

Title: PSI 2017/2018 - Relativistic Quantum Information - Lecture 8

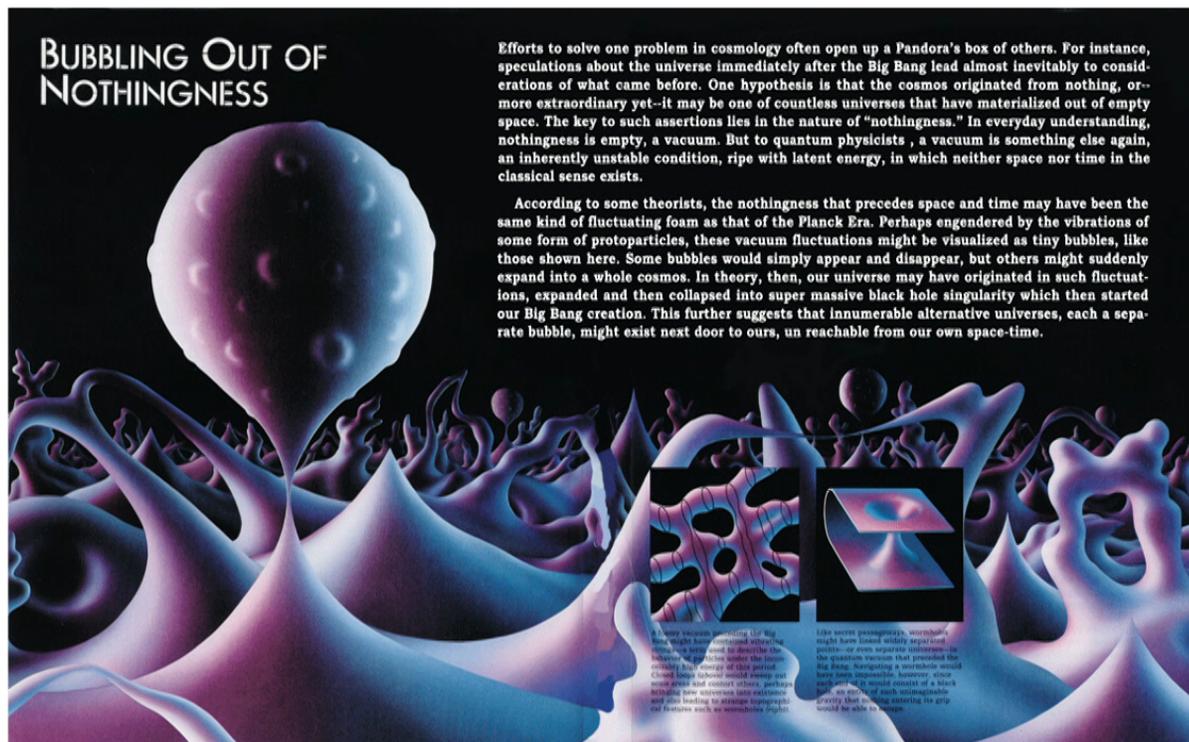
Date: Mar 28, 2018 09:00 AM

URL: <http://pirsa.org/18030035>

Abstract:

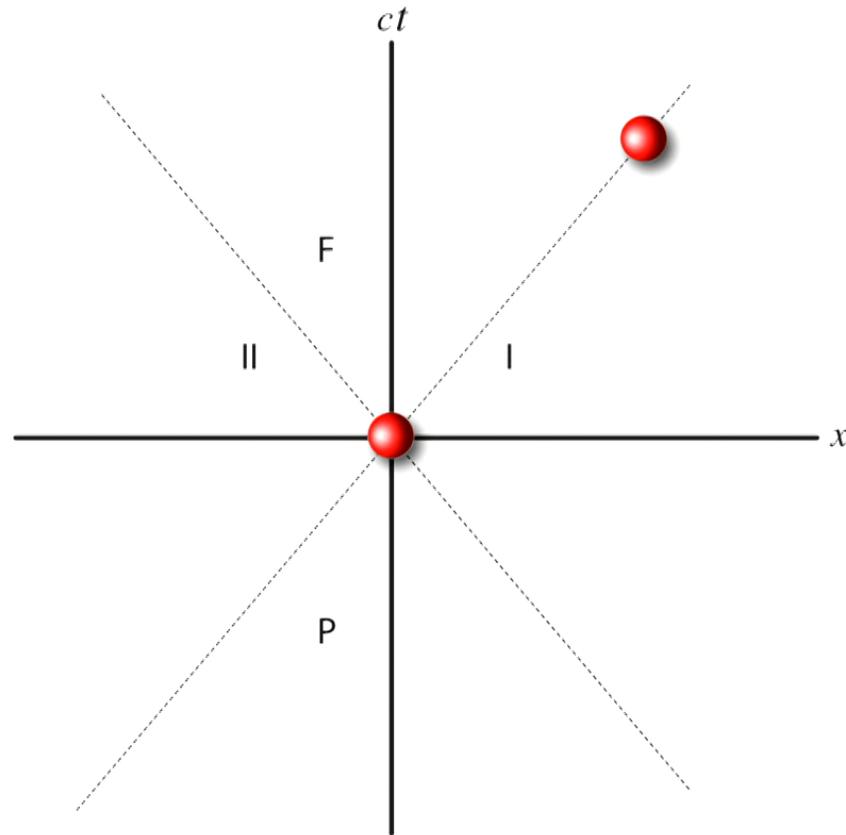
# The Quantum Vacuum

## The Vacuum is not empty

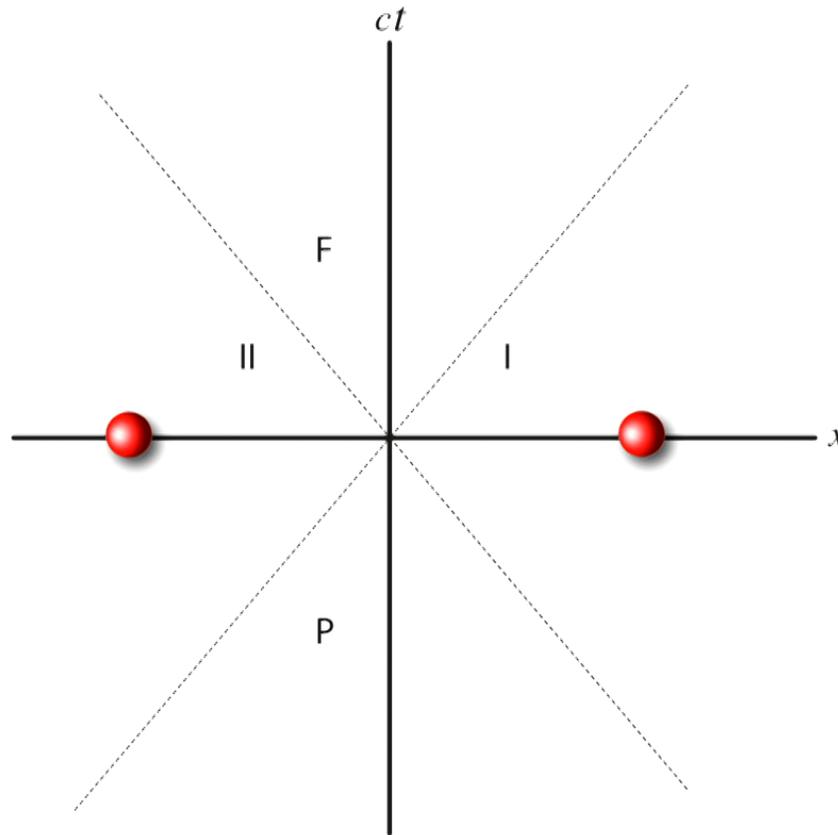




# Entanglement Harvesting

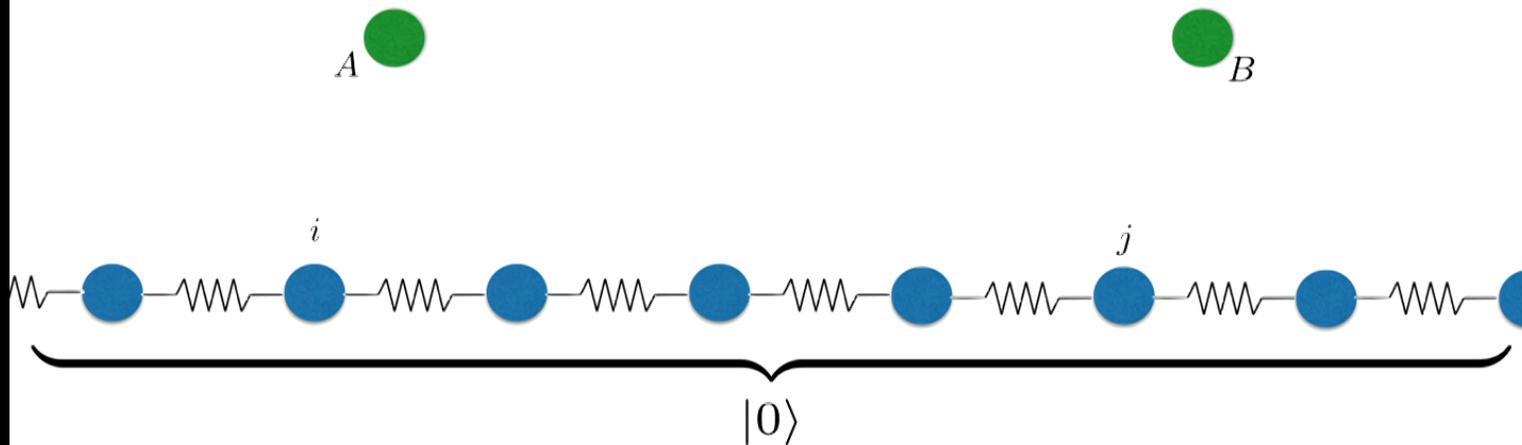


# (Spacelike) Entanglement Harvesting



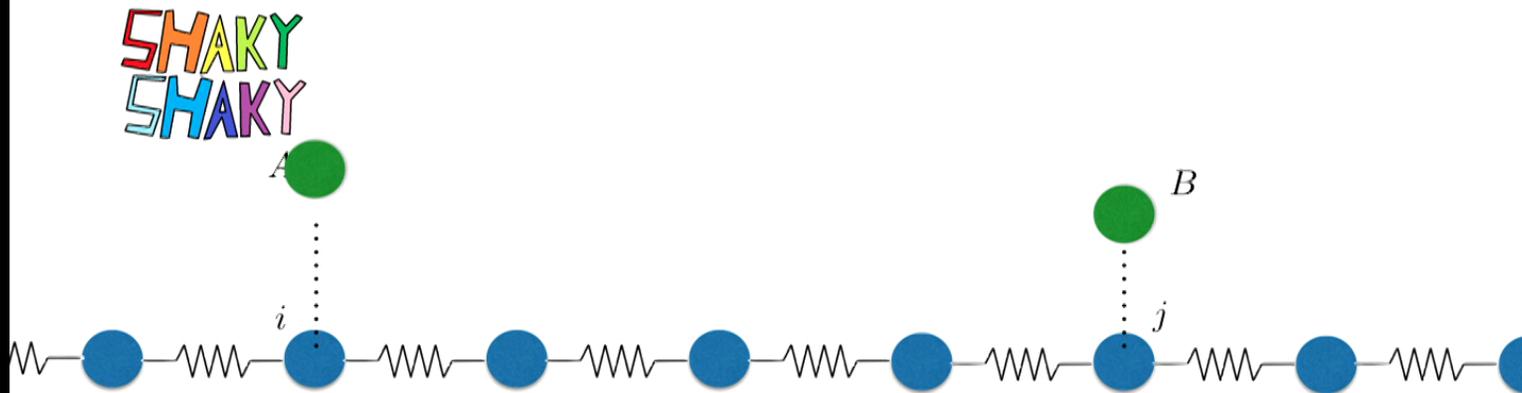
# 1-D Harmonic lattice in the Ground state

How do we get two systems entangled by means of local interactions with a lattice in the ground state?



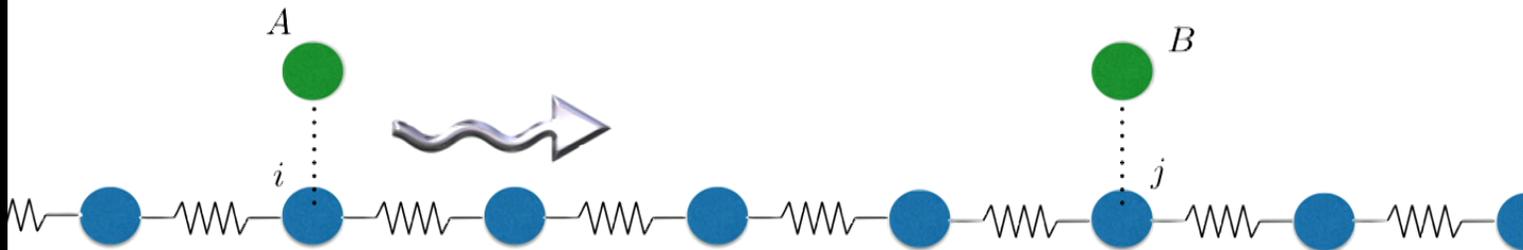
Two possible mechanisms.

