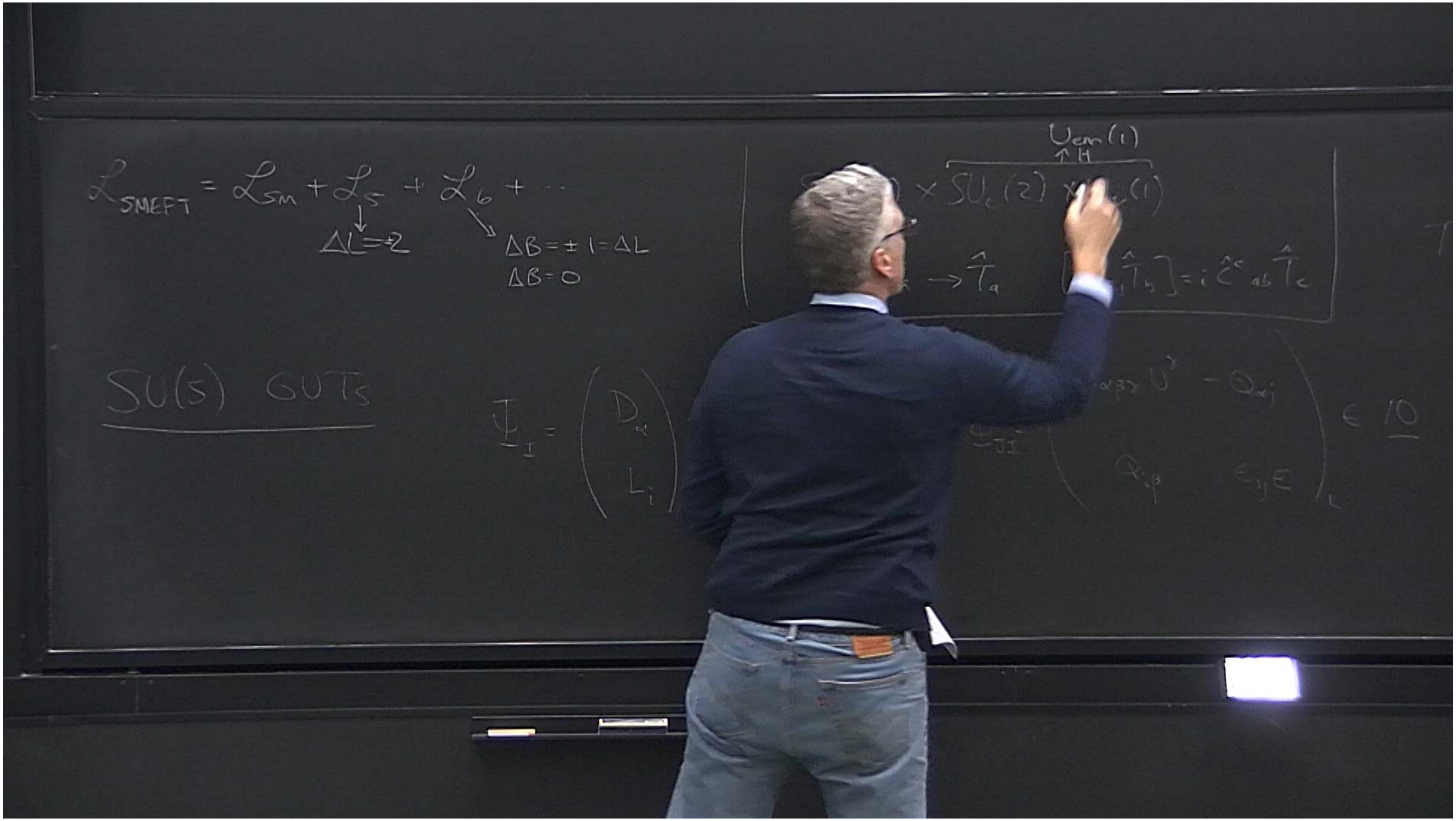
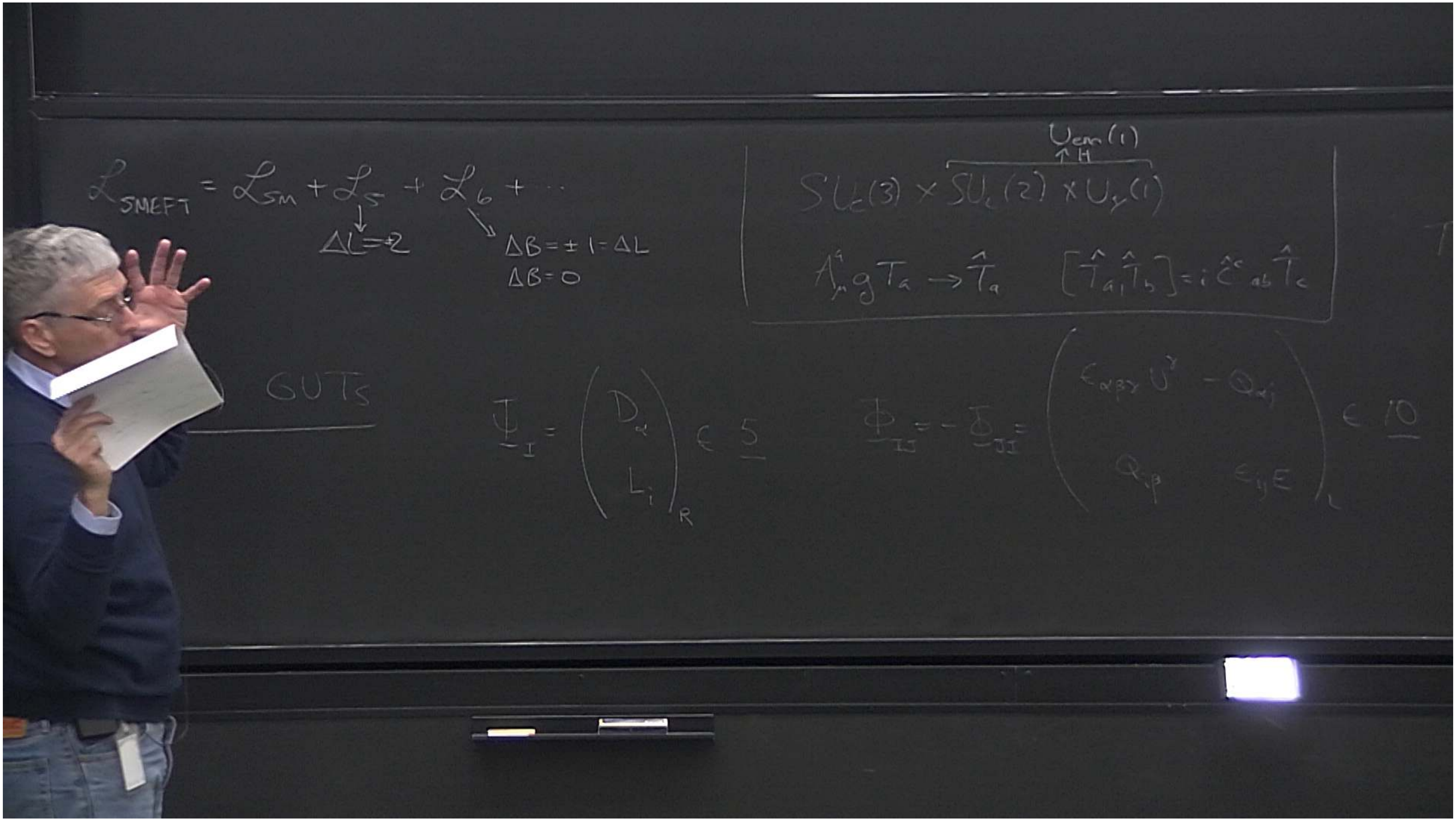
Title: PSI 2017/2018 - Beyond Standard Model - Lecture 4

Date: Mar 02, 2018 09:00 AM

URL: http://pirsa.org/18030075

Abstract:


L_SMEFT = L_SM + L_5 + L_6 + ...
ΔL = ±2
ΔB = ±1 = ΔL
ΔB = 0
SU(5) GUTs
U_em(1)
∈ 10

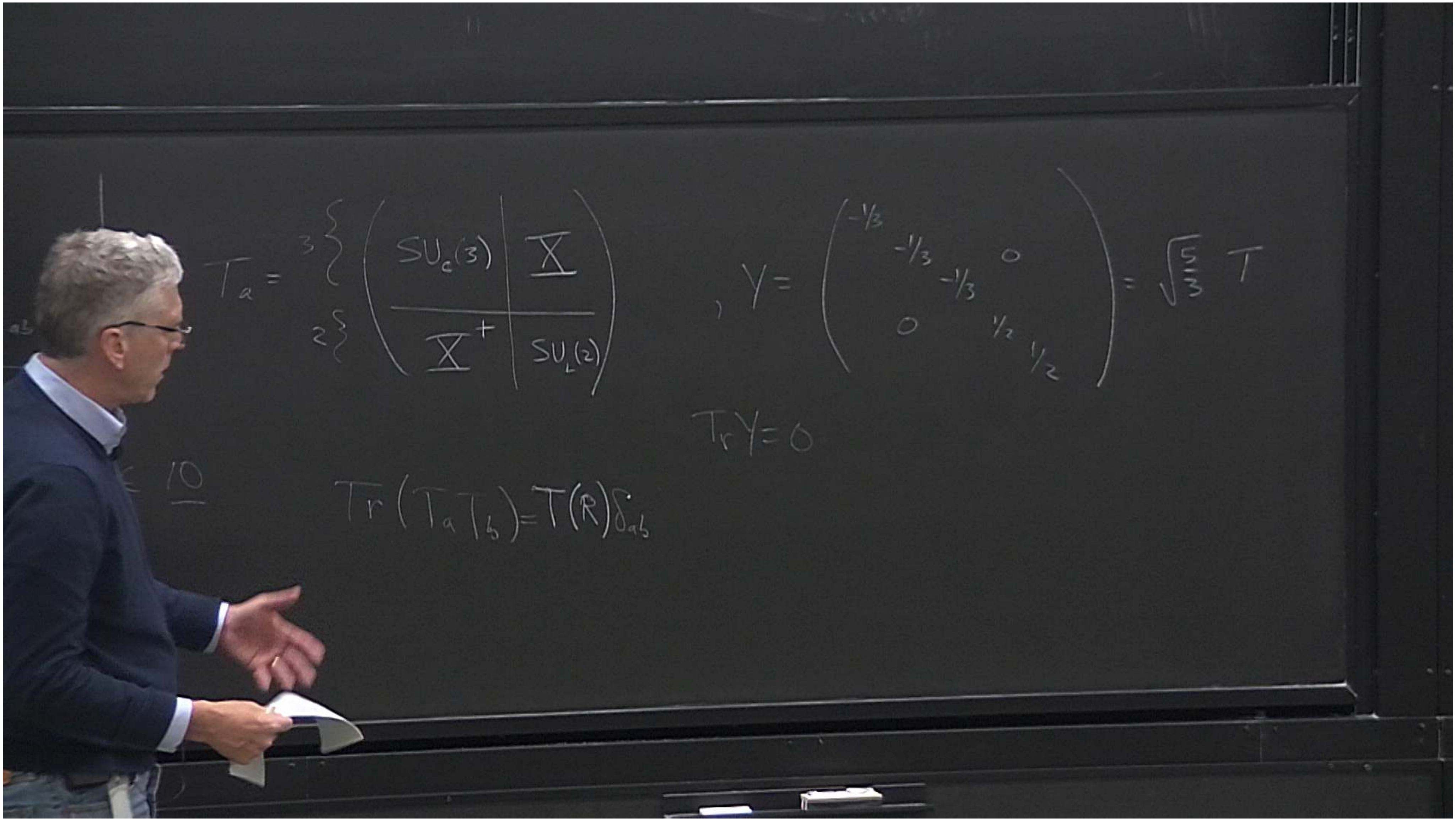

$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \mathcal{L}_5 + \mathcal{L}_6 + \cdots$
$\Delta L = 2$
$\Delta B = \pm 1 = \Delta L$
$\Delta B = 0$
$U_{em}(1)$
$SU_c(3) \times SU_L(2) \times U_Y(1)$
$A^a_\mu g T_a \to \hat{T}_a$
$[\hat{T}_a, \hat{T}_b] = i \hat{C}^c{}_{ab} \hat{T}_c$
GUTs
$\Psi_I = \begin{pmatrix} D_\alpha \\ L_i \end{pmatrix}_R \in \underline{5}$
$\Phi_{IJ} = -\Phi_{JI} = \begin{pmatrix} \epsilon_{\alpha\beta\gamma} U^\gamma & -Q_{\alpha j} \\ Q_{i\beta} & \epsilon_{ij} E \end{pmatrix}_L \in \underline{10}$

$\hat{T}_c$

$\in \underline{10}$

$$T_a = \begin{matrix} 3\{ \\ 2\{ \end{matrix} \left(\begin{array}{c|c} SU_c(3) & \underline{X} \\ \hline \underline{X}^+ & SU_L(2) \end{array} \right), \quad Y = \begin{pmatrix} -1/3 & & & & \\ & -1/3 & & 0 & \\ & & -1/3 & & \\ & 0 & & 1/2 & \\ & & & & 1/2 \end{pmatrix} = \sqrt{\frac{5}{3}}\, T$$

$$\mathrm{Tr}\, Y = 0$$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$Tr Y = 0$
$Tr(T_a T_b) = T(R)\delta_{ab}$
10

$$T_a = \begin{array}{c} 3\{ \\ 2\{ \end{array} \left(\begin{array}{c|c} SU_c(3) & X \\ \hline X^{\dagger} & SU_L(2) \end{array} \right), \quad Y = \begin{pmatrix} -1/3 & & & & \\ & -1/3 & & 0 & \\ & & -1/3 & & \\ & 0 & & 1/2 & \\ & & & & 1/2 \end{pmatrix} = \sqrt{\tfrac{5}{3}}\, T$$

