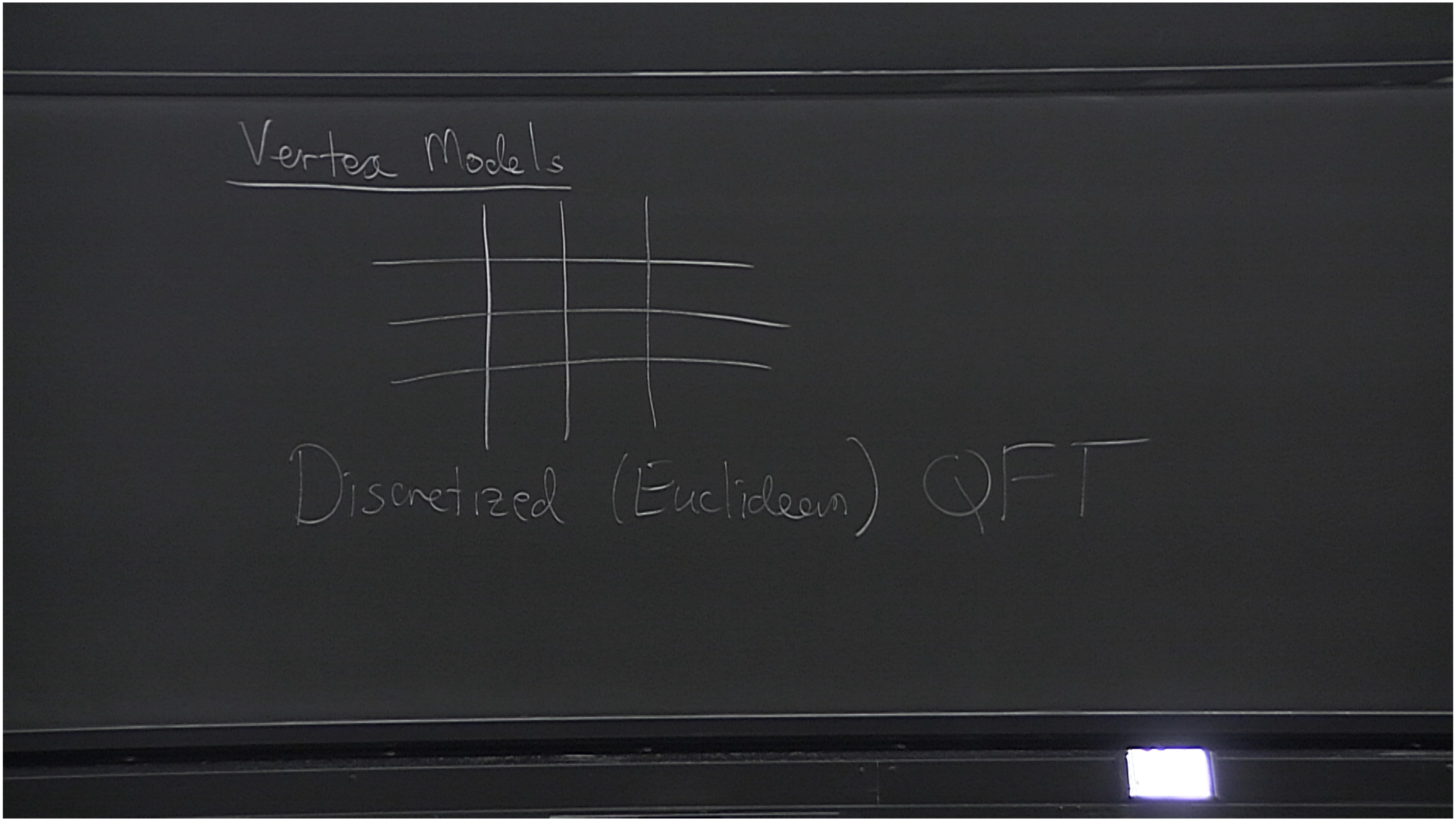
Title: PSI 2017/2018 - Quantum Integrable Models - Lecture 4

