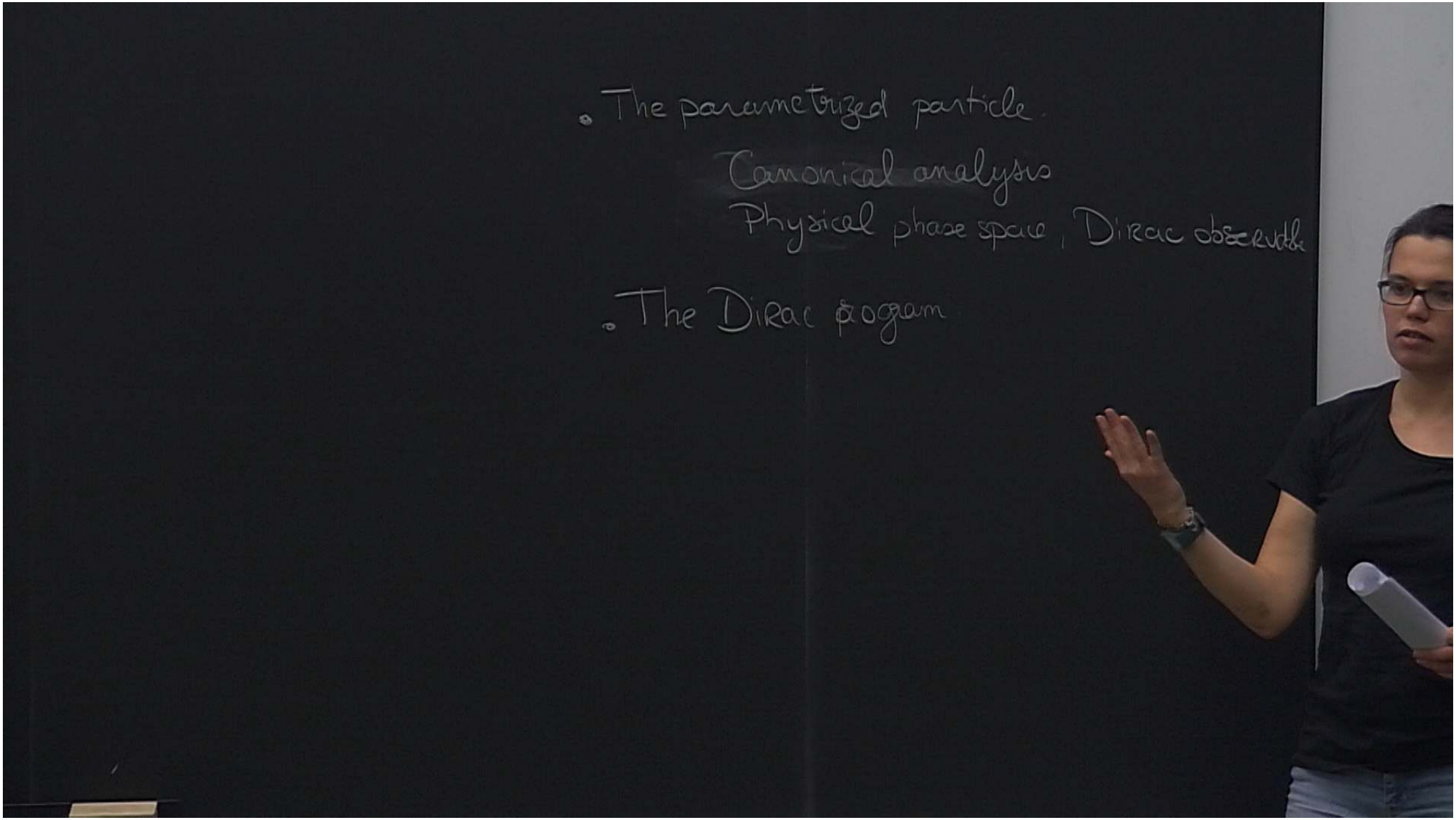
Title: PSI 2017/2018 - Quantum Gravity - Lecture 3

Date: Mar 21, 2018 10:15 AM

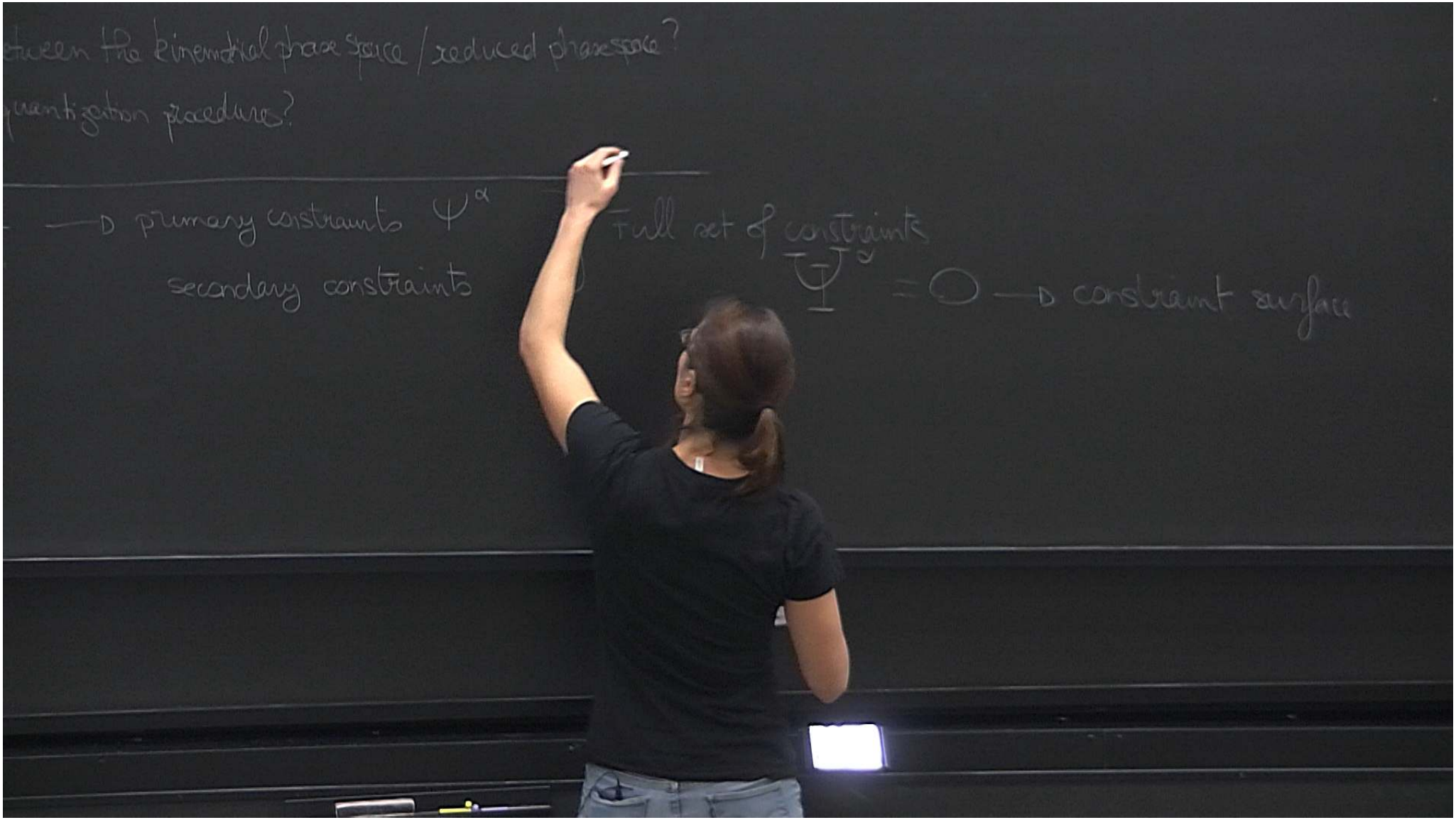
URL: http://pirsa.org/18030040

Abstract:


. The parametrized particle
Canonical analysis
Physical phase space, Dirac observable
. The Dirac program

- Distinction between the kinematical phase space / reduced phase space?
- Canonical quantization procedure?

$p_i = \frac{\partial L}{\partial \dot{q}_i}$ → primary constraints Ψ^α
secondary constraints
] Full set of constraints $\Psi^\alpha = 0$ →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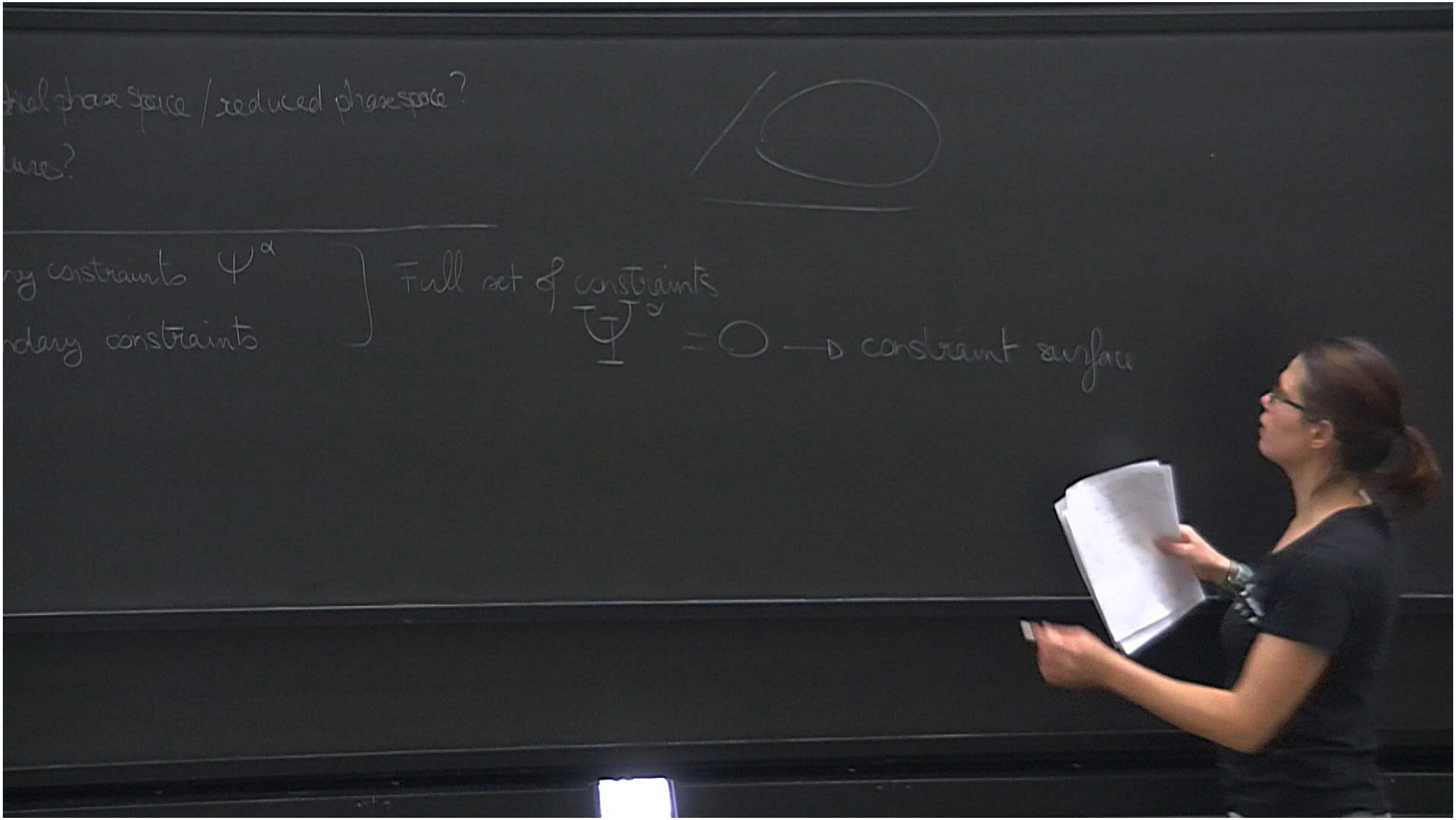
uantization procedures?
→ primary constraints ψ^α
secondary constraints
Full set of constraints
$\Psi^\alpha = 0$ → constraint surface


phase space / reduced phasespace?
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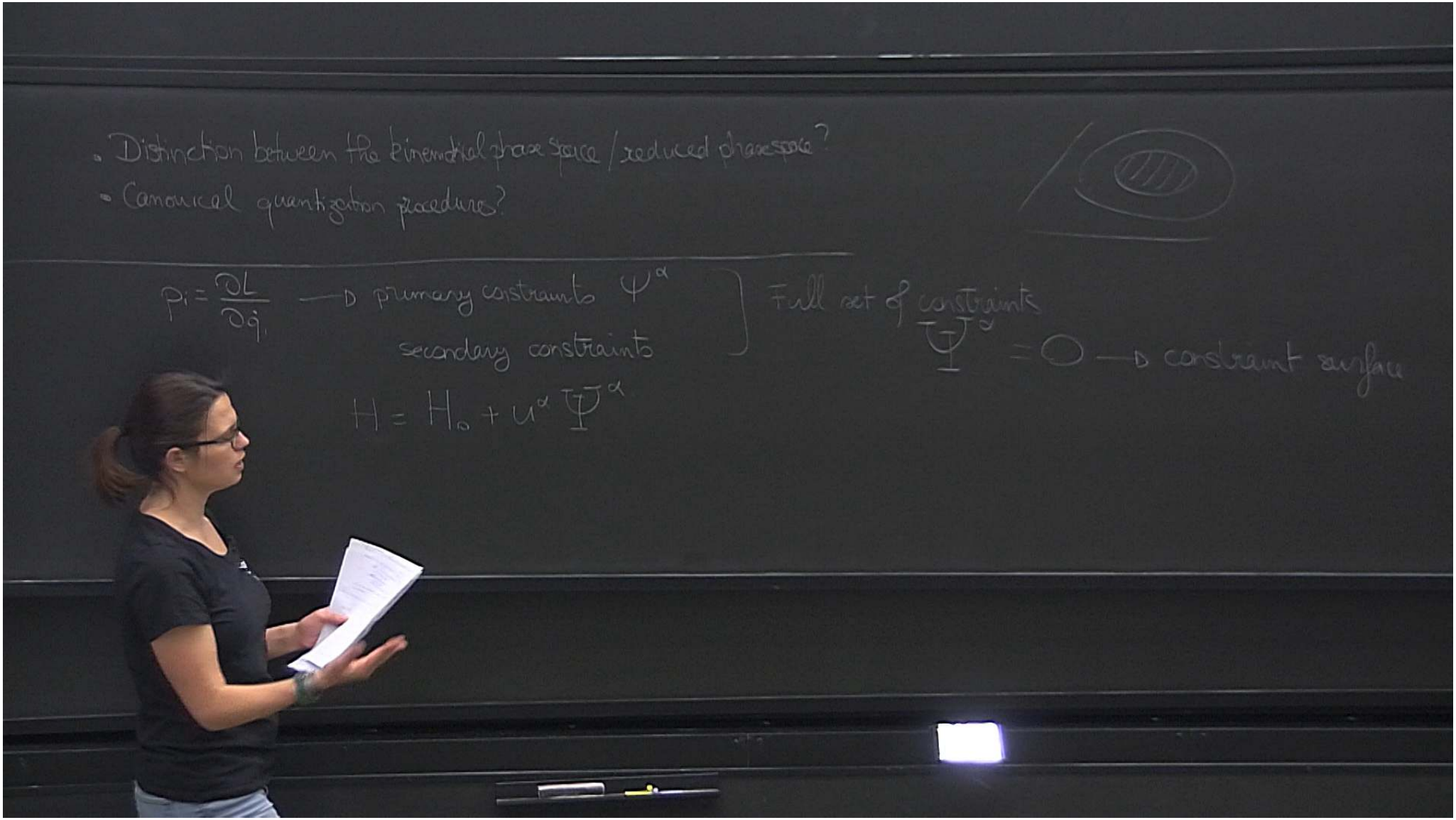
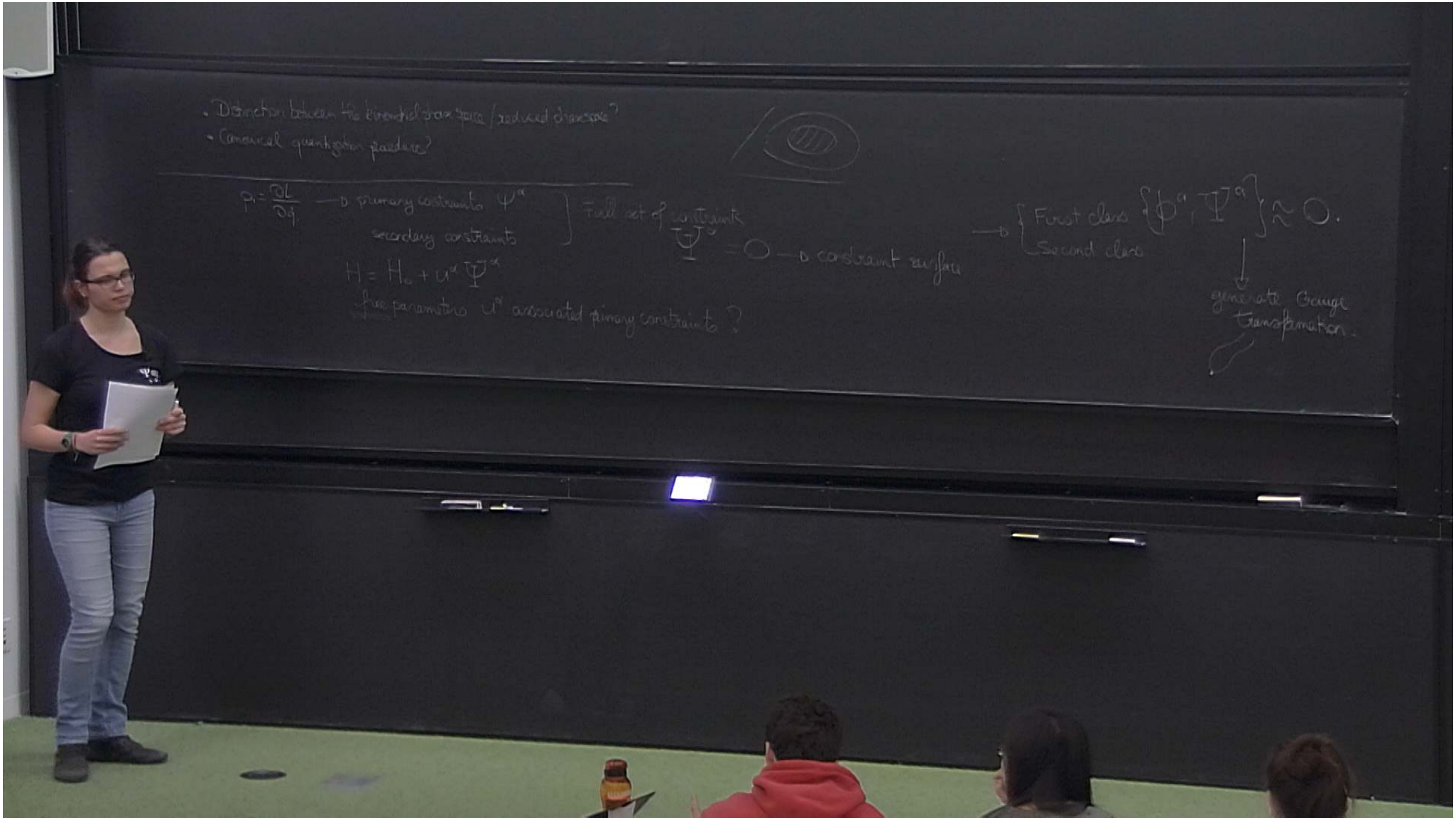

