Title: Dual gauge field theory of quantum liquid crystals

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Abstract: <p>Already in their early papers, Kosterlitz and Thouless envisaged the melting of solids by the unbinding of the topological defects associated with translational order: dislocations. Later it was realized that the resulting phases have translational symmetry but rotational rigidity: they are liquid crystals.</p>

<p> </p>

<p>We consider the topological melting of solids as a zero-temperature quantum phase transition. In a generalization of particle-vortex duality, the Goldstone modes of the solid, phonons, map onto gauge bosons which mediate long-range interactions between dislocations. The phase transition is achieved by a Bose-Einstein condensation of dislocations, restoring translational symmetry and destroying shear rigidity. The dual gauge fields become massive due to the Anderson-Higgs mechanism. In this sense, the liquid crystal is a "stress superconductor".</p>

<p> </p>

<p>We have developed this dual gauge field theory both in 2+1D, where dislocations are particle-like and phonons are vector bosons, and 3+1D where dislocations are string-like and phonons are Kalb-Ramond gauge fields. Focussing mostly on the theoretical formalism, I will discuss the relevance to recent experiments on helium monolayers, which show evidence for a quantum hexatic phase.</p>

<p> </p>

<p>References:</p>

<p>2+1D : Physics Reports 683, 1 (2017)</p>

<p>3+1D : Physical Review B 96, 1651115 (2017)</p>

Dual gauge field theory of quantum liquid crystals

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Perimeter Institute, March 26$^{\text{th}}$ 2018

Physics Reports 683, 1 (2017) – arXiv:1603.04254
Physical Review B 96, 165115 (2017) – arXiv:1703.03157

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Kai Wu	Stanford	Zohar Nussinov	St. Louis
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MEXT-Supported Program for the Strategic Research Foundation
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Keio University
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1858


Keio University
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~ 30000 students, 6 main campuses
Yukichi Fukuzawa
Keio University
Hiyoshi campus
Keio University

Outline

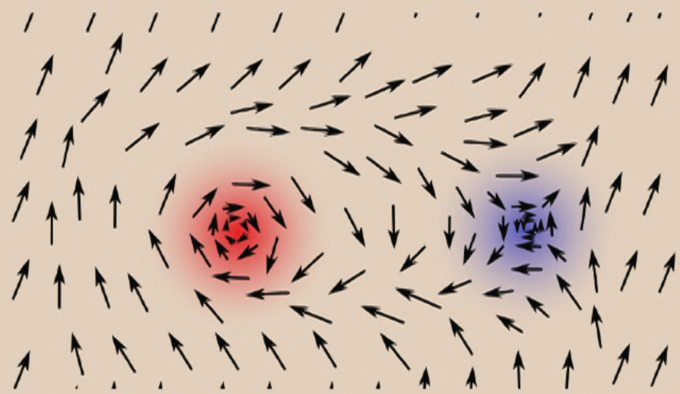
- from BKT to quantum liquid crystals
- vortex–boson duality
- quantum liquid crystals in 2+1D
- quantum liquid crystals in 3+1D
- quantum hexatic phase in helium monolayers

Introduction:

Dislocation-mediated melting

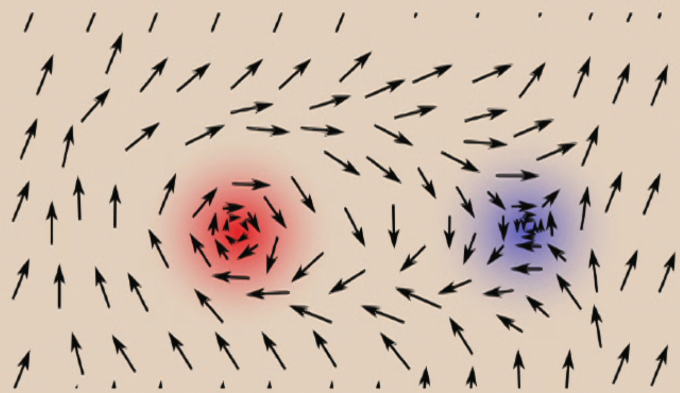
Quantum phase transitions

BKT phase transition



- In two dimensions, vortices are pointlike topological defects.
- Energy of a single vortex $E \propto \ln \frac{L}{a_{\text{core}}}$.
- Entropy of a single vortex $S \propto \ln \frac{L^2}{a^2_{\text{small}}} \propto 2 \ln \frac{L}{a_{\text{small}}}$.
- Free energy $F = E - TS$.
- At large L
  - Low T: energy dominates, no free vortices, only bound pairs;
  - High T: entropy dominates, unbound vortices.

BKT phase transition



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We did not use any special properties of superfluids or its vortices. This argument is general and should hold for many two-dimensional systems.

Dislocation-mediated melting

Input: DVI - 1920x1080p@60Hz
Output: SDI - 1920x1080i@60Hz

J. Phys. C: Solid State Phys., Vol. 5, 1972. Printed in Great Britain. © 1972

LETTER TO THE EDITOR

Long range order and metastability in two dimensional solids and superfluids

J M KOSTERLITZ and D J THOULESS

Department of Mathematical Physics, University of Birmingham, Birmingham B15 2TT

MS received 11 April 1972

Abstract. Dislocation theory is used to define long range order for two dimensional solids. An ordered state exists at low temperatures, and the rigidity modulus is nonzero at the transition temperature. Similar arguments show that the superfluid density is nonzero at the transition temperature of a two dimensional superfluid.

$$S = k_B \ln (A/a^2) \tag{2}$$

At temperatures which satisfy the inequality

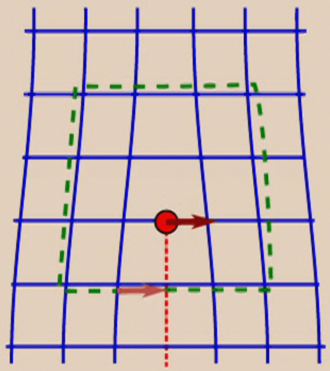
$$k_B T < k_B T_c = \eta a^2 (1 + \tau)/4\pi \tag{3}$$

the logarithmically large energy dominates, and no isolated dislocation can be formed, so the system is rigid, but once this inequality is violated there are free dislocations in the equilibrium state, and viscous flow can occur.

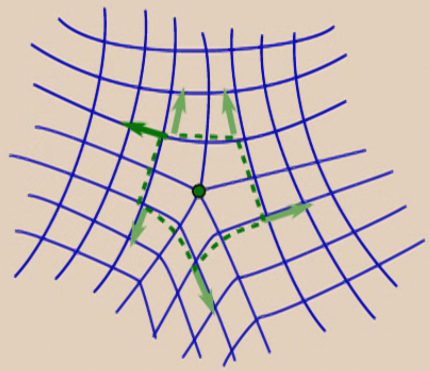
Although isolated dislocations cannot occur at low temperatures in a large system (except near the boundary), pairs of dislocations of equal and opposite Burgers vector have finite energy and must occur. Such pairs can respond to an applied stress and so reduce the rigidity modulus. When the inequality (3) is violated the largest pairs become unstable under an applied shearing stress, and produce a viscous response to the shear. We have worked out the behaviour of these pairs in some detail, and the results will be

Dislocation-mediated melting

Kosterlitz and Thouless did not realize there are two types of topological defects.