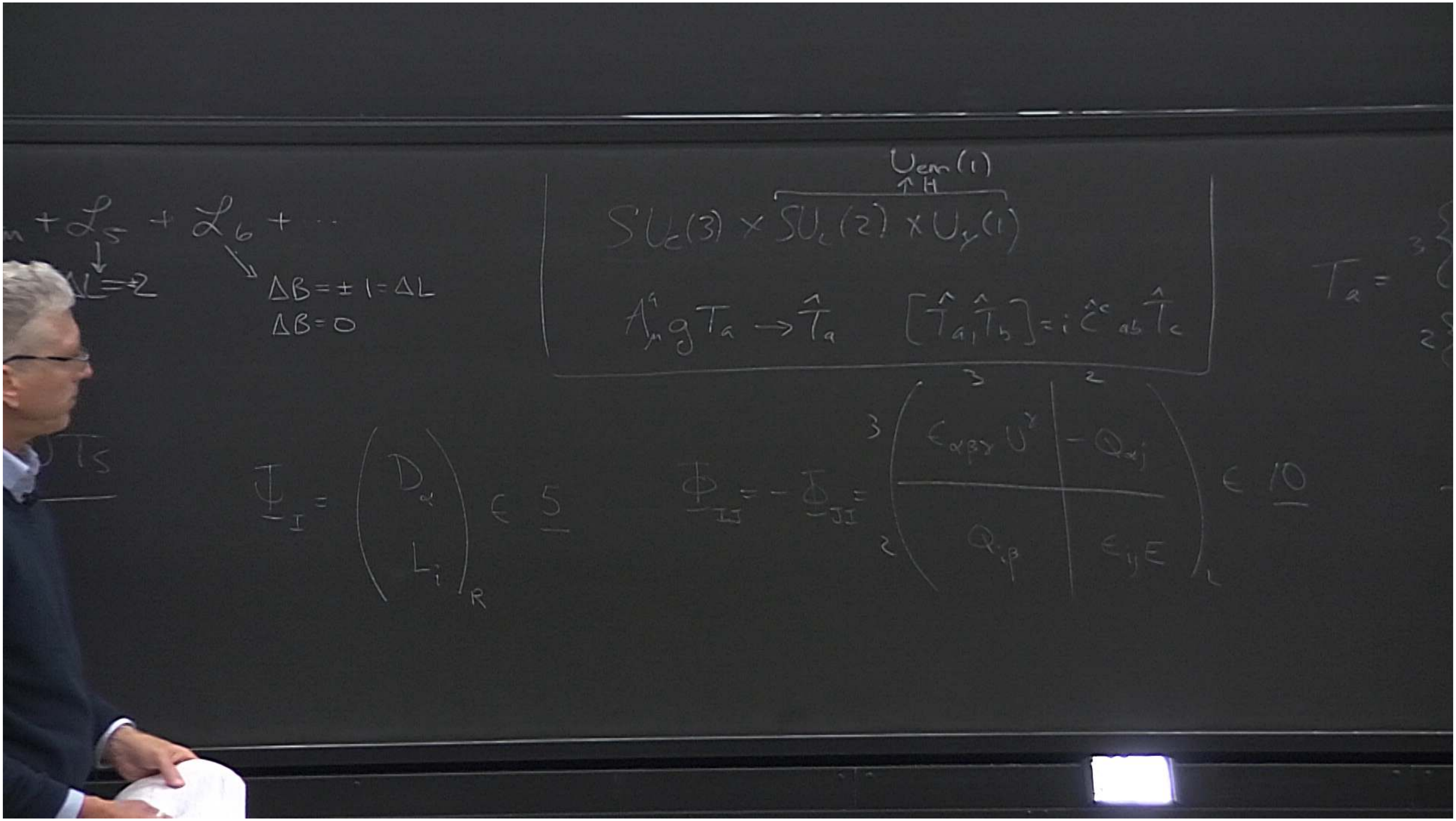
$$\mathrm{Tr}\, Y = 0$$

$$\in \underline{10}$$

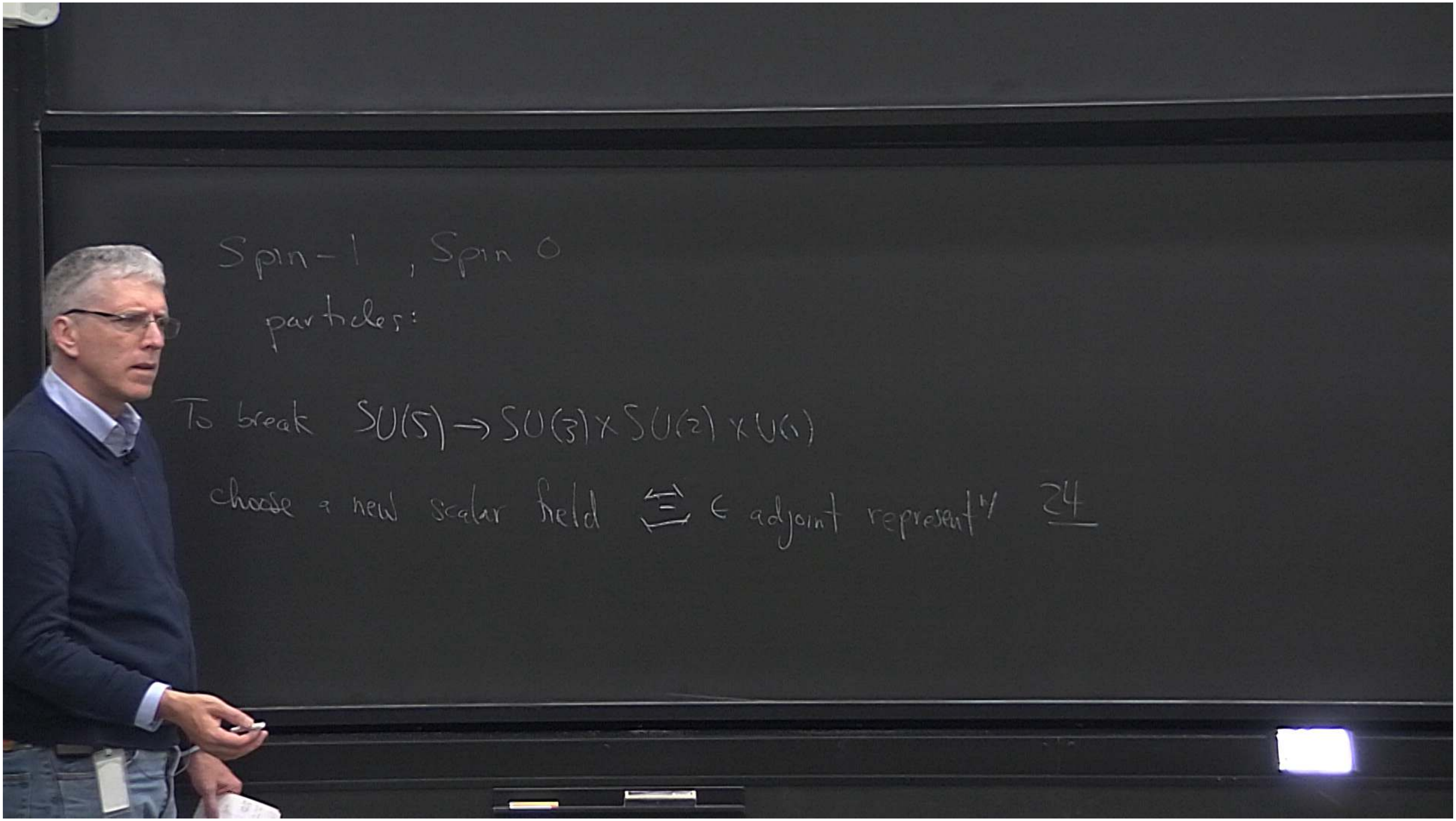
$$\mathrm{Tr}(T_a T_b) = T(R)\,\delta_{ab}$$

$$\tau_a = \tfrac{1}{2}\sigma_a$$

$$T(F) = 1/2$$



$+ \mathcal{L}_5 + \mathcal{L}_6 + \cdots$
$\Delta L = 2$
$\Delta B = \pm 1 = \Delta L$
$\Delta B = 0$
$U_{em}(1)$
$SU_c(3) \times SU_L(2) \times U_Y(1)$
$A^a_\mu g T_a \to \hat{T}_a \quad [\hat{T}_a, \hat{T}_b] = i \hat{C}^c{}_{ab} \hat{T}_c$
$T_a =$
$\Phi_I = \begin{pmatrix} D_\alpha \\ L_i \end{pmatrix}_R \in \underline{5}$
$\Phi_{IJ} = -\Phi_{JI} = \begin{pmatrix} \epsilon_{\alpha\beta\gamma} U^\gamma & -Q_{\alpha j} \\ Q_{i\beta} & \epsilon_{ij} E \end{pmatrix}_L \in \underline{10}$


Spin-1 , Spin 0
particles:
To break SU(5) → SU(3) x SU(2) x U(1)
choose a new scalar field Ξ ∈ adjoint represent^n 24

Spin-1, Spin 0

particles:

To break $SU(5) \to SU(3) \times SU(2) \times U(1)$

choose a new scalar field $\Xi \in$ adjoint represent'n $\underline{24}$

$\delta \Xi = i[\omega^a T_a, \Xi]$ $\quad \langle \Xi \rangle \neq 0$ $\quad$ Choose V s.t. $\langle \Xi \rangle = \begin{pmatrix} w & & & & 0 \\ & w & & & \\ & & w & & \\ & & & -\frac{3}{2}w & \\ 0 & & & & -\frac{3}{2}w \end{pmatrix} \propto$

Spin-1, Spin 0 particles:

Choose

To break $SU(5) \to SU(3) \times SU(2) \times U(1)$

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Choose $H \in \underline{5} = \begin{pmatrix} \chi_\alpha \\ \hline \phi_i \end{pmatrix}^{3}$

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \end{pmatrix}$$

$v \simeq 246$ GeV

$\omega \gg v$.

$$\begin{pmatrix} 0 \\ \tfrac{3}{2}\omega \quad -\tfrac{3}{2}\omega \end{pmatrix} \propto Y$$

Choose $H \in \underline{5} = \begin{pmatrix} \chi_\alpha \\ \hline \phi_i \end{pmatrix}^{3} \leftarrow$ mass $\sim \omega$

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$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \end{pmatrix}$$

$$\mathbb{X}_\mu = \begin{pmatrix} X \\ Y \end{pmatrix}$$

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$$\mathbb{X}_\mu = \begin{pmatrix} X \\ Y \end{pmatrix}$$

$$\delta \mathbb{X} = [\, iY, \mathbb{X} \,] \longrightarrow y_X = 5/6$$

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$$\mathbb{X}_\mu = \begin{pmatrix} X \\ Y \end{pmatrix}$$

$$\delta \mathbb{X} = [iY, \mathbb{X}] \longrightarrow y_{\mathbb{X}} = 5/6$$

$$Q(X) = 5/6 + 1/2 = 4/3$$

$$Q(Y) = 5/6 - 1/2 = 2/3$$

$$\mathcal{L} \supset \bar{\Psi} \not{D} \Psi = \bar{\Psi}\left(\not{\partial} - i g A_\mu^A T_A\right)\Psi$$

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$$\bar{D}\left(\gamma\, g X_\mu\right) L + \bar{L}\, g X^\dagger D + \cdots$$

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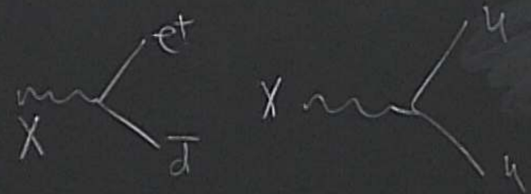
$$\bar{D}\left(\overset{\alpha}{g}\, X_\mu\right) L + \bar{L}\, g\, X^\dagger D + \cdots$$



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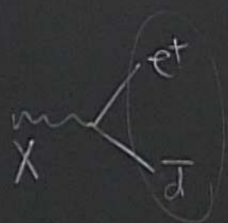
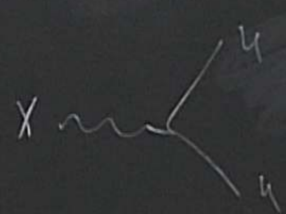




