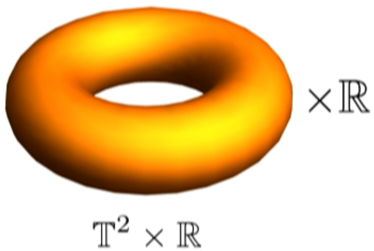
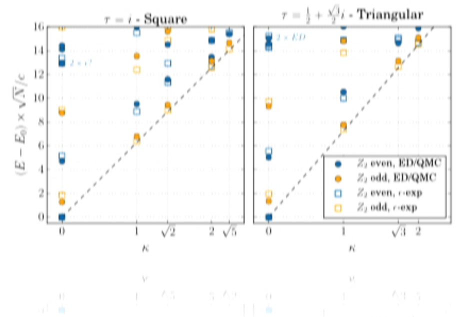
Title: Computational Spectroscopy of Quantum Field Theories

Date: Mar 14, 2018 02:00 PM

URL: http://pirsa.org/18030058

Abstract: <p>Quantum field theories play an important role in many condensed matter systems for their description at low energies and long length scales. In 1+1 dimensional critical systems the energy spectrum and the spectrum of scaling dimensions are intimately related in the presence of conformal symmetry. In higher space-time dimensions this relation is more subtle and not well explored numerically. In this talk we motivate and review our recent effort to characterize 2+1 dimensional quantum field theories using computational techniques 2+targetting the energy spectrum on a spatial torus. We discuss several examples ranging from the O(N) Wilson Fisher theories and Gross-Neveu-Yukawa theories to deconfinement-confinement transitions in the context of topological ordered systems. We advocate a phenomenological picture that provides insight into the operator content of the critical field theories.</p>




# Computational Spectroscopy of Quantum Field Theories


Andreas Läuchli
Institute for Theoretical Physics
University of Innsbruck, Austria

andreas.laeuchli@uibk.ac.at
http://www.uibk.ac.at/th-physik/laeuchli-lab


universität innsbruck
Theoretische Physik


Support: FWF SFB FoQuS Foundations and Applications of Quantum Science FOR 1807

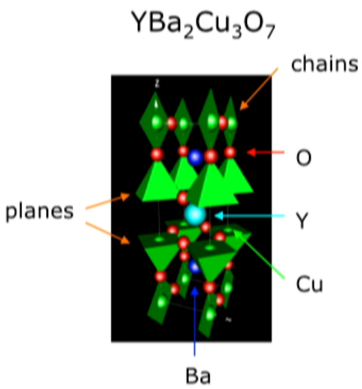

Colloquium, Perimeter Institute, $\pi$ Day 2018

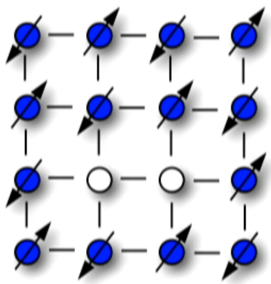
# Outline of this talk

- Introduction: Quantum Many Body Systems / Spectroscopy
- Torus Energy Spectra and CFT ?
- Spectrum of the standard 2+1D Ising transition (Ising)
- Spectrum of the “$Z_2$ confinement” transition (Ising*)
- Spectrum of the 3D XY* Transition
- Outlook

# Quantum Matter:
# Strongly correlated electrons in solids

- High Tc superconductors & Quantum Magnets

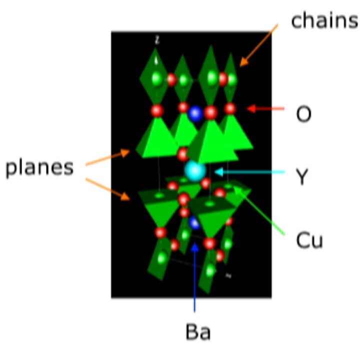


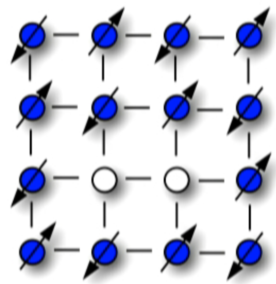



# Quantum Matter:
# Strongly correlated electrons in solids

- High Tc superconductors & Quantum Magnets

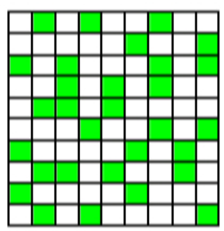
$YBa_2Cu_3O_7$

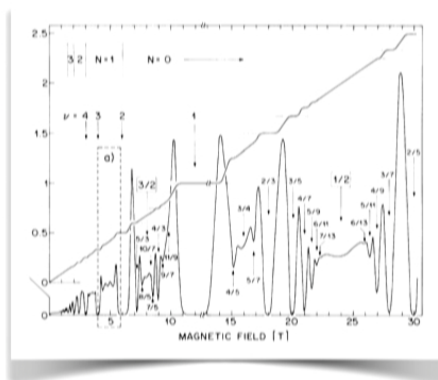


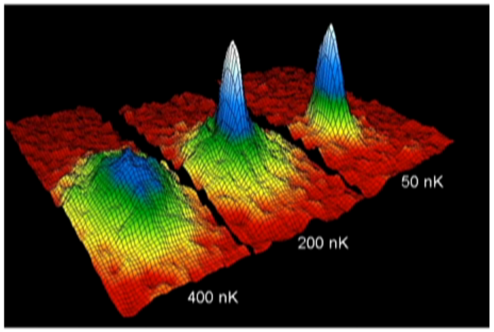
- Fractional Quantum Hall Effect

$$\nu = 1/3$$



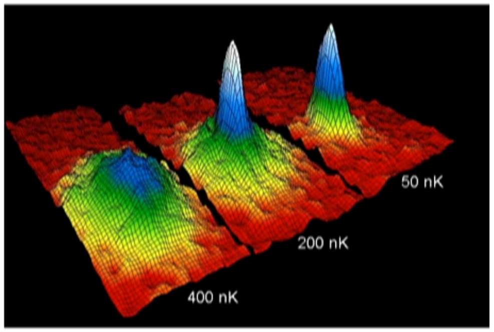


# Quantum Matter:
# Ultracold atomic gases




From weakly interacting Bose gases

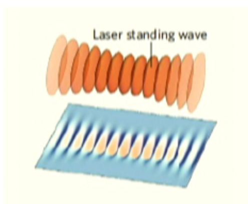
# Quantum Matter: Ultracold atomic gases

