Title: PSI 2017/2018 - Relativistic Quantum Information - Lecture 8

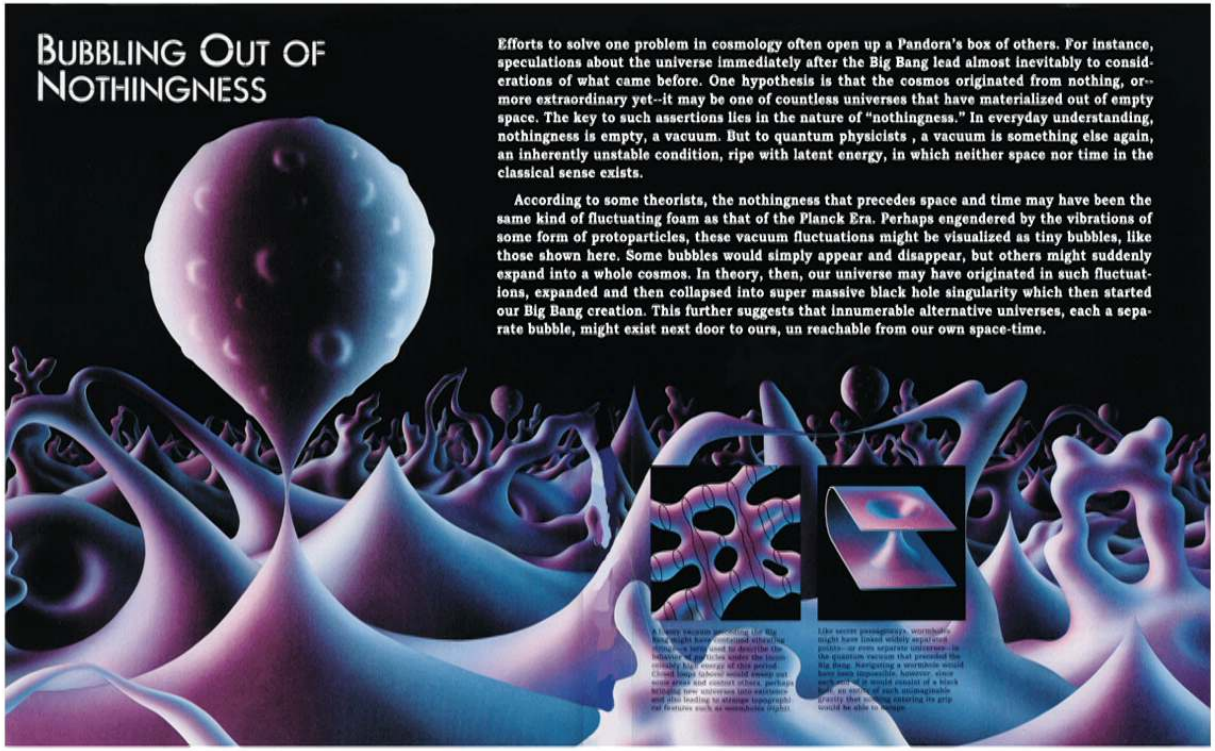
Date: Mar 28, 2018 09:00 AM

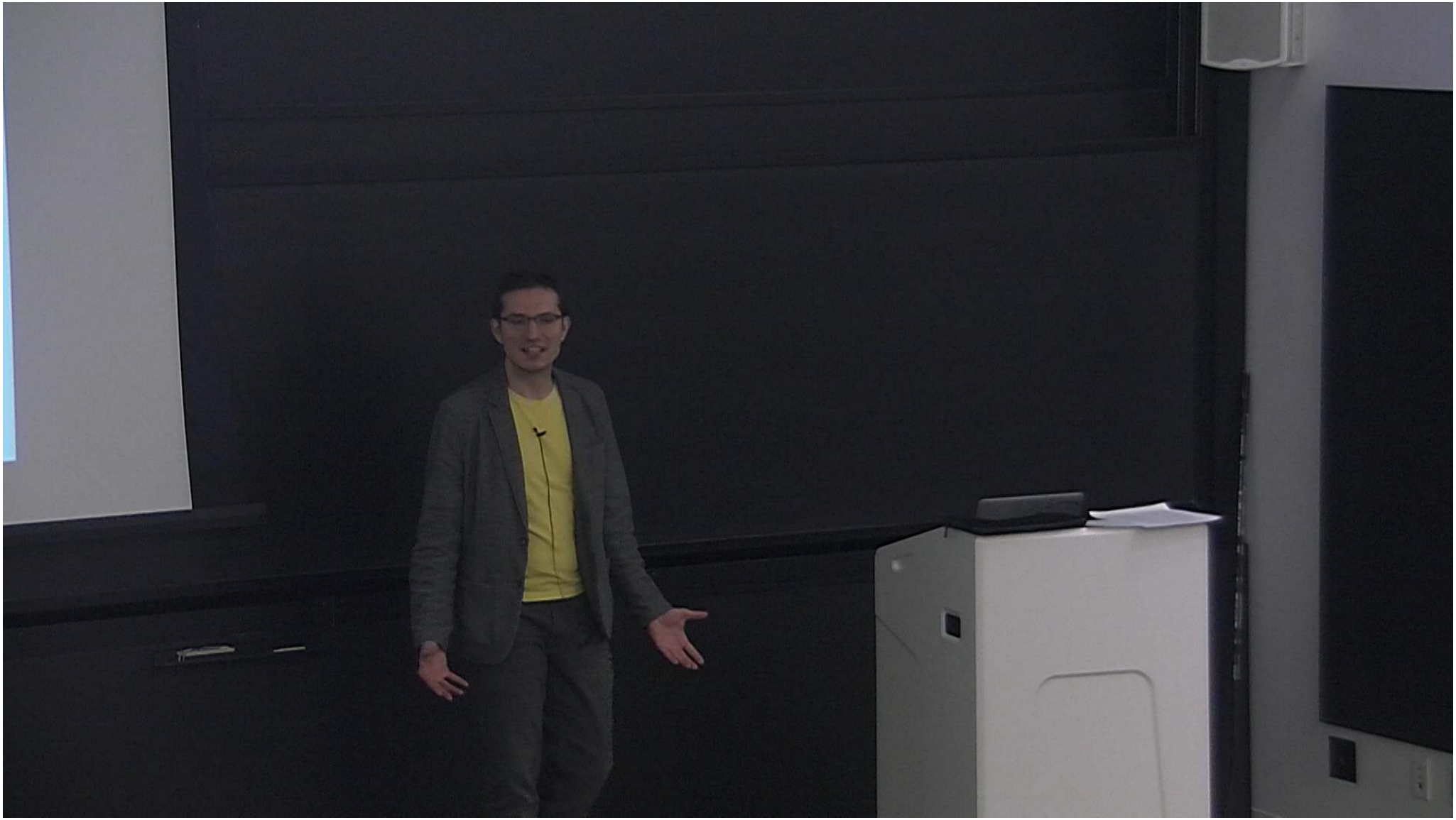
URL: http://pirsa.org/18030035

Abstract:

