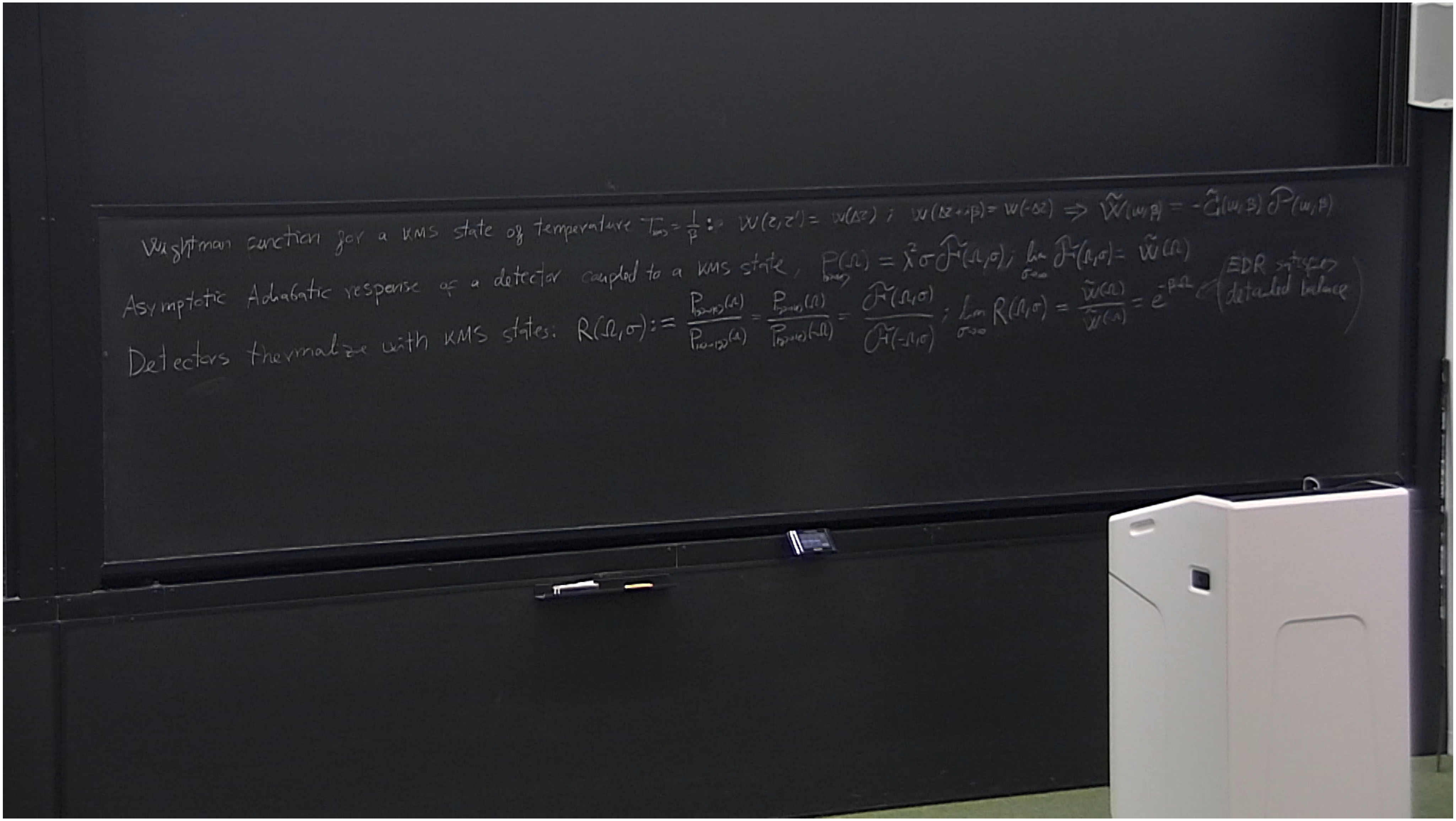
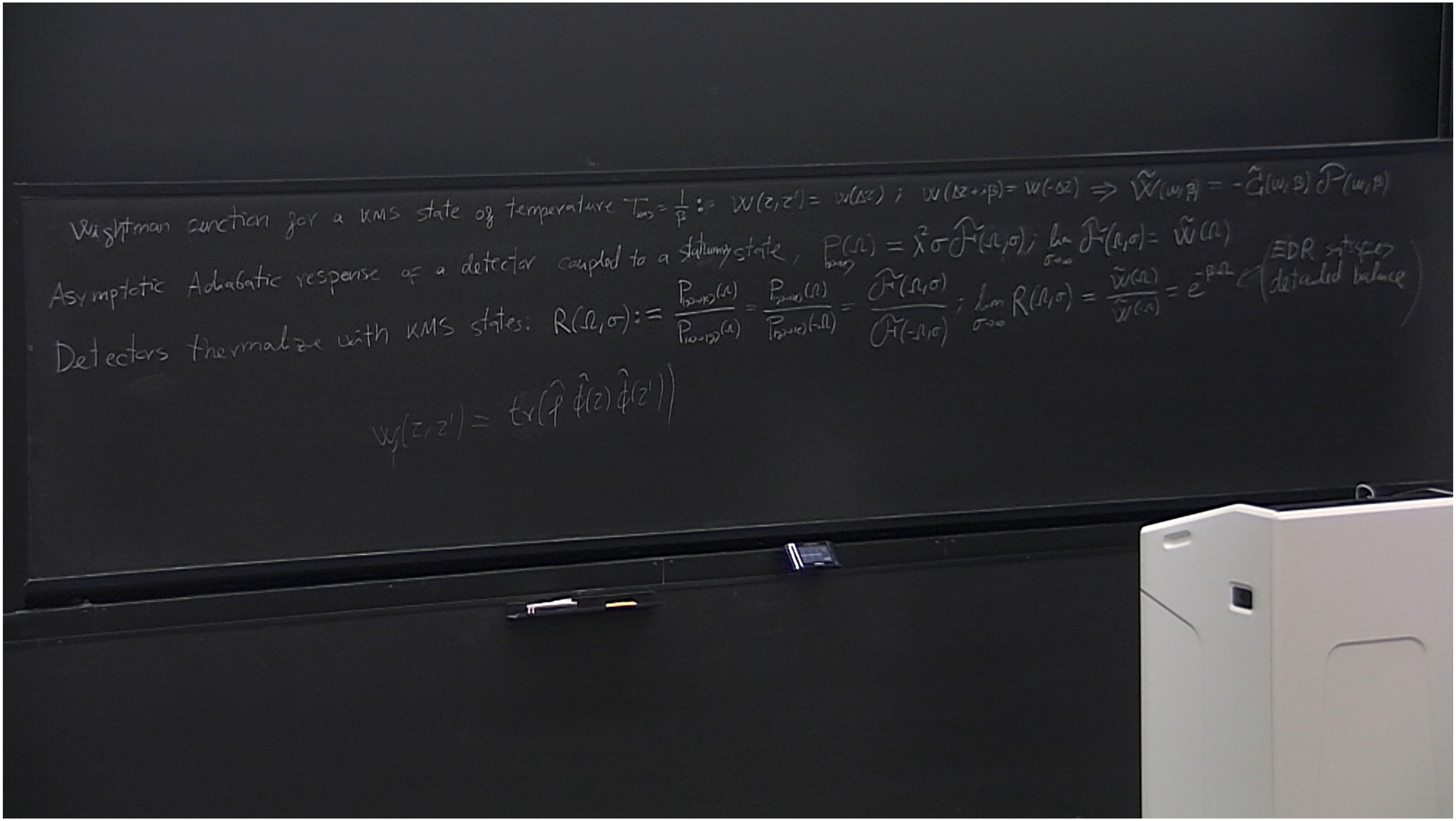
Title: PSI 2017/2018 - Relativistic Quantum Information - Lecture 7

