Title: PSI 2017/2018 - Relativistic Quantum Information - Lecture 9

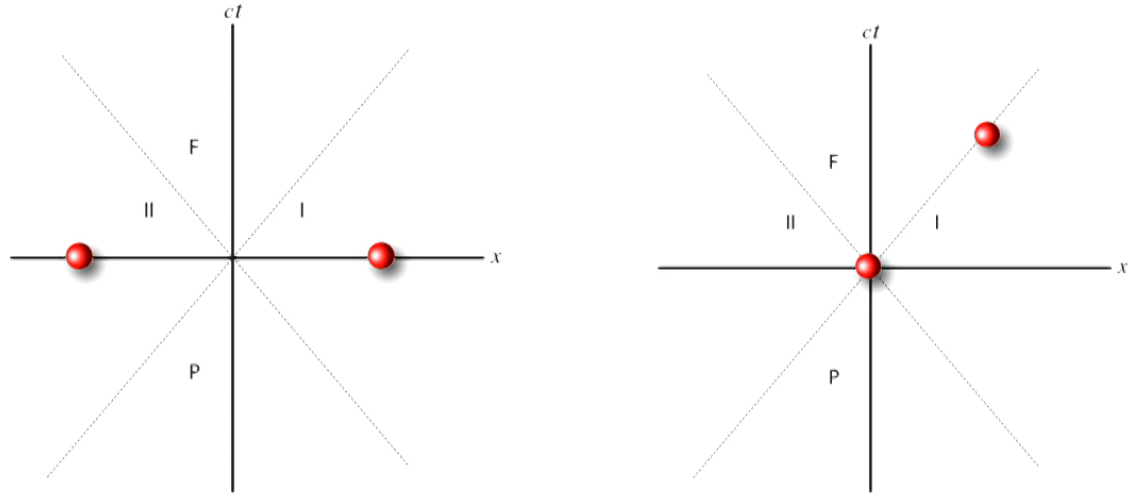
Date: Mar 29, 2018 09:00 AM

URL: http://pirsa.org/18030036

Abstract:

Quantum Fields

A 1D quantum field can be thought as the 'continuum limit' of such a lattice

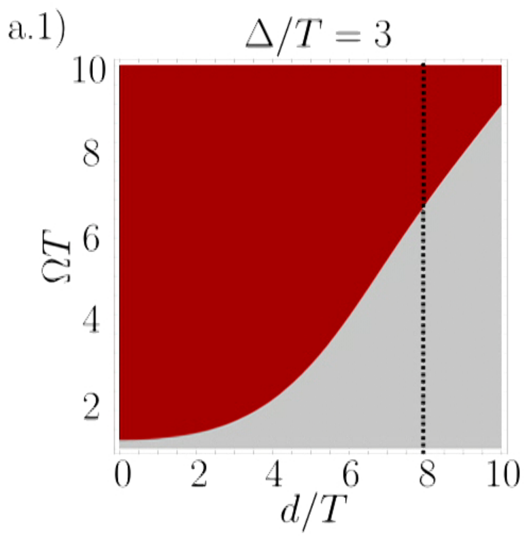


Two mechanisms to get 'atoms' entangled via interaction with quantum fields:

1) Via exchange of real field quanta

2) Swapping vacuum entanglement

Quantum Fields



ΩT Detector Gap (In units of coupling time)

d/T Detector separation (In units of coupling time)

Δ/T Detector time delay (In units of coupling time)

A. Pozas, E. M-M, Phys. Rev. D 92, 064042 (2015)

Can we extract vacuum entanglement?

Scalar fields and Unruh-DeWitt detectors:

- A. Valentini, Phys. Lett. A,153, 321 (1991)
- B. Reznik, Found. Phys. 33, 167 (2003)
- A. Pozas-Kerstjens and E. Martín-Martínez, Phys. Rev. D 92, 064042 (2015)

Electromagnetic fields and atoms:

- A. Pozas-Kerstjens and E. Martín-Martínez, Phys. Rev. D 94, 064074 (2016).

Sensitivity to spacetime geometry:

- G. V. Steeg and N. C. Menicucci, Phys. Rev. D 79, 044027 (2009)

Sensitivity to spacetime topology:

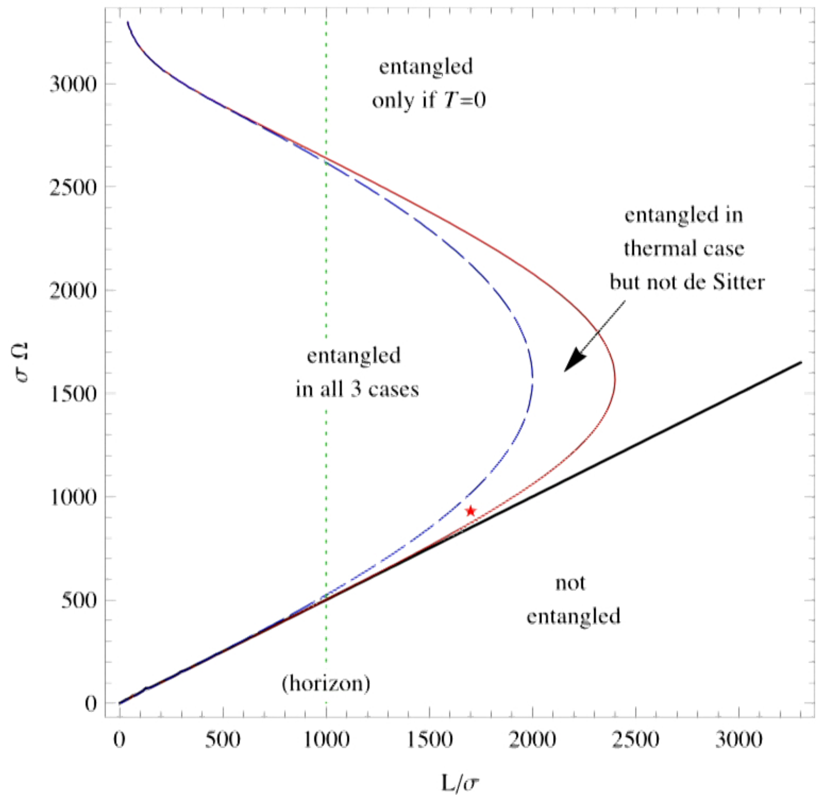
- E. Martín-Martínez, A. R. H. Smith and D. R. Terno, Phys. Rev. D, 93, 044001 (2016)