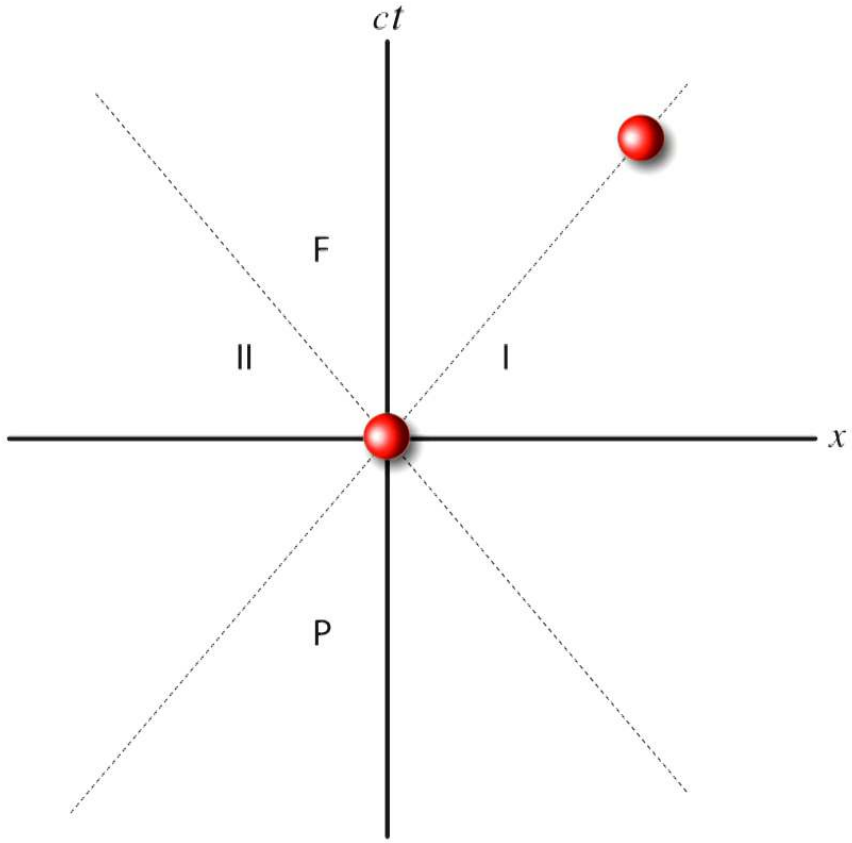
The Quantum Vacuum

The Vacuum is not empty

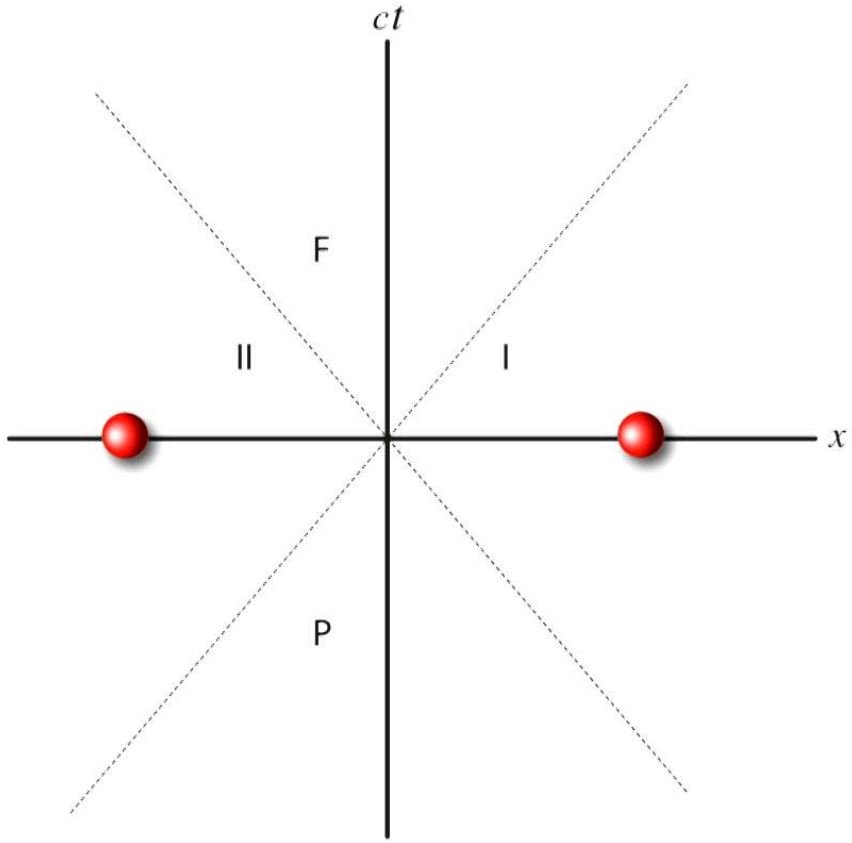




Entanglement Harvesting

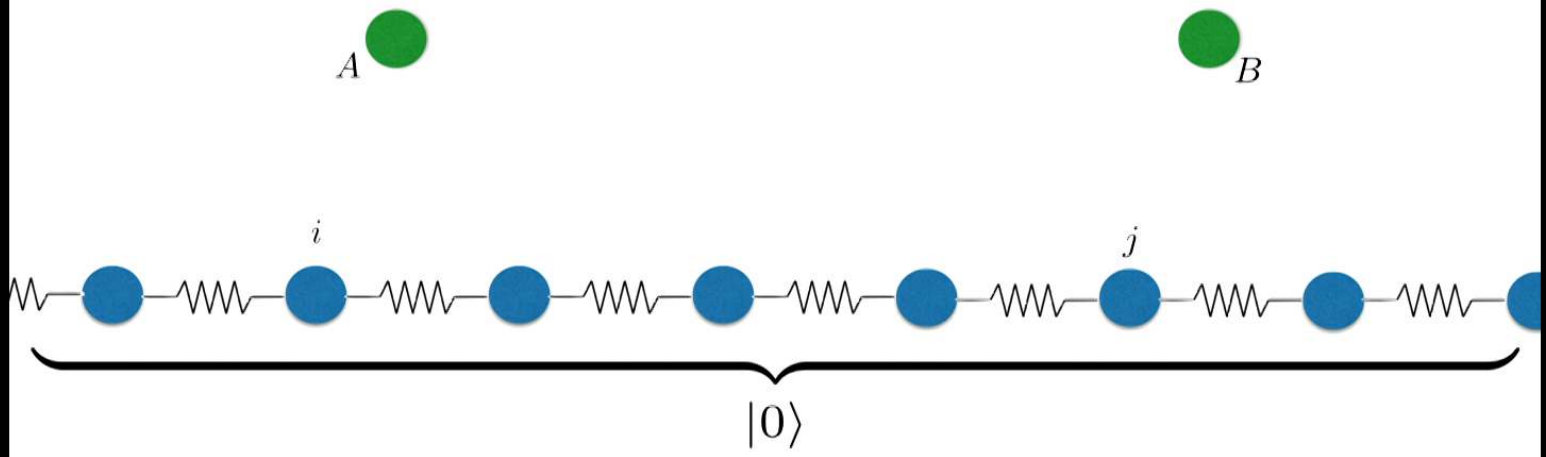


(Spacelike) Entanglement Harvesting



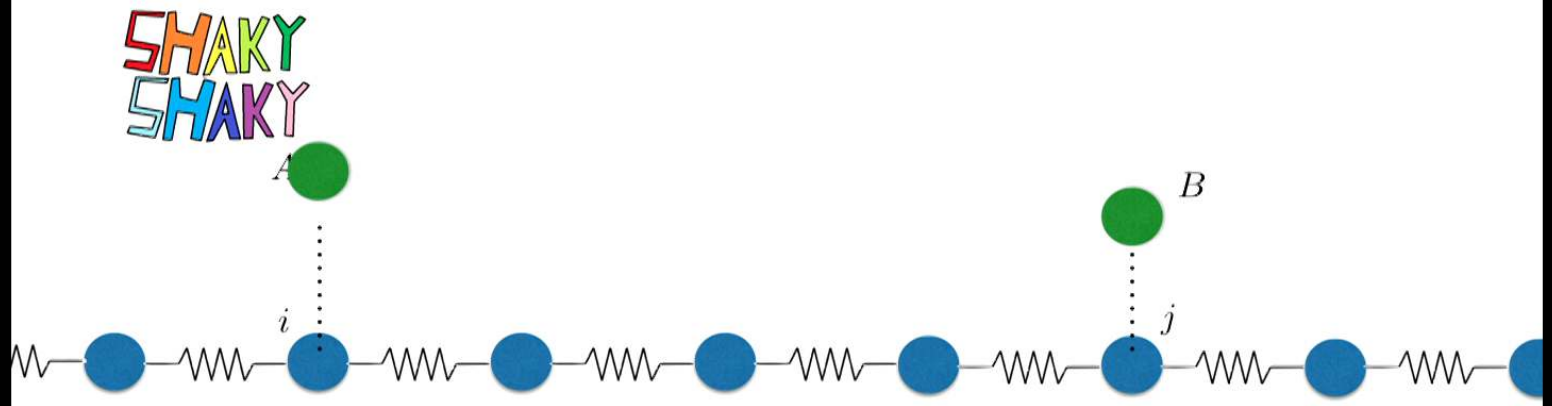
1-D Harmonic lattice in the Ground state

How do we get two systems entangled by means of local interactions with a lattice in the ground state?



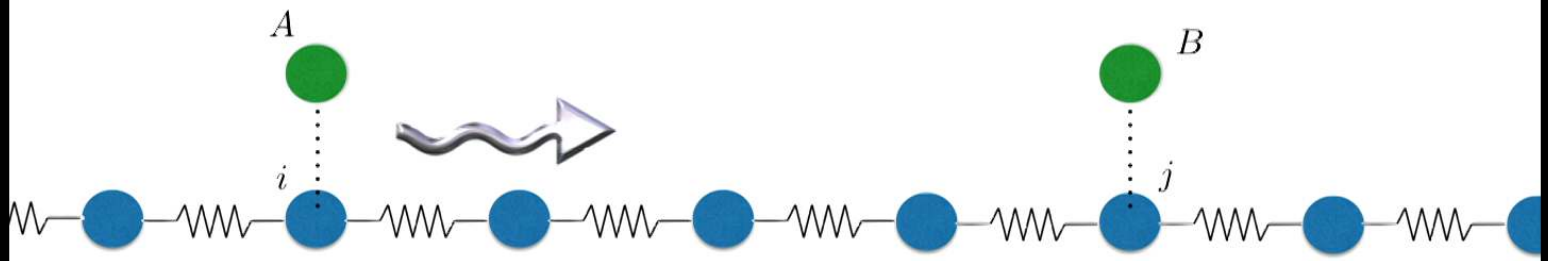
Two possible mechanisms.

1-D Harmonic lattice in the Ground state



1) Communication via phonons

1-D Harmonic lattice in the Ground state



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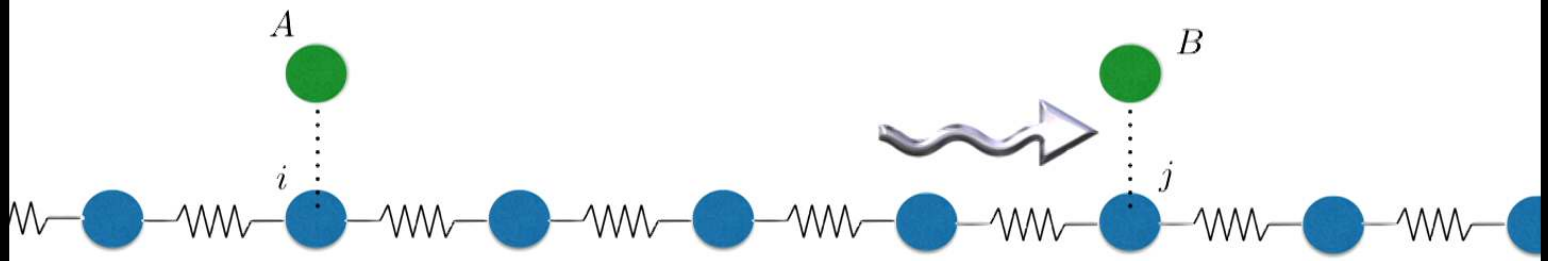
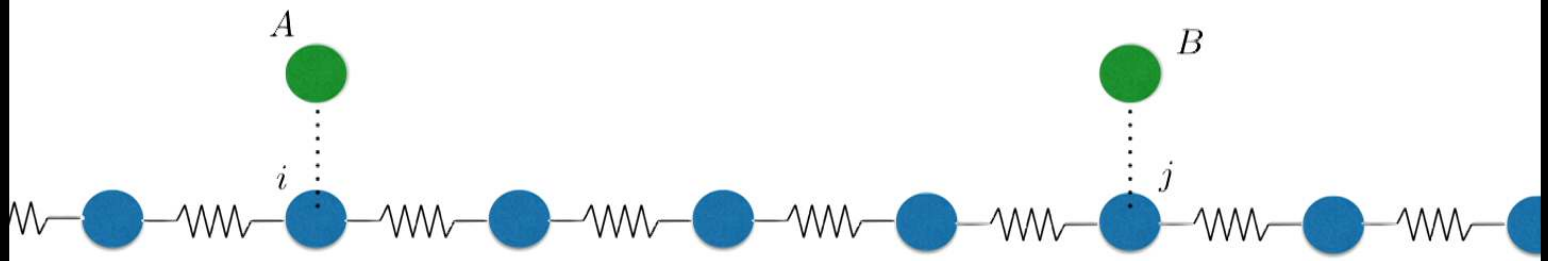


1) Communication via phonons

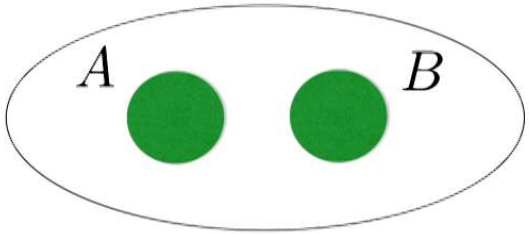
1-D Harmonic lattice in the Ground state



1) Communication via phonons

1-D Harmonic lattice in the Ground state

1) Communication via phonons

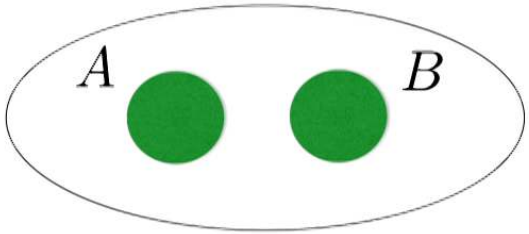


$$\rho_{AB} \neq \sum_i p_i \rho_A \otimes \rho_B$$

Limited by the speed of 'sound'

1-D Harmonic lattice in the Ground state

1) Communication via phonons



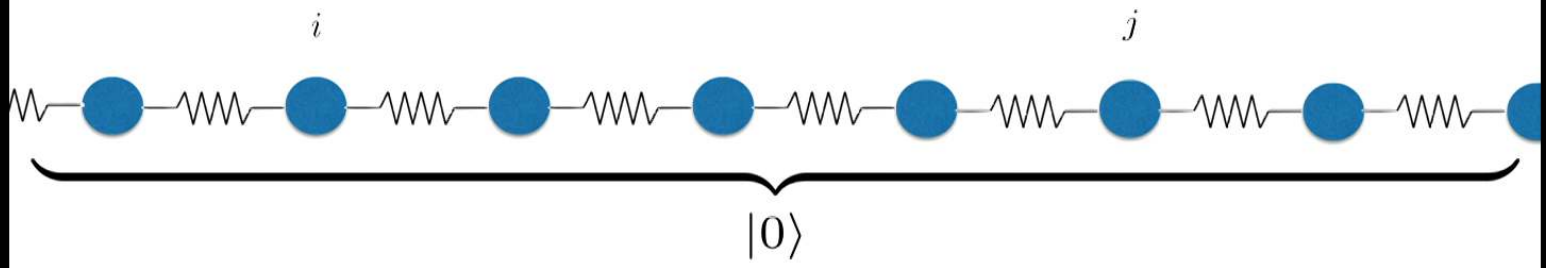
$$\rho_{AB} \neq \sum_i p_i \rho_A \otimes \rho_B$$

Limited by the speed of 'sound'

1-D Harmonic lattice in the Ground state

There's another possibility:

