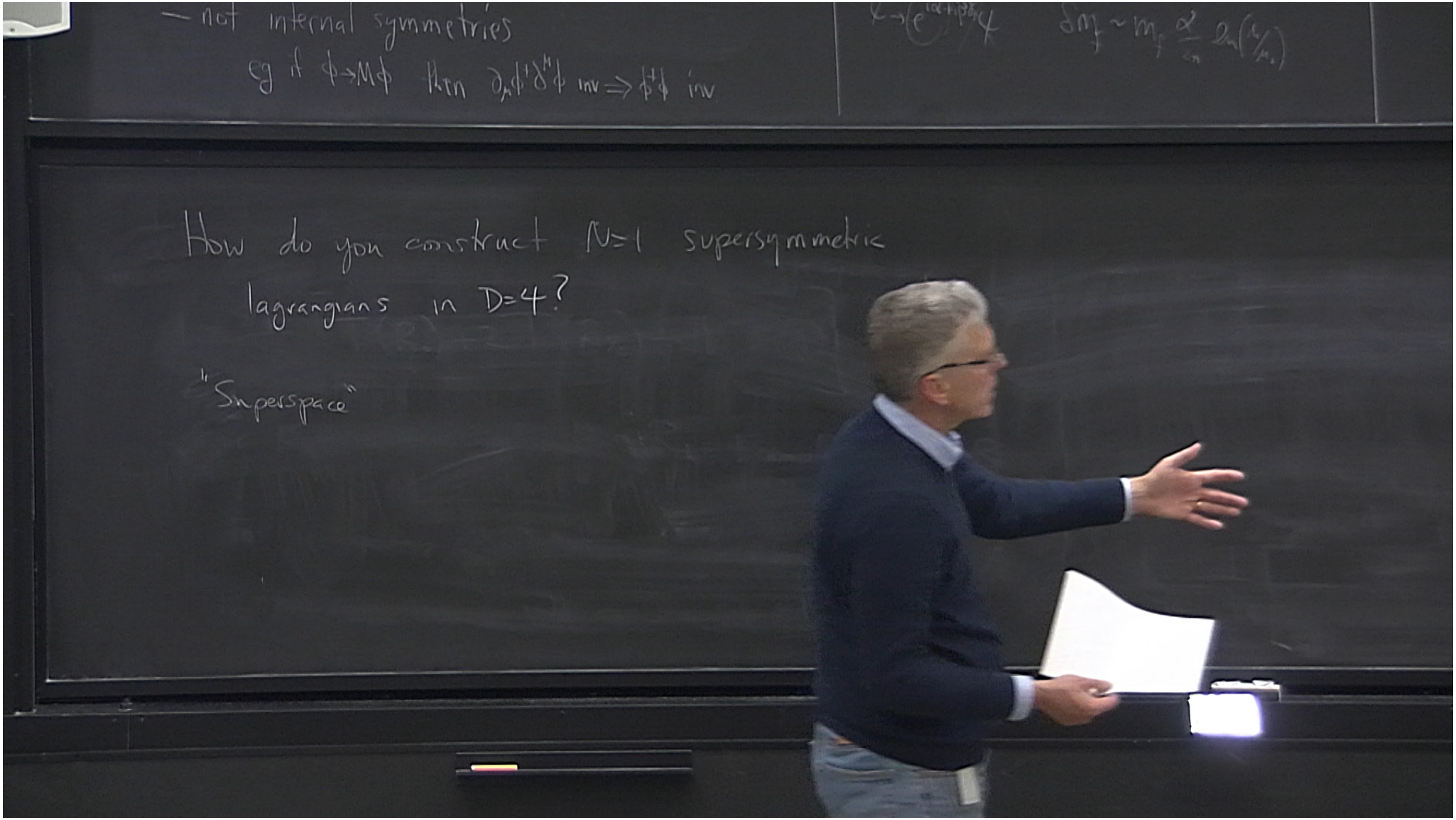
is $\mathcal{L}_y = \frac{\partial^2 W}{\partial f^2} (\bar{f} \lambda_- f) + \frac{\partial^2 W}{\partial f \partial \phi} (\bar{f} \lambda_+ f)$ + c.c.

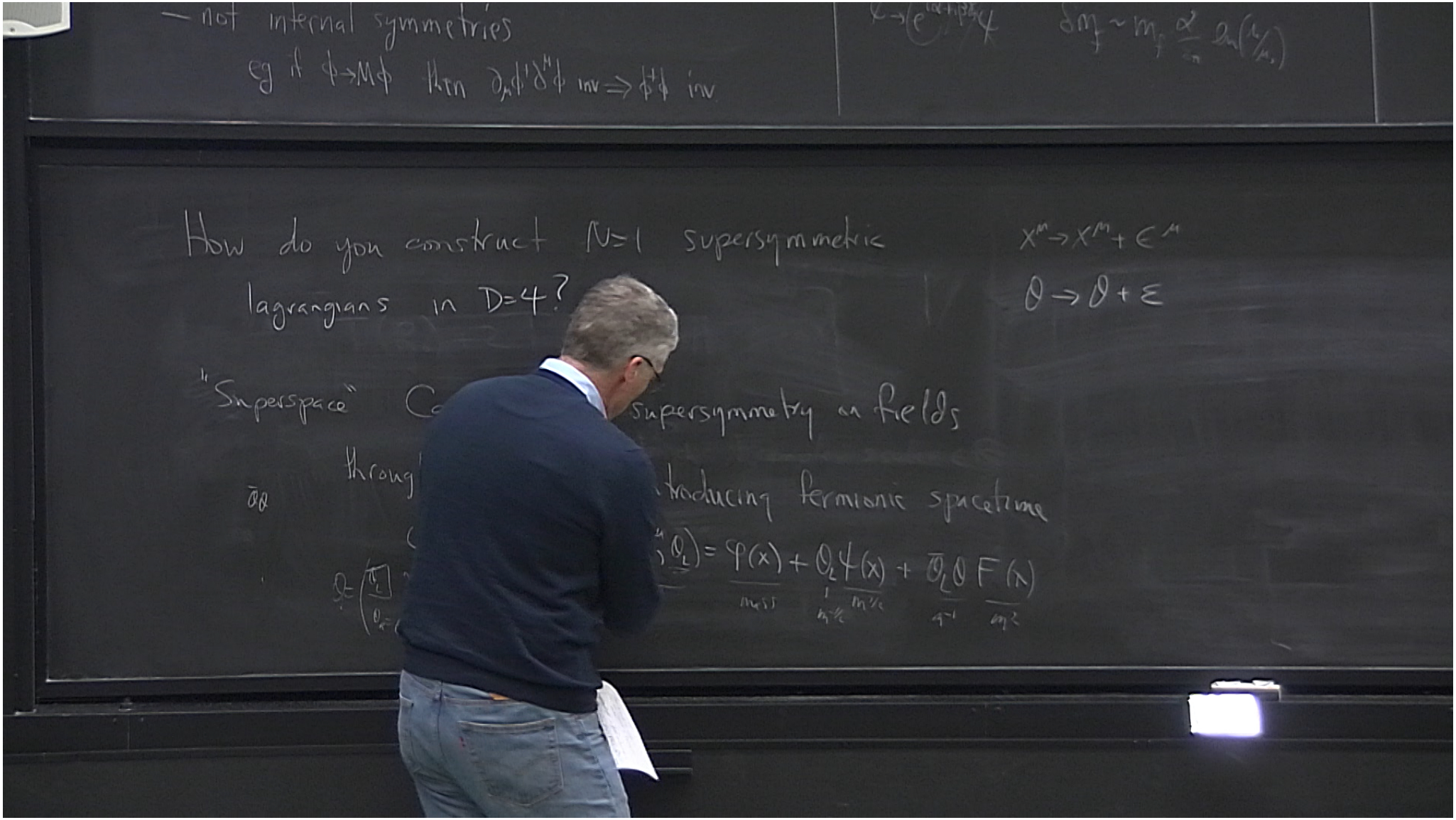
$V = \left|\frac{\partial W}{\partial f}\right|^2 + \left|\frac{\partial W}{\partial \phi}\right|^2$ $\quad W = \mu \phi^2$ $\quad \frac{\partial W}{\partial \phi} = m^2 \phi$

$V = m^2 \phi^\dagger \phi$


$= 0$ if $m_f = m_f$
and $\lambda = y^2$
$\frac{\partial^2 W}{\partial f^2} \bar{\psi} \chi_L \psi + c.c.$
m
$\frac{\partial^2 W}{\partial f^2} (\bar{f} \chi_L f) + \frac{\partial^2 W}{\partial f \partial \phi} (\bar{f} \chi_L \psi) + c.c.$
$V = \left|\frac{\partial W}{\partial f}\right|^2 + \left|\frac{\partial W}{\partial \phi}\right|^2$
$W = m\phi^2$ $\frac{\partial W}{\partial \phi} = m\phi$
$V = m^2 \phi^* \phi$


