Title: Topological line operators and renormalization group flows in two dimensions

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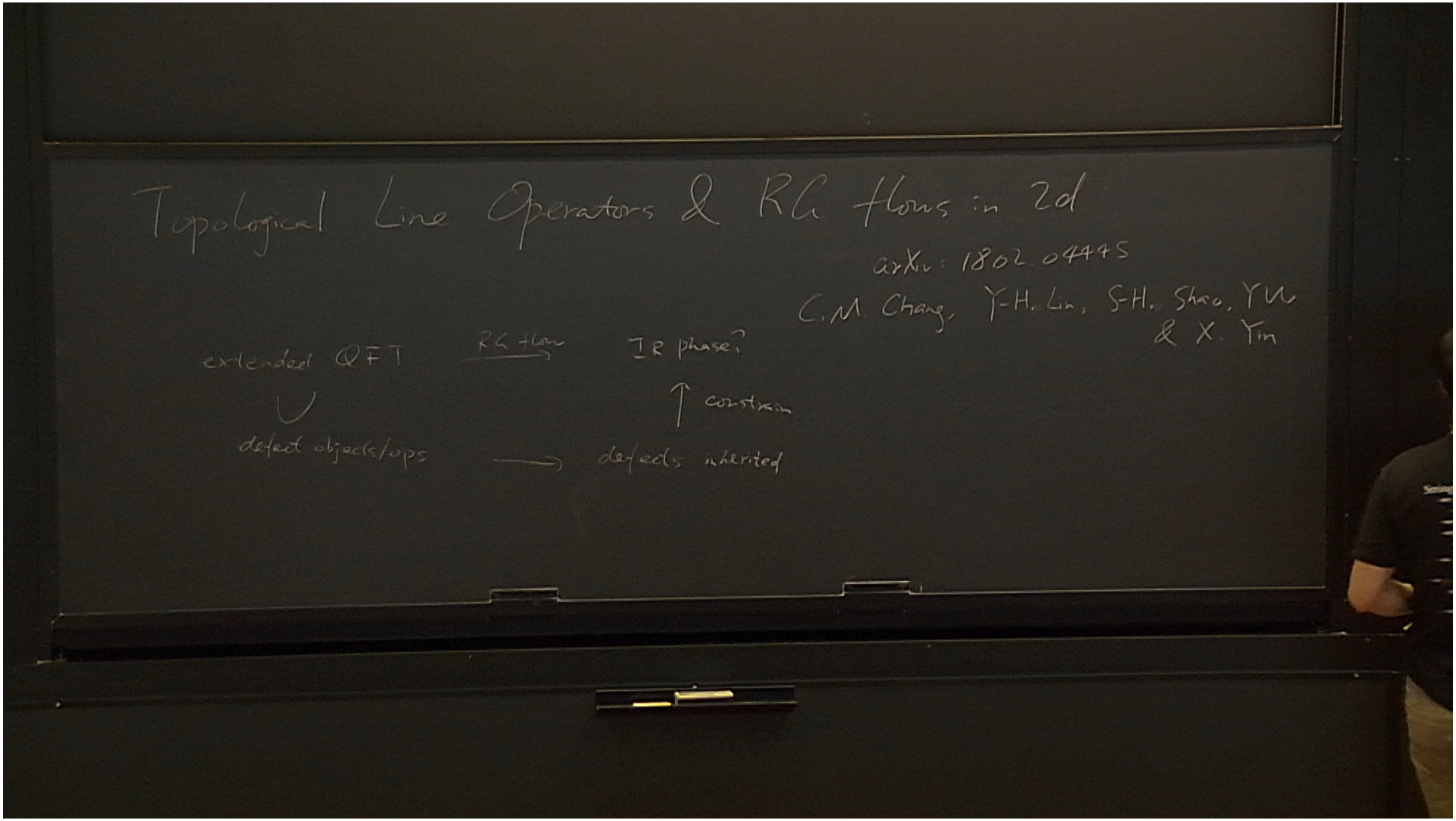
Abstract: <p>Topological defect operators are extended operators in a quantum field theory (QFT) whose correlation functions are independent of continuous changes of the ambient space. They satisfy nontrivial fusion relations and put nontrivial constraints on the QFT itself and its deformations (such as renormalization group (RG) flows). Canonical examples of topological defect operators include generators of (higher-form) global symmetries whose constraints on the QFT have been well-studied. However they constitute just a tip of a much richer framework which we explore in detail in two spacetime dimensions. </p>

<p>More specifically, we study the crossing relations of general topological defect lines (TDL) in two dimensions, discuss their relation to the usual 't Hooft anomalies associated to global symmetries, and use them to constrain RG flows to either conformal fixed points or topological quantum field theories (TQFTs). We show that if certain non-invertible TDLs are preserved along a RG flow, then the vacuum cannot be a non-degenerate gapped state. For various massive flows, we determine the infrared TQFTs completely from the consideration of TDLs together with modular invariance.</p>

Topological Line Operators & RG flows in 2d

arXiv: 1802.04445

C.M. Chang, Y-H. Lin, S-H. Shao, YW
& X. Yin



Topological Line Operators & RG flows in 2d
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C.M. Chang, Y-H. Lin, S-H. Shao, YW
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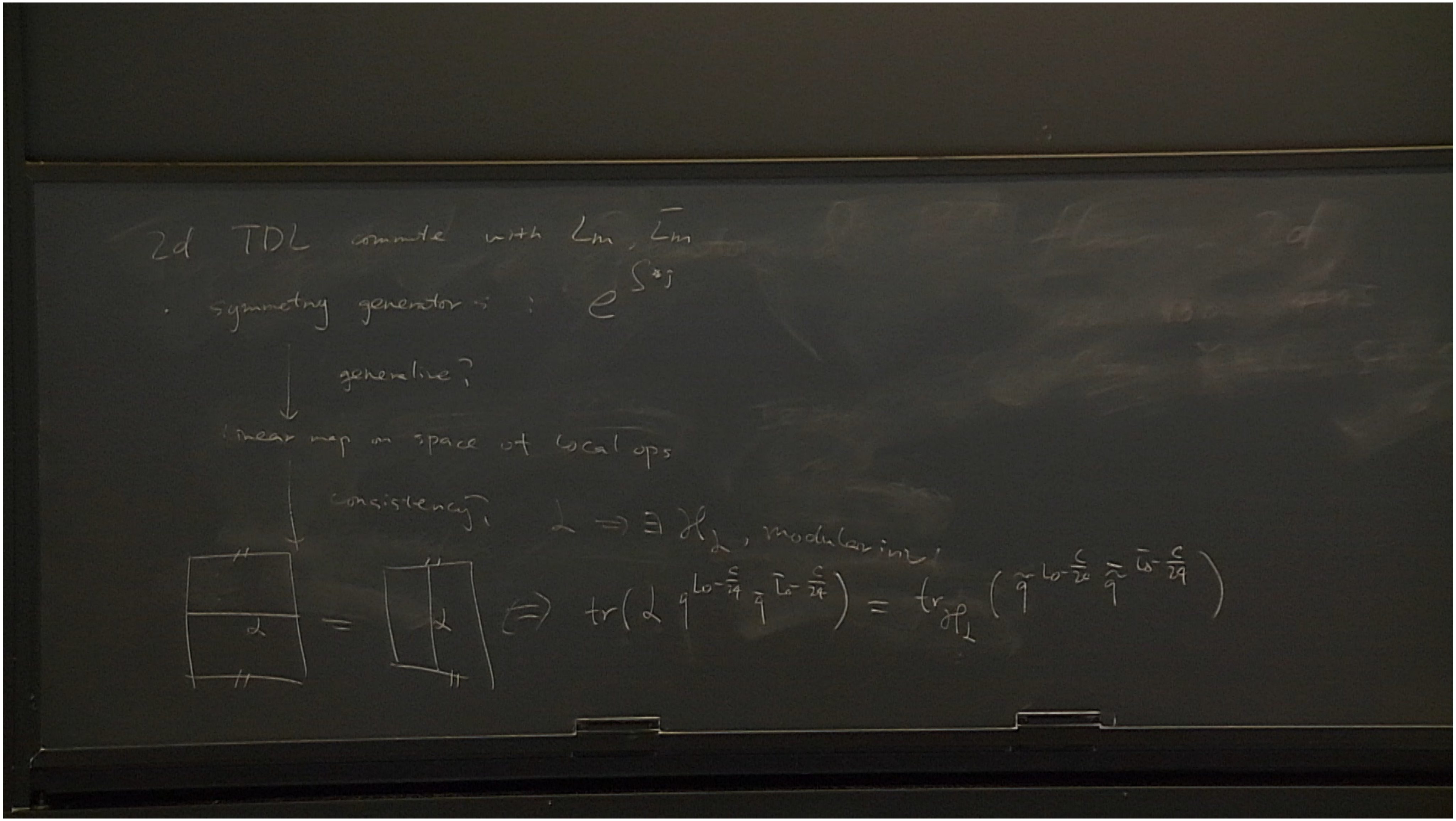
extended QFT
RG flow
IR phase?
constrain
defect objects/ops
defects inherited