Take advantage of pre-existent entanglement

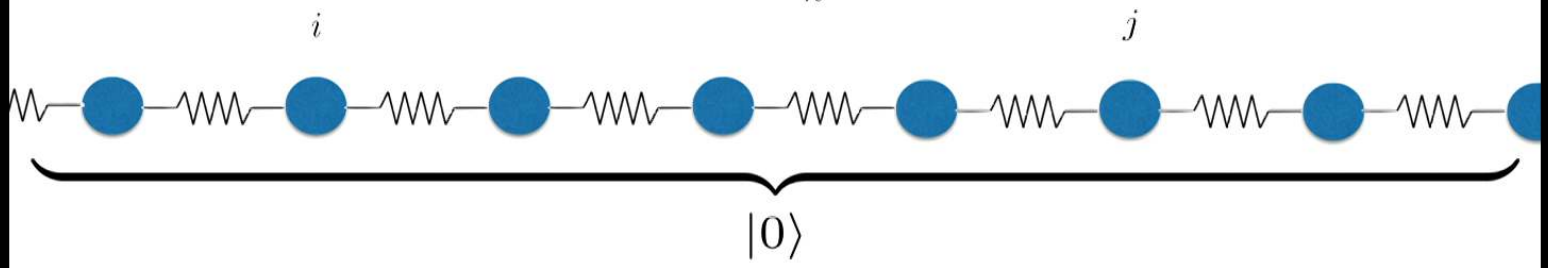


1-D Harmonic lattice in the Ground state

‘Non-local’ basis: Normal modes $|0\rangle, |1\rangle, |2\rangle, \ldots$

‘Local’ basis: individual number states $\{|n_1, \ldots, n_i, \ldots, n_j, \ldots\rangle\}$

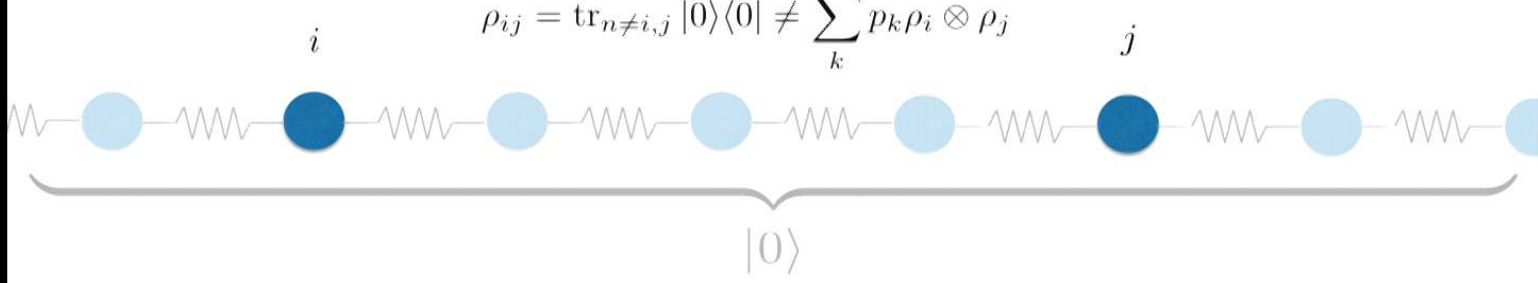
$$|0\rangle \neq \bigotimes_n |0_n\rangle$$



1-D Harmonic lattice

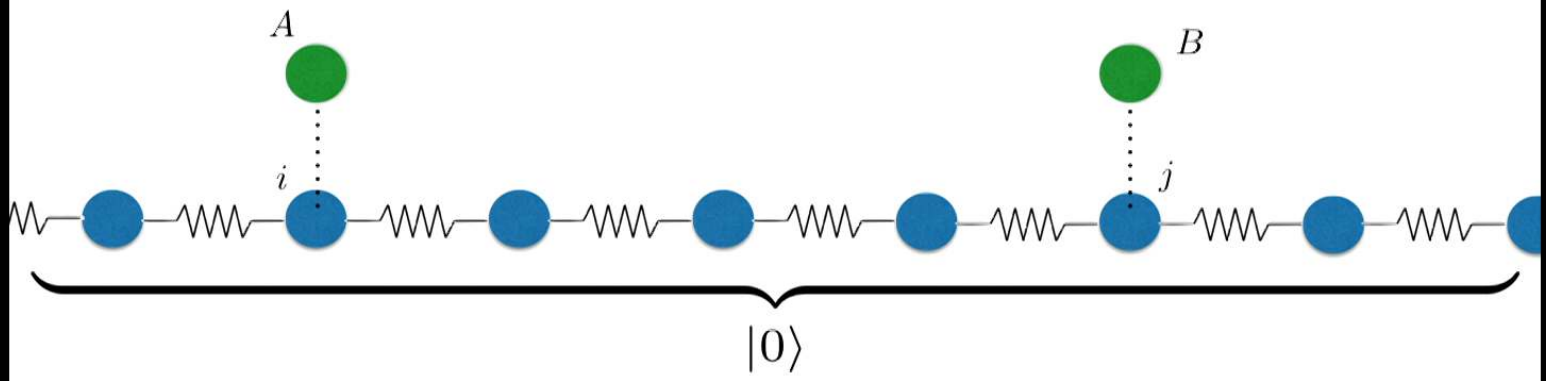
$$|0\rangle \neq \bigotimes_n |0_n\rangle$$

$$\rho_{ij} = \mathrm{tr}_{n\neq i,j}\, |0\rangle\langle 0| \neq \sum_k p_k \rho_i \otimes \rho_j$$



1-D Harmonic lattice in the Ground state

2) Swapping ground state entanglement



1-D Harmonic lattice in the Ground state

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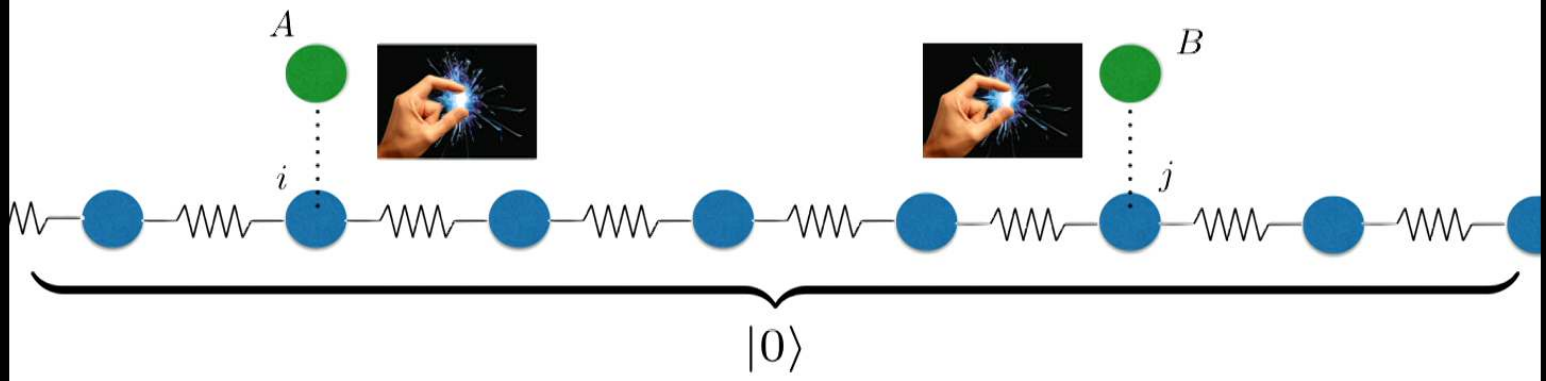
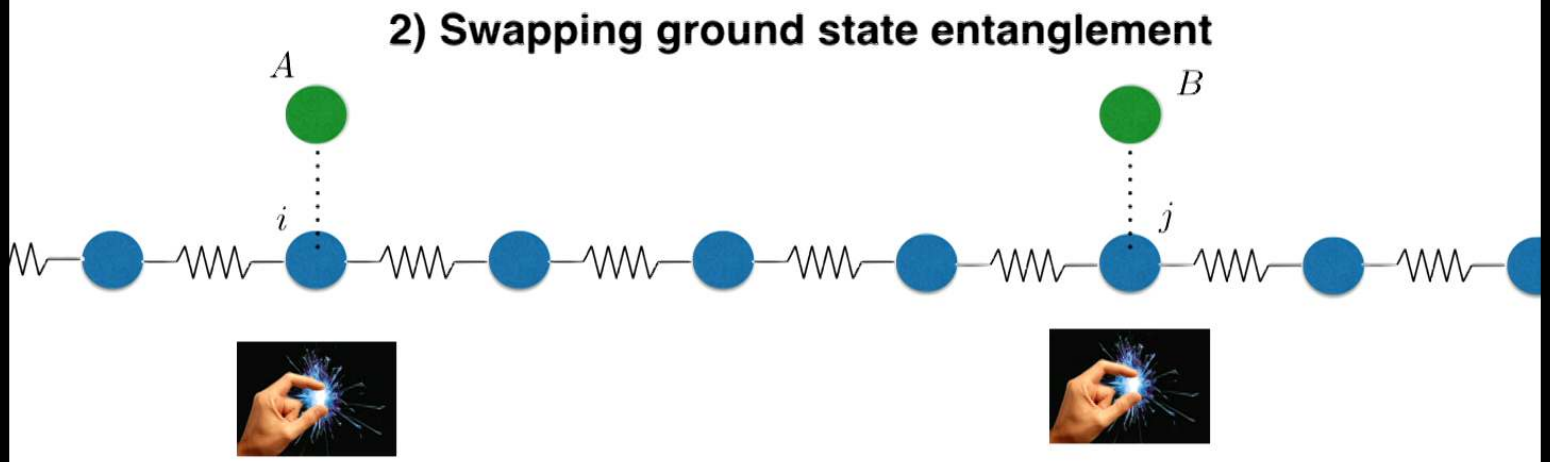
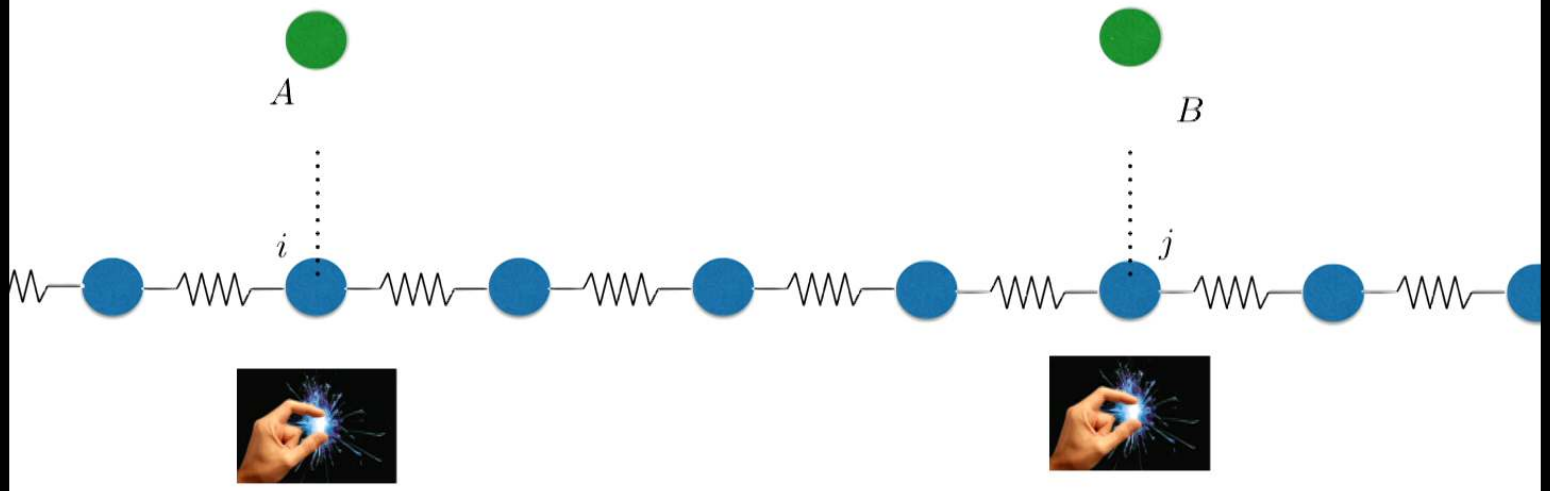