$= 0$ if $m_f = m_f$
and $\lambda = y^2$
computing yukawa + Potential terms.
$\frac{\partial^2 W}{\partial f^2}(\bar{f}\chi_L f) + \frac{\partial^2 W}{\partial f \partial \phi}(\bar{f}\chi_L \psi) + c.c.$
$V = \left|\frac{\partial W}{\partial f}\right|^2 + \left|\frac{\partial W}{\partial \phi}\right|^2$
$W = m\phi^2 \quad \frac{\partial W}{\partial \phi} = m\phi$
$V = m^2 \phi^\dagger \phi$
$\frac{\partial^2 W}{m}\bar{\psi}\chi_L\psi + c.c.$
$W = y\phi f^2 \quad \frac{\partial^2 W}{\partial f^2} = y\phi$
$\frac{\partial^2 W}{\partial \phi \partial f} = 2yf$
$V = |y\phi f|^2 + |yf^2|^2$
$= y^2|\phi^\dagger f^\dagger f| +$


— not internal symmetries
eg if $\phi \to M\phi$ then $\partial_\mu \phi^\dagger \bar{\sigma}^\mu \phi$ inv $\Rightarrow \phi^\dagger \phi$ inv
How do you construct $N=1$ supersymmetric lagrangians in $D=4$?
"Superspace"


— not internal symmetries
eg if $\phi \to M\phi$ then $\partial_\mu \phi^\dagger \gamma^\mu \phi$ inv $\Rightarrow \phi^\dagger \phi$ inv
How do you construct $N=1$ supersymmetric lagrangians in $D=4$?
$x^\mu \to x^\mu + \epsilon^\mu$
$\theta \to \theta + \epsilon$
"Superspace"
supersymmetry on fields
introducing fermionic spacetime
$\Phi(x,\theta) = \varphi(x) + \theta\psi(x) + \theta\theta F(x)$

— not internal symmetries

eg if $\phi \to M\phi$ then $\partial_\mu \phi^\dagger \gamma^\mu \phi$ inv $\Rightarrow \phi^\dagger \phi$ inv

$\phi \to e^{i\alpha + i\beta\gamma_5}\phi$ $\quad \delta m_f \sim m_f \frac{\alpha^2}{\Lambda^2} \ln\left(\frac{\Lambda}{\mu}\right)$

How do you construct $N=1$ supersymmetric lagrangians in $D=4$?

$x^\mu \to x^\mu + \epsilon^\mu$

$\theta \to \theta + \epsilon$

"Superspace" Can represent supersymmetry on fields

or

through the trick of introducing fermionic spacetime coordinates $\Phi(x^\mu, \theta_L) = \varphi(x) + \theta_L \psi(x) + \bar{\theta}_L \theta F(x)$

mass $\quad m^{-1/2} \quad m^{3/2} \quad m^{-1} \quad m^2$

$\theta = \begin{pmatrix} \theta_L \\ \theta_R = \theta^\dagger \end{pmatrix}$

rem: no massless interacting particles with $4 > 2$

ψ_L spinor (or a majorana spinor)

$\{\phi, \psi\}$ "chiral multiplet"

es:

$\mu \lesssim$ TeV

$\frac{\alpha}{2\pi} \ln(\Lambda/\mu)$

$\delta\phi = i\omega^a T_a \phi$

$\delta\psi = i\omega^a T_a \psi$

$\delta\phi = \bar{\varepsilon}\gamma_L\psi$ (mass: $-\frac{1}{2}$ + $\frac{3}{2}$)

ϕ: scalar; ψ: spin-$\frac{1}{2}$ fermion

$\delta\psi_L = \gamma^\mu \varepsilon \partial_\mu \phi + F\varepsilon$ (fermionic spinor; mass: $\frac{3}{2}$ + $-\frac{1}{2}$ + mass)

$\delta A^a_\mu = \bar{\varepsilon}\gamma_\mu \lambda^a$ (λ: fermion)

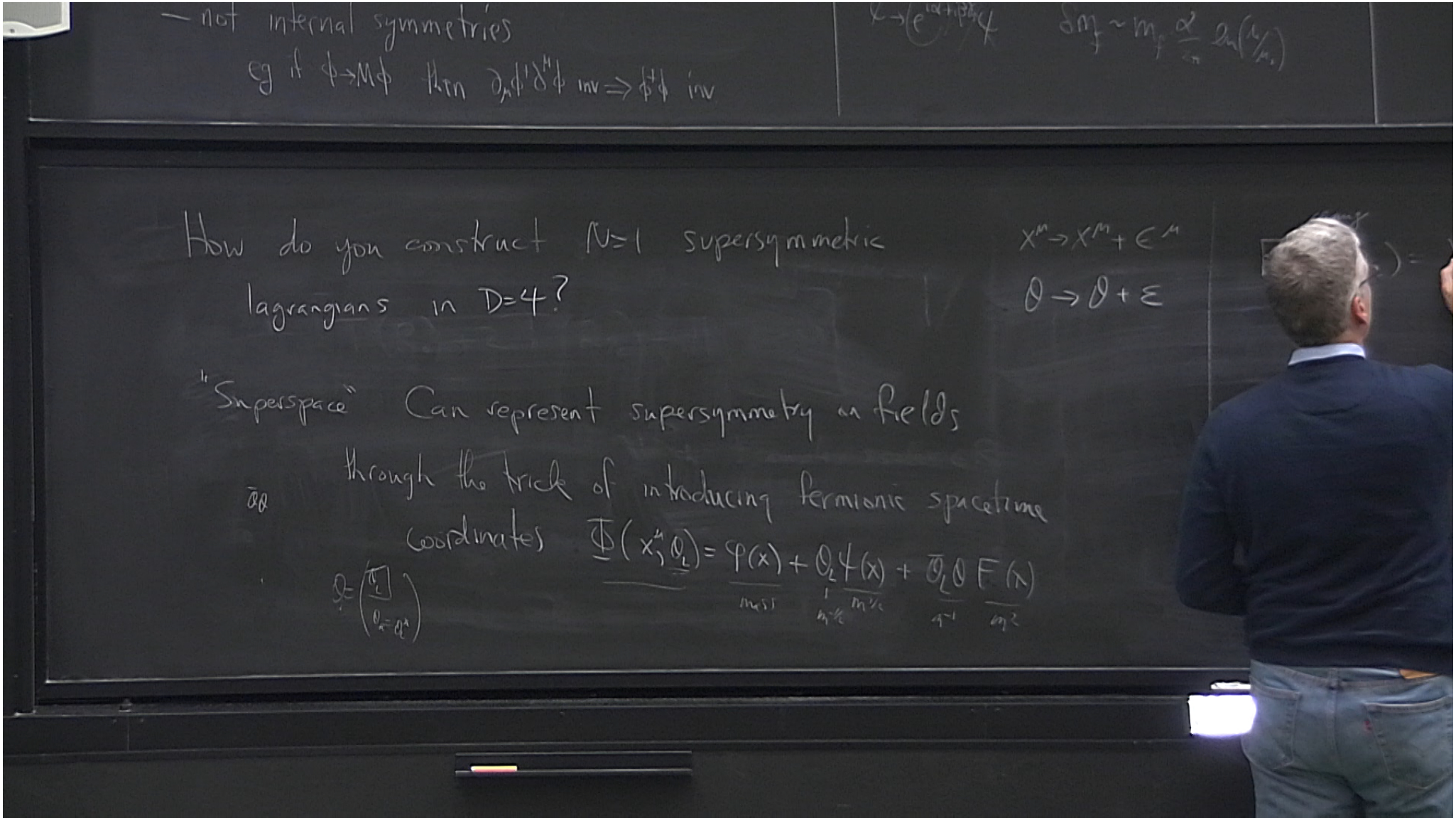
spin 1 spin $\frac{1}{2}$

$\delta\lambda = \gamma^{\mu\nu}\varepsilon F_{\mu\nu}$ $\quad \gamma_L = \frac{1}{2}(1+\gamma_5)$

$\delta_2\delta_1\phi = \bar{\varepsilon}_1\gamma_L\delta\psi = \bar{\varepsilon}_1\gamma_L\gamma^\mu\varepsilon_2\partial_\mu\phi$

$[\delta_2\delta_1 - \delta_1\delta_2]\phi = (\bar{\varepsilon}_1\gamma^\mu\varepsilon_2)\partial_\mu\phi$

$\delta\phi(x) = \epsilon^\mu\partial_\mu\phi$ SUSY = $\sqrt{\text{translation}}$