$B = -1/3$ $B - L = 2/3$

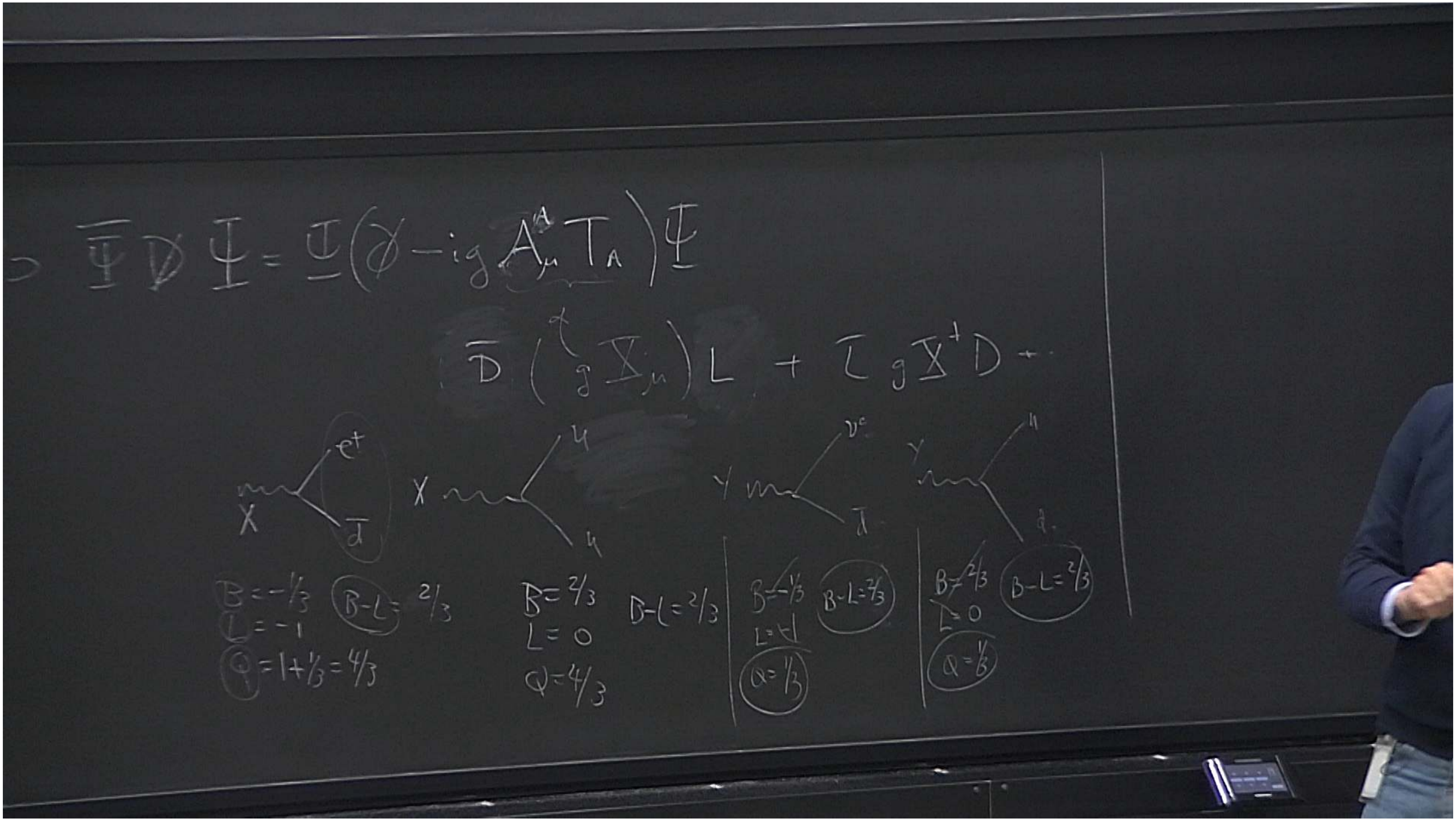
$L = -1$

$Q = 1 + 1/3 = 4/3$

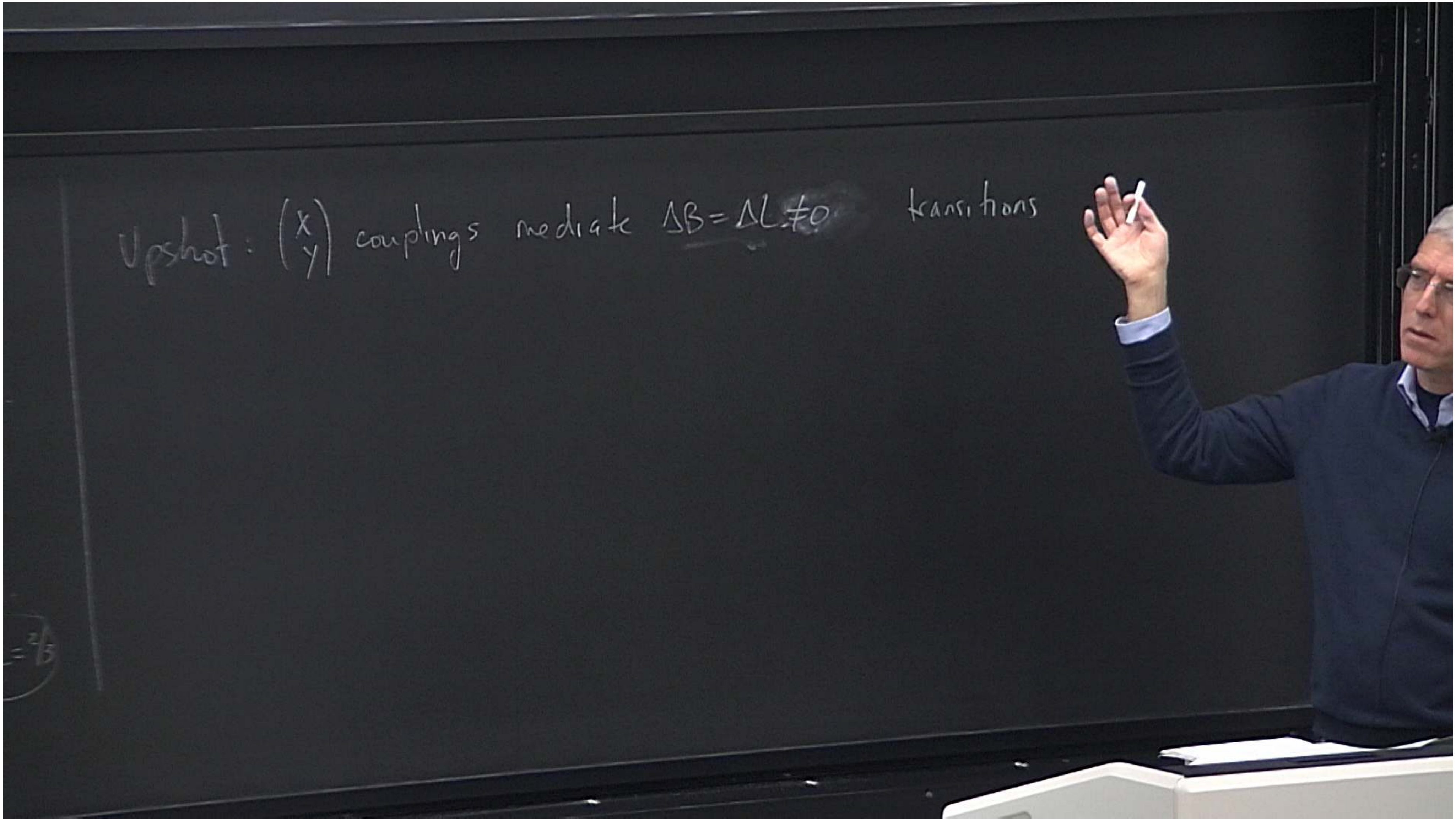
$B = 2/3$ $B - L = 2/3$

$L = 0$

$Q = 4/3$



$\bar{\Psi} \not{D} \Psi = \bar{\Psi}(\not{\partial} - i g A_\mu^A T_A)\Psi$
$\bar{D}(g X_\mu) L + \bar{L} g X^+ D + \ldots$
X
e^+
$\bar{d}$
$B = -1/3$ $B-L = 2/3$
$L = -1$
$Q = 1 + 1/3 = 4/3$
X
u
u
$B = 2/3$ $B-L = 2/3$
$L = 0$
$Q = 4/3$
Y
ν^c
$\bar{d}$
$B = -1/3$ $B-L = 2/3$
$L = +1$
$Q = 1/3$
Y
u
d
$B = 2/3$ $B-L = 2/3$
$L = 0$
$Q = 1/3$


Upshot: (X Y) couplings mediate ΔB = ΔL ≠ 0 transitions

Upshot: $\begin{pmatrix} X \\ Y \end{pmatrix}$ couplings mediate $\Delta B = \Delta L \neq 0$ transitions