• Distinction between the kinematical phase space / reduced phase space?
• Canonical quantization procedures?
$p_i = \frac{\partial L}{\partial \dot{q}_i}$ → primary constraints Ψ^α
secondary constraints
Full set of constraints
$\Psi^\alpha = 0$ → constraint surface
$H = H_0 + u^\alpha \Psi^\alpha$


• Distinction between the Hamiltonian phase space / reduced phase space?
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$p_i = \frac{\partial L}{\partial \dot{q}_i}$ → primary constraints Ψ^α
secondary constraints
Full set of constraints
$\Upsilon^\alpha = 0$ → constraint surface
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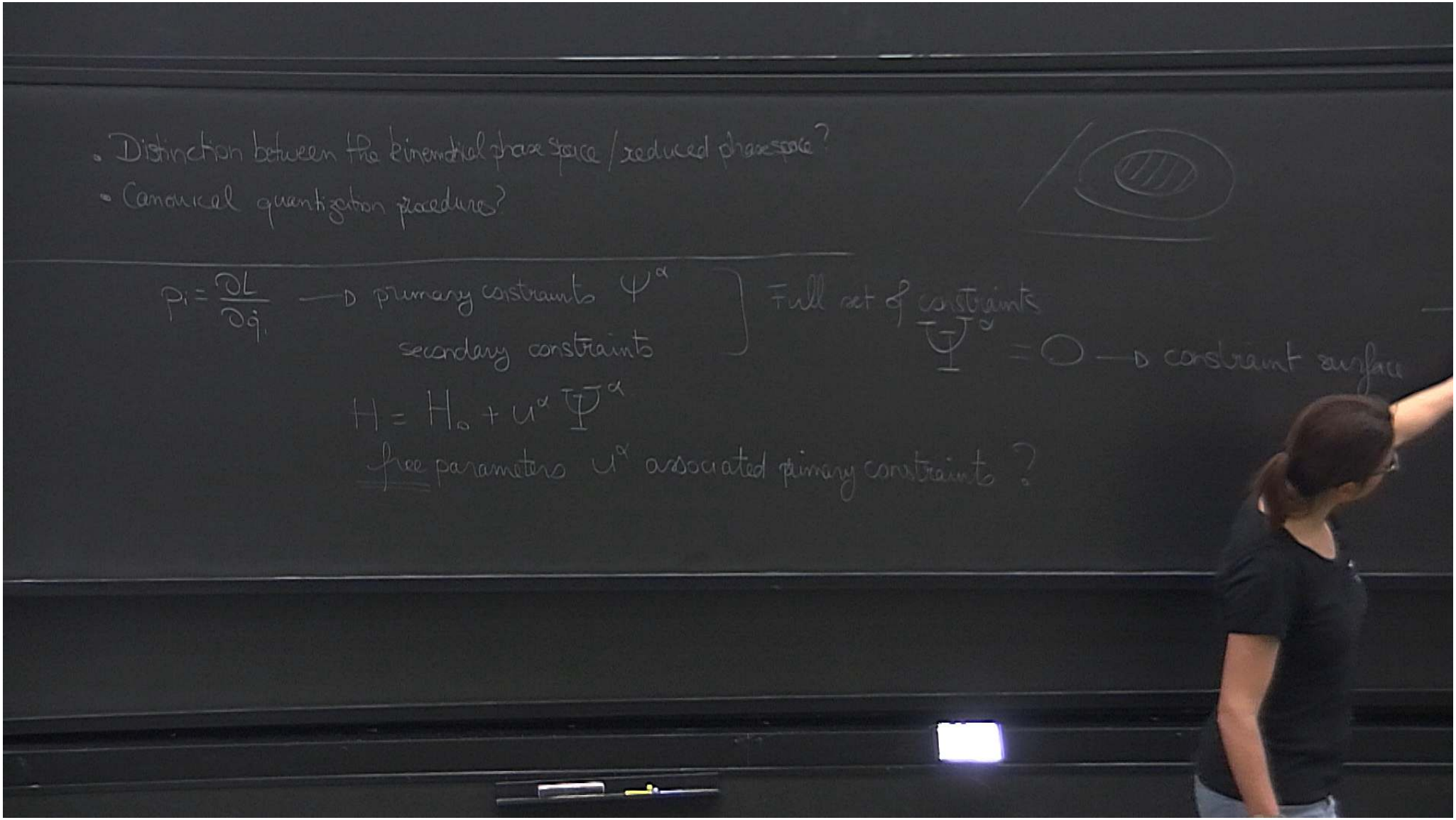
→ { First class
Second class
$\{\phi^\alpha, \Psi^\beta\} \approx 0$.
generate Gauge transformation.
u^α associated primary constraints ?


• Distinction between the kinematical phase space / reduced phase space?
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$p_i = \frac{\partial L}{\partial \dot{q}_i}$ → primary constraints Ψ^α
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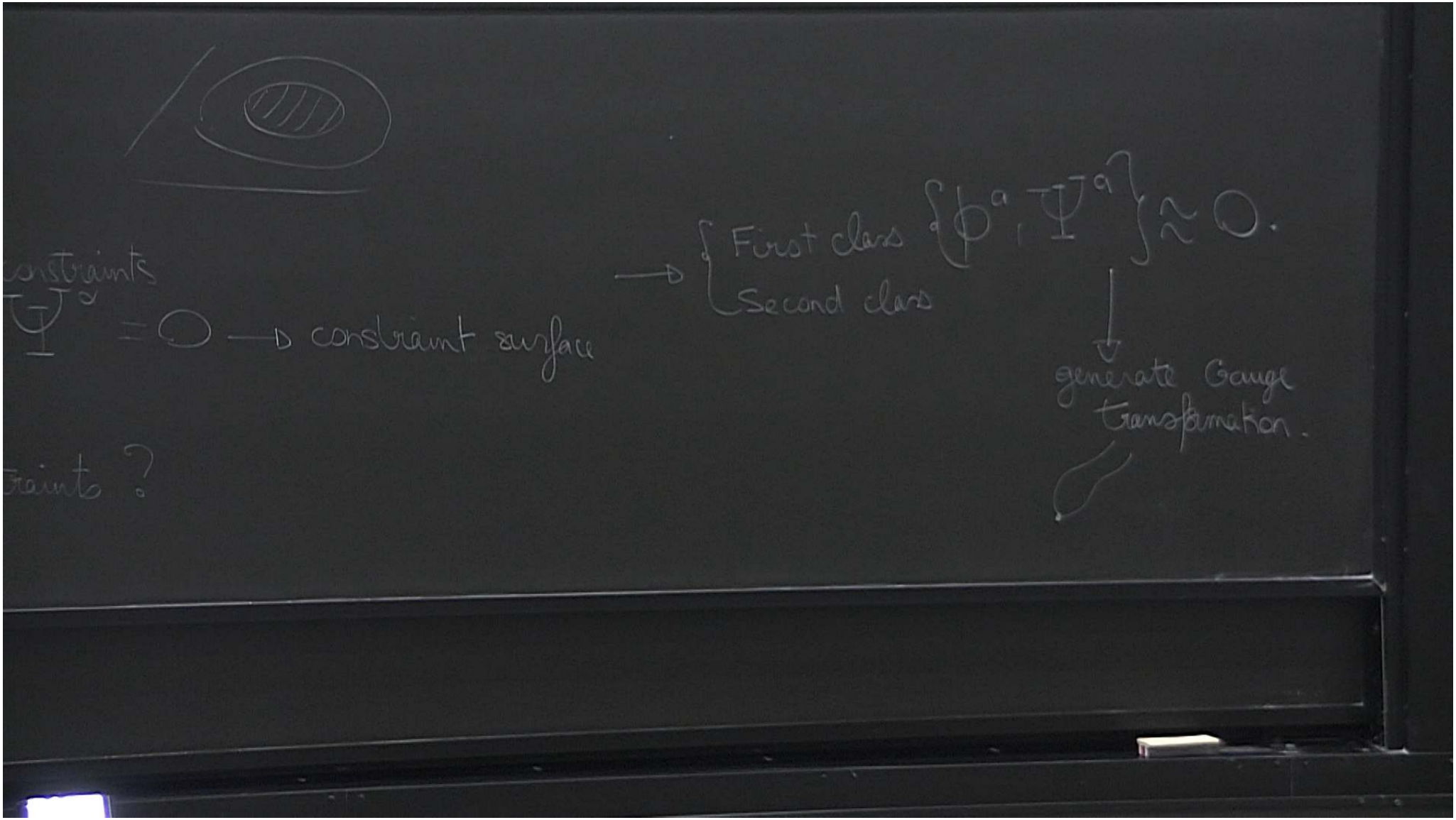
Full set of constraints
$\Psi^\alpha = 0$ → constraint surface
$H = H_0 + u^\alpha \Psi^\alpha$
free parameters u^α associated primary constraints ?


constraints
$\Psi^a = 0$ → constraint surface
→ { First class, Second class
$\{\phi^a, \Psi^a\} \approx 0$.
generate Gauge transformation.

Parametrized particle

$$S[q] = \int dt\, \frac{1}{2} m \dot{q}^2 \longrightarrow S_p[q(s), t(s)] = \int \left(\frac{1}{2} m \frac{q'^2}{t'} \right) ds \qquad q' = \frac{dq}{ds}$$

Canonical analysis

$$\left\{ \begin{array}{l} P_t = -m \dfrac{q'^2}{t'^2} \\ P_q = m \dfrac{q'}{t'} \end{array} \right\} \longrightarrow \boxed{C = P_t + \frac{P_q^2}{2m} =}$$

ametrized particle

$$\Rightarrow \quad S_p[q(s), t(s)] = \int \left(\frac{1}{2} m \frac{q'^2}{t'}\right) ds \qquad q' = \frac{dq}{ds}$$

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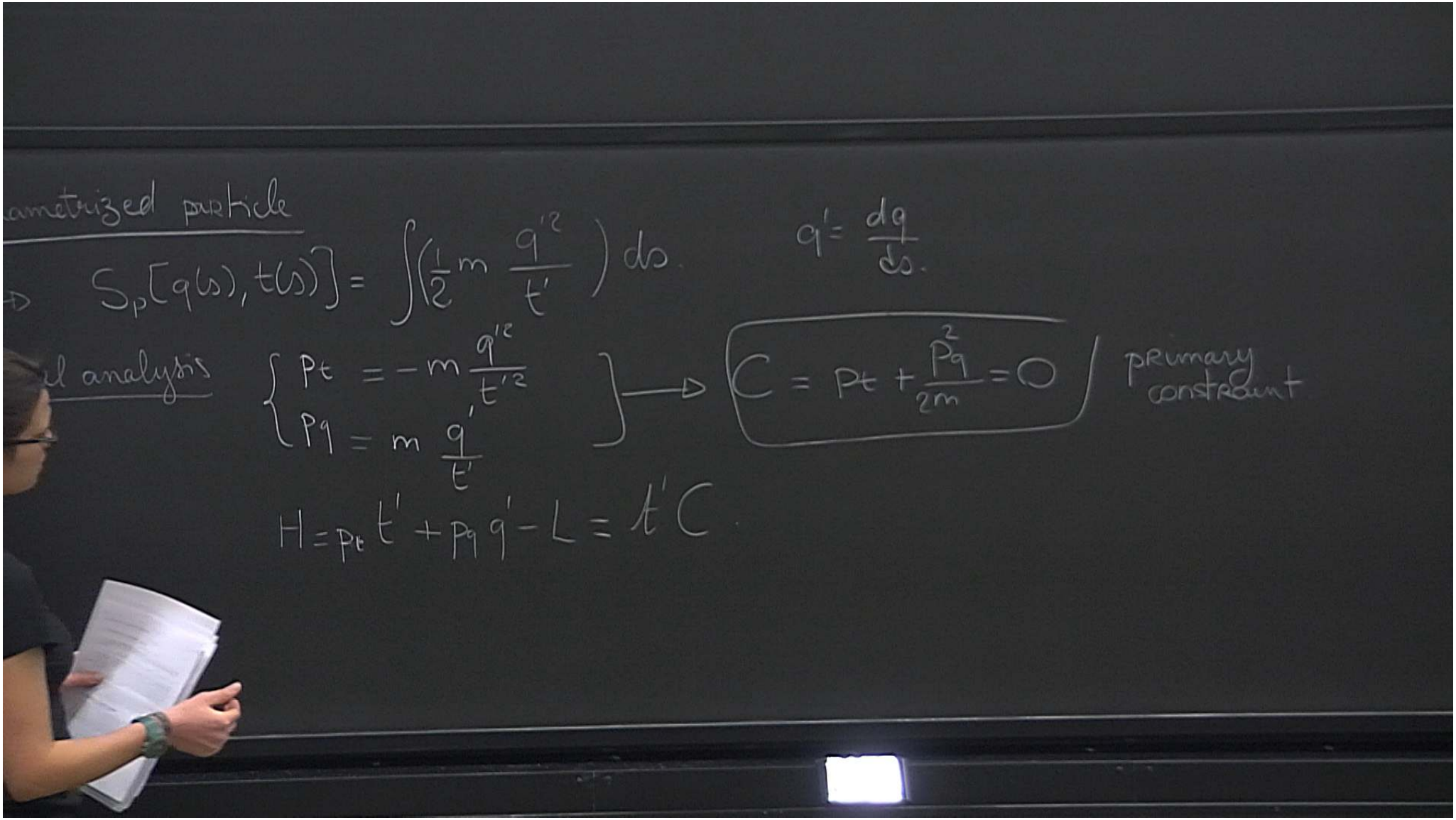
primary constraint


ametrized particle
$S_p[q(s), t(s)] = \int \left(\frac{1}{2} m \frac{q'^2}{t'}\right) ds$
$q' = \frac{dq}{ds}$
l analysis
$\begin{cases} p_t = -m \frac{q'^2}{t'^2} \\ p_q = m \frac{q'}{t'} \end{cases} \longrightarrow C = p_t + \frac{p_q^2}{2m} = 0$
primary constraint
$H = p_t t' + p_q q' - L = t' C$

parameters of associated primary constraints :

ametrized particle

$$\Rightarrow \quad S_p[q(s), t(s)] = \int \left(\frac{1}{2} m \frac{q'^2}{t'} \right) ds. \qquad q' = \frac{dq}{ds}.$$

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$$H = N(s)\, C$$

parameters of associated primary constraints :

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ical analysis

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primary constraint

$$H = p_t t' + p_q q' - L = t' C.$$

$$H = N(s)\, C$$

Lapse function.