— not internal symmetries
eg if $\phi \to M\phi$ then $\partial_\mu \phi^\dagger \gamma^\mu \phi$ inv $\Rightarrow \phi^\dagger \phi$ inv
How do you construct $N=1$ supersymmetric lagrangians in $D=4$?
$x^\mu \to x^\mu + \epsilon^\mu$
$\theta \to \theta + \varepsilon$
"Superspace" Can represent supersymmetry on fields
or through the trick of introducing fermionic spacetime coordinates
$\Phi(x^\mu, \theta_L) = \varphi(x) + \theta_L \psi(x) + \bar{\theta}_L \theta F(x)$
$\theta = \begin{pmatrix} \theta_L \\ \theta_R = \theta^\dagger \end{pmatrix}$

translation

$$F^a(x,\theta_L) = \lambda^a(x) + (D^a + F^a_{\mu\nu}\gamma^{\mu\nu})\theta_L$$

$$+ (\not{D}\lambda)^a \bar{\theta}_L \theta_L$$

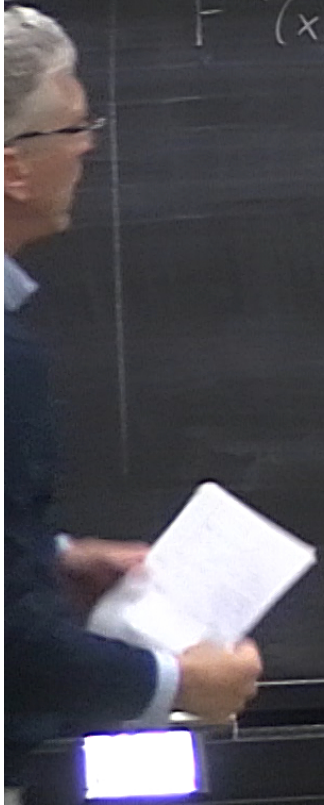
$N=4$ | $N=2$ matter

$+$ | $N=2$ gauge $\in A$

$b_{N=4} = 0$.

Better UV properties because

in loops bosons & fermions cancel



translation

$$F^a(x,\theta_L) = \lambda^a(x) + (D^a + F^a_{\mu\nu}\gamma^{\mu\nu})\theta_L$$

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Invariant actions in superspace:

$N=4$ | $N=2$ matter

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translation

F^a (gaugs) $(x, \theta_L) = \lambda^a(x) + (D^a + F^a_{\mu\nu}\gamma^{\mu\nu})\theta_L$

$+ (\not{D}\lambda)^a \bar{\theta}_L \theta_L$

Invariant actions in superspace:

"D term" $\mathcal{L} = \int d^2\theta_L d^2\theta_R \, K(\Phi, \Phi^*)$

$N=4$ | $N=2$ matter

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"D term" $\mathcal{L} = \int d^2\theta_L d^2\theta_R \, K(\Phi, \Phi^*)$ Kähler pot.

"F term"

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(gauge)

Invariant actions in superspace:

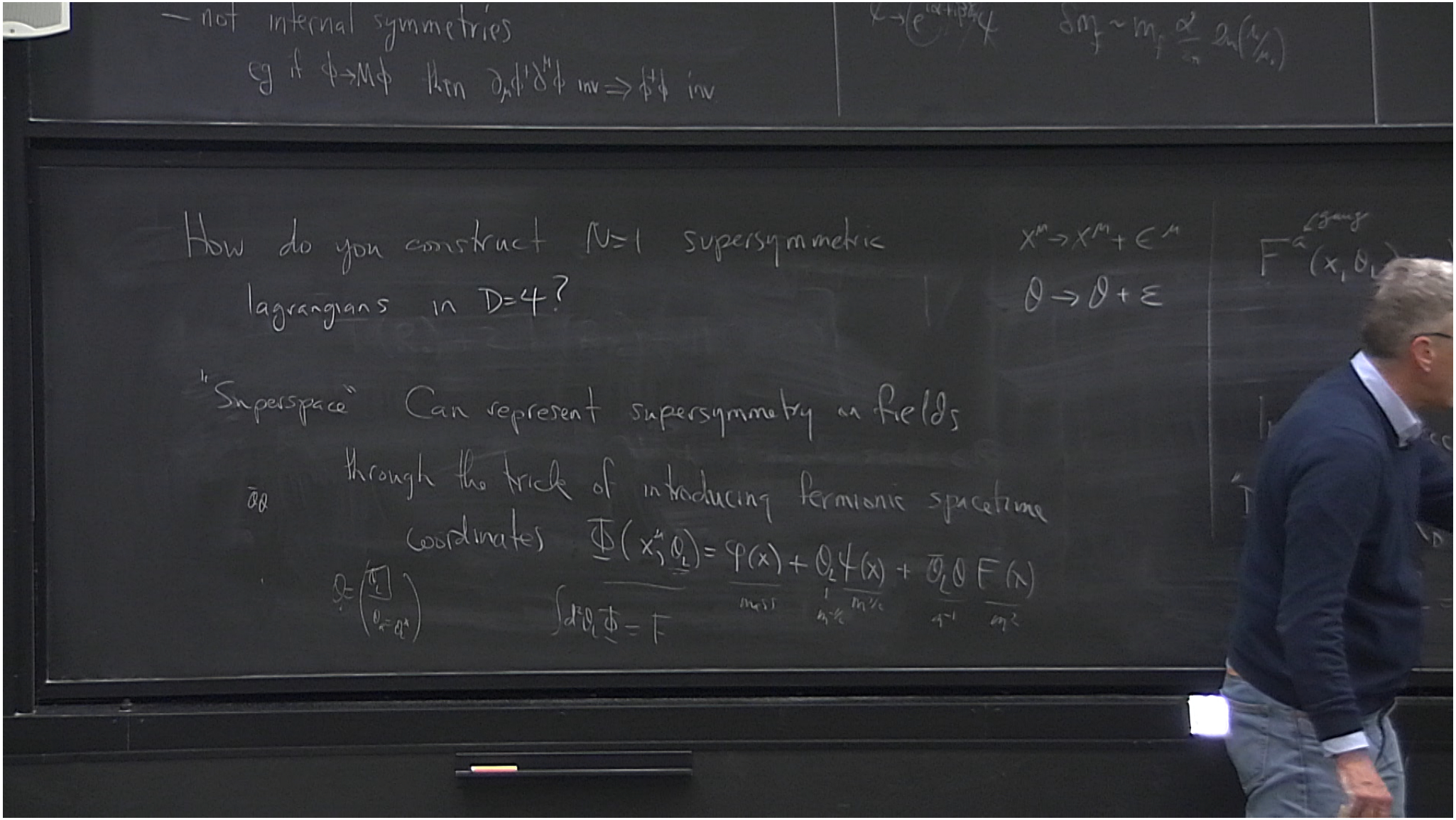
"D term" $\mathcal{L}_D = \int d^2\theta_L d^2\theta_R \, K(\Phi, \Phi^*)$ Pot. (Kähler)

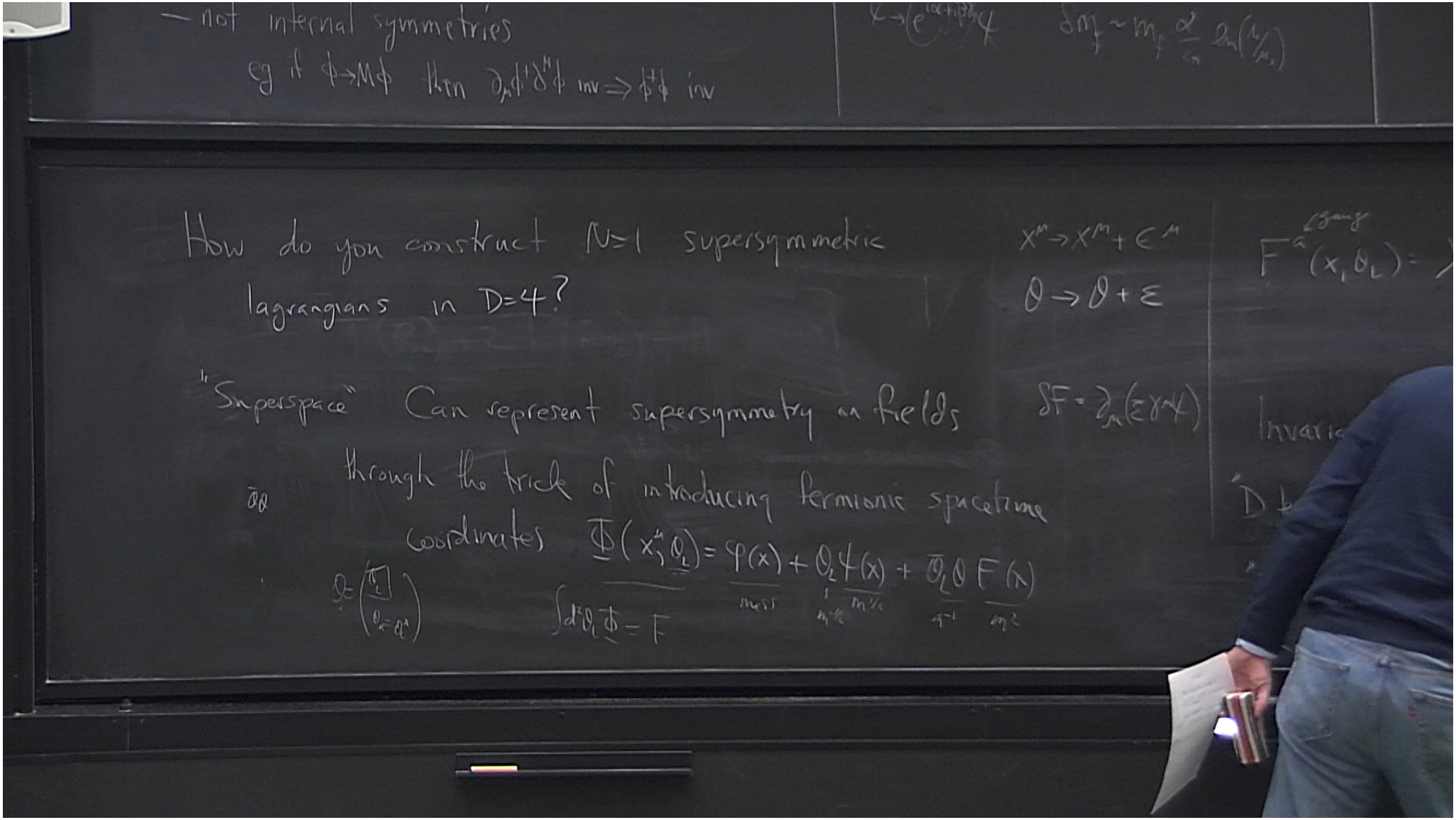
"F term" $\mathcal{L}_F = \int d^2\theta_L \left[W(\Phi) + \bar{F}^i \gamma_L F^b f_{ab}(\Phi) \right]$ (superpotential; gauge coupl function)