 $\sim \boxed{\frac{g^2}{M_X^2}} (\ \ )(\ \ ) \subset \mathcal{L}_6$

proton lifetime

$$\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$$

$$\delta \Phi = [iY, \Phi]$$

$$v \simeq 246 \text{ GeV}$$

$$\omega \gg v.$$

$$M_X = M_Y = g\omega$$

$$Q(X) = 5/$$

$$Q(Y) = 5/$$

$\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$

$\delta \Phi = [iY, \Phi] \longrightarrow$

$v \simeq 246 \text{ GeV}$

$w \gg v.$

$M_X^2 = M_Y^2 = g^2 w^2 \frac{25}{8}$

$Q(X) = 5/6 + 1/2$

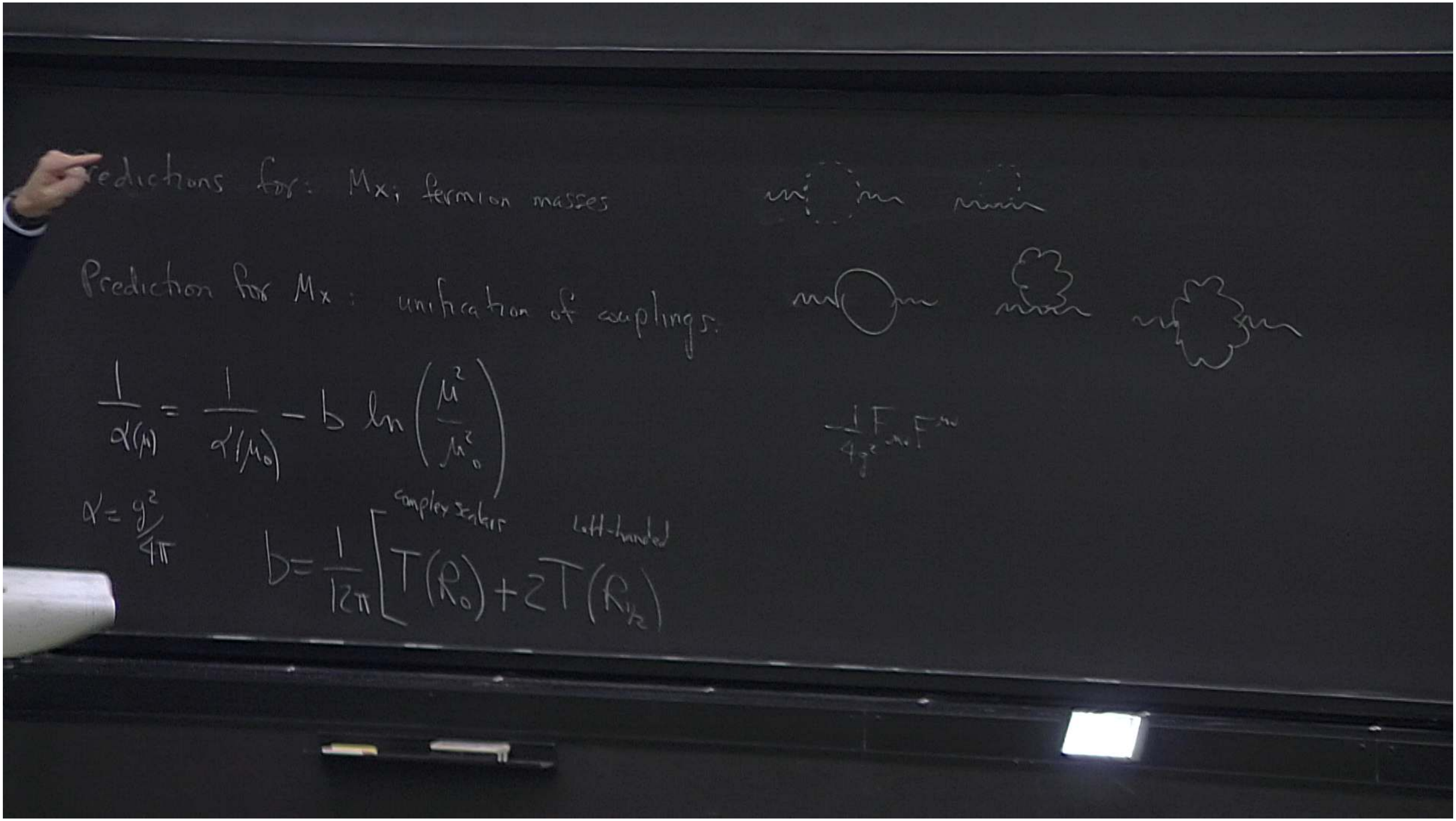
$Q(Y) = 5/6 - 1/2$

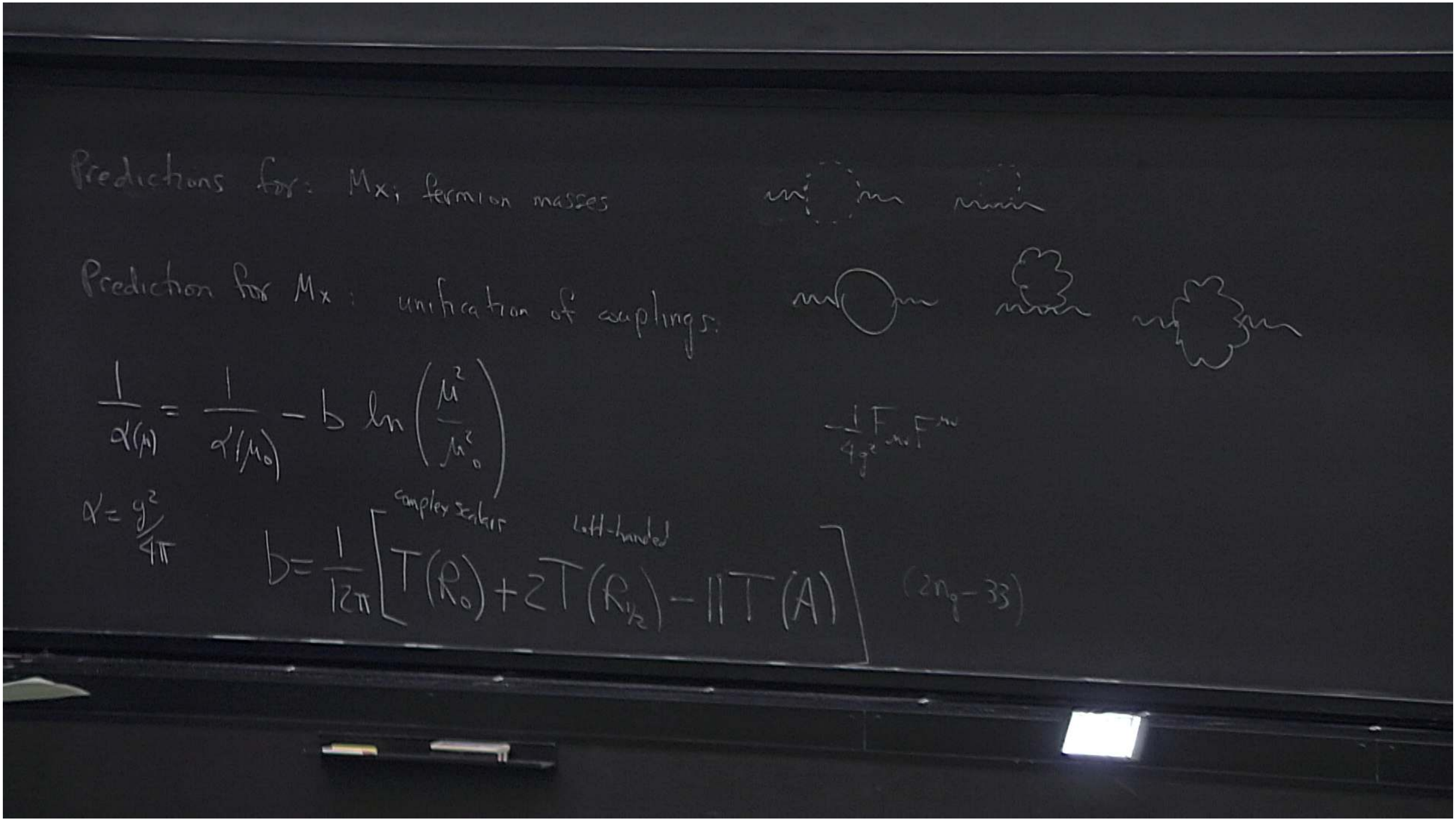
Upshot: $\begin{pmatrix} X \\ Y \end{pmatrix}$ couplings mediate $\Delta B = \Delta L \neq 0$ transitions

fixed by $\alpha_1\ \alpha_2\ \alpha_3$ of SM

 $\sim \boxed{\dfrac{g^2}{M_X^2}} (\quad)(\quad) \subset \mathcal{L}_6$

proton lifetime $\Rightarrow M_X > 10^{16}$ GeV (more precise value possible here)


Predictions for: M_X; fermion masses
Prediction for M_X: unification of couplings.
$\frac{1}{\alpha(\mu)} = \frac{1}{\alpha(\mu_0)} - b \ln\left(\frac{\mu^2}{\mu_0^2}\right)$
$-\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu}$
$\alpha = g^2/4\pi$
complex scalar
Left-handed
$b = \frac{1}{12\pi}\left[T(R_0) + 2T(R_{1/2})\right.$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$\alpha = g^2/4\pi$
complex scalar
Left-handed
$b = \frac{1}{12\pi}\left[T(R_0) + 2T(R_{1/2}) - 11T(A)\right]$
$(2n_g - 33)$