1-D Harmonic lattice in the Ground state



1-D Harmonic lattice in the Ground state

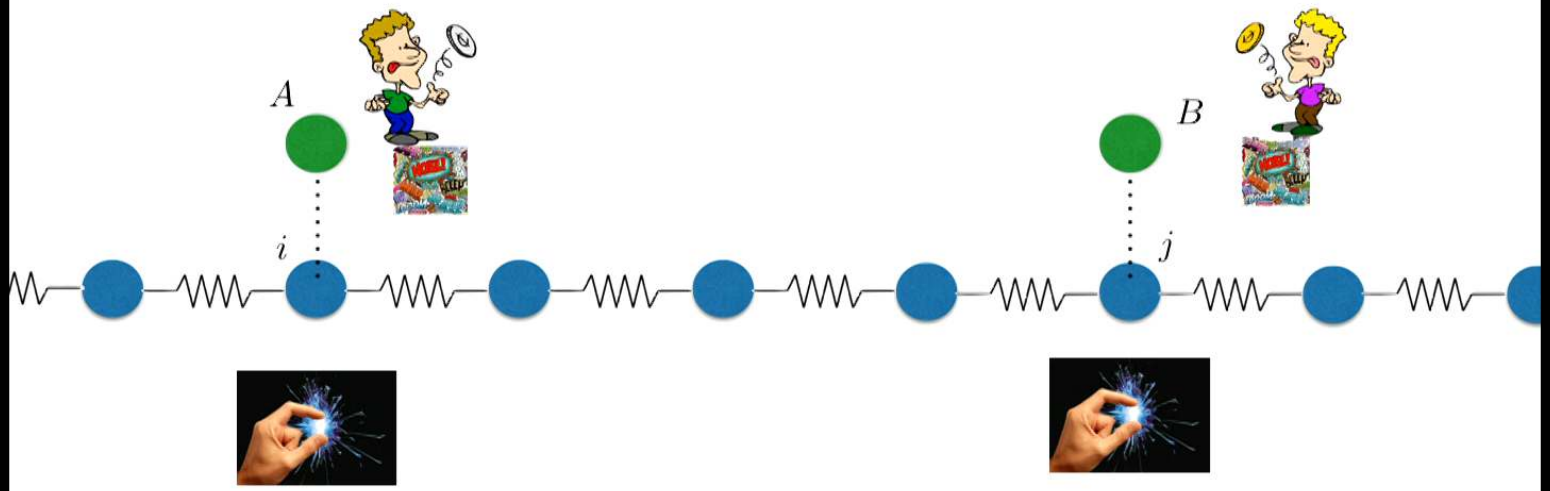
Local coupling to the vacuum: Observed fluctuations are correlated



2) Swapping ground state entanglement

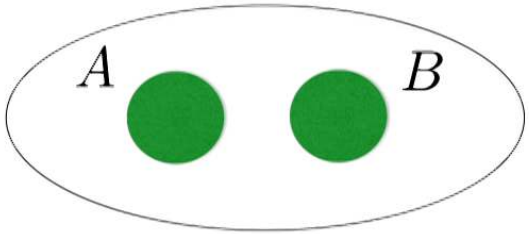
1-D Harmonic lattice in the Ground state

2) Swapping ground state entanglement



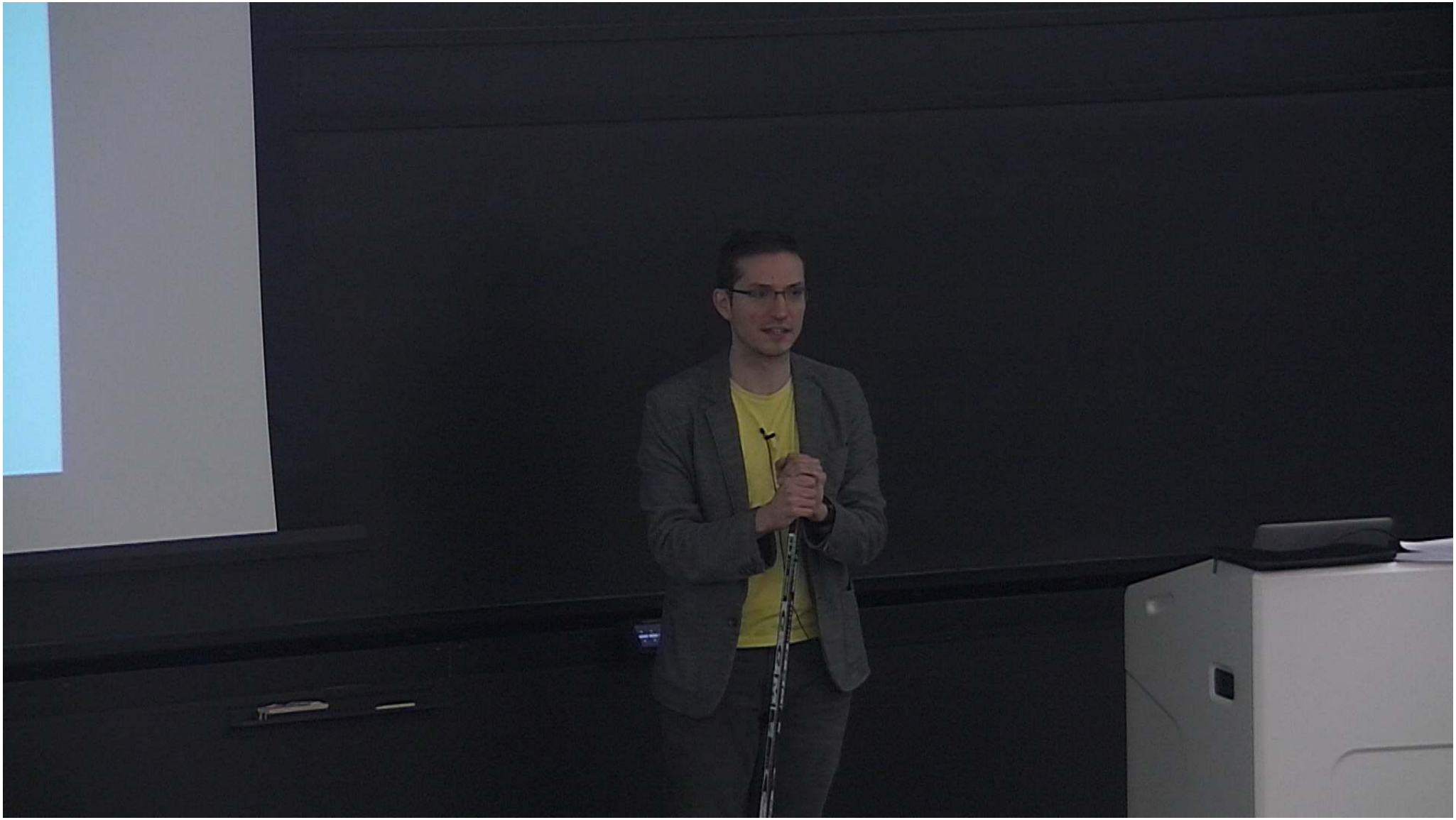
1-D Harmonic lattice in the Ground state

2) Swapping ground state entanglement



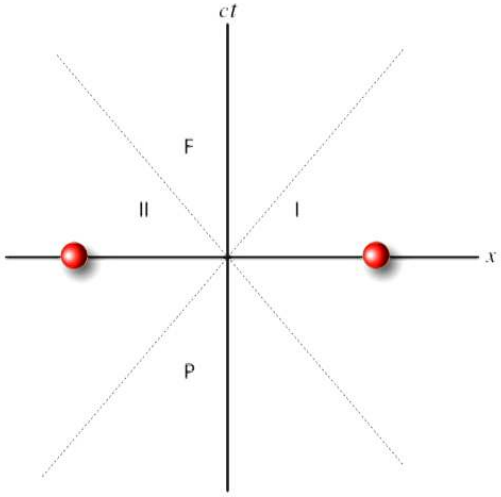
$$\rho_{AB} \neq \sum_i p_i \rho_A \otimes \rho_B$$

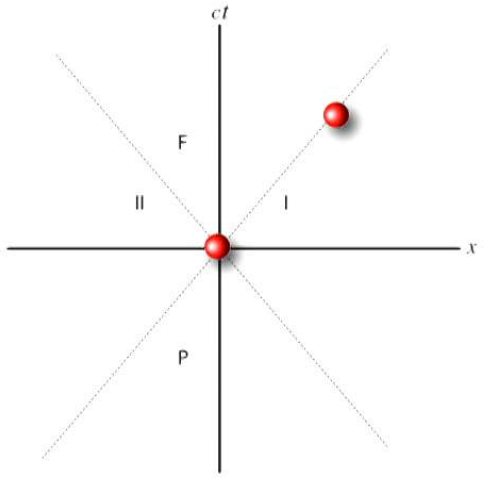
NOT Limited by the speed of 'sound'