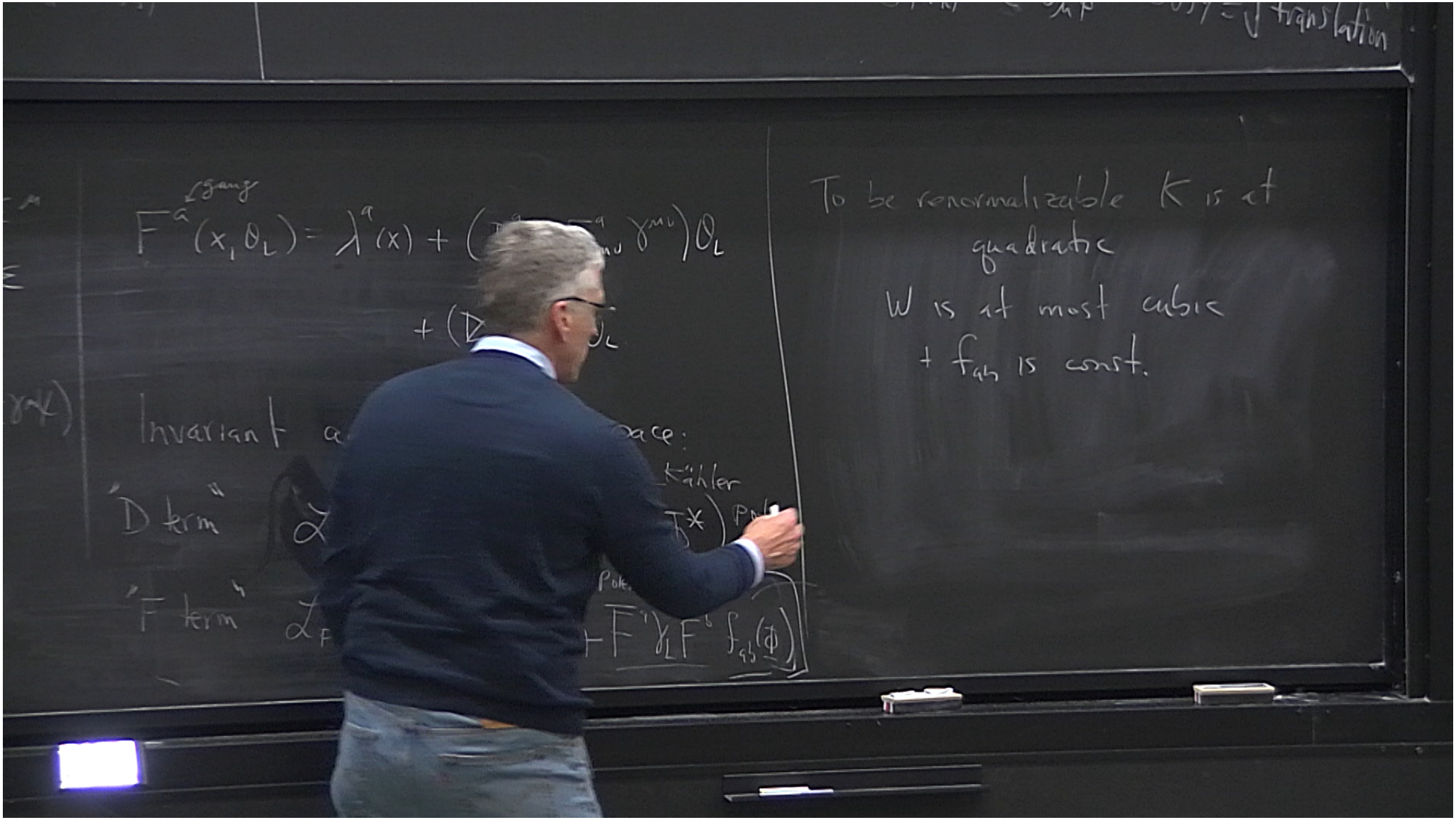
$N=4$ | $N=2$ matter + | $N=2$ gauge $\in A$

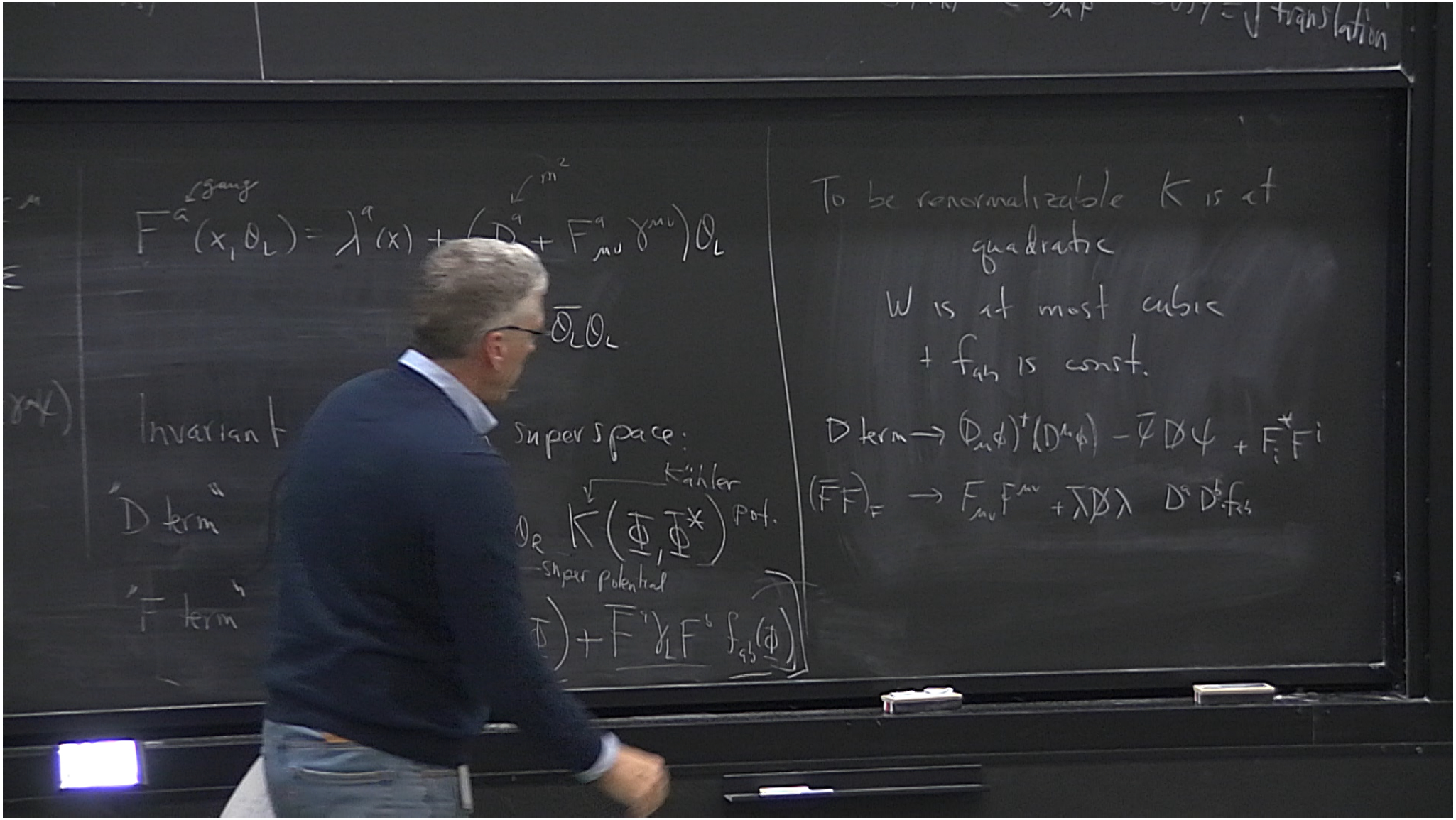
$b_{N=4} = 0$.

