Title: PSI 2017/2018 - Quantum Integrable Models - Lecture 7

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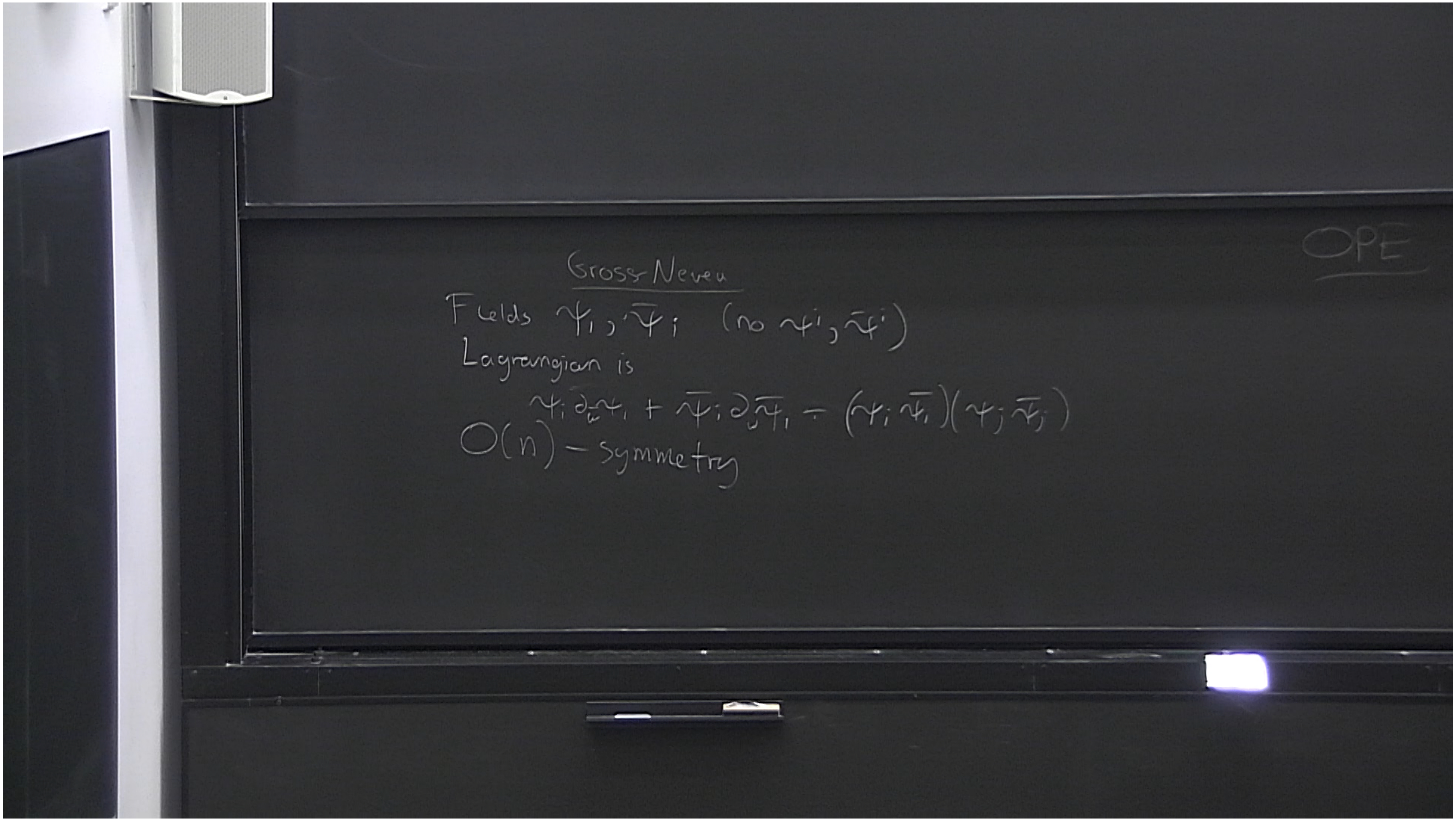
Abstract:

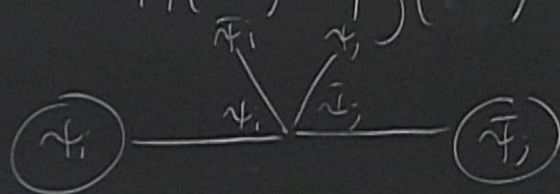

Gross-Neveu
Fields $\psi_i, \bar\psi_i$ (no $\psi^i, \bar\psi^i$)
Lagrangian is
$\psi_i \partial_{\bar z} \psi_i + \bar\psi_i \partial_z \bar\psi_i - (\psi_i \bar\psi_i)(\psi_j \bar\psi_j)$
$O(n)$ - symmetry
OPE