Date: Mar 22, 2018 11:30 AM

URL: http://pirsa.org/18030062

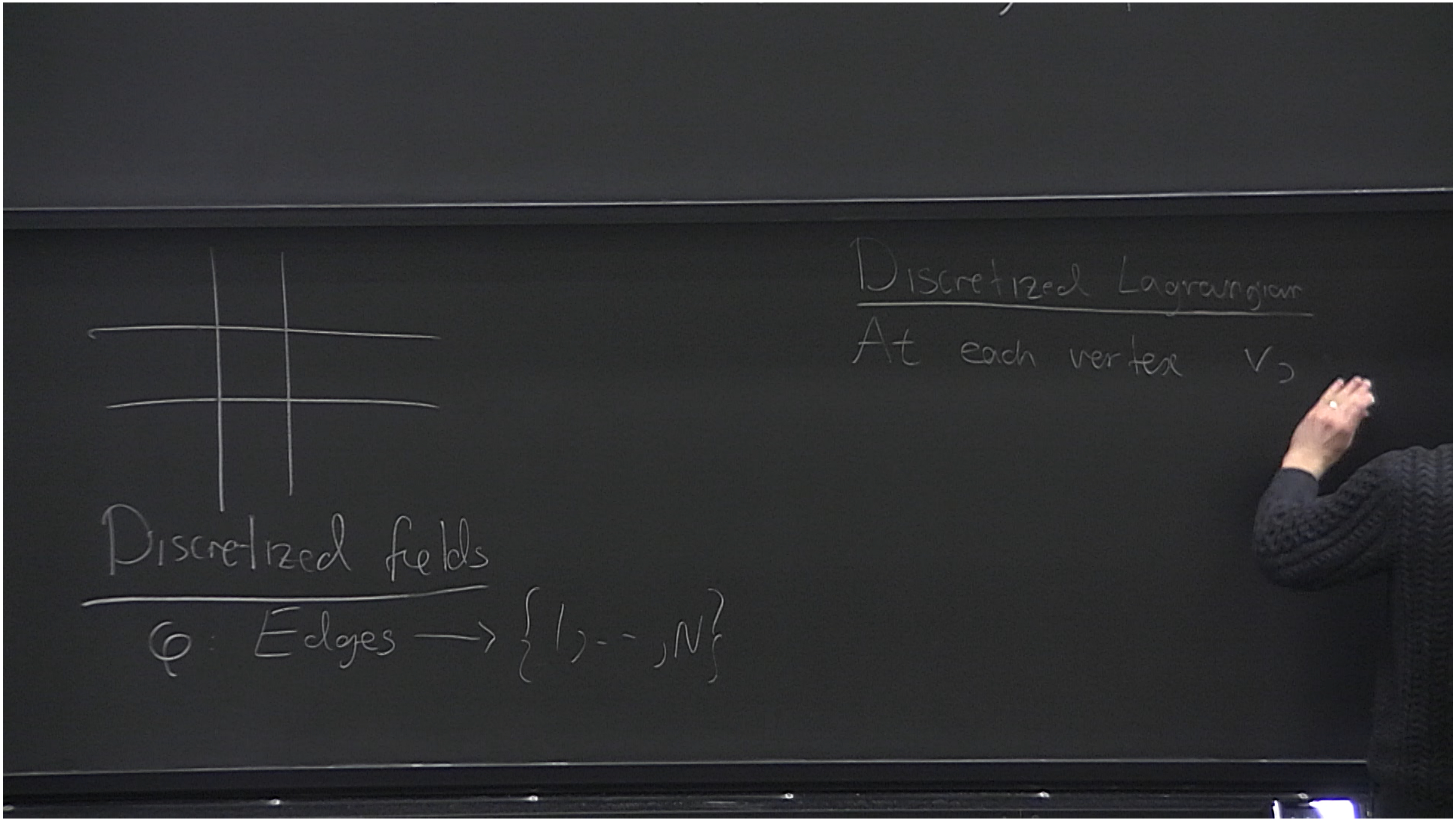
Abstract:


Vertex Models
Discretized (Euclidean) QFT

Ordinary QFT

Field $\varphi(x)$, $x \in$ Space-time $(\mathbb{R}^2)$

$$\int_{\varphi} e^{-\int_{\mathbb{R}^2} \mathcal{L}(\varphi)}$$


Discretized Lagrangian
At each vertex v,
Discretized fields
φ : Edges ⟶ {1, ..., N}

Discretized Lagrangian Fix some B^{ij}_{kl} $i,j,k,l : 1 \to N$

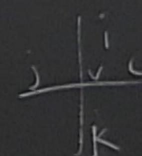
At each vertex v, we let

$$\mathcal{L}(\varphi, v)$$

lidean) QFT

<u>Discretized Lagrangian</u> Fix some B^{ij}_{kl} $i,j,k,l : 1 \to N$

At each vertex v, we let

$$\mathcal{L}(\varphi, v) = -\log B^{ij}_{kl}$$ if φ looks like  at this vertex

...clidean) QFT

φ

<u>Discretized Lagrangian</u> Fix some B^{ij}_{kl} i,j,k,l : $1 \to N$

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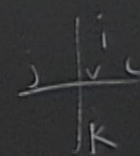
$$\mathcal{L}(\varphi, v) = -\log B^{ij}_{kl}$$ if φ looks like  at this vertex

Discretized path integral is

$$\sum_{\varphi} e^{-\sum_v \mathcal{L}(\varphi, v)} = \sum_{\varphi} \prod_v B^{ij}_{kl}$$