Better UV properties because in loops bosons & fermions cancel.


— not internal symmetries
eg if $\psi \to M\psi$ then $\partial_\mu \psi^\dagger \gamma^\mu \psi$ inv $\Rightarrow \psi^\dagger \psi$ inv
How do you construct $\mathcal{N}=1$ supersymmetric lagrangians in D=4?
$x^\mu \to x^\mu + \epsilon^\mu$
$\theta \to \theta + \varepsilon$
"Superspace" Can represent supersymmetry on fields through the trick of introducing fermionic spacetime coordinates
$\Phi(x^\mu, \theta_L) = \varphi(x) + \theta_L \psi(x) + \bar{\theta}_L \theta F(x)$
$\theta = \begin{pmatrix} \theta_L \\ \theta_R = \bar{\theta}^{\dot{\alpha}} \end{pmatrix}$
$\int d^2\theta_L \Phi = F$


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"Superspace" Can represent supersymmetry on fields
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$\Phi(x^\mu, \theta_L) = \varphi(x) + \theta_L \psi(x) + \overline{\theta_L} \theta F(x)$
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Invari
"D


translation
gauge
To be renormalizable K is at most quadratic
W is at most cubic
+ f_{ab} is const.
Invariant
Kähler
"D term"
"F term"


translation
gauge
m^2
$F^a(x,\theta_L) = \lambda^a(x) + (D^a + F^a_{\mu\nu}\gamma^{\mu\nu})\theta_L$
$\bar{\theta}_L\theta_L$
Invariant
superspace:
"D term"
Kähler
θ_R $K(\Phi,\Phi^*)$ pot.
-super Potential
"F term"
$\bar{F}^i\gamma_L F^b f_{ab}(\Phi)$
To be renormalizable K is at quadratic
W is at most cubic
+ f_{ab} is const.
D term $\rightarrow (D_\mu\phi)^\dagger(D^\mu\phi) - \bar{\psi}\not{D}\psi + F_i^\dagger F^i$
$(FF)_F \rightarrow F_{\mu\nu}F^{\mu\nu} + \bar{\lambda}\not{D}\lambda$ $D^a D^b f_{ab}$

translation

gauge

$$F^a(x,\theta_L) = \lambda^a(x) + (D^a + F^a_{\mu\nu}\gamma^{\mu\nu})\theta_L$$

$$+ (\not{\partial}\lambda)^a \bar{\theta}_L \theta_L$$

m^2

Invariant actions in superspace:

"D term" $\mathcal{L}_D = \int d^2\theta_L d^2\theta_R \, K(\Phi, \Phi^*)$ Kähler Pot.

"F term" $\mathcal{L}_F = \int d^2\theta_L \left[W(\Phi) + \bar{F}^i \gamma_L F^b f_{ab}(\Phi) \right]$ Super Potential

To be renormalizable K is at quadratic

W is at most cubic

+ f_{ab} is const.

D term $\rightarrow (D_\mu \phi)^\dagger (D^\mu \phi) - \bar{\psi} \not{D} \psi + F_i^* F^i$

$(FF)_F \rightarrow F_{\mu\nu} F^{\mu\nu} + \bar{\lambda} \not{D} \lambda \quad D^a D^b f_{ab}$

$W_F \rightarrow \frac{\partial^2 W}{\partial \phi^i \partial \phi^j} \bar{\psi}^i \gamma_L \psi^j + \frac{\partial W}{\partial \phi_i} F^i + \text{c.c.}$

Gaussian integral $\Rightarrow$ saddle point for F, D

$F^i = \left(\frac{\partial W}{\partial \phi^i}\right)^*$ $\qquad D^a = f^{abc}\,\phi^\dagger T_b \phi$

$\mu^2 \phi^\dagger \phi$

$\mathcal{L} = y\,\bar{f} f \phi - \lambda\,\phi^\dagger \phi\, \bar{f} f$

Rule for compu

is $\quad \alpha_y = \frac{\partial^2 W}{\partial f^2}(\bar{f}, f)$

$V =$

translation

$$F^a(x,\theta_L) = \lambda^a(x) + \left(D^a + F^a_{\mu\nu}\gamma^{\mu\nu}\right)\theta_L + (\not{D}\lambda)^a\,\bar{\theta}_L\theta_L$$

(gauge; m^2)

To be renormalizable K is at quadratic

W is at most cubic

+ f_{ab} is const.

Invariant actions in superspace:

"D term" $\mathcal{L}_D = \int d^2\theta_L d^2\theta_R\, K(\Phi,\Phi^*)$ (Kähler pot.)

"F term" $\mathcal{L}_F = \int d^2\theta_L \left[W(\Phi) + \bar{F}^i\gamma_L F^b f_{ab}(\Phi)\right]$ (super potential)

D term $\rightarrow (D_\mu\phi)^\dagger(D^\mu\phi) - \bar{\psi}\not{D}\psi + F_i^\dagger F^i$ ($D^a\phi^\dagger T_a\phi$)

$(FF)_F \rightarrow F_{\mu\nu}F^{\mu\nu} + \bar{\lambda}\not{D}\lambda$ $D^a D^b f_{ab}$

$W_F \rightarrow \frac{\partial^2 W}{\partial\phi^a\partial\phi^b}\bar{\psi}^i\gamma_L\psi^j + \frac{\partial W}{\partial\phi_i}F^i + \text{c.c.}$

Gaussian integral ⇒ saddle point for F, D

$$F^i = \left(\frac{\partial W}{\partial \phi^i}\right)^* \qquad D^a = f^{ab} \phi^\dagger T_b \phi$$

$$V = \sum_i \left|\frac{\partial W}{\partial \phi^i}\right|^2 + \sum_a (\phi^\dagger T_a \phi)(\phi^\dagger T_b \phi) f^{ab}$$