# 1-D Harmonic lattice in the Ground state



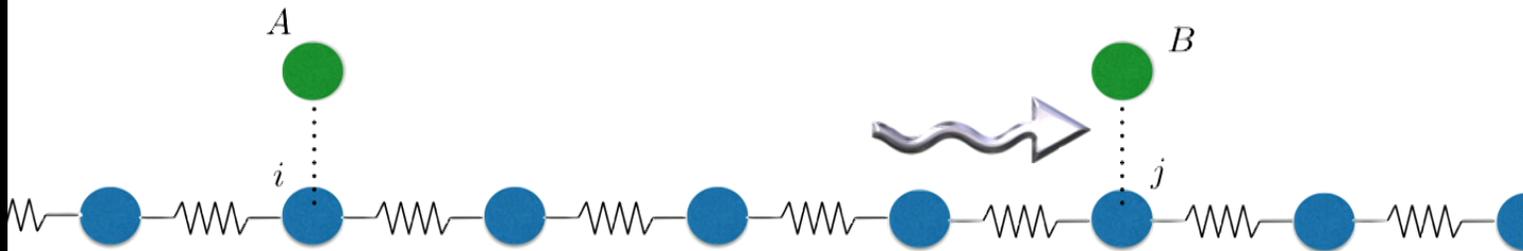
## 1) Communication via phonons

# 1-D Harmonic lattice in the Ground state



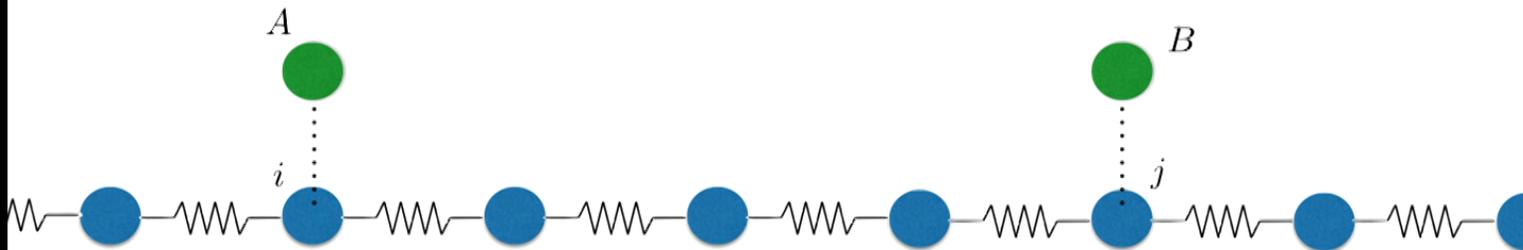
## 1) Communication via phonons

# 1-D Harmonic lattice in the Ground state



## 1) Communication via phonons

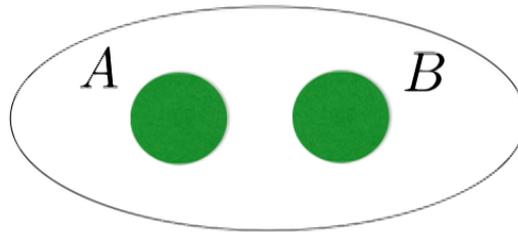
# 1-D Harmonic lattice in the Ground state



## 1) Communication via phonons

# 1-D Harmonic lattice in the Ground state

## 1) Communication via phonons

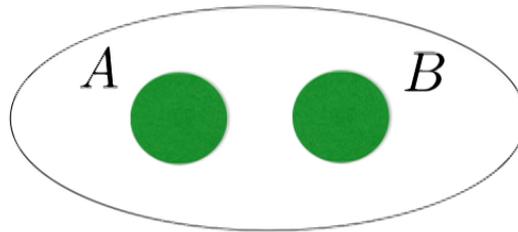


$$\rho_{AB} \neq \sum_i p_i \rho_A \otimes \rho_B$$

Limited by the speed of 'sound'

# 1-D Harmonic lattice in the Ground state

## 1) Communication via phonons



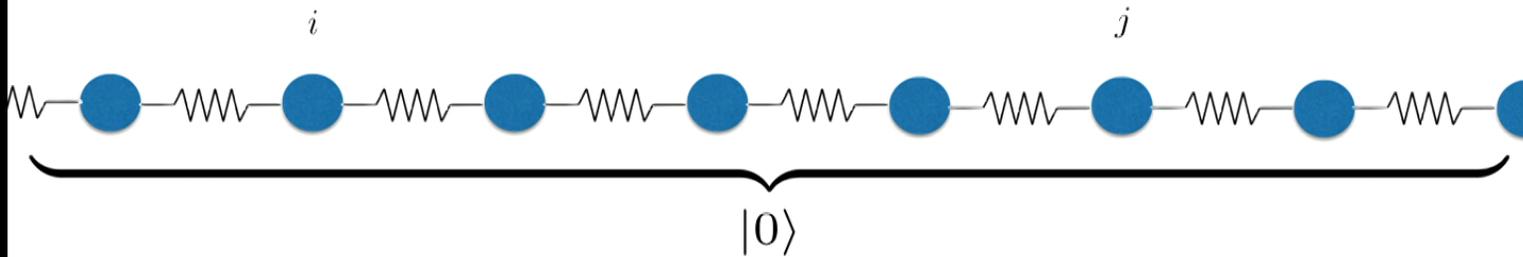
$$\rho_{AB} \neq \sum_i p_i \rho_A \otimes \rho_B$$

Limited by the speed of 'sound'

# 1-D Harmonic lattice in the Ground state

There's another possibility:

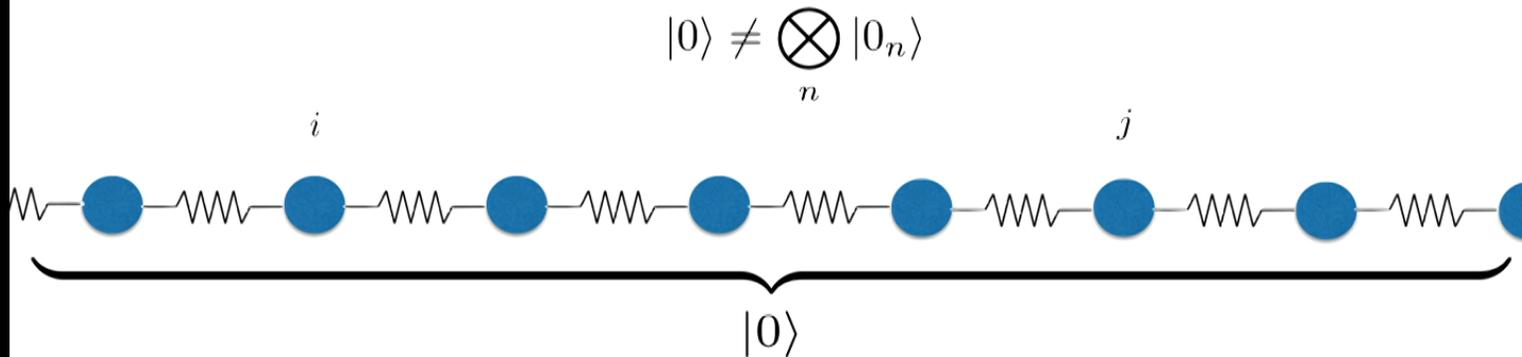
Take advantage of pre-existent entanglement