φ

clidean) QFT

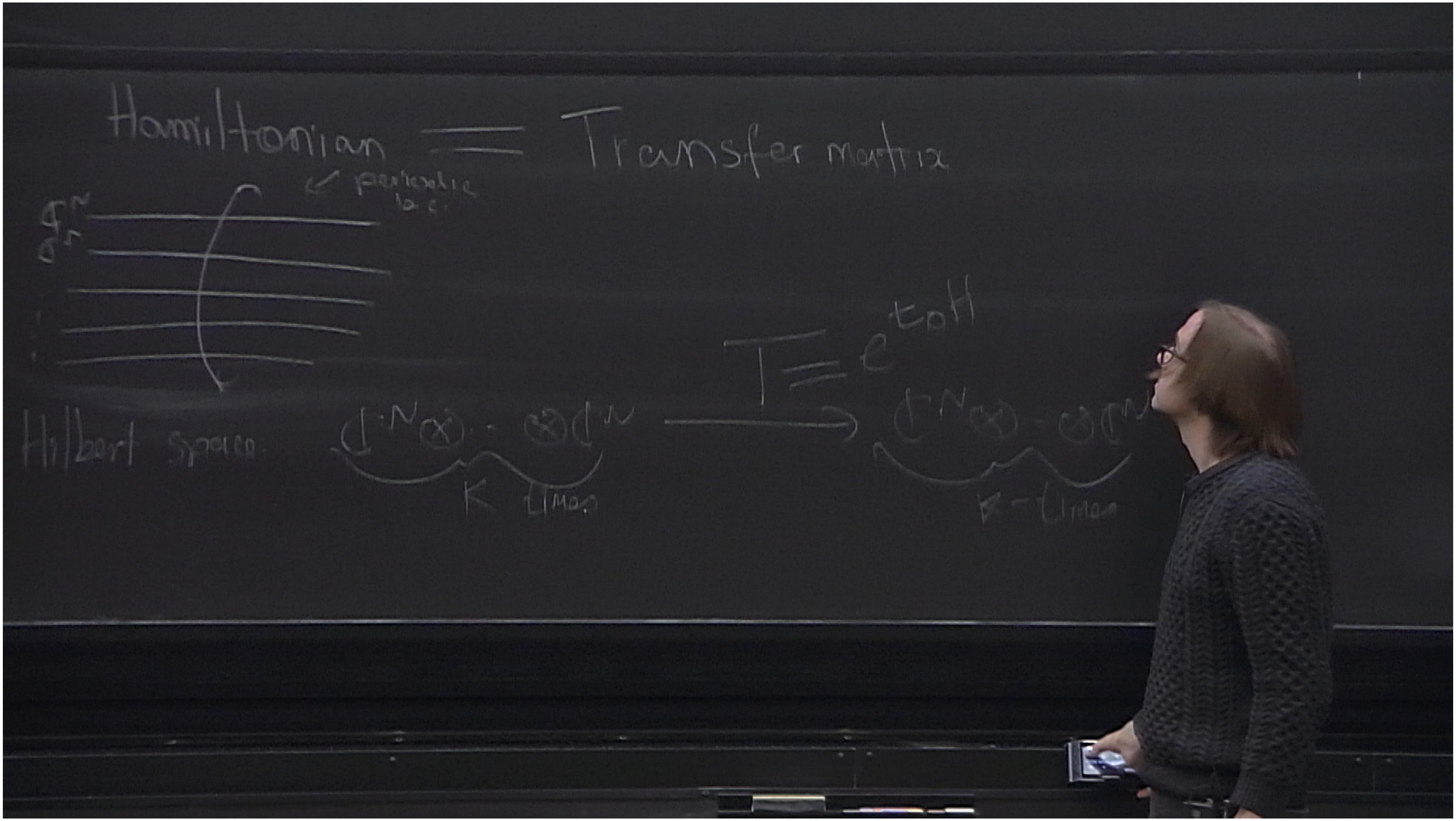
Discretized Lagrangian Fix some B^{ij}_{kl} i,j,k,l : $1 \to N$