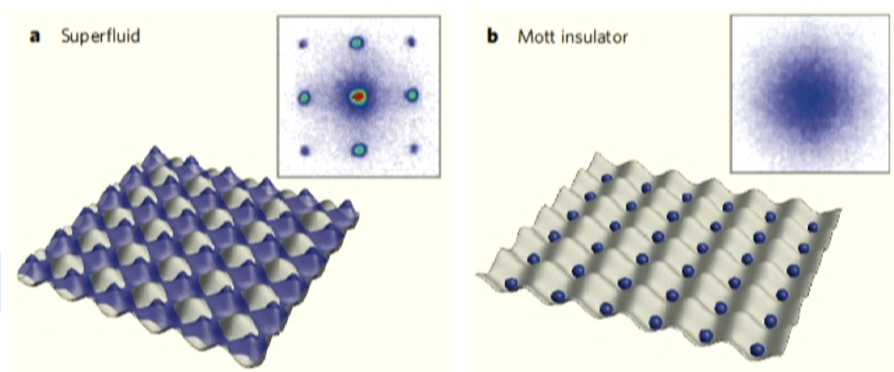



From weakly interacting Bose gases

to strongly interacting gases in optical lattices

M. Greiner et al., Nature (2002)

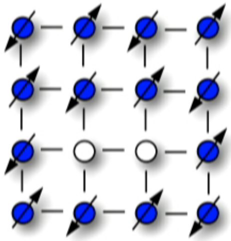


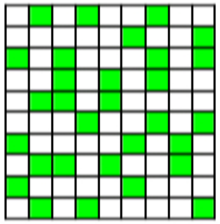



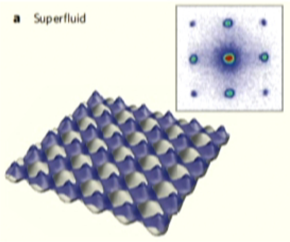

$$H = -J\sum_{\langle i,j\rangle}(b_i^\dagger b_j + h.c.) + \frac{U}{2}\sum_i n_i(n_i - 1)$$

D. Jaksch et al., PRL (1998)

# Quantum Matter





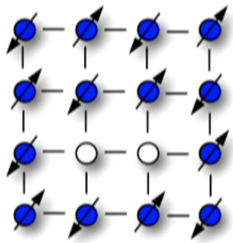


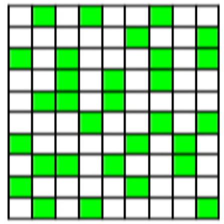

- We would like to understand phase diagrams of complex systems, but whose Hamiltonians are often reasonably well known.

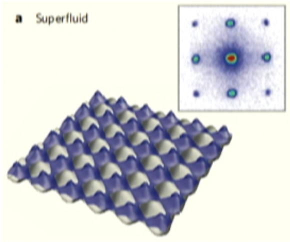
- Quantum phase transitions occur. What is their universality class & field theoretical description ?

- New tools welcome to diagnose/characterize QFTs at phase transitions

# Quantum Matter



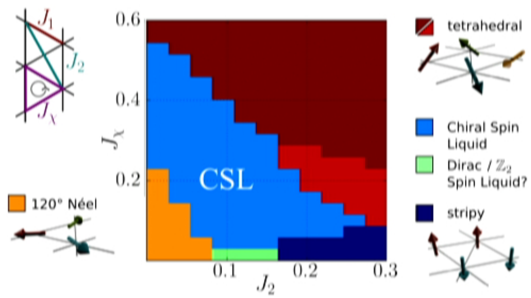




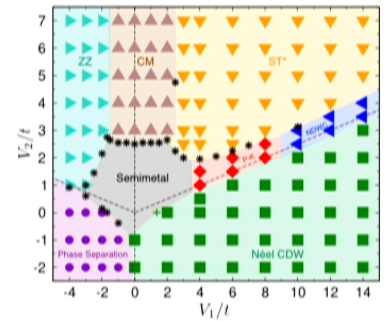

- We would like to understand phase diagrams of complex systems, but whose Hamiltonians are often reasonably well known.
- Quantum phase transitions occur. What is their universality class & field theoretical description ?
- New tools welcome to diagnose/characterize QFTs at phase transitions

# Example of Microscopic Condensed Matter Models

- From microscopic models:




Phys. Rev. B. **95**, 035141 (2017)



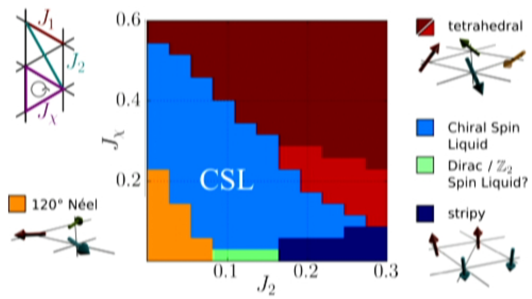

Phys. Rev. B **92**, 085146 (2015)