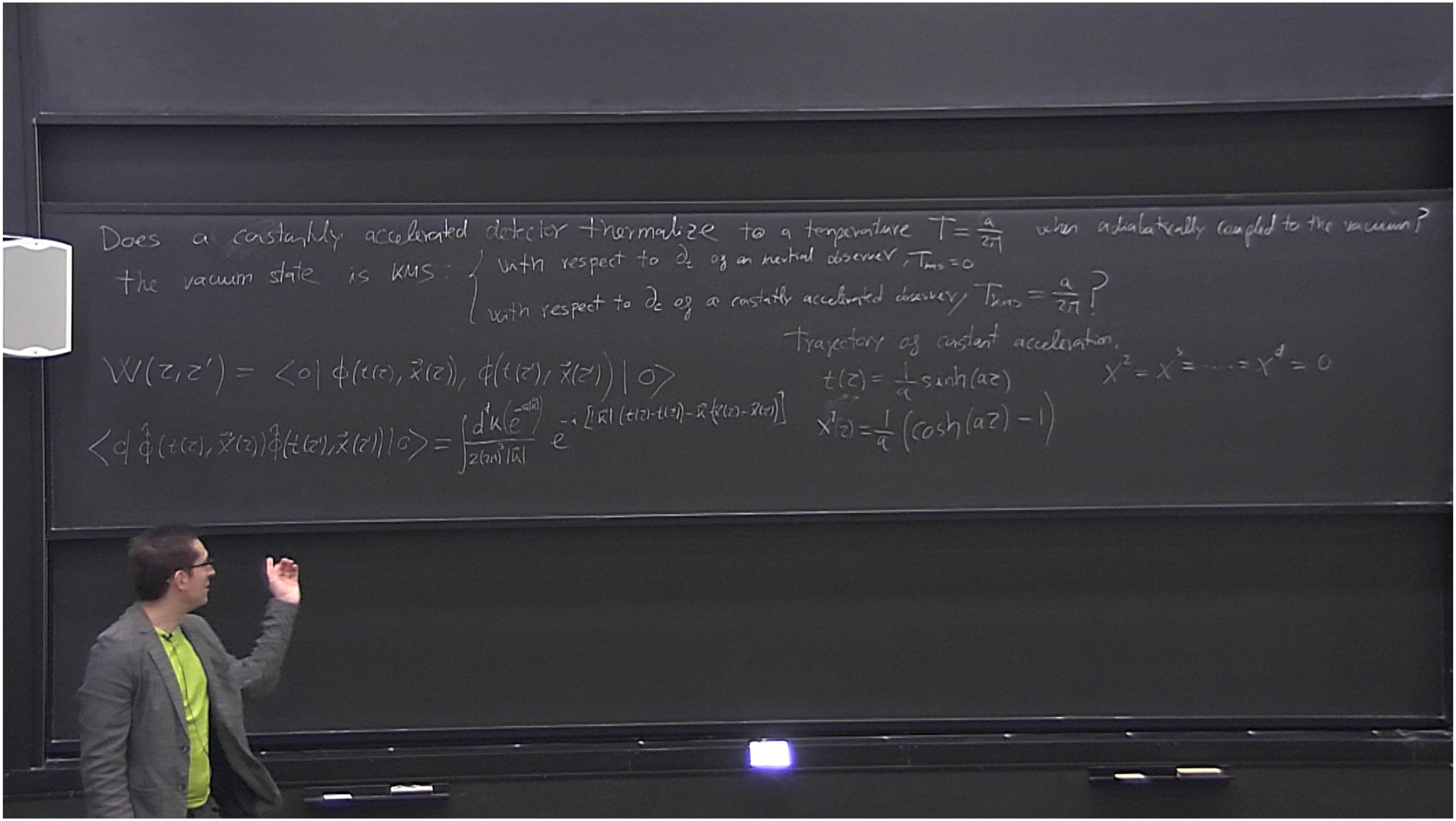
Date: Mar 27, 2018 09:00 AM

URL: http://pirsa.org/18030034

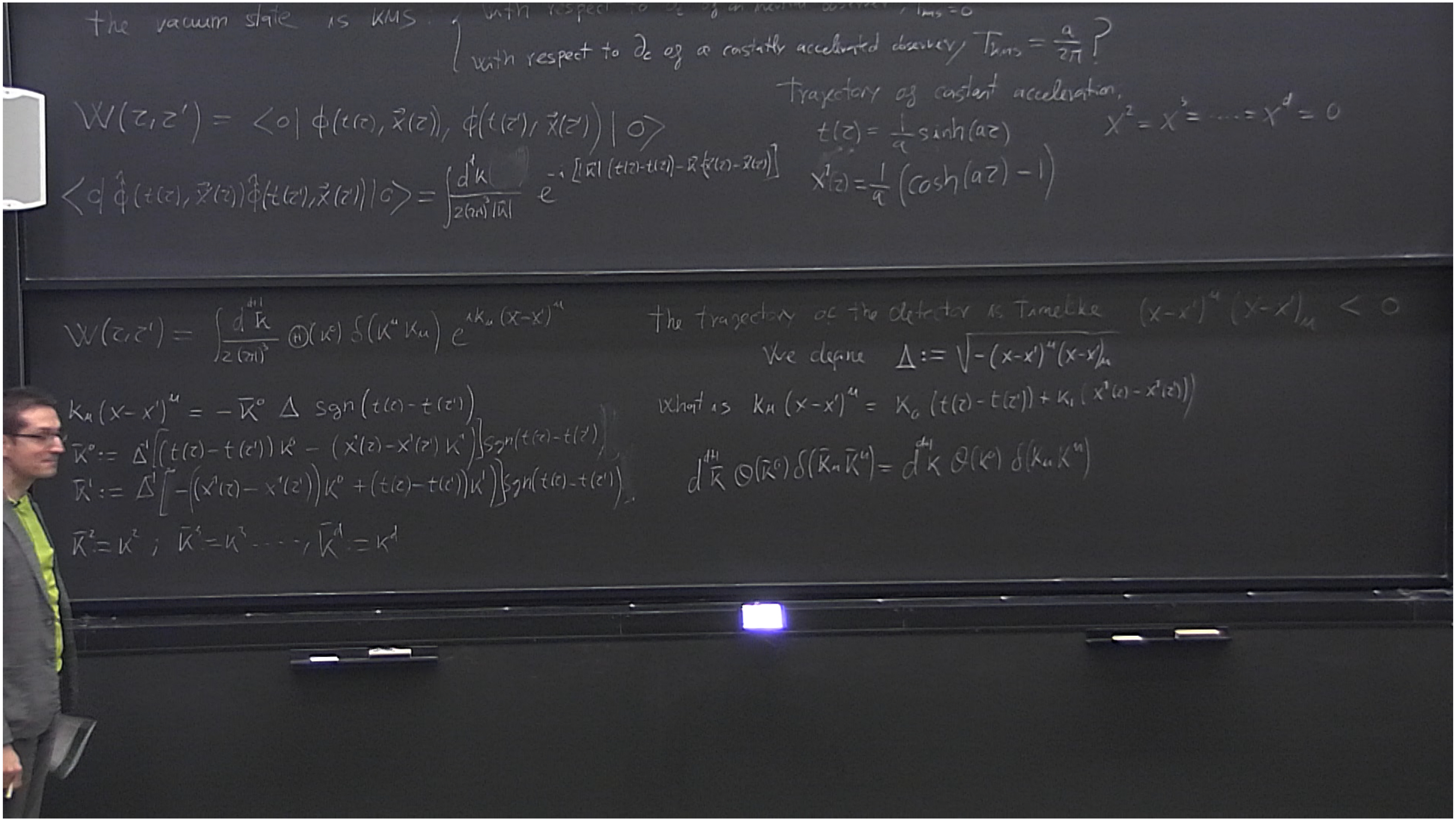
Abstract:

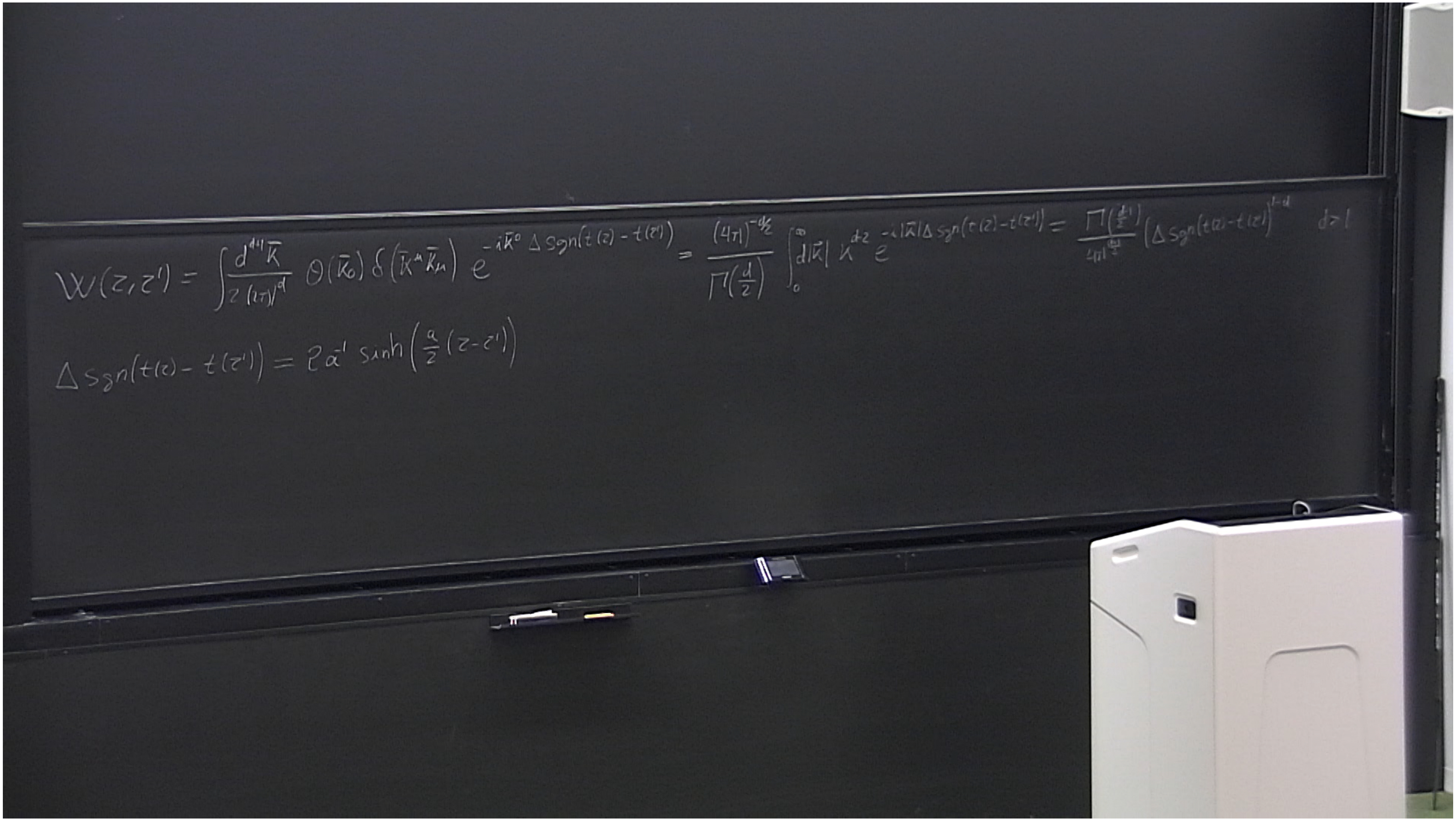

Wightman function for a KMS state of temperature $T_{KMS} = \frac{1}{\beta}$: $W(\tau,\tau') = W(\Delta\tau)$; $W(\Delta\tau + i\beta) = W(-\Delta\tau) \Rightarrow \tilde{W}(\omega,\beta) = -\tilde{G}(\omega,\beta)\,\mathcal{P}(\omega,\beta)$
Asymptotic Adiabatic response of a detector coupled to a KMS state, $P(\Omega) = \lambda^2 \sigma \tilde{\mathcal{F}}(\Omega,\sigma)$; $\lim_{\sigma\to\infty} \tilde{\mathcal{F}}(\Omega,\sigma) = \tilde{W}(\Omega)$
Detectors thermalize with KMS states: $R(\Omega,\sigma) := \frac{P(\Omega)}{P(-\Omega)} = \frac{\tilde{\mathcal{F}}(\Omega,\sigma)}{\tilde{\mathcal{F}}(-\Omega,\sigma)}$; $\lim_{\sigma\to\infty} R(\Omega,\sigma) = \frac{\tilde{W}(\Omega)}{\tilde{W}(-\Omega)} = e^{-\beta\Omega}$
EDR satisfies detailed balance