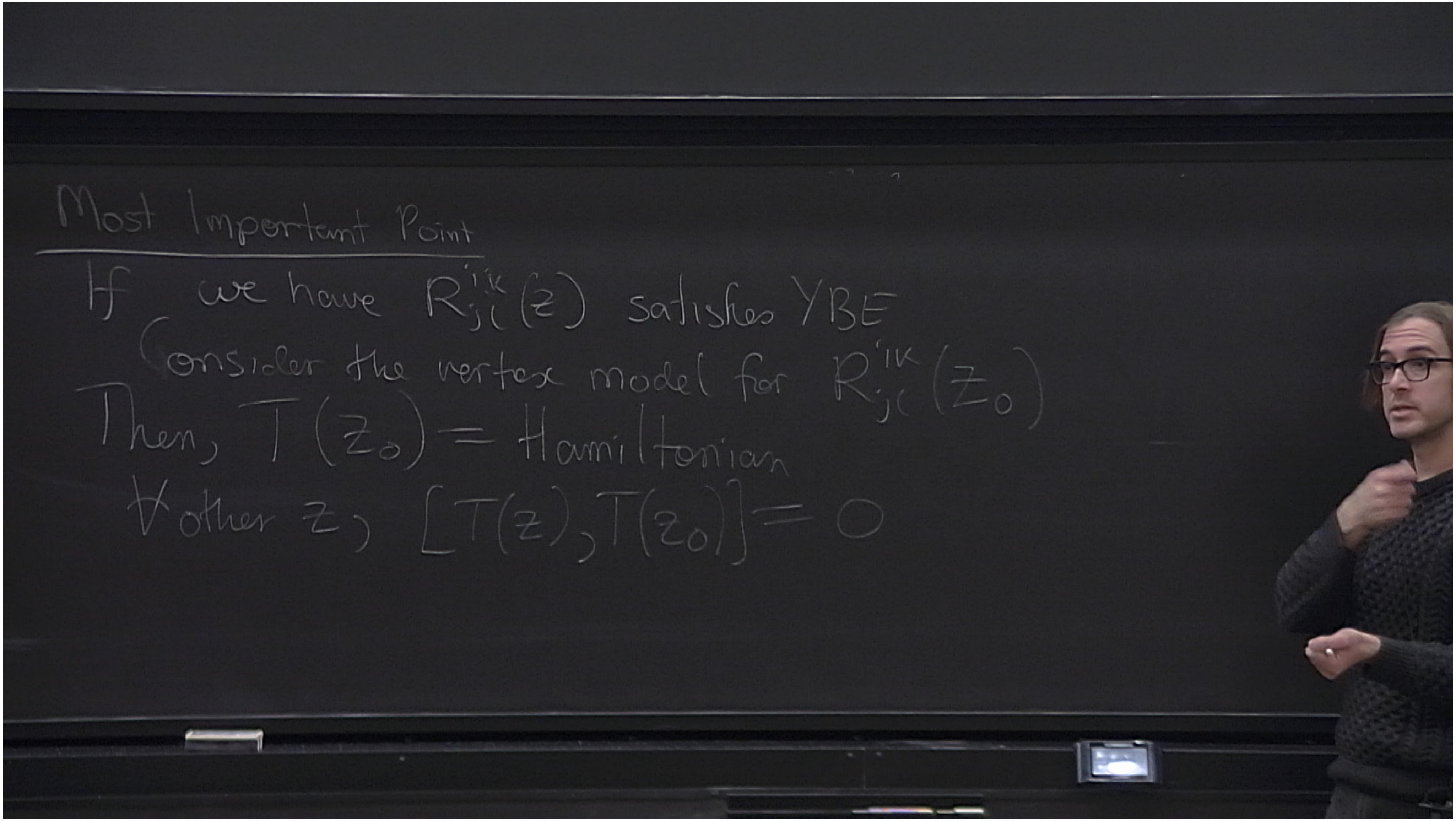
At each vertex v, we let

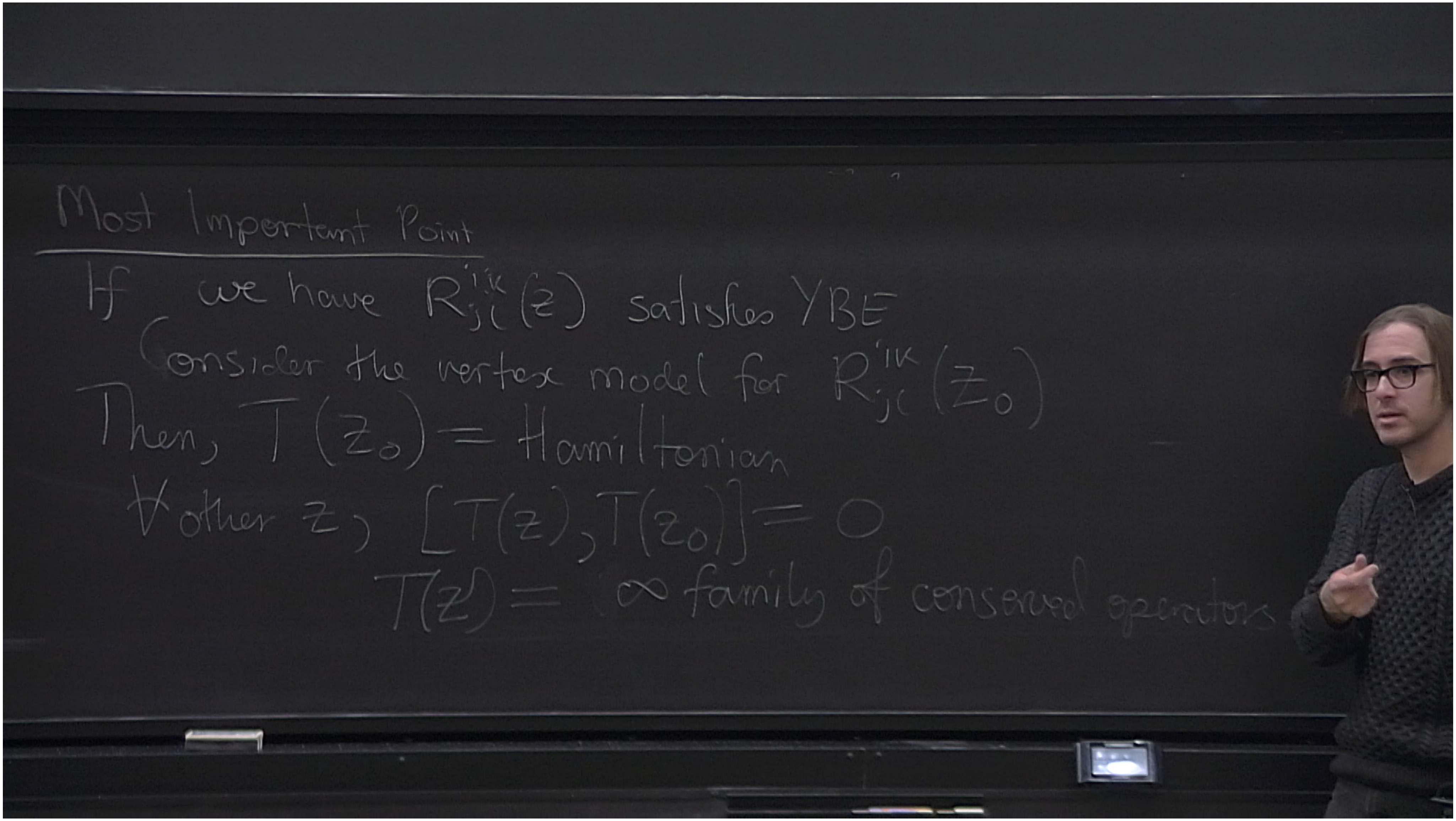
$$\mathcal{L}(\varphi, v) = -\log B^{ij}_{kl}$$ if φ looks like i, j, k, l at this vertex

Discretized path integral is

$$\sum_{\varphi} e^{-\sum_v \mathcal{L}(\varphi, v)} = \sum_{\varphi} \prod_v B^{ij}_{kl} = Z$$, partition fn of statistical system.