2d TDL commute with L_m, $\bar{L}_m$
· symmetry generators : $e^{\int * j}$
generalize?
linear map on space of local ops
consistency : $\mathcal{L} \Rightarrow \exists\, \mathcal{H}_{\mathcal{L}}$, modular inv!
$\mathcal{L}$
$\mathcal{L}$
$\Leftrightarrow \mathrm{tr}(\mathcal{L}\, q^{L_0 - \frac{c}{24}} \bar{q}^{\bar{L}_0 - \frac{c}{24}}) = \mathrm{tr}_{\mathcal{H}_{\mathcal{L}}}(\tilde{q}^{L_0 - \frac{c}{24}} \bar{\tilde{q}}^{\bar{L}_0 - \frac{c}{24}})$

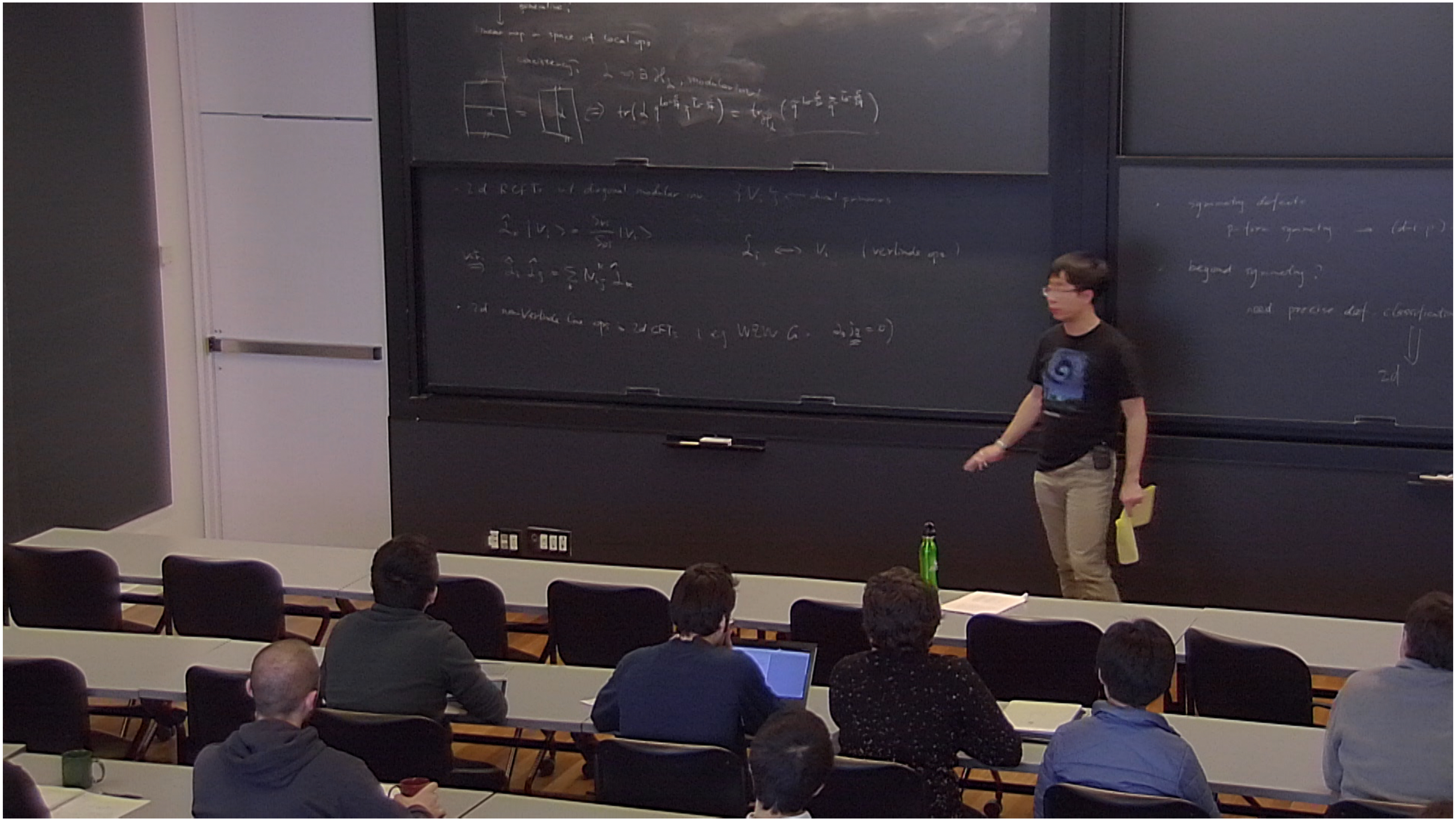
$$\Longleftrightarrow \quad \mathrm{tr}\left(\mathcal{L}\, q^{L_0-\frac{c}{24}}\, \bar{q}^{\bar{L}_0-\frac{c}{24}}\right) = \mathrm{tr}_{\mathcal{H}_\mathcal{L}}\left(\tilde{q}^{L_0-\frac{c}{24}}\, \bar{\tilde{q}}^{\bar{L}_0-\frac{c}{24}}\right)$$

· 2d RCFTs w/ diagonal modular inv. $\{V_i\} \leftarrow$ chiral primaries

$$\hat{\mathcal{L}}_k |V_i\rangle = \frac{S_{ki}}{S_{0i}} |V_i\rangle$$

$\mathcal{L}_i \longleftrightarrow V_i$ (verlinde ops)

$$\overset{\text{V.F.}}{\Longrightarrow} \quad \hat{\mathcal{L}}_i \hat{\mathcal{L}}_j = \sum_k N_{ij}^{k} \hat{\mathcal{L}}_k$$



... modular inv!