(a) dislocation – translational



(b) disclination – rotational

Dislocation-mediated melting

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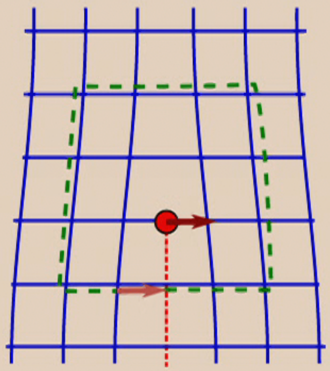
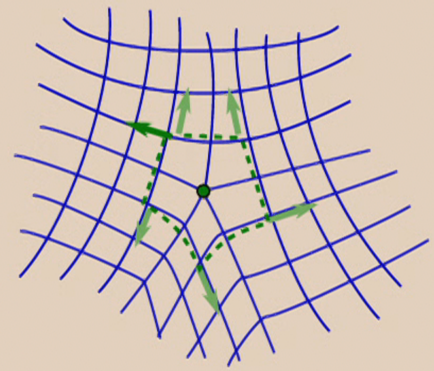
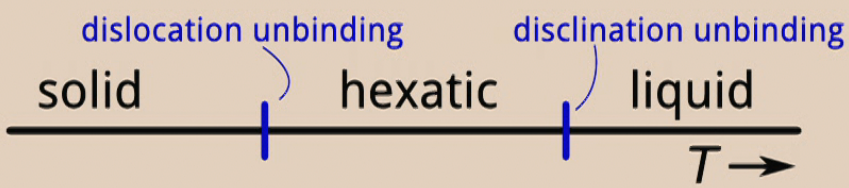


(c) dislocation – translational



(d) disclination – rotational

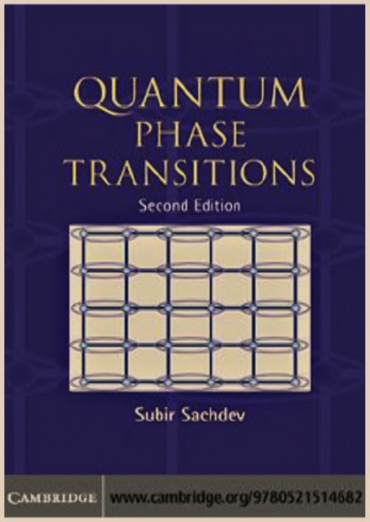


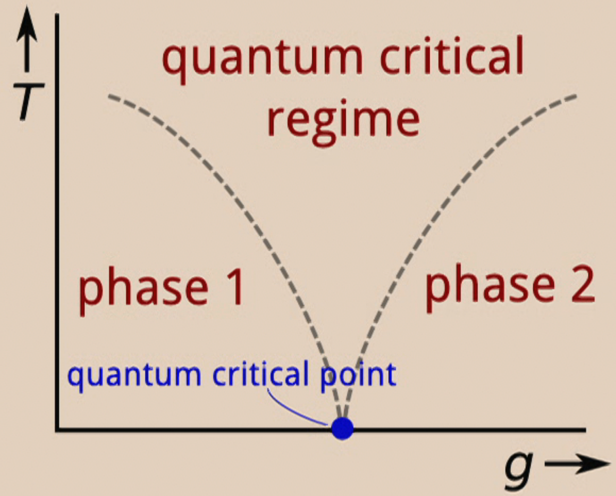
B. Halperin, D. Nelson, Phys. Rev. Lett. 41 (1978) 121
D. Nelson, B. Halperin, Phys. Rev. B 19 (1979) 2457

A. Young, Phys. Rev. B 19 (1979) 1855

Going quantum

- Zero temperature: quantum phase transition, as function of parameter g e.g. density, magnetic field, chemical doping ...





Order–disorder quantum phase transitions

- Ginzburg–Landau paradigm: formation of order is symmetry breaking.
- Alternatively: symmetry restoration by proliferation of topological defects.

Order–disorder quantum phase transitions

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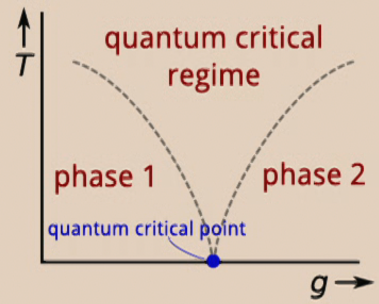

Order–disorder quantum phase transitions

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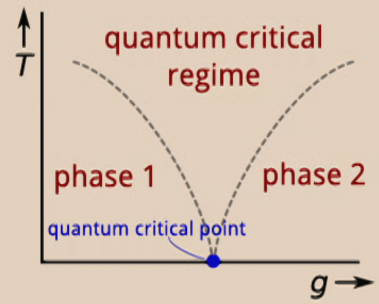
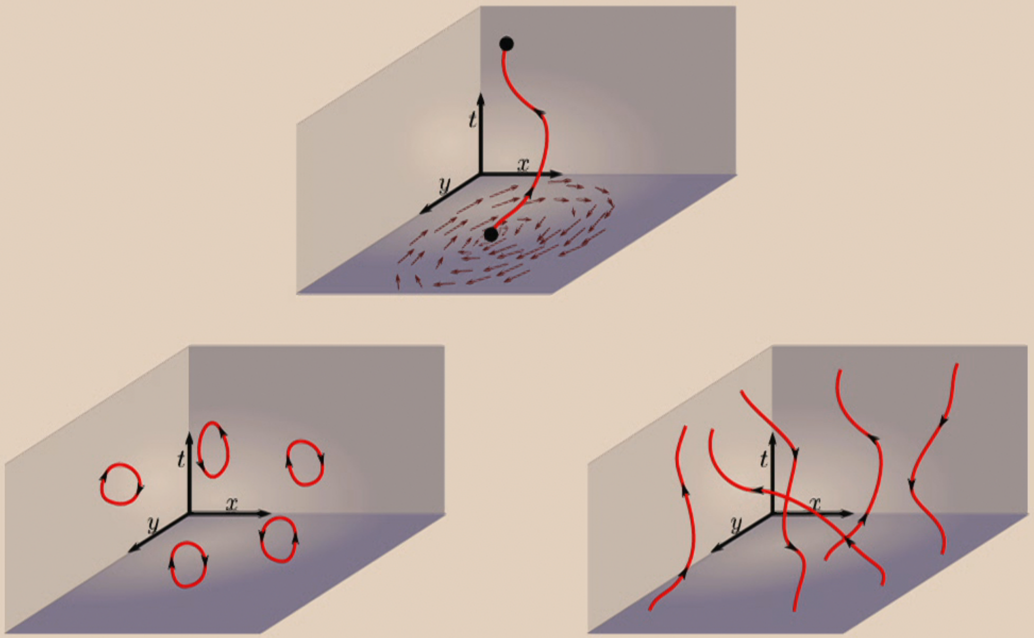


- At zero temperature, 'all' states are somehow ordered.
- Order–disorder transition depends on point of view $\rightarrow$ **duality**
- Disordering = Bose–Einstein condensation of topological defects

Quantum field theory of topological defects

- D-dimensional quantum field theory $\leftrightarrow$ $D+1$-dimensional statistical physics, e.g. 2D superfluid–insulator QPT is in the 3D XY universality class.
- Time axis is the additional dimension. Statistical physics of *worldlines*.