Experimental proposals:

- S. J. Olson and T. C. Ralph, Phys. Rev. Lett. 106, 110404 (2011).
- C. Sabín, B. Peropadre, M. del Rey & E. Martín-Martínez, Phys. Rev. Lett. 109, 033602 (2012)

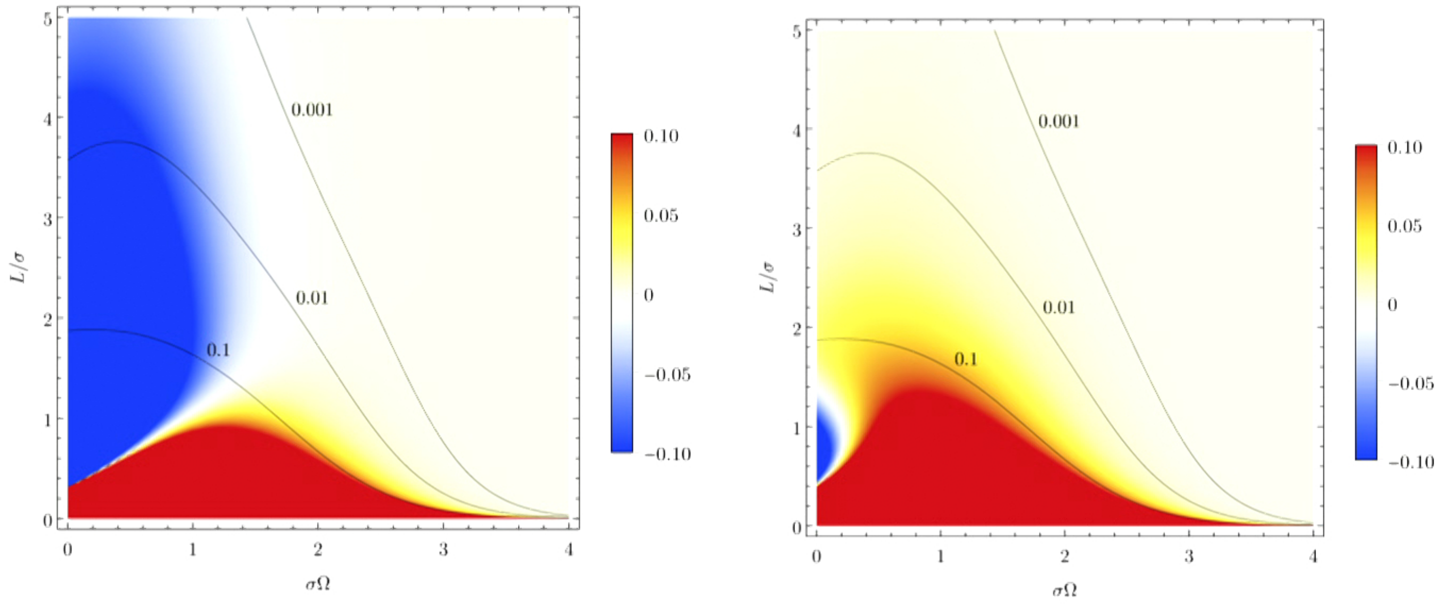
Quantum Fields

dS conformal vacuum Vs Minkowski thermal state



G. V. Steeg, N.C. Menicucci, Phys. Rev. D 79, 044027 (2009)

Quantum Fields



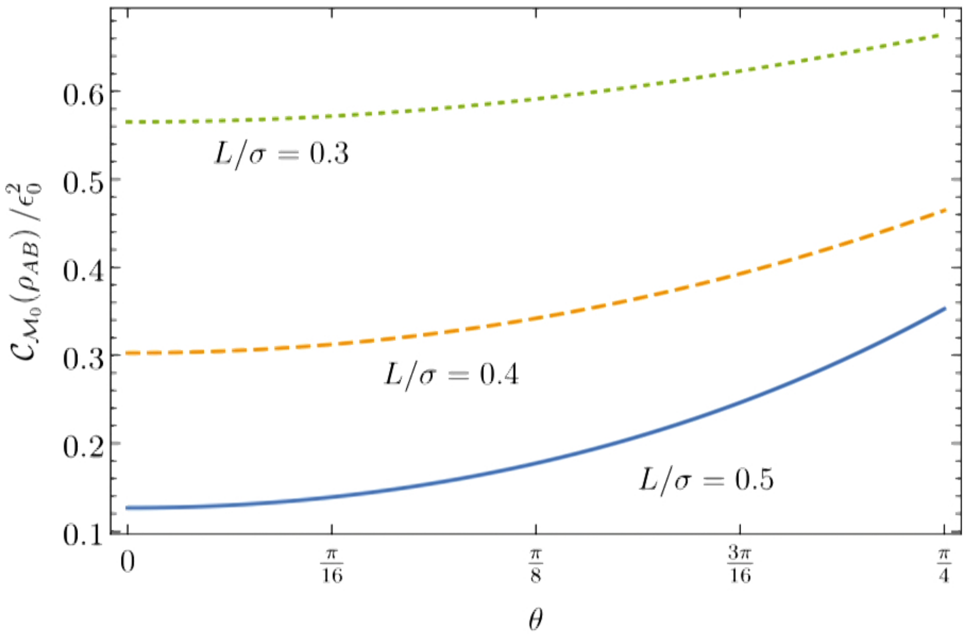
(b) Difference in corr_{AB} between detectors in $\mathcal{M}$ and $\mathcal{M}_-$