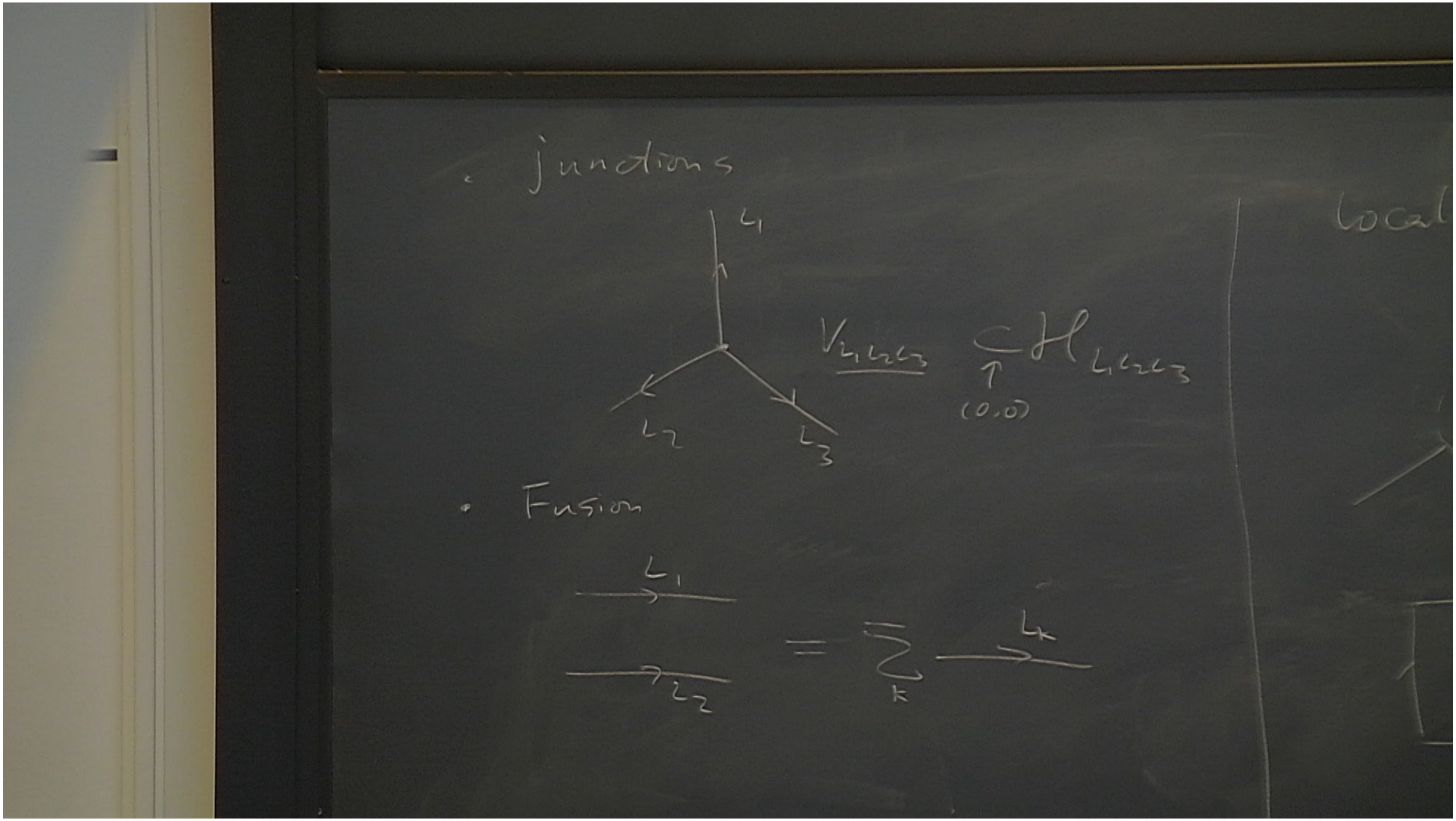
$\Leftrightarrow \; \mathrm{tr}\left(\mathcal{L}\, q^{L_0 - \frac{c}{24}}\, \bar{q}^{\bar{L}_0 - \frac{c}{24}}\right) = \mathrm{tr}_{\mathcal{H}_{\mathcal{L}}}\left(\tilde{q}^{L_0 - \frac{c}{24}}\, \bar{\tilde{q}}^{\bar{L}_0 - \frac{c}{24}}\right)$

• 2d RCFTs w/ diagonal modular inv. $\{V_i\} \leftarrow$ chiral primaries

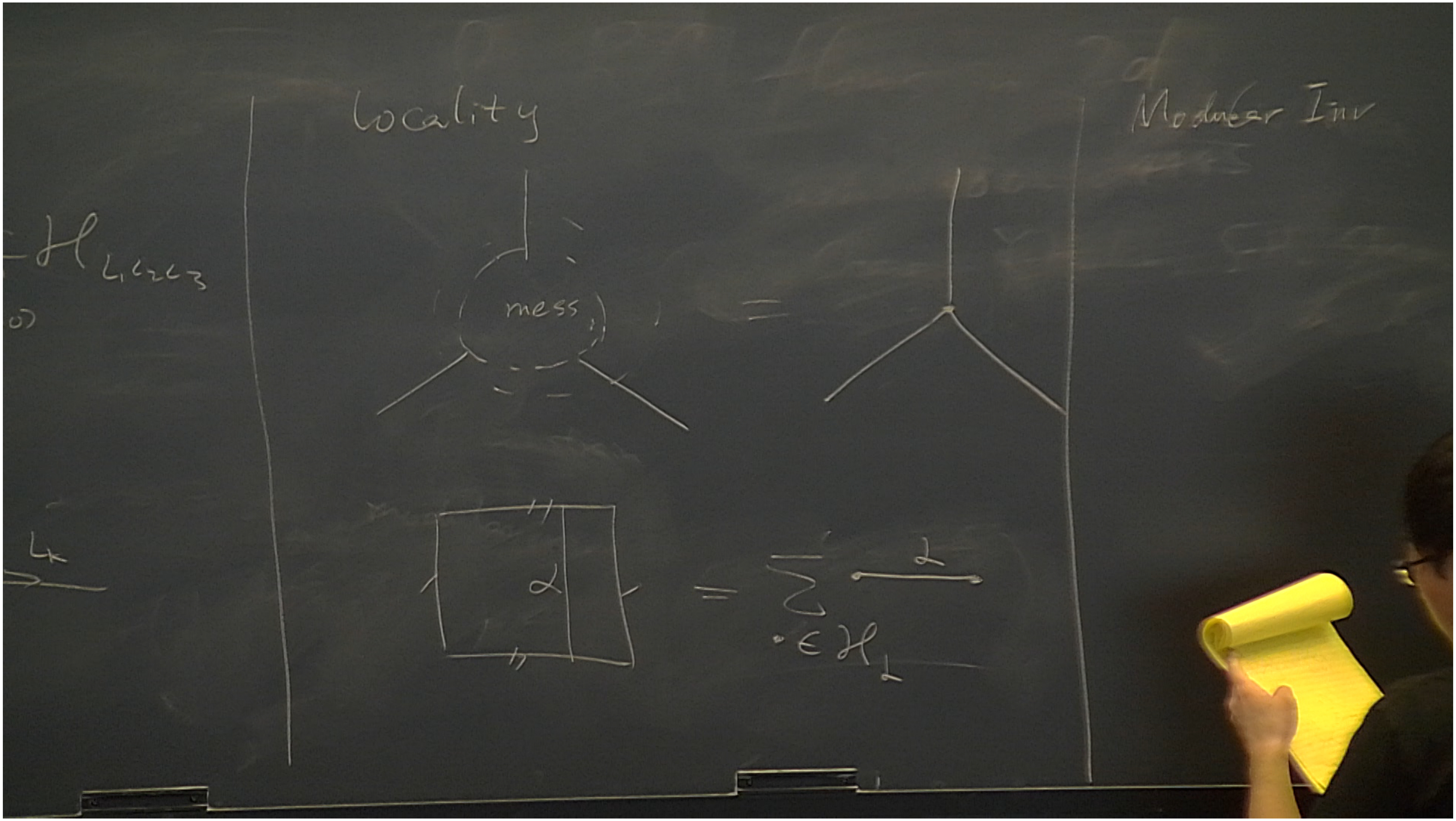
$$\hat{\mathcal{L}}_k |V_i\rangle = \frac{S_{ki}}{S_{0i}} |V_i\rangle \qquad \mathcal{L}_i \longleftrightarrow V_i \quad \text{(verlinde ops)}$$

