Title: PSI 2017/2018 - Relativistic Quantum Information - Lecture 6

Date: Mar 26, 2018 09:00 AM

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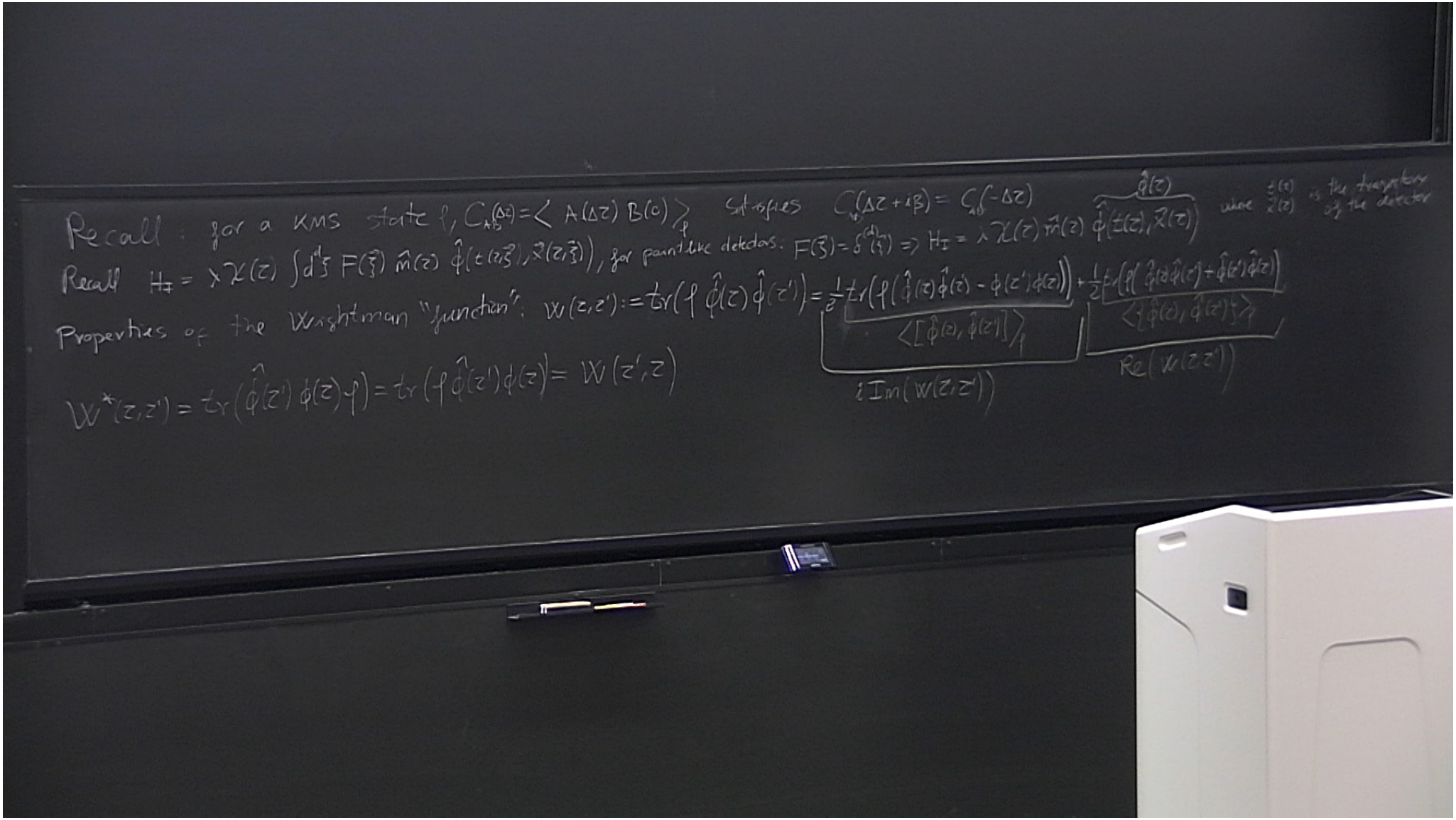
Abstract:

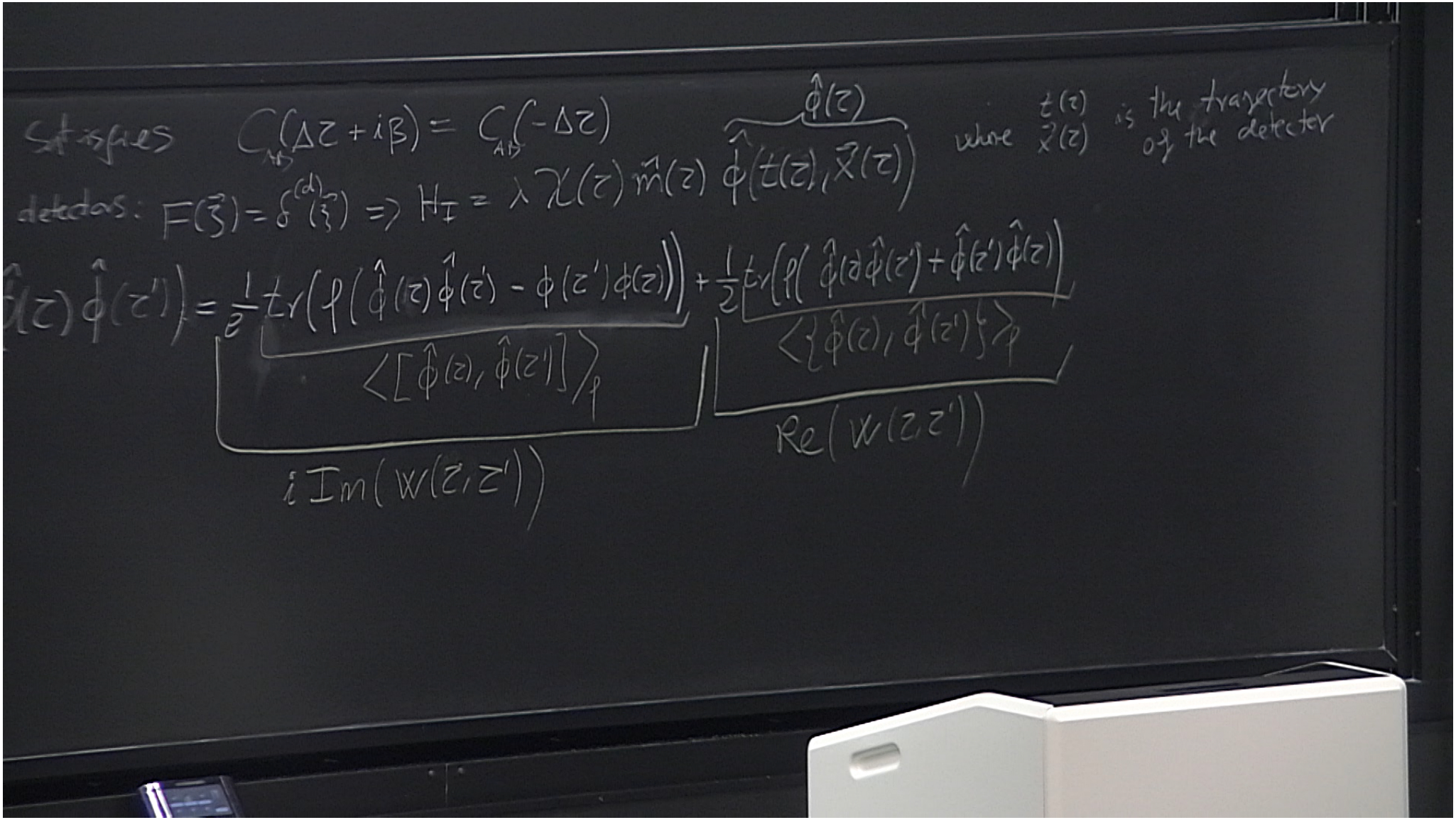

Recall: for a KMS state ρ, $C_{AB}(\Delta\tau) = \langle A(\Delta\tau)\, B(0)\rangle_\rho$ satisfies $C_{AB}(\Delta\tau + i\beta) = C_{BA}(-\Delta\tau)$
Recall $H_I = \lambda \chi(\tau) \int d^d\xi\, F(\vec{\xi})\, \hat{m}(\tau)\, \hat{\phi}(t(\tau,\vec{\xi}), \vec{x}(\tau,\vec{\xi}))$, for pointlike detectors: $F(\vec{\xi}) = \delta^{(d)}(\vec{\xi}) \Rightarrow H_I = \lambda \chi(\tau) \hat{m}(\tau) \hat{\phi}(t(\tau), \vec{x}(\tau))$
$\hat{\phi}(\tau)$
where $t(\tau)$, $\vec{x}(\tau)$ is the trajectory of the detector
Properties of the Wightman "function": $W(\tau,\tau') := \mathrm{tr}(\rho\, \hat{\phi}(\tau)\hat{\phi}(\tau')) = \frac{1}{2}\mathrm{tr}(\rho(\hat{\phi}(\tau)\hat{\phi}(\tau') - \hat{\phi}(\tau')\hat{\phi}(\tau))) + \frac{1}{2}\mathrm{tr}(\rho(\hat{\phi}(\tau)\hat{\phi}(\tau') + \hat{\phi}(\tau')\hat{\phi}(\tau)))$
$\langle[\hat{\phi}(\tau), \hat{\phi}(\tau')]\rangle_\rho$
$\langle\{\hat{\phi}(\tau), \hat{\phi}(\tau')\}\rangle_\rho$
$i\,\mathrm{Im}(W(\tau,\tau'))$
$\mathrm{Re}(W(\tau,\tau'))$
$W^*(\tau,\tau') = \mathrm{tr}(\hat{\phi}(\tau')\hat{\phi}(\tau)\rho) = \mathrm{tr}(\rho\hat{\phi}(\tau')\hat{\phi}(\tau)) = W(\tau',\tau)$

Recall: for a KMS state ρ, $C_{AB}(\Delta z) = \langle A(\Delta z)\, B(0) \rangle_\rho$ satisfies