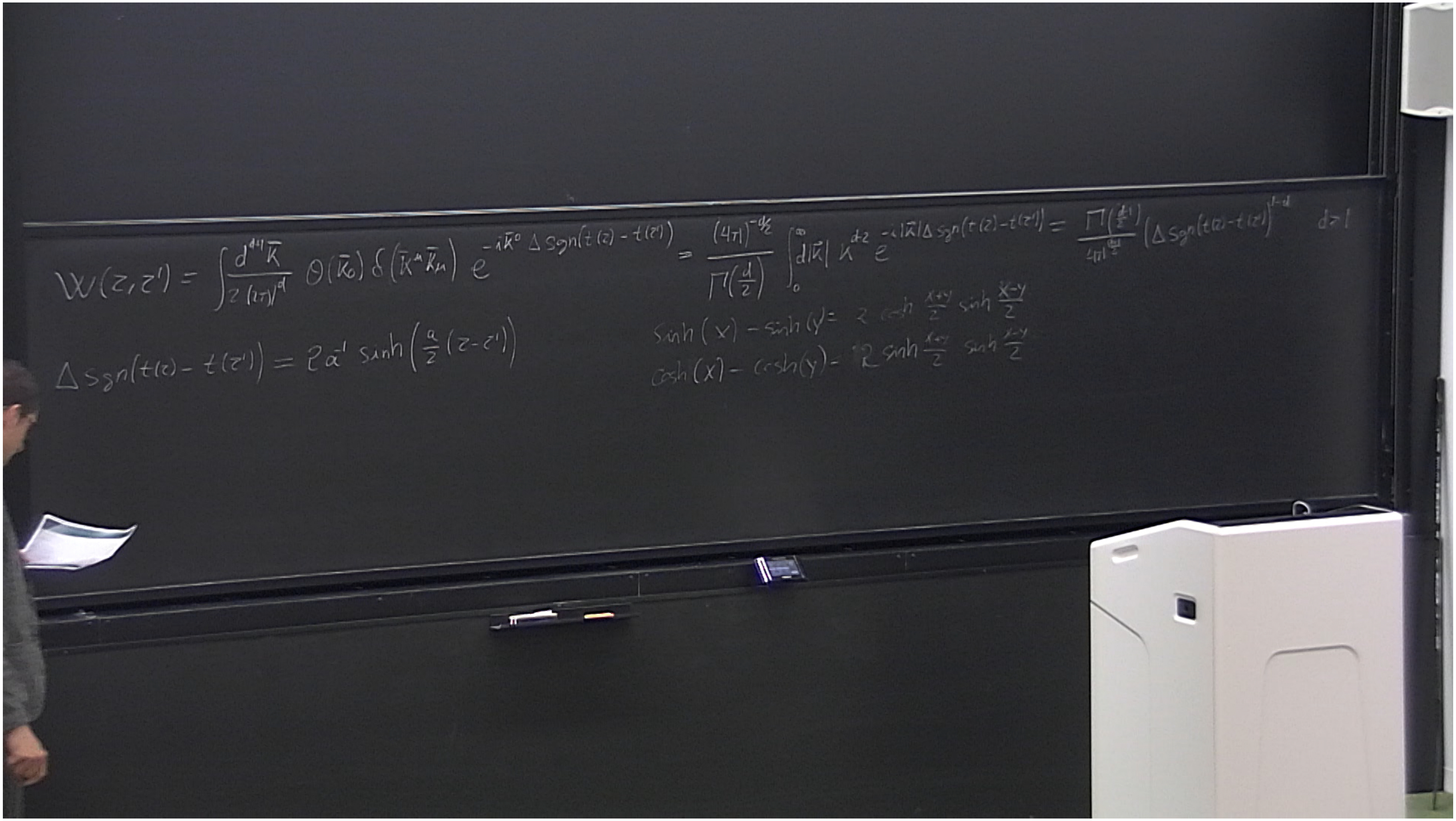

Wightman function for a KMS state of temperature T_KMS = 1/β : W(τ,τ') = W(Δτ) ; W(Δτ + iβ) = W(−Δτ) ⇒ W̃(ω,β) = −G̃(ω,β) P(ω,β)
Asymptotic Adiabatic response of a detector coupled to a stationary state, P(Ω) = λ²σ F̃(Ω,σ) ; lim F̃(Ω,σ) = W̃(Ω)
EDR satisfies (detailed balance)
Detectors thermalize with KMS states: R(Ω,σ) := = F̃(Ω,σ) / F̃(−Ω,σ) ; lim R(Ω,σ) = W̃(Ω) / W̃(−Ω) = e^{−βΩ}
W_ρ(τ,τ') = tr(ρ̂ φ̂(τ) φ̂(τ'))

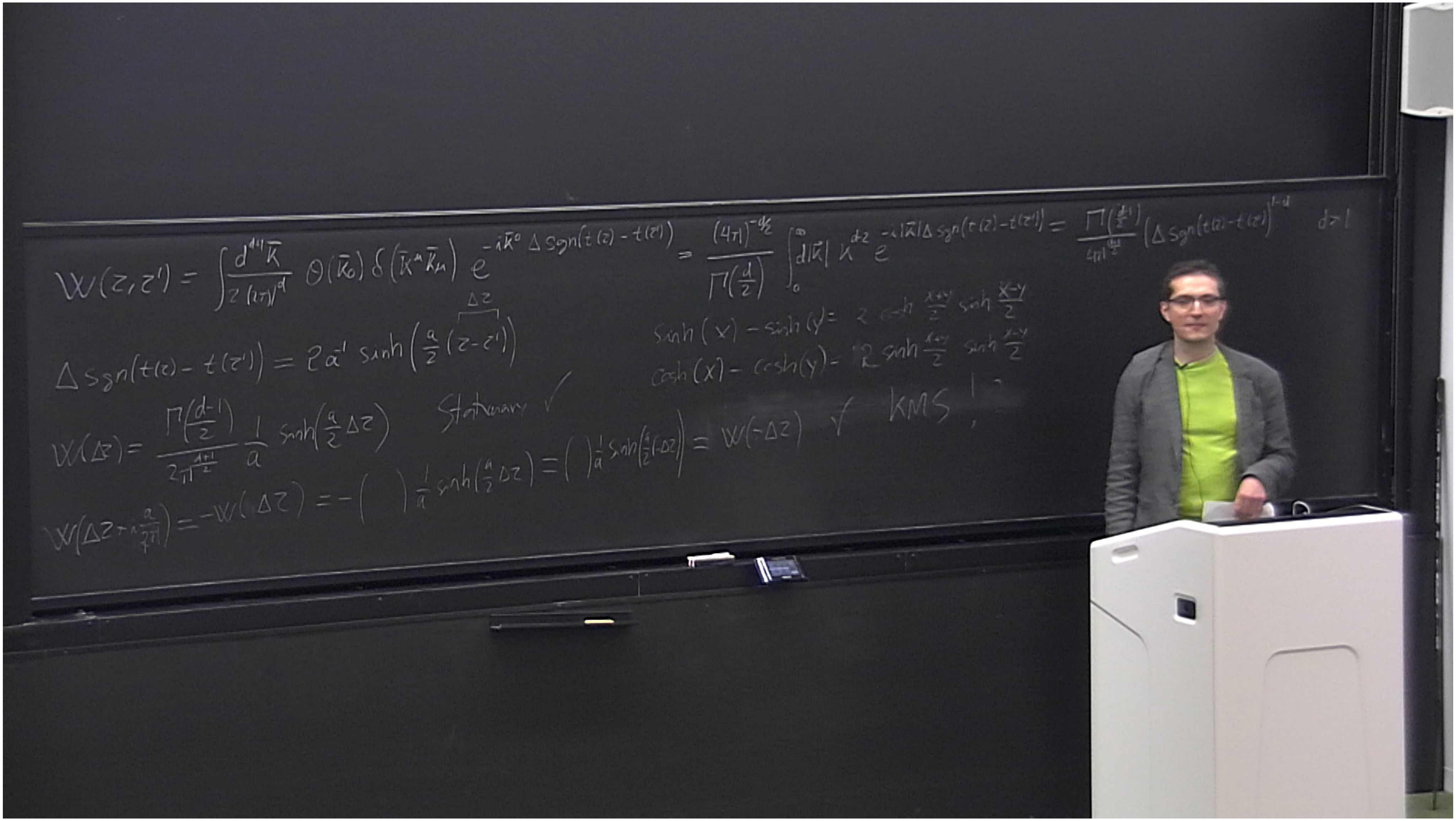


Does a constantly accelerated detector thermalize to a temperature $T = \frac{a}{2\pi}$ when adiabatically coupled to the vacuum?
the vacuum state is KMS: with respect to ∂_t of an inertial observer, $T_{KMS} = 0$
with respect to ∂_τ of a constantly accelerated observer, $T_{KMS} = \frac{a}{2\pi}$?!
Trajectory of constant acceleration,
$W(\tau,\tau') = \langle 0 | \phi(t(\tau), \vec{x}(\tau)), \phi(t(\tau'), \vec{x}(\tau')) | 0 \rangle$
$t(\tau) = \frac{1}{a}\sinh(a\tau)$
$x^2 = x^3 = \dots = x^d = 0$
$\langle 0 | \hat{\phi}(t(\tau), \vec{x}(\tau)) \hat{\phi}(t(\tau'), \vec{x}(\tau')) | 0 \rangle = \int \frac{d^d k \,(e^{-\epsilon|\vec{k}|})}{2(2\pi)^3 |\vec{k}|} e^{-i[|\vec{k}|(t(\tau)-t(\tau')) - \vec{k}\cdot(\vec{x}(\tau)-\vec{x}(\tau'))]}$
$x^1(\tau) = \frac{1}{a}(\cosh(a\tau) - 1)$