OPE

$$\psi_i(0)\,\psi_j(w,\bar{w}) \sim \frac{1}{2\pi i}\,\frac{1}{w}\,\delta_{ij}$$

$$\bar{\psi}_i(0)\,\bar{\psi}_j(w,\bar{w}) \sim -\frac{1}{2\pi i}\,\frac{1}{\bar{w}}\,\delta_{ij}$$

$$\psi_i(0)\,\bar{\psi}_j(w,\bar{w}) \sim C\,\log|w|^2\,\bar{\psi}_i(0)\,\psi_j(0)$$

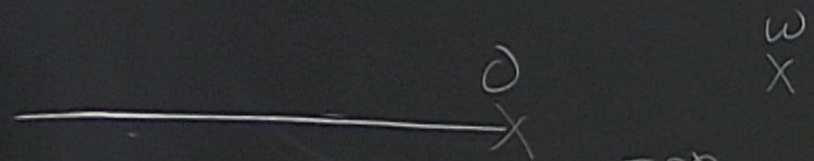


$$\int_{w'} \frac{1}{\bar{w}'}\,\frac{1}{w-w'}\,dw'\,d\bar{w}'$$

Trick

$$\frac{d\bar{w}'}{\bar{w}'} = d\log \bar{w}'$$

$$\int \partial_{\bar{w}'}\left(\log \bar{w}' \, \frac{1}{\omega-\omega'} \right)$$



$$= \int_{x=0}^{-\infty} \frac{1}{\omega - x} \pm \log \overline{\omega}$$

What do we learn from the OPE?

Classical theory is conformal — $\bar{\psi}_i, \psi_i$ have dim

$\int \mathcal{L}$ is scale invariant (dim'n 0)

But: $\psi_i(0)\bar{\psi}_j(w) \sim C \log|w|^2 \bar{\psi}_i(0)\psi_j(0)$

is not scale invariant

$\psi_i(0)\bar{\psi}_j(\lambda w)$ $\qquad C \log|\lambda w|^2 \bar{\psi}_i(0)\psi_j(0)$

What do we learn from the OPE?