Recall $H_I = \lambda \chi(z) \int d^d\xi\, F(\vec{\xi})\, \hat{m}(z)\, \hat{\phi}\big(t(z,\vec{\xi}), \vec{x}(z,\vec{\xi})\big)$, for pointlike detectors: $F(\vec{\xi}) = \delta$

Properties of the Wightman "function": $W(z,z') := \mathrm{tr}(\rho\, \hat{\phi}(z)\, \hat{\phi}(z')) = \frac{1}{Z} \mathrm{tr}$

$$W^*(z,z') = \mathrm{tr}\big(\hat{\phi}(z')\, \phi(z)\, \rho\big) = \mathrm{tr}\big(\rho\, \hat{\phi}(z')\, \phi(z)\big) = W(z',z)$$


Stieltjes $C_{AB}(\Delta\tau + i\beta) = C_{AB}(-\Delta\tau)$
detectors: $F(\vec{\xi}) = \delta^{(d)}(\vec{\xi}) \Rightarrow H_I = \lambda\, \chi(\tau)\, m(\tau)\, \hat{\phi}(t(\tau), \vec{x}(\tau))$
$\hat{\phi}(\tau)$
where $t(\tau)$, $\vec{x}(\tau)$ is the trajectory of the detector
$\hat{\phi}(\tau)\hat{\phi}(\tau') = \frac{1}{2}\mathrm{tr}(\rho(\hat{\phi}(\tau)\hat{\phi}(\tau') - \hat{\phi}(\tau')\hat{\phi}(\tau))) + \frac{1}{2}\mathrm{tr}(\rho(\hat{\phi}(\tau)\hat{\phi}(\tau') + \hat{\phi}(\tau')\hat{\phi}(\tau)))$
$\langle[\hat{\phi}(\tau), \hat{\phi}(\tau')]\rangle_\beta$
$\langle\{\hat{\phi}(\tau), \hat{\phi}(\tau')\}\rangle_\beta$
$i\,\mathrm{Im}(W(\tau,\tau'))$
$\mathrm{Re}(W(\tau,\tau'))$

Recall: for a KMS state ρ, $C_{A,B}(\Delta z) = \langle \hat{A}(\Delta z)\, \hat{B}(0) \rangle_\rho$ satisfies

Recall $H_\mathcal{I} = \lambda \chi(z) \int d^d\xi\, F(\vec{\xi})\, \hat{m}(z)\, \hat{\phi}\big(t(z,\vec{\xi}), \vec{x}(z,\vec{\xi})\big)$, for pointlike detectors: $F(\vec{\xi}) = \delta$

Properties of the Wightman "function": $W(z,z') := \mathrm{tr}(\rho\, \hat{\phi}(z)\, \hat{\phi}(z')) = \frac{1}{\mathcal{Z}} \mathrm{tr}$

$$W^*(z,z') = \mathrm{tr}(\hat{\phi}(z')\, \phi(z)\, \rho) = \mathrm{tr}(\rho\, \hat{\phi}(z')\, \phi(z)) = W(z',z)$$

Recall: for a KMS state ρ, $C_{AB}(\Delta\tau) = \langle \hat{A}(\Delta\tau)\, \hat{B}(0) \rangle_\rho$ satisfies

Recall $H_I = \lambda \chi(\tau) \int d^d\xi\, F(\vec{\xi})\, \hat{m}(\tau)\, \hat{\phi}\big(t(\tau,\vec{\xi}), \vec{x}(\tau,\vec{\xi})\big)$, for point-like detectors: $F(\vec{\xi}) = \delta$