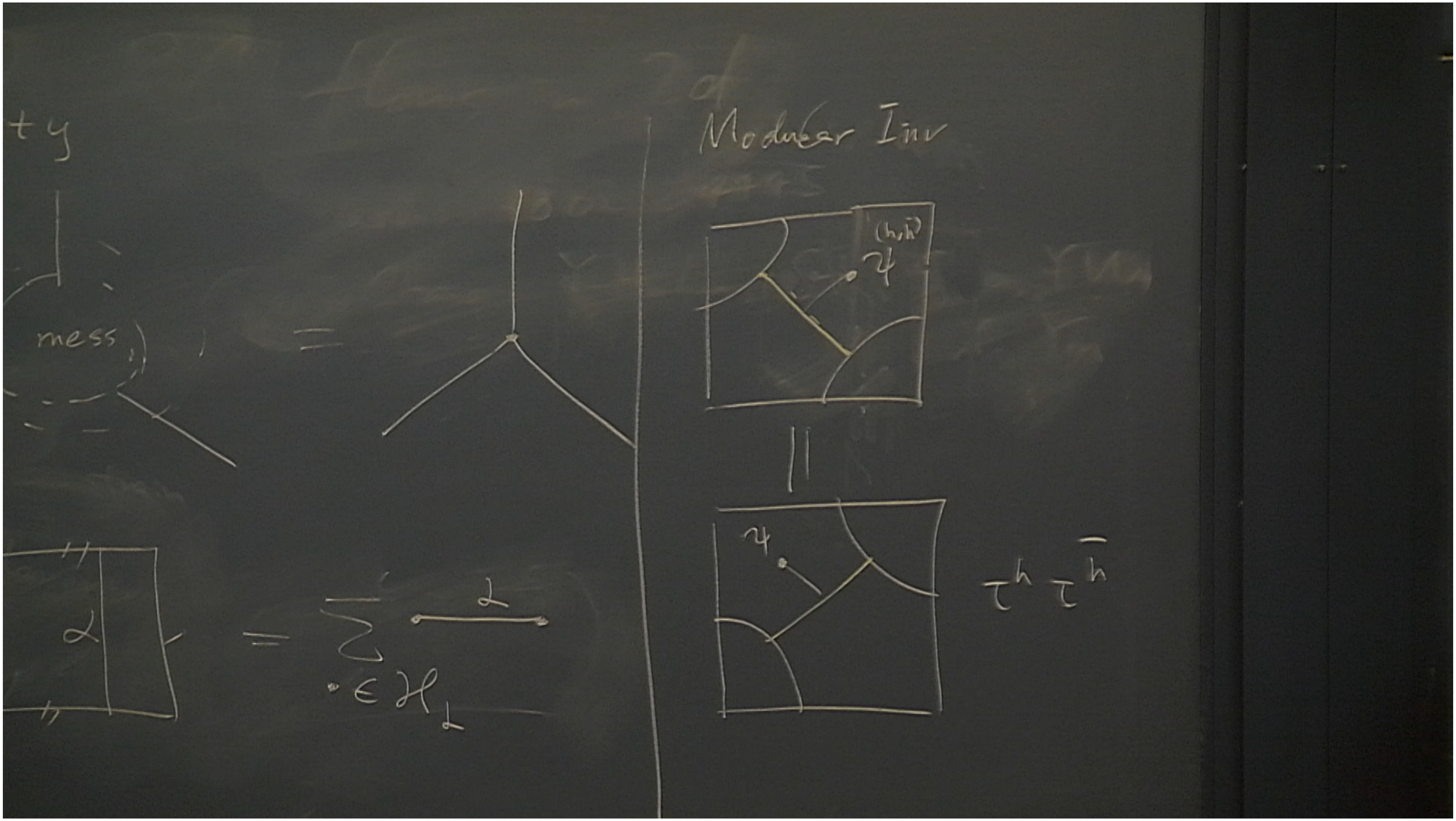
$$\overset{\text{V.F.}}{\Longrightarrow} \quad \hat{\mathcal{L}}_i \hat{\mathcal{L}}_j = \sum_k N_{ij}^{k} \hat{\mathcal{L}}_k$$

• 2d non-Verlinde line ops in 2d CFTs (e.g. WZW G, $\partial_z \underline{\underline{j_{\bar{z}}}} = 0$)