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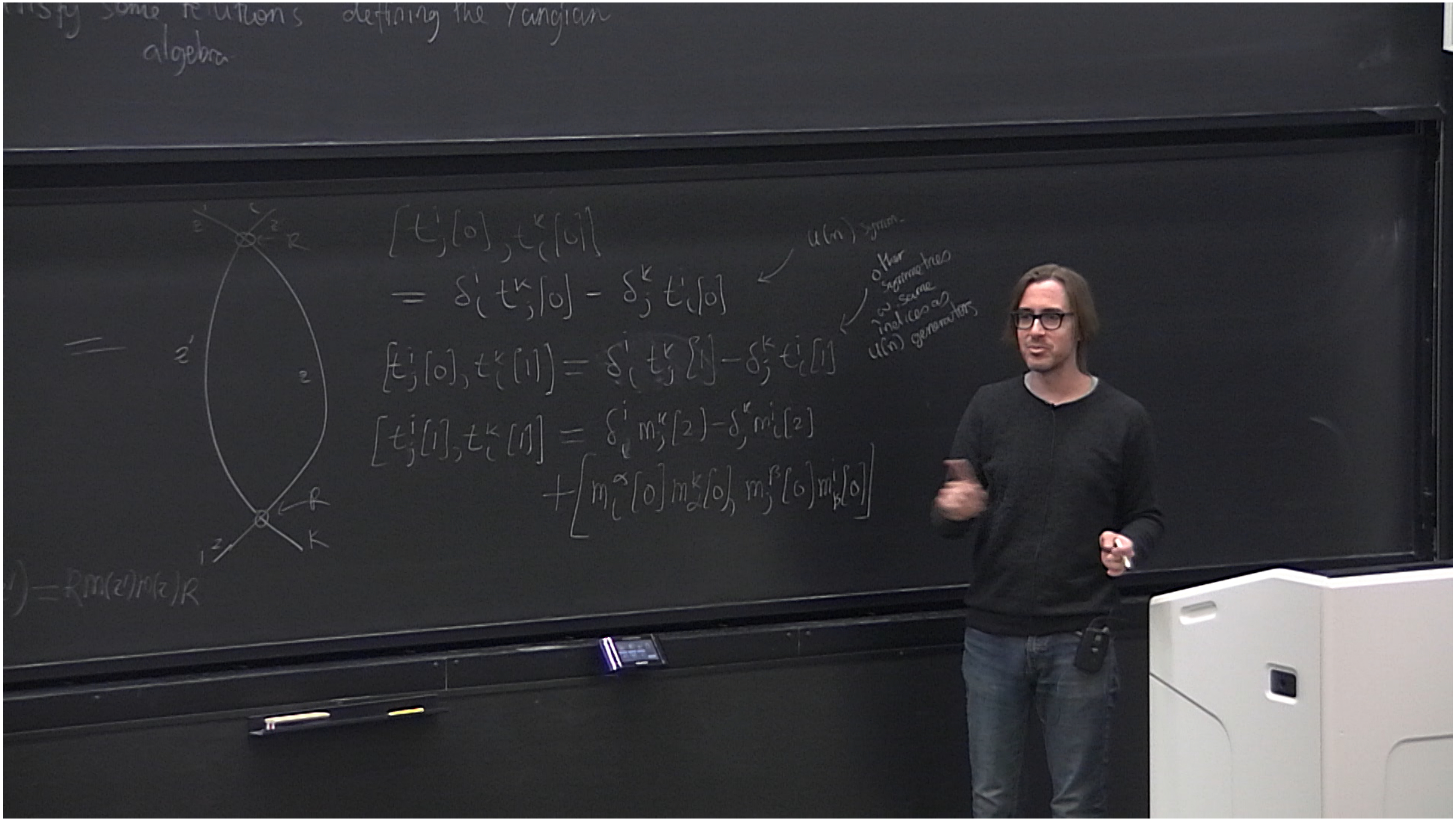
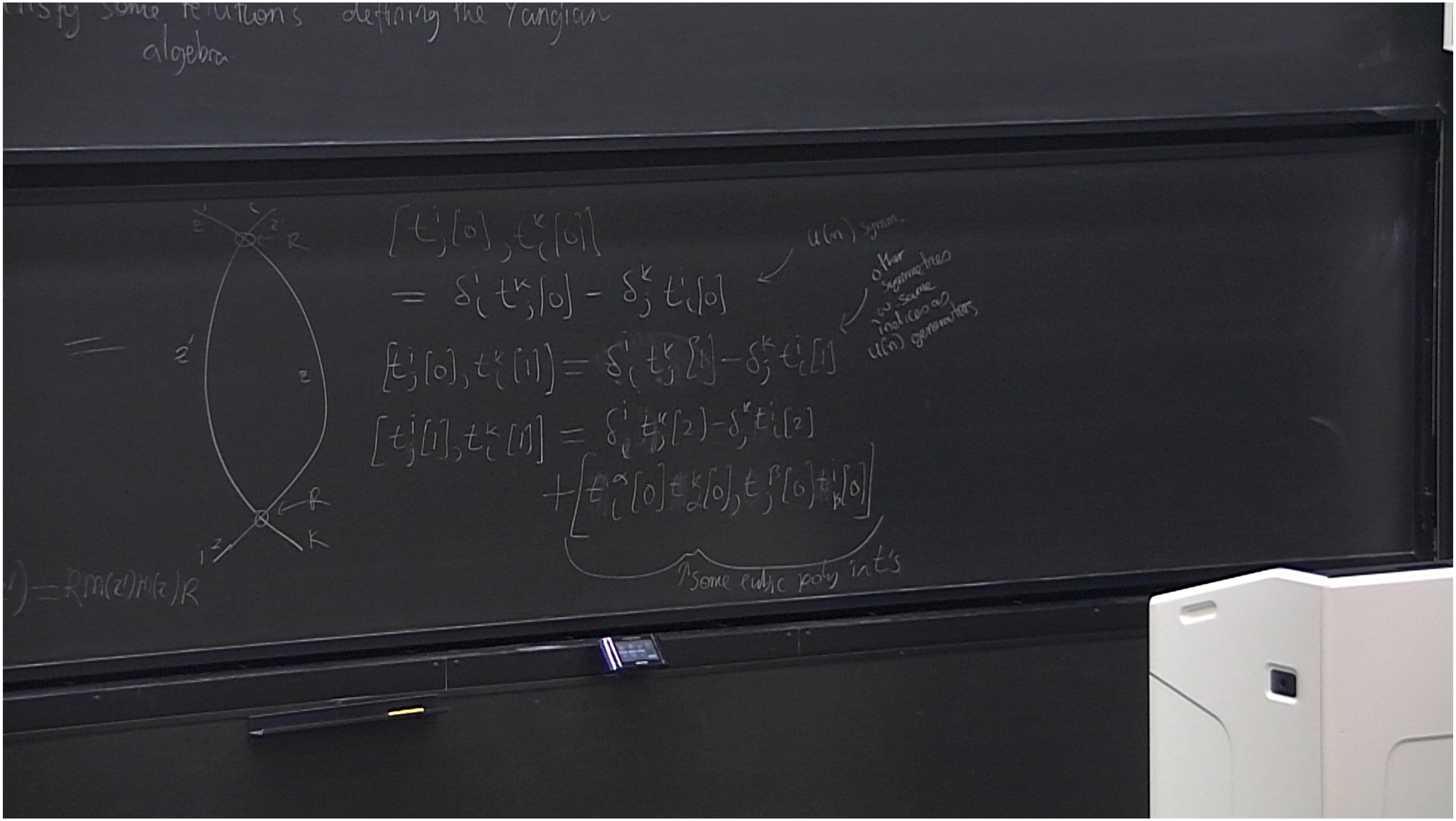
$$\log \lambda^2|w|^2 = \log\lambda^2 + \log|w|^2$$