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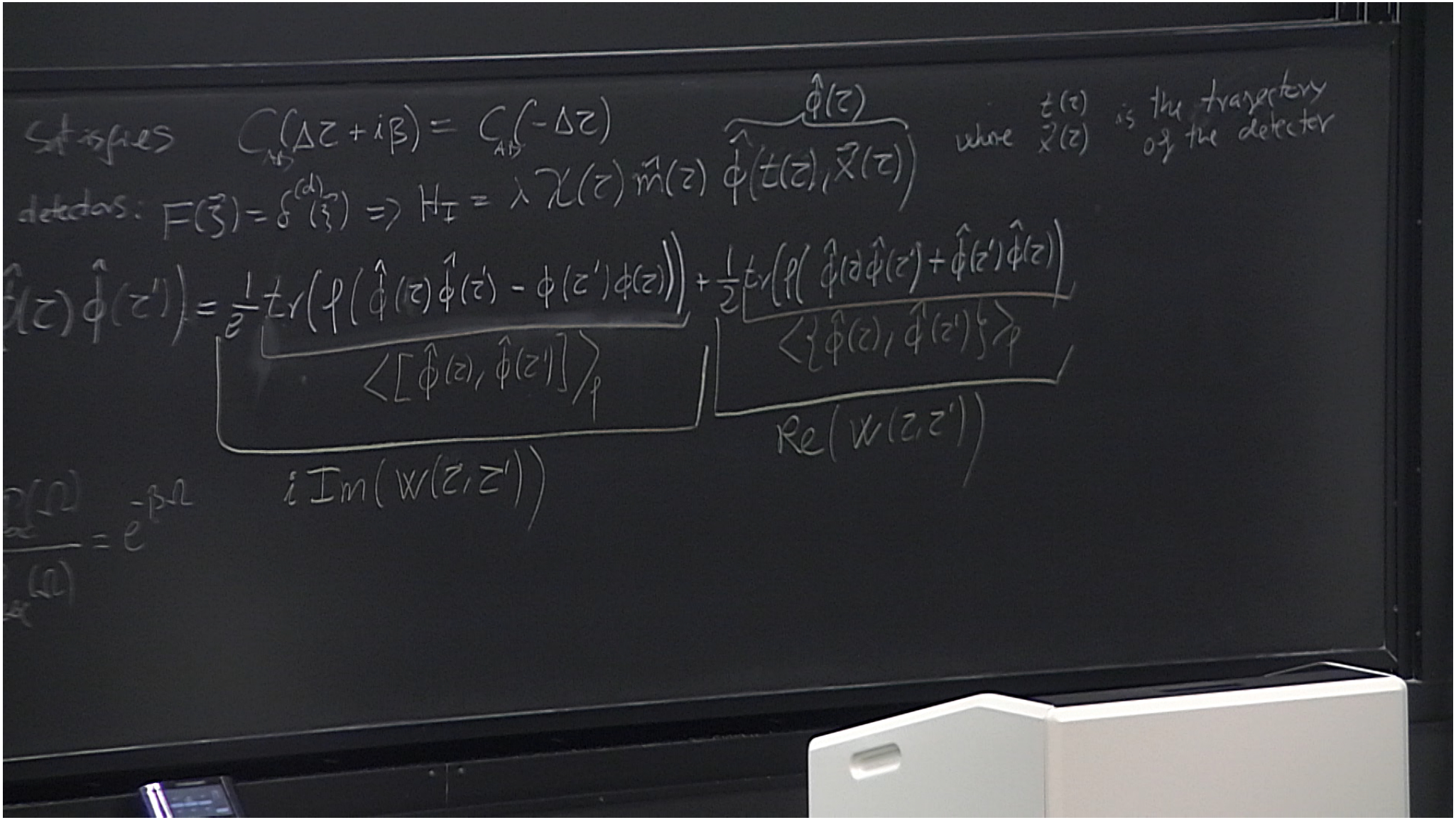
$$W^*(\tau,\tau') = \mathrm{tr}\big(\hat{\phi}(\tau')\,\phi(\tau)\,\rho\big) = \mathrm{tr}\big(\rho\,\hat{\phi}(\tau')\,\phi(\tau)\big) = W(\tau',\tau)$$

Thermality expectation $\Rightarrow$ detector state evolves to $\rho = \frac{1}{Z} e^{-\beta H_0} \Rightarrow \frac{P_{\text{exc}}(\Omega)}{P_{\text{deex}}(\Omega)} = e^{-\beta\Omega}$


Satisfies $C_{AB}(\Delta\tau + i\beta) = C_{AB}(-\Delta\tau)$
detectors: $F(\xi) = \delta(\xi)$ $\Rightarrow H_I = \lambda\, \chi(\tau)\, m(\tau)\, \hat{\phi}(t(\tau), \vec{x}(\tau))$
$\hat{\phi}(\tau)$
where $t(\tau)$, $\vec{x}(\tau)$ is the trajectory of the detector
$\hat{\phi}(\tau)\hat{\phi}(\tau') = \frac{1}{2}\mathrm{tr}(\rho(\hat{\phi}(\tau)\hat{\phi}(\tau') - \hat{\phi}(\tau')\hat{\phi}(\tau))) + \frac{1}{2}\mathrm{tr}(\rho(\hat{\phi}(\tau)\hat{\phi}(\tau') + \hat{\phi}(\tau')\hat{\phi}(\tau)))$
$\langle[\hat{\phi}(\tau), \hat{\phi}(\tau')]\rangle_\rho$
$\langle\{\hat{\phi}(\tau), \hat{\phi}(\tau')\}\rangle_\rho$
$i\,\mathrm{Im}(W(\tau,\tau'))$
$\mathrm{Re}(W(\tau,\tau'))$
$= e^{-\beta\Omega}$

Recall: for a KMS state ρ, $C_{AB}(\Delta\tau) = \langle \hat{A}(\Delta\tau)\, \hat{B}(0) \rangle_\rho$ satisfies

Recall $H_I = \lambda \chi(\tau) \int d^d\xi\, F(\vec{\xi})\, \hat{m}(\tau)\, \hat{\phi}\big(t(\tau,\vec{\xi}), \vec{x}(\tau,\vec{\xi})\big)$, for pointlike detectors: $F(\vec{\xi}) = \delta$