$[T_a, T_b] = i f_{ab}{}^c T_c$

$$\Phi_{IJ} = -\Phi_{JI} = \begin{array}{c|cc} & 3 & 2 \\ \hline 3 & \epsilon_{\alpha\beta\gamma} U^\gamma & -Q_{\alpha j} \\ 2 & Q_{i\beta} & \epsilon_{ij} E \end{array}_L \in \underline{10}$$

$2\Big\{ \left(X^+ \;\middle|\; SU_L(2) \right)$ $\qquad 0 \qquad 1/2$

$\mathrm{Tr}\, Y = 0$

$$\mathrm{Tr}(T_a T_b) = \boxed{T(R)}\, \delta_{ab}$$

$T_a = \frac{1}{2}\sigma_a$

$T(F) = 1/2$

$\Downarrow$

$T(A) = N$ for $SU(N)$

ngs:

$-\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu}$

$T(F)=\frac{1}{2}$

$\Downarrow$

$T(A)=N$ for $SU(N)$

For $SU(5)$ the $\underline{24}$ gauge bosons give:

$$b_5 = -\frac{11}{12\pi}(5) = -\frac{55}{12\pi}$$

Repeat for b_3, b_2 using the field content of the $\underline{24}$

$$b_3 = \frac{-11}{12\pi}\Big[3$$

$T(F) = 1/2$

$\Downarrow$

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$$b_5 = -\frac{11}{12\pi}(5) = -\frac{55}{12\pi}$$

Repeat for b_3, b_2 using the field content of the $\underline{24}$

$$b_3 = \frac{-11}{12\pi}\left[3 + \overset{\text{complex}}{2} \times \overset{SU(2)}{2} \times 1/2\right] = \frac{-11}{12\pi}(5) \checkmark$$

$T(\Box) = \frac{1}{2}$

$\Downarrow$

$T(A) = N$ for $SU(N)$

For $SU(5)$ the $\underline{24}$ gauge bosons give:

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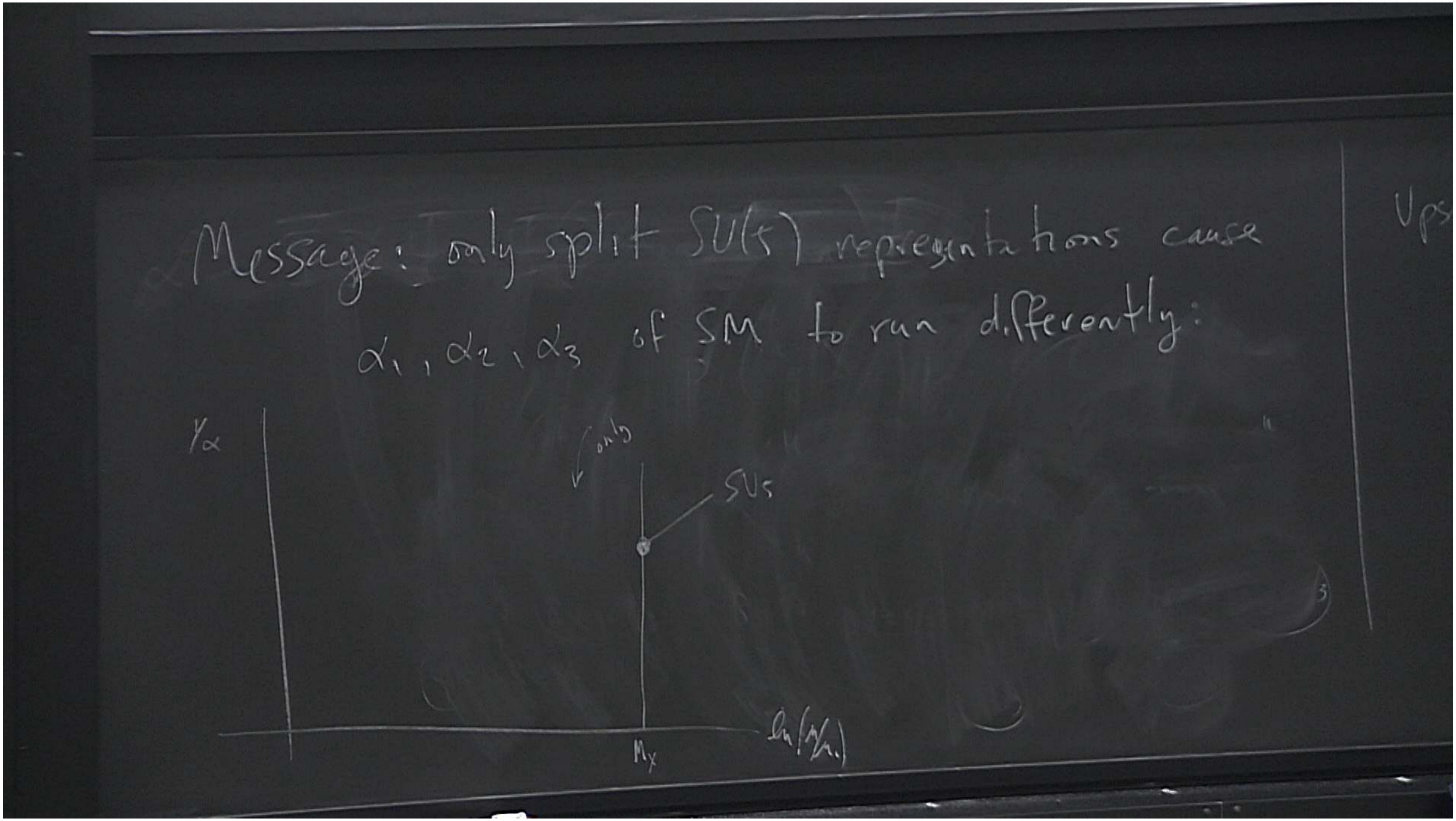
$$b_3 = \frac{-11}{12\pi}\left[3 + \overset{\text{complex}}{2} \times \overset{SU(2)}{2} \times \frac{1}{2}\right] = \frac{-11}{12\pi}(5) \checkmark$$

$$b_2 = -\frac{11}{12\pi}\left[2 + 2 \times 3 \times \frac{1}{2}\right] = \frac{-11}{12\pi}(5) \checkmark$$

Message: only split SU(5) representations cause

$\alpha_1, \alpha_2, \alpha_3$ of SM to run differently:

Ups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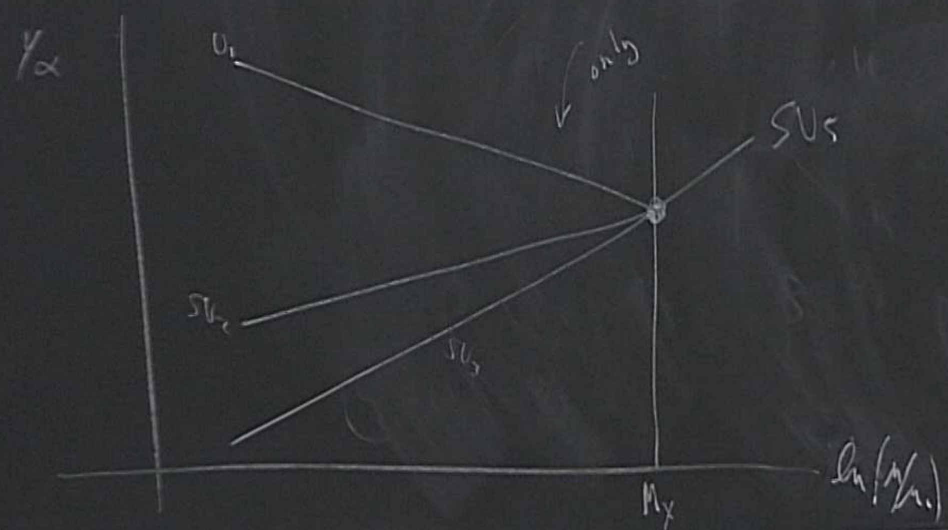
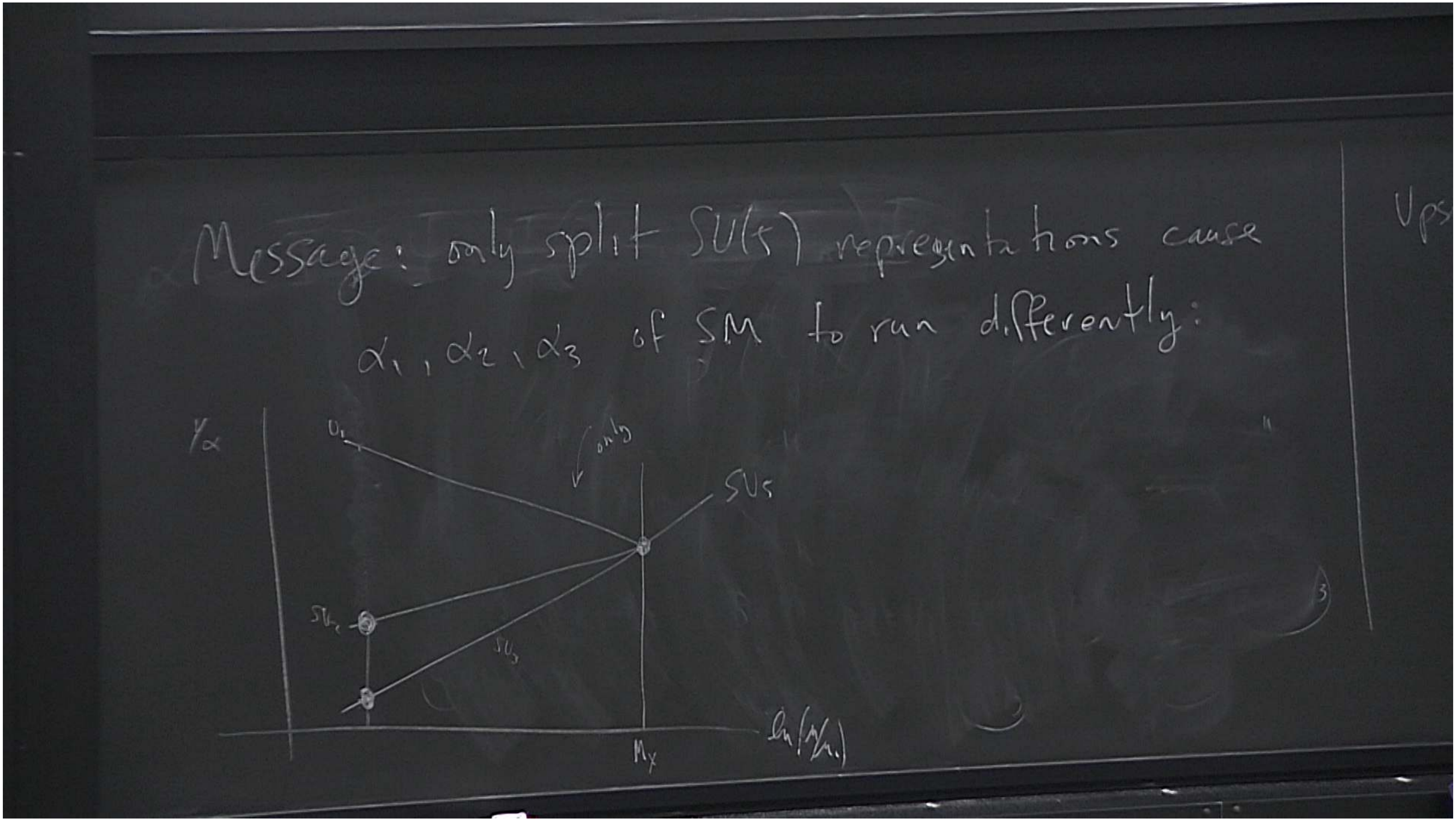
$1/\alpha$
only
SU5
M_X
$\ln(\mu/\mu_0)$
Ups