parameters of associated primary constraints :

ametrized particle

$$\Rightarrow \quad S_p[q(s), t(s)] = \int \left(\frac{1}{2} m \frac{q'^2}{t'} \right) ds. \qquad q' = \frac{dq}{ds}.$$

ical analysis

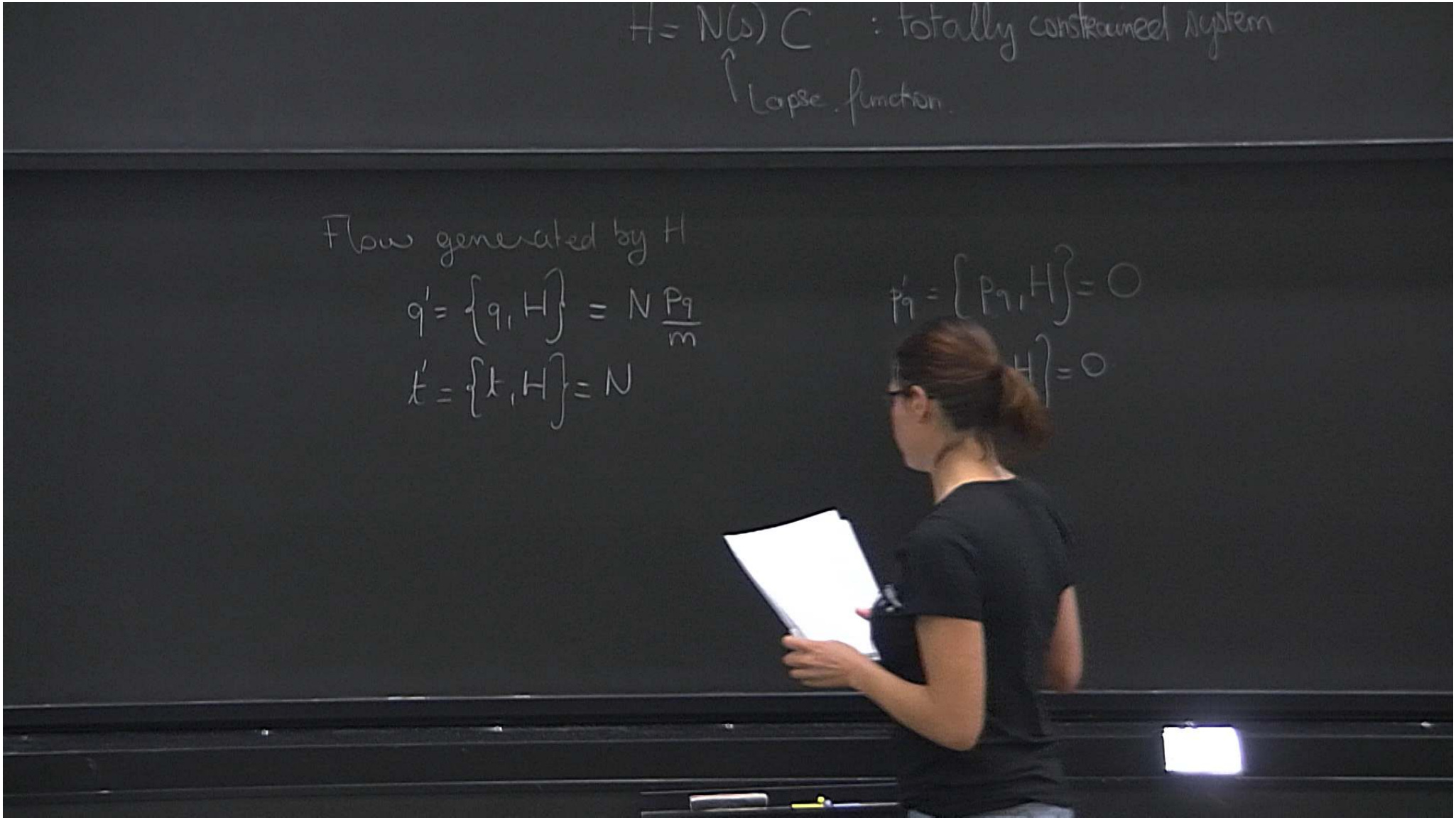
$$\left\{ \begin{array}{l} p_t = -m \dfrac{q'^2}{t'^2} \\ p_q = m \dfrac{q'}{t'} \end{array} \right\} \longrightarrow \boxed{C = p_t + \frac{p_q^2}{2m} = 0}$$

primary constraint

$$H = p_t t' + p_q q' - L = t' C.$$

$$H = N(s)\, C \quad : \text{totally constrained system}$$

Lapse function


H = N(s) C : totally constrained system
lapse function
Flow generated by H
q' = {q, H} = N p_q / m
t' = {t, H} = N
p'_q = {p_q, H} = 0
H} = 0

$H = N(s)\, C$: totally constrained system

↑ Lapse function

Flow generated by H

$$q' = \{q, H\} = N \frac{p_q}{m} \qquad p_q' = \{p_q, H\} = 0$$

$$t' = \{t, H\} = N \qquad p_t' = \{p_t, H\} = 0$$

↳ starting from a given initial condition

$\Rightarrow$ different solutions (different values of N)

$H = N(s)\, C$: totally constrained system

↑ Lapse function

Flow generated by H

$$q' = \{q, H\} = N \frac{p_q}{m} \qquad p_q' = \{p_q, H\} = 0$$

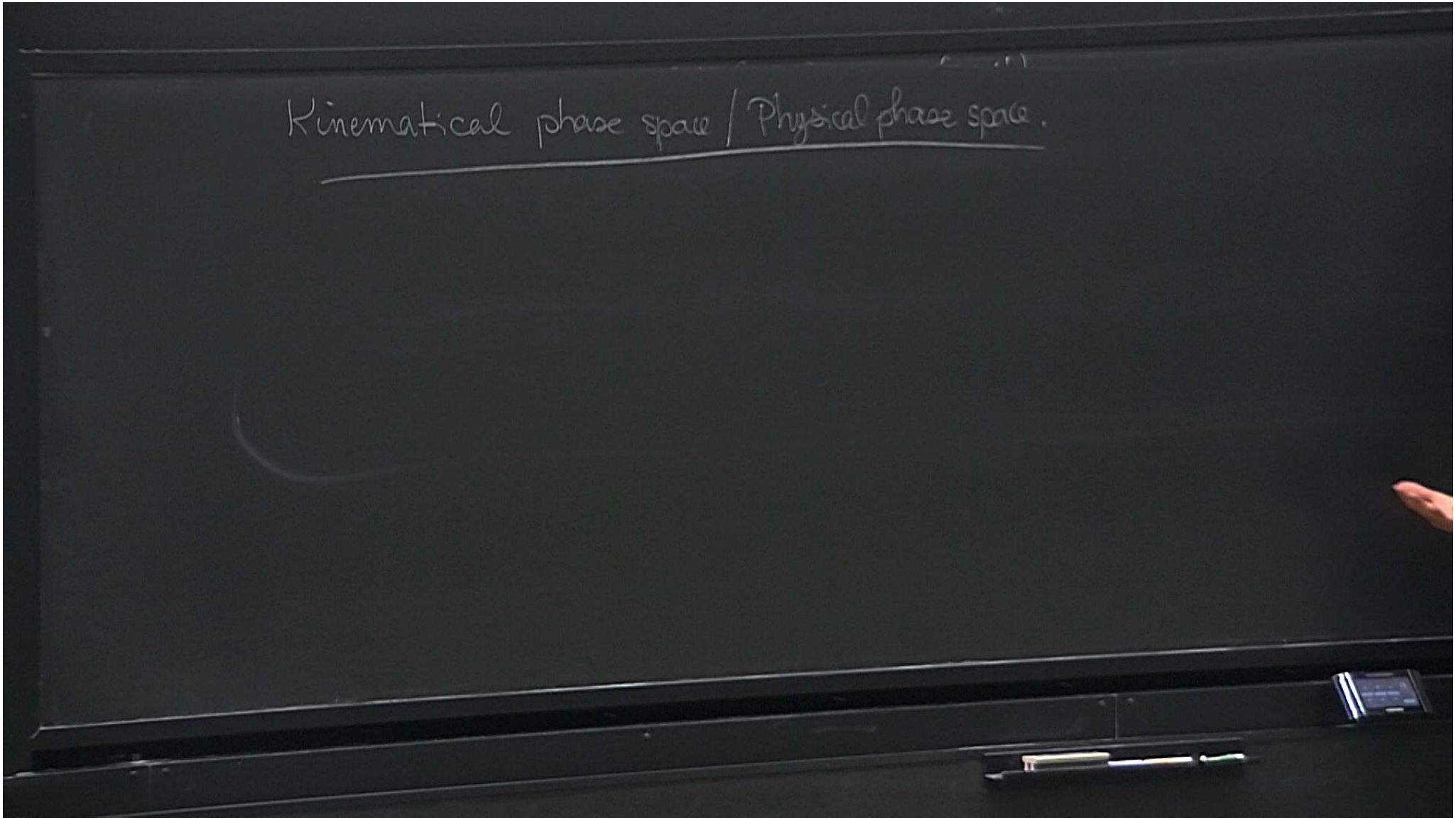
$$t' = \{t, H\} = N \qquad p_t' = \{p_t, H\} = 0$$

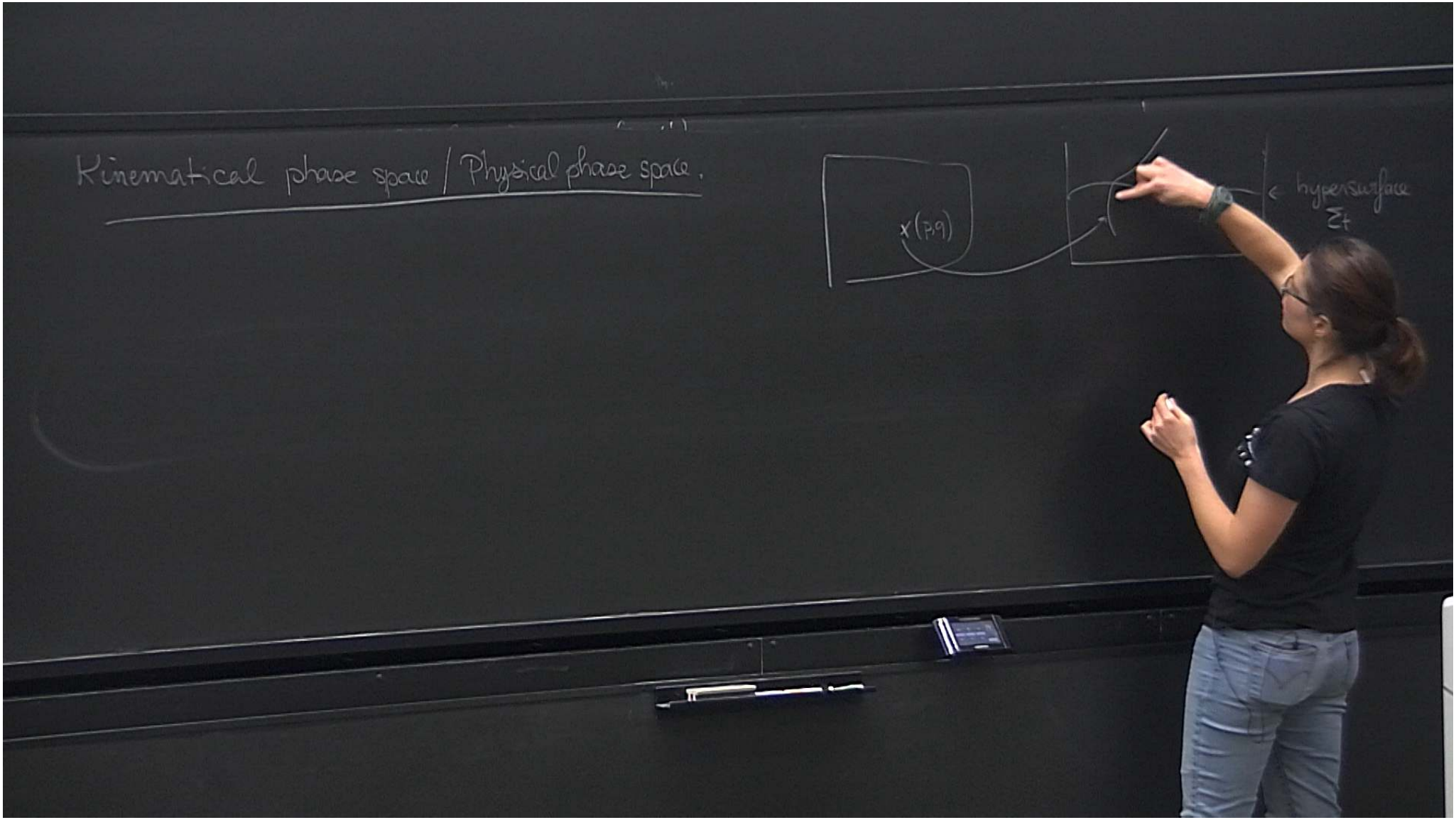
↳ starting from a given initial condition

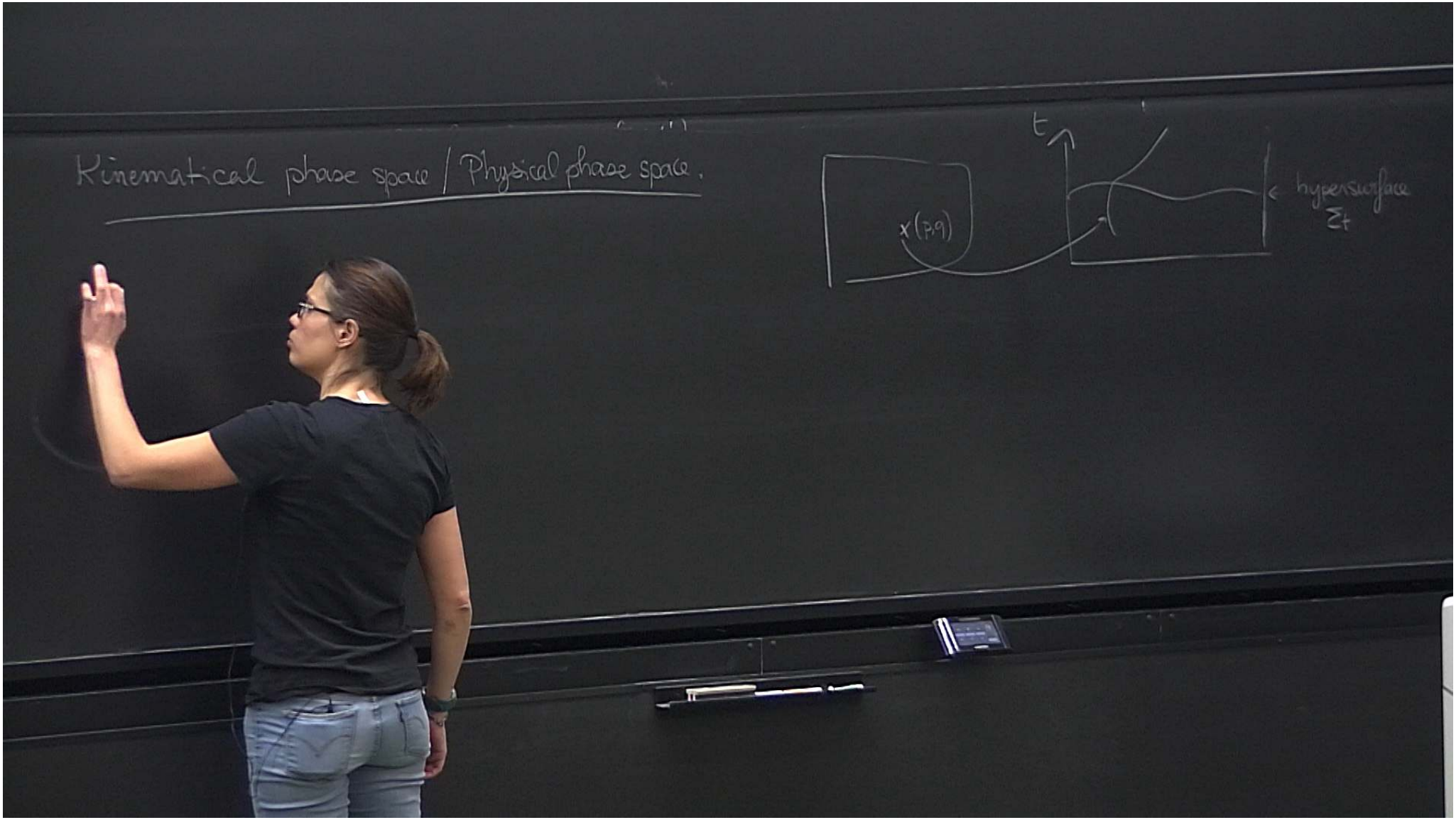
$\Rightarrow$ different solutions (different values of N)

Relevant Q: relational question

What is the value of $q(s)$ when $t(s) = 3$?


Kinematical phase space / Physical phase space.


Kinematical phase space / Physical phase space.
x(p,q)
hypersurface
Σ_t


Kinematical phase space / Physical phase space.
t
x(p,q)
hypersurface
Σ_t

Kinematical phase space / Physical phase space.