Quantum Fields

A 1D quantum field can be thought as the 'continuum limit' of such a lattice

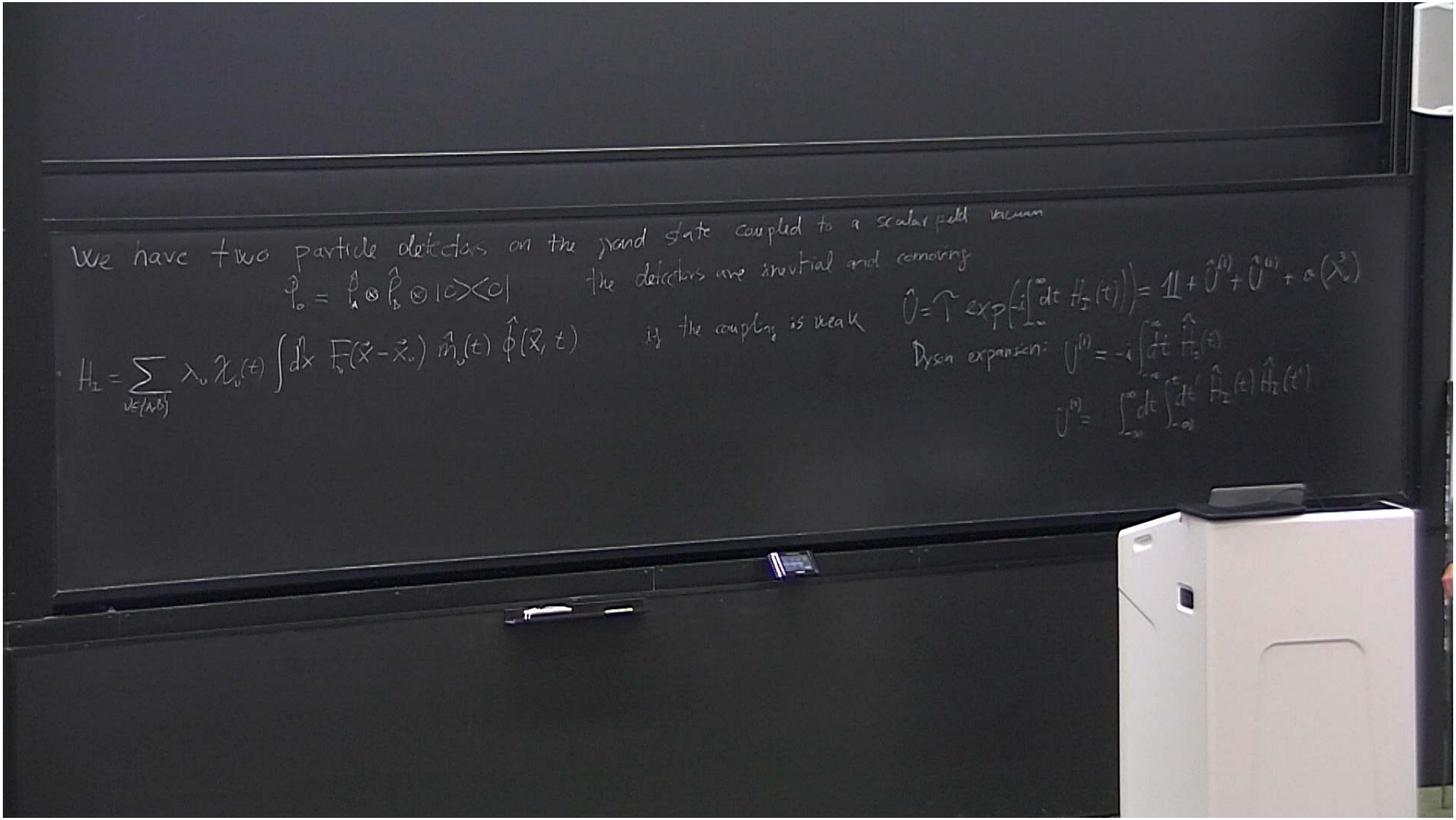


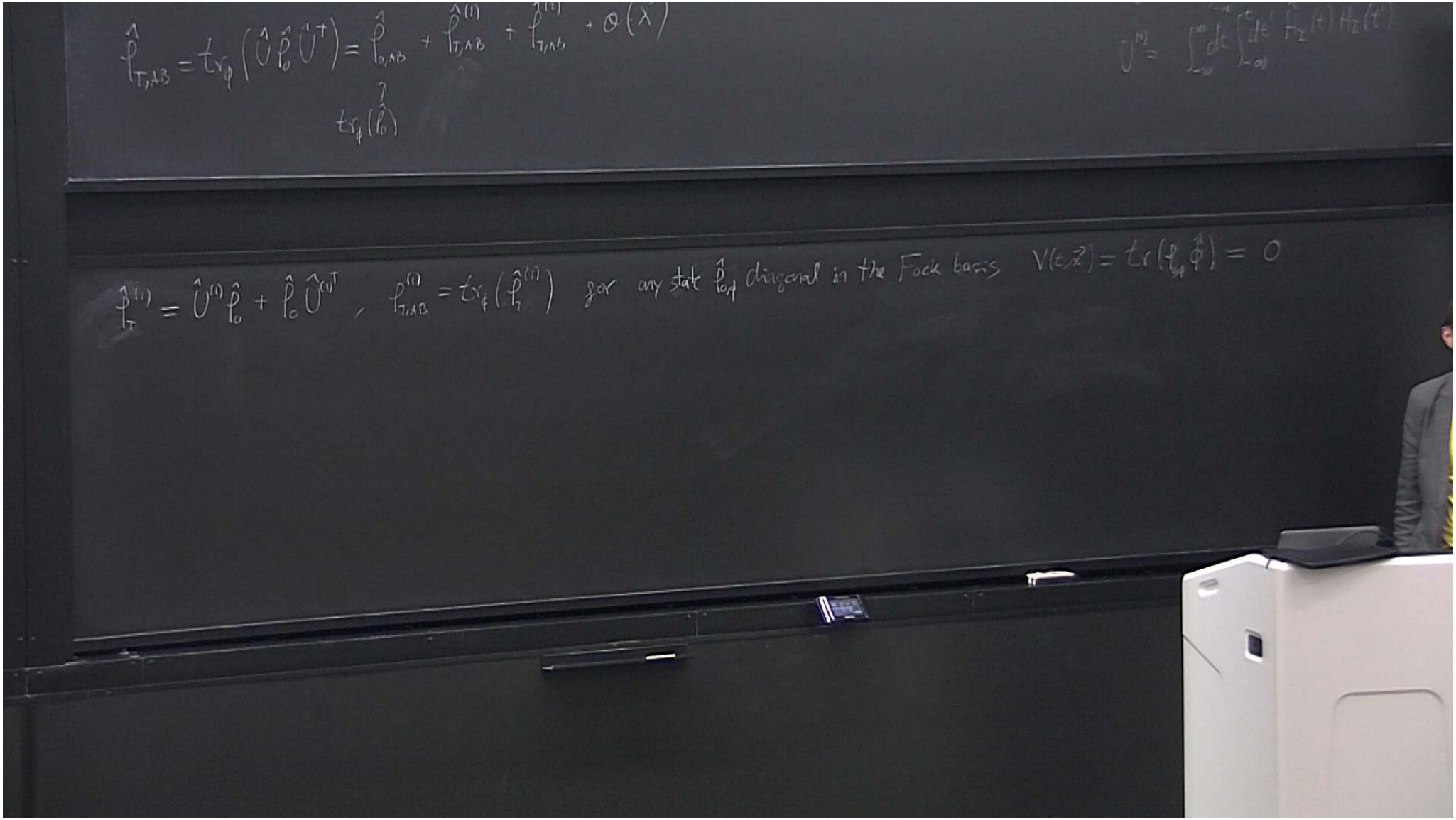


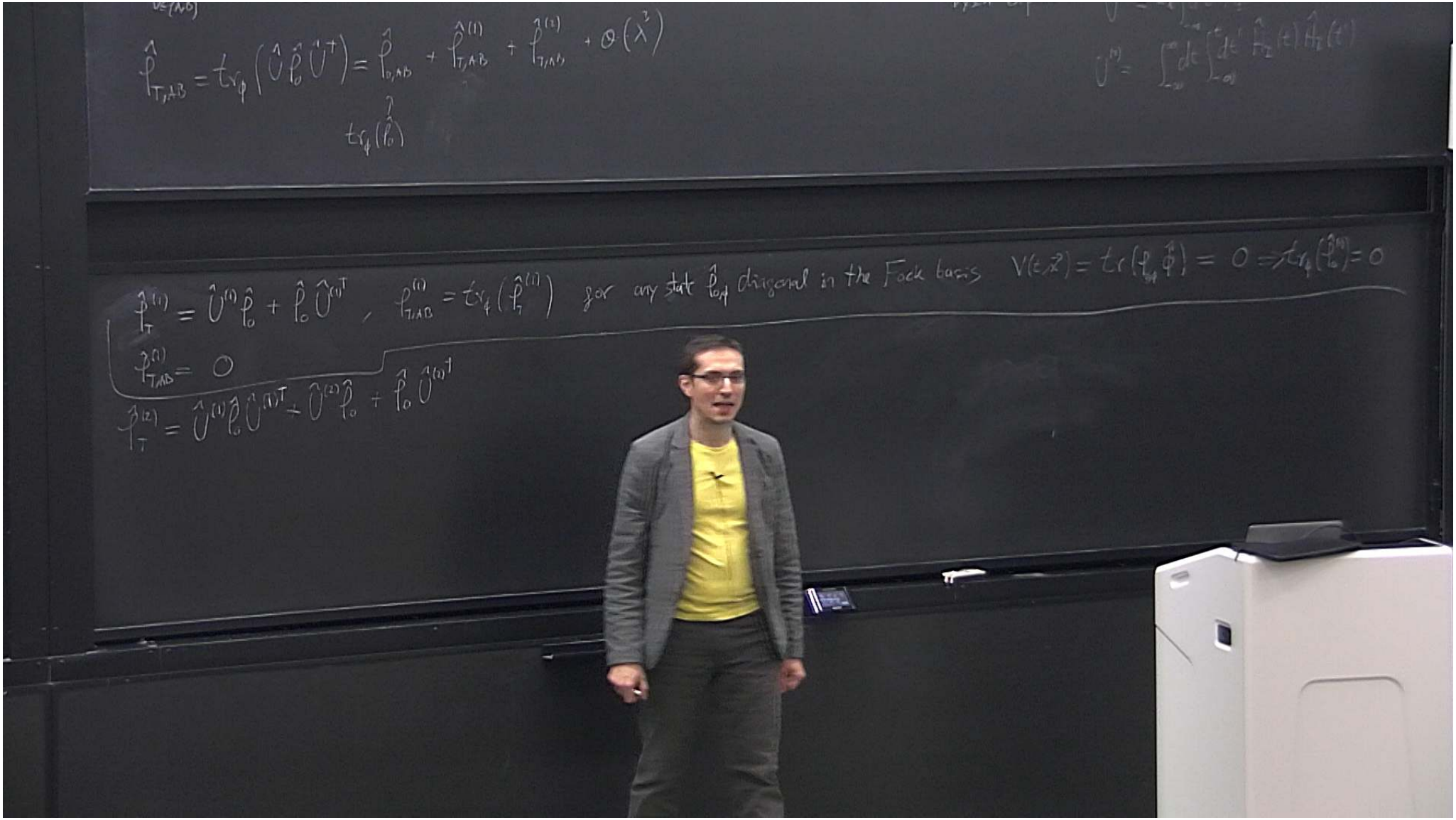
Two mechanisms to get 'atoms' entangled via interaction with quantum fields:

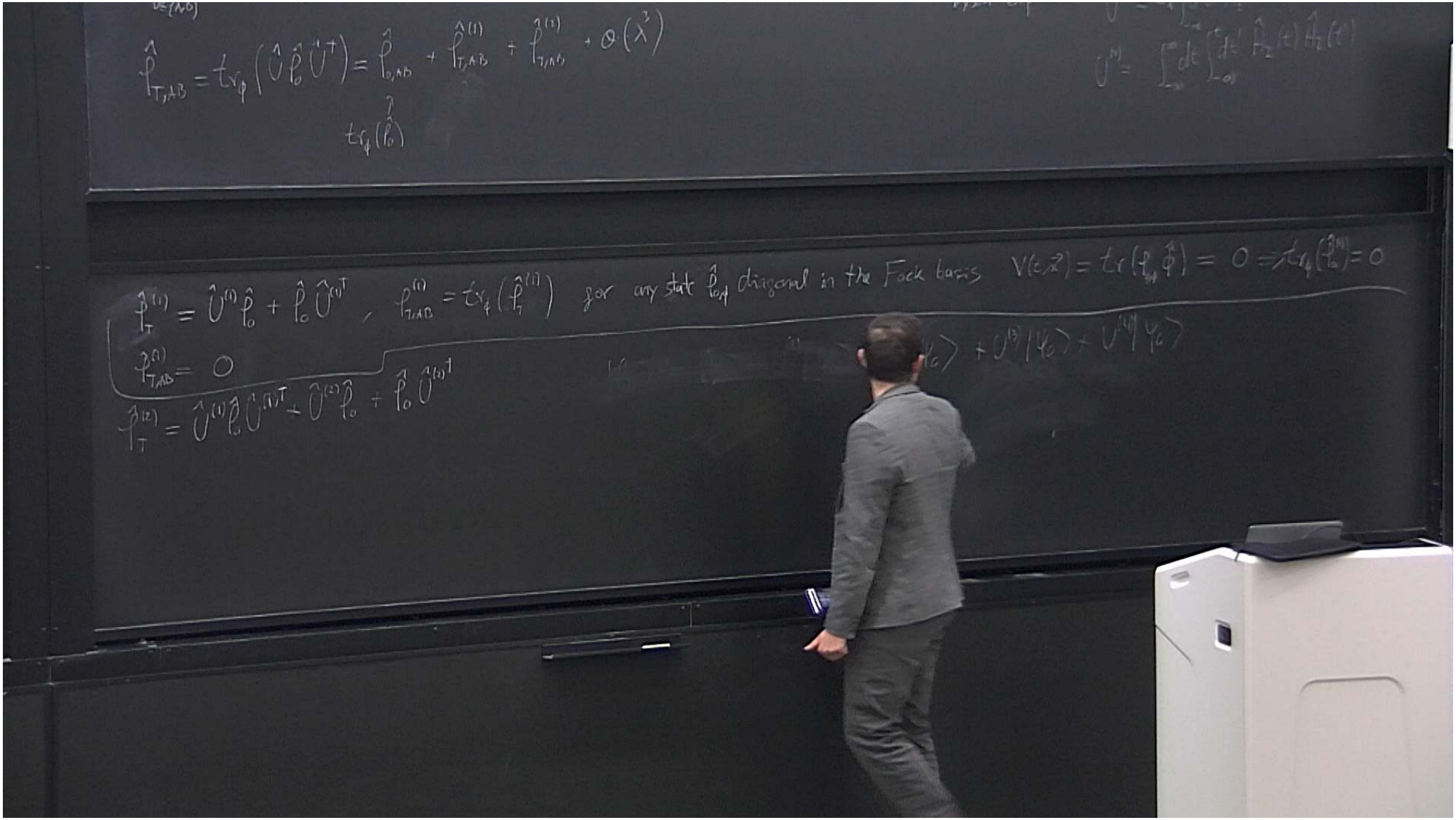
1) Via exchange of real field quanta

2) Swapping vacuum entanglement


We have two particle detectors on the ground state coupled to a scalar field vacuum
$\hat{\rho}_0 = \hat{\rho}_A \otimes \hat{\rho}_B \otimes |0\rangle\langle 0|$
the detectors are inertial and comoving
$H_I = \sum_{\nu \in \{A,B\}} \lambda_\nu \chi_\nu(t) \int d^3x \, F_\nu(\vec{x} - \vec{x}_\nu) \, \hat{m}_\nu(t) \, \hat{\phi}(\vec{x}, t)$
if the coupling is weak
$\hat{U} = \mathcal{T} \exp\left(-i \int_{-\infty}^{\infty} dt \, H_I(t)\right) = \mathbb{1} + \hat{U}^{(1)} + \hat{U}^{(2)} + o(\lambda_\nu^3)$
Dyson expansion: $U^{(1)} = -i \int_{-\infty}^{\infty} dt \, \hat{H}_I(t)$
$U^{(2)} = \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \, \hat{H}_I(t) \hat{H}_I(t')$


$\hat{\rho}_{T,AB} = tr_\phi\left(\hat{U}\hat{\rho}_0\hat{U}^\dagger\right) = \hat{\rho}_{0,AB} + \hat{\rho}^{(1)}_{T,AB} + \hat{\rho}^{(2)}_{T,AB} + \mathcal{O}(\lambda)$
$tr_\phi(\hat{\rho}_0)$
$U^{(1)} = \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \hat{H}_2(t) H_I(t')$
$\hat{\rho}^{(1)}_T = \hat{U}^{(1)}\hat{\rho}_0 + \hat{\rho}_0\hat{U}^{(1)\dagger}$, $\rho^{(1)}_{T,AB} = tr_\phi(\hat{\rho}^{(1)}_T)$ for any state $\hat{\rho}_{0A}$ diagonal in the Fock basis $V(t,\vec{x}) = tr(\hat{\rho}_{0\phi}\hat{\phi}) = 0$


$\hat{\rho}_{T,AB} = tr_\phi(\hat{U}\hat{\rho}_0\hat{U}^\dagger) = \hat{\rho}_{0,AB} + \hat{\rho}^{(1)}_{T,AB} + \hat{\rho}^{(2)}_{T,AB} + O(\lambda^3)$
$tr_\phi(\hat{\rho}_0)$
$U^{(1)} = \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \hat{H}_I(t)\hat{H}_I(t')$
$\hat{\rho}^{(1)}_T = \hat{U}^{(1)}\hat{\rho}_0 + \hat{\rho}_0\hat{U}^{(1)\dagger}$, $\rho^{(1)}_{T,AB} = tr_\phi(\hat{\rho}^{(1)}_T)$ for any state $\hat{\rho}_{0A}$ diagonal in the Fock basis $V(t,\vec{x}) = tr(\rho_{0A}\hat{\phi}) = 0 \Rightarrow tr_\phi(\hat{\rho}^{(1)}_0) = 0$
$\hat{\rho}^{(1)}_{T,AB} = 0$
$\hat{\rho}^{(2)}_T = \hat{U}^{(1)}\hat{\rho}_0\hat{U}^{(1)\dagger} + \hat{U}^{(2)}\hat{\rho}_0 + \hat{\rho}_0\hat{U}^{(2)\dagger}$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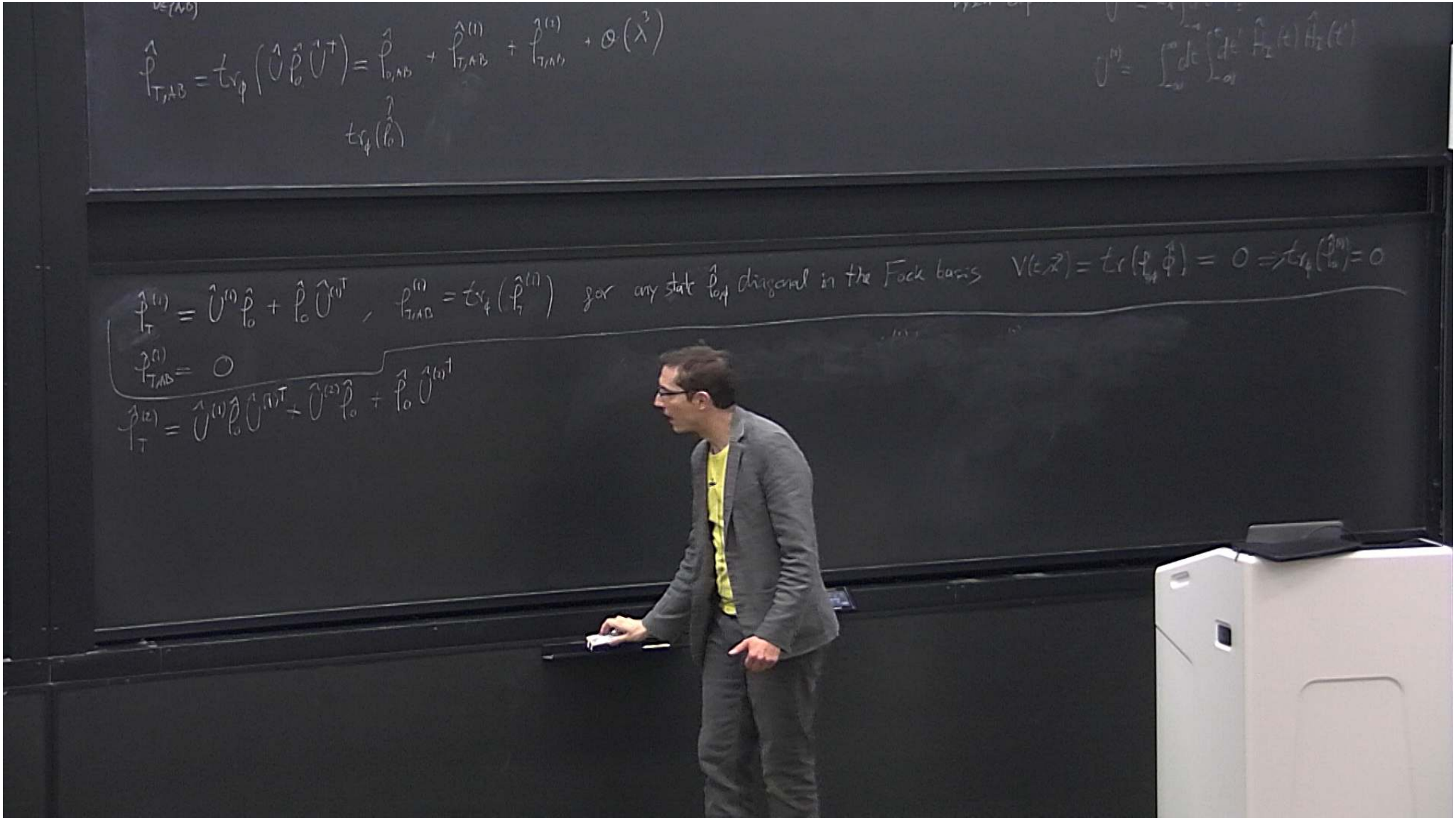
$\hat{\rho}^{(1)}_T = \hat{U}^{(1)}\hat{\rho}_0 + \hat{\rho}_0\hat{U}^{(1)\dagger}$, $\rho^{(1)}_{T,AB} = tr_\phi(\hat{\rho}^{(1)}_T)$ for any state $\hat{\rho}_0$ diagonal in the Fock basis
$V(\vec{x}) = tr(\hat{\rho}_\phi\hat{\phi}) = 0 \Rightarrow tr_\phi(\hat{\rho}^{(1)}_0) = 0$
$\hat{\rho}^{(1)}_{T,AB} = 0$
$\hat{\rho}^{(2)}_T = \hat{U}^{(1)}\hat{\rho}_0\hat{U}^{(1)\dagger} + \hat{U}^{(2)}\hat{\rho}_0 + \hat{\rho}_0\hat{U}^{(2)\dagger}$
$U^{(2)} = \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \hat{H}_2(t)\hat{H}_2(t')$


$\hat{\rho}_{T,AB} = tr_q(\hat{U}\hat{\rho}_0\hat{U}^\dagger) = \hat{\rho}_{0,AB} + \hat{\rho}^{(1)}_{T,AB} + \hat{\rho}^{(2)}_{T,AB} + O(\lambda^3)$
$tr_q(\hat{\rho}_0)$
$\hat{\rho}^{(1)}_T = \hat{U}^{(1)}\hat{\rho}_0 + \hat{\rho}_0\hat{U}^{(1)\dagger}$, $\rho^{(1)}_{T,AB} = tr_q(\hat{\rho}^{(1)}_T)$ for any state $\hat{\rho}_{0,q}$ diagonal in the Fock basis $V(t,\vec{x}) = tr(\hat{\rho}_q\hat{\phi}) = 0 \Rightarrow tr_q(\hat{\rho}^{(1)}_0) = 0$
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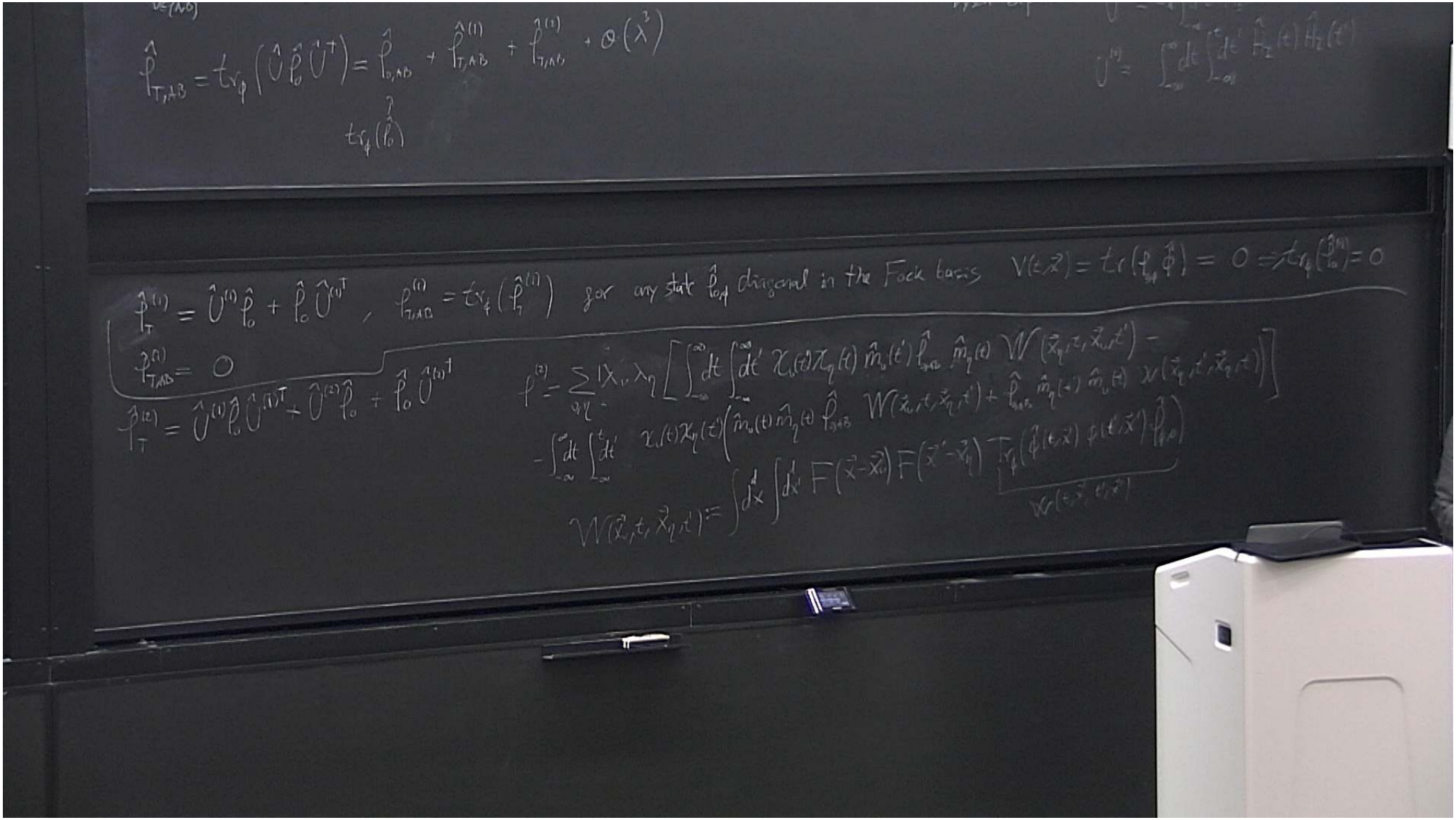