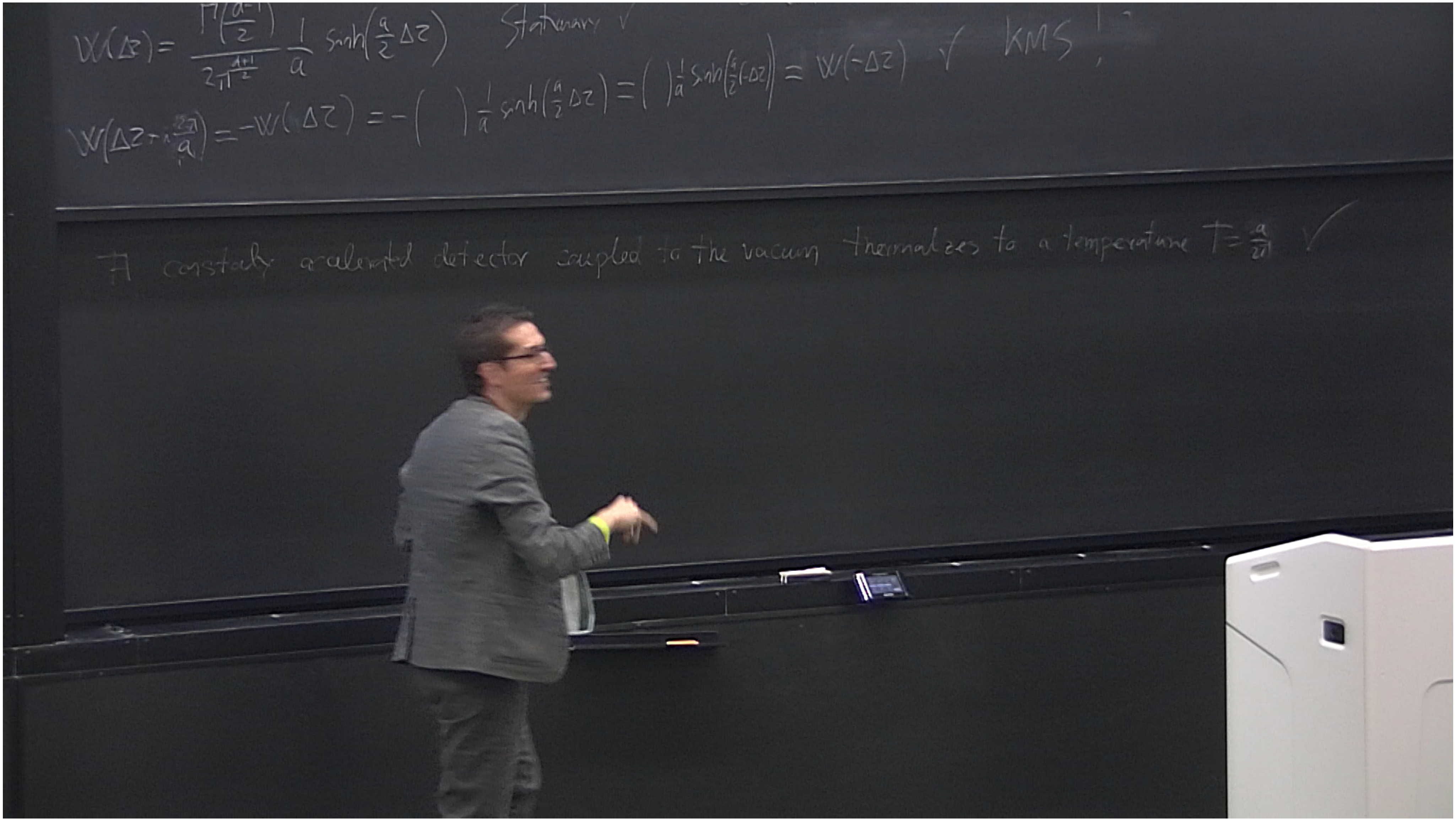

the vacuum state is KMS
with respect to ∂τ of a constantly accelerated observer, T_KMS = a/2π ?
Trajectory of constant acceleration,
W(τ,τ') = <0| φ(t(τ), x(τ)), φ(t(τ'), x(τ')) |0>
t(τ) = 1/a sinh(aτ)
x^2 = x^3 = ... = x^d = 0
x^1(τ) = 1/a (cosh(aτ) − 1)
the trajectory of the detector is Timelike (x−x')^μ (x−x')_μ < 0
We define Δ := √(−(x−x')^μ (x−x')_μ)
What is k_μ (x−x')^μ = k_0 (t(τ) − t(τ')) + k_1 (x^1(τ) − x^1(τ'))
k_μ (x−x')^μ = −k̄^0 Δ sgn(t(τ) − t(τ'))
k̄^2 = k^2 ; k̄^3 = k^3 . . . , k̄^d = k^d


$W(z,z') = \int \frac{d^{d+1}\bar{k}}{2(2\pi)^d}\, \theta(\bar{k}_0)\, \delta(\bar{k}^\mu \bar{k}_\mu)\, e^{-i\bar{k}^0 \Delta\, \mathrm{sgn}(t(z)-t(z'))} = \frac{(4\pi)^{-d/2}}{\Gamma(\frac{d}{2})} \int_0^\infty d|\vec{k}|\, k^{d-2}\, e^{-i|\vec{k}|\Delta\, \mathrm{sgn}(t(z)-t(z'))} = \frac{\Gamma(\frac{d-1}{2})}{(4\pi)^{\frac{d+1}{2}}} \left(\Delta\, \mathrm{sgn}(t(z)-t(z'))\right)^{1-d} \quad d>1$
$\Delta\, \mathrm{sgn}(t(z)-t(z')) = 2a^{-1} \sinh\left(\frac{a}{2}(z-z')\right)$


$W(z,z') = \int \frac{d^{d+1}\bar{k}}{2(2\pi)^d} \theta(\bar{k}_0) \delta(\bar{k}^a \bar{k}_a) e^{-i\bar{k}^0 \Delta \mathrm{sgn}(t(z) - t(z'))} = \frac{(4\pi)^{-d/2}}{\Gamma(\frac{d}{2})} \int_0^\infty d|\bar{k}| \, k^{d-2} e^{-i|\bar{k}| \Delta \mathrm{sgn}(t(z)-t(z'))} = \frac{\Gamma(\frac{d-1}{2})}{(4\pi)^{\frac{d+1}{2}}} (\Delta \mathrm{sgn}(t(z)-t(z')))^{1-d}$ $d>1$
$\Delta \mathrm{sgn}(t(z) - t(z')) = 2\alpha' \sinh(\frac{a}{2}(z-z'))$
$\sinh(x) - \sinh(y) = 2\cosh\frac{x+y}{2} \sinh\frac{x-y}{2}$
$\cosh(x) - \cosh(y) = 2\sinh\frac{x+y}{2} \sinh\frac{x-y}{2}$



sinh(x) - sinh(y) = 2 cosh (x+y)/2 sinh (x-y)/2
cosh(x) - cosh(y) = 2 sinh (x+y)/2 sinh (x-y)/2
Stationary ✓
KMS !
d>1