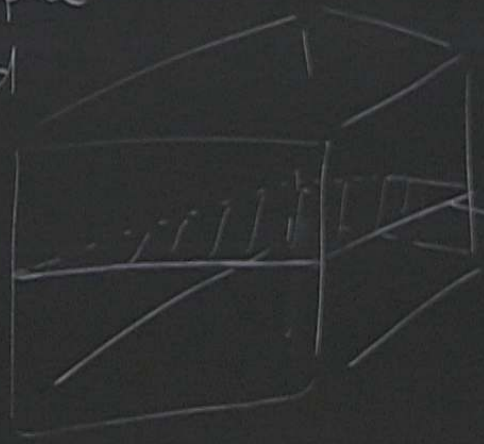
↳ initial phase space

parametrized particle : Lid

(q, t, p_q, p_t).

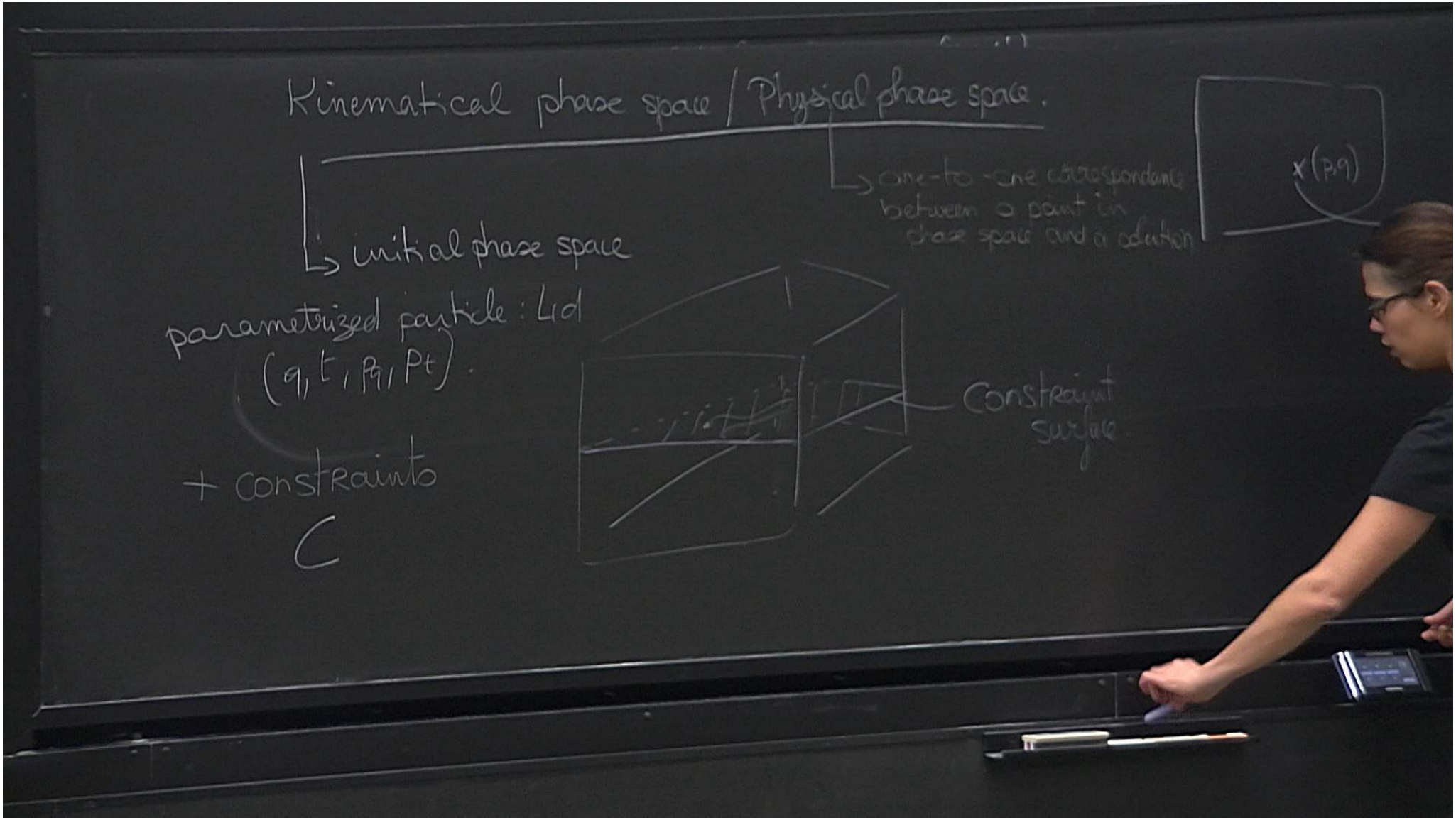
+ constraints

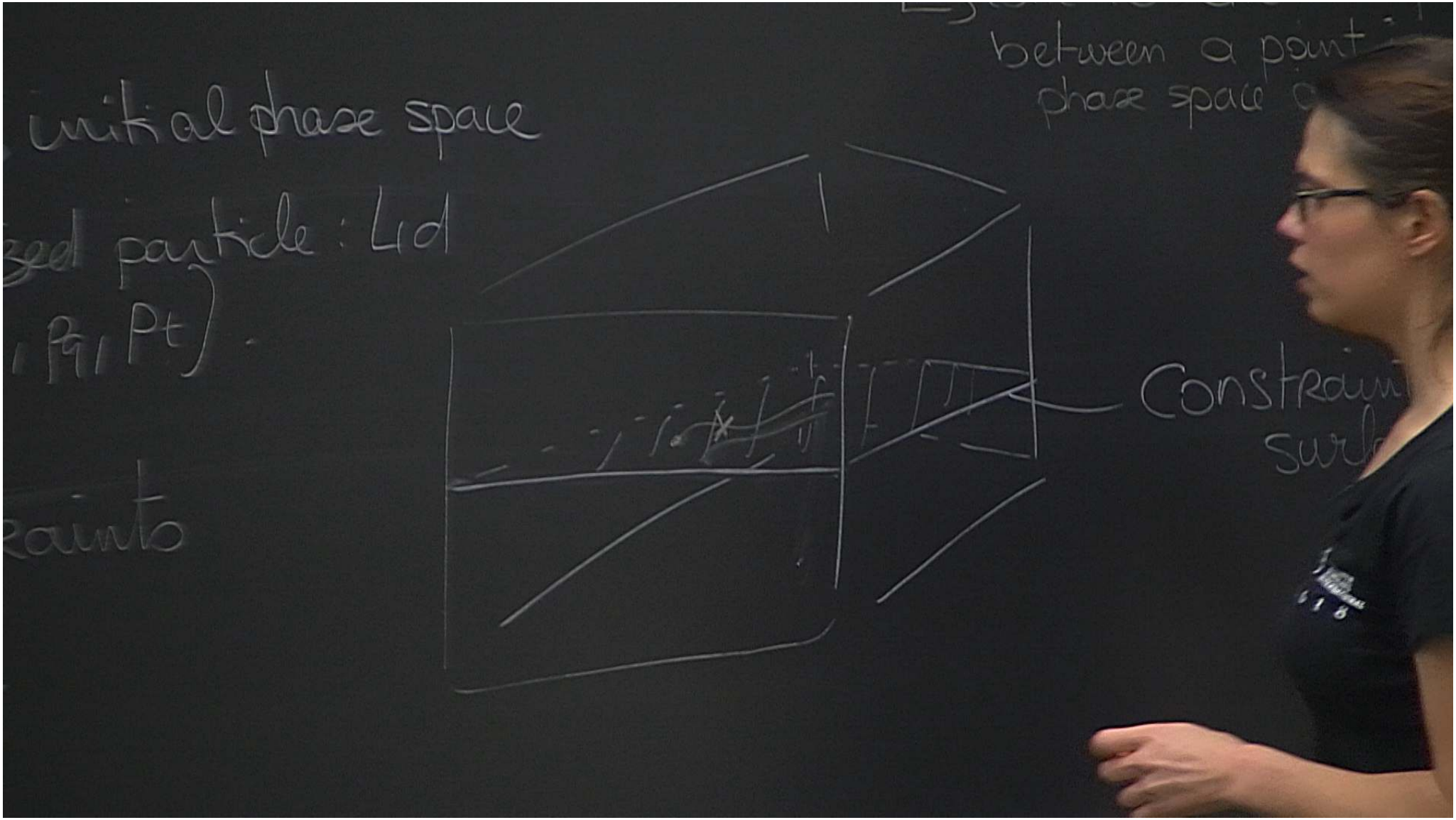
C



Constraint surface

$x(p,q)$


Kinematical phase space / Physical phase space.
initial phase space
parametrized particle : Lid
(q, t, pq, pt)
+ constraints
C
one-to-one correspondance between a point in phase space and a solution
x(p,q)
Constraint surface


between a pair
phase space
Constrain
surf

Kinematical phase space / Physical phase space.

↳ initial phase space

↳ one-to-one correspondence between a point in phase space and a solution

$x(p,q)$

parametrized particle : Lol

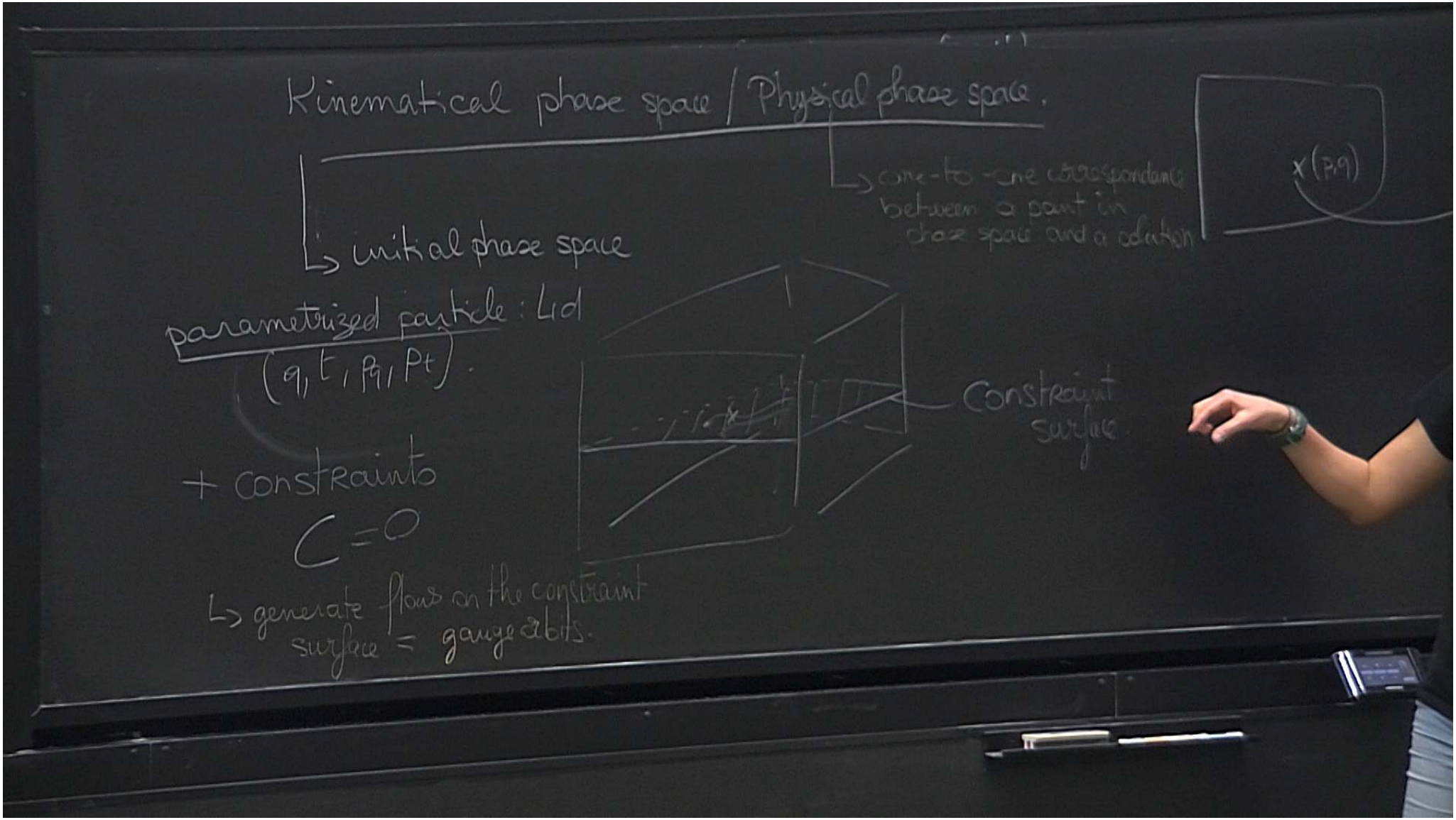
(q, t, p_q, p_t).

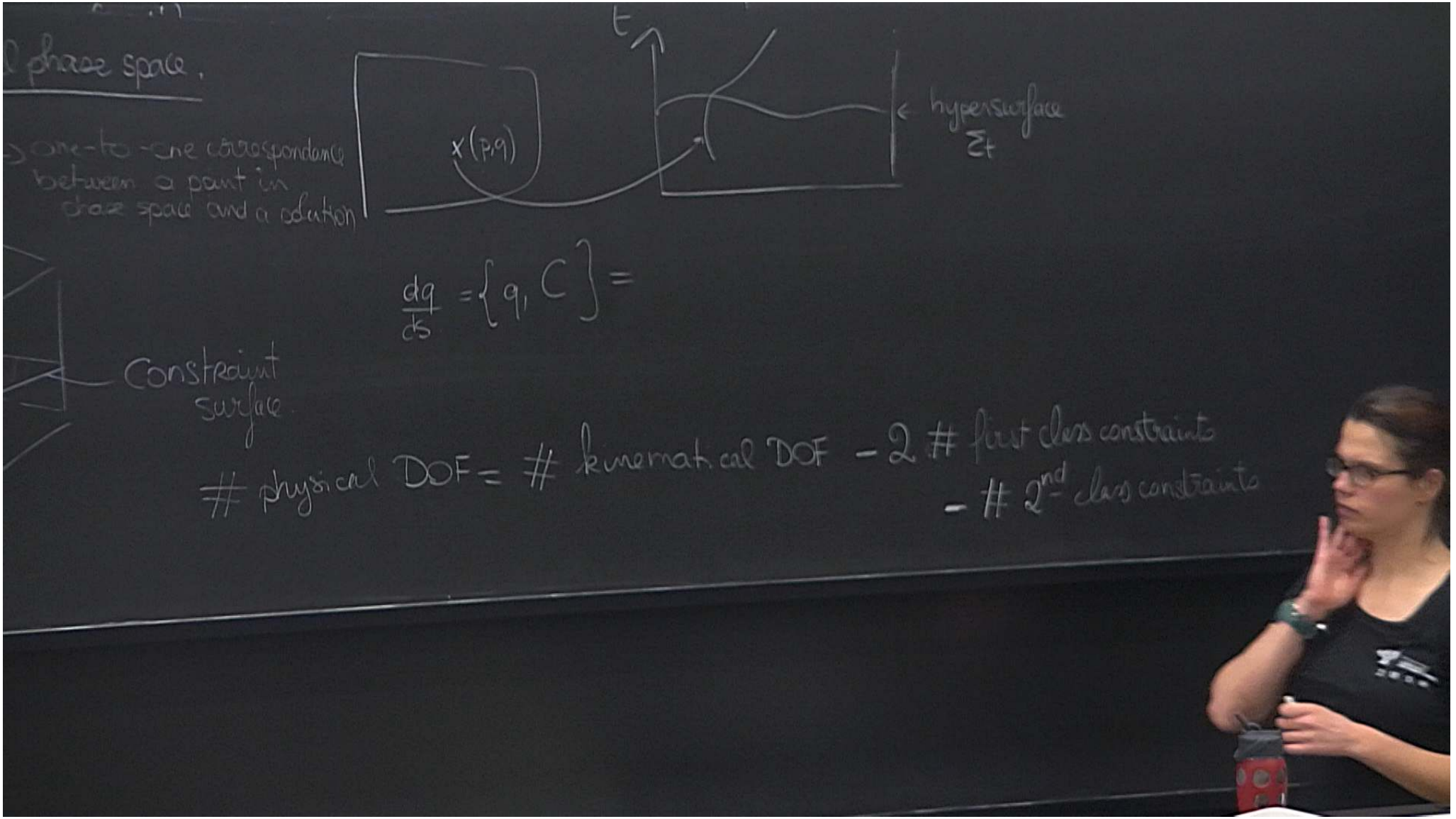
+ constraints

C

Constraint surface

↳ generate flows on the constraint surface = gauge dofs.


Kinematical phase space / Physical phase space.
↳ one-to-one correspondance between a point in phase space and a solution
x(P,Q)
↳ initial phase space
parametrized particle : Lid
(q, t, pq, pt).
+ constraints
C = 0
Constraint surface
↳ generate flows on the constraint surface = gauge dofs.


phase space.
one-to-one correspondance between a point in phase space and a solution
x(p,q)
t
hypersurface Σt
$\frac{dq}{ds} = \{q, C\} =$
Constraint surface
physical DOF = # kinematical DOF − 2 # first class constraints − # 2nd class constraints

> generate flows on the constraint surface = gauge orbits.