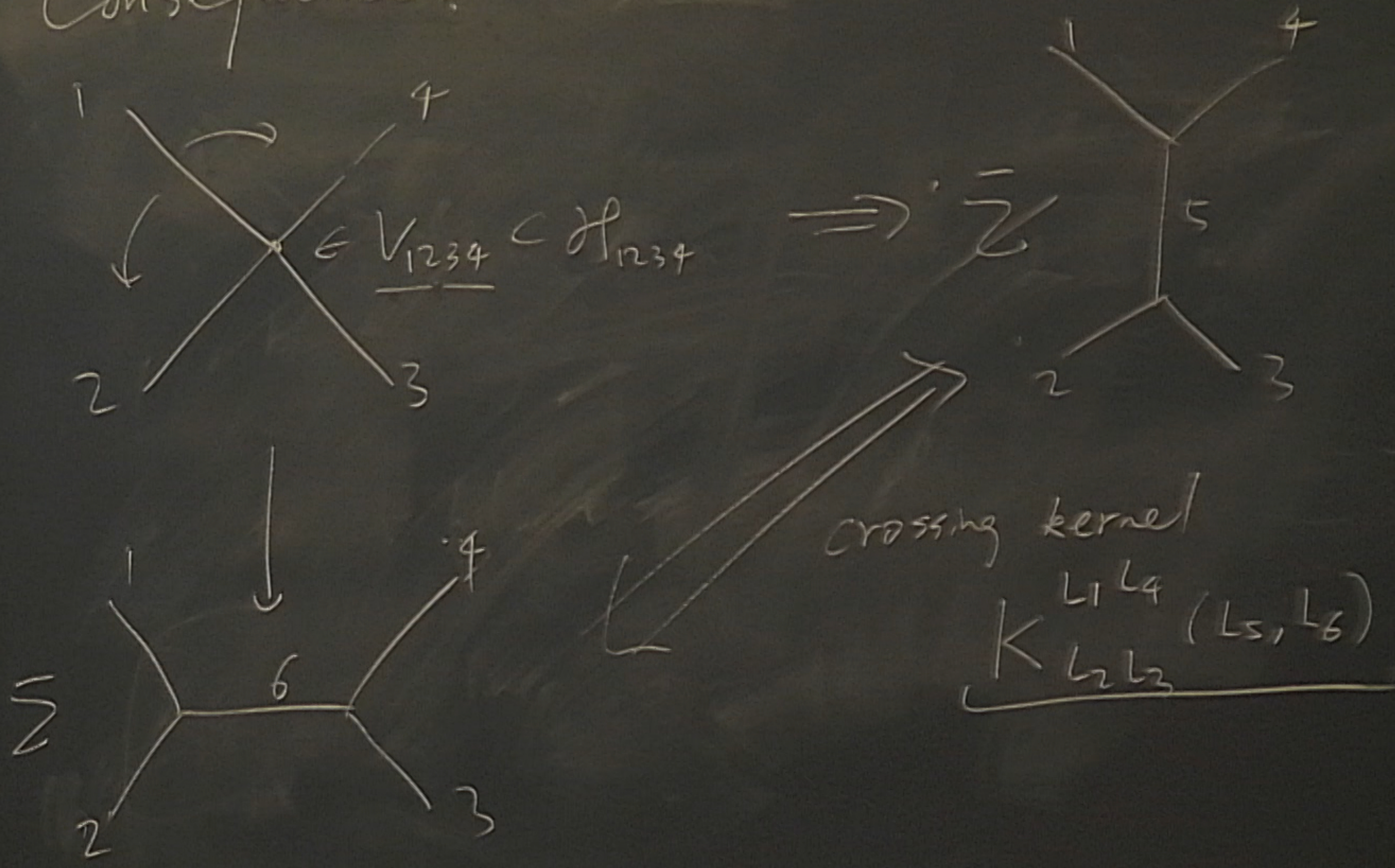


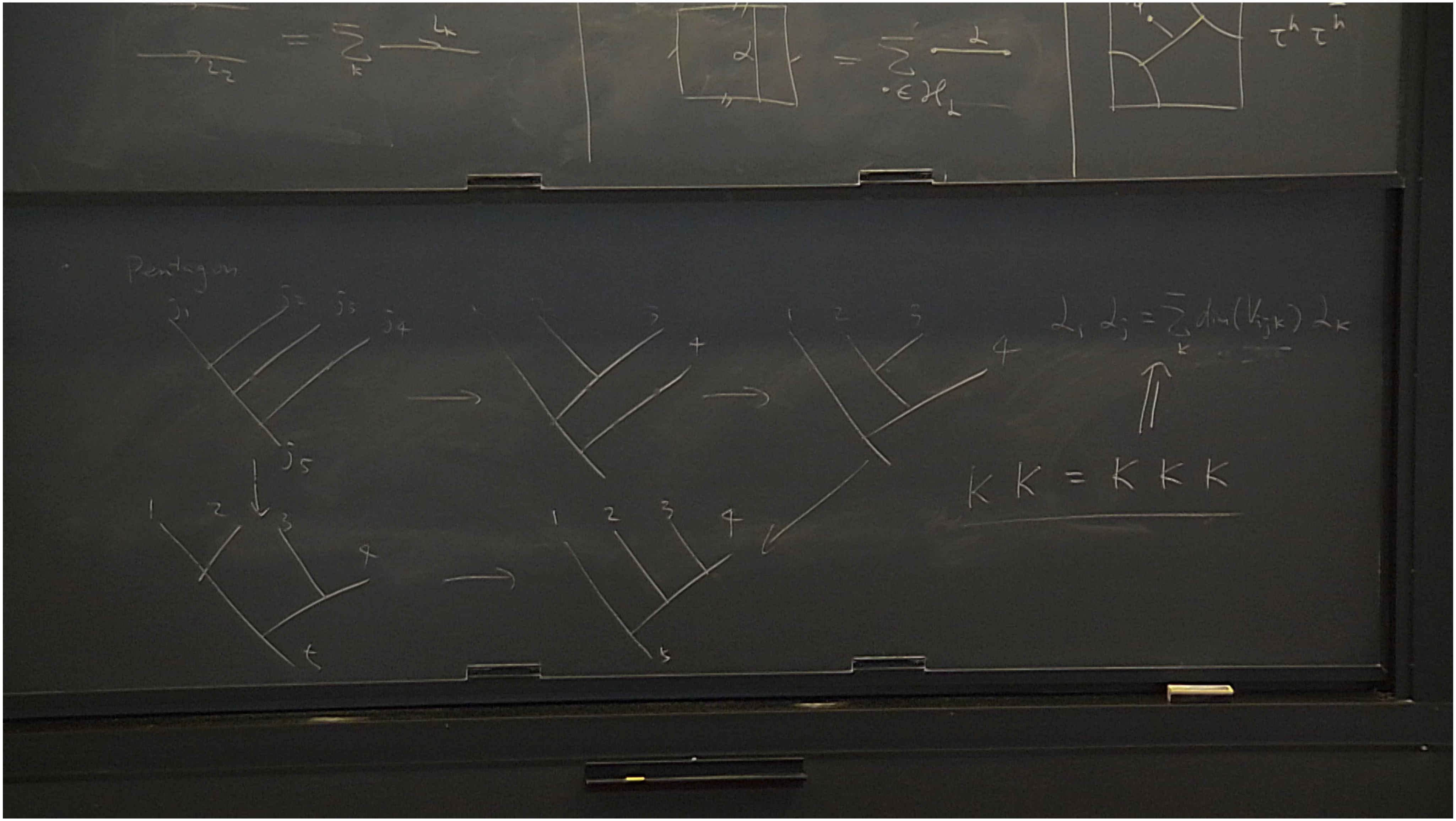
junctions
L1
V_{L1L2L3} ⊂ H_{L1L2L3}
(0,0)
L2
L3
Fusion
L1
L2
= Σ_k L_k
local