# 1-D Harmonic lattice in the Ground state

**'Non-local' basis: Normal modes**  $|0\rangle, |1\rangle, |2\rangle, \dots$

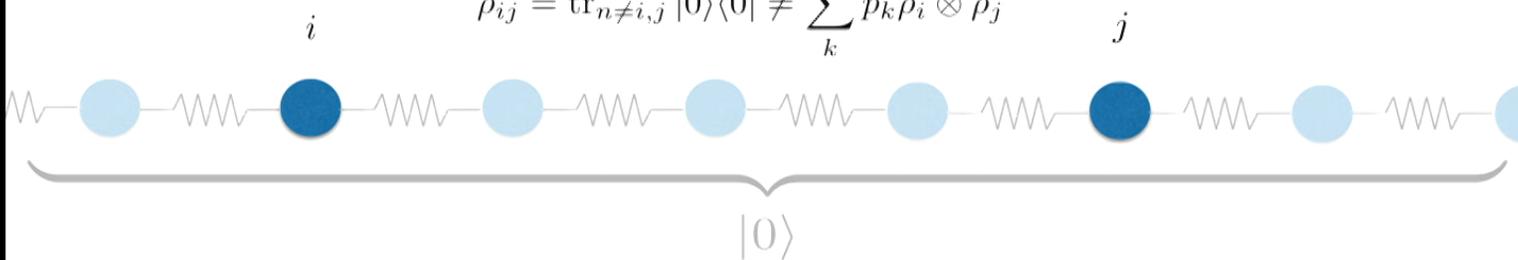
**'Local' basis: individual number states**  $\{|n_1, \dots, n_i, \dots, n_j, \dots\rangle\}$



# 1-D Harmonic lattice

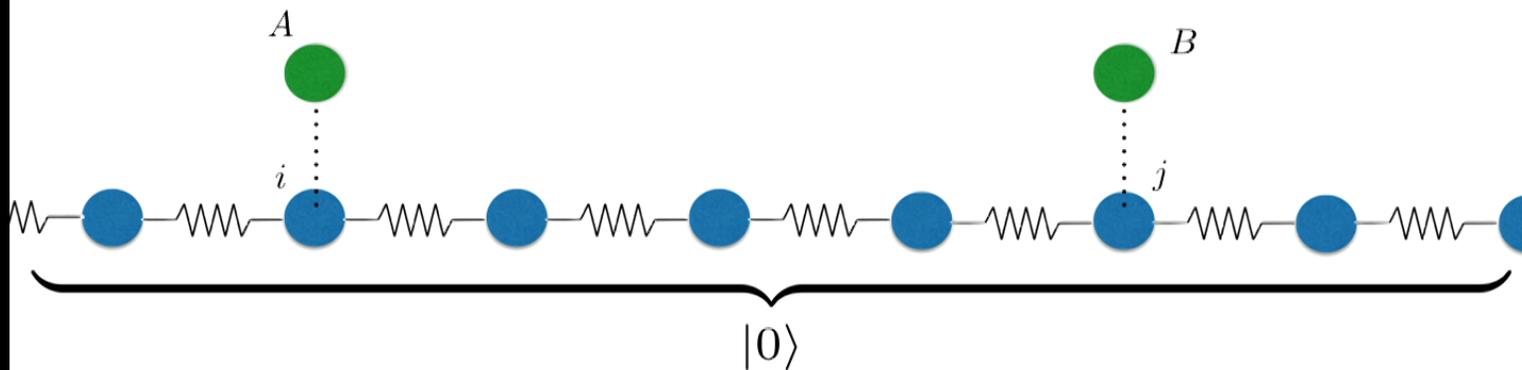
$$|0\rangle \neq \bigotimes_n |0_n\rangle$$

$$\rho_{ij} = \text{tr}_{n \neq i,j} |0\rangle\langle 0| \neq \sum_k p_k \rho_i \otimes \rho_j$$



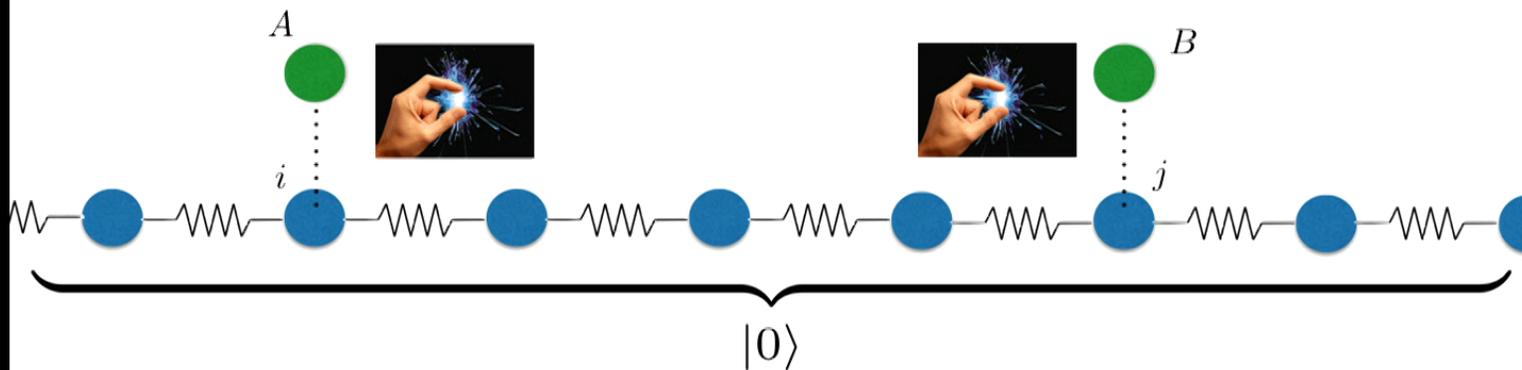
# 1-D Harmonic lattice in the Ground state