Hamiltonian = Transfer matrix
periodic b.c.
Hilbert space:
k times
k times


Most Important Point
If we have $R^{ik}_{jl}(z)$ satisfies YBE
Consider the vertex model for $R^{ik}_{jl}(z_0)$
Then, $T(z_0)$ = Hamiltonian
$\forall$ other z, $[T(z), T(z_0)] = 0$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Consider the vertex model for $R^{ik}_{jl}(z_0)$
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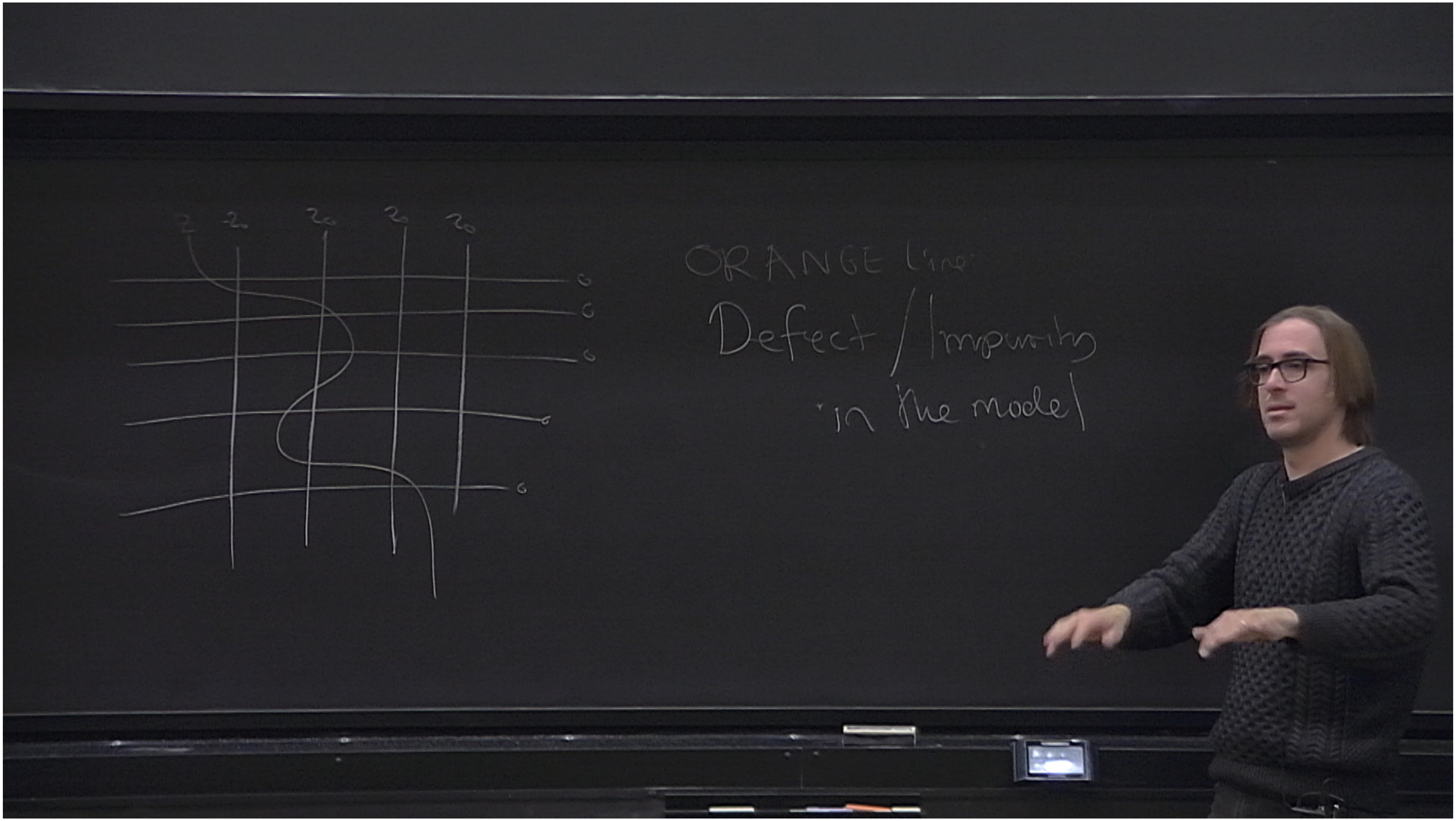
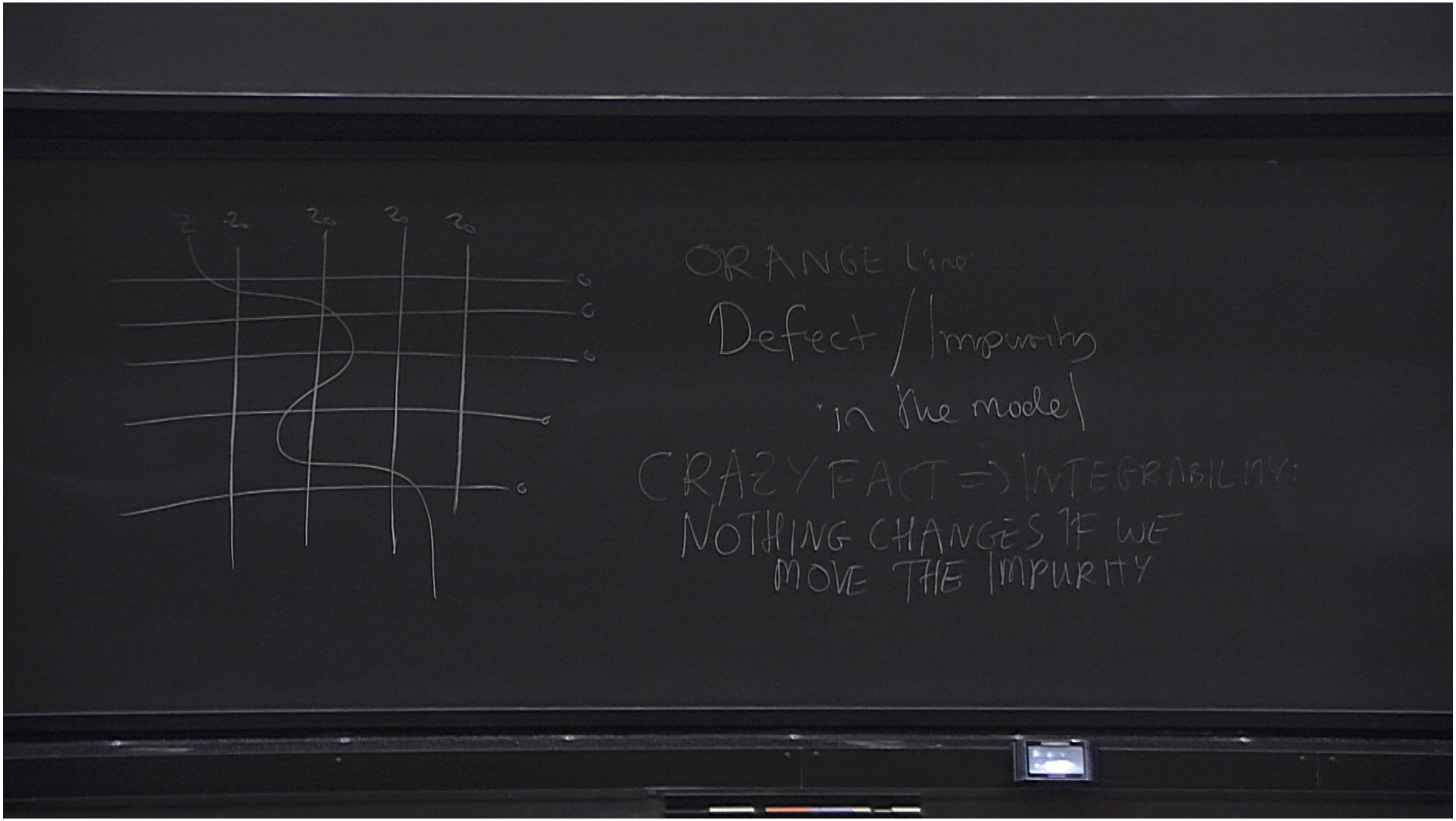
$T(z) =$ ∞ family of conserved operators