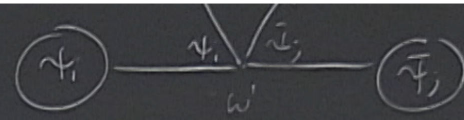
$\log|w|^2$ not of dim^n 0.


some relations defining the Yangian
algebra
u(n) symm.
other symmetries w same indices as u(n) generators


isfy some relations defining the Yangian
algebra
[t^i_j[0], t^k_l[0]]
= δ^i_l t^k_j[0] − δ^k_j t^i_l[0]
u(n) symm.
other symmetries w. same indices as u(n) generators
[t^i_j[0], t^k_l[1]] = δ^i_l t^k_j[1] − δ^k_j t^i_l[1]
[t^i_j[1], t^k_l[1]] = δ^i_l t^k_j[2] − δ^k_j t^i_l[2]
+ [t^α_l[0] t^k_α[0], t^β_j[0] t^i_β[0]]
some cubic poly in t's
= R m(z') m(z) R
R
R
K

If i, j, k, l are distinct

$$[t^i_j[1], t^k_l[1]] = t^i_l[0]\, t^k_\beta[0]\, t^\beta_j[0] + \cdots \text{ (other cubic expressions)}$$



$$\int_{\omega'} \frac{1}{\bar\omega'}\,\frac{1}{\omega-\omega'}\, d\omega' d\bar\omega'$$

$\ldots[0]\, t^k_\beta[0]\, t^\beta_j[0]$

---- (other cubic expressions)

Field theory (back to Thirring model ψ_i, ψ^i ..)