Properties of the Wightman "function": $W_\rho(\tau,\tau') := \mathrm{tr}\big(\rho\, \hat{\phi}(\tau)\, \hat{\phi}(\tau')\big) = \frac{1}{Z}\mathrm{tr}$

$$W_\rho^*(\tau,\tau') = \mathrm{tr}\big(\hat{\phi}(\tau')\, \phi(\tau)\, \rho\big) = \mathrm{tr}\big(\rho\, \hat{\phi}(\tau')\, \phi(\tau)\big) = W_\rho(\tau',\tau)$$

Hermaliti, expectation $\Rightarrow$ detector state evolves to $\rho = \frac{1}{Z} e^{-\beta H_0} \Rightarrow \frac{P_{exc}(\Omega)}{P_{deex}(\Omega)} = e^{-\beta\Omega}$

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Recall $H_I = \lambda \chi(\tau) \int d^d\xi\, F(\vec{\xi})\, \hat{m}(\tau)\, \hat{\phi}\big(t(\tau,\vec{\xi}), \vec{x}(\tau,\vec{\xi})\big)$, for pointlike detectors: $F(\vec{\xi}) = \delta$

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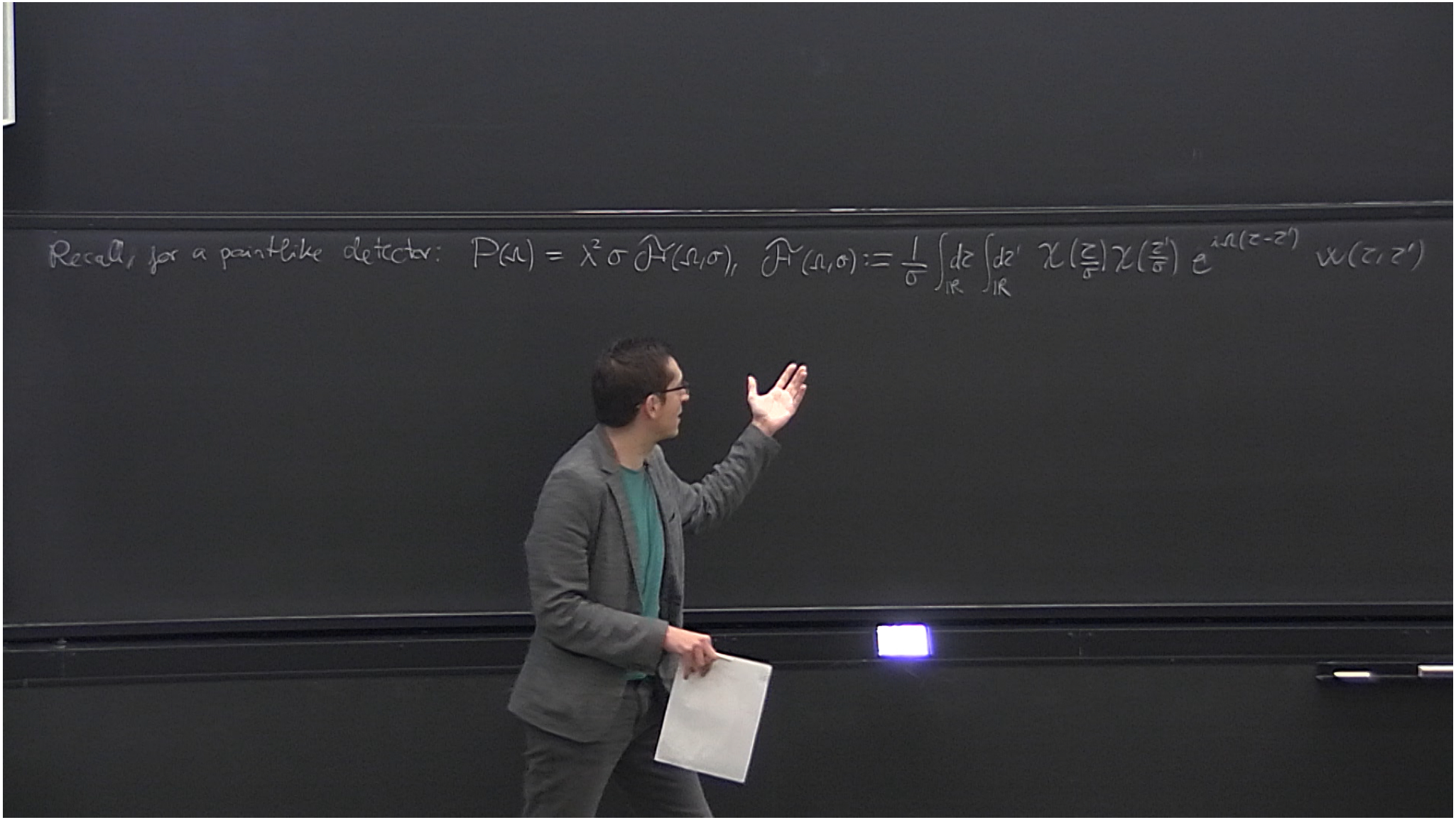
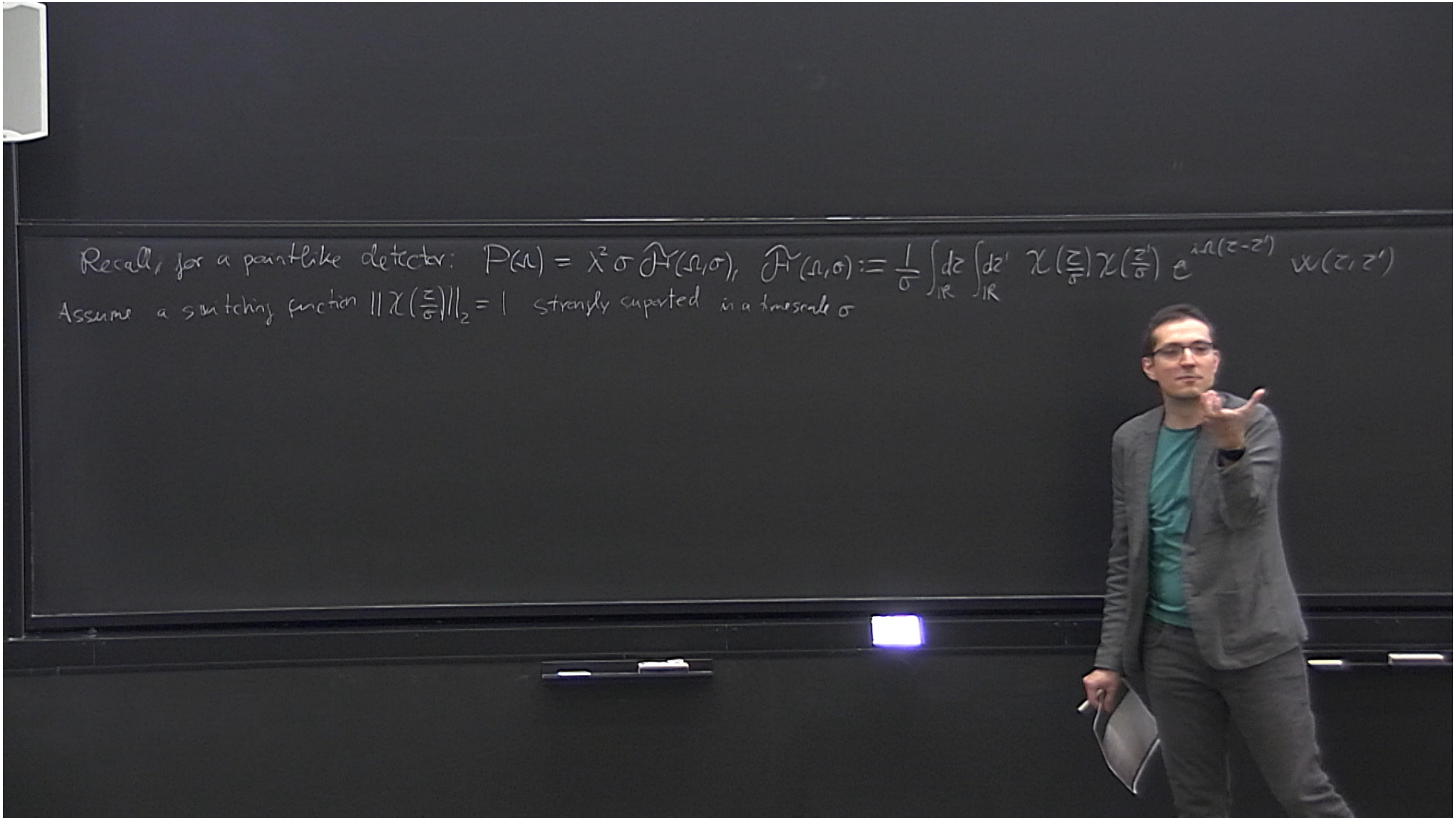
$W_\rho^*(\tau,\tau') = \mathrm{tr}\big(\hat{\phi}(\tau')\, \hat{\phi}(\tau)\, \rho\big) = \mathrm{tr}\big(\rho\, \hat{\phi}(\tau')\, \hat{\phi}(\tau)\big) = W_\rho(\tau',\tau)$