Quantum liquid crystals

quantum liquid crystal: quantum system with symmetry of liquid crystal

- Strong phase fluctuations and "stripes" in high-T_c superconductors
- 1998: Kivelson Emery Fradkin
- 2004: Zaanen Nussinov Mukhin
  - strong coupling
  - Bose–Einstein condensation of dislocations
  - $SO(2) \to C_n$ symmetry breaking
- "Quantum nematics"
  - weak coupling
  - spontaneous anisotropy of the Fermi surface
  - $C_4 \to C_2$ symmetry breaking

Letters to Nature

Nature **393**, 550-553 (11 June 1998) | doi:10.1038/31177; Received 21 July 1997; Accepte

Electronic liquid-crystal phases of a doped Mott insulator

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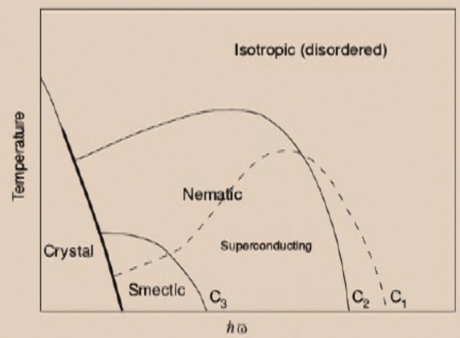
2. Department of Physics, University of Illinois, Urbana, Illinois 61801-3080, USA
3. Brookhaven National Laboratory, Upton, New York 11973-5000, USA

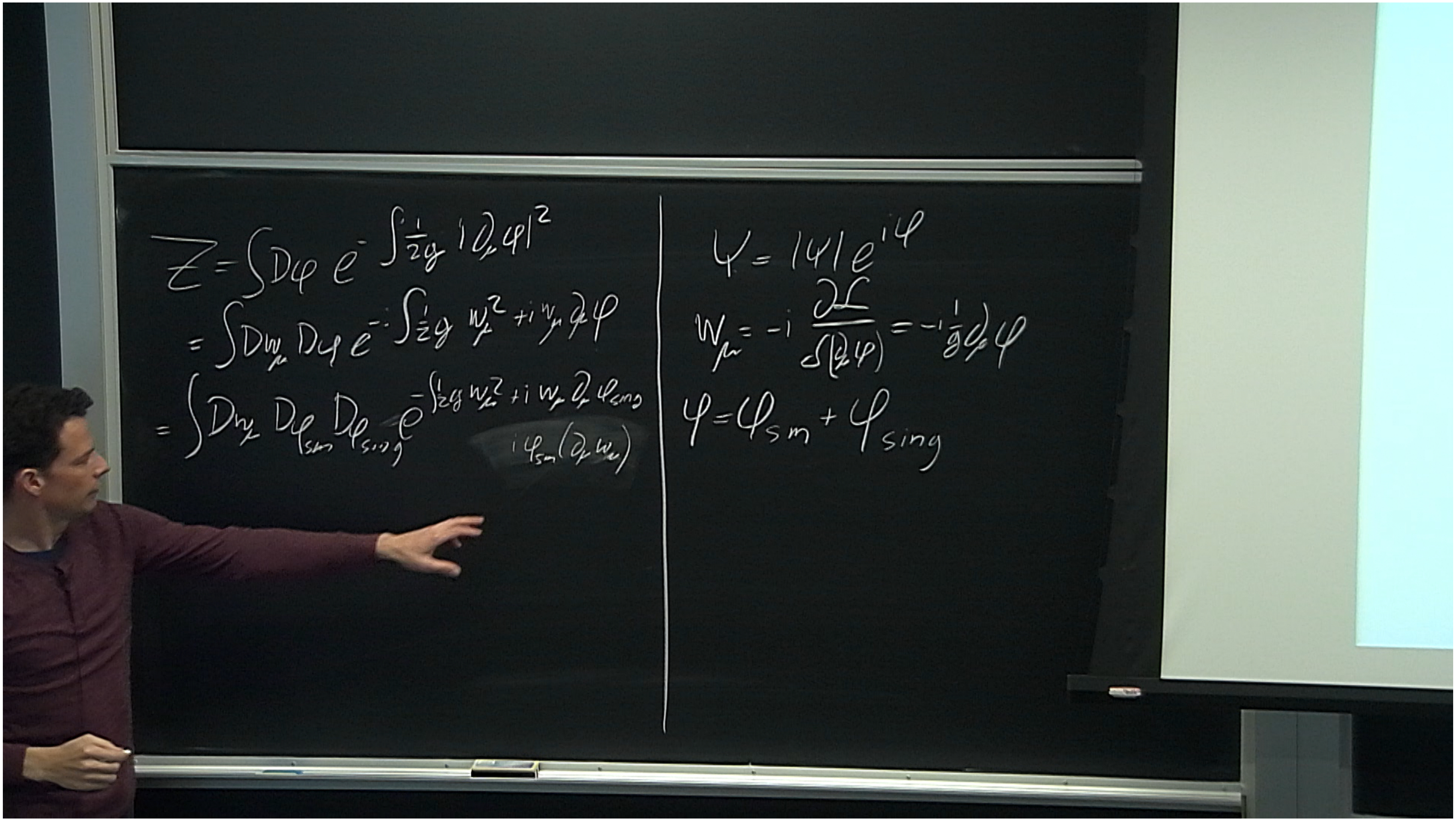
Correspondence to: V. J. Emery[3] Correspondence and requests for materials should be addressed to V.J.E. (e-mail: Email: emery@cmth.phy.bnl.gov).