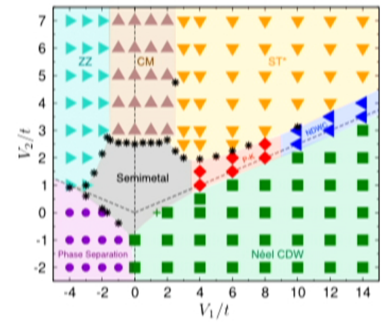
$$\mathcal{H} = J_1 \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \vec{S}_i \cdot \vec{S}_j + J_\chi \sum_{i,j,k \in \triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$

$$\mathcal{H} = -t \sum_{\langle ij \rangle} (c_i^\dagger c_j + h.c.) + V_1 \sum_{\langle ij \rangle} (n_i - 1/2)(n_j - 1/2) + V_2 \sum_{\langle\langle ij \rangle\rangle} (n_i - 1/2)(n_j - 1/2)$$

- To quantum phase transitions: Wilson Fisher CFTs, $\text{QED}_3$, Gross Neveu, …

# Quantum Matter



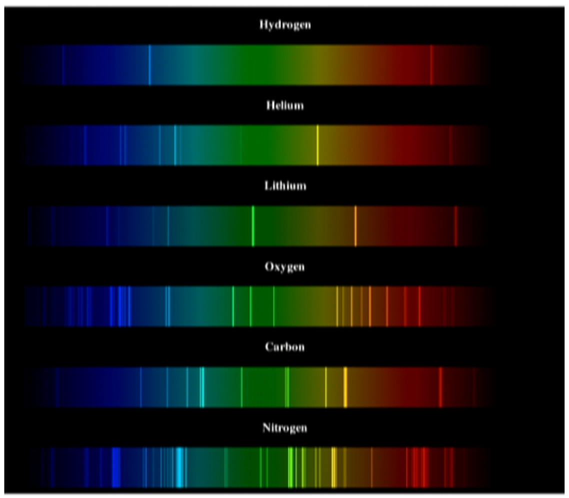




- Standard Approach: Simulate system on a computer, calculate correlation functions, order parameter, and determine critical exponents. Can work very well, but does not have to…

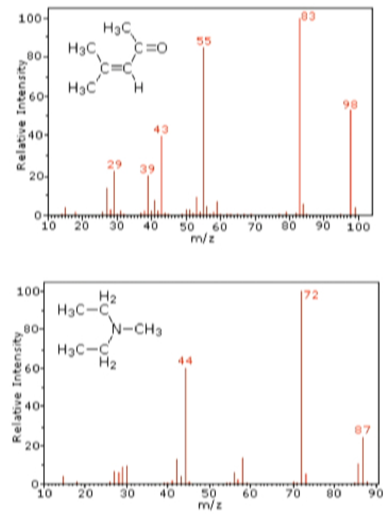
- Here want to investigate whether the Energy Spectrum of a quantum many body system at criticality reveals its universality class (Spectroscopy) ?

# Spectroscopy in other areas:

- For example in optics and and mass spectroscopy one measures spectra, and then compares with a catalogue of known spectra to infer the nature of an “unknown” substance.




http://www.astro.rug.nl




https://www2.chemistry.msu.edu

- Can we do the same with Quantum Field Theories ?

# “Can one hear the shape of a drum” ?

- Can one infer the shape of a domain from the spectrum of the Laplacian ?
  (not unambiguously, there are non-congruent shapes with the same spectrum)

CAN ONE HEAR THE SHAPE OF A DRUM?

MARK KAC, The Rockefeller University, New York

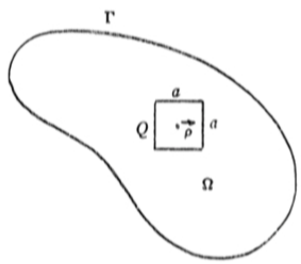
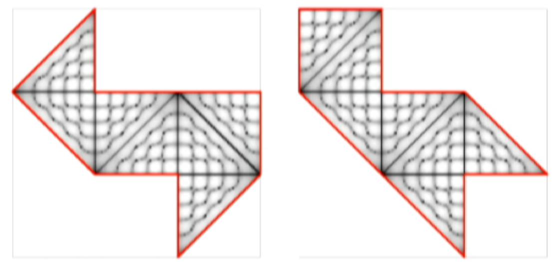



FIG. 1

Amer. Math. Monthly 73, 1-23, 1966.



http://mathworld.wolfram.com/IsospectralManifolds.html

- We would ask a related, but slightly different question:
  Given a shape, can we “hear” the nature of the (massless) field theory embedded on this shape ?

# Outline of this talk

- Introduction: Quantum Many Body Systems / Spectroscopy
- Torus Energy Spectra and CFT ?
- Spectrum of the standard 2+1D Ising transition (Ising)
- Spectrum of the “$Z_2$ confinement” transition (Ising*)
- Spectrum of the 3D XY* Transition
- Outlook

# Operator spectrum in conformal field theories

- A local operator has a scaling dimension:

$$\mathcal{O}_i \to \Delta_i = \text{scaling dimension}$$

- The scaling dimension determines the decay of the 2-point correlation function:

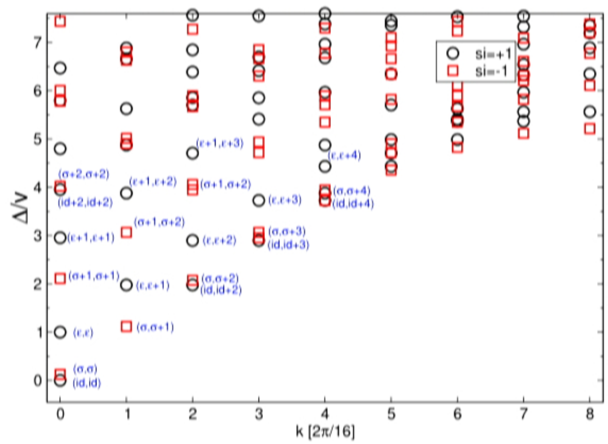
$$\langle \mathcal{O}_i(x) \mathcal{O}_i(0) \rangle = \frac{c}{|x|^{2\Delta_i}}$$

- It seems interesting and important to know the various fields with their corresponding scaling dimensions.