$$m^i_j(z) = P\,\mathrm{Exp}^{\infty}_{x-\infty}\, \mathcal{L}(z)_x$$

$$\mathcal{L}^i_j(z)_\omega = \frac{1}{z}\,\psi_j\psi^i$$

$$\mathcal{L}^i_j(z)_{\bar\omega} = \frac{1}{1-z}\,\bar\psi_j\bar\psi^i$$

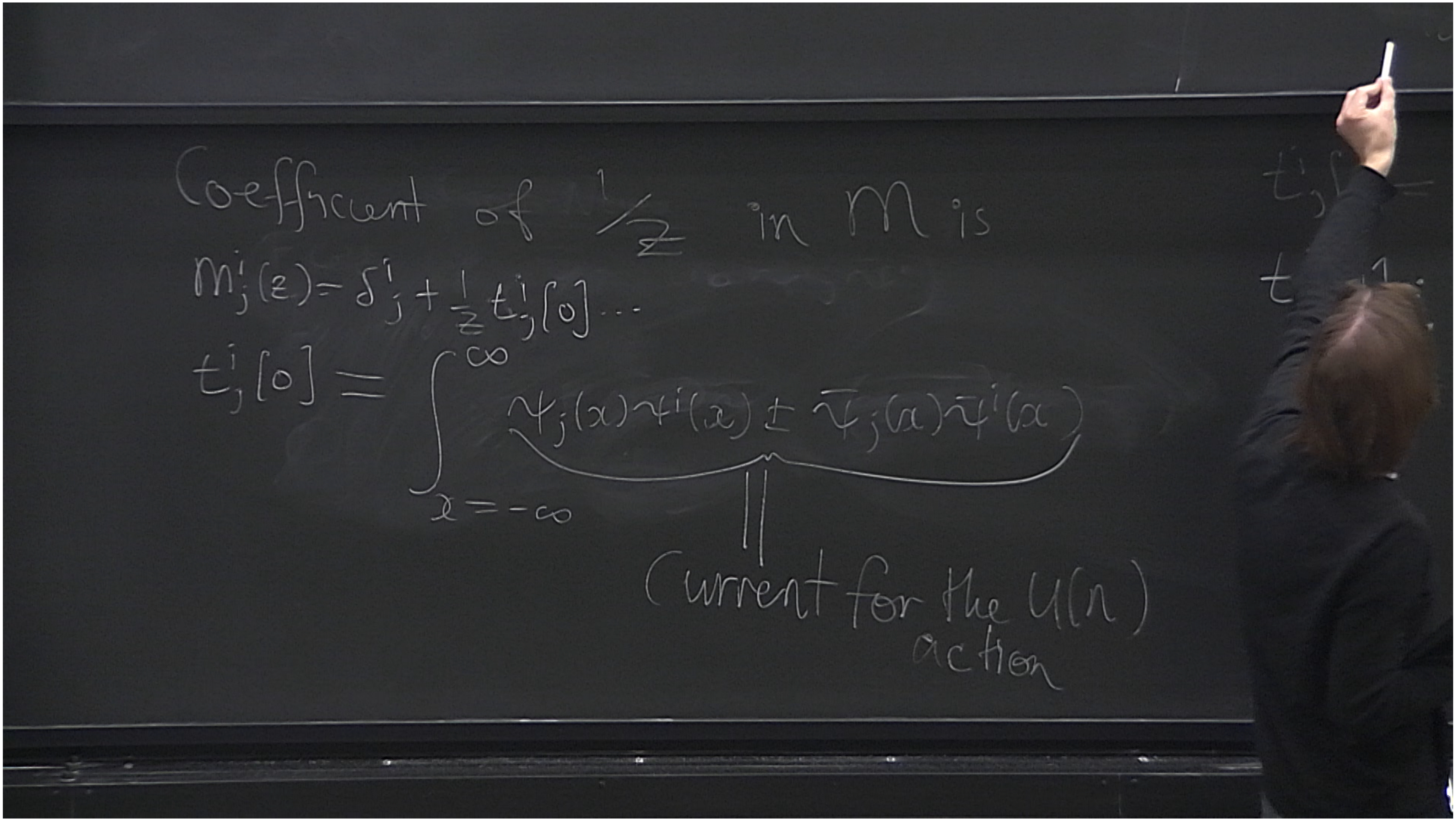
Field theory (back to Thirring model $\psi, \bar\psi$ ...)

$$m^i_j(z) = \mathrm{PExp}^{\infty}_{x-\infty}\, \mathcal{L}(z)_x$$

$$\mathcal{L}^i_j(z)_\omega = \frac{1}{z}\,\psi_j\psi^i$$

$$\mathcal{L}^i_j(z)_{\bar\omega} = \frac{1}{1-z}\,\bar\psi_j\bar\psi^i = \frac{z^{-1}}{z^{-1}-1}\,(\bar\psi_j\bar\psi^i) = \frac{-1}{z}\,\bar\psi_j\bar\psi^i + \frac{1}{z^2}\,\bar\psi_j\bar\psi^i \dots$$

ubic
pressions)


Coefficient of $1/z$ in M is
$m^i_j(z) = \delta^i_j + \frac{1}{z} t^i_j[0] \ldots$
$t^i_j[0] = \int_{x=-\infty}^{\infty} \underbrace{\psi_j(x)\psi^i(x) \pm \bar{\psi}_j(x)\bar{\psi}^i(x)}$
Current for the $U(n)$ action

$L^i_j(z)_{\bar{\omega}} = \frac{1}{1-z}\bar{\psi}_j \psi^i = \frac{z}{z^{-1}-1}(\psi_j \psi^i) \ldots$

$/z$ in $\mathcal{M}$ is

$\underbrace{)\psi^i(x) \pm \bar{\psi}_j(x)\psi^i(x)}$

$\|$

Current for the $U(n)$ action

$t^i_j[0]$ = charge for conserved current.