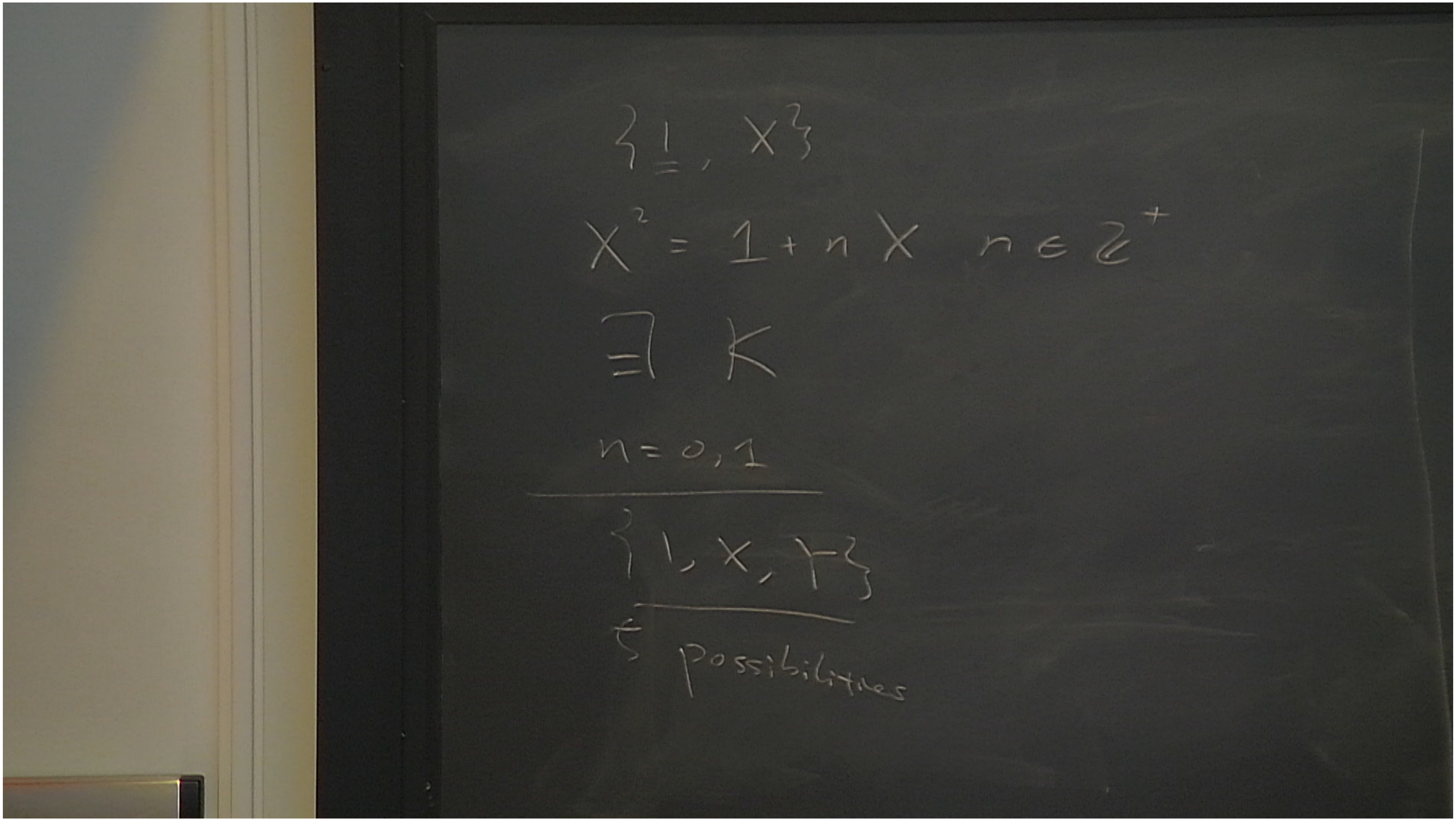

Locality
mess
Modular Inv

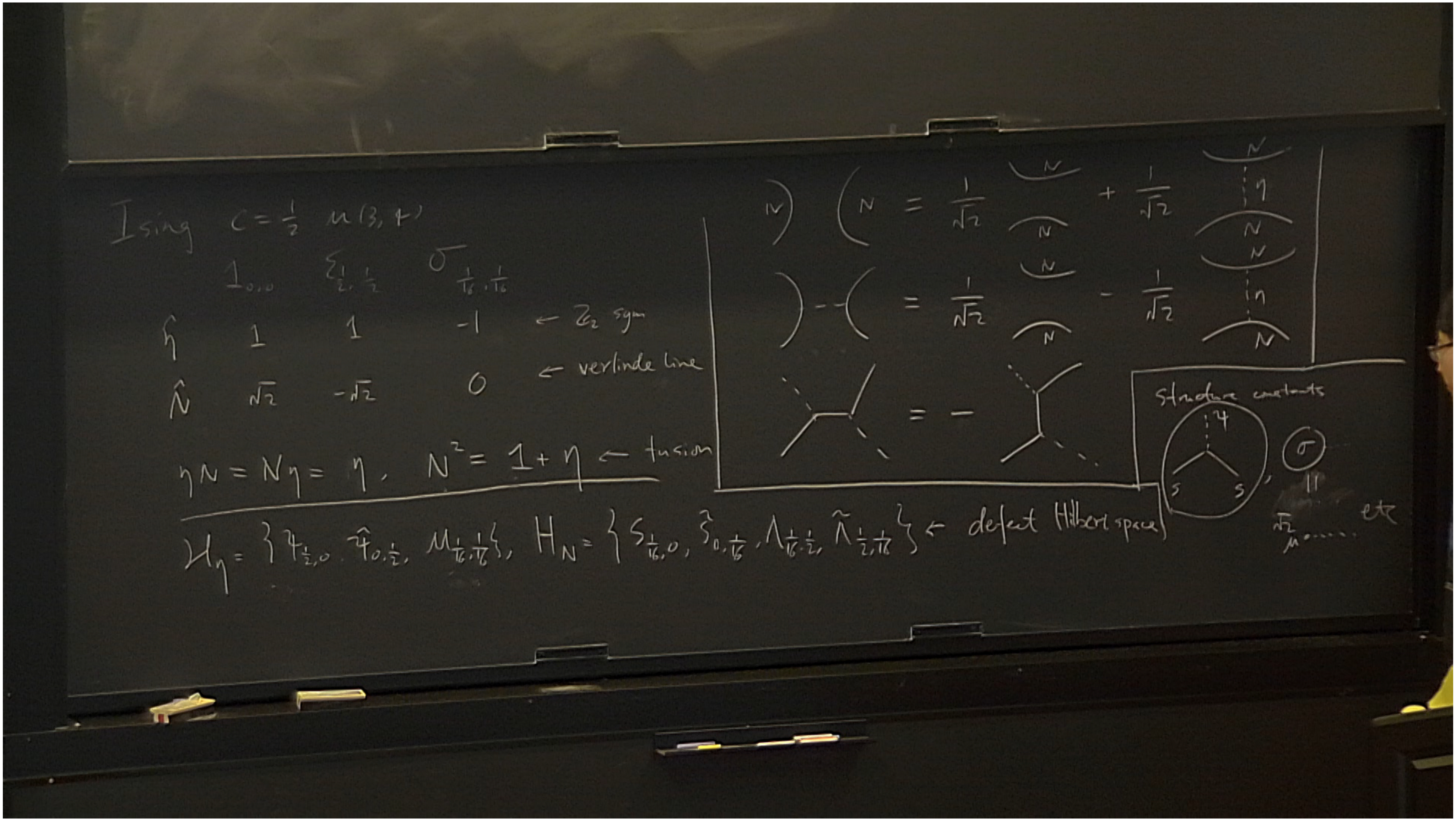

ty
mess
Modular Inv
(h,h̄)
τ^h τ̄^h̄
∈ H_α

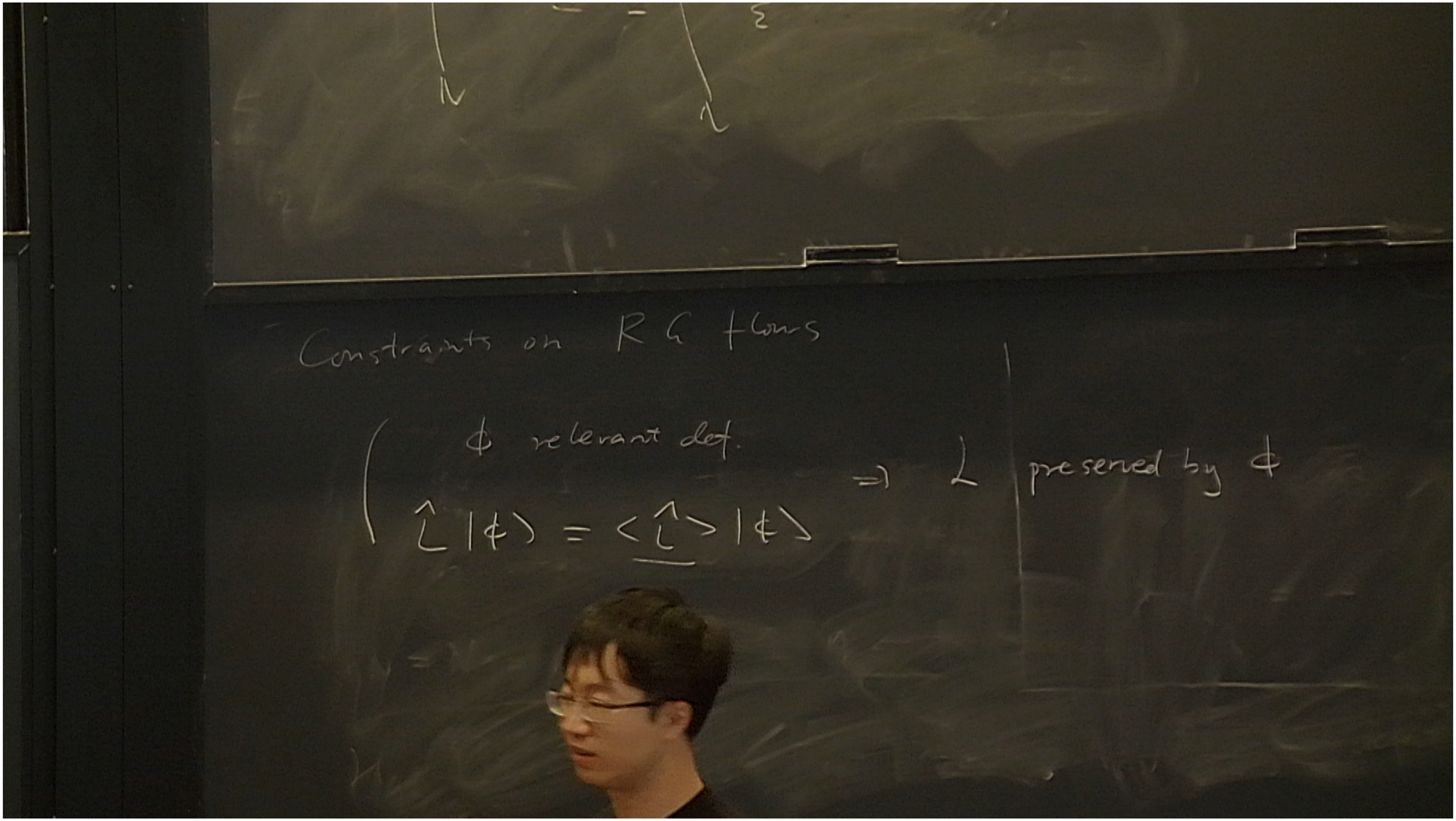
Consequences:

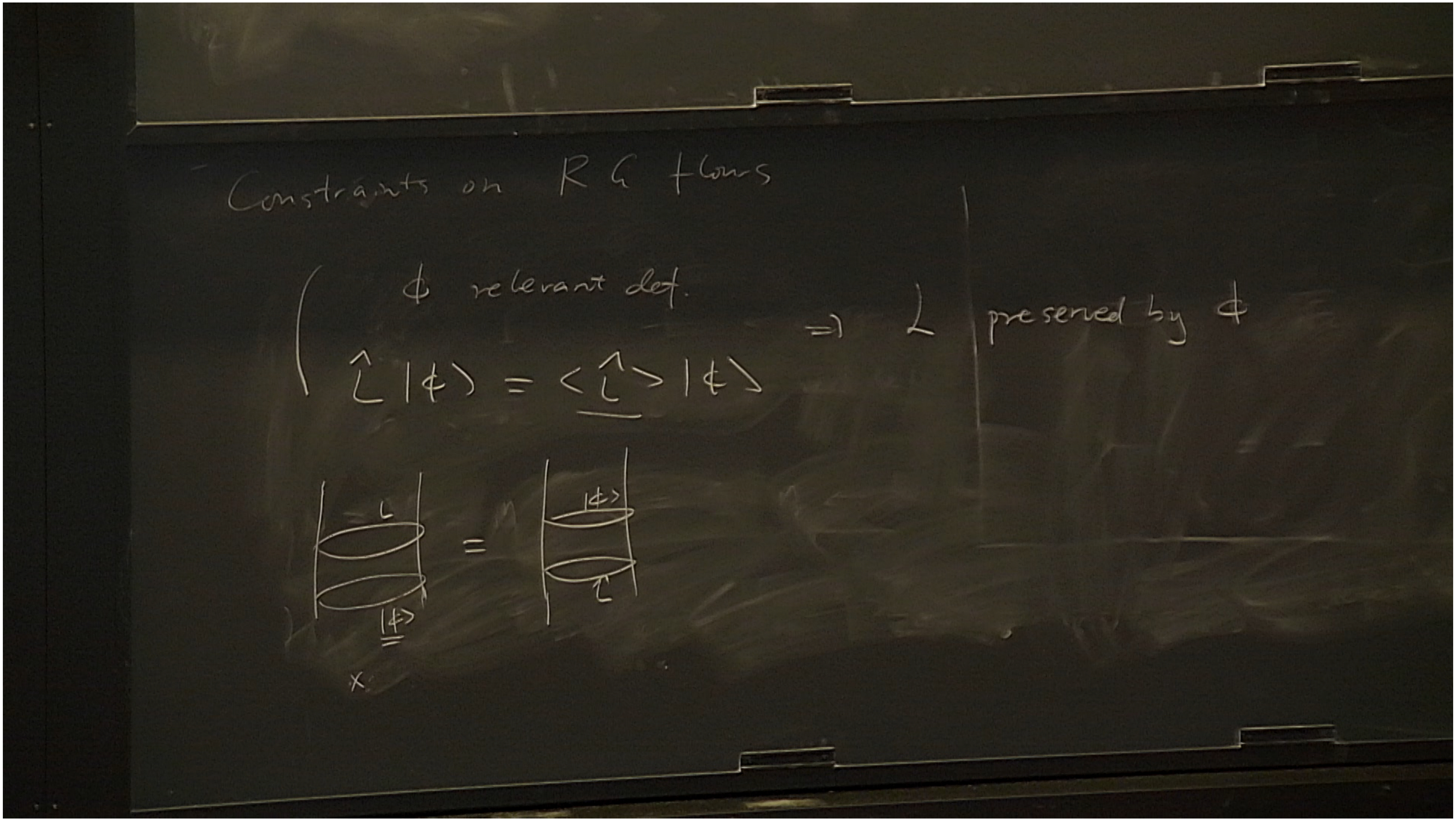


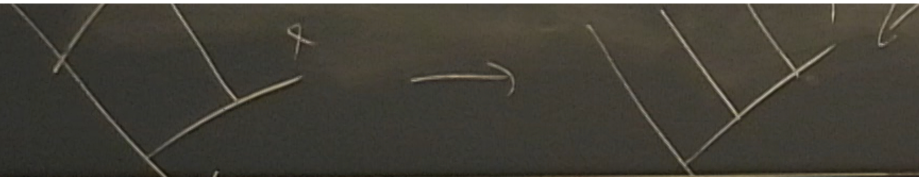

Pentagon
$L_i L_j = \sum_k \dim(V_{ijk}) L_k$
KK = KKK

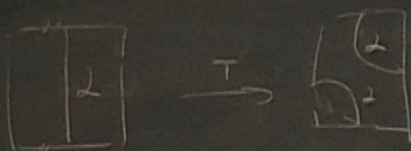

$\{\underline{1}, X\}$
$X^2 = 1 + nX \quad n \in \mathbb{Z}^+$
$\exists K$
$n = 0, 1$
$\{1, X, Y\}$
5 possibilities


Ising $c=\frac{1}{2}$ $M(3,4)$
$1_{0,0}$ $\varepsilon_{\frac{1}{2},\frac{1}{2}}$ $\sigma_{\frac{1}{16},\frac{1}{16}}$
$\hat{\eta}$ 1 1 -1 ← $\mathbb{Z}_2$ sym
$\hat{N}$ $\sqrt{2}$ $-\sqrt{2}$ 0 ← verlinde line
$\eta N = N\eta = \eta,\ N^2 = 1+\eta$ ← fusion
$\mathcal{H}_\eta = \{\psi_{\frac{1}{2},0}, \tilde{\psi}_{0,\frac{1}{2}}, \mu_{\frac{1}{16},\frac{1}{16}}\}$, $H_N = \{s_{\frac{1}{16},0}, \xi_{0,\frac{1}{16}}, \Lambda_{\frac{1}{16},\frac{1}{2}}, \tilde{\Lambda}_{\frac{1}{2},\frac{1}{16}}\}$ ← defect Hilbert space
$= \frac{1}{\sqrt{2}}$ $+ \frac{1}{\sqrt{2}}$
$= \frac{1}{\sqrt{2}}$ $- \frac{1}{\sqrt{2}}$
N
η
= −
Structure constants
ψ
s s
σ
$\sqrt{2}$ μ
etc


Constraints on RG flows
φ relevant def.
⇒ L preserved by φ
L̂ |φ⟩ = ⟨L̂⟩ |φ⟩


Constraints on RG flows
ϕ relevant def.
L̂ |ϕ⟩ = ⟨L̂⟩ |ϕ⟩
⇒ L preserved by ϕ
L
|ϕ⟩
=
|ϕ⟩
L





$= \mathrm{tr}_{\mathcal{H}_\alpha}$

$\xrightarrow{\text{crossing}}$ constrain $e^{2\pi i s n}$

↑ spin selection rule

Thm 1: if $\exists$ spin selection rule that forbids $s=0$ in $\mathcal{H}_\alpha$ & α preserved by RG