The character of the ground state of an antiferromagnetic insulator is fundamentally altered following addition of even a small amount of charge[1]. The added charge is concentrated into domain walls across which a π phase shift

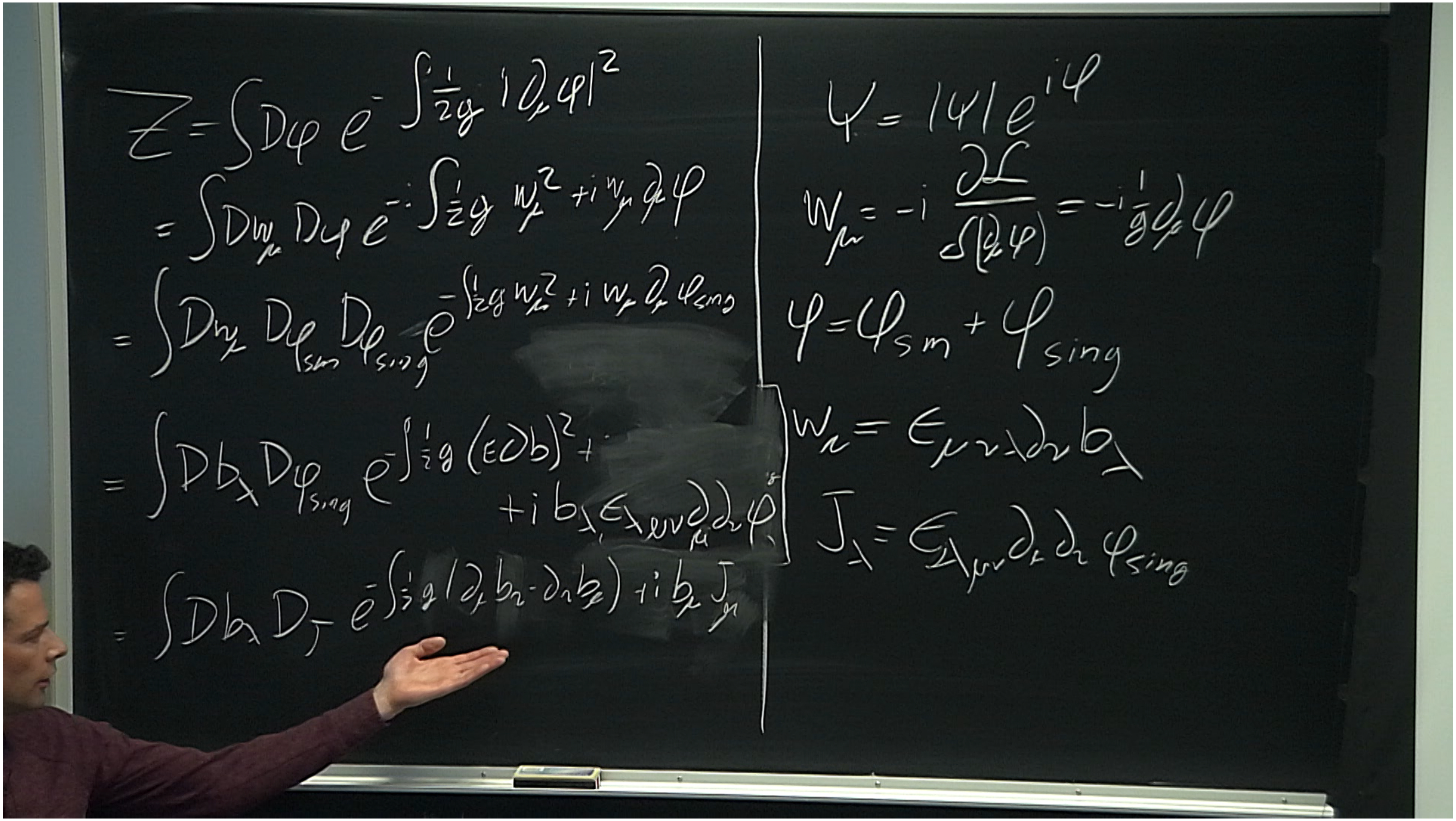
Top



Vortex–boson duality in 2+1D



$Z = \int D\varphi \, e^{-\int \frac{1}{2g} |\partial_\mu \varphi|^2}$
$= \int Dw_\mu D\varphi \, e^{-\int \frac{1}{2}g \, w_\mu^2 + i w_\mu \partial_\mu \varphi}$
$= \int Dw_\mu D\varphi_{sm} D\varphi_{sing} \, e^{-\int \frac{1}{2}g \, w_\mu^2 + i w_\mu \partial_\mu \varphi_{sing}}$
$i\varphi_{sm}(\partial_\mu w_\mu)$
$\Psi = |\Psi| e^{i\varphi}$
$w_\mu = -i \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} = -i \frac{1}{g} \partial_\mu \varphi$
$\varphi = \varphi_{sm} + \varphi_{sing}$



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$= \int Dw_\mu D\varphi_{sm} D\varphi_{sing} \, e^{-\int \frac{1}{2} g \, w_\mu^2 + i w_\mu \partial_\mu \varphi_{sing}}$
$= \int Db_\lambda D\varphi_{sing} \, e^{-\int \frac{1}{2} g (\epsilon\partial b)^2 + i b_\lambda \epsilon_{\lambda\mu\nu} \partial_\mu \partial_\nu \varphi}$
$= \int Db_\lambda D_J \, e^{-\int \frac{1}{2} g (\partial_\mu b_\nu - \partial_\nu b_\mu) + i b_\mu J_\mu}$
$\Psi = |\Psi| e^{i\varphi}$
$w_\mu = -i \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} = -i \frac{1}{g} \partial_\mu \varphi$
$\varphi = \varphi_{sm} + \varphi_{sing}$
$w_\mu = \epsilon_{\mu\nu\lambda} \partial_\nu b_\lambda$
$J_\lambda = \epsilon_{\lambda\mu\nu} \partial_\mu \partial_\nu \varphi_{sing}$

Vortex–boson duality

$$\partial_\mu w_\mu = 0 \qquad \Rightarrow \qquad w_\mu = \epsilon_{\mu\nu\lambda}\partial_\nu b_\lambda$$

Nambu–Goldstone mode $\varphi \to$ dual gauge field b_λ
invariant under $b_\lambda(x) \to b_\lambda(x) + \partial_\lambda \varepsilon(x)$

$$Z = \int \mathcal{D}\varphi_{\text{sing}} \mathcal{D}b_\lambda \, \exp\Big[- \int \frac{g}{2}(\epsilon_{\mu\nu\lambda}\partial_\nu b_\lambda)^2 + \mathrm{i} b_\lambda \epsilon_{\lambda\nu\mu}\partial_\nu\partial_\mu\varphi_{\text{sing}} \Big] \tag{3}$$

$$2\pi N = \oint_{\partial\mathcal{S}} \mathrm{d}x^\mu \partial_\mu \varphi_{\text{sing}} = \int_{\mathcal{S}} \mathrm{d}S^\lambda \epsilon_{\lambda\mu\nu}\partial_\mu\partial_\nu\varphi_{\text{sing}} = \int_{\mathcal{S}} \mathrm{d}S^\lambda J_\lambda \tag{4}$$

vortex worldline J_λ

$$Z \equiv \int \mathcal{D}J_\lambda \mathcal{D}b_\lambda \, \exp\Big[- \int \frac{g}{2}(\partial_\mu b_\nu - \partial_\nu b_\mu)^2 + \mathrm{i} b_\lambda J_\lambda \Big]$$

The superfluid is a 'Coulomb vacuum' for 'dual photons' b_λ which mediate interactions between vortex 'charges' J_λ.