$$\hat{\rho}_{T,AB} = tr_\phi\left(\hat{U}\hat{\rho}_0\hat{U}^\dagger\right) = \hat{\rho}_{0,AB} + \hat{\rho}^{(1)}_{T,AB} + \hat{\rho}^{(2)}_{T,AB} + \mathcal{O}(\lambda^3)$$

$$tr_\phi(\hat{\rho}_0)$$

$$U^{(2)} = \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \, \hat{H}_I(t)\hat{H}_I(t')$$

$$\hat{\rho}^{(1)}_T = \hat{U}^{(1)}\hat{\rho}_0 + \hat{\rho}_0\hat{U}^{(1)\dagger}, \quad \rho^{(1)}_{T,AB} = tr_\phi\left(\hat{\rho}^{(1)}_T\right)$$

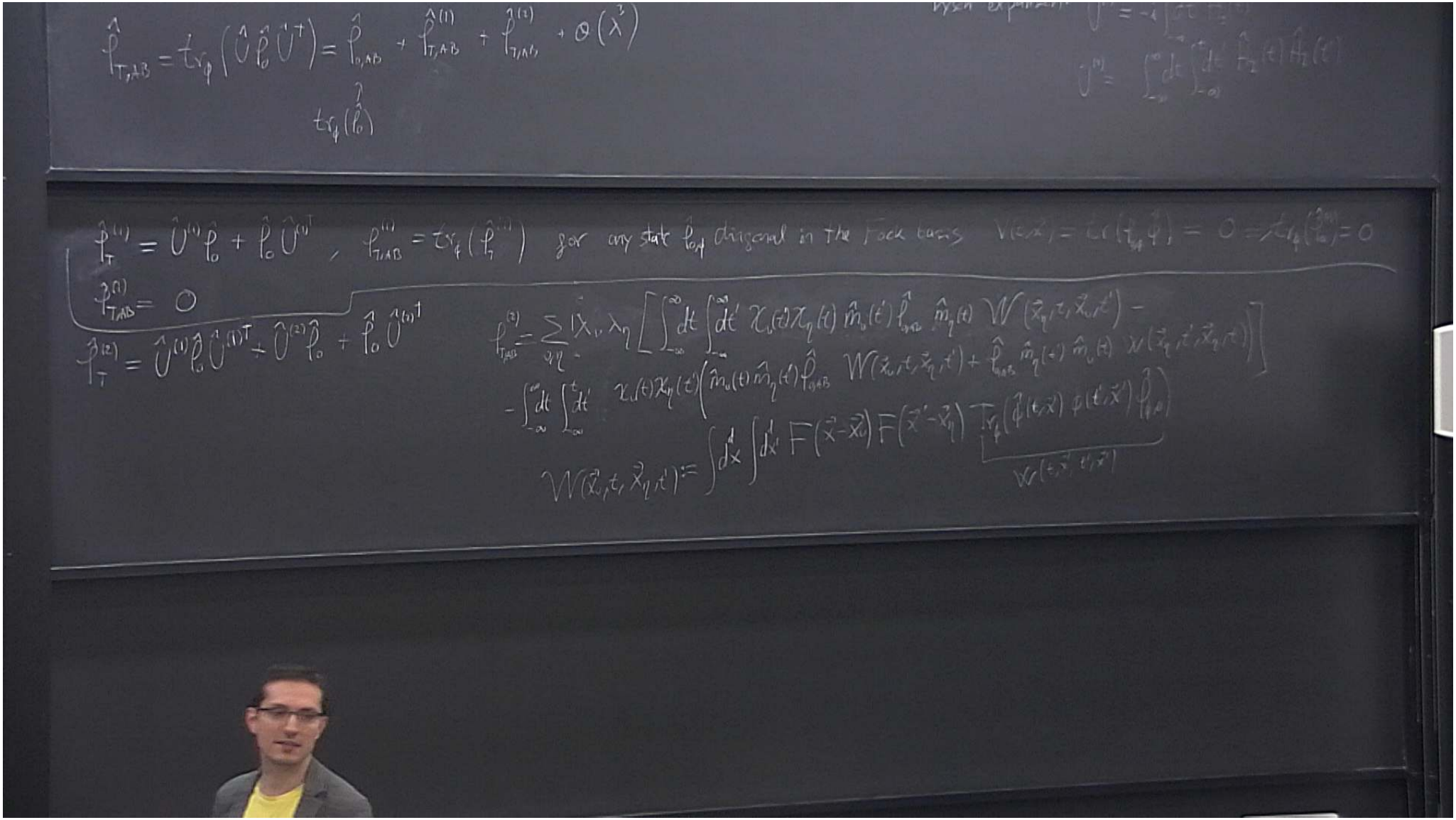
for any state $\hat{\rho}_{0,\phi}$ diagonal in the Fock basis $V(t,\vec{x}) = tr(\rho_{0,\phi}\hat{\phi}) = 0 \Rightarrow tr_\phi(\hat{\rho}^{(1)}_0) = 0$

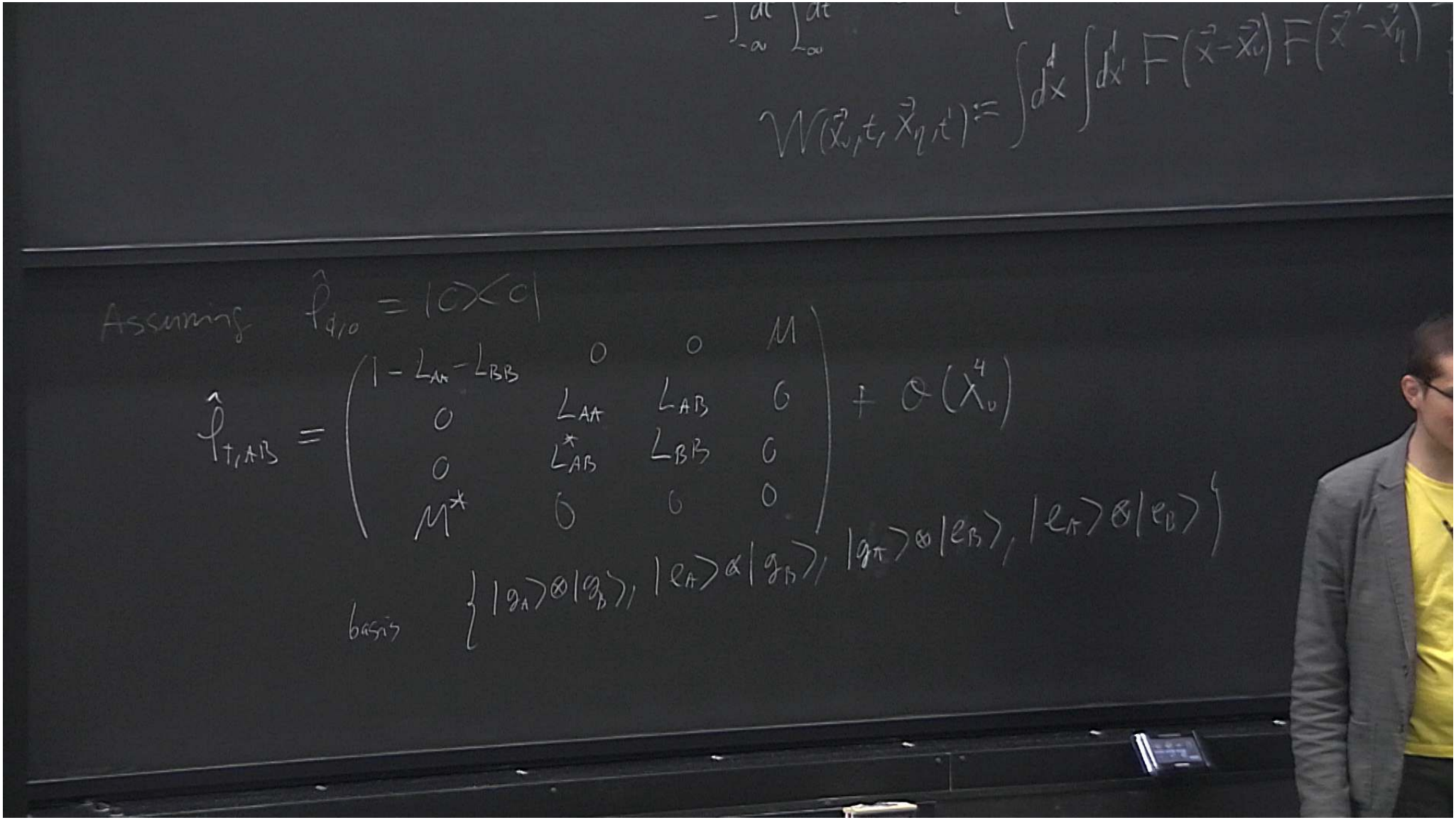
$$\hat{\rho}^{(1)}_{T,AB} = 0$$

$$\hat{\rho}^{(2)}_T = \hat{U}^{(1)}\hat{\rho}_0\hat{U}^{(1)\dagger} + \hat{U}^{(2)}\hat{\rho}_0 + \hat{\rho}_0\hat{U}^{(2)\dagger}$$