V_F V_D

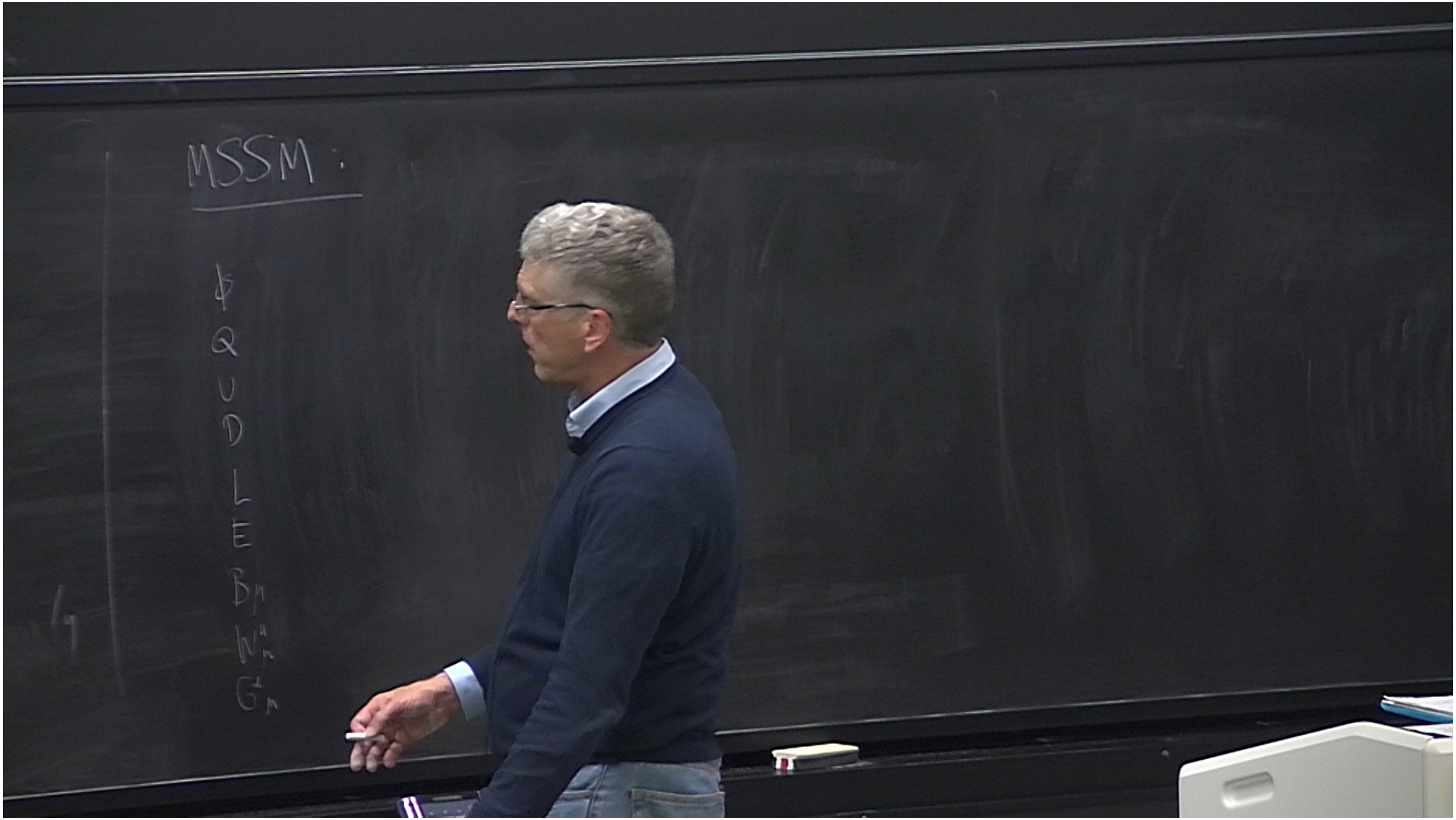
$\mu^2 \phi^\dagger \phi$

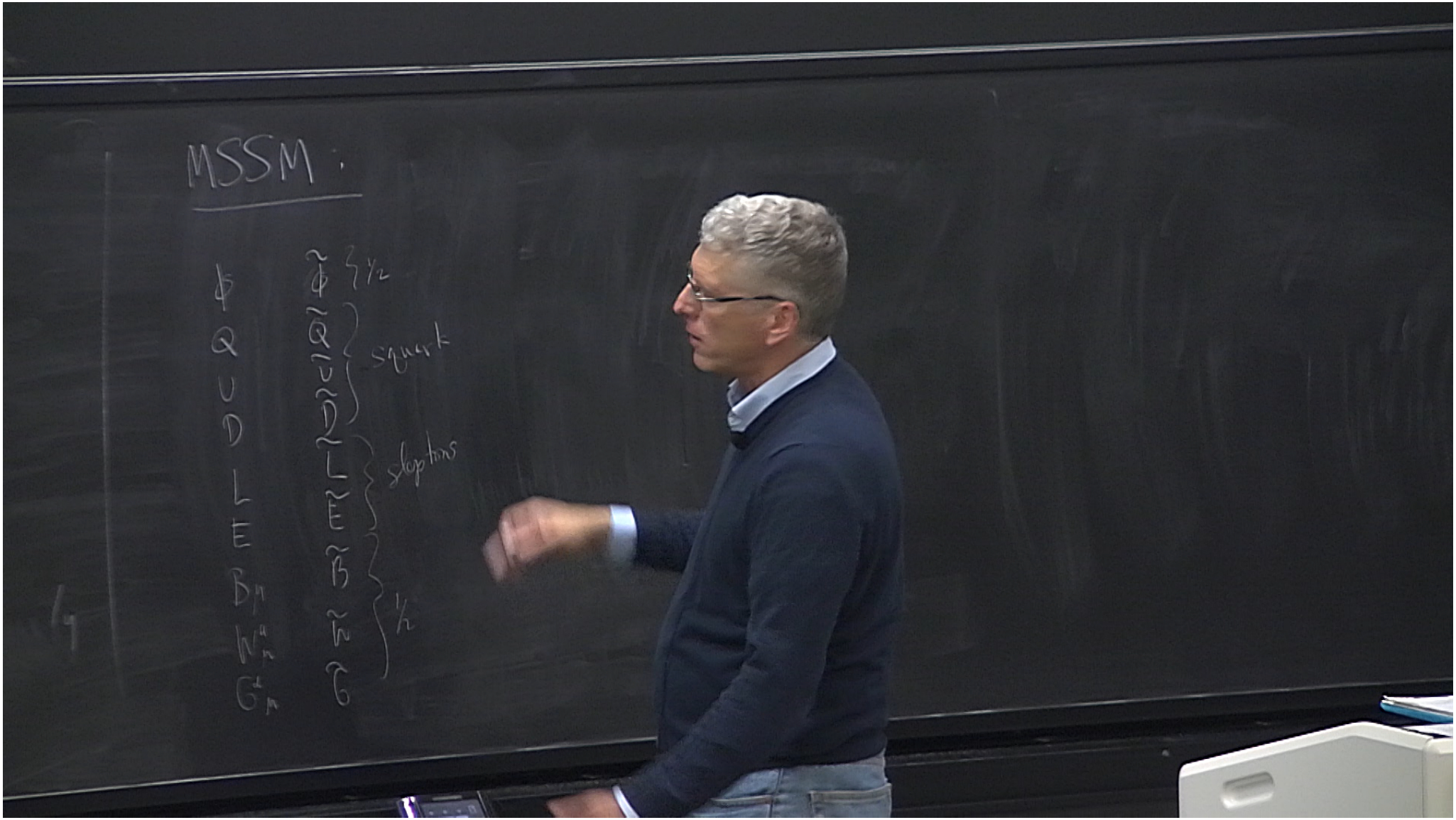
$\mathcal{L} = y \bar{f} f \phi - \lambda \phi^\dagger \phi \bar{f}^\dagger f$