- path integral over J_λ is over isolated vortices only, not a grand canonical ensemble

Disorder field theory

tangle of worldlines is described by *collective* field Φ
heuristic correspondence

$$J_\lambda \leftrightarrow \mathrm{i}(\partial_\lambda \Phi^*)\Phi - \mathrm{i}\Phi^*(\partial_\lambda \Phi) \tag{5}$$

- J_λ is only well-defined in superfluid, Φ is only well-defined in insulator
- order field theory relativistic $\Rightarrow$ vortex worldlines relativistic dynamics

grand-canonical partition function of non-interacting vortex worldlines

$$\Xi = \mathrm{e}^{Z_{1\mathrm{loop}}} = \int \mathcal{D}\Phi^* \mathcal{D}\Phi \mathrm{e}^{\int \mathcal{L}_{\mathrm{GLW}}} \tag{6}$$

$$\mathcal{L}_{\mathrm{GLW}} = \tfrac{1}{2}|\partial_\mu \Phi|^2 + \tfrac{1}{2}\alpha|\Phi|^2 + \tfrac{1}{4}\beta|\Phi|^4 \tag{7}$$

lattice derivation : $\alpha \propto (\mathrm{e}^\epsilon - 2D)$, ϵ line tension

(see e.g. H. Kleinert *Gauge fields in condensed matter* 1989)

Insulator as dual superconductor

vortices are charged under the dual gauge field

$$Z = \int \mathcal{D}\Phi \mathcal{D}b_\lambda \; \exp\Big[- \int \frac{g}{2}(\partial_\mu b_\nu - \partial_\nu b_\mu)^2 + |(\partial_\mu - \mathrm{i} b_\mu)\Phi|^2 + V(|\Phi|) \Big]$$

precisely the action of a (relativistic) superconductor

The Bose-Mott insulator is a 'superconductor' for 'dual photons' b_λ, which get a mass through the Anderson–Higgs mechanism.

Insulator as dual superconductor

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precisely the action of a (relativistic) superconductor

The Bose-Mott insulator is a 'superconductor' for 'dual photons' b_λ, which get a mass through the Anderson–Higgs mechanism.

- dual photons b_λ are 'Meissner screened'
- long-range phase rigidity in superfluid phase φ is lost due to vortex condensate

Duality dictionary

real side	dual side	EM equivalent
vortex	vortex worldline	charged particle
Goldstone modes	gauge bosons	photons
superfluid	vortex gas	Coulomb gas
Bose-Mott insulator	vortex condensate	superconductor
Mott gap	massive bosons	superconducting gap
insulator	current expulsion	Meissner effect
	current vortex †	Abrikosov vortex
	current quantum	flux quantum

† AJB & J. Zaanen, PRB 86, 125129 (2012)

2+1D quantum liquid crystals

Dual elasticity

- Crystal in D dimensions
- Breaking of translational & rotational symmetry — D phonons
- Shear rigidity — $D-1$ transverse phonons
- Topological defects: dislocations with Burgers vector B^a

Dual elasticity

- Dual gauge fields: D vector bosons b^a_λ
- Dislocations restore translational symmetry
- Transverse phonons are gapped
- Glide constraint (mass conservation): longitudinal phonon is protected

Dual elasticity

Recall:

	superfluid	solid
NG modes	phase mode	phonons
canonical momentum	supercurrent	stress
topological defects	vortices	dislocations
dual Higgs mechanism	insulating gap	shear stress gap

Stress 'tensor' $\sigma^a_\mu = \dfrac{\partial \mathcal{L}}{\partial(\partial_\mu u^a)}$ $\left(\mathcal{L} = \frac{1}{2}\partial_\mu u^a C_{\mu\nu ab}\partial_\nu u^b\right)$

σ^a_τ	kinetic energy	D
$P^{(0)}_{mnab}\sigma^b_n$	compression stress	1
$P^{(1)}_{mnab}\sigma^b_n$	rotational stress (absent)	$\frac{1}{2}D(D-1)$
$P^{(2)}_{mnab}\sigma^b_n$	shear stress	$\frac{1}{2}D(D-1)-1$

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Dual elasticity

$$\mathcal{Z} = \int \mathcal{D}\sigma^a_\mu \mathcal{D}u^b \ \exp\left[\int_0^\beta \mathrm{d}\tau \mathrm{d}^D\mathbf{x} \ \tfrac{1}{2}\sigma^a_\mu C^{-1}_{\mu\nu ab}\sigma^b_\nu + \mathrm{i}\sigma^a_\mu \partial_\mu u^a\right] \tag{8}$$

$$u^a = u^a_{\mathrm{smooth}} + u^a_{\mathrm{singular}} \tag{9}$$

$$\partial_\mu \sigma^a_\mu = 0 \qquad\qquad \sigma^a_\mu = \epsilon_{\mu\nu\lambda}\partial_\nu b^a_\lambda \tag{10}$$

b^a_λ : dual stress gauge field
- b^a_τ : static (instantaneous) force
- b^a_{L} : gauge freedom
- b^a_{T} : propagating mode 'stress photon'

▶ at this point we specialize to 2+1D

Dislocation currents

$$\begin{aligned}
\mathcal{L}_{\text{int}} &= \mathrm{i}\,\sigma^a_\mu\,\partial_\mu u^a_{\text{singular}} && (11)\\
&= \mathrm{i}\,(\epsilon_{\mu\nu\lambda}\partial_\nu b^a_\lambda)\,\partial_\mu u^a_{\text{singular}} && (12)\\
&= \mathrm{i}\,b^a_\lambda\,(\epsilon_{\lambda\mu\nu}\partial_\mu\partial_\nu u^a_{\text{singular}}) && (13)\\
&\equiv \mathrm{i}\,b^a_\lambda\,J^a_\lambda. && (14)
\end{aligned}$$

Dislocation current $J^{a \leftarrow \text{Burgers index}}_{\lambda \leftarrow \text{worldline index}}$

$$\mathcal{Z} = \int \mathcal{D}b^a_\lambda \text{“}\mathcal{D}J^a_\lambda\text{”}\mathcal{F}(b)\ \exp\left[\int \tfrac{1}{2}(\epsilon_{\mu\kappa\lambda}\partial_\kappa b^a_\lambda)C^{-1}_{\mu\nu ab}(\epsilon_{\nu\rho\sigma}\partial_\rho b^b_\sigma) + \mathrm{i}b^a_\lambda J^a_\lambda\right]$$

The solid is a ‘Coulomb vacuum’ for ‘stress photons’ b^a_λ which mediate interactions between dislocation ‘charges’ J^a_λ.