$= (\mathrm{tr}(\rho\, \hat{\phi}(\tau)\, \hat{\phi}(\tau')))^*$

thermality expectation $\Rightarrow$ detector state evolves to $\rho = \frac{1}{Z} e^{-\beta H_0} \Rightarrow \frac{P_{\mathrm{exc}}(\Omega)}{P_{\mathrm{vac}}(\Omega)} = e^{-\beta\Omega}$


Recall, for a pointlike detector: $P(\Omega) = \lambda^2 \sigma \tilde{\mathcal{F}}(\Omega,\sigma)$, $\tilde{\mathcal{F}}(\Omega,\sigma) := \frac{1}{\sigma}\int_{\mathbb{R}} dx \int_{\mathbb{R}} dx' \; \chi(\frac{x}{\sigma})\chi(\frac{x'}{\sigma})\, e^{i\Omega(x-x')} \; W(x,x')$


Recall, for a pointlike detector: $P(\Omega) = \lambda^2 \sigma \widetilde{\mathcal{F}}(\Omega,\sigma)$, $\widetilde{\mathcal{F}}(\Omega,\sigma) := \frac{1}{\sigma}\int_{\mathbb{R}} dt \int_{\mathbb{R}} dt' \, \chi\left(\frac{t}{\sigma}\right)\chi\left(\frac{t'}{\sigma}\right) e^{i\Omega(t-t')} W(t,t')$
Assume a switching function $\|\chi\left(\frac{t}{\sigma}\right)\|_2 = 1$ strongly supported in a timescale σ

Recall, for a pointlike detector: $P(\Omega) = \lambda^2 \sigma \tilde{\mathcal{F}}(\Omega,\sigma)$, $\tilde{\mathcal{F}}(\Omega,\sigma) := \frac{1}{\sigma}\int_{\mathbb{R}} dz \int_{\mathbb{R}} dz'\ \chi\left(\frac{z}{\sigma}\right)\chi\left(\frac{z'}{\sigma}\right) e^{i\Omega(z-z')} W(z,z')$

Assume a switching function $\|\chi(\frac{z}{\sigma})\|_2 = 1$ strongly supported in a timescale σ

$$\left.\begin{aligned}\chi\left(\tfrac{z}{\sigma}\right) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} du\ \tilde{\chi}(u)\, e^{-iu\frac{z}{\sigma}} \\ \chi\left(\tfrac{z}{\sigma}\right) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} du\ \tilde{\chi}^*(u)\, e^{iu\frac{z}{\sigma}}\end{aligned}\right\} \Rightarrow \tilde{\mathcal{F}}(\Omega,\sigma) = \frac{1}{4\pi^2\sigma}\int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} dz' \int_{-\infty}^{\infty} du \int_{-\infty}^{\infty} du'\ \tilde{\chi}^*(u)\,\tilde{\chi}(u')\, e^{i\left(\frac{uz}{\sigma} - \frac{u'z'}{\sigma}\right)} W(z,z')\, e^{i(z-z')\Omega}$$

$$= \frac{1}{\sigma} \int_{\mathbb{R}} dz \int_{\mathbb{R}} dz' \; \chi\left(\frac{z}{\sigma}\right) \chi\left(\frac{z'}{\sigma}\right) e^{i\Omega(z-z')} \; W(z,z')$$

$$\tilde{\chi}^*(\omega)\, \tilde{\chi}(\omega') \, e^{i\left(\frac{\omega z}{\sigma} - \frac{\omega' z'}{\sigma}\right)} \; W(z,z') \, e^{i(z-z')\Omega}$$

if the field is stationary w.r.t ∂_z $\Rightarrow W(z,z') = W(z-z')$

$$= \frac{1}{\sigma} \int_{\mathbb{R}} dz \int_{\mathbb{R}} dz' \; \chi\left(\frac{z}{\sigma}\right) \chi\left(\frac{z'}{\sigma}\right) e^{i\Omega(z-z')} \; W(z,z')$$

$$\tilde{\chi}^*(w)\, \tilde{\chi}(w')\, e^{i\left(\frac{wz}{\sigma} - \frac{w'z'}{\sigma}\right)} \; W(z,z')\, e^{i(z-z')\Omega}$$

if the field is stationary w.r.t ∂_z $\Rightarrow W(z,z') = W(z-z')$

Change of variables: $u = z - z'$

$v = z + z'$

Recall, for a pointlike detector: $P(\Omega) = \lambda^2 \sigma \tilde{\mathcal{F}}(\Omega,\sigma)$, $\tilde{\mathcal{F}}(\Omega,\sigma) := \frac{1}{\sigma}\int_{\mathbb{R}} dz \int_{\mathbb{R}} dz'\ \chi\left(\frac{z}{\sigma}\right)$

Assume a switching function $\|\chi(\frac{z}{\sigma})\|_2 = 1$ strongly supported in a timescale σ

$$\chi\left(\frac{z}{\sigma}\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty} du\ \tilde{\chi}(u)\, e^{-iu\frac{z}{\sigma}}$$

$$\chi\left(\frac{z}{\sigma}\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty} du\ \tilde{\chi}^*(u)\, e^{iu\frac{z}{\sigma}}$$

$$\Rightarrow \tilde{\mathcal{F}}(\Omega,\sigma) = \frac{1}{4\pi^2\sigma}\int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} dz' \int_{-\infty}^{\infty} du \int_{-\infty}^{\infty} du'\ \tilde{\chi}^*(u)\,\tilde{\chi}(u')\, e^{i\left(\frac{u}{\sigma}z - \right.}$$

$$\tilde{\mathcal{F}}(\Omega,\sigma) = \frac{1}{8\pi^2\sigma}\int_{-\infty}^{\infty} du \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} du' \int_{-\infty}^{\infty} dv'\ \tilde{\chi}^*(u)\,\tilde{\chi}(u')\, e^{\frac{i}{2\sigma}(u+u')u}\ e^{\frac{i}{2\sigma}(}$$

$$= \frac{1}{\sigma} \int_{\mathbb{R}} dz \int_{\mathbb{R}} dz' \; \chi\left(\frac{z}{\sigma}\right) \chi\left(\frac{z'}{\sigma}\right) e^{i\Omega(z-z')} \; W(z, z')$$

$$\tilde{\chi}^*(\omega)\, \tilde{\chi}(\omega')\, e^{i\left(\frac{\omega z}{\sigma} - \frac{\omega' z'}{\sigma}\right)} \; W(z, z')\, e^{i(z-z')\Omega}$$