- $\mathcal{H}_\alpha^{IR}$ empty
- $\mathcal{H}_\alpha^{IR}$ empty $\rightarrow$ IR phase is a CFT satisfying the spin selection rule

Obs:

In the case when IR phase is gapped

$\Rightarrow$ all structure const determined by TDL spin selection rule that forbids $s=0$ in $\mathcal{H}_L$

& L preserved by RG

- $\mathcal{H}_L^{IR}$ empty

- $\mathcal{H}_L^{IR}$ empty $\Longrightarrow$ IR phase is a CFT satisfying the spin selection rule

$M(4,5)$ tricritical Ising $c = \frac{7}{10}$

RG: ε, σ'

	1	ε	ε'	ε''	σ	σ'
		$\frac{1}{10}$	$\frac{3}{5}$	$\frac{3}{2}$	$\frac{3}{80}$	$\frac{7}{16}$
$\mathbb{Z}_2 \to \hat{\eta}$	1	1	1	1	-1	-1
$\hat{W}$	ζ	$-\zeta^{-1}$	$-\zeta^{-1}$	ζ	$-\zeta^{-1}$	ζ
$\hat{N}$	$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$	0	0

simple lines: $1, \eta, W, N \ldots$

$\zeta = \frac{1+\sqrt{5}}{2}$

$W^2 = 1 + W$ (LY)

ε preserving

$N^2 = 1 + \eta$ (Ising)

σ'

preserved by RG

- $\mathcal{H}_{\perp}^{IR}$ empty
- $\mathcal{H}_{\perp}^{IR}$ nonempty $\Longrightarrow$ IR phase is a CFT satisfying the spin selection rule

$M(7,5)$ tricritical ising $c = \frac{7}{10}$

	1	ε	ε'	ε''	σ	σ'
		$\frac{1}{10}$	$\frac{3}{5}$	$\frac{3}{2}$	$\frac{3}{80}$	$\frac{7}{16}$
simple lines:		$1,\ \eta,\ W,\ N \cdots$				
$\mathbb{Z}_2 \to \hat{\eta}$	1	1	1	1	-1	-1
$\hat{W}$	ζ	$-\zeta^{-1}$	$-\zeta^{-1}$	ζ	$-\zeta^{-1}$	ζ
$\hat{N}$	$\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$	0	0

$$\zeta = \frac{1+\sqrt{5}}{2}$$

RG: $\varepsilon,\ \sigma'$

$\mathcal{H}_W: s \in \mathbb{Z} \pm \{0, \frac{2}{5}\}$

$\mathcal{H}_N: s \in \frac{\mathbb{Z}}{2} \pm \frac{1}{16}$

$W^2 = 1 + W$ (LY)

σ' preserving

$\downarrow$

1 phase

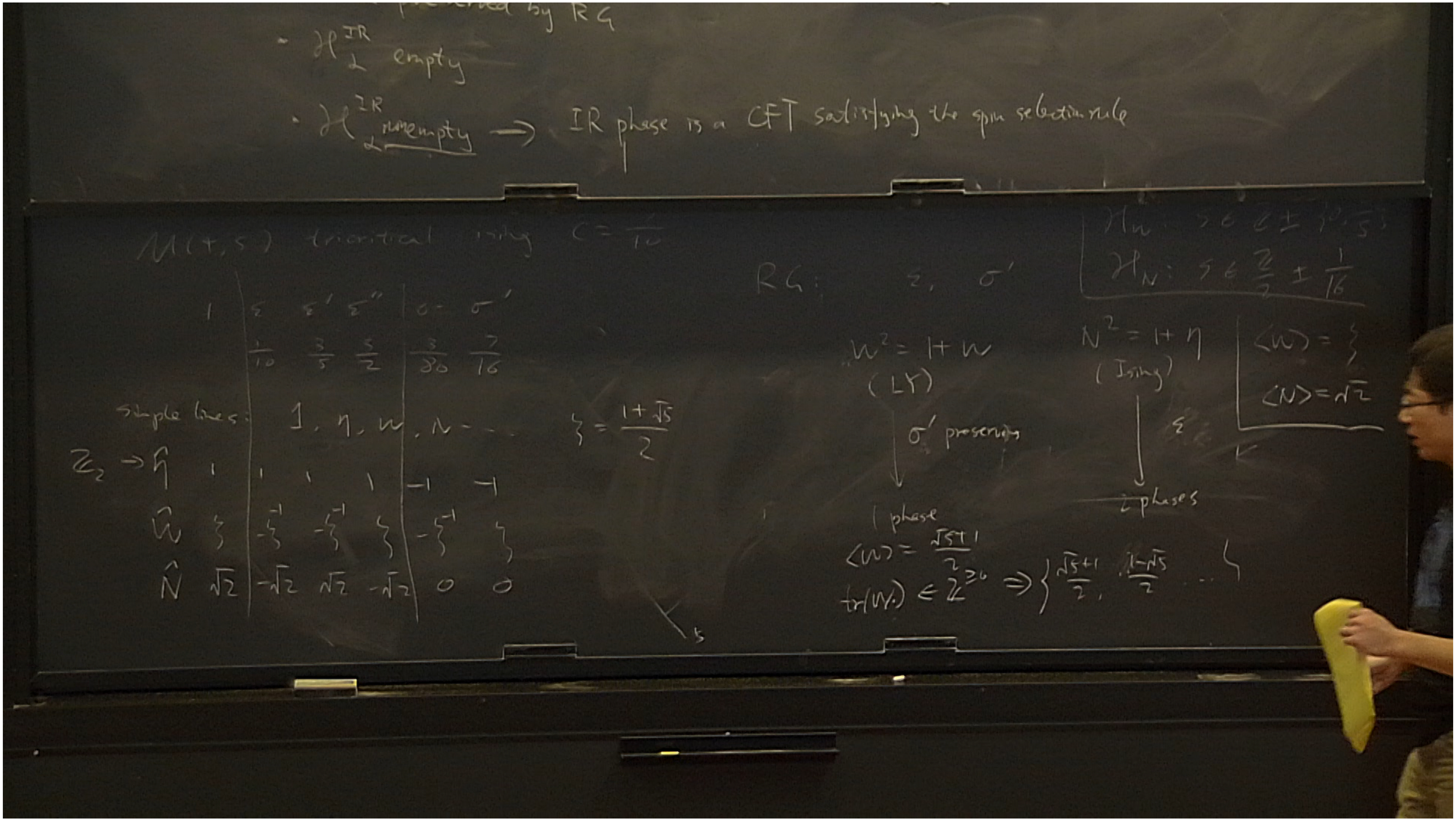
$N^2 = 1 + \eta$ (Ising)

$\downarrow \varepsilon$

2 phases

$\langle W \rangle = \zeta$

$\langle N \rangle = \sqrt{2}$


preserved by RG
$\mathcal{H}_{\perp}^{IR}$ empty
$\mathcal{H}_{\perp}^{IR}$ nonempty ⟹ IR phase is a CFT satisfying the spin selection rule
M(4,5) tricritical Ising
RG: ε, σ'
$\mathcal{H}_N$: $s \in \frac{\mathbb{Z}}{2} \pm \frac{1}{16}$
simple lines: 1, η, W, N
$\zeta = \frac{1+\sqrt{5}}{2}$
$W^2 = 1 + W$
(LY)
σ' preserving
1 phase
$\langle W \rangle = \frac{\sqrt{5}+1}{2}$
$\mathrm{tr}(W_\cdot) \in \mathbb{Z}^{\geq 0}$
$N^2 = 1 + \eta$
(Ising)
2 phases
$\langle W \rangle = \zeta$
$\langle N \rangle = \sqrt{2}$