$t^i_j[1]$: coeff. of z^{-2}

"Local" term: $\int_{x=-\infty}^{\infty} \bar{\psi}_i \bar{\psi}^j$

$t^i_j[0]$ = charge for conserved current.

$t^i_j[1]$: coeff. of z^{-2}

"Local" term: $\int_{x=-\infty}^{\infty} \tilde{\psi}_i \tilde{\psi}^j$

"Bilocal" terms where we integrate 2 L's

$\int_{x_1 < x_2} \psi_j(x_1)\psi^\alpha(x_1)\psi_\alpha(x_2)\psi^i(x_2)$ + other similar terms

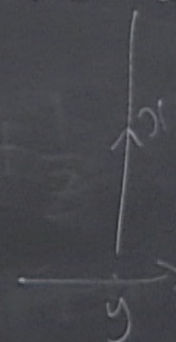
$\times \mathcal{L}_w + \mathcal{L}_{\bar{w}}$

$\times \mathcal{L}_w + \mathcal{L}_{\bar{w}}$

n)

We want to compute $[t[1], t[1]]$

<u>Commutator</u> $m^i_j(z,y)$ where we integrate at y



$$\partial_y m^i_j(z,y)=0$$

Consider OPE

$$m^i_j(z,0)\, m^k_l(z',\varepsilon)$$

$$=\sum F(\varepsilon)\,\mathcal{O}$$

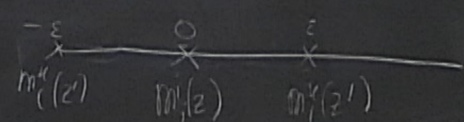
$$\partial_\varepsilon\left(m^i_j(z,0)\, m^k_l(z',\varepsilon)\right)=0$$