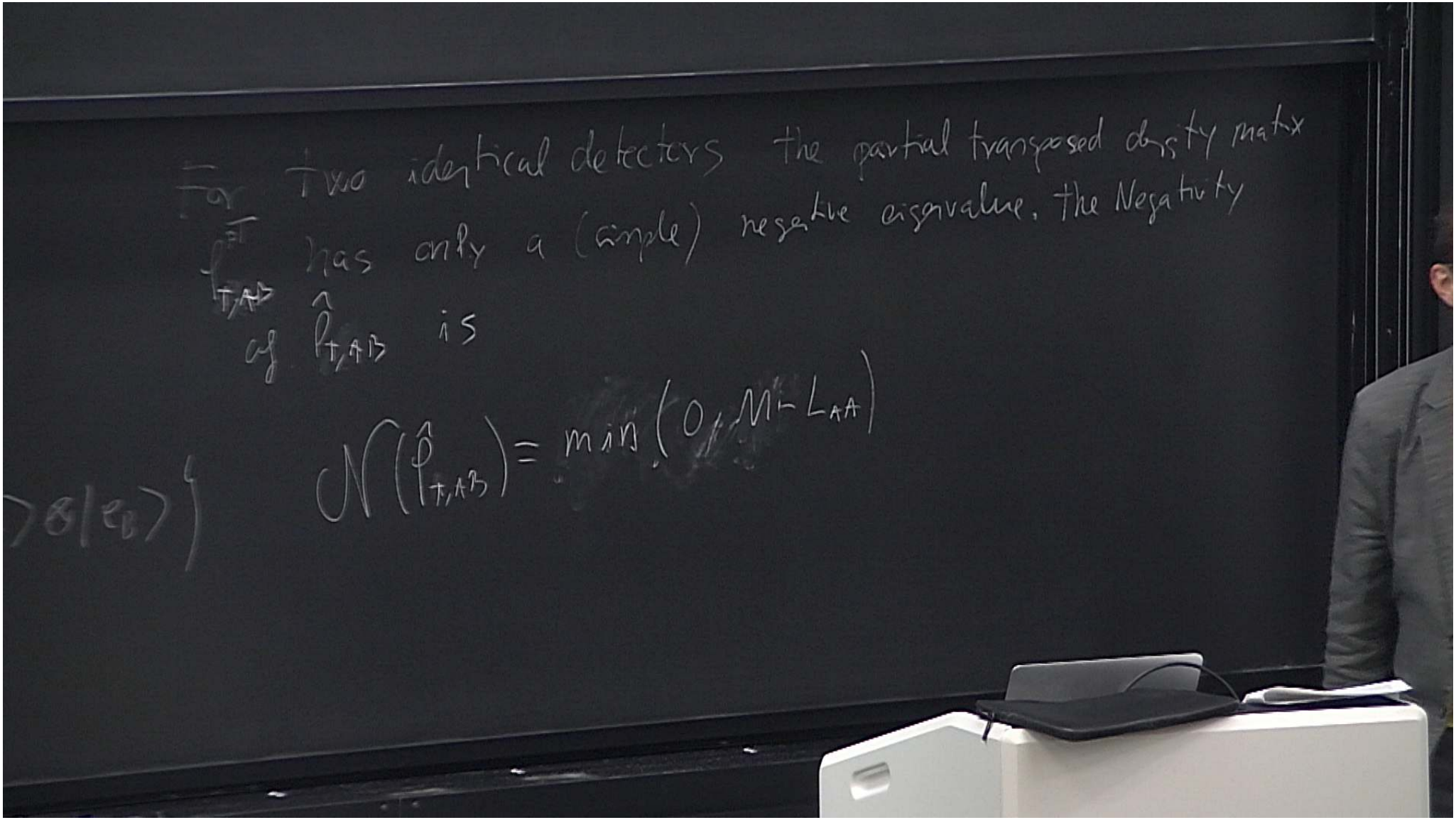
$$\rho^{(2)} = \sum_{\nu,\eta} \lambda_\nu \lambda_\eta \Big[\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \, \chi_\nu(t)\chi_\eta(t)\, \hat{m}_\nu(t')\hat{\rho}_{0,AB}\, \hat{m}_\eta(t)\, W(\vec{x}_\eta, t, \vec{x}_\nu, t') - \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \, \chi_\nu(t)\chi_\eta(t') \Big(\hat{m}_\nu(t)\hat{m}_\eta(t)\hat{\rho}_{0,AB}\, W(\vec{x}_\nu, t, \vec{x}_\eta, t') + \hat{\rho}_{0,AB}\, \hat{m}_\eta(t')\hat{m}_\nu(t)\, W(\vec{x}_\eta, t', \vec{x}_\eta, t) \Big) \Big]$$

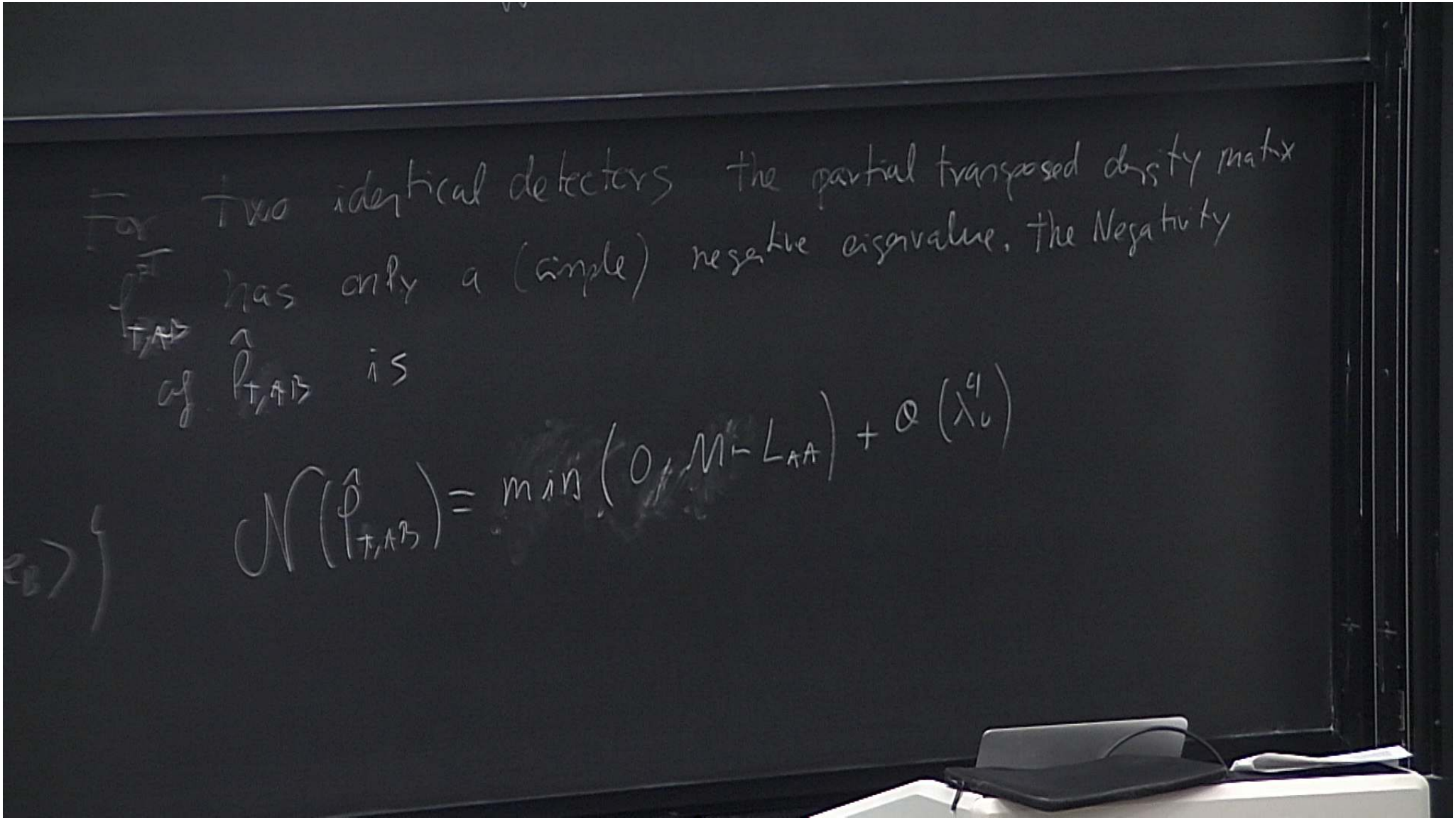
$$W(\vec{x}_\nu, t, \vec{x}_\eta, t') := \int d^dx \int d^dx' \, F(\vec{x} - \vec{x}_\nu) F(\vec{x}' - \vec{x}_\eta) \underbrace{Tr_\phi\left(\hat{\phi}(t,\vec{x})\, \phi(t',\vec{x}')\, \hat{\rho}_{\phi,0}\right)}_{W(t,\vec{x},t',\vec{x}')}$$


$\hat{\rho}_{T,AB} = tr_\phi(\hat{U}\hat{\rho}_0\hat{U}^\dagger) = \hat{\rho}_{0,AB} + \hat{\rho}^{(1)}_{T,AB} + \hat{\rho}^{(2)}_{T,AB} + O(\lambda^3)$
$tr_\phi(\hat{\rho}_0)$
$\hat{\rho}^{(1)}_T = \hat{U}^{(1)}\hat{\rho}_0 + \hat{\rho}_0\hat{U}^{(1)\dagger}$, $\rho^{(1)}_{T,AB} = tr_\phi(\hat{\rho}^{(1)}_T)$ for any state $\hat{\rho}_{0,\phi}$ diagonal in the Fock basis
$\hat{\rho}^{(1)}_{T,AB} = 0$
$\hat{\rho}^{(2)}_T = \hat{U}^{(1)}\hat{\rho}_0\hat{U}^{(1)\dagger} + \hat{U}^{(2)}\hat{\rho}_0 + \hat{\rho}_0\hat{U}^{(2)\dagger}$
$\rho^{(2)}_{T,AB} = \sum_{\nu,\eta} \lambda_\nu \lambda_\eta \Big[\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \, \chi_\nu(t)\chi_\eta(t)\, \hat{m}_\nu(t')\hat{\rho}_{0,AB}\, \hat{m}_\eta(t)\, W(\vec{x}_\eta, t, \vec{x}_\nu, t') -$
$- \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \, \chi_\nu(t)\chi_\eta(t') \big(\hat{m}_\nu(t)\hat{m}_\eta(t')\hat{\rho}_{0,AB}\, W(\vec{x}_\nu, t, \vec{x}_\eta, t') + \hat{\rho}_{0,AB}\, \hat{m}_\eta(t')\hat{m}_\nu(t)\, W(\vec{x}_\eta, t', \vec{x}_\eta, t) \big) \Big]$
$W(\vec{x}_\nu, t, \vec{x}_\eta, t') := \int d^dx \int d^dx' \, F(\vec{x} - \vec{x}_\nu) F(\vec{x}' - \vec{x}_\eta) \underbrace{Tr_\phi(\hat{\phi}(t,\vec{x})\, \phi(t',\vec{x}')\, \hat{\rho}_{\phi,0})}_{W(t,\vec{x}',t',\vec{x}')}$


$W(\vec{x}_u, t, \vec{x}_\eta, t') := \int d^4x \int dx' F(\vec{x}-\vec{x}_u) F(\vec{x}'-\vec{x}_\eta)$
Assuming $\hat{\rho}_{q/0} = |0\rangle\langle 0|$
$\hat{\rho}_{t,AB} = \begin{pmatrix} 1-L_{AA}-L_{BB} & 0 & 0 & M \\ 0 & L_{AA} & L_{AB} & 0 \\ 0 & L^*_{AB} & L_{BB} & 0 \\ M^* & 0 & 0 & 0 \end{pmatrix} + \mathcal{O}(x_u^4)$
basis $\{|g_A\rangle\otimes|g_B\rangle, |e_A\rangle\otimes|g_B\rangle, |g_A\rangle\otimes|e_B\rangle, |e_A\rangle\otimes|e_B\rangle\}$

$$\rho_{AB} = \rho_A \otimes \rho_B$$


For two identical detectors the partial transposed density matrix
$\hat{\rho}^{T}_{T,AB}$ has only a (single) negative eigenvalue. The Negativity
of $\hat{\rho}_{T,AB}$ is
$\mathcal{N}(\hat{\rho}_{T,AB}) = \min(0, M - L_{AA})$


For Two identical detectors the partial transposed density matrix
has only a (single) negative eigenvalue, the Negativity
of $\hat{\rho}_{T,AB}$ is
$$\mathcal{N}(\hat{\rho}_{T,AB}) = \min(0, M - L_{AA}) + O(\lambda_0^4)$$