ORANGE line:
Defect / Impurity
in the model


ORANGE line:
Defect / Impurity in the model
CRAZY FACT ⇒ INTEGRABILITY:
NOTHING CHANGES IF WE MOVE THE IMPURITY

Continuum Models

2d chiral + antichiral fermions
+ quartic coupling

Chiral ψ_i, ψ^i

A. Chiral $\bar{\psi}_i, \bar{\psi}^i$

Lagrangian is

$$\psi_i \partial_{\bar{w}} \psi^i + \bar{\psi}_i \partial_w \bar{\psi}^i - \psi_i \psi^j \bar{\psi}^i \bar{\psi}_j$$

U(N) symmetry - Thirring model

U(N) symmetry - Thirring model

CURRENTS for U(N) symmetry

$\partial_{\bar{w}} \longrightarrow$ cov derivative $\partial_{\bar{w}} + A^{i}_{j\bar{w}}$

Find the term

$$\psi_i \partial_{\bar{w}} \psi^i + \psi_i A^{i}_{j\bar{w}} \psi^j$$

Conclude current is

$$J^{j}_{i\bar{w}} = \psi_i \psi^j$$

$$\bar{J}^{j}_{iw} = \bar{\psi}_i \bar{\psi}^j$$

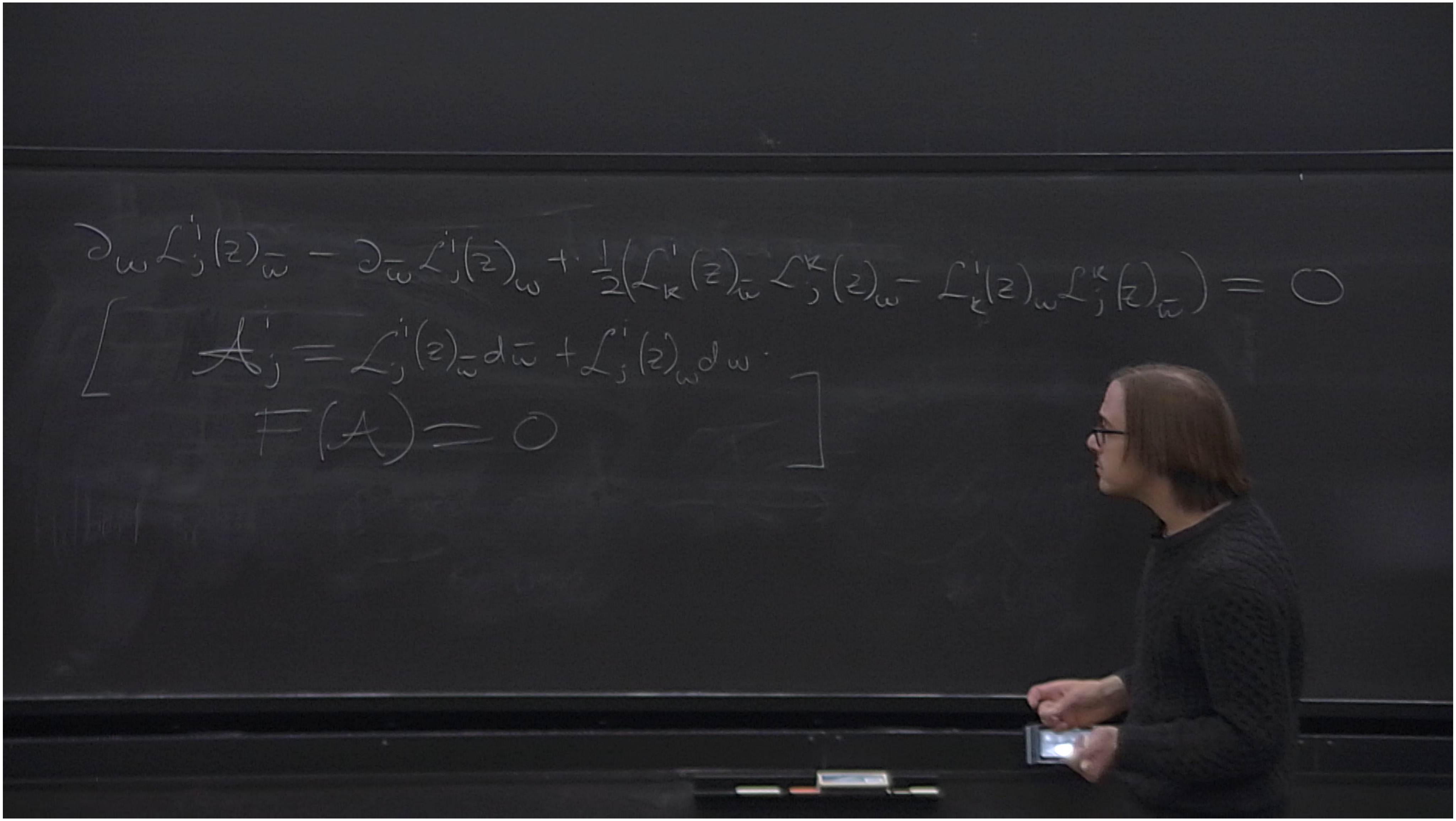
Define the Lax operator by

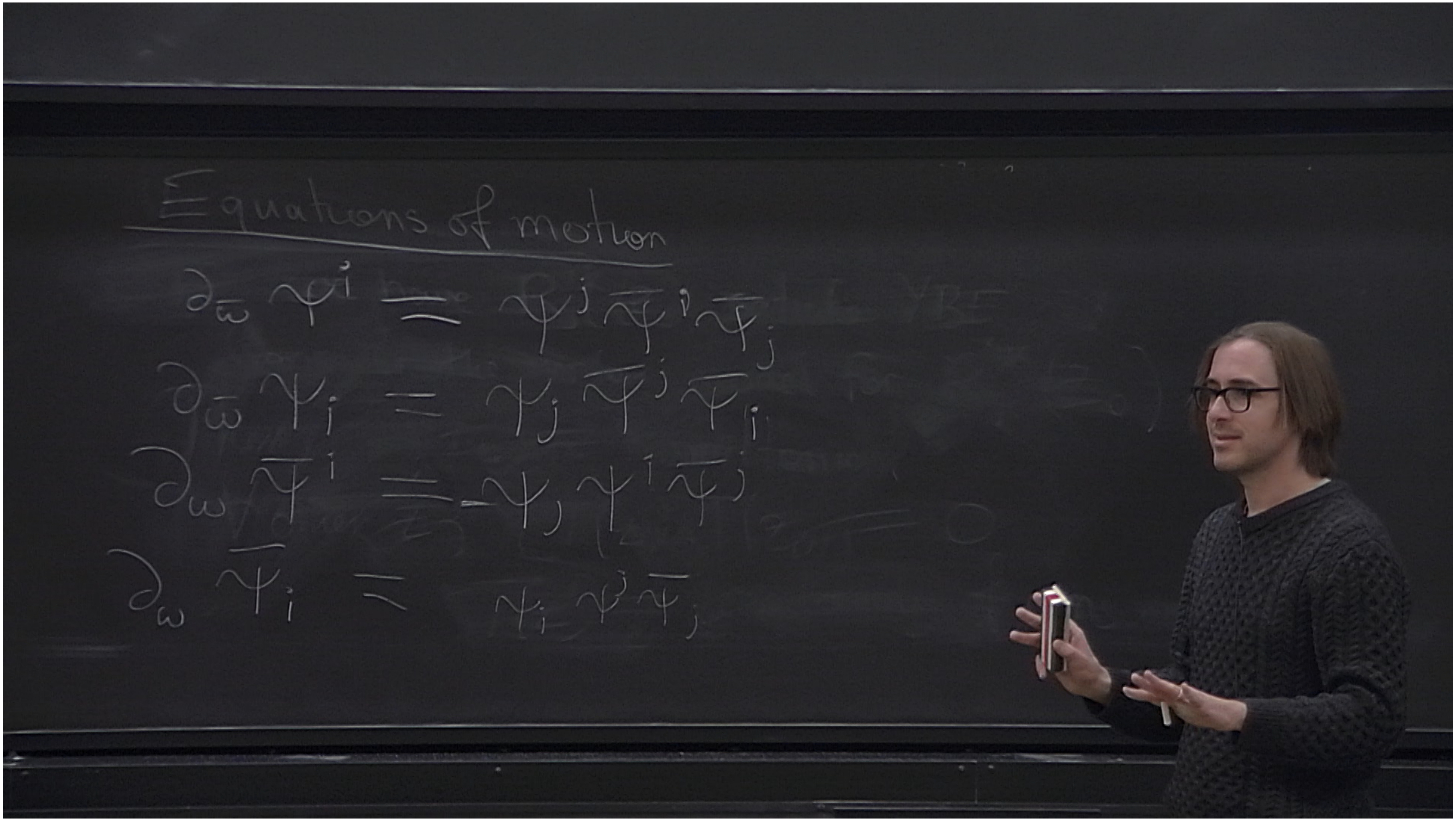
$$\mathcal{L}^i_j(z)_{\bar{\omega}} = \frac{1}{z} J^i_{j\bar{\omega}}$$

$$\mathcal{L}^i_j(z)_{\omega} = \frac{1}{1-z} J^i_{j\omega}$$

Lemma

These operators satisfy the Lax eq^n (zero-curvature eq^n)