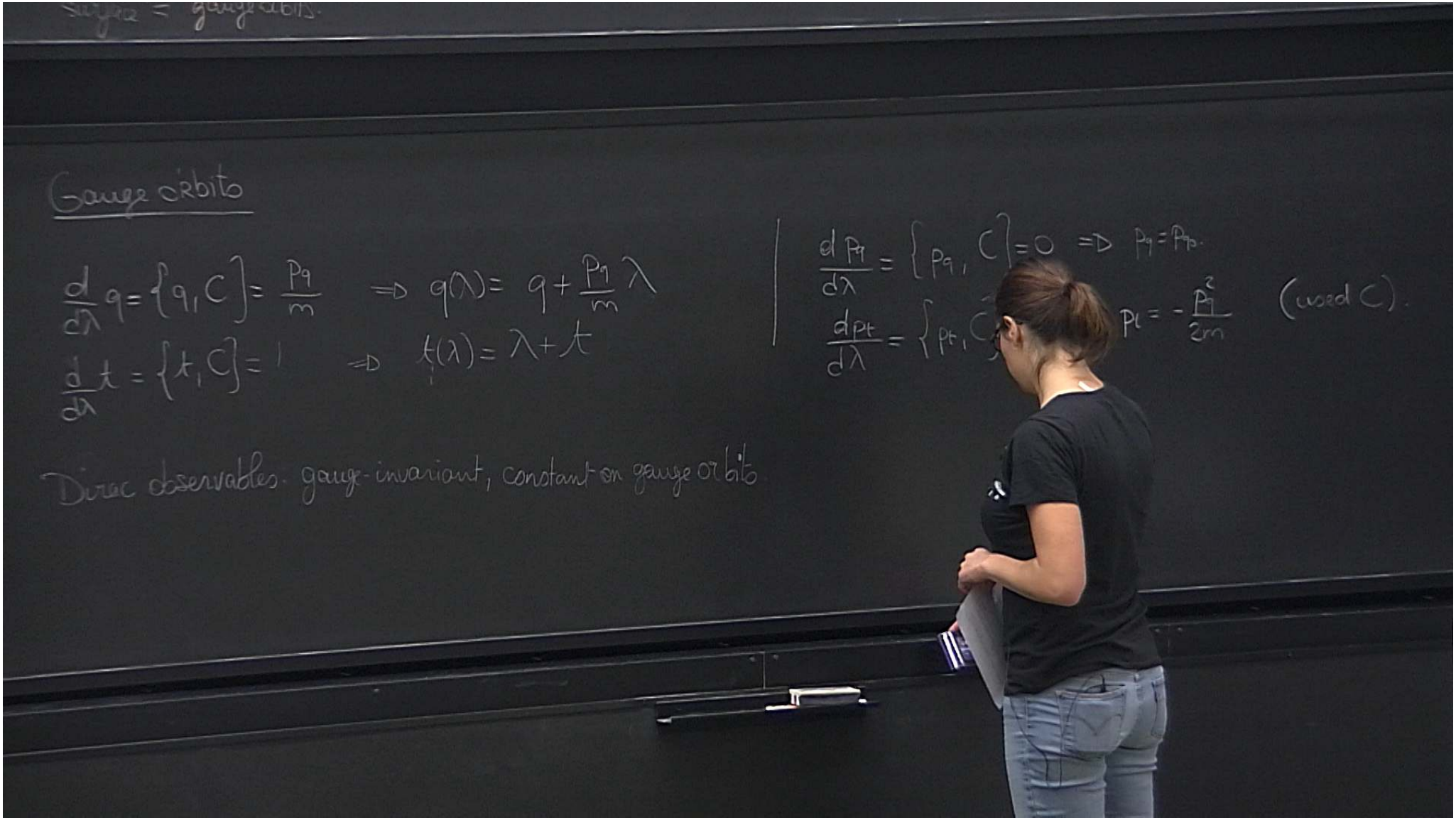
Gauge orbits

$$\frac{d}{d\lambda} q = \{q, C\} = \frac{p_q}{m} \Rightarrow q(\lambda) = q + \frac{p_q}{m}\lambda$$

$$\frac{d}{d\lambda} t = \{t, C\} = 1 \Rightarrow t(\lambda) = \lambda + t$$

$$\frac{dp_q}{d\lambda} = \{p_q, C\} = 0 \Rightarrow p_q = p_{q_0}.$$

$$\frac{dp_t}{d\lambda} = \{p_t, C\} = 0 \Rightarrow p_t = -\frac{p_q^2}{2m} \quad \text{(used } C\text{)}.$$


Gauge orbits
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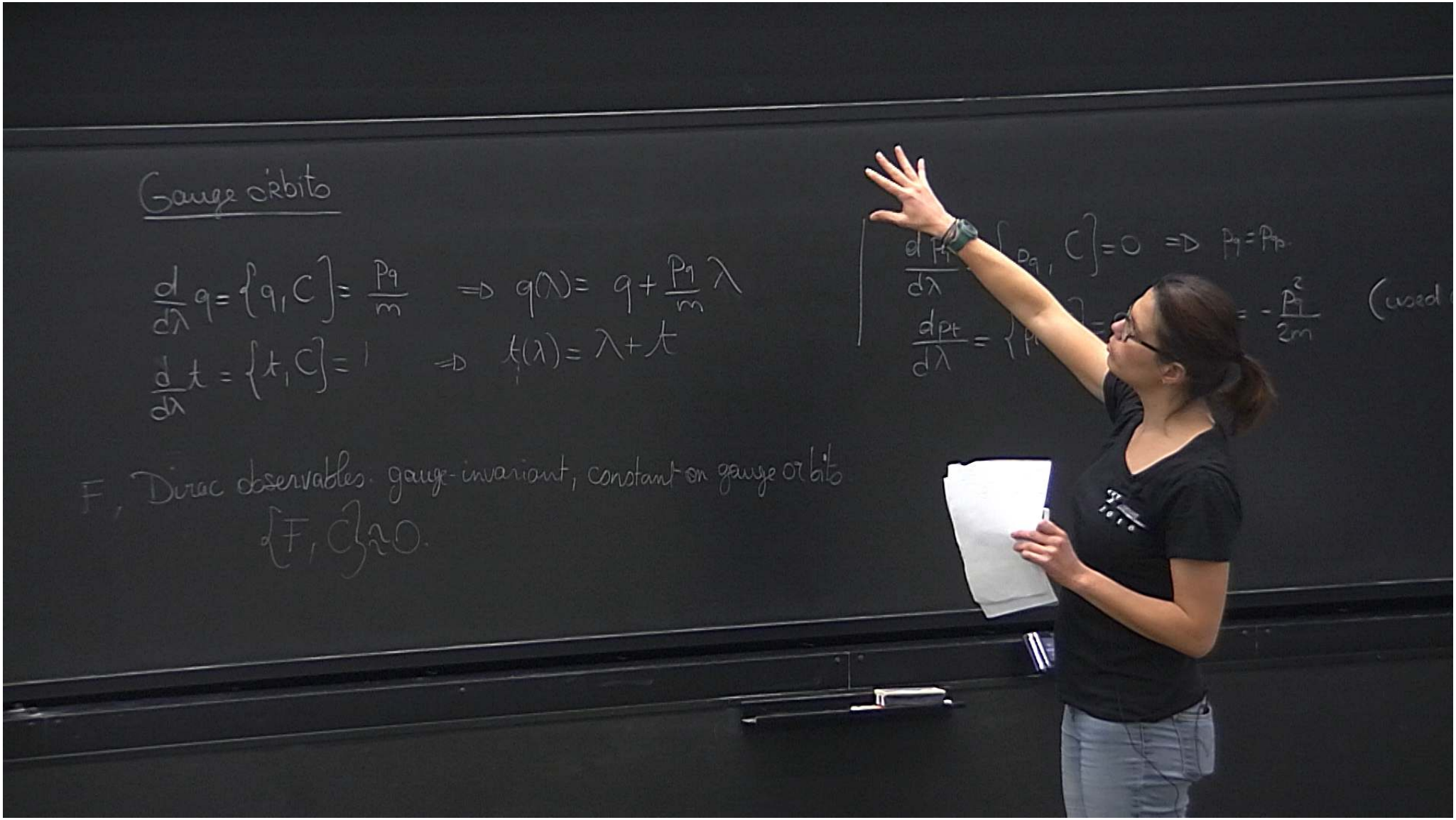
$\frac{d}{d\lambda} t = \{t, C\} = 1 \Rightarrow t(\lambda) = \lambda + t$
$\frac{dp_q}{d\lambda} = \{p_q, C\} = 0 \Rightarrow p_q = p_{q_0}$.
$p_t = -\frac{p_q^2}{2m}$ (used C).
Dirac observables. gauge-invariant, constant on gauge orbits


Gauge orbits
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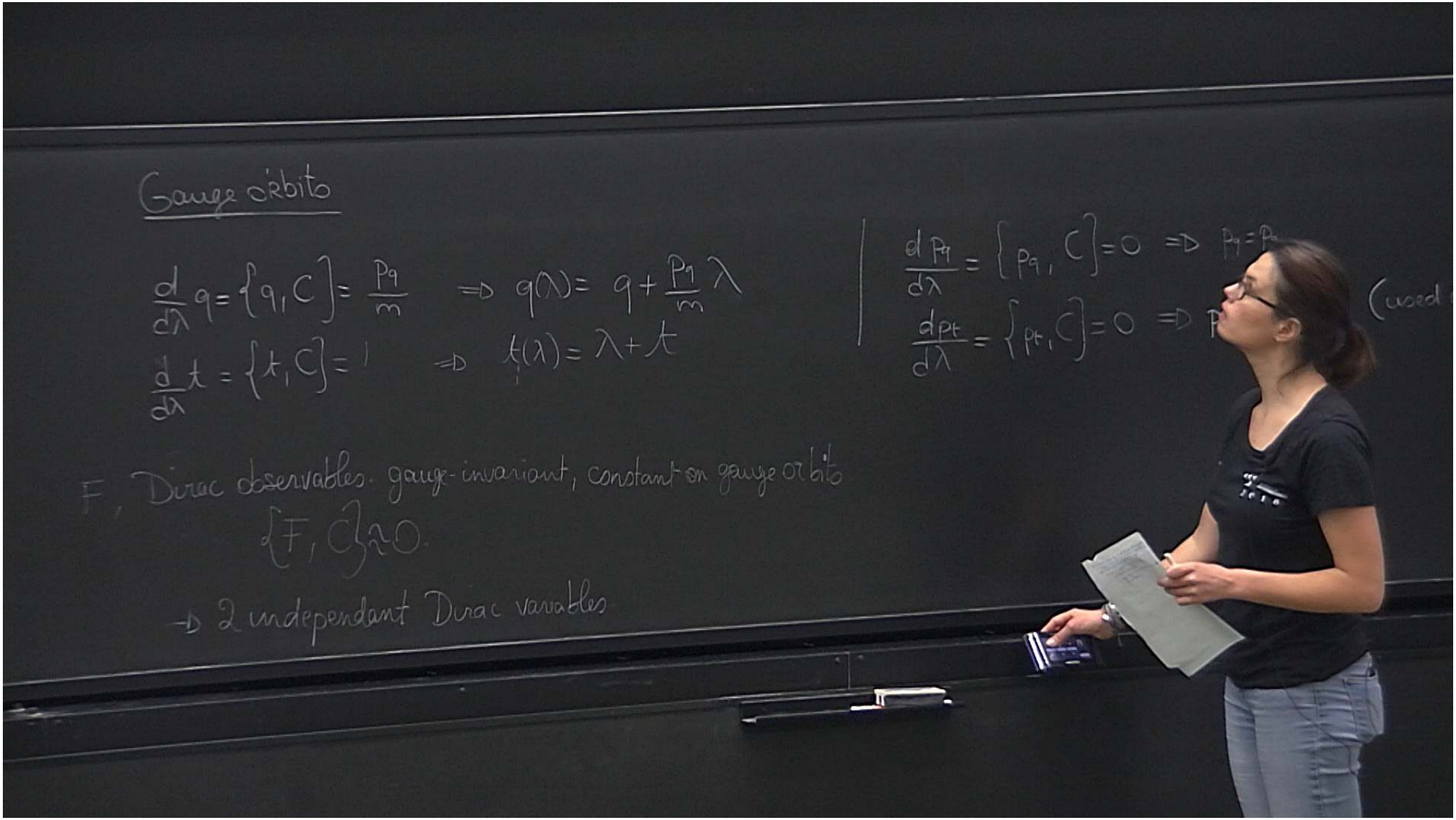
F, Dirac observables. gauge-invariant, constant on gauge orbits
$\{F, C\} \approx 0$.
$\frac{dp_q}{d\lambda} = \{p_q, C\} = 0 \Rightarrow p_q = p_{q_0}$
$\frac{dp_t}{d\lambda} = \{p_t$
$= -\frac{p_q^2}{2m}$
(used


Gauge orbits
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$\frac{d p_q}{d\lambda} = \{p_q, C\} = 0 \Rightarrow p_q = p_q$
$\frac{d p_t}{d\lambda} = \{p_t, C\} = 0 \Rightarrow$
(used
F, Dirac observables. gauge-invariant, constant on gauge orbits
$\{F, C\} \approx 0$.
→ 2 independant Dirac variables

F, Dirac observables. gauge-invariant, constant on gauge orbits

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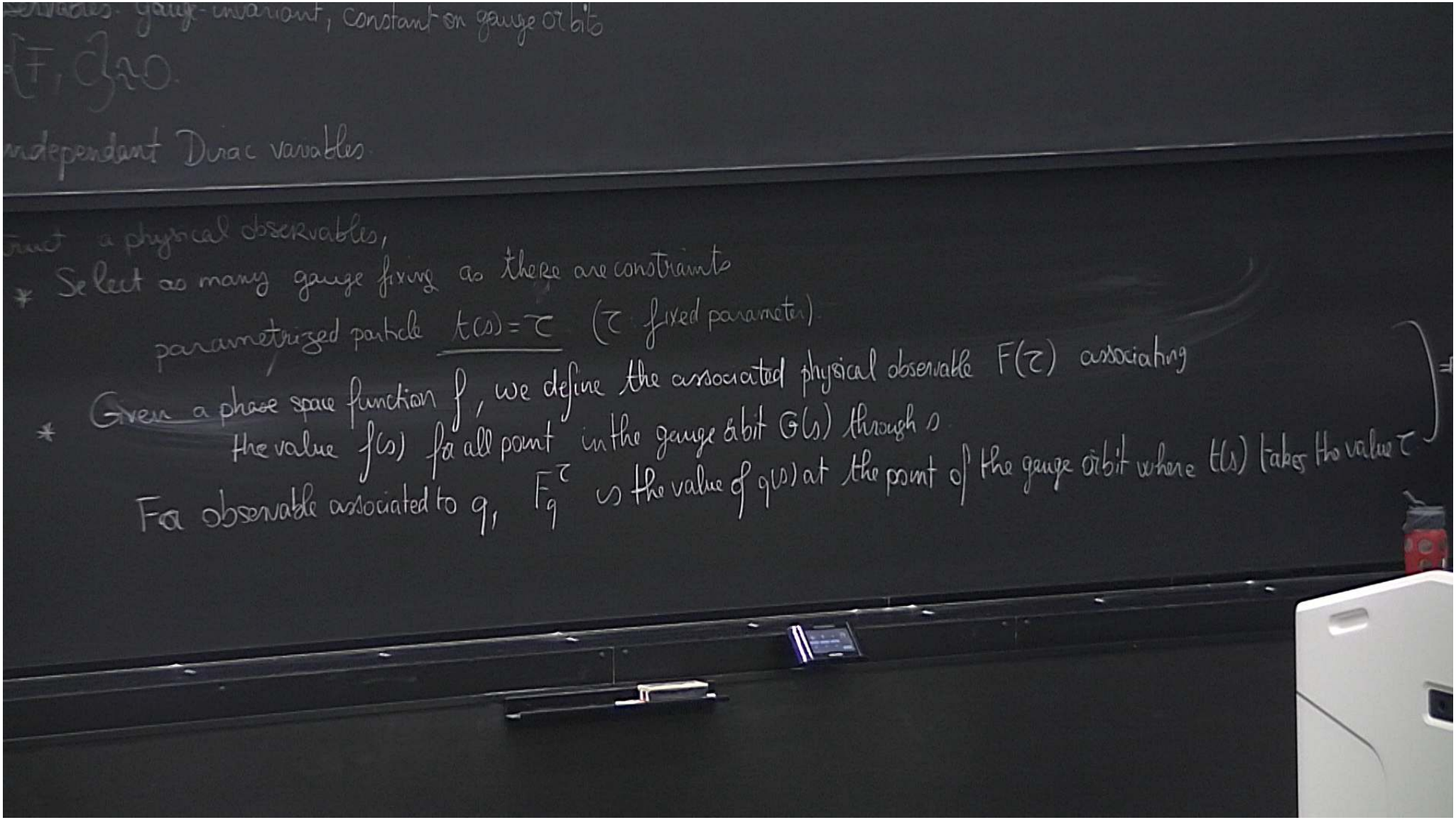
→ 2 independant Dirac variables

. To construct a physical observables,

* Select as many gauge fixing as there are constraints



...ervables: gauge-invariant, constant on gauge orbits

$\{F, C\} \approx 0$.

...ndependant Dirac variables.