(a) Difference in corr_{AB} between detectors in $\mathcal{M}$ and $\mathcal{M}_0$

Sensitive to topology

E. M-M, A. R. H. Smith, D. R. Terno Phys. Rev. D 93, 044001 (2016)

Quantum Fields



Sensitive to topology

E. M-M, A. R. H. Smith, D. R. Terno Phys. Rev. D 93, 044001 (2016)

What are the limits?

Can we repeat the process cyclically?

Is the vacuum entanglement in a cavity replenishable?

Is there a ‘Carnot-like’ optimal extraction cycle?

Can we do it sustainably and realiably?

Not with the swapping mechanism alone…

Entanglement resources get exhausted: Entropy increase: Heating, mixedness,…

E. Martin-Martinez, E. G. Brown, W. Donnelly, A. Kempf. Phys. Rev. A 88, 052310 (2013)

Can we do it sustainably and realiably?

Not with the swapping mechanism alone…

…But yes combining swapping and communication

E. Martin-Martinez, E. G. Brown, W. Donnelly, A. Kempf. Phys. Rev. A 88, 052310 (2013)

How do we do it?

Requirement: Go beyond the usual approximations in Quantum Optics.

The Light-Matter interaction

In a fully relativistic approach, the usual approximations break down

-Rotating Wave Approximation

-Single Mode Approximation

-Perturbative Approximation

There are effects not predicted by the approximated theory:

Example: “Dynamical Casimir Effect” Chris Wilson et al. Nature 479, 376–379, 2011

Let us get some insight into these approximations

The usual 'non-relativistic' Q.O. fails

-Rotating wave approximation ✗

-Single (or few) mode approximation ✗

-Perturbation Theory ✗

How do we do non-perturbative calculations in Q.O.?

- Harmonic model.
- Gaussian methods
- Delta-coupling

Gaussian methods in QM

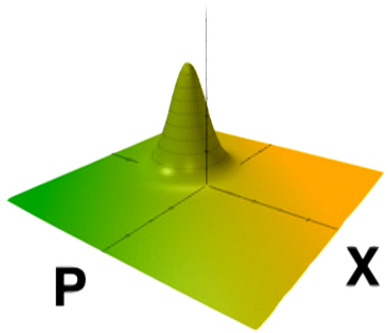
Quantum Mechanics is computationally difficult

- Set of N harmonic oscillators
- Density operators are infinite dimensional

Not the whole Hilbert space is needed here.

Gaussian States

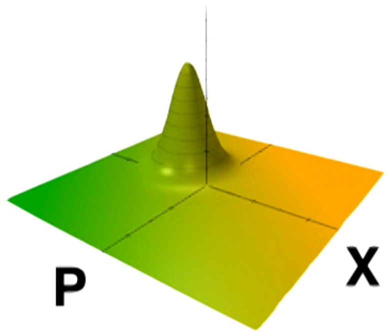
G.S. are states whose Wigner function is Gaussian



- Thermal states
- Coherent states
- Squeezed states
- Squeezed thermal states

Gaussian States

G.S. are states whose Wigner function is Gaussian



- Thermal states
- Coherent states
- Squeezed states
- Squeezed thermal states
- The vacuum state

Gaussian States

A zero mean Gaussian state can be characterized by its first and second moments

$$\sigma_{ij} \equiv \langle \hat{x}_i \hat{x}_j + \hat{x}_j \hat{x}_i \rangle - 2\langle \hat{x}_i \rangle \langle \hat{x}_j \rangle$$

Set of M+N harmonic oscillators

$$\hat{\mathbf{x}} := (\hat{q}_{d_1}, \ldots, \hat{q}_{d_M}, \hat{q}_1, \ldots, \hat{q}_N, \hat{p}_{d_1}, \ldots, \hat{p}_{d_M}, \hat{p}_1, \ldots, \hat{p}_N)^{\mathrm{T}} ,$$

$$\hat{q}_i = \frac{1}{\sqrt{2}}(\hat{a}_i + \hat{a}_i^\dagger), \quad \hat{p}_i = \frac{\mathrm{i}}{\sqrt{2}}(\hat{a}_i^\dagger - \hat{a}_i)$$

Gaussian Evolution

Evolution under quadratic Hamiltonians

$$\hat{H} = \hat{\mathbf{x}}^{\mathrm{T}}\mathbf{F}(t)\hat{\mathbf{x}} = (\hat{\mathbf{a}}^{\dagger})^{\mathrm{T}}\mathbf{w}(t)\hat{\mathbf{a}} + (\hat{\mathbf{a}}^{\dagger})^{\mathrm{T}}\mathbf{g}(t)\hat{\mathbf{a}}^{\dagger} + \hat{\mathbf{a}}^{\mathrm{T}}\mathbf{g}(t)^{\mathrm{H}}\hat{\mathbf{a}}$$