Message: only split SU(5) representations cause $\alpha_1, \alpha_2, \alpha_3$ of SM to run differently:




Message: only split SU(5) representations cause
α1, α2, α3 of SM to run differently:
1/α
U1
only
SU5
SU2
SU3
Mx
ln(μ/μ0)

If only Higgs 5, 24, gauge 24 contribute below M_X,

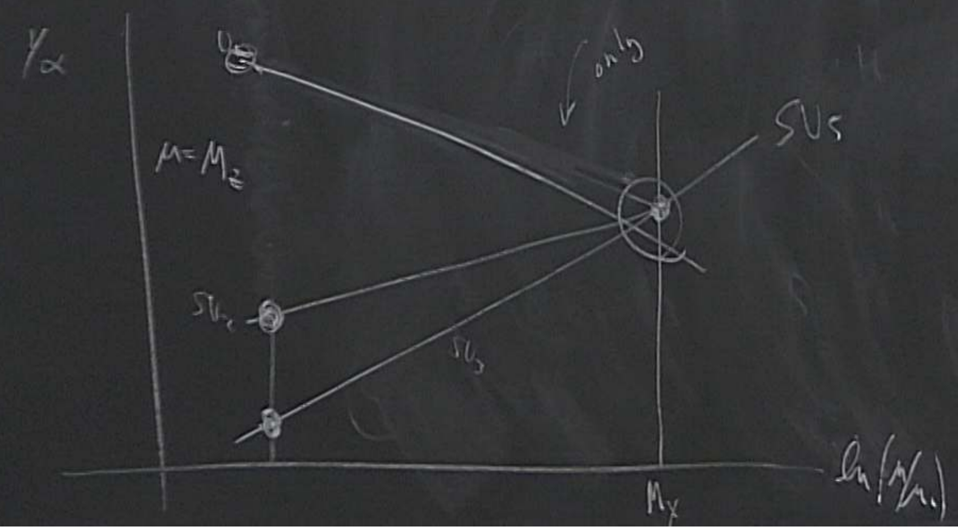
then:

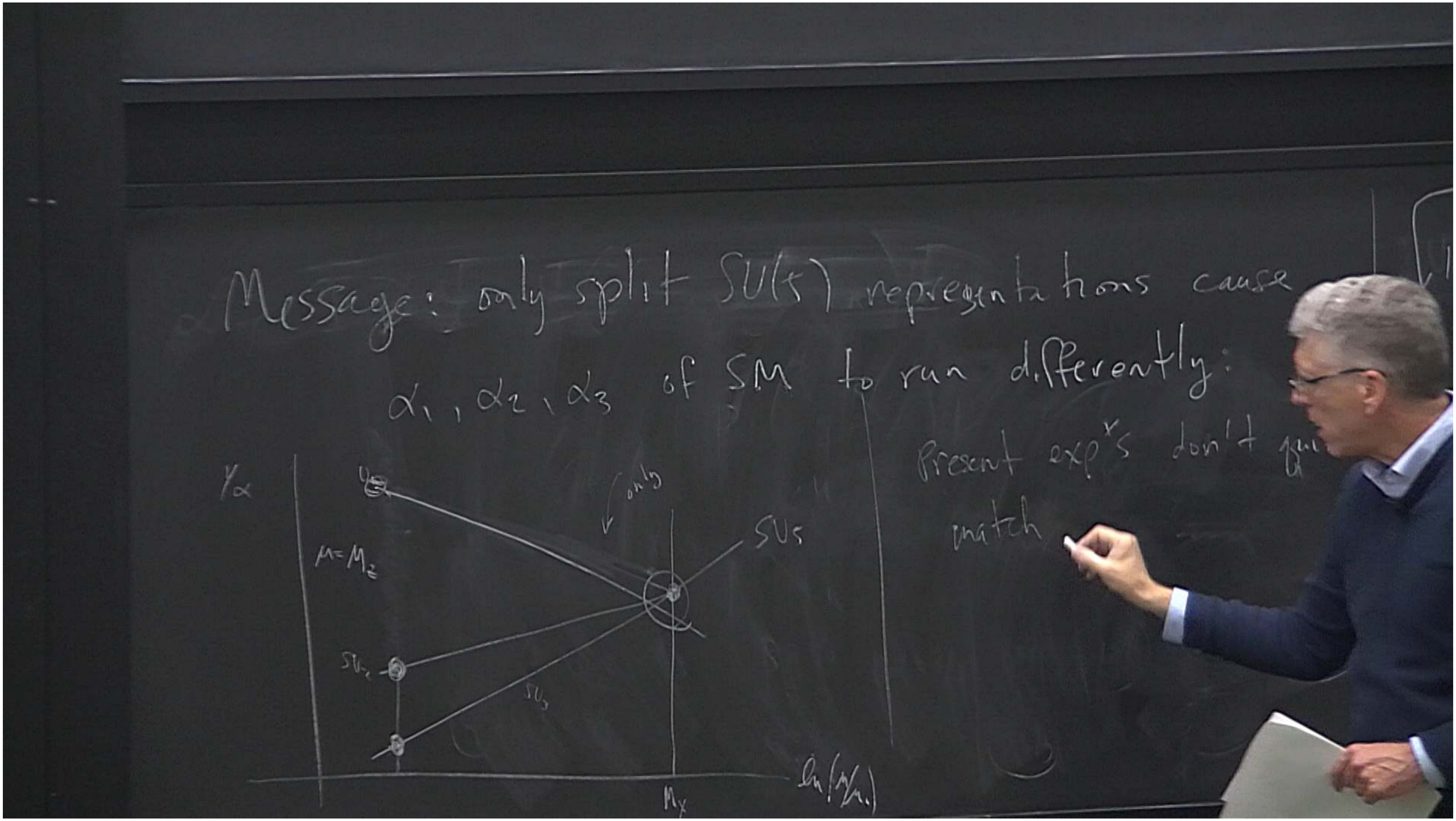
If only Higgs 5, 24, gauge 24 contribute below M_X,

$$\left(\frac{1}{\alpha_3}-\frac{1}{\alpha_2}\right)_\mu = -(b_3-b_2)\ln\left(\mu^2/\mu_0^2\right) \quad \text{if } \alpha_3(\mu_0)=\alpha_2(\mu_0)$$

$$M_X=\mu\exp\left[\frac{30\pi}{201}\left(\frac{3}{5\alpha_{em}(\mu)}-\frac{8}{5\alpha_3(\mu)}\right)\right]=\boxed{2\times10^{15}\ \mathrm{GeV}}$$