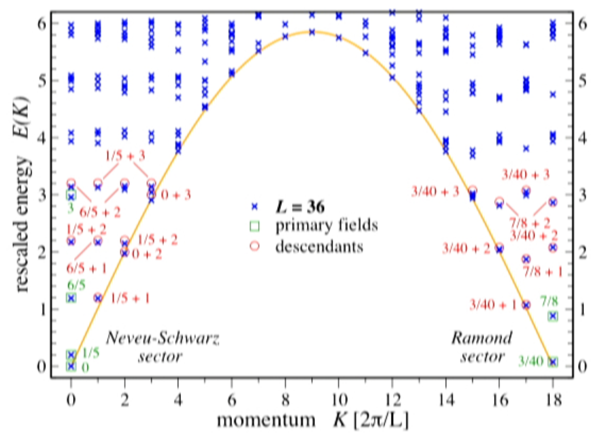
- Where can we find those in numerics ?

# 1D Torus (Circle) Energy Spectra

- For CFTs energy spectra of finite size (1+1D) systems arrange into conformal towers !



TFI chain L=16
2D Ising CFT Spectrum



A. Feiguin et al. PRL 2007
tricritical Ising CFT Spectrum in anyon chains


$$\mathbb{R}^2 \leftrightarrow \mathbb{S}^1 \times \mathbb{R}$$

- Spectrum of scaling dimensions of CFT maps to Hamiltonian spectrum on a circle.

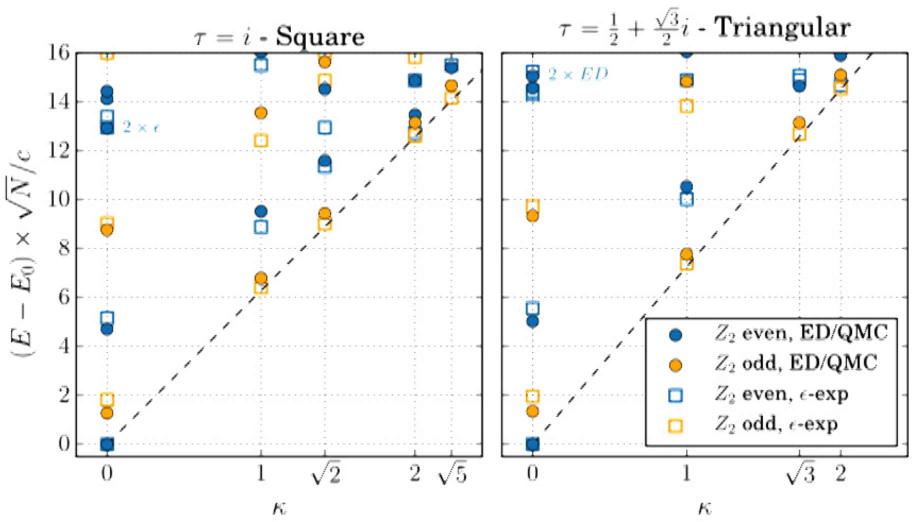
# Analytical approach: (4-epsilon)-expansion

- Work done by S. Withsett and S. Sachdev. Lowest non-trivial order in epsilon.

$$\mathcal{H} = \int d^d x \left[ \frac{1}{2}\Pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{s}{2}\phi^2 + \frac{u}{4!}\phi^4 \right]$$

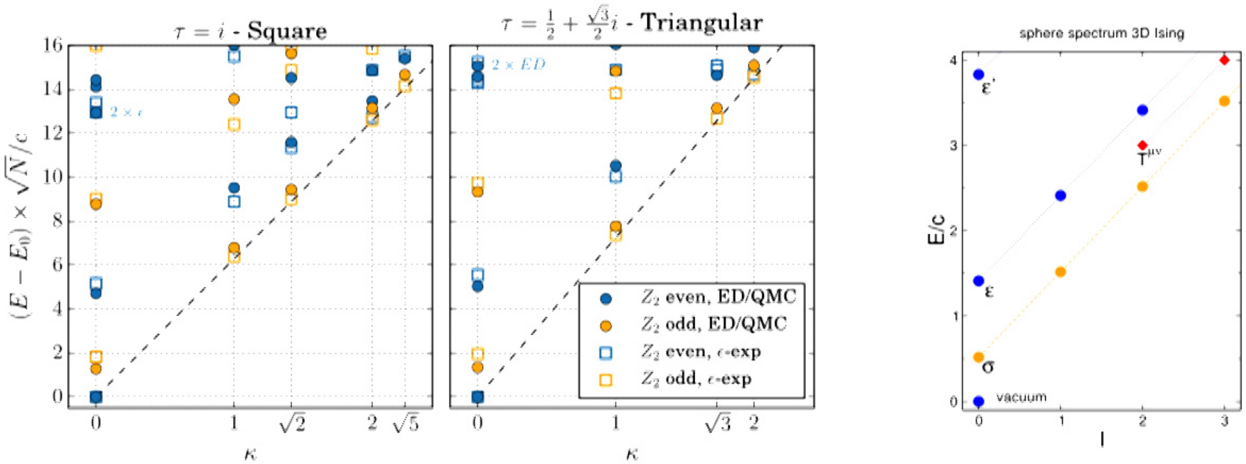
- Rather good agreement between analytics and numerics.

- Zero-mode is most important in (4-epsilon)-expansion, anharmonic oscillator.