Preserves Gaussianity

$$\hat{\mathbf{a}} := (\hat{a}_{d_1}, \ldots, \hat{a}_{d_M}, \hat{a}_1, \ldots \hat{a}_N)^{\mathrm{T}},$$
$$\hat{\mathbf{a}}^{\dagger} := (\hat{a}^{\dagger}_{d_1}, \ldots, \hat{a}^{\dagger}_{d_M}, \hat{a}^{\dagger}_1, \ldots \hat{a}^{\dagger}_N)^{\mathrm{T}}.$$

$$\hat{\mathbf{x}} := (\hat{q}_{d_1}, \ldots, \hat{q}_{d_M}, \hat{q}_1, \ldots, \hat{q}_N, \hat{p}_{d_1}, \ldots, \hat{p}_{d_M}, \hat{p}_1, \ldots, \hat{p}_N)^{\mathrm{T}} ,$$
$$\hat{q}_i = \frac{1}{\sqrt{2}}\big(\hat{a}_i + \hat{a}_i^{\dagger}\big), \quad \hat{p}_i = \frac{\mathrm{i}}{\sqrt{2}}\big(\hat{a}_i^{\dagger} - \hat{a}_i\big)$$

E. G. Brown, E. Martín-Martínez, N. C. Menicucci, R. B. Mann. Phys. Rev. D 87, 084062 (2013)

Gaussian Evolution (no displacements)

Unitary transformations in Hilbert space

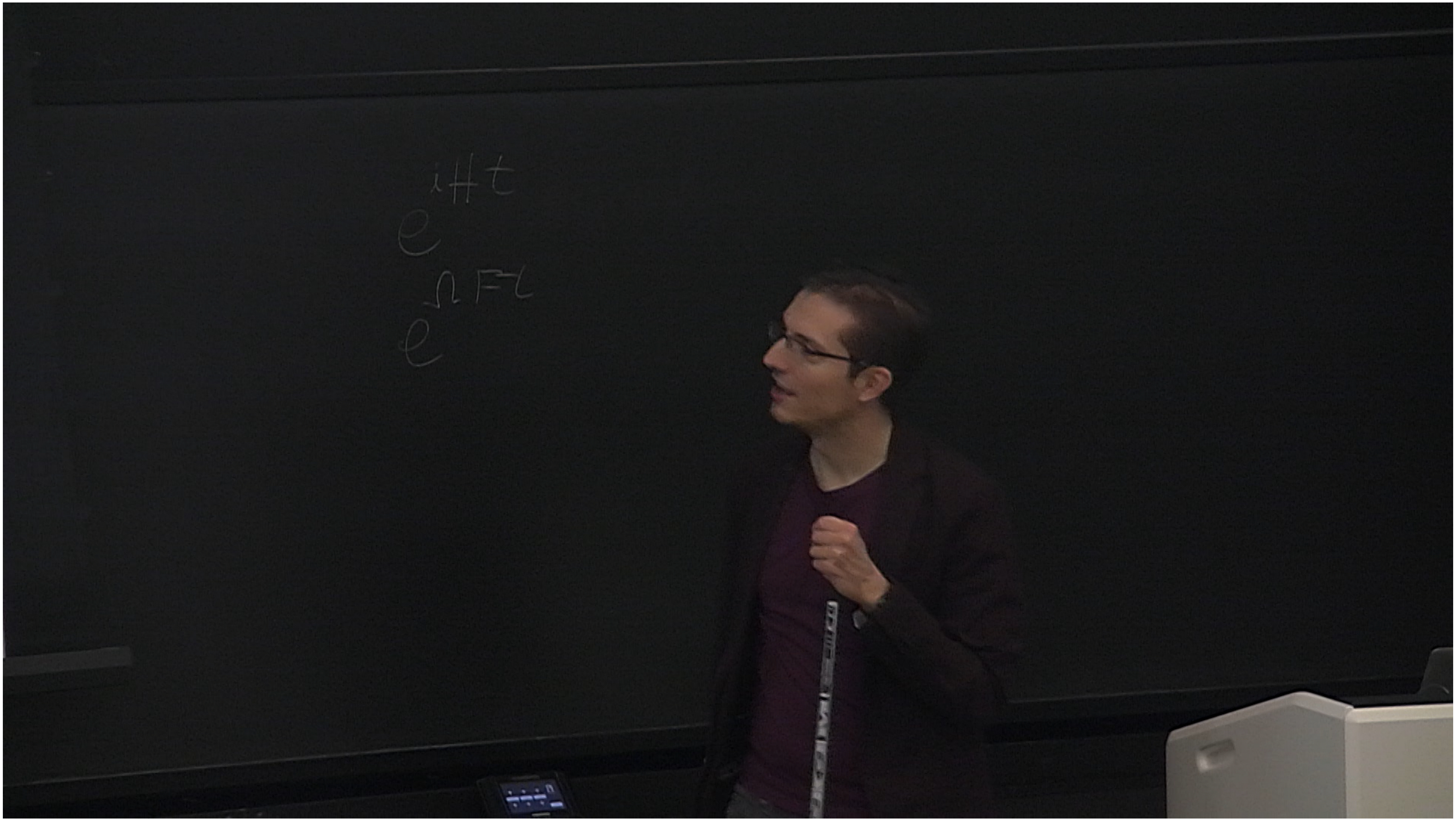
$$\hat{\mathbf{x}}(t) = \hat{U}^\dagger(t)\,\hat{\mathbf{x}}_0\,\hat{U}(t) = \mathbf{S}(t)\,\hat{\mathbf{x}}_0$$