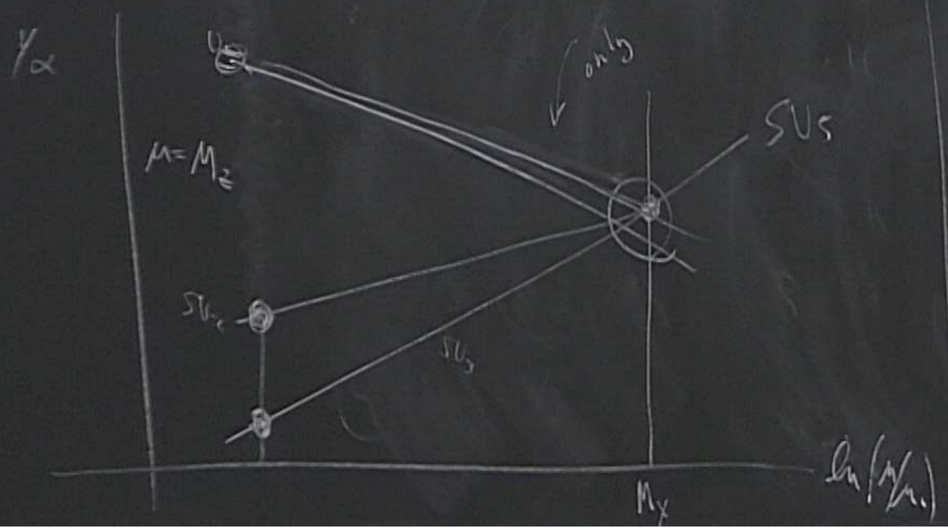
Message: only split SU(5) representations cause $\alpha_1, \alpha_2, \alpha_3$ of SM to run differently:




Message: only split SU(5) representations cause
α1, α2, α3 of SM to run differently:
Present exp's don't
match
1/α
μ=M_Z
only
SU5
SU2
SU3
M_X
ln(μ/μ0)

Message: only split SU(5) representations cause

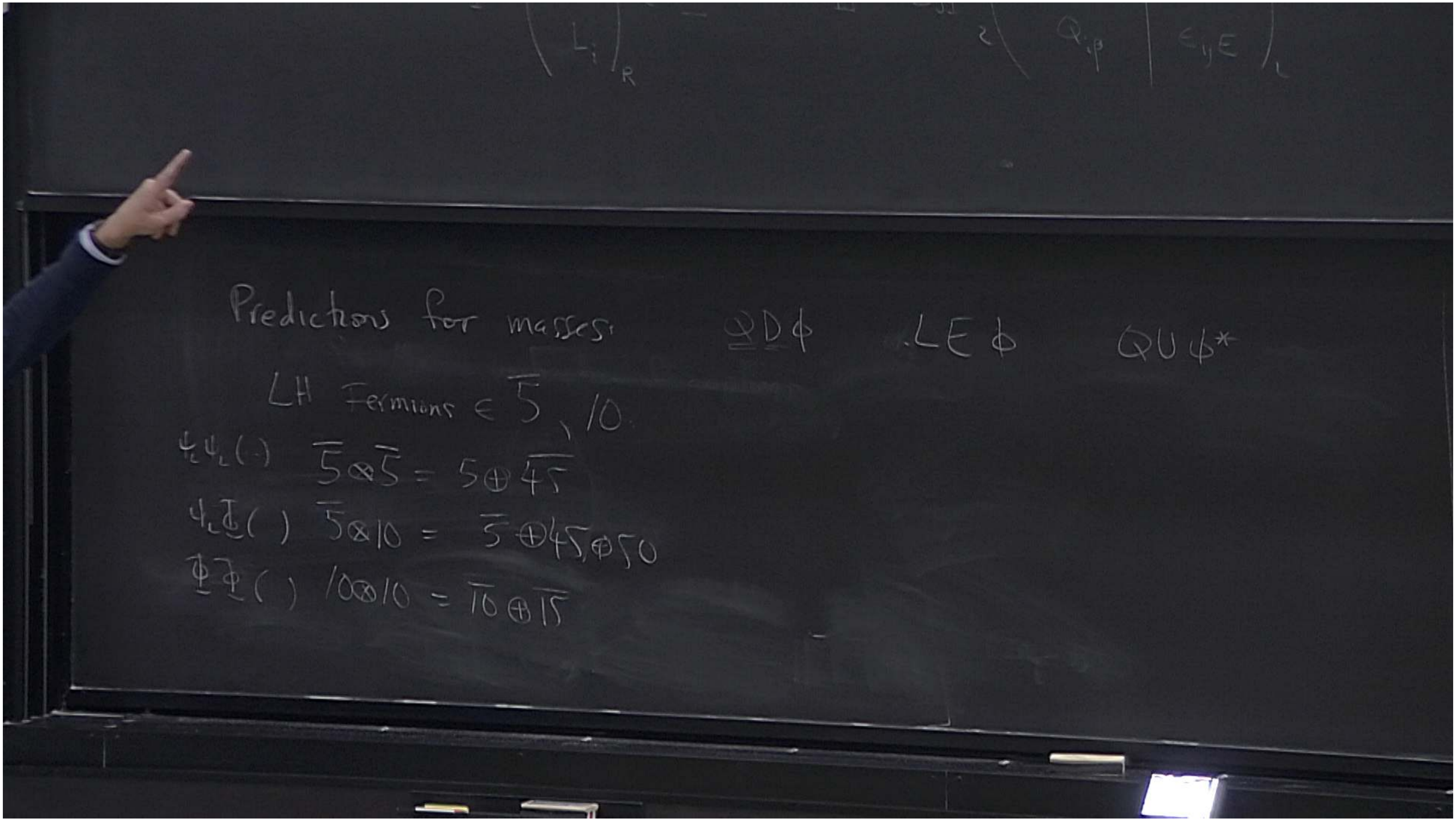
$\alpha_1, \alpha_2, \alpha_3$ of SM to run differently:



Present exp's don't quite match:

but they would IF

the SM spectrum is replaced by the MSSM at TeV scale


Predictions for masses: $\Phi D \phi$ $L E \phi$ $Q U \phi^*$
LH Fermions $\in \bar{5}, 10$.
$\psi_L \psi_L ( )$ $\bar{5} \otimes \bar{5} = 5 \oplus \overline{45}$
$\psi_L \Phi ( )$ $\bar{5} \otimes 10 = \bar{5} \oplus 45 \oplus 50$
$\Phi \Phi ( )$ $10 \otimes 10 = \overline{10} \oplus \overline{15}$

Predictions for masses: $\Psi D \phi$ $\quad L E \phi$ $\quad Q U \phi^*$

LH Fermions $\in \bar{5}, 10$.

$\psi_L \psi_L (\cdot)$ $\bar{5} \otimes \bar{5} = 5 \oplus \overline{45}$

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$\Phi \Phi (\ )$ $10 \otimes 10 = \overline{10} \oplus \overline{15}$

If only 5 higgs contributes:

$m_d = m_e, \quad m_s = m_\mu, \quad m_b = m_\tau.$

$(L_i)_R$ $2(Q_{i\beta} \mid \epsilon_{ij} E)_L$

Predictions for masses:

LH Fermions $\in \bar{5}, 10$

$\psi_L\psi_L(\cdot)$ $\quad \bar{5}\otimes\bar{5} = 5 \oplus \overline{45}$

$\psi_L\Phi(\;)$ $\quad \bar{5}\otimes 10 = \bar{5} \oplus 45 \oplus 50$

$\Phi\Phi(\;)$ $\quad 10\otimes 10 = \overline{10} \oplus \overline{15}$

$QD\phi \qquad LE\phi \qquad QU\phi^*$

If only 5 higgs contributes:

$m_d = m_e, \quad m_s = m_\mu, \quad \boxed{m_b = m_\tau}$

$$\left[\frac{m_b(\mu)}{m_\tau(\mu)}\right] = \left[\frac{m_b(M_X)}{m_\tau(M_X)}\right]\left[\frac{\alpha_3(\mu)}{\alpha_3(M_X)}\right]^{\frac{4}{11-\frac{4}{3}n_g}}\left[\frac{\alpha_1(\mu)}{\alpha_1(M_X)}\right]^{-3/2n_g}$$

m_τ given $\to m_b \approx 5.5$

$T(F) = 1/2$

$\Downarrow$

$T(A) = N$ for $SU(N)$

$$\frac{m_d}{m_s} = \frac{m_e}{m_\mu}$$

$1/24$ $\quad$ $\frac{1}{207}$

m_τ

$/2m_b$

m_τ given $\to m_b \simeq 5.5$ GeV

t of the $\underline{24}$

$= 2 \times$

$b_2 = \quad \frac{}{12\pi^2}$

$= \frac{-11}{12\pi^2}(5)$