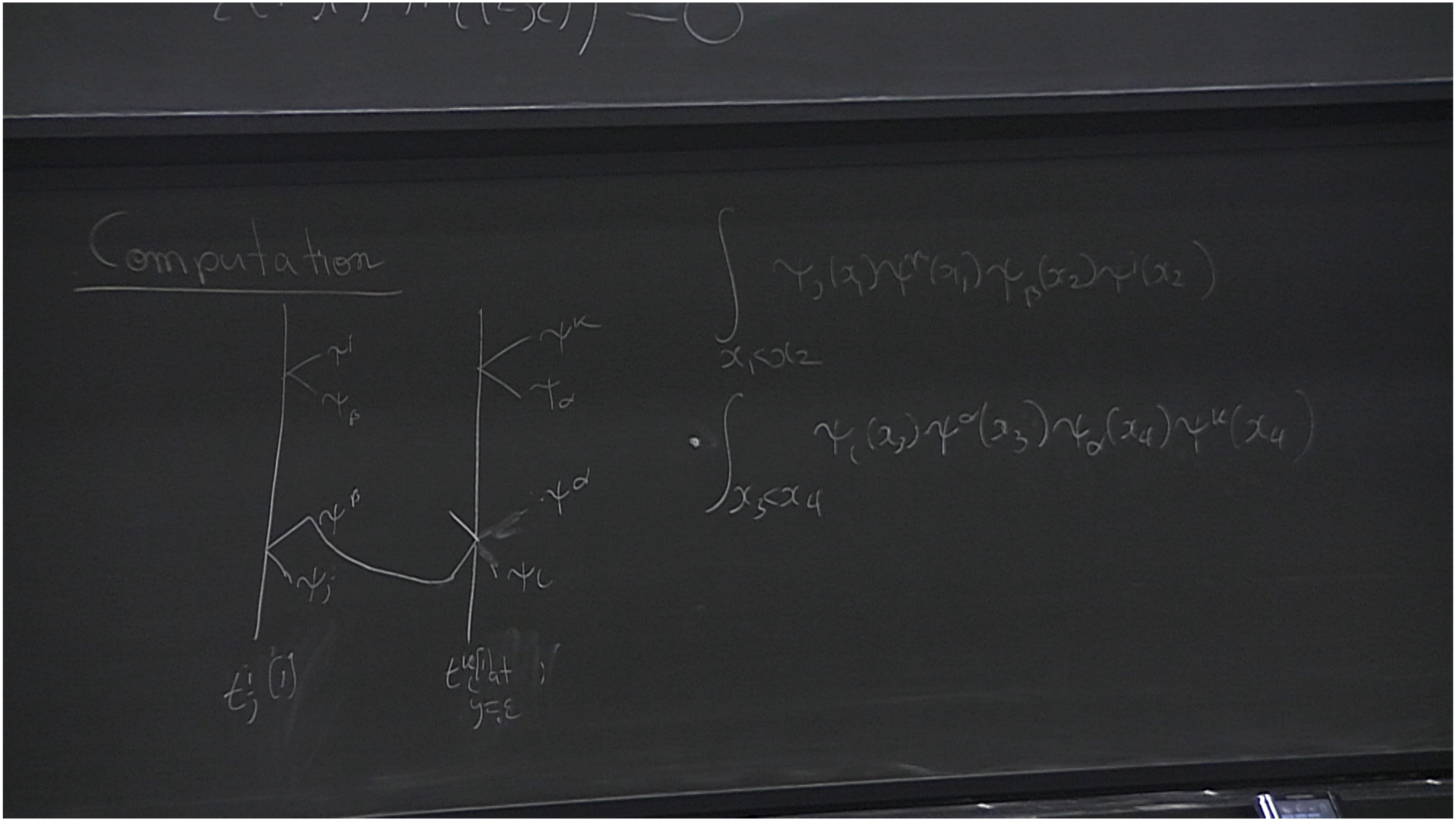
$$= \sum F(\varepsilon)\, \Theta$$

$$\partial_\varepsilon \left(m^i_j(z,0)\, m^k_l(z',\varepsilon) \right) = 0$$

$$m^i_j(z,0)\, m^k_l(z',\varepsilon) \sim \mathrm{Sign}(\varepsilon)\, \Theta$$

Θ is the commutator of $m^i_j(z)$ with $m^k_l(z')$




Computation
$\int_{x_1<x_2} \psi_j(x_1)\psi^k(x_1)\psi_\beta(x_2)\psi^i(x_2)$
$\int_{x_3<x_4} \psi_l(x_3)\psi^\alpha(x_3)\psi_\alpha(x_4)\psi^k(x_4)$
$t^i_j(1)$
$t^{k[1]}_l$ at
$y=\varepsilon$