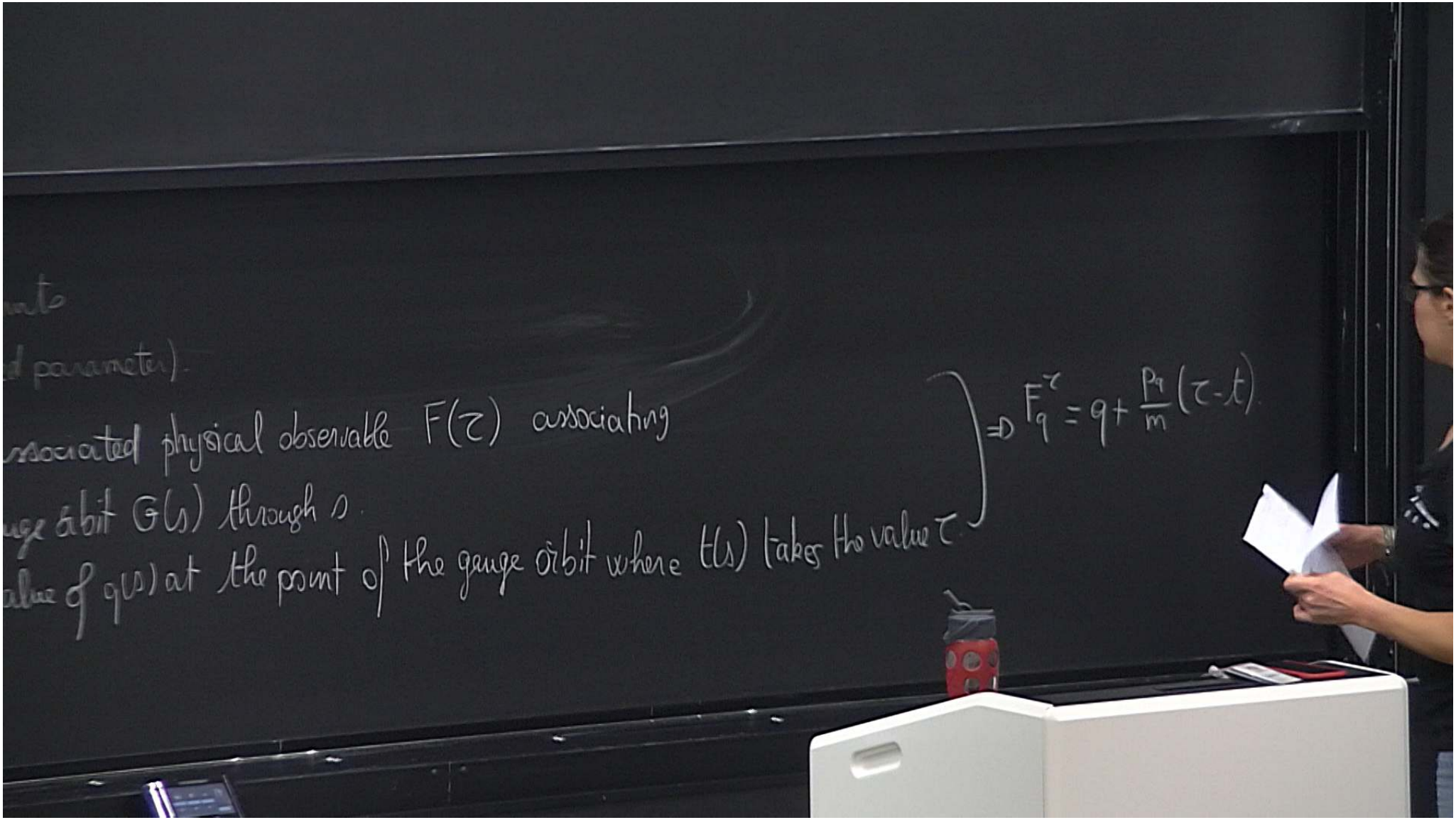
...ruct a physical observables,

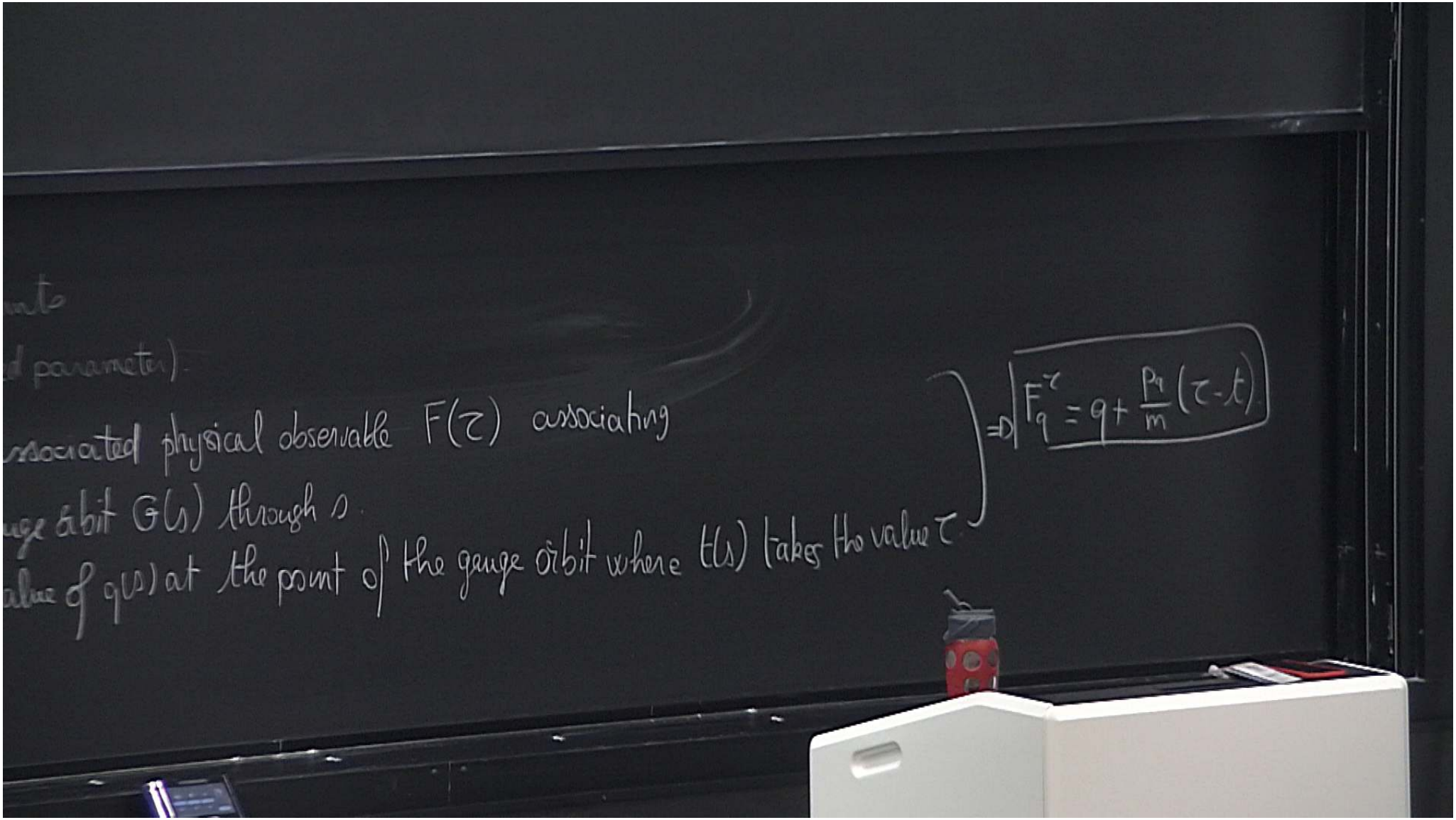
* Select as many gauge fixing as there are constraints

parametrized particle $t(s) = \tau$ (τ fixed parameter).

* Given a phase space function f, we define the associated physical observable $F(\tau)$ associating the value $f(s)$ for all point in the gauge orbit $G(s)$ through s

For observable associated to q, F_q^τ is the value of $q(s)$ at the point of the gauge orbit where $t(s)$ takes the value τ


nt
d parameter).
ssociated physical observable F(τ) associating
uge orbit G(s) through s.
alue of q(s) at the point of the gauge orbit where t(s) takes the value τ.
⇒ $F_q^\tau = q + \frac{p_q}{m}(\tau - t)$.