## 2) Swapping ground state entanglement



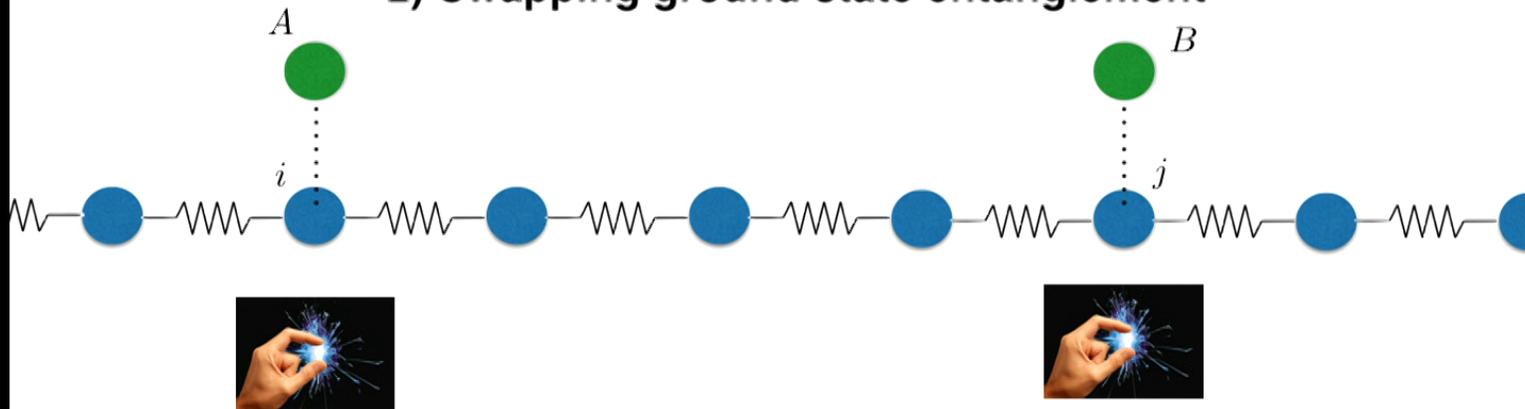
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## 2) Swapping ground state entanglement



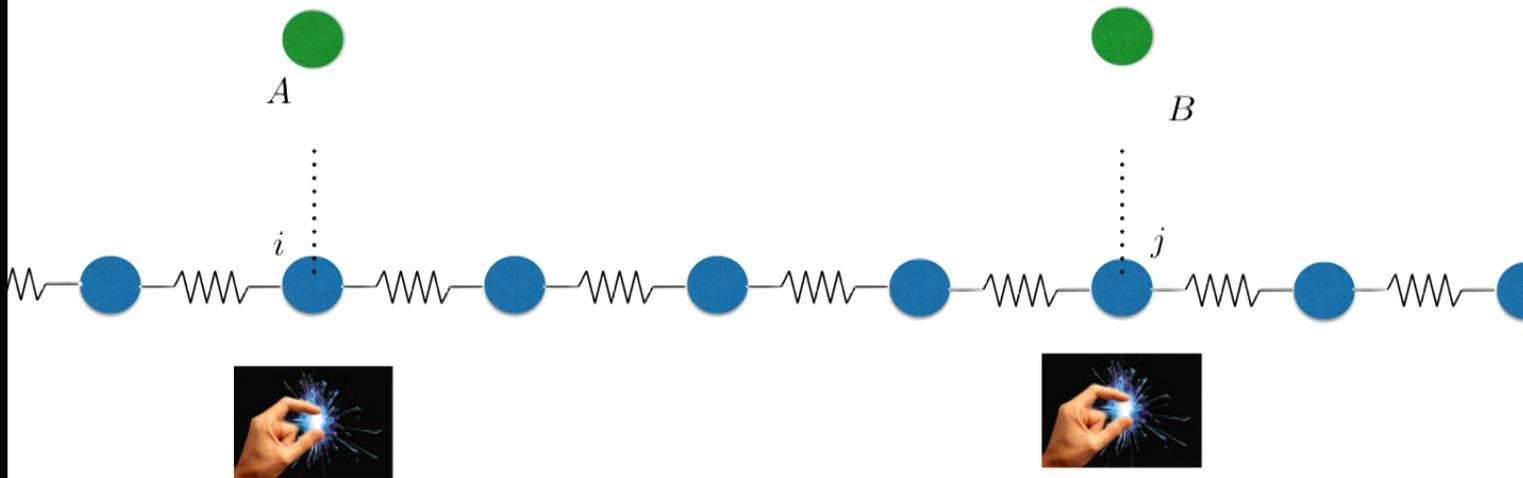
# 1-D Harmonic lattice in the Ground state

## 2) Swapping ground state entanglement



# 1-D Harmonic lattice in the Ground state

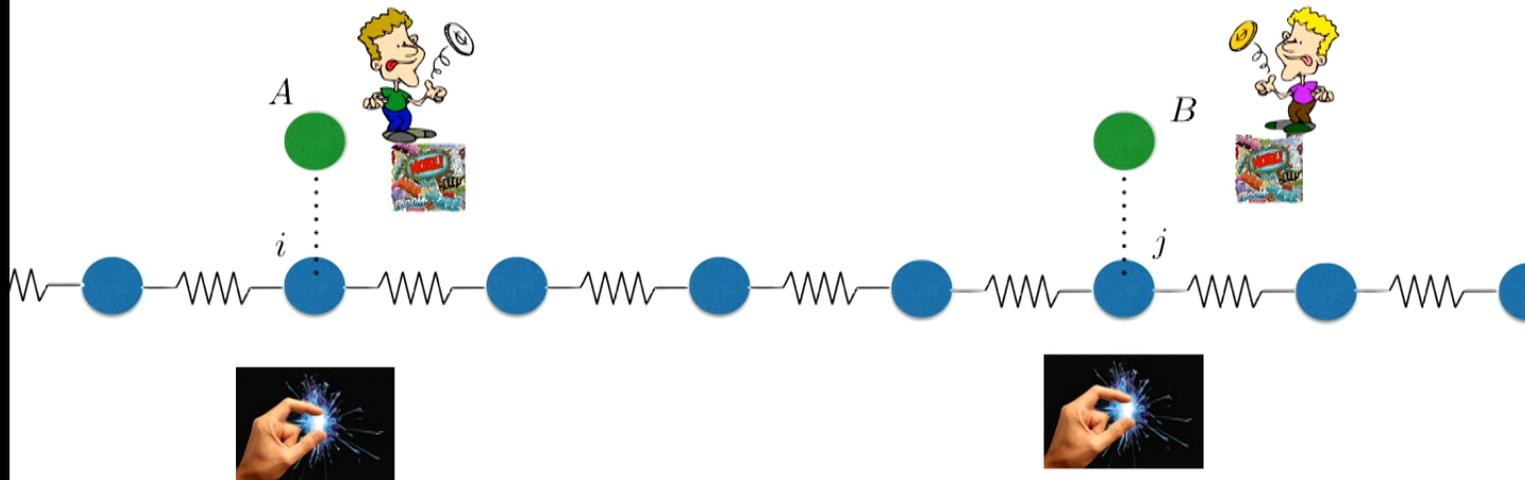
Local coupling to the vacuum: Observed fluctuations are correlated