Symplectic transformations in phase space

$$\boldsymbol{\sigma}(t) = \mathbf{S}(t)\boldsymbol{\sigma}_0\mathbf{S}(t)^{\mathrm{T}},$$

$$\frac{d}{dt}\hat{\mathbf{x}}(t) = i\big[\hat{H}(t), \hat{\mathbf{x}}(t)\big] \Rightarrow \quad \frac{d}{dt}\mathbf{S}(t) = \mathbf{\Omega}\mathbf{F}^{\mathrm{sym}}(t)\mathbf{S}(t).$$

$$[\hat{\mathbf{x}}, \hat{\mathbf{x}}^{\mathrm{T}}] = i\begin{pmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{I} & \mathbf{0} \end{pmatrix} =: i\mathbf{\Omega} \qquad \mathbf{F}^{\mathrm{sym}} = (\mathbf{F} + \mathbf{F}^{\mathrm{T}})$$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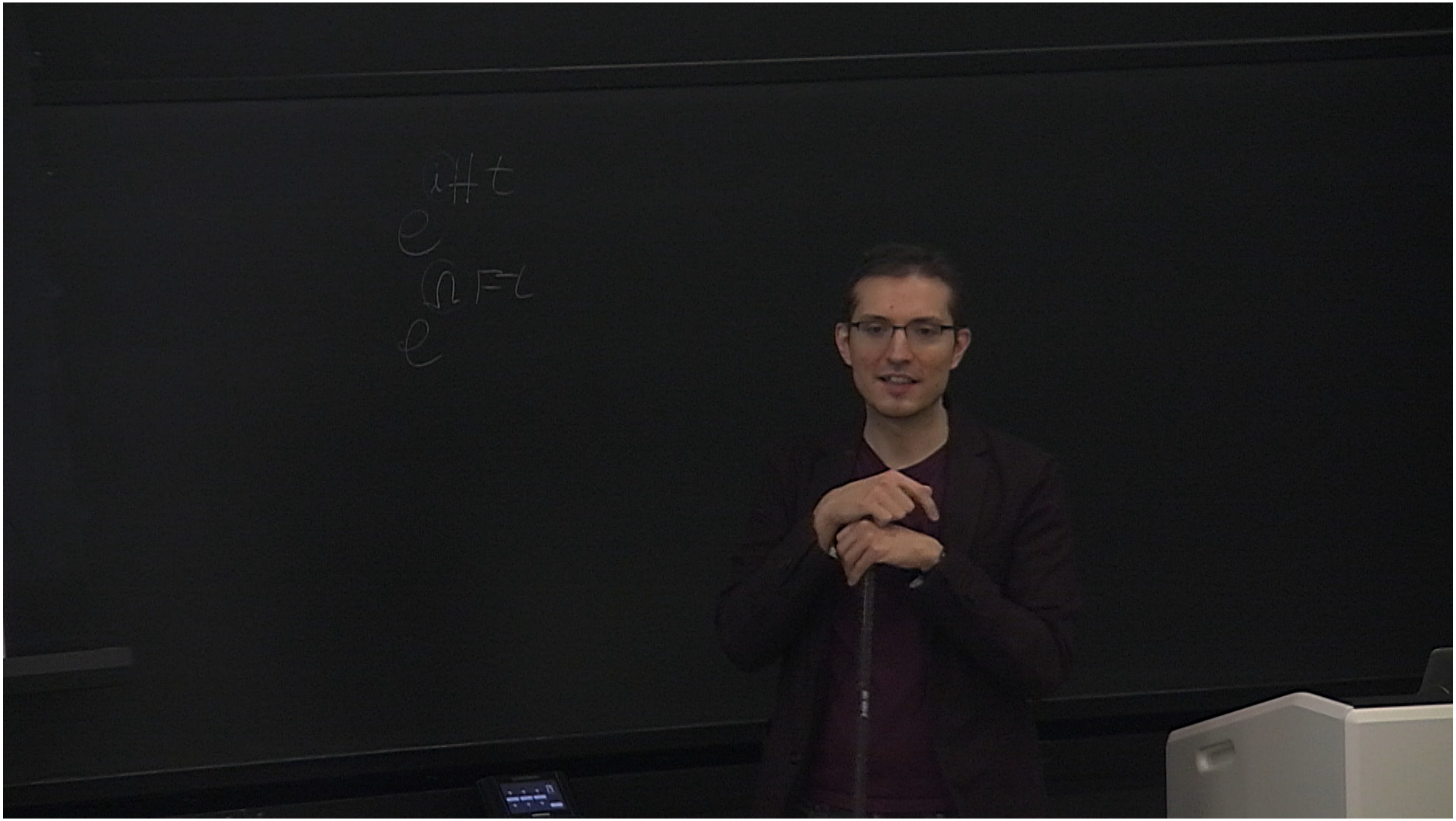
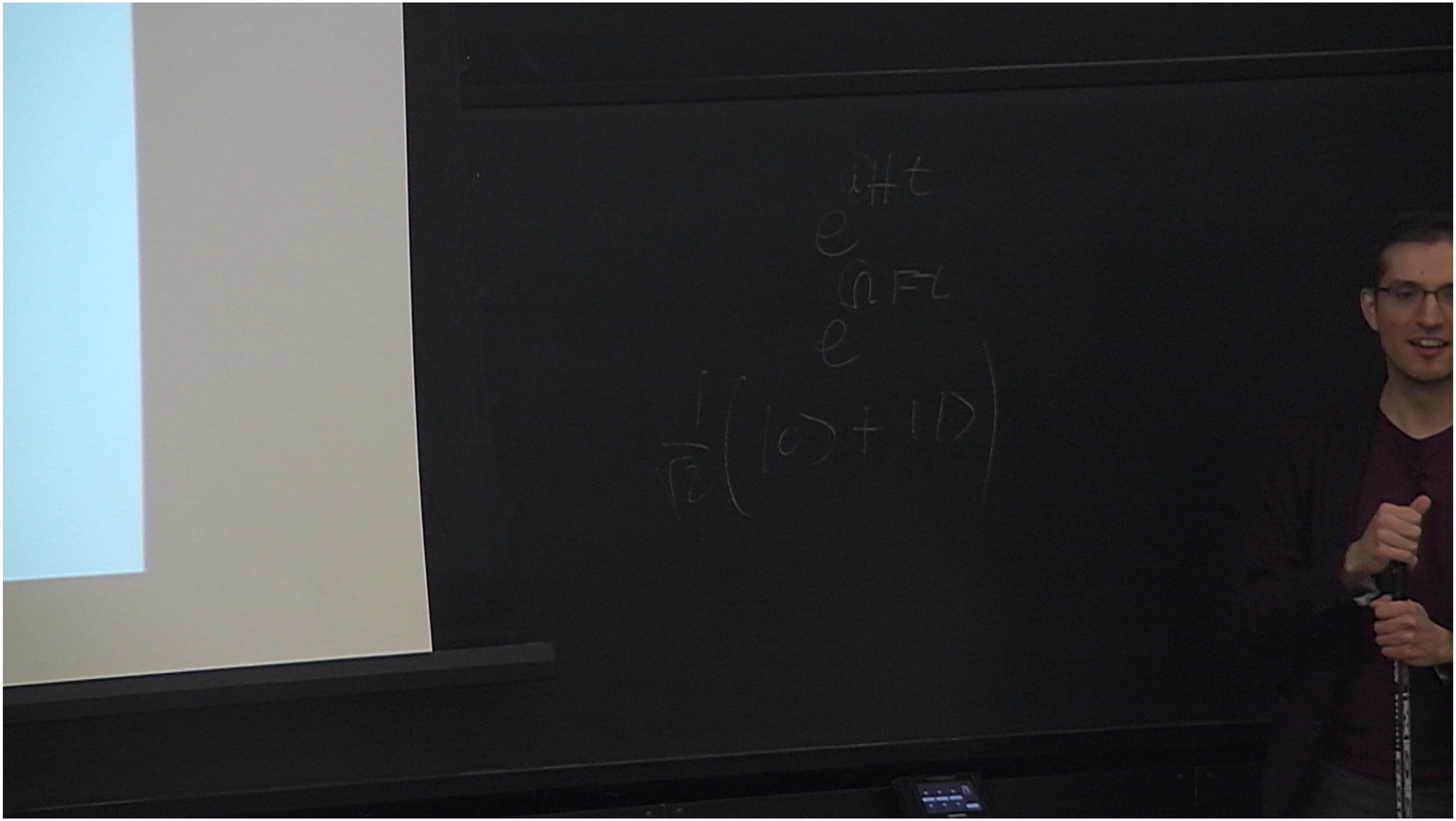