nt
d parameter).
ssociated physical observable F(τ) associating
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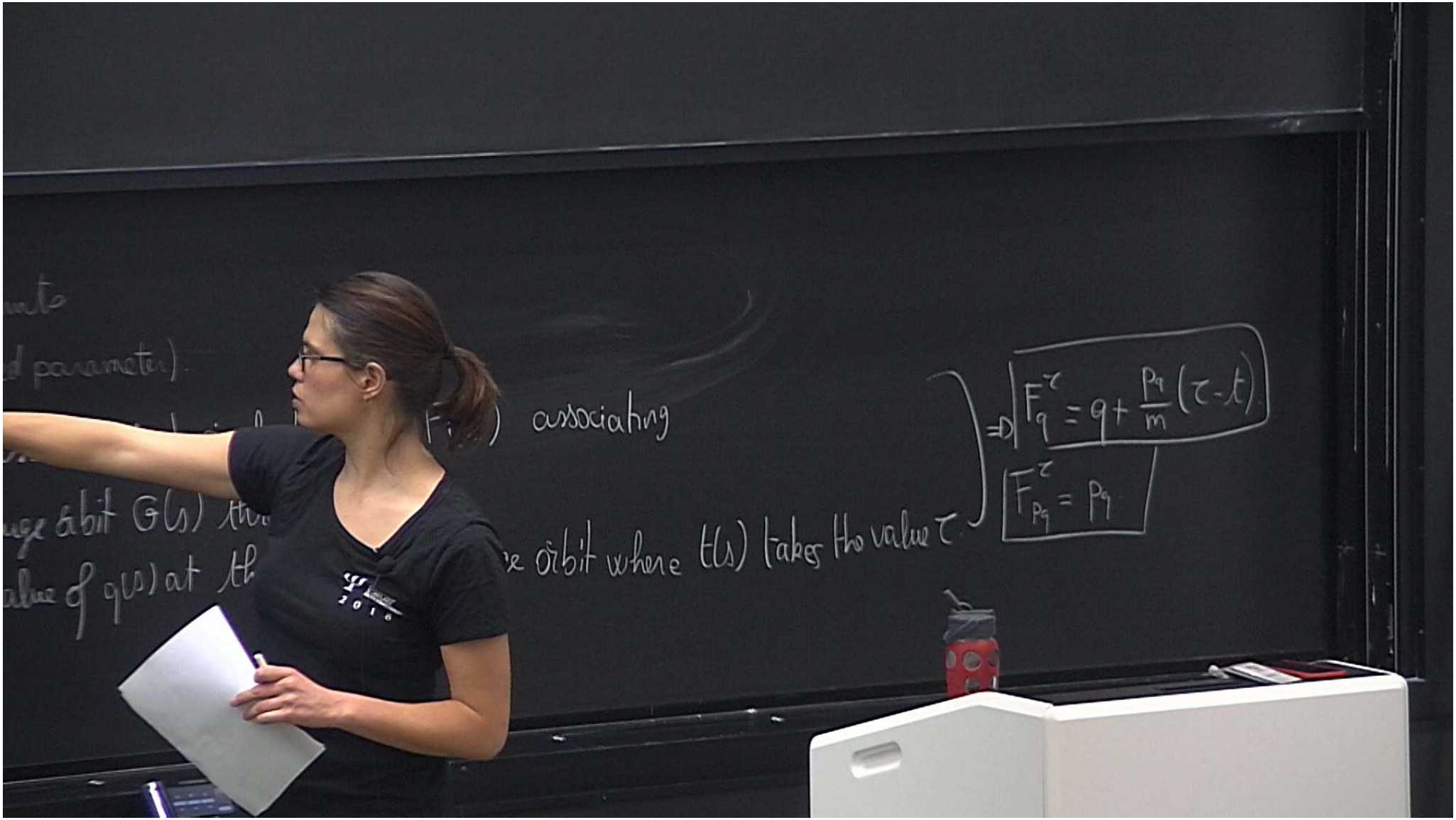
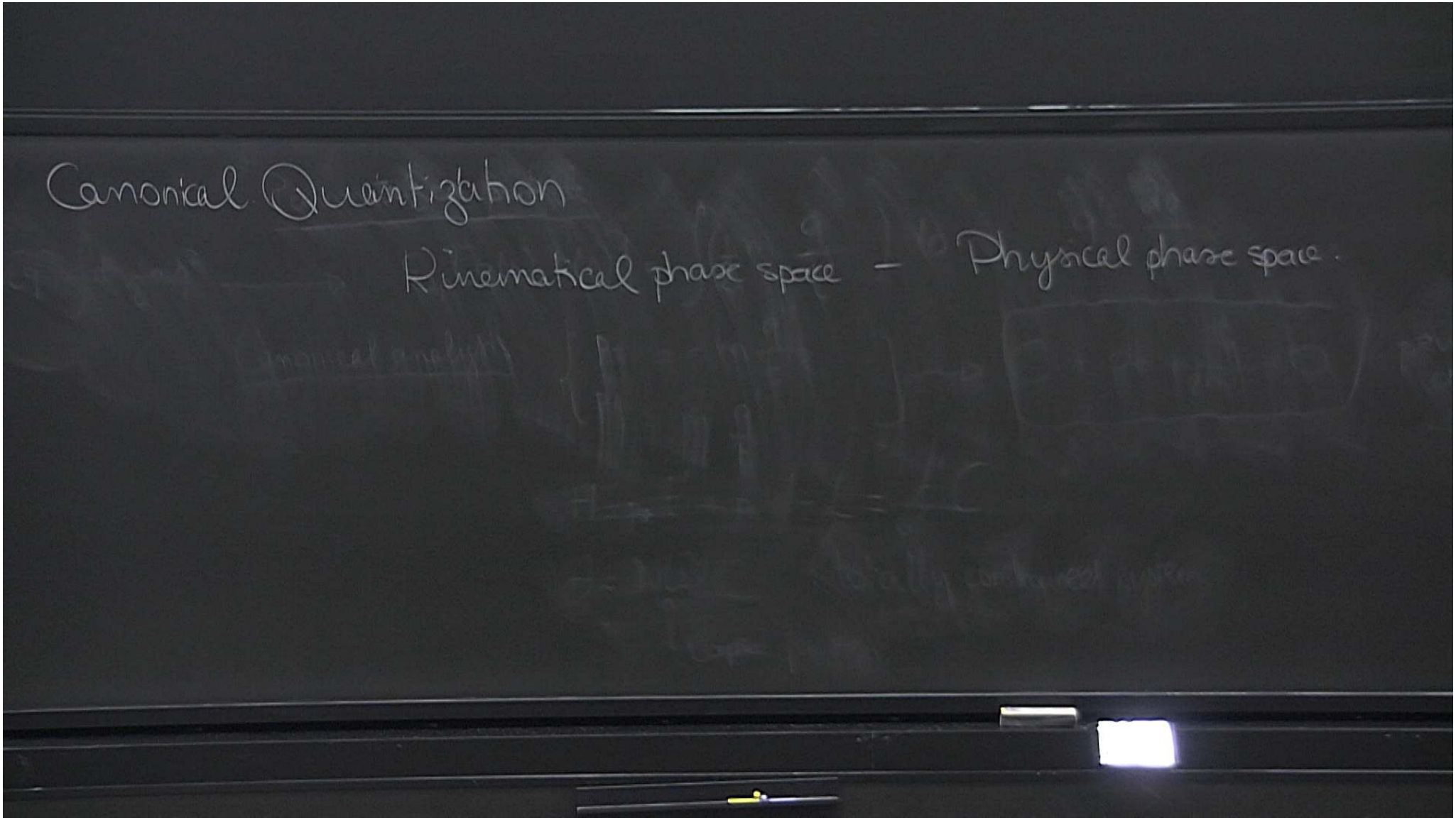
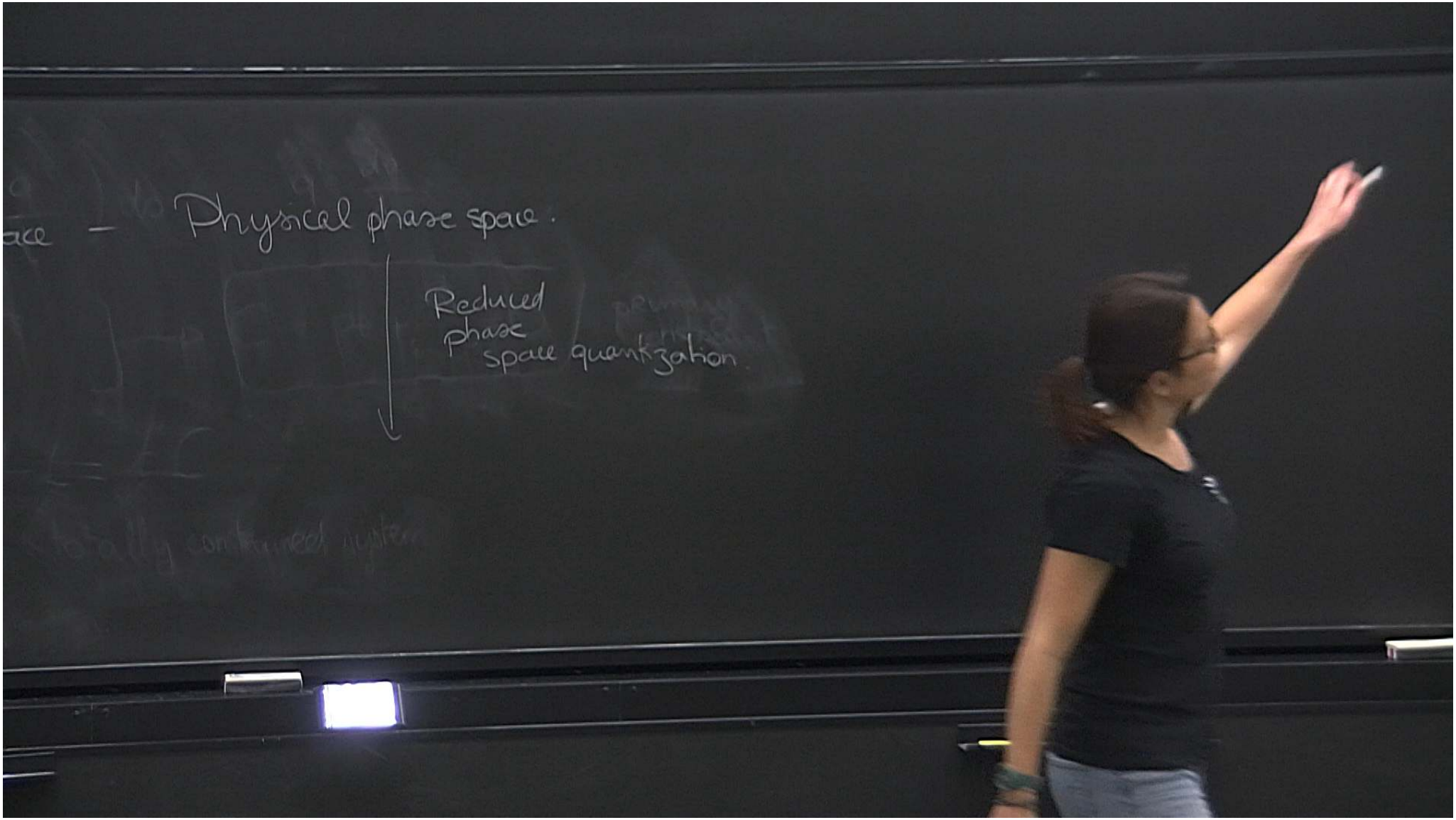
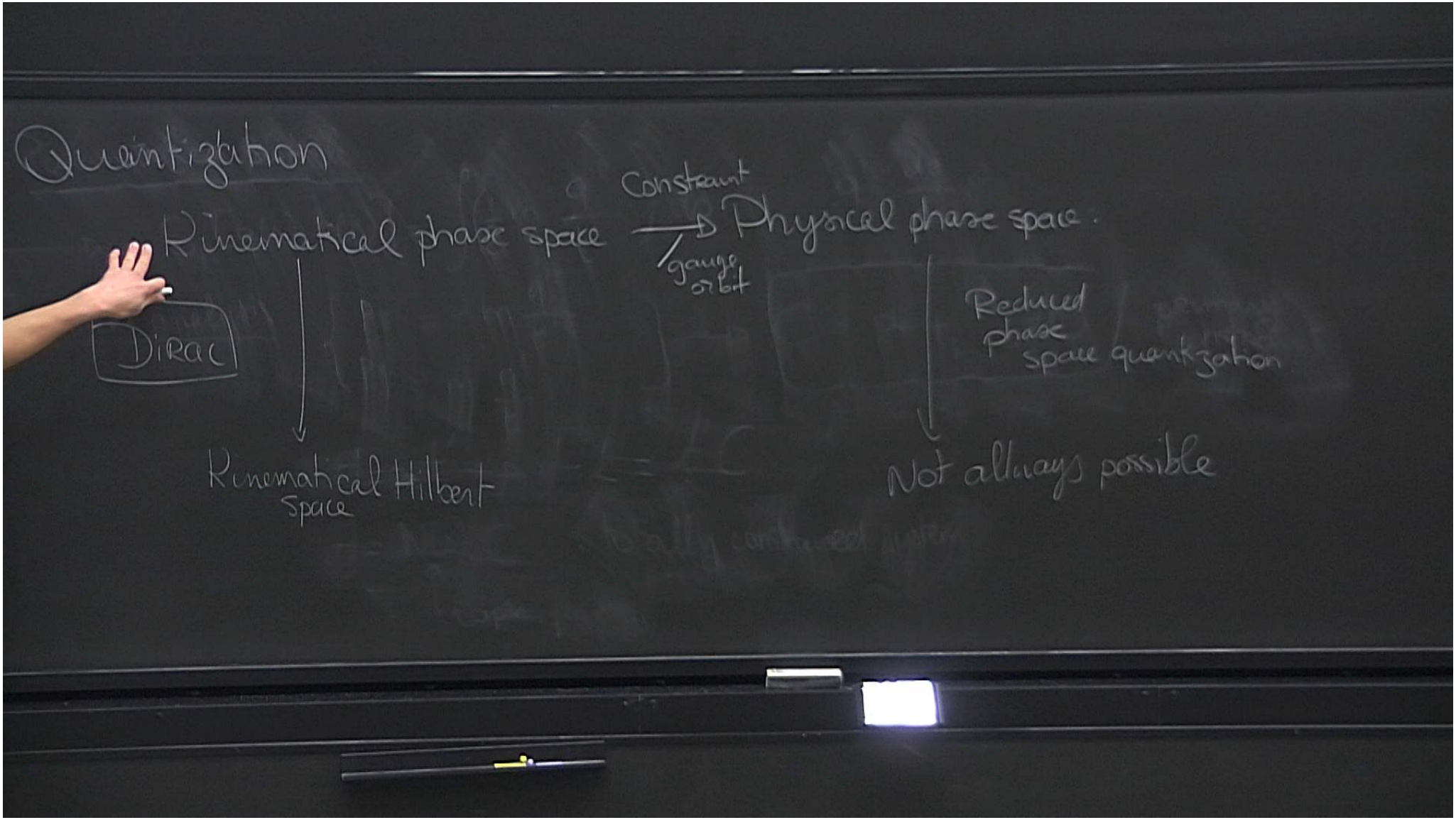
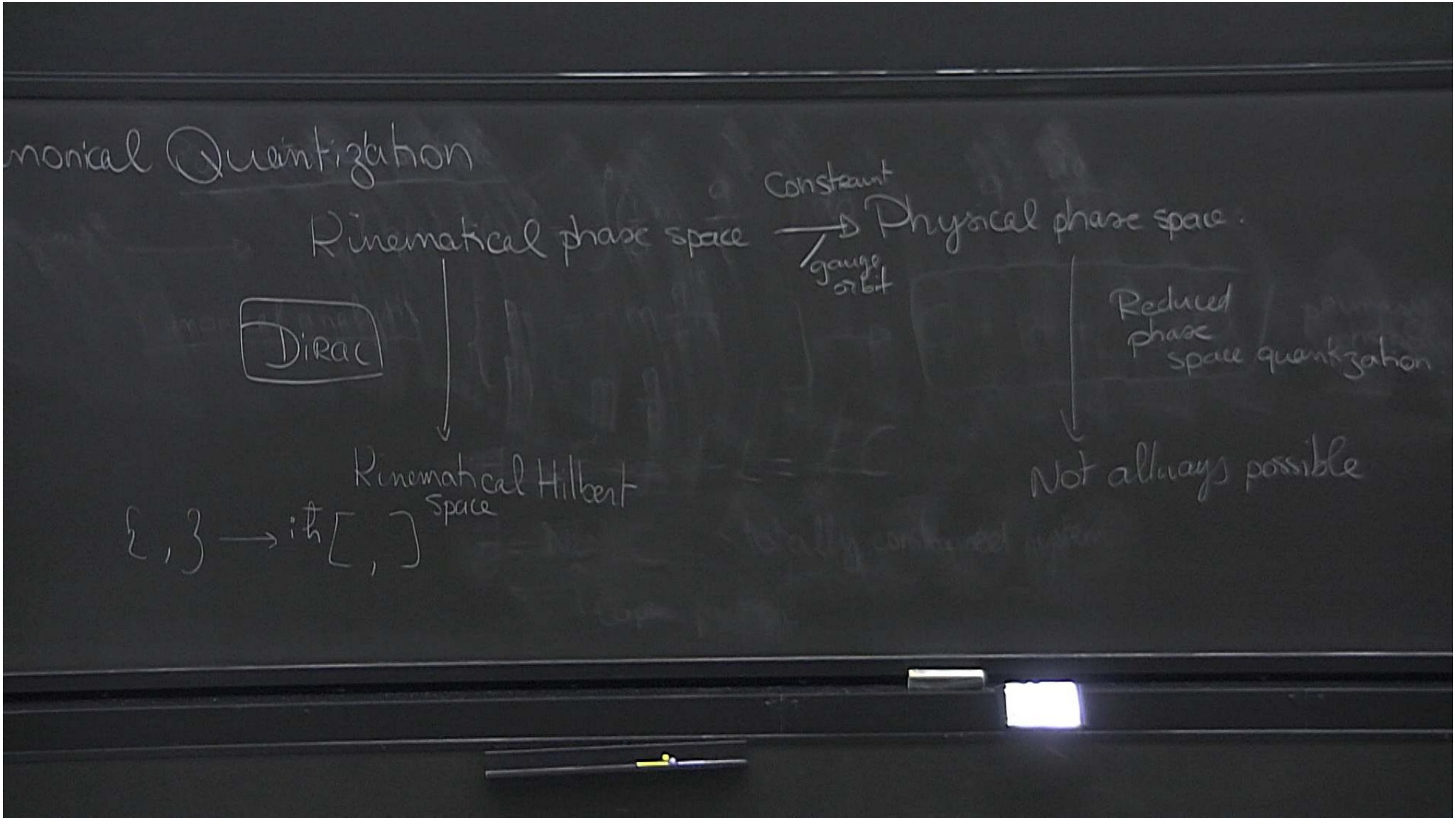
alue of q(s) at the point of the gauge orbit where t(s) takes the value τ
$\Rightarrow F_q^\tau = q + \frac{p_q}{m}(\tau - t)$


associating
orbit where t(s) takes the value τ
$F_q^\tau = q + \frac{p_q}{m}(\tau - t)$
$F_{p_q}^\tau = p_q$


Canonical Quantization
Kinematical phase space – Physical phase space.


Physical phase space.
Reduced
phase
space quantization


Quantization
Kinematical phase space
Constraint
gauge orbit
Physical phase space.
Dirac
Reduced phase space quantization
Kinematical Hilbert space
Not always possible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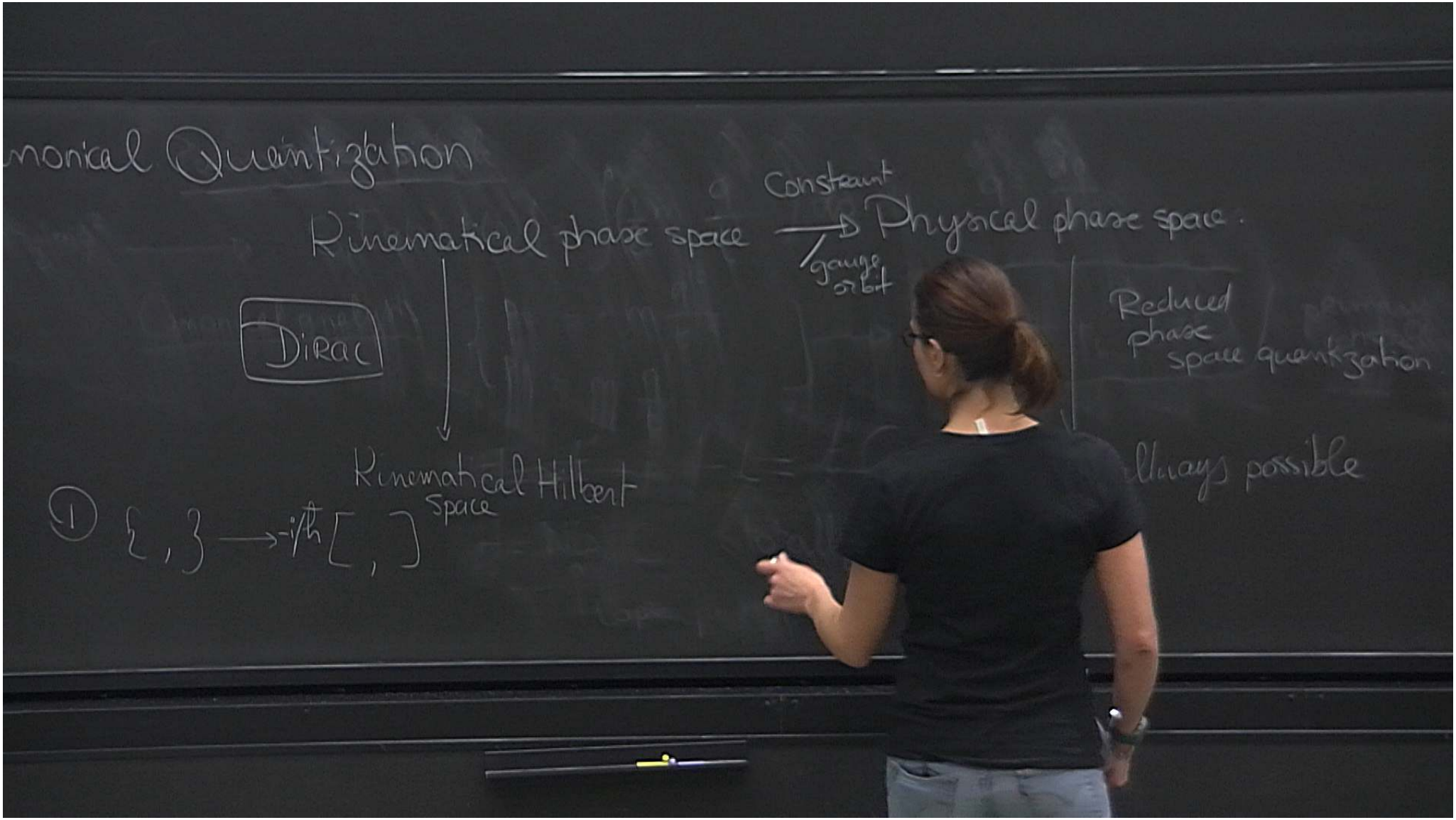
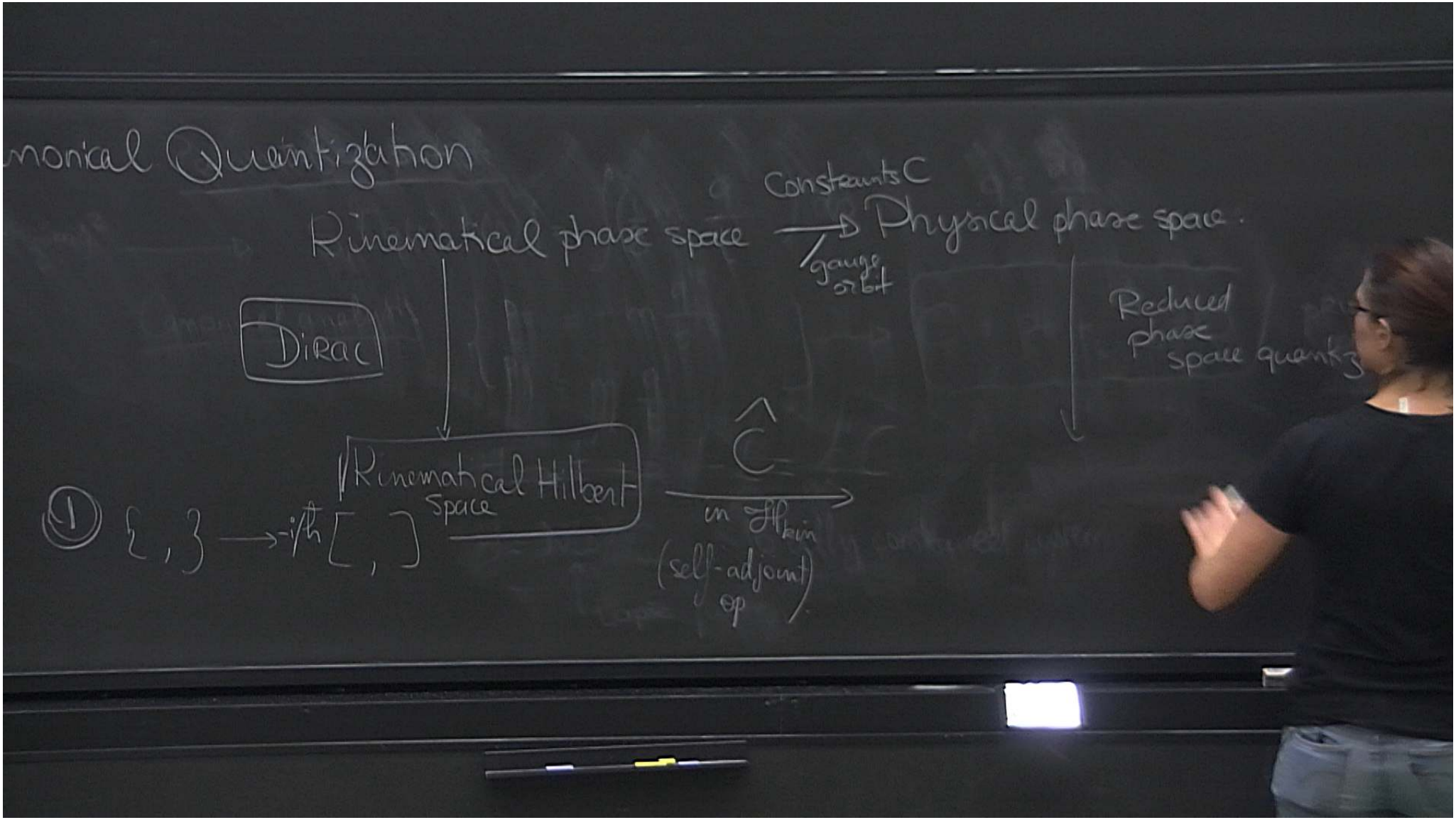
Kinematical phase space
Constraint
Physical phase space.
gauge orbit
Dirac
Reduced phase space quantization
Kinematical Hilbert space
Not allways possible