2) Swapping ground state entanglement

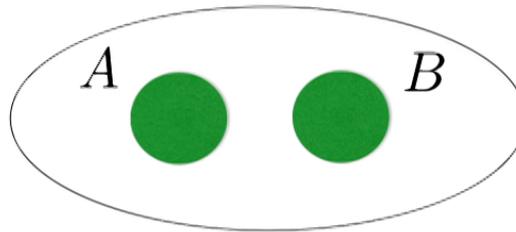
# 1-D Harmonic lattice in the Ground state

## 2) Swapping ground state entanglement



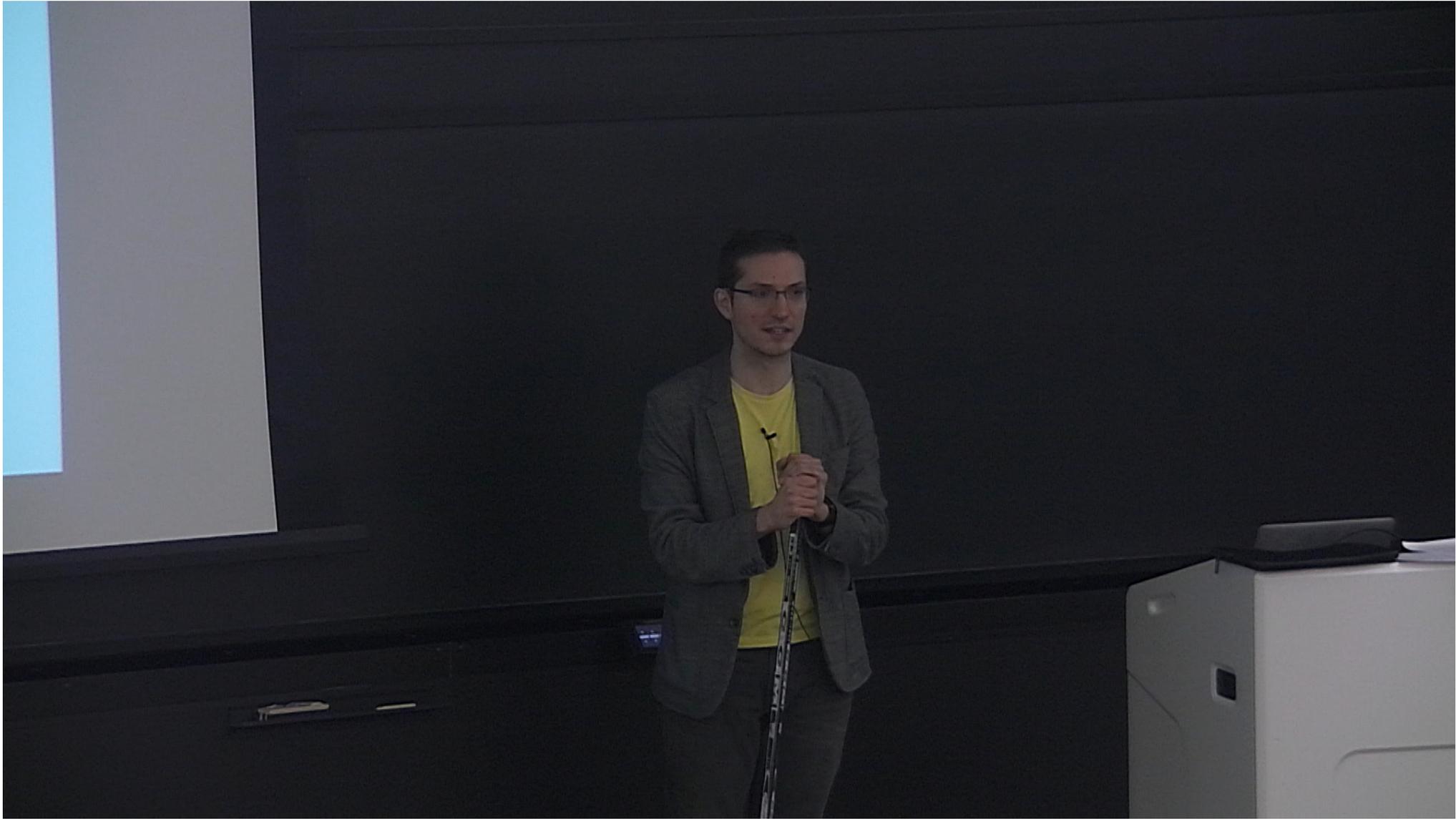
# 1-D Harmonic lattice in the Ground state

## 2) Swapping ground state entanglement



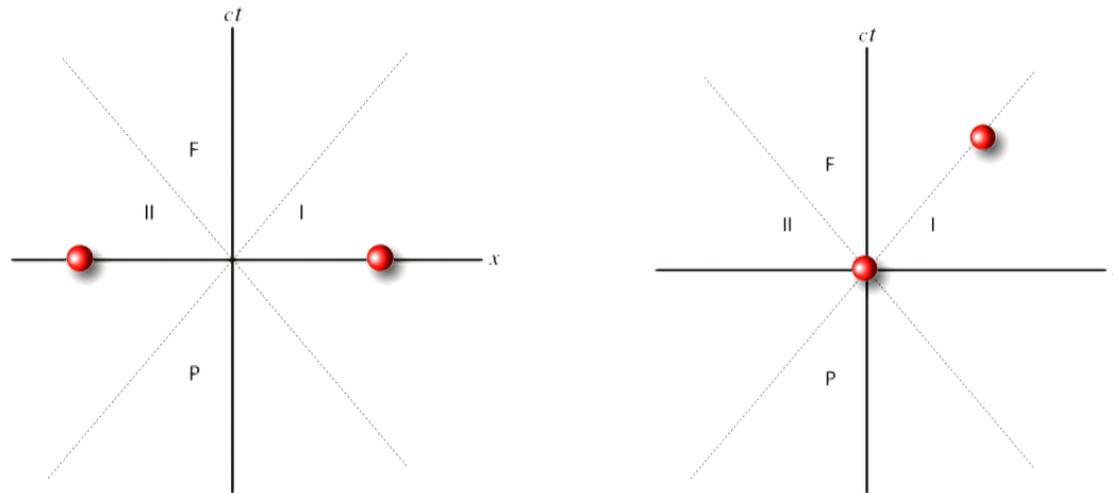
$$\rho_{AB} \neq \sum_i p_i \rho_A \otimes \rho_B$$

**NOT Limited by the speed of 'sound'**



# Quantum Fields

A 1D quantum field can be thought as the 'continuum limit' of such a lattice



Two mechanisms to get 'atoms' entangled via interaction with quantum fields:

- 1) Via exchange of real field quanta
- 2) Swapping vacuum entanglement

We have two particle detectors on the ground state coupled to a scalar field vacuum  
 $\hat{\rho}_0 = \hat{\rho}_a \otimes \hat{\rho}_b \otimes |0\rangle\langle 0|$  the detectors are inertial and comoving

$$H_I = \sum_{\nu \in \{A, B\}} \lambda_\nu \chi_\nu(t) \int d\vec{x} F_\nu(\vec{x} - \vec{x}_\nu) \hat{m}_\nu(t) \hat{\phi}(\vec{x}, t)$$

if the coupling is weak

$$\hat{U} = \mathcal{T} \exp\left(-i \int_{-\infty}^{\infty} dt \hat{H}_I(t)\right) = \mathbb{1} + \hat{U}^{(1)} + \hat{U}^{(2)} + \mathcal{O}(\lambda^3)$$

Dyson expansion:  $\hat{U}^{(1)} = -i \int_{-\infty}^{\infty} dt \hat{H}_I(t)$   
 $\hat{U}^{(2)} = \int_{-\infty}^{\infty} dt \int_{-\infty}^t dt' \hat{H}_I(t) \hat{H}_I(t')$

$$\hat{\rho}_{T,AB} = \text{tr}_F(\hat{U} \hat{\rho}_0 \hat{U}^\dagger) = \hat{\rho}_{0,AB} + \hat{\rho}_{T,AB}^{(1)} + \hat{\rho}_{T,AB}^{(2)} + \mathcal{O}(\lambda^3)$$

$$\text{tr}_F(\hat{\rho}_0)$$

$$j^{(n)} = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' H_2(t) H_2(t')$$

$$\hat{\rho}_T^{(1)} = \hat{U}^{(1)} \hat{\rho}_0 + \hat{\rho}_0 \hat{U}^{(1)\dagger}, \quad \rho_{T,AB}^{(1)} = \text{tr}_F(\hat{\rho}_T^{(1)}) \quad \text{for any state } \hat{\rho}_0 \text{ diagonal in the Fock basis} \quad V(t, \vec{x}) = \text{tr}(\hat{\rho}_0 \hat{\phi}) = 0$$

$$\hat{\rho}_{T,AB}^{(1)} = \text{tr}_q(\hat{U} \hat{\rho}_0 \hat{U}^\dagger) = \hat{\rho}_{0,AB} + \hat{\rho}_{T,AB}^{(1)} + \hat{\rho}_{T,AB}^{(2)} + \mathcal{O}(\lambda^3)$$

$$\hat{U}^{(1)} = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \hat{H}_2(t) \hat{H}_2(t')$$

$$\hat{\rho}_T^{(1)} = \hat{U}^{(1)} \hat{\rho}_0 + \hat{\rho}_0 \hat{U}^{(1)\dagger}, \quad \rho_{T,AB}^{(1)} = \text{tr}_q(\hat{\rho}_T^{(1)}) \text{ for any state } \hat{\rho}_0 \text{ diagonal in the Fock basis}$$

$$V(\epsilon, \vec{r}) = \text{tr}(\hat{\rho}_q \hat{\phi}) = 0 \Rightarrow \text{tr}_q(\hat{\rho}_0^{(1)}) = 0$$

$$\rho_{T,AB}^{(1)} = 0$$

$$\hat{\rho}_T^{(2)} = \hat{U}^{(1)} \hat{\rho}_0 \hat{U}^{(1)\dagger} + \hat{U}^{(2)} \hat{\rho}_0 + \hat{\rho}_0 \hat{U}^{(2)\dagger}$$



$$\hat{\rho}_{T,AB}^{(1)} = \text{tr}_p(\hat{U} \hat{\rho}_0 \hat{U}^\dagger) = \hat{\rho}_{0,AB}^{(1)} + \hat{\rho}_{T,AB}^{(1)} + \hat{\rho}_{T,AB}^{(2)} + \mathcal{O}(\lambda^3)$$

$$\text{tr}_p(\hat{\rho}_0)$$

$$U^{(n)} = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \hat{H}_I(t) \hat{H}_I(t')$$

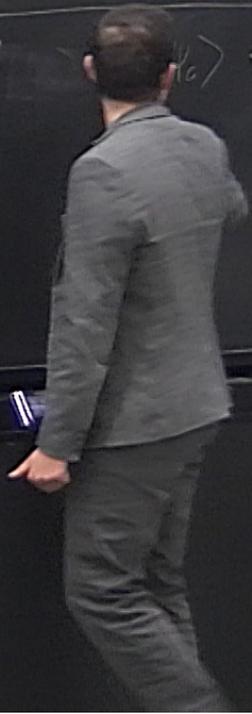
$$\hat{\rho}_T^{(1)} = \hat{U}^{(1)} \hat{\rho}_0 + \hat{\rho}_0 \hat{U}^{(1)\dagger}, \quad \hat{\rho}_{T,AB}^{(1)} = \text{tr}_p(\hat{\rho}_T^{(1)}) \text{ for any state } \hat{\rho}_0 \text{ diagonal in the Fock basis}$$

$$V(\mathbf{r}, \mathbf{r}') = \text{tr}(\hat{\rho}_0 \hat{\phi}) = 0 \Rightarrow \text{tr}_p(\hat{\rho}_0^{(1)}) = 0$$

$$\hat{\rho}_{T,AB}^{(1)} = 0$$

$$\hat{\rho}_T^{(2)} = \hat{U}^{(1)} \hat{\rho}_0 \hat{U}^{(1)\dagger} + \hat{U}^{(2)} \hat{\rho}_0 + \hat{\rho}_0 \hat{U}^{(2)\dagger}$$

$$|\psi_0\rangle + U^{(1)}|\psi_0\rangle + U^{(2)}|\psi_0\rangle$$



$$\hat{\rho}_{T,AB}^{(1)} = \text{tr}_q(\hat{U} \hat{\rho}_0 \hat{U}^\dagger) = \hat{\rho}_{0,AB}^{(1)} + \hat{\rho}_{T,AB}^{(1)} + \hat{\rho}_{T,AB}^{(2)} + \mathcal{O}(\lambda^3)$$

$$\text{tr}_q(\hat{\rho}_0)$$

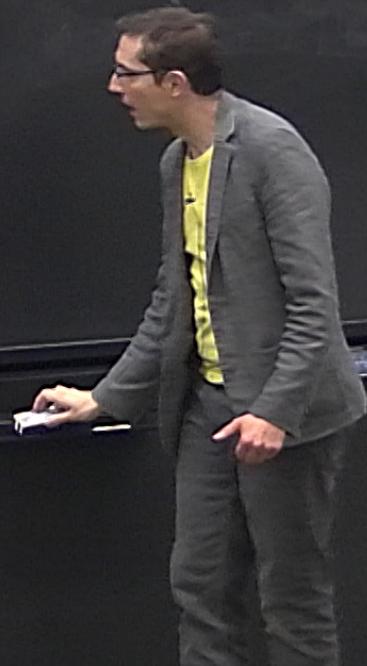
$$\hat{U}^{(1)} = \int_{-a}^a dt \int_{-a}^a dt' \hat{H}_2(t) \hat{H}_2(t')$$