# Comparison between torus and sphere spectra

- Torus spectra at low energy per sector resemble the spectrum on the sphere:

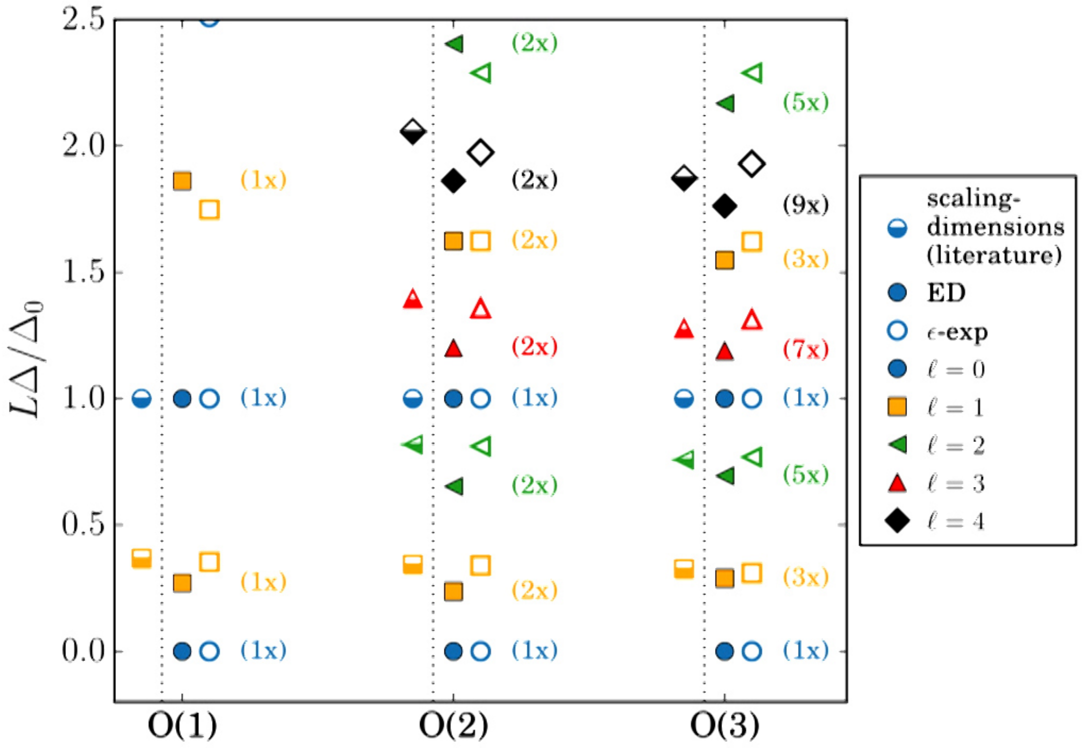



- We believe this handwaving resemblance might be more generally the case: "light states on the sphere have a light analogon on the torus"

- But likely *no* state operator correspondence on the torus.

# Wilson-Fisher $Z_2$ / O(2) / O(3) Results

- Torus spectra at low energy (still) resemble the spectrum on the sphere:



# Outline of this talk

- Introduction: Quantum Many Body Systems / Spectroscopy
- Torus Energy Spectra and CFT ?
- Spectrum of the standard 2+1D Ising transition (Ising)
- Spectrum of the "$Z_2$ confinement" transition (Ising*)

  M. Schuler, S. Whittsitt, L.-P. Henry, S. Sachdev & AML
  Phys. Rev. Lett. 2016
- Spectrum of the 3D XY* Transition
- Outlook

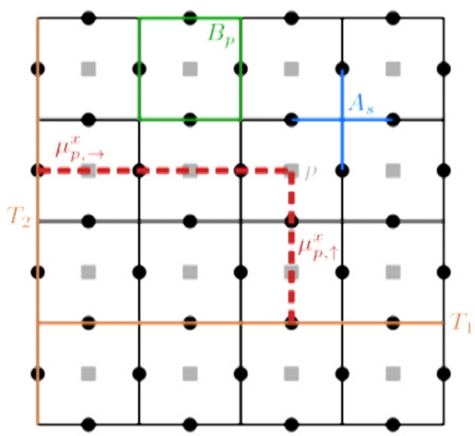
# Confinement transition

- $Z_2$ spin liquids are rather fashionable these days.
- The are phases with a four-fold ground state degeneracy on a torus, but the degeneracy is topological, and not related to symmetry breaking.
- One of the simplest incarnations of this phase appears in the Toric Code model by Kitaev.
- By an appropriate perturbation the topological phase ("deconfined") gives way to a simple paramagnetic phase ("confined"). The transition is a confinement transition and is expected to be in the 2+1D = 3D Ising universality class.
- Q: Is the torus spectrum at criticality identical to the symmetry breaking case ?

# Toric code in a magnetic field

- We study the following microscopic model
  (but results will be independent of specific model):

- Toric code with a longitudinal magnetic field (S. Trebst et al., …):




$$H = -J\sum_s A_s - J\sum_p B_p - h\sum_i \sigma_i^x$$

$$A_s = \prod_{i\in s}\sigma_i^x,\ B_p = \prod_{i\in p}\sigma_i^z$$

$$\mu_p^z = B_p$$

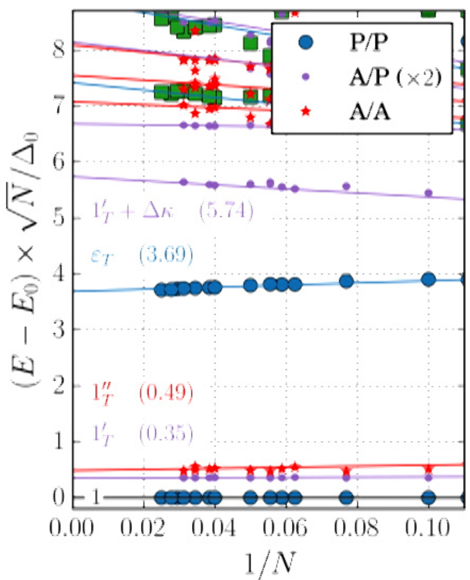
$$\mu^x_{p,\rightarrow(\uparrow)} = \prod_{i\in c_{p\rightarrow(\uparrow)}} \sigma_i^x$$