Applied to the light-matter interaction?

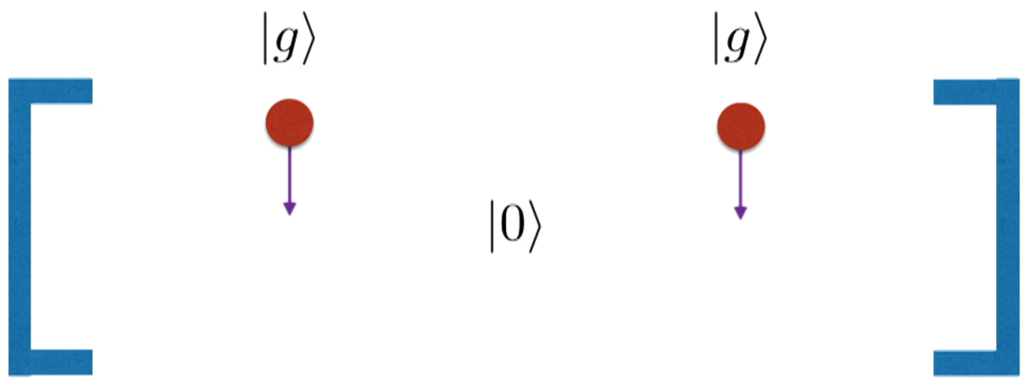
The first M harmonic oscillators are atoms

The next $N \to \infty$ harmonic oscillators are modes of a quantum field in an optical cavity

- Non-perturbative
- Generically time-dependent problems
- Ideal for relativistic approaches
- Ideal for cases where you need to relax approximations

Towards Entanglement Farming

Let us consider the following setting:



Two atoms going through an optical cavity prepared in the vacuum

E. Martin-Martinez, E. G. Brown, W. Donnelly, A. Kempf. Phys. Rev. A 88, 052310 (2013)

Towards Farming

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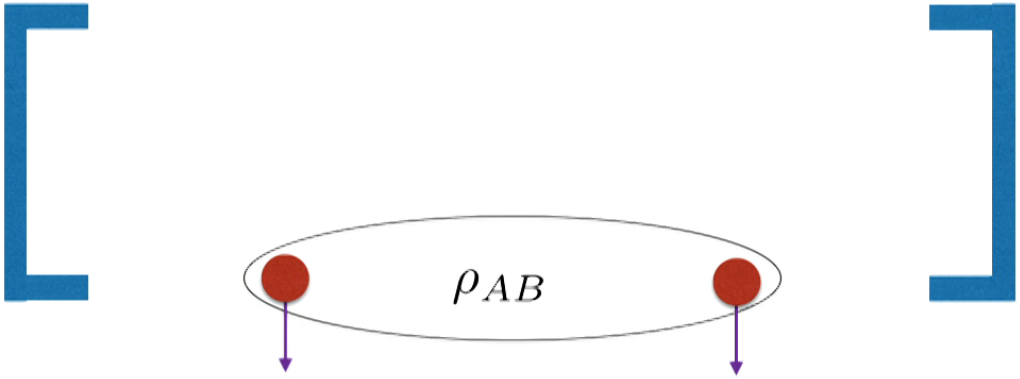


The atoms get slightly entangled

If they spend enough time, both mechanisms in place

Still the effect is non-rotating wave

E. Martin-Martinez, E. G. Brown, W. Donnelly, A. Kempf. Phys. Rev. A 88, 052310 (2013)

Towards Farming

Let us consider the following setting:

$$\rho_{AB} = \sum_i p_i \, \rho_A \otimes \rho_B$$

The atoms will NOT get entangled

Too much 'local' noise

E. Martin-Martinez, E. G. Brown, W. Donnelly, A. Kempf. Phys. Rev. A 88, 052310 (2013)

Towards Farming

It would seem this setting is not so great to get atoms entangled

Not robust under finite temperatures:

The amount of entanglement extracted **vanishes** quickly as the temperature increases

What if we repeat the process iteratively?

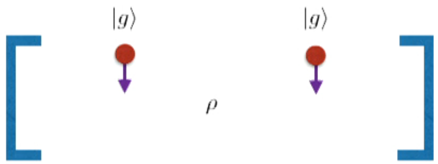
We send many pairs of atoms initialized in the ground state

Let us analyze the dynamics of this process

E. Martin-Martinez, E. G. Brown, W. Donnelly, A. Kempf. Phys. Rev. A 88, 052310 (2013)

Towards Farming

Step 1a: field in arbitrary state. Atoms in the ground state



$$\sigma_0 = \sigma_{A,g} \oplus \sigma_{B,g} \oplus \sigma_{\text{f}}$$

E. Martin-Martinez, E. G. Brown, W. Donnelly, A. Kempf. Phys. Rev. A 88, 052310 (2013)

Towards Farming

Step 1a: field in arbitrary state. Atoms in the ground state

$$\sigma_0 = \sigma_{A,g} \oplus \sigma_{B,g} \oplus \sigma_{\mathrm{f}}$$

$$\downarrow$$

$$\sigma_1 = \mathbf{S}\sigma_0\mathbf{S}^{\mathrm{T}} = \begin{pmatrix} \sigma_{A,1} & I_{AB} & I_{AF} \\ I^*_{AB} & \sigma_{B,1} & I_{BF} \\ I^*_{AF} & I^*_{BF} & \sigma_{f,1} \end{pmatrix}$$

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Step 1b: Remove the atoms and prepare a fresh pair

E. Martin-Martinez, E. G. Brown, W. Donnelly, A. Kempf. Phys. Rev. A 88, 052310 (2013)

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Step 1b: Remove the atoms and prepare a fresh pair

$$\sigma_{1\mathrm{b}} = \begin{pmatrix} \sigma_{A,g} & 0 & 0 \\ 0 & \sigma_{B,g} & 0 \\ 0 & 0 & \sigma_{f,1} \end{pmatrix}$$

E. Martin-Martinez, E. G. Brown, W. Donnelly, A. Kempf. Phys. Rev. A 88, 052310 (2013)

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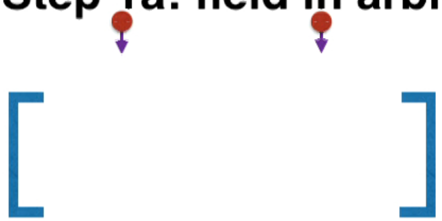

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Step 1b: Remove the atoms and prepare a fresh pair

$$\sigma_{1\mathrm{b}} = \begin{pmatrix} \sigma_{A,g} & 0 & 0 \\ 0 & \sigma_{B,g} & 0 \\ 0 & 0 & \sigma_{f,1} \end{pmatrix}$$

Step 2: Repeat the process

$$\downarrow$$

$$\sigma_2 = \mathbf{S}\sigma_{1\mathrm{b}}\mathbf{S}^{\mathrm{T}}$$

Step 3: Iterate to obtain $\sigma_3, \sigma_4, \sigma_5, \ldots$

E. Martin-Martinez, E. G. Brown, W. Donnelly, A. Kempf. Phys. Rev. A 88, 052310 (2013)

Fixed point

Is there a fixed point in this iterative process?