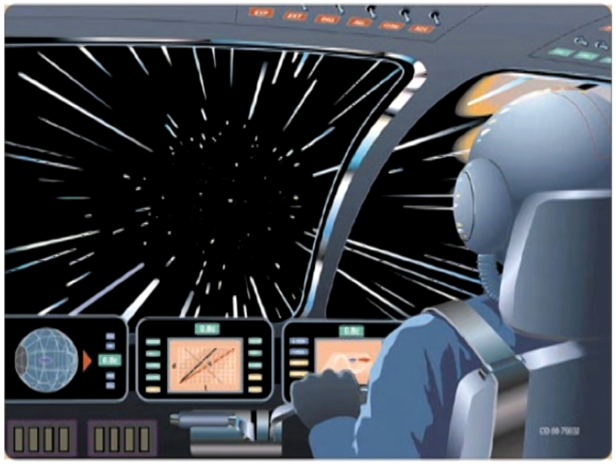


Stationary ✓
KMS !
∃ constantly accelerated detector coupled to the vacuum thermalizes to a temperature T = a/2π ✓

The Unruh Effect

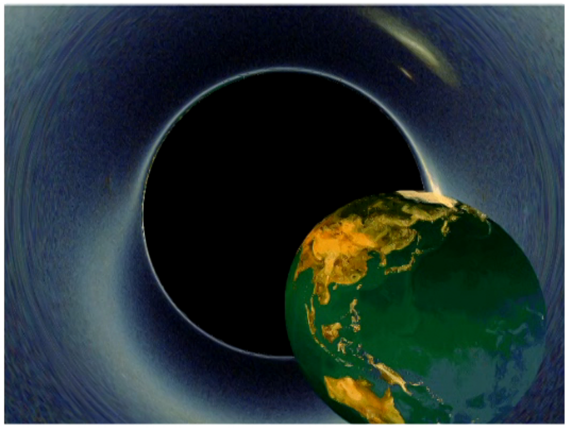
Inertial frame

Accelerated frame



- Alice Observes the field vacuum.
- Bob observes a thermal bath of temperature $T_{\mathrm{U}} \propto a$

The Black Hole Information Paradox



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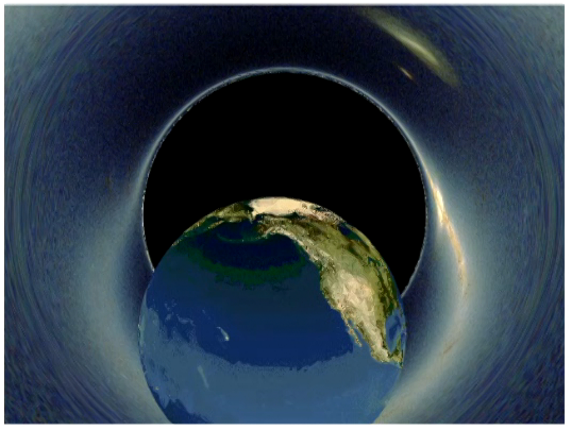

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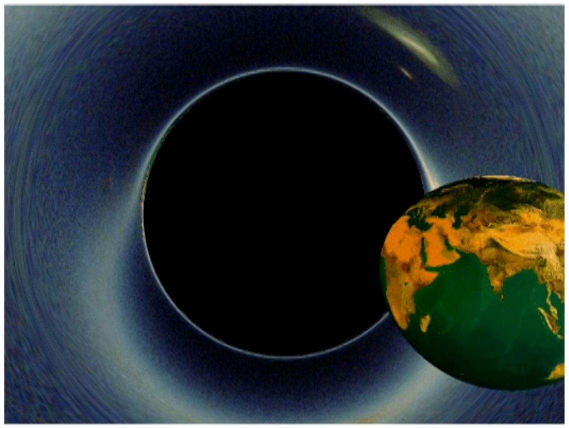


Entanglement in a Stellar Collapse

Once upon a time...

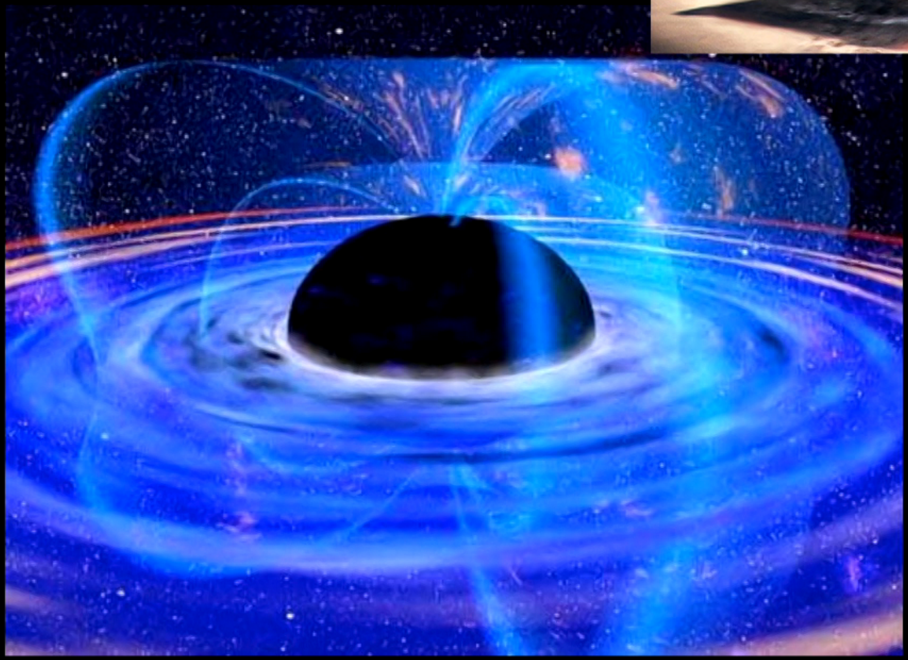
There was... NADA $\Psi_0 = |0\rangle$

Entanglement in a Stellar Collapse

Once upon a time...

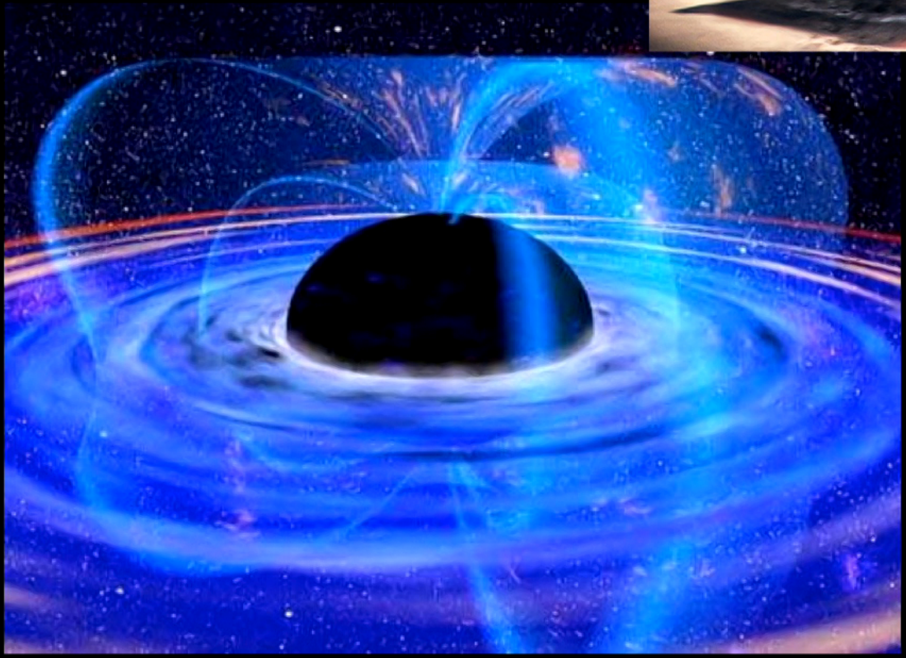
There was... NADA $\quad \Psi_0 = |0\rangle$

But then...