Dislocation condensation — full Lagrangian

$$\mathcal{L}_{\mathrm{QLC}} = \mathcal{L}_{\mathrm{solid}} + \mathcal{L}_{|\Phi|} + \mathcal{L}_{\mathrm{coupling}}$$

$$\mathcal{L}_{\mathrm{solid}} = \tfrac{1}{2}(\epsilon_{\mu\kappa\lambda}\partial_\kappa b^a_\lambda) C^{-1}_{\mu\nu ab} (\epsilon_{\nu\rho\sigma}\partial_\rho b^b_\sigma)$$

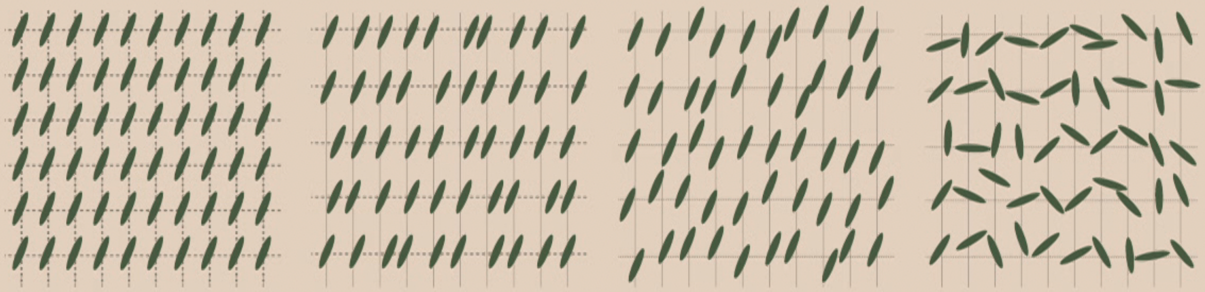
$$\mathcal{L}_{|\Phi|} = \tfrac{1}{2}\alpha_x|\Phi^x|^2 + \tfrac{1}{2}\alpha_y|\Phi^y|^2 + \tfrac{1}{4}\beta_x|\Phi^x|^4 + \tfrac{1}{4}\beta_y|\Phi^y|^4 + \tfrac{1}{2}\gamma|\Phi^x|^2|\Phi^y|^2$$

$$\mathcal{L}_{\mathrm{coupling}} = \tfrac{1}{2}\sum_{a=x,y} \left| \left(\partial_\mu - \mathrm{i} b^a_\mu - \mathrm{i}\lambda\epsilon_{\tau\mu a}\right)\Phi^a \right|^2$$

$$\to \tfrac{1}{2}\sum_{a=x,y} (\Omega^a)^2 (\partial_\mu \phi^a - b^a_\mu - \lambda\epsilon_{\tau m a})^2$$

$C^{-1}_{\mu\nu ab}$: contains elastic moduli (coupling constants)

Classical liquid crystals

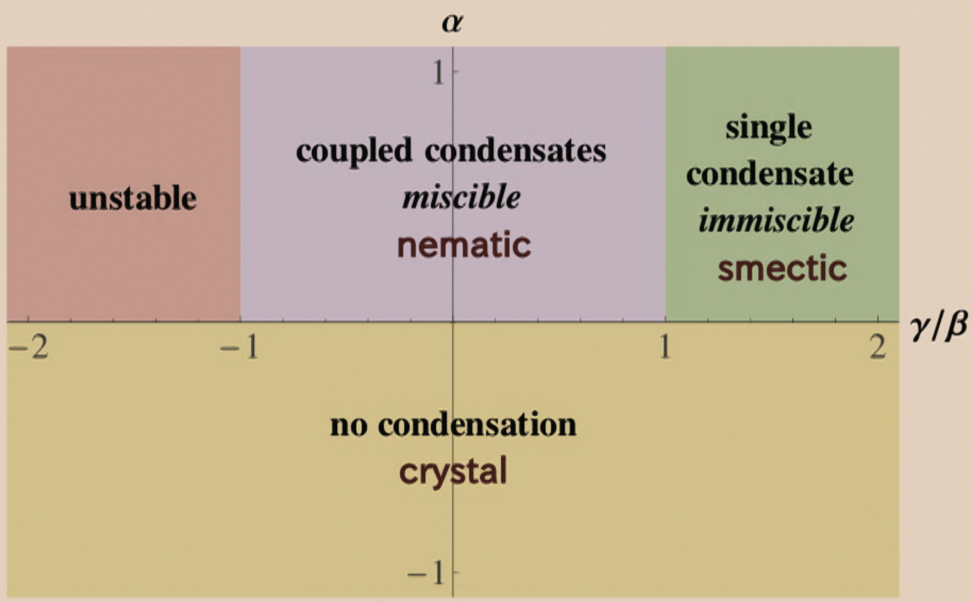


solid – $\mathbb{Z}^2 \rtimes P$ smectic – $(\mathbb{R} \times \mathbb{Z}) \rtimes P$ nematic – $\mathbb{R}^2 \rtimes P$ liquid – $\mathbb{R}^2 \rtimes O(2)$

$U(1) \times U(1)$ condensation phase diagram

$$\mathcal{L}_{|\Phi|} = \tfrac{1}{2}\alpha_x|\Phi^x|^2 + \tfrac{1}{2}\alpha_y|\Phi^y|^2 + \tfrac{1}{4}\beta_x|\Phi^x|^4 + \tfrac{1}{4}\beta_y|\Phi^y|^4 + \tfrac{1}{2}\gamma|\Phi^x|^2|\Phi^y|^2.$$

identical to *two-component Bose–Einstein condensate* (Roberts Ueda 2006)
for isotropic solid, assume $\alpha_x = \alpha_y \equiv \alpha$ and $\beta_x = \beta_y \equiv \beta$:



Assumptions and limitations

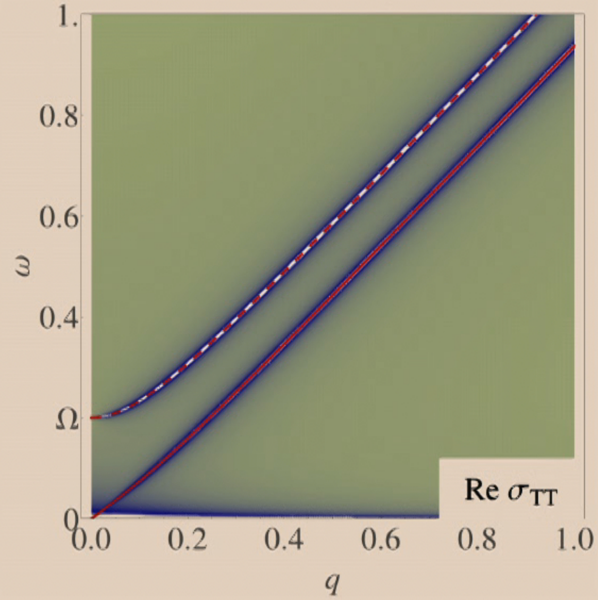
- Zero temperature
- Ginzburg–Landau $\rightarrow$ only near the phase transition
- London limit, phase fluctuations only
- Maximal crystalline correlations (collective physics only)
- No interstitials
- No disclinations
- Bosons only but 4-He and 3-He experiments similar
- Isotropic solid only