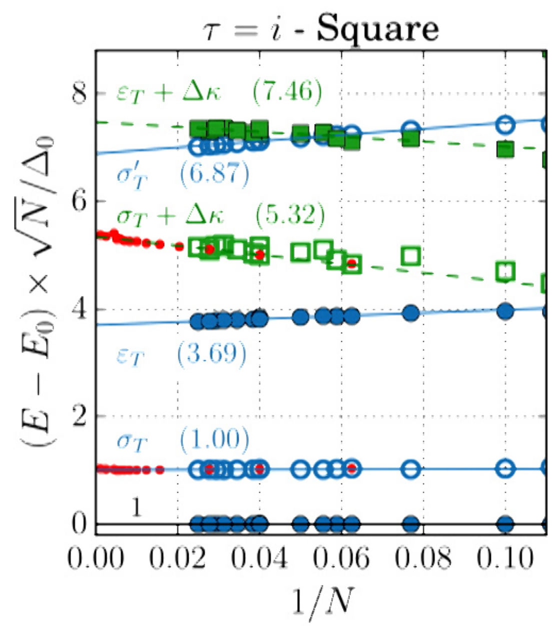
TFI boundary conditions imposed by $T_1$,$T_2$ loops !

$$H_{TFI} = -h\sum_{\langle p,q\rangle}\mu_p^x\mu_q^x - J_p\sum_p \mu_q^z + const.$$

# Numerics at criticality

- Left: data for the TC at criticality, Right: Symmetry breaking




- The spectra at criticality do not agree ! What is going on ?

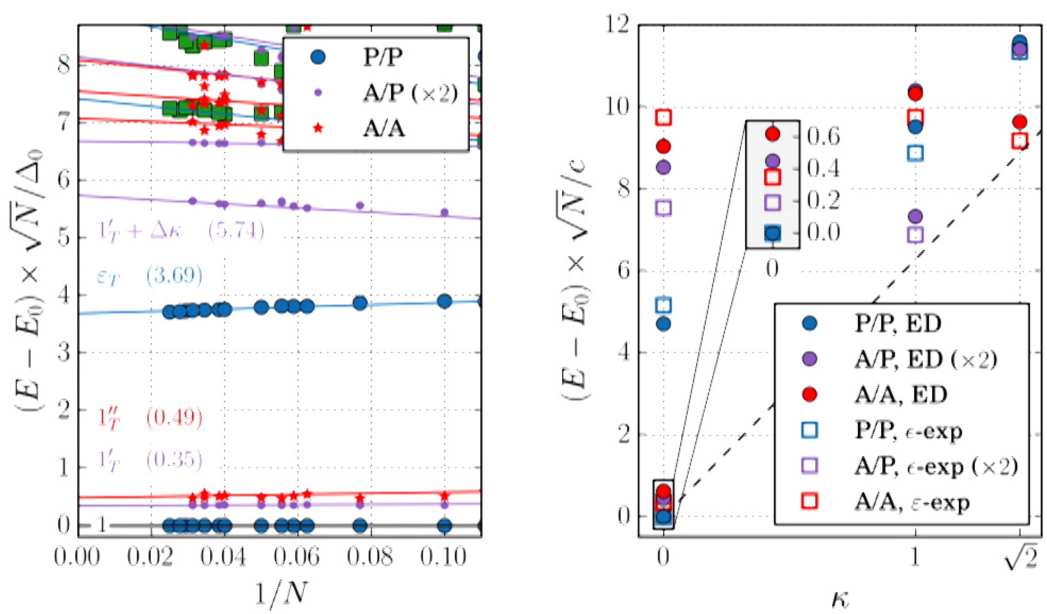
# The Ising* transition

- The explanation is that the operator content of the two transitions are different:
- In the $Z_2$ symmetry breaking case we have $Z_2$ even and odd levels and only one set of boundary conditions (fixed by the lattice model).
- In the confinement transition (Ising*), only $Z_2$ even levels are allowed, and for periodic boundary conditions in the Toric Code, four different boundary conditions of the CFT become simultaneously apparent.
- This can be understood at the microscopic level in the Toric Code Hamiltonian and is supported by general field theoretical considerations.
- In the Ising* case the magnetic sector is completely absent, and the torus energy spectrum reflects this fact.

# The Ising* transition

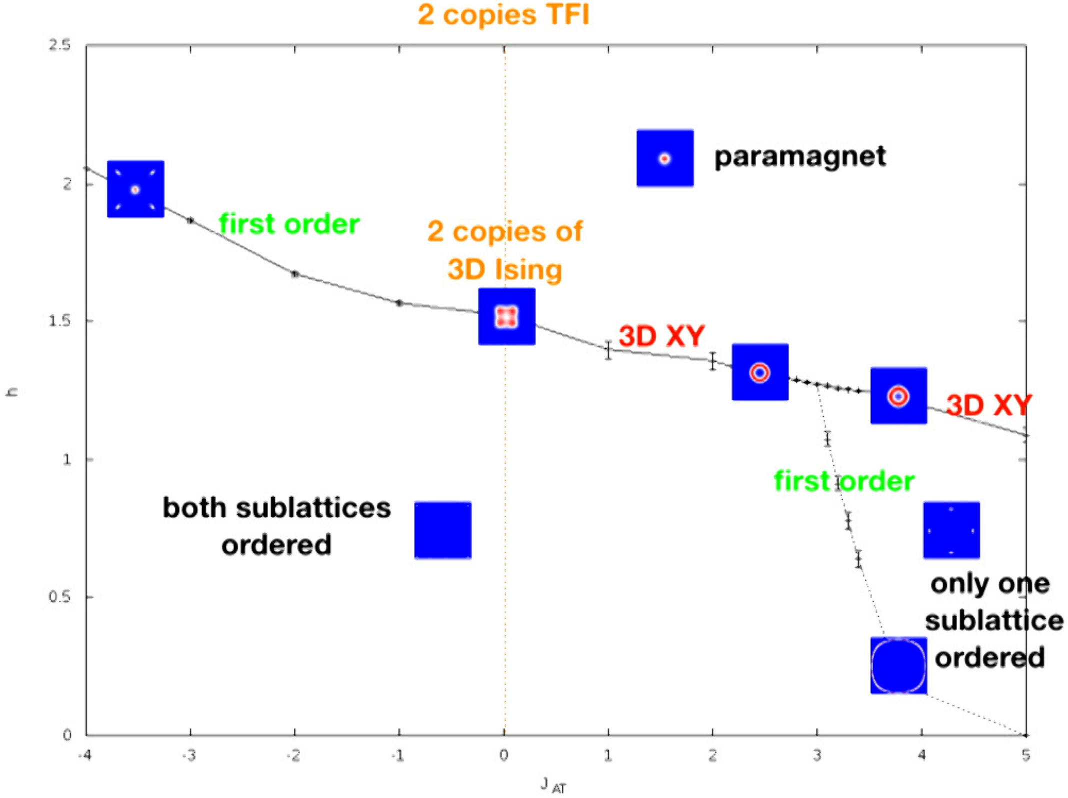
- comparison between numerics and epsilon-expansion:

- At criticality the 4 “topological sectors” scale also as 1/L , but are much closer together than the next level above them.



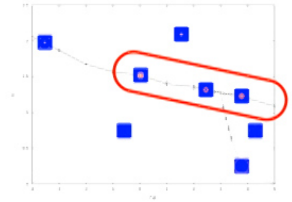
# Phase diagram of the Quantum Ashkin-Teller model

- Rather poorly studied in the past, so here we perform a new QMC study:

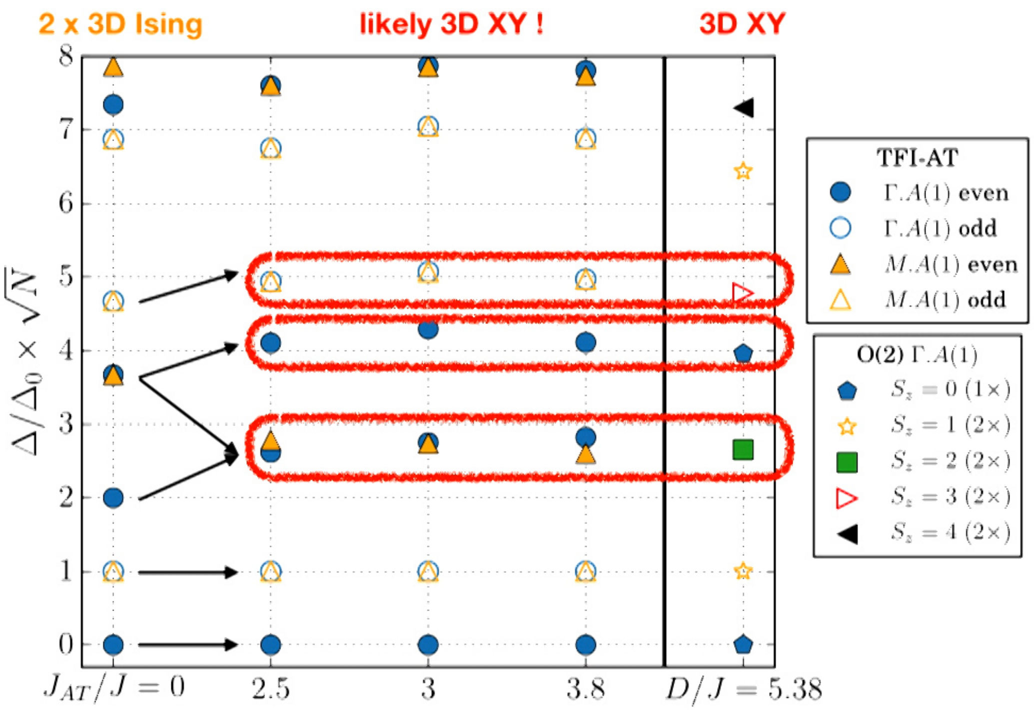



- Phase structure in agreement with QFT results of $N_c$=2 $\phi^4$ theory with cubic anisotropy.

# Spectroscopy of QCP

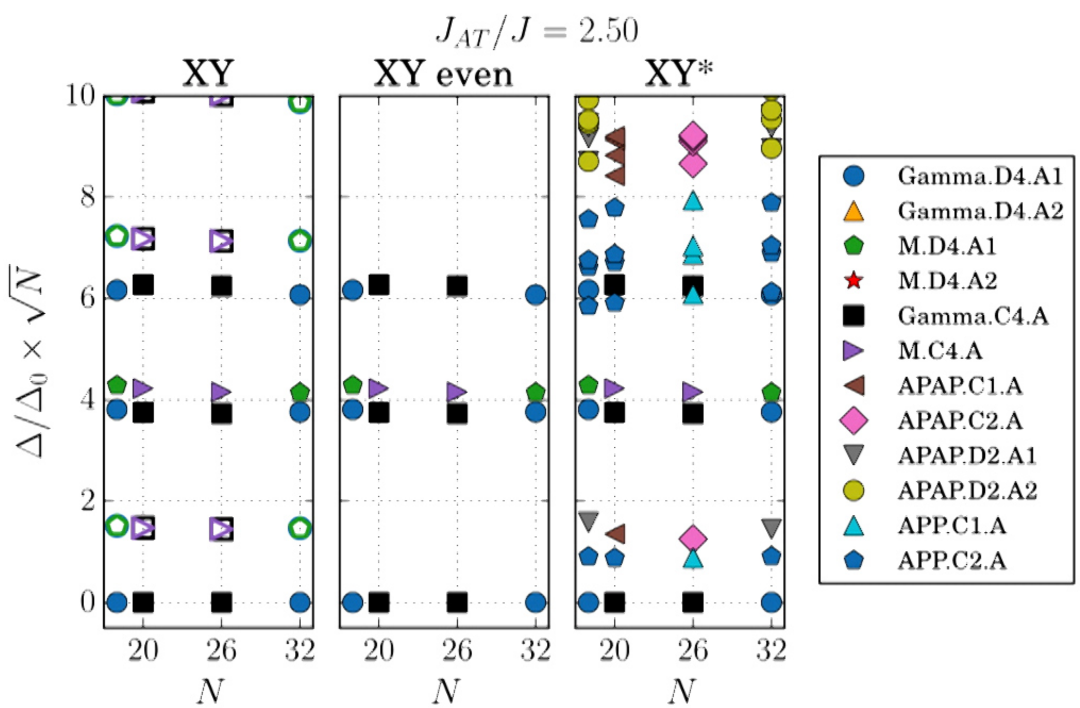


- ED Torus Spectra in the Quantum Ashkin-Teller model at criticality:



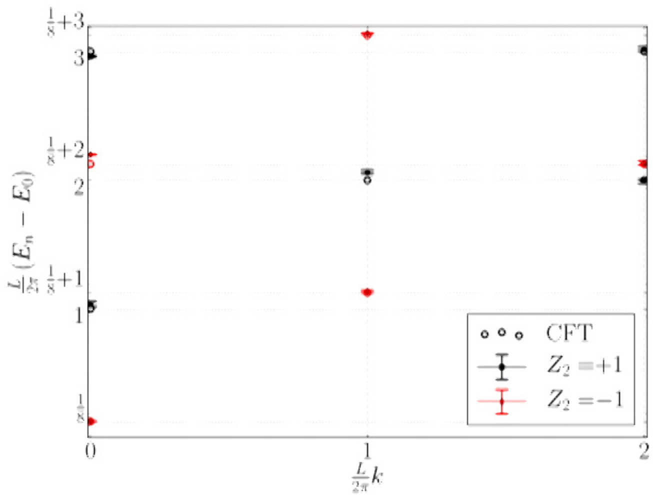
# Torus energy spectrum of 3D XY*

- Remove all odd charge sectors in 3D XY but add all 4 BC PP/PA/AP/AA sectors:

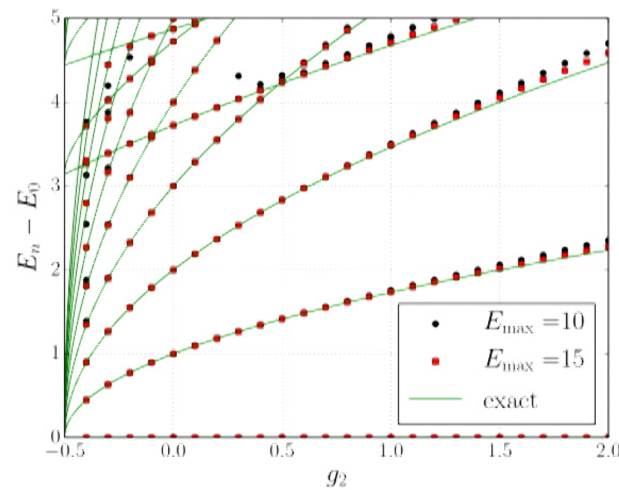


# Hamiltonian truncation of interacting real scalar field
# Energy spectra in 1+1D and 2+1D

- (truncated) Hilbert space of non-interacting massive theory S. Rychkov & L. Vitale, PRD 2015




1+1D $\phi^4$ critical point

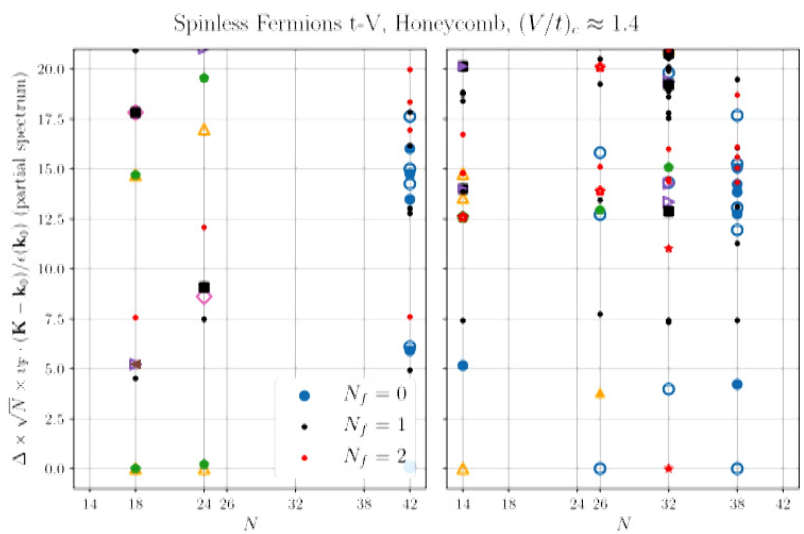



2+1D $\phi^2$ massive phase

C. Pernul & AML , to appear

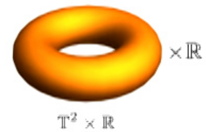
# Gross-Neveu-Yukawa

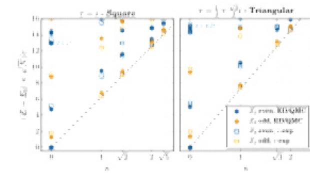
- Spinless fermions on a honeycomb lattice: massless Dirac fermions ↔ charge density wave
- Gross-Neveu-Yukawa $N_f$=4 CFT?




M. Schuler, T.C. Lang & AML
unpublished

# Conclusion / Outlook




- We have shown that the universal torus energy spectrum of the CFT describing quantum critical points is accessible numerically.

- The torus energy spectrum contains valuable information on the "operator content". It is e.g. able to discriminate the Ising from the Ising* universality class, and 2 x Ising from 3D XY

- We have results for O(2)/O(3) Wilson-Fisher fixed points and some preliminary results for Gross-Neveu-Yukawa critical points.

- Results from CFT side ?