But then…

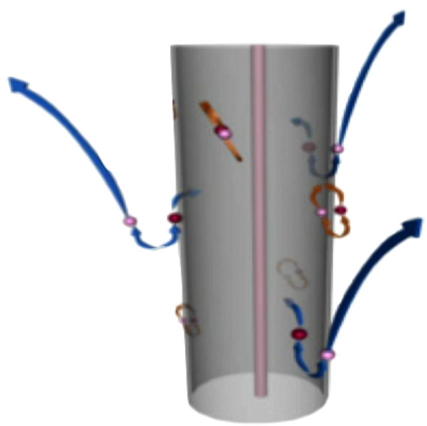
What happened to the field!?



Entanglement in a Stellar Collapse

Vacuum in the far past evolves into two mode squeezed state between infalling and outgoing modes in the far future

$$|0\rangle \to \bigotimes_{\omega} \frac{1}{\cosh r} \sum \tanh^n r_\omega \, |n_\omega\rangle_{\text{hor}} \, |n_\omega\rangle_{\text{out}}$$



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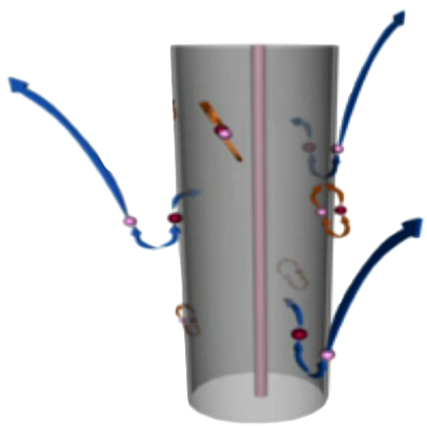
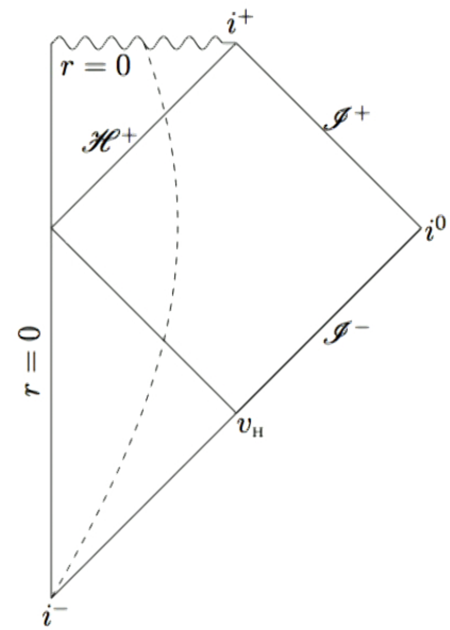


What do we really see if we look at the black hole?

Hawking Radiation

We need to write the annihilation operators of field modes in the asymptotic past in terms of the corresponding creation and annihilation operators defined in terms of modes in the future:



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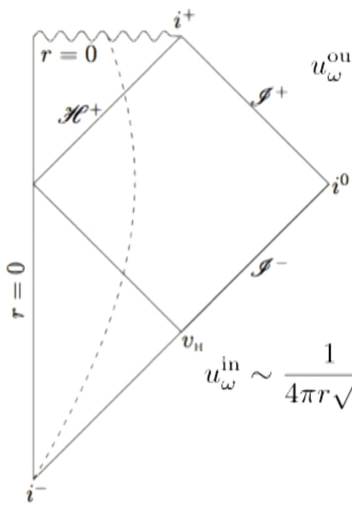
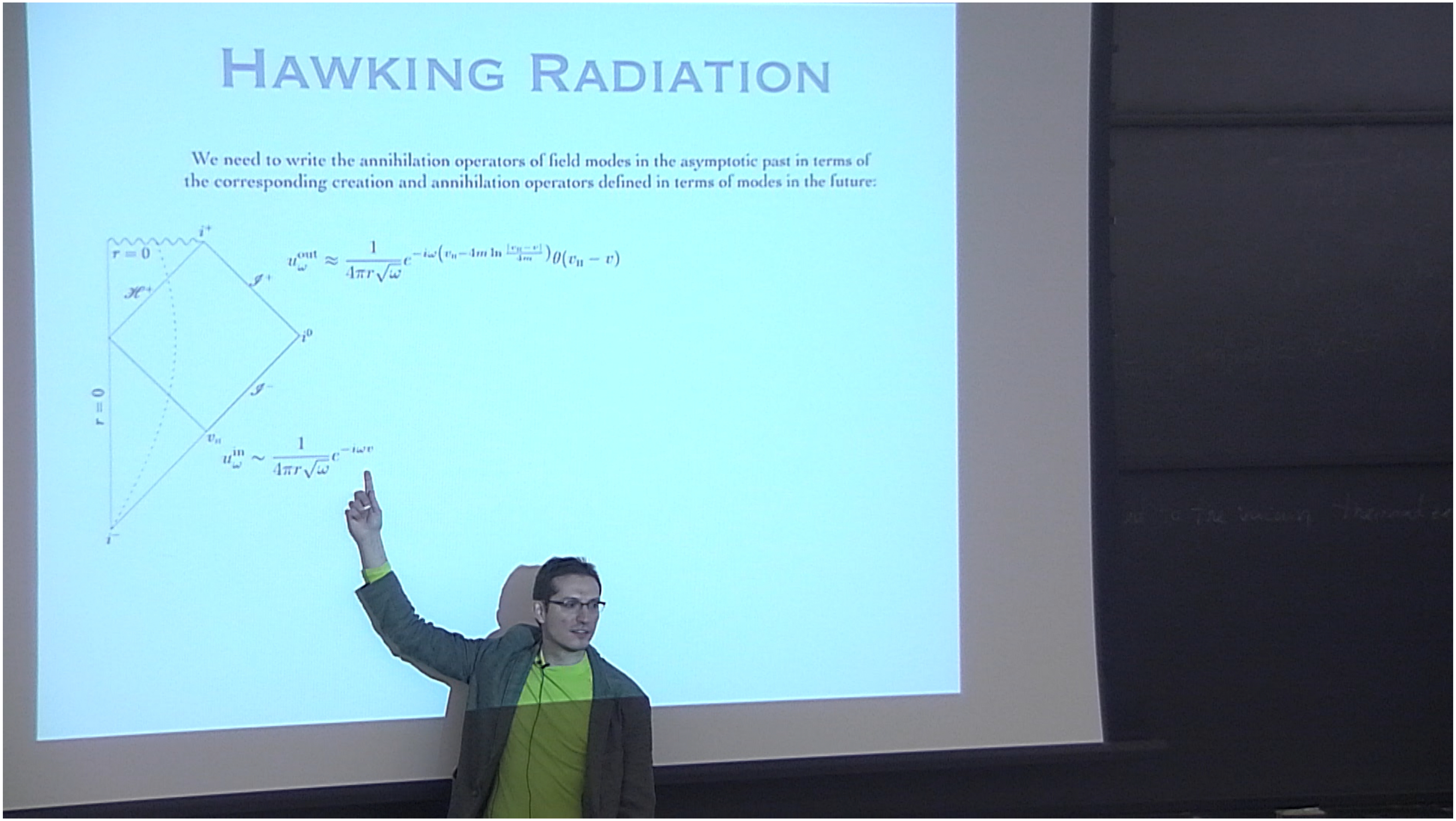


$$u_\omega^{\text{out}} \approx \frac{1}{4\pi r\sqrt{\omega}} e^{-i\omega\left(v_{\text{H}} - 4m \ln \frac{|v_{\text{H}} - v|}{4m}\right)} \theta(v_{\text{H}} - v)$$

$$u_\omega^{\text{in}} \sim \frac{1}{4\pi r\sqrt{\omega}} e^{-i\omega v}$$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i^+
$r = 0$
$\mathscr{H}^+$
$\mathscr{I}^+$
i^0
$\mathscr{I}^-$
$r = 0$
v_0
i^-
$u_\omega^{\rm in} \sim \frac{1}{4\pi r\sqrt{\omega}} e^{-i\omega v}$