Spectrum of the quantum nematic

- Longitudinal
  - gapless phonon, purely compressional
  - gapped dislocation condensate mode

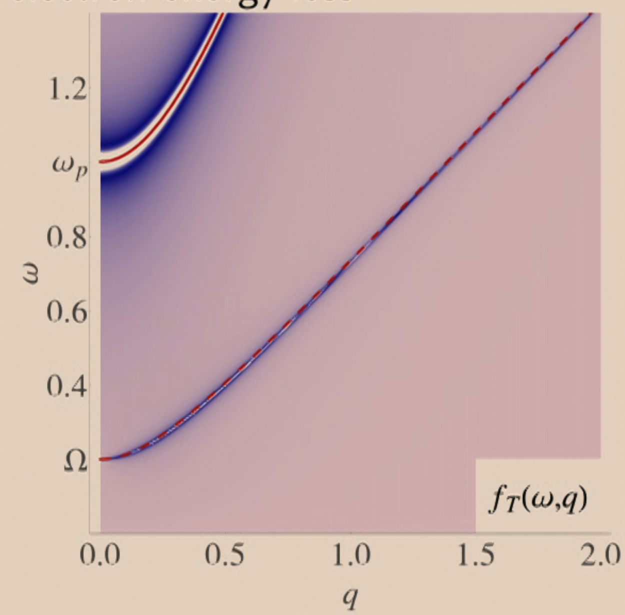
Transverse optical conductivity quantum nematic

transverse conductivity



gapped shear phonon
rotational Goldstone mode
perfect Drude pole at $\omega = 0$

electron-energy loss



plasmon
dislocation condensate mode

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3+1D quantum liquid crystals

Duality in 3+1D

does the duality construction work in 3+1D?

Duality in 3+1D

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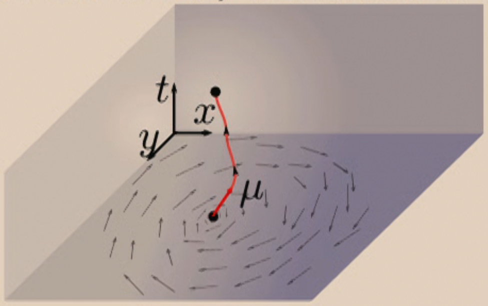
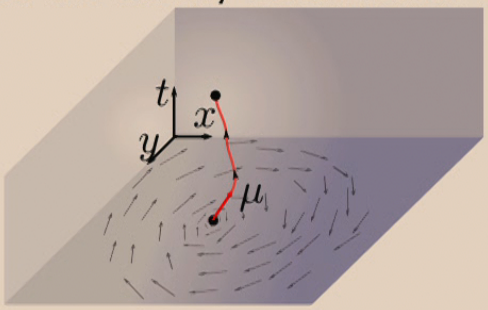
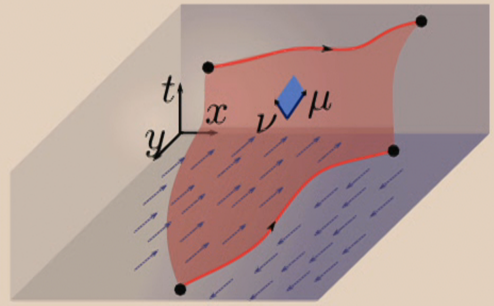


- dislocation worldline
- $J^a_\mu = \epsilon_{\mu\kappa\lambda}\partial_\kappa\partial_\lambda u^a_{\text{sing}}$

Duality in 3+1D

does the duality construction work in 3+1D?





- dislocation worldline
- $J^a_\mu = \epsilon_{\mu\kappa\lambda}\partial_\kappa\partial_\lambda u^a_{\text{sing}}$

- dislocation worldsheet
- $J^a_{\mu\nu} = \epsilon_{\mu\nu\kappa\lambda}\partial_\kappa\partial_\lambda u^a_{\text{sing}}$

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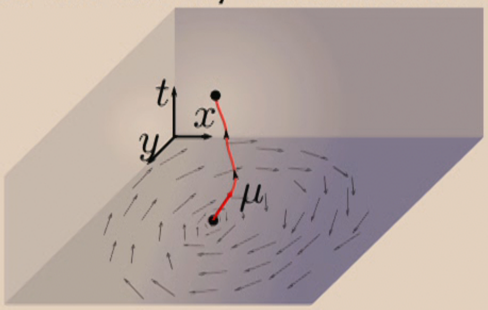

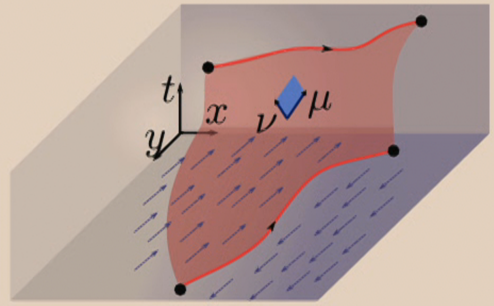

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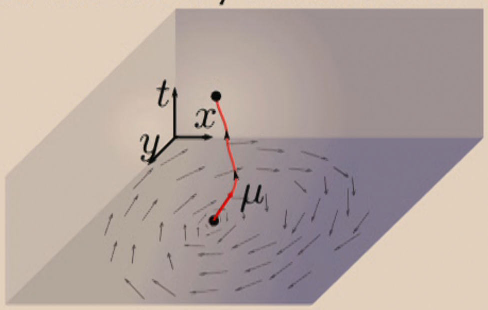

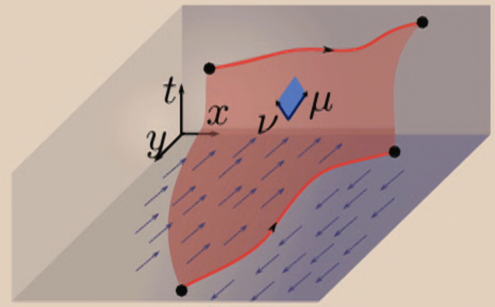

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The dual gauge fields are now two-form fields

$$\sigma^a_\mu = \epsilon_{\mu\nu\kappa\lambda}\partial_\nu b^a_{\kappa\lambda}$$

Dual elasticity in 3+1D

stress tensor $\sigma^a_m \to \sigma^a_\mu$, where $a = x, y, z$ and $\mu = t, x, y, z$