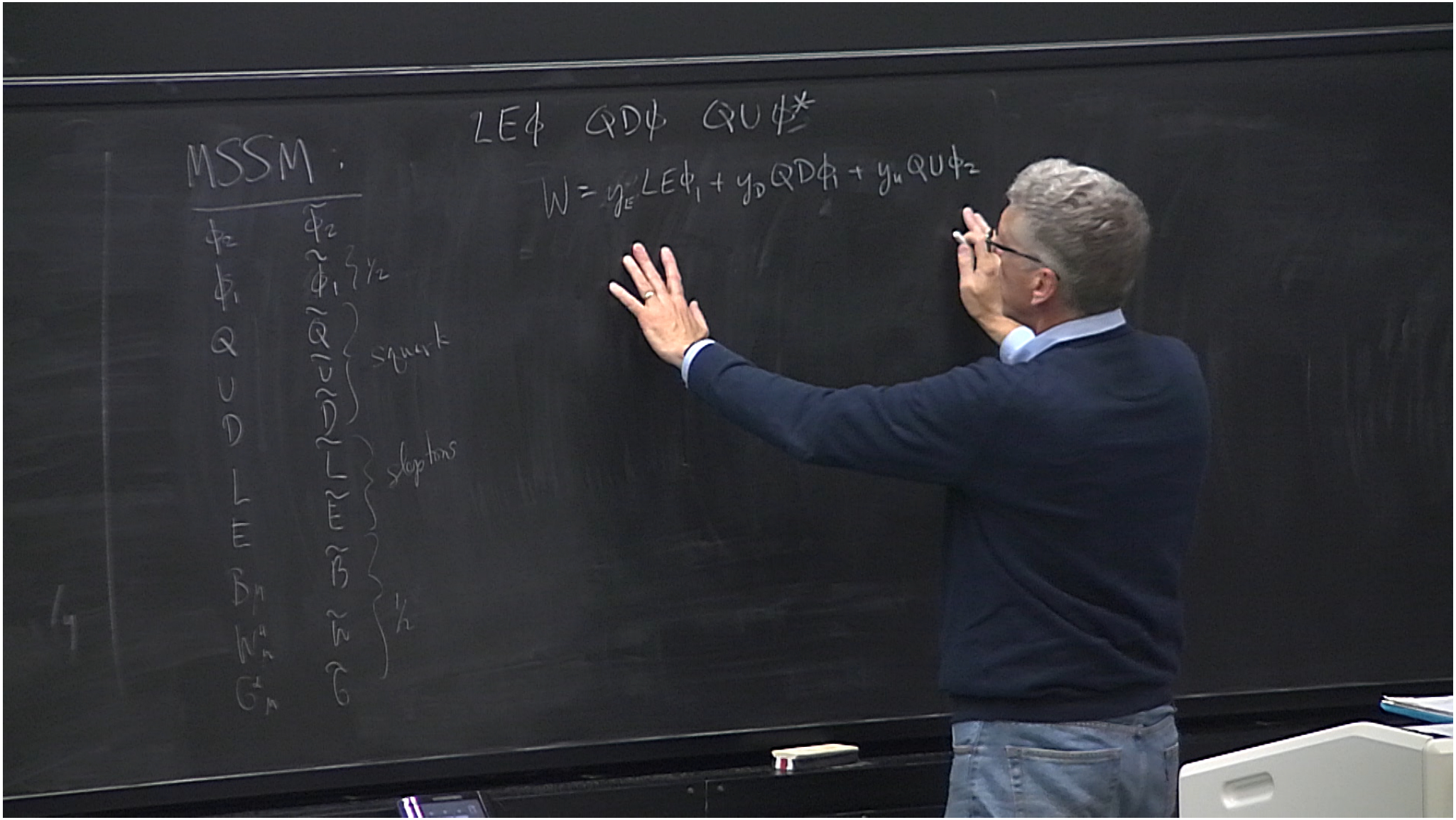
Rule for compu

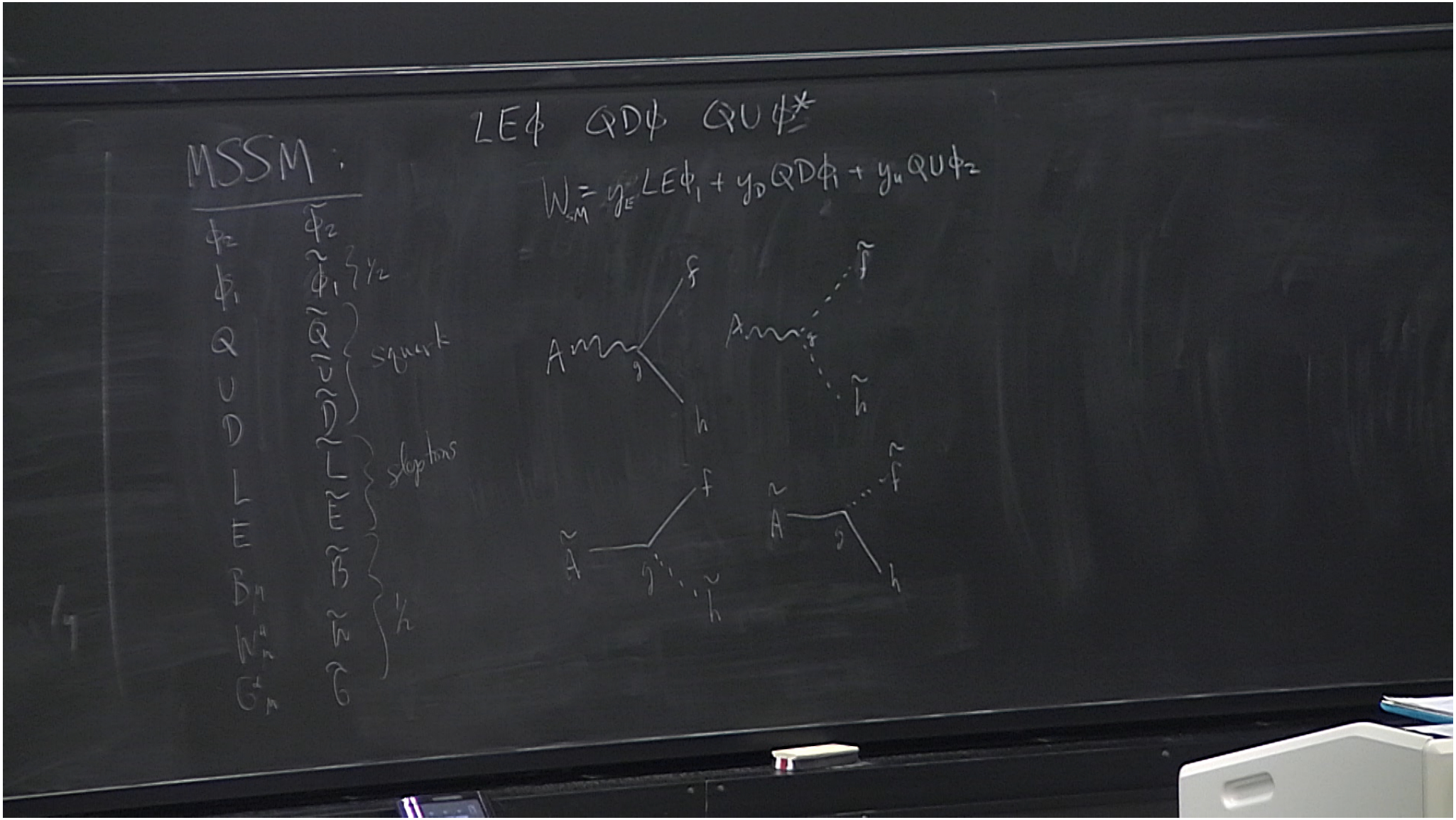
is $\alpha_y = \frac{\partial^2 W}{\partial f^2} (\bar{f}_L f)$

$V =$


MSSM:


MSSM:
squark
sleptons


MSSM:
LEϕ QDϕ QUϕ*
W = y_E LEϕ_1 + y_D QDϕ_1 + y_u QUϕ_2
squark
sleptons
½


MSSM:
$LE\phi_1$ $QD\phi_1$ $QU\phi_2^*$
$W_{MSSM} = y_E LE\phi_1 + y_D QD\phi_1 + y_u QU\phi_2$
squark
sleptons

translation

F^a (gauge) $(x, \theta_L) = \lambda^a(x) + (D^a + F^a_{\mu\nu}\gamma^{\mu\nu})\theta_L$ (m^2)

$+ (\not{\partial}\lambda)^\dagger \bar{\theta}_L \theta_L$

Invariant actions in superspace:

"D term" $\mathcal{L}_D = \int d^2\theta_L d^2\theta_R \, K(\Phi, \Phi^*)$ (Kähler pot.)

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To be renormalizable K is at quadratic