monical Quantization
Kinematical phase space
Constraint
Physical phase space.
gauge orbit
Dirac
Reduced phase space quantization
Kinematical Hilbert space
always possible
1
{ , } → -i/ħ [,]


monical Quantization
Kinematical phase space
Constraints C
Physical phase space.
gauge orbit
Dirac
Reduced phase space quantis
Kinematical Hilbert space
in $\mathcal{H}_{kin}$
(self-adjoint op)
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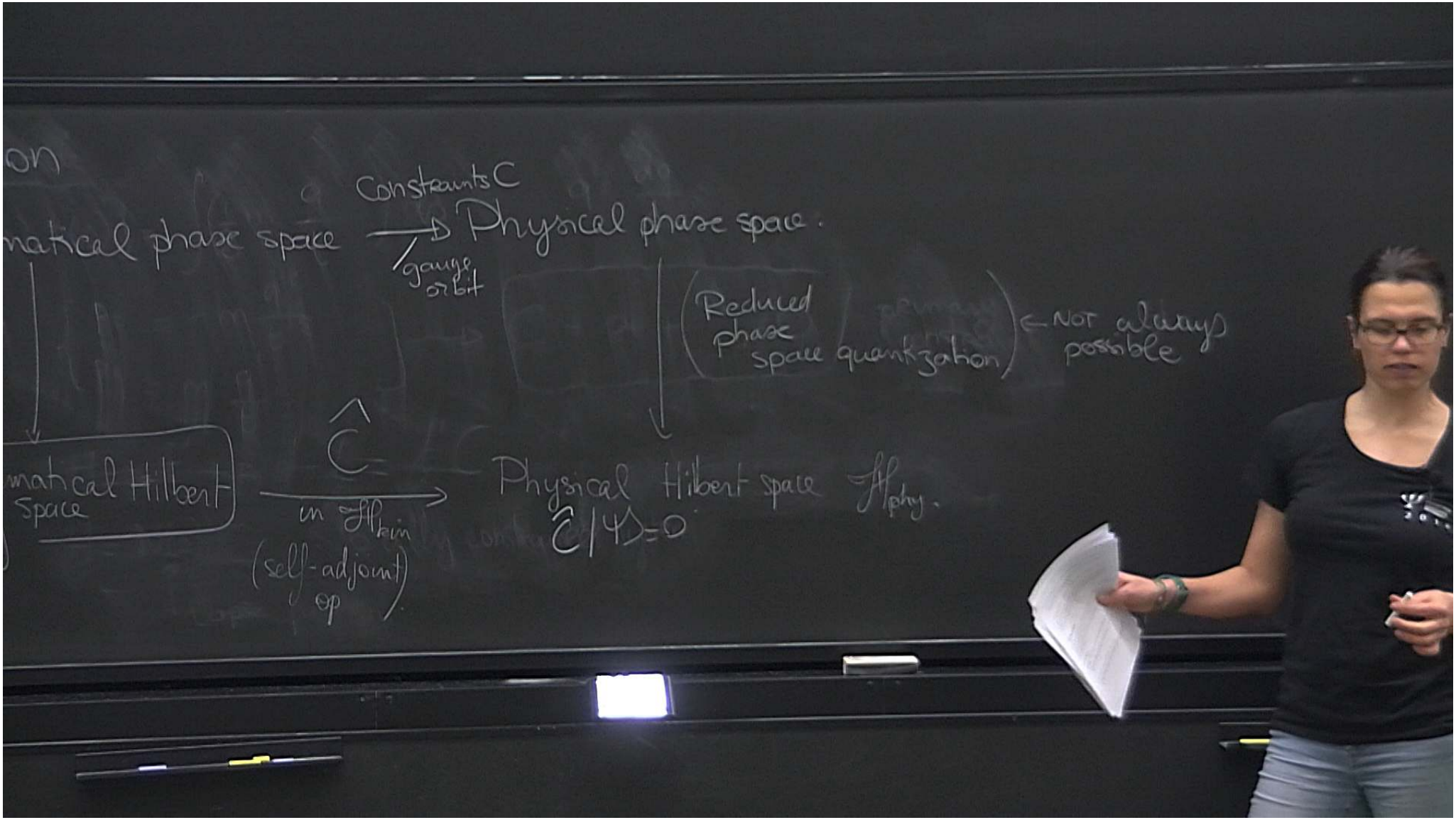
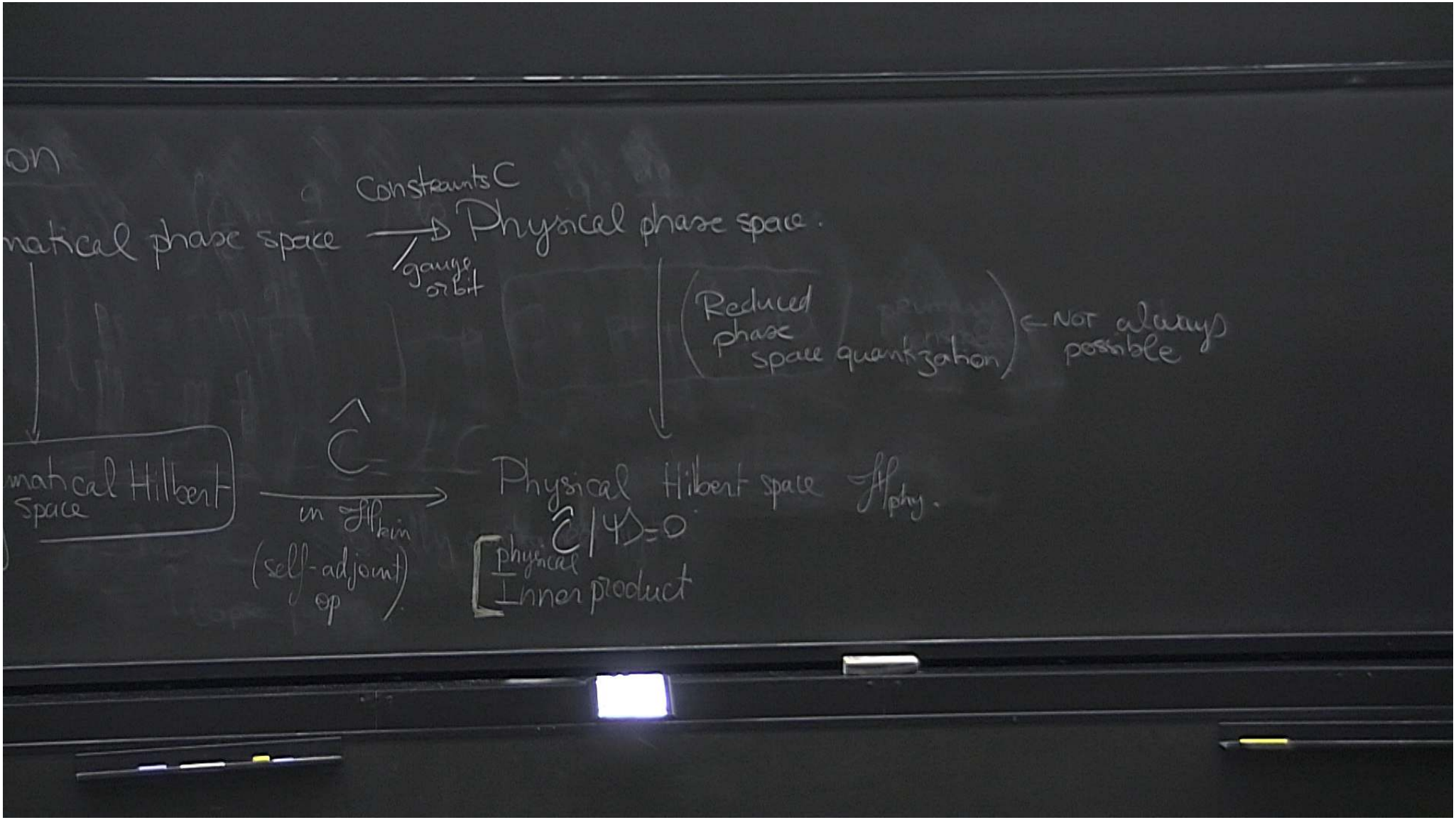
{ , } → -i/ħ [,]


on
Constraints C
matical phase space
Physical phase space.
gauge orbit
Reduced phase space quantization
← NOT always possible
matical Hilbert space
Physical Hilbert space $\mathcal{H}_{phy}$.
$\hat{C}|\psi\rangle = 0$
in $\mathcal{H}_{kin}$
(self-adjoint) op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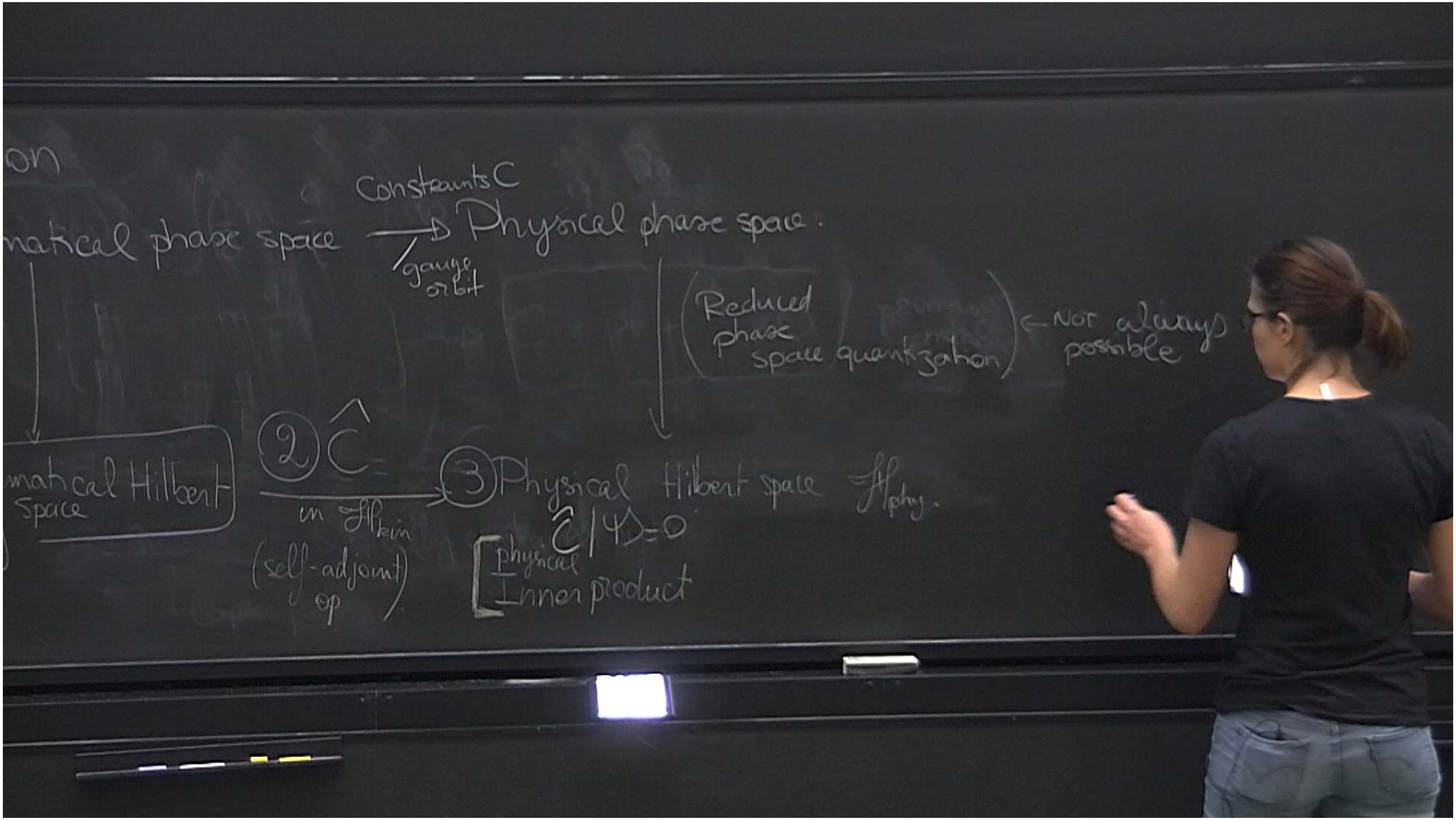
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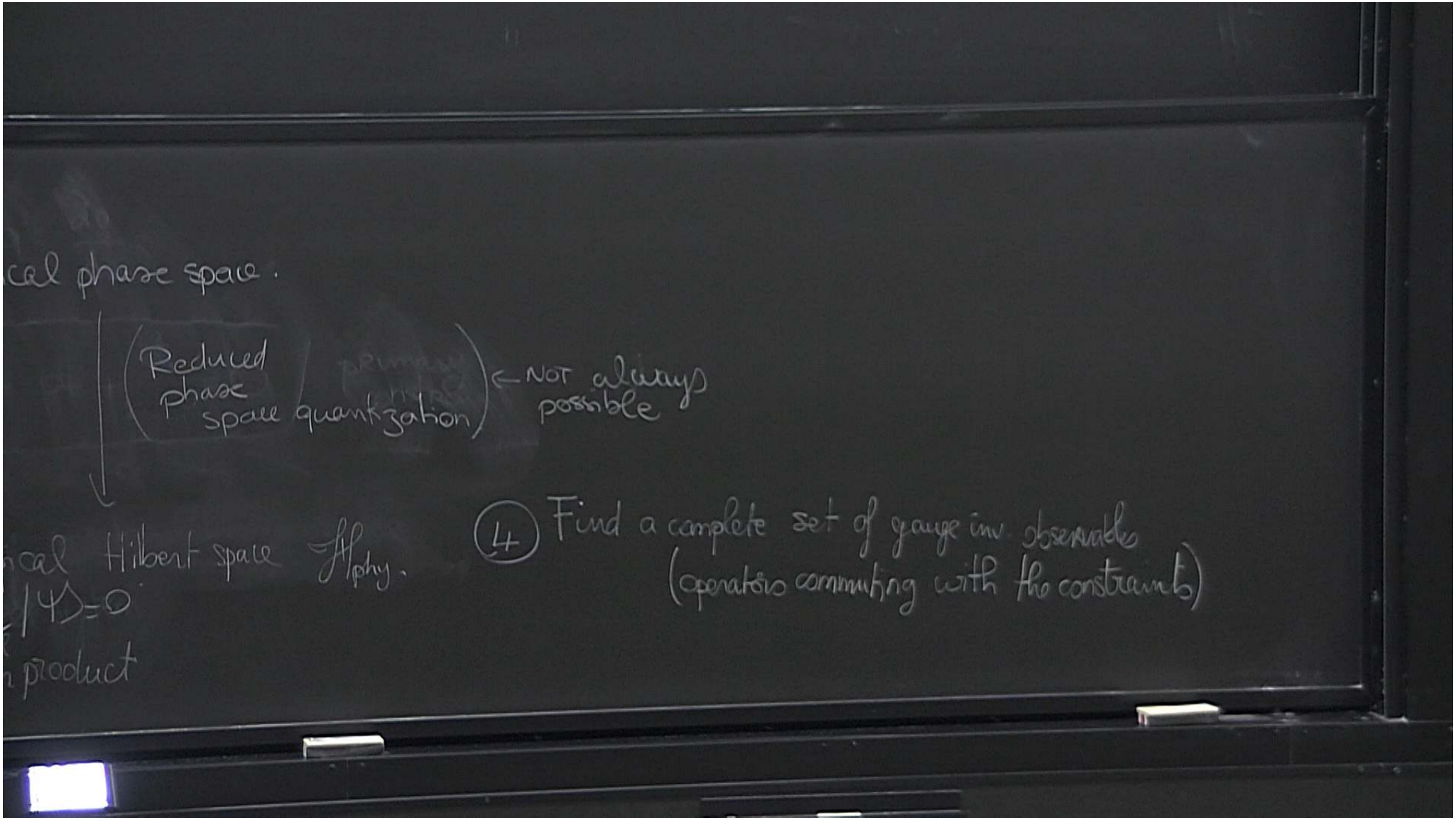
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Reduced phase space quantization
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$|\psi\rangle = 0$
product
(4) Find a complete set of gauge inv. observables
(operators commuting with the constraints)