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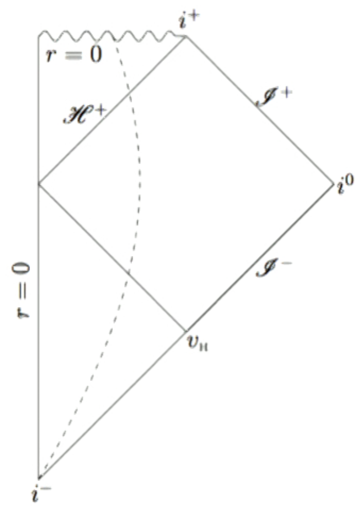

$$u_\omega^{\rm in} \sim \frac{1}{4\pi r\sqrt{\omega}} e^{-i\omega v} \qquad u_\omega^{\rm out} \approx \frac{1}{4\pi r\sqrt{\omega}} e^{-i\omega\left(v_{\rm H} - 4m\ln\frac{|v_{\rm H}-v|}{4m}\right)}\theta(v_{\rm H} - v)$$

$$u_\omega^{\rm hor} \sim \frac{1}{4\pi r\sqrt{\omega}} e^{i\omega\left(v_{\rm H} - 4m\ln\frac{|v_{\rm H}-v|}{4m}\right)}\theta(v - v_{\rm H})$$

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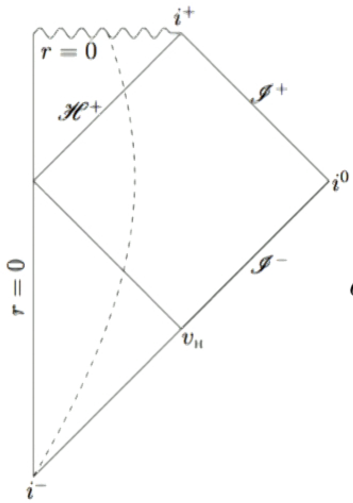

$$u_\omega^{\rm in} \sim \frac{1}{4\pi r\sqrt{\omega}} e^{-i\omega v} \qquad u_\omega^{\rm out} \approx \frac{1}{4\pi r\sqrt{\omega}} e^{-i\omega\left(v_{\rm H} - 4m \ln \frac{|v_{\rm H} - v|}{4m}\right)} \theta(v_{\rm H} - v)$$

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$$a_{\omega'}^{\rm in} = \int d\omega \left[\alpha_{\omega\omega'}^{*} \left(a_\omega^{\rm out} - \tanh r_\omega \, a_\omega^{\rm hor\,\dagger}\right) + \alpha_{\omega\omega'} e^{i\varphi} \left(a_\omega^{\rm hor} - \tanh r_\omega \, a_\omega^{\rm out\,\dagger}\right)\right]$$

HAWKING RADIATION

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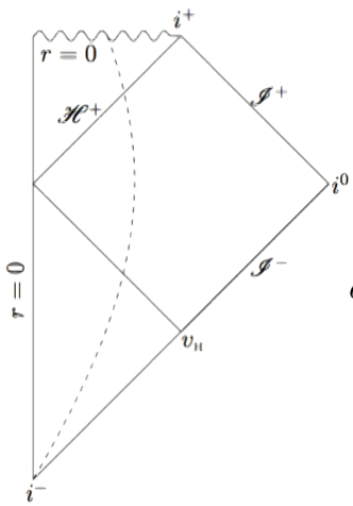

$$u_\omega^{\text{in}} \sim \frac{1}{4\pi r\sqrt{\omega}} e^{-i\omega v} \qquad u_\omega^{\text{out}} \approx \frac{1}{4\pi r\sqrt{\omega}} e^{-i\omega\left(v_H - 4m \ln \frac{|v_H - v|}{4m}\right)} \theta(v_H - v)$$

$$u_\omega^{\text{hor}} \sim \frac{1}{4\pi r\sqrt{\omega}} e^{i\omega\left(v_H - 4m \ln \frac{|v_H - v|}{4m}\right)} \theta(v - v_H)$$

$$a_{\omega'}^{\text{in}} = \int d\omega \left[\alpha^*_{\omega\omega'} \left(a_\omega^{\text{out}} - \tanh r_\omega \, a_\omega^{\text{hor}\dagger}\right) + \alpha_{\omega\omega'} e^{i\varphi} \left(a_\omega^{\text{hor}} - \tanh r_\omega \, a_\omega^{\text{out}\dagger}\right)\right]$$

$$|0\rangle_{\text{in}} = \prod_\omega \frac{1}{\cosh r_\omega} \sum_{n=0}^{\infty} (\tanh r_\omega)^n \, |n_\omega\rangle_{\text{hor}} \, |n_\omega\rangle_{\text{out}}$$

Black holes are not that black

We see outgoing radiation

$$\rho_{\text{out}} = \text{Tr}_{\text{hor}}\left(|0\rangle\langle 0|\right) = \bigotimes_{\omega} \frac{1}{\cosh^2 r} \sum \tanh^{2n} r_\omega \, |n_\omega\rangle_{\text{out}} \langle n_\omega|_{\text{out}}$$

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$$\text{Tr}\left(N_\omega \rho_{\text{out}}\right) = \frac{1}{e^{\hbar\omega/K_{\text{B}}T_{\text{H}}} - 1} \qquad T_{\text{H}} = \frac{1}{8\pi G}\frac{\hbar c^3}{m K_{\text{B}}}$$

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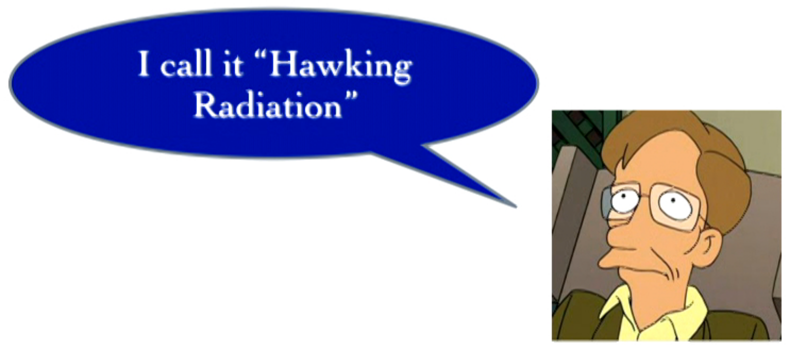

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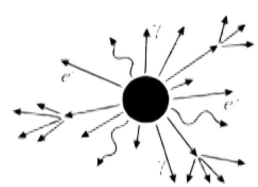

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Black hole Information loss problem

If we believe in quantum theory, information cannot be lost…

After corrections, the outflow may not be entirely thermal…

Like when a piece of charcoal burns