$$\hat{\rho}_T^{(1)} = \hat{U}^{(1)} \hat{\rho}_0 + \hat{\rho}_0 \hat{U}^{(1)\dagger}, \quad \hat{\rho}_{T,AB}^{(1)} = \text{tr}_q(\hat{\rho}_T^{(1)}) \text{ for any state } \hat{\rho}_0 \text{ diagonal in the Fock basis}$$

$$V(\vec{r}, \vec{r}') = \text{tr}(\hat{\rho}_q \hat{\phi}) = 0 \Rightarrow \text{tr}_q(\hat{\rho}_0) = 0$$

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$$\hat{\rho}_T^{(2)} = \sum_{\alpha\beta} \lambda_\alpha \lambda_\beta \left[ \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \chi_\alpha(t) \chi_\beta(t') \hat{m}_\alpha(t) \hat{\rho}_{\alpha\beta} \hat{m}_\beta(t') W(\vec{x}_\alpha, t, \vec{x}_\beta, t') - \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \chi_\alpha(t) \chi_\beta(t') (\hat{m}_\alpha(t) \hat{m}_\beta(t') \hat{\rho}_{\alpha\beta} W(\vec{x}_\alpha, t, \vec{x}_\beta, t') + \hat{\rho}_{\alpha\beta} \hat{m}_\alpha(t) \hat{m}_\beta(t') W(\vec{x}_\beta, t, \vec{x}_\alpha, t')) \right]$$

$$W(\vec{x}_\alpha, t, \vec{x}_\beta, t') = \int d\vec{x} \int d\vec{x}' F(\vec{x} - \vec{x}_\alpha) F(\vec{x}' - \vec{x}_\beta) \underbrace{\text{Tr}_q(\hat{\phi}(t, \vec{x}) \hat{\phi}(t', \vec{x}'))}_{W(\vec{x}, t, \vec{x}', t')}$$

$$\hat{P}_{TAB} = t_{\psi_f} (\hat{U} \hat{P}_0 \hat{U}^\dagger) = \hat{P}_{TAB} + \hat{P}_{TAB}^{(1)} + \hat{P}_{TAB}^{(2)} + \mathcal{O}(\lambda^3)$$

$t_{\psi_f}(\hat{P}_0)$

by expansion  $\hat{U} = 1 - i \int_{-\infty}^t \hat{H}_I(t') dt'$

$$\hat{U}^{(1)} = \int_{-\infty}^t dt' \left[ \hat{H}_I(t') \hat{H}_I(t) \right]$$

$\hat{P}_T^{(1)} = \hat{U}^{(1)} \hat{P}_0 + \hat{P}_0 \hat{U}^{(1)}$ ,  $\hat{P}_{TAB}^{(1)} = t_{\psi_f}(\hat{P}_T^{(1)})$  for any state  $\hat{P}_0$  diagonal in the Fock basis  $V(\vec{x}) = t_{\psi_f}(\hat{P}_0) = 0 \Rightarrow t_{\psi_f}(\hat{P}_0) = 0$

$$\hat{P}_{TAB}^{(1)} = 0$$

$$\hat{P}_T^{(2)} = \hat{U}^{(1)} \hat{P}_0 \hat{U}^{(1)\dagger} + \hat{U}^{(2)} \hat{P}_0 + \hat{P}_0 \hat{U}^{(2)\dagger}$$

$$\hat{P}_{TAB}^{(2)} = \sum_{\lambda\eta} |\chi_\lambda\rangle \langle \chi_\eta| \left[ \int_{-\infty}^t dt \int_{-\infty}^t dt' \chi_\lambda(t) \chi_\eta(t') \hat{m}_\alpha(t) \hat{P}_{TAB} \hat{m}_\beta(t') W(\vec{x}_\lambda, t, \vec{x}_\eta, t') - \int_{-\infty}^t dt \int_{-\infty}^t dt' \chi_\lambda(t) \chi_\eta(t') \left( \hat{m}_\alpha(t) \hat{m}_\beta(t') \hat{P}_{TAB} W(\vec{x}_\lambda, t, \vec{x}_\eta, t') + \hat{P}_{TAB} \hat{m}_\alpha(t) \hat{m}_\beta(t') W(\vec{x}_\lambda, t, \vec{x}_\eta, t') \right) \right]$$

$$W(\vec{x}_\lambda, t, \vec{x}_\eta, t') = \int d\vec{x} \int d\vec{x}' F(\vec{x} - \vec{x}_\lambda) F(\vec{x}' - \vec{x}_\eta) \underbrace{T_{\psi_f}(\hat{\phi}(t, \vec{x}) \hat{\phi}(t', \vec{x}'))}_{W(t, \vec{x}, t', \vec{x}')}$$



$$W(\vec{x}_1, t_1, \vec{x}_2, t_2) = \int_{-\infty}^{t_1} dt \int_{-\infty}^{t_2} dt' F(\vec{x} - \vec{x}_0) F(\vec{x}' - \vec{x}_1)$$

Assuming  $\hat{p}_{q/0} = |0\rangle\langle 0|$

$$\hat{p}_{T,AB} = \begin{pmatrix} 1 - L_{AA} - L_{BB} & 0 & 0 & M \\ 0 & L_{AA} & L_{AB} & 0 \\ 0 & L_{AB}^* & L_{BB} & 0 \\ M^* & 0 & 0 & 0 \end{pmatrix} + \mathcal{O}(\lambda_0^4)$$

basis  $\left\{ |g_A\rangle \otimes |g_B\rangle, |e_A\rangle \otimes |g_B\rangle, |g_A\rangle \otimes |e_B\rangle, |e_A\rangle \otimes |e_B\rangle \right\}$

$$\rho_{AB} = \rho_A \otimes \rho_B$$

For two identical detectors the partial transposed density matrix  $\rho_{TAB}^{PT}$  has only a (single) negative eigenvalue. The Negativity of  $\rho_{TAB}$  is

$$N(\rho_{TAB}) = \min(0, M - L_{AA})$$

For two identical detectors the partial transposed density matrix  $\hat{\rho}_{T,AB}$  has only a (simple) negative eigenvalue. The Negativity of  $\hat{\rho}_{T,AB}$  is

$$N(\hat{\rho}_{T,AB}) = \min(0, M - L_{AA}) + \mathcal{O}(\lambda_0^q)$$