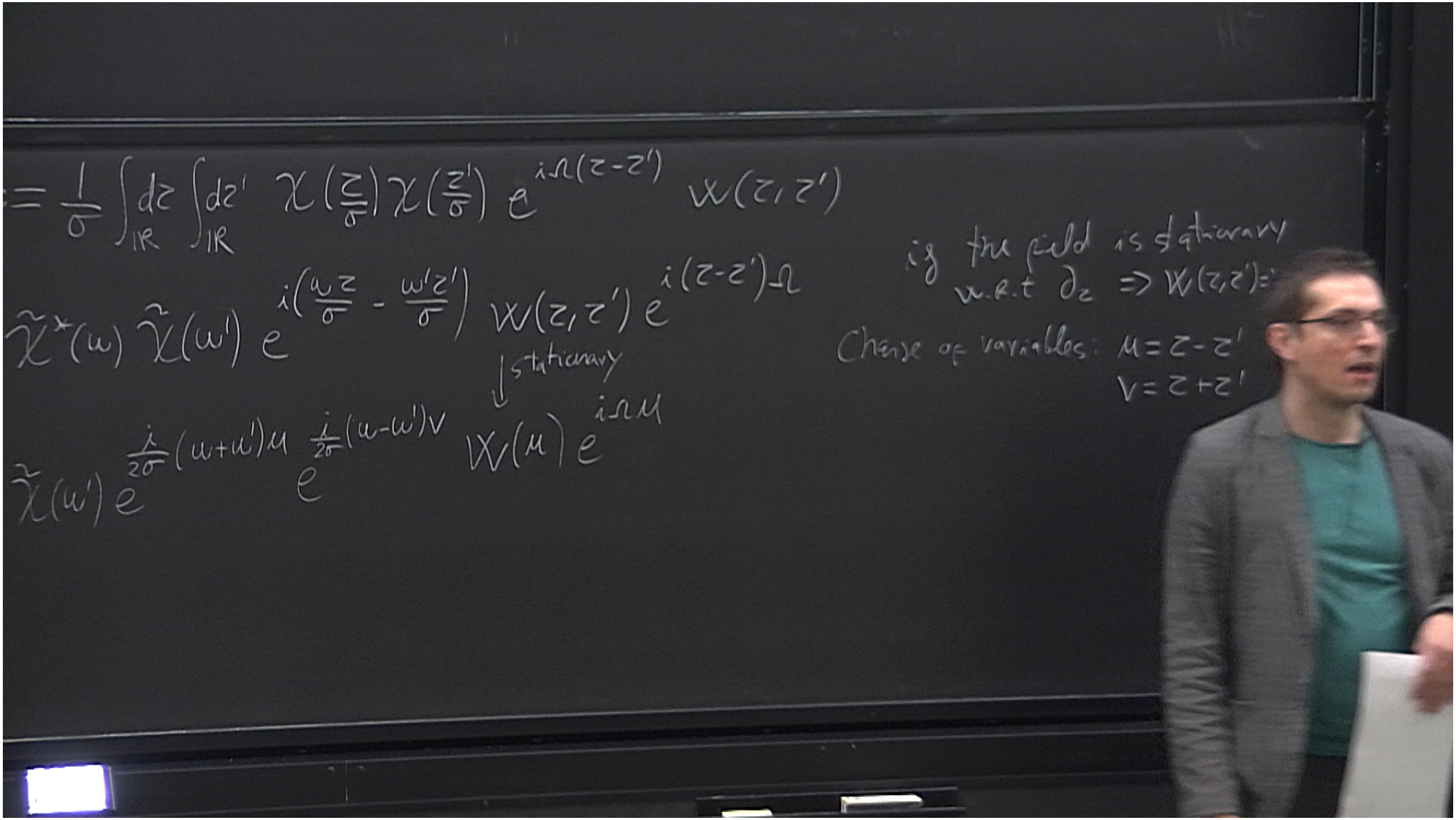
↓ stationary

$$\tilde{\chi}(\omega')\, e^{\frac{i}{2\sigma}(\omega+\omega')u}\, e^{\frac{i}{2\sigma}(\omega-\omega')v}\, W(u)$$

if the field is stationary w.r.t $\partial_z \Rightarrow W(z,z') = W(z-z')$

Change of variables: $u = z - z'$

$v = z + z'$


= 1/σ ∫dz ∫dz' χ(z/σ) χ(z'/σ) e^{iΩ(z-z')} W(z,z')
χ̃*(w) χ̃(w') e^{i(wz/σ - w'z'/σ)} W(z,z') e^{i(z-z')Ω}
stationary
χ̃(w') e^{i/2σ (w+w')μ} e^{i/2σ (w-w')ν} W(μ) e^{iΩμ}