$\partial_w \mathcal{L}^i_j(z)_{\bar w} - \partial_{\bar w} \mathcal{L}^i_j(z)_w + \frac{1}{2}\left(\mathcal{L}^i_k(z)_{\bar w} \mathcal{L}^k_j(z)_w - \mathcal{L}^i_k(z)_w \mathcal{L}^k_j(z)_{\bar w}\right) = 0$
$\Big[\; A^i_j = \mathcal{L}^i_j(z)_{\bar w}\, d\bar w + \mathcal{L}^i_j(z)_w\, dw .$
$F(A) = 0 \;\Big]$


Equations of motion
$\partial_{\bar\omega}\psi^i = \psi^j\bar\psi^i\bar\psi_j$
$\partial_{\bar\omega}\psi_i = \psi_j\bar\psi^j\bar\psi_i$
$\partial_\omega\bar\psi^i = -\psi_j\psi^i\bar\psi^j$
$\partial_\omega\bar\psi_i = \psi_i\psi^j\bar\psi_j$

$$\mathcal{L}^i_j(z)_{\bar{w}} = \frac{1}{z}\psi_j\psi^i$$

$$\mathcal{L}^i_j(z)_w = \frac{1}{1-z}\bar{\psi}_j\bar{\psi}^i$$

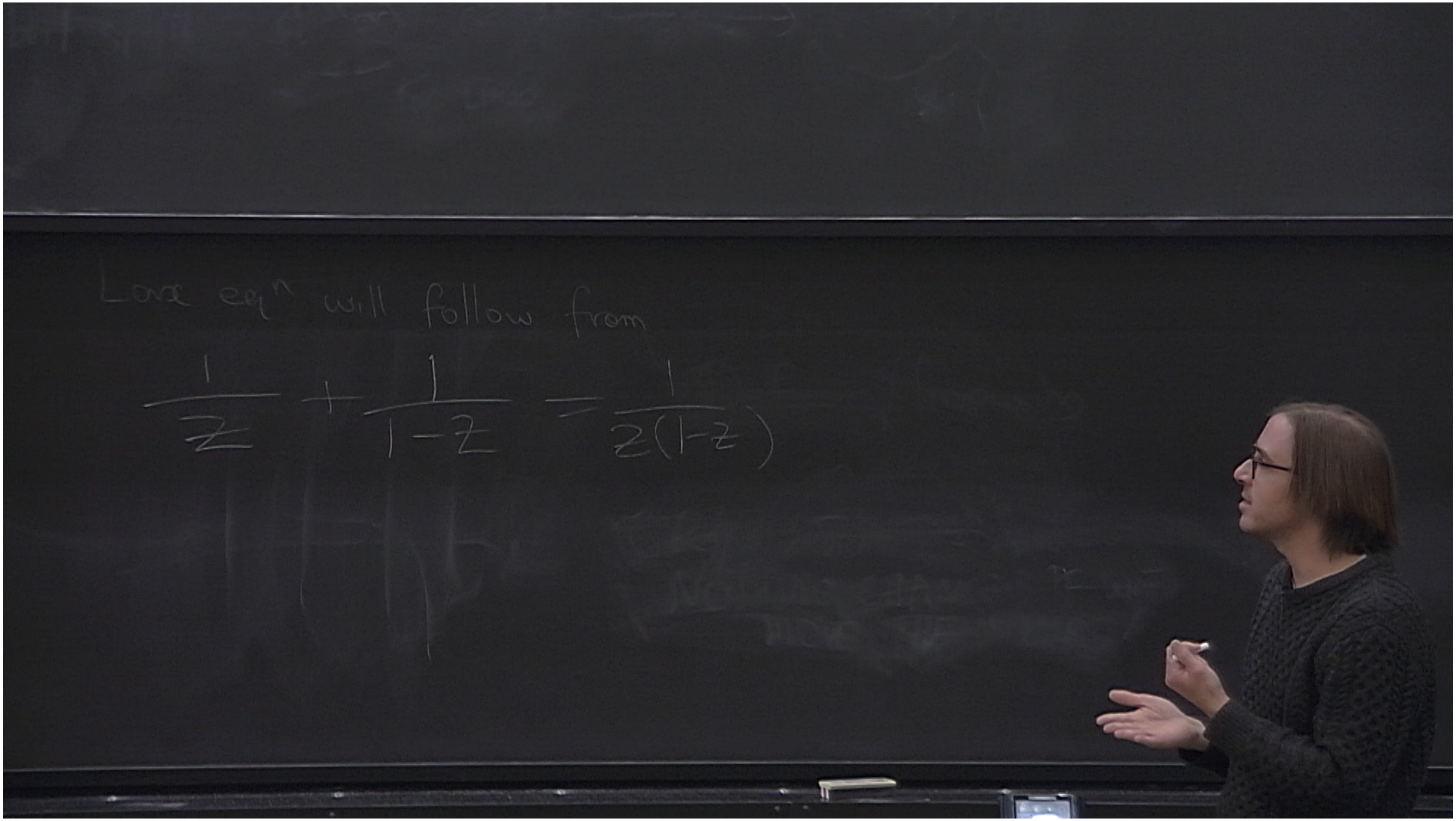
<u>Idea</u>

$$\partial_w \mathcal{L}^i_j(z) = \frac{1}{z}\psi\psi\bar{\psi}\bar{\psi}$$

some indices

$$\partial_{\bar{w}} \mathcal{L}^i_j(z) = \frac{1}{1-z}\psi\psi\bar{\psi}\bar{\psi}$$

$$\mathcal{L}^i_k(z)_{\bar{w}}\mathcal{L}^k_j(z)_w = \frac{1}{z}\frac{1}{1-z}\psi\psi\bar{\psi}\bar{\psi}$$


Love eqn will follow from
1/z + 1/(1−z) = 1/z(1−z)