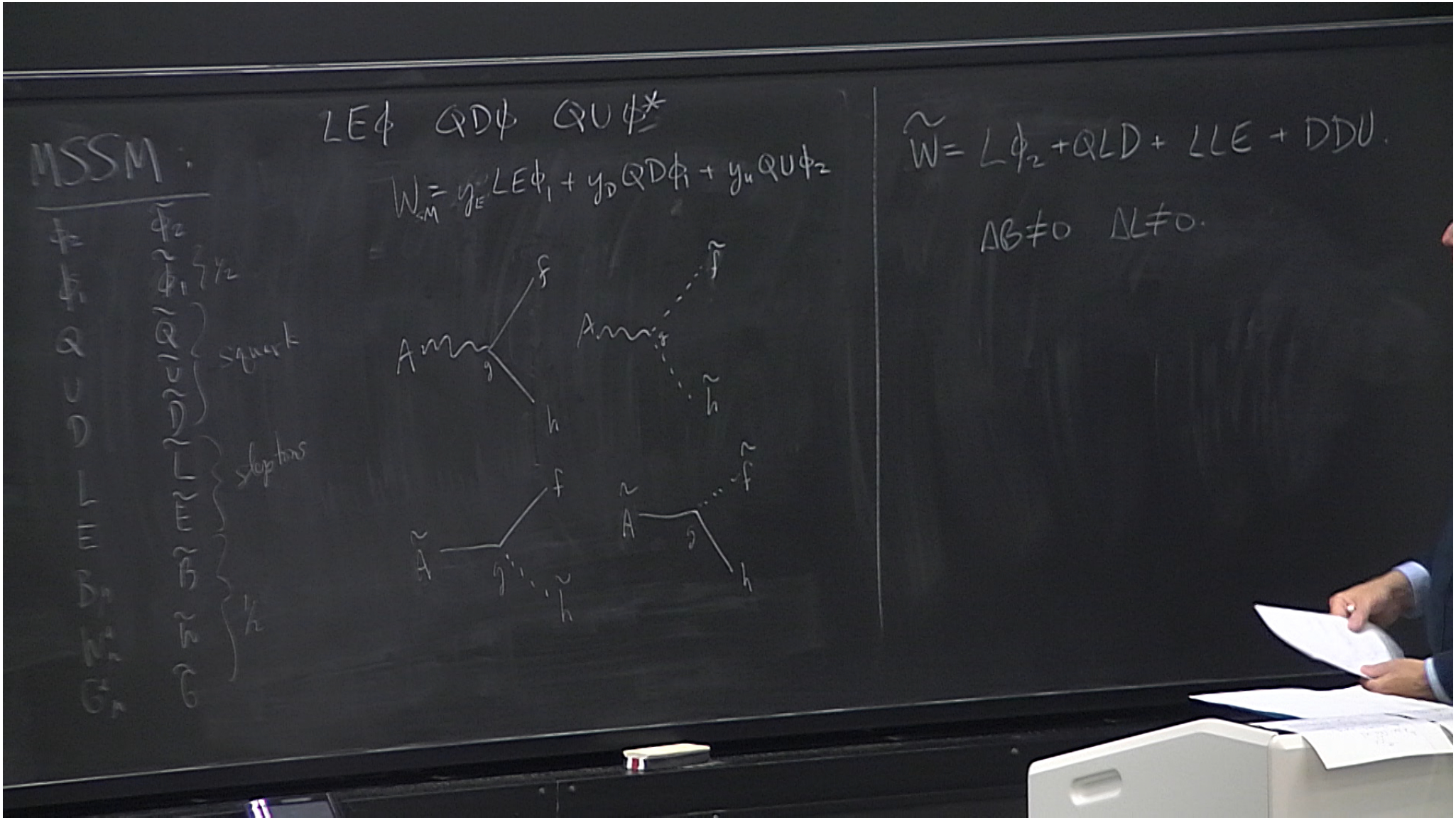
W is at most cubic

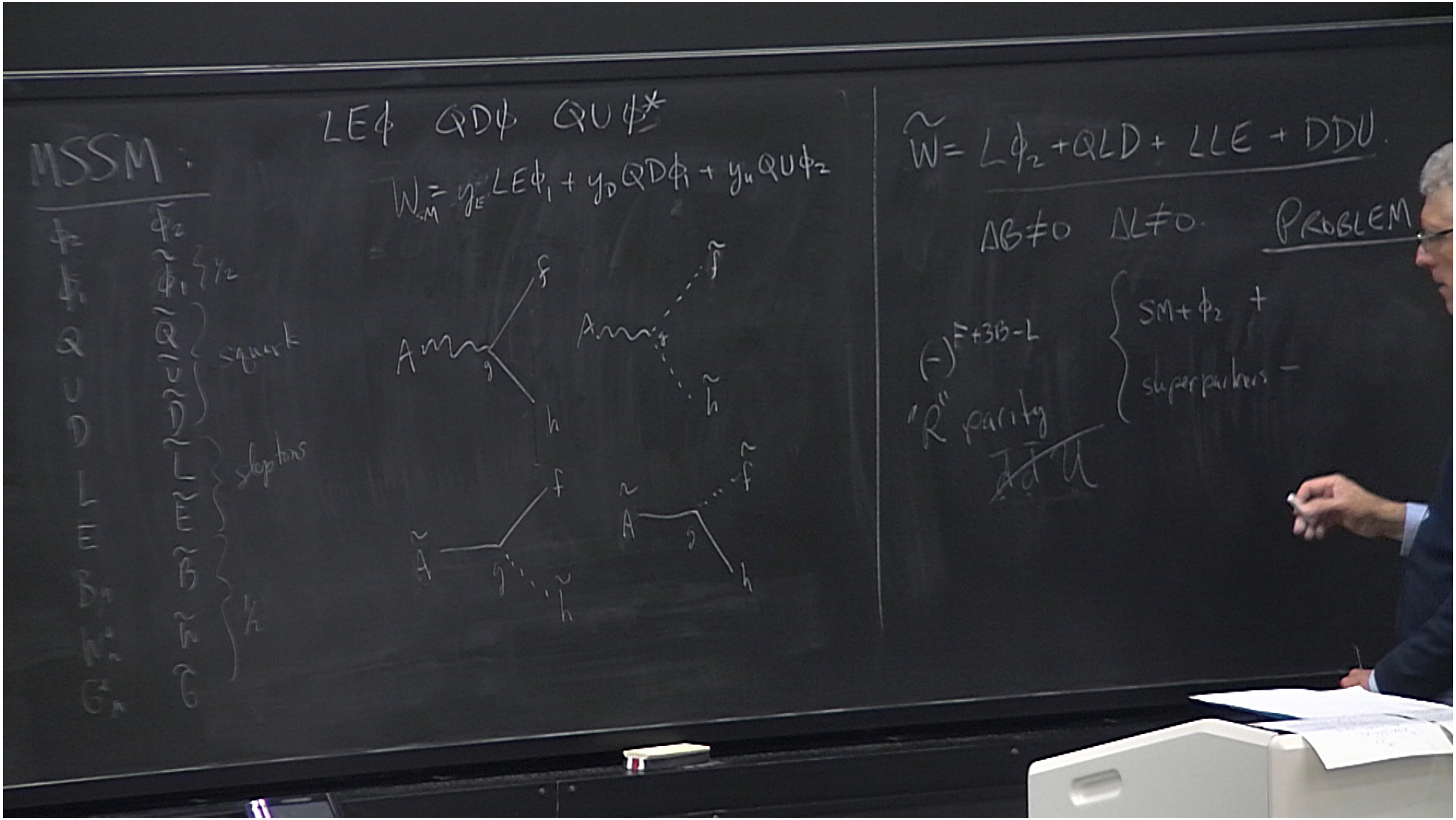
+ f_{ab} is const.

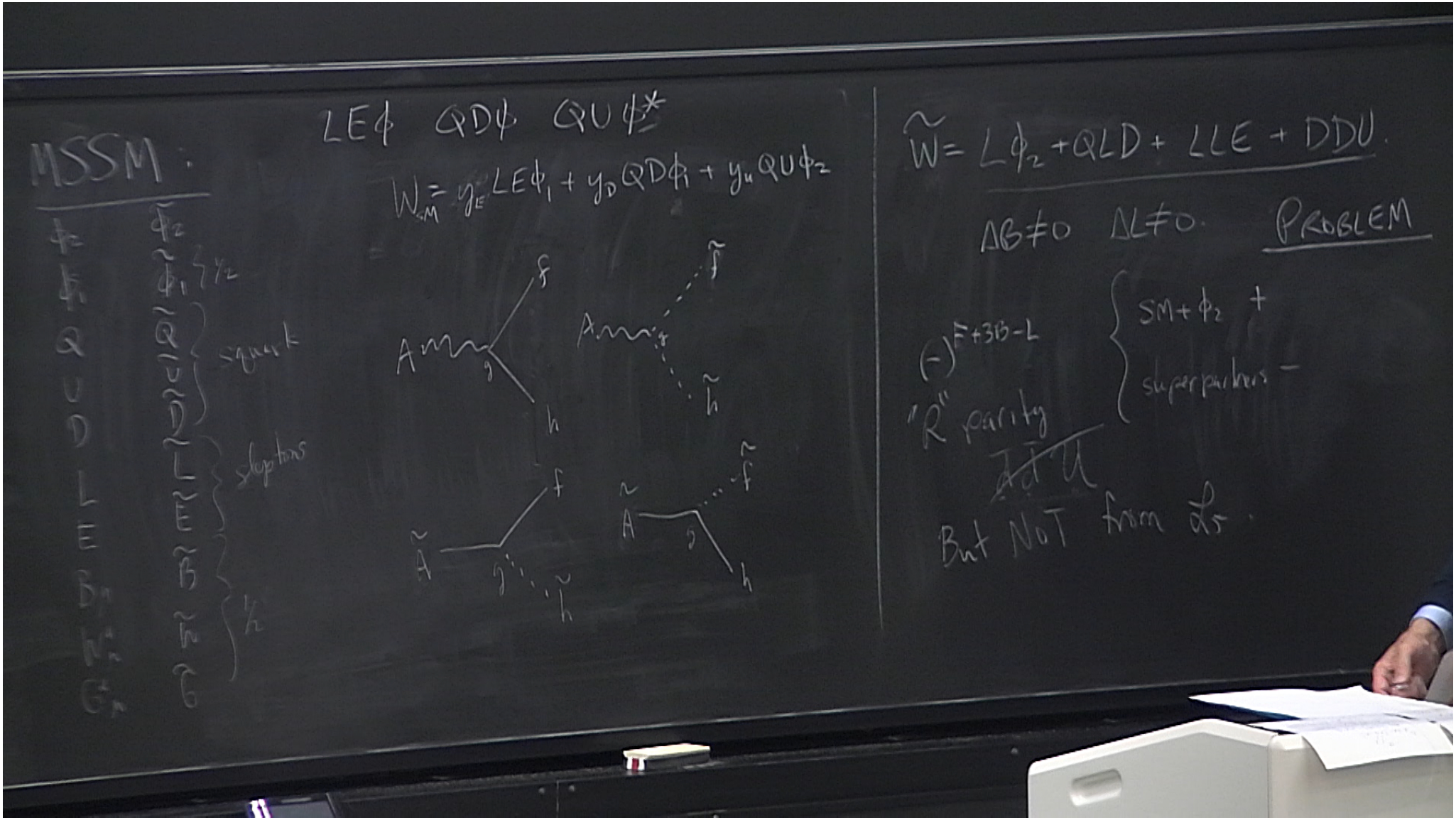
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MSSM:
LEϕ QDϕ QUϕ*
$W_M = y_E LE\phi_1 + y_D QD\phi_1 + y_u QU\phi_2$
squark
sleptons
$\tilde{W} = L\phi_2 + QLD + LLE + DDU$
$\Delta B \neq 0$ $\Delta L \neq 0$.


MSSM:
LEϕ QDϕ QUϕ*
$W_M = y_E LE\phi_1 + y_D QD\phi_1 + y_u QU\phi_2$
squark
sleptons
$\tilde{W} = L\phi_2 + QLD + LLE + DDU$
ΔB≠0 ΔL≠0. PROBLEM
$(-)^{F+3B-L}$
"R" parity
SM + ϕ_2 +
superpartners


MSSM:
LEϕ QDϕ QUϕ*
$W_M = y_E LE\phi_1 + y_D QD\phi_1 + y_u QU\phi_2$
squark
sleptons
$\tilde{W} = L\phi_2 + QLD + LLE + DDU$
$\Delta B \neq 0$ $\Delta L \neq 0$. PROBLEM
$(-)^{F+3B-L}$
"R" parity
SM + ϕ_2 +
superpartners −
But NOT from $\mathcal{L}_S$.