One term in OPE will be

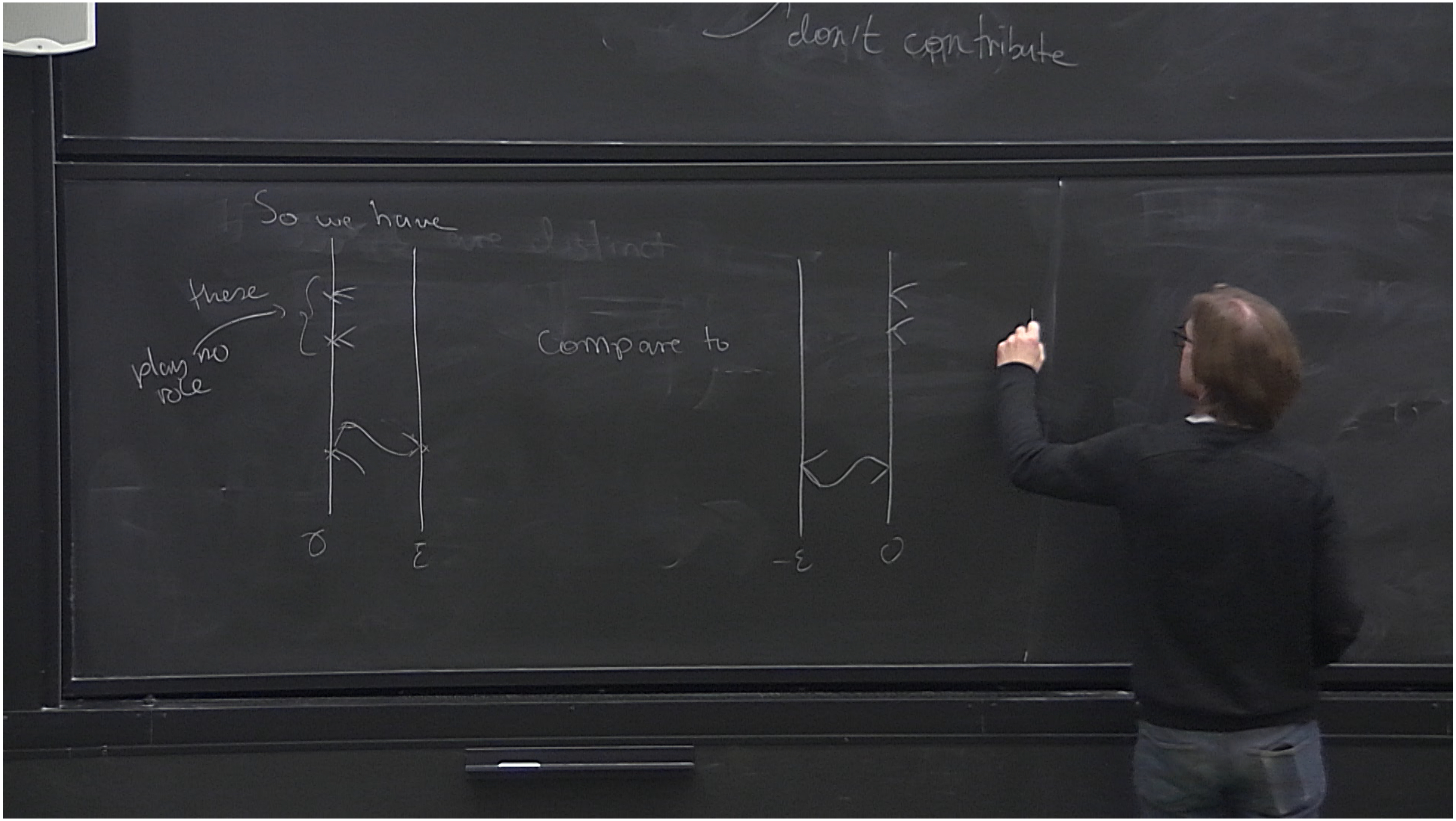
$$\int_{\substack{x_1<x_2 \\ x_3<x_4}} \psi_j(x_{1,0})\,\psi_l(x_{2,0})\,\psi^j(x_{2,0})\,\psi^\alpha(x_{3,\varepsilon})\,\psi_\alpha(x_{4,\varepsilon})\,\psi^l(x_{4,\varepsilon})$$

One term in OPE will be

$$\int_{\substack{x_1<x_2 \\ x_3<x_4}} \psi_j(x_1,0)\,\psi_l(x_2,0)\,\psi^j(x_2,0)\,\psi^\alpha(x_3,\varepsilon)\,\psi_\alpha(x_4,\varepsilon)\,\psi^l(x_4,\varepsilon)\,\frac{1}{x_1-x_3+i\varepsilon}$$

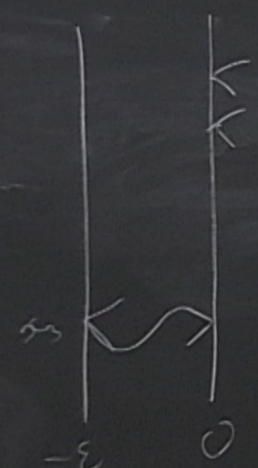
$$\psi^\alpha(x_3,\varepsilon) = \psi^\alpha(x_3,0) + \varepsilon\,\partial_y\psi^\alpha(x_3,0) + \dots$$

don't contribute

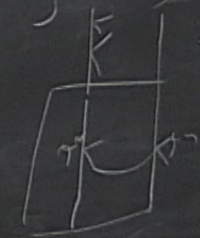

don't contribute
So we have
these
play no role
Compare to
0
ε
−ε
0

don't contribute

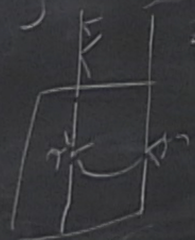
...mpare to



the difference between two sides is (up to order ε terms) a contour integral:
Fix x_1, x_2, x_4, $\int x_3$ in a contour around x_2



...ence between two sides is
...order ε terms) a contour integral:
... x_2, x_4, $\int$ x_3 in a contour
...d x_1.



Residue theorem:

The result is

$$\int_{x_1, x_3, x_4} \psi_j(x_1)\psi^\alpha(x_1)\psi_i(x_2)\psi^i(x_2)\,\psi_\alpha(x_4)\psi^k(x_4)$$

This has 3 copies of
things like $\psi_{i}(x)\psi^{i}(x)$

So it looks like $t[0]t[0]t[0]$

So, we find,

"$[t[1], t[1]] = t[0]^3$" as desired.