Consider the symplectic matrix $$\mathbf{S} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}$$

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Fixed point

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Consider the symplectic matrix $\mathbf{S} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}$

The partial state of the field at the step k+1 becomes

$$\boldsymbol{\sigma}_{f,(k+1)} = \mathbf{D}\boldsymbol{\sigma}_{f,k}\mathbf{D}^T + \mathbf{C}\mathbf{C}^T.$$

Recast the problem in a more familiar form

$$\mathbf{v}^{(k+1)} = (\mathbf{D} \otimes \mathbf{D})\mathbf{v}^{(k)} + \mathbf{c}$$

E. Martin-Martinez, E. G. Brown, W. Donnelly, A. Kempf. Phys. Rev. A 88, 052310 (2013)

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Recast the problem in a more familiar form

$$\mathbf{v}^{(k+1)} = (\mathbf{D}\otimes\mathbf{D})\mathbf{v}^{(k)} + \mathbf{c}$$

If the eigenvalues of D are within the unit circle, there is a fixed point

$$\mathbf{v}_{\text{fixed}} = (\mathbf{I}-\mathbf{D}\otimes\mathbf{D})^{-1}\mathbf{c}.$$

E. Martin-Martinez, E. G. Brown, W. Donnelly, A. Kempf. Phys. Rev. A 88, 052310 (2013)

Fixed point

- **How common is that fixed point?**

  If the interaction time is long enough there always exists a fixed point

E. Martin-Martinez, E. G. Brown, W. Donnelly, A. Kempf. Phys. Rev. A 88, 052310 (2013)

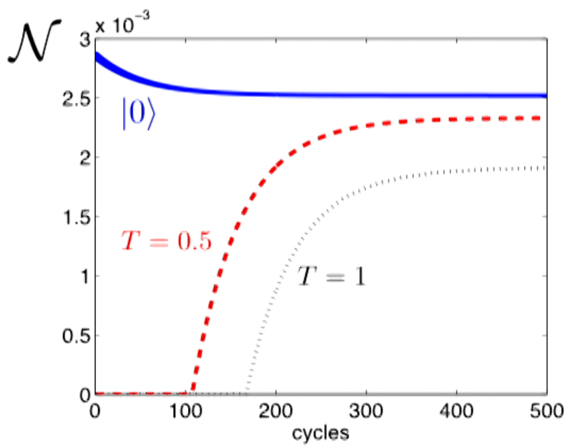
Fixed point

- How common is that fixed point?

  If the interaction time is long enough there always exists a fixed point

- Can we extract entanglement from that fixed point?
- How fast do we go towards the fixed point depending on the initial state?

Entanglement can be extracted!



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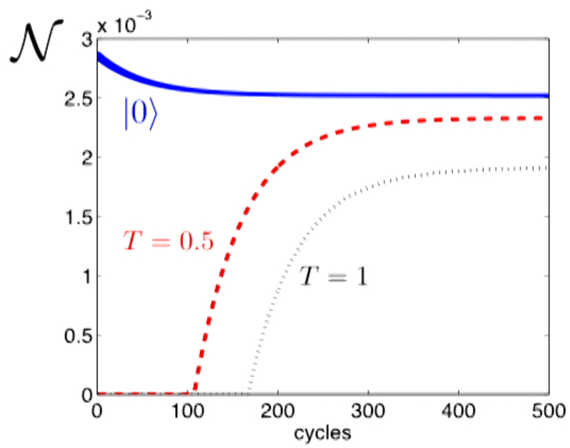

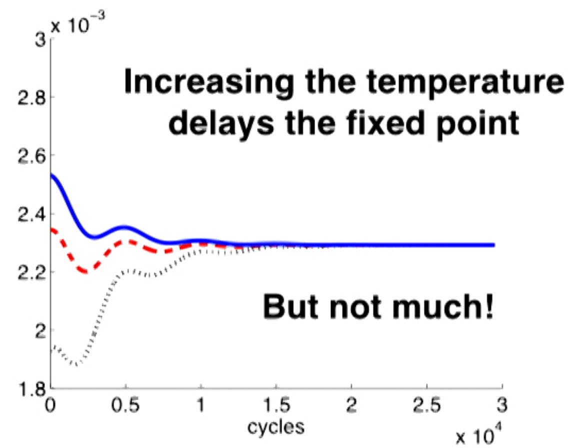

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The usual 'non-relativistic' Q.O. fails

-Rotating wave approximation ✗

-Single (or few) mode approximation ✗

The usual 'non-relativistic' Q.O. fails

- -Rotating wave approximation ✗
- -Single (or few) mode approximation ✗

-Perturbation Theory

Non-perturbative / Non-RWA / Non-SM Relativistic light-matter interaction

PHYSICAL REVIEW D **87,** 084062 (2013)

Detectors for probing relativistic quantum physics beyond perturbation theory

Eric G. Brown,[1] Eduardo Martín-Martínez,[1,2,3] Nicolas C. Menicucci,[4] and Robert B. Mann[1,3]

We develop a general formalism for a nonperturbative treatment of harmonic-oscillator particle detectors in relativistic quantum field theory using continuous-variable techniques. By means of this we forgo perturbation theory altogether and reduce the complete dynamics to a readily solvable set of first-order, linear differential equations. The formalism applies unchanged to a wide variety of physical setups, including arbitrary detector trajectories, any number of detectors, arbitrary time-dependent quadratic couplings, arbitrary Gaussian initial states, and a variety of background spacetimes. As a first set of concrete results, we prove nonperturbatively—and without invoking Bogoliubov transformations—that an accelerated detector in a cavity evolves to a state that is very nearly thermal with a temperature proportional to its acceleration, allowing us to discuss the universality of the Unruh effect. Additionally we quantitatively analyze the problems of considering single-mode approximations in cavity field theory and show the emergence of causal behavior when we include a sufficiently large number of field modes in the analysis. Finally, we analyze how the harmonic particle detector can harvest entanglement from the vacuum. We also study the effect of noise in time-dependent problems introduced by suddenly switching on the interaction versus ramping it up slowly (adiabatic activation).