- three phonons (1 longitudinal, 2 transverse)
- three shear forces
- (three rotational forces)

dual stress photons

$$\partial_\mu \sigma^a_\mu = 0, \qquad \to \qquad \sigma^a_\mu = \epsilon_{\mu\nu\kappa\lambda} \partial_\nu b^a_{\kappa\lambda} \tag{15}$$

momentum space directions $(\tau, \mathrm{L}, \mathrm{R}, \mathrm{S})$ and $(0, 1, \mathrm{R}, \mathrm{S})$

$b^a_{\tau m}$: static (instantaneous) force

$b^a_{\mathrm{L}m}$: gauge freedom

b^a_{RS} : propagating mode 'stress photon'

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earlier proposals from string theory: the vortex condensate has several degrees of freedom

$$\Phi[X(\sigma)] = |\Phi| \mathrm{e}^{\mathrm{i} \int \mathrm{d}X^\mu(\sigma)\phi_\mu(\sigma)}$$

$$\text{minimal coupling} : (\partial_\kappa \phi_\lambda - \partial_\lambda \phi_\kappa - b_{\kappa\lambda})|\Phi|$$

Marshall & Ramond (1975), Rey (1989), Franz (2007)

bottom line : all gauge field degrees of freedom become Higgsed

3D quantum liquid crystals

$$\mathcal{L}_{\text{dual}} = \frac{g}{2}(\epsilon_{\mu\nu\kappa\lambda}\partial_\nu b^a_{\kappa\lambda})^2 + \mathrm{i} b^a_{\kappa\lambda} J^a_{\kappa\lambda}$$
$$\leftrightarrow \frac{g}{2}(\epsilon_{\mu\nu\kappa\lambda}\partial_\nu b^a_{\kappa\lambda})^2 + \frac{1}{2}\sum_a |\Phi^a|^2 (\partial_\mu \phi^a_\nu - \partial_\nu \phi^a_\mu - b^a_{\mu\nu})^2$$

3D quantum liquid crystals

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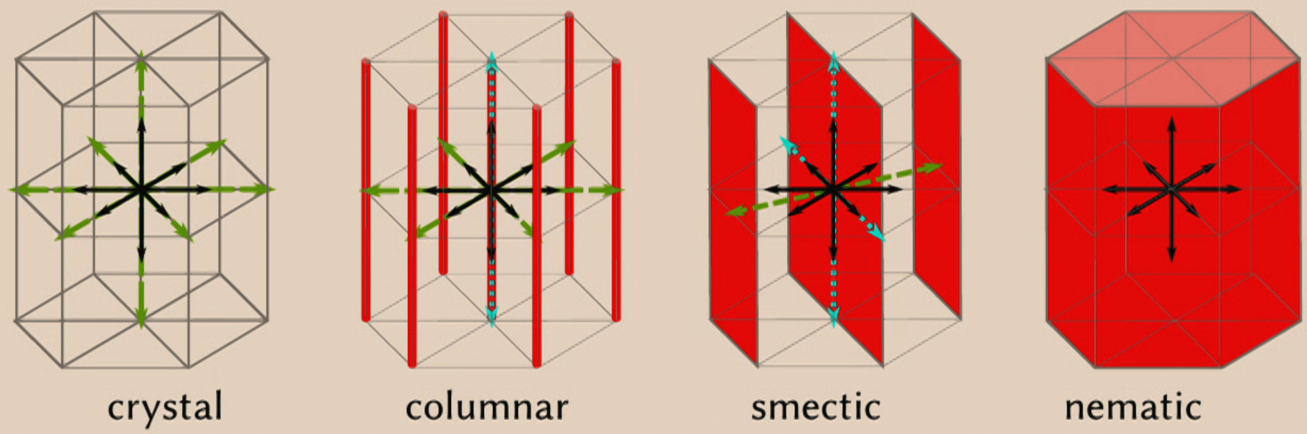
\end{aligned}
$$

$$
\begin{aligned}
V(|\Phi^a|) = {} & \tfrac{1}{2}\alpha_x|\Phi^x|^2 + \tfrac{1}{2}\alpha_y|\Phi^y|^2 + \tfrac{1}{2}\alpha_z|\Phi^z|^2 \\
& + \tfrac{1}{4}\beta_x|\Phi^x|^4 + \tfrac{1}{4}\beta_y|\Phi^y|^4 + \tfrac{1}{4}\beta_z|\Phi^z|^4 \\
& + \tfrac{1}{2}\gamma_{xy}|\Phi^x|^2|\Phi^y|^2 + \tfrac{1}{4}\gamma_{xz}|\Phi^x|^2|\Phi^z|^2 + \tfrac{1}{4}\gamma_{yz}|\Phi^y|^2|\Phi^z|^2 .
\end{aligned}
$$

cf. multicomponent BECs : Roberts and Ueda, Phys. Rev. A 73, 053611 (2006)

condense 1, 2 or 3 components

Sequential melting



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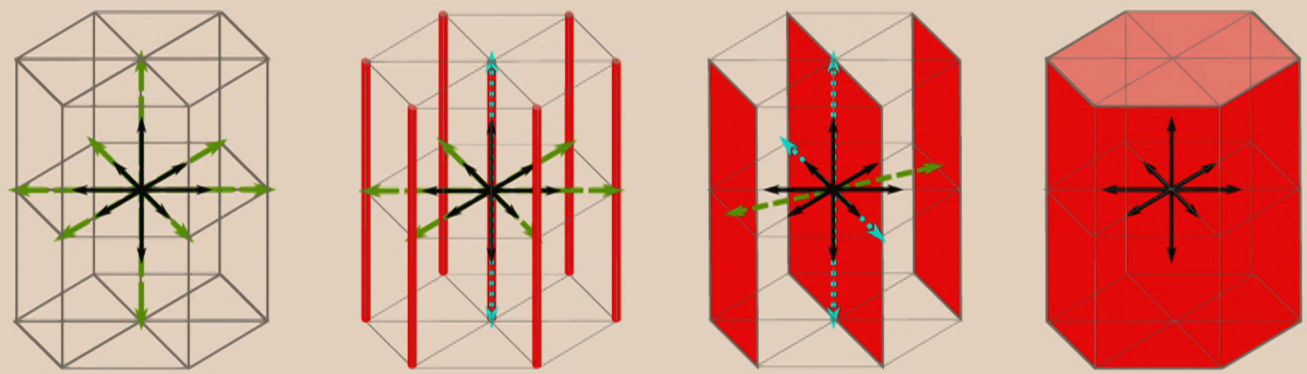

Quantum liquid crystal phenomenology

- phonons are two-form gauge bosons
- transverse phonons (shear) become gapped, propagating mode
- longitudinal phonon remains massless
- rotational Goldstone modes emerge in planes with translational symmetry
- superconductivity (Meissner effect) in directions with translational symmetry
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Goldstone modes
phonons
rotational

	solid	columnar	smectic	nematic
2+1D	2 / 0	–	1 / 0	0 / 1
3+1D	3 / 0	2 / 0	1 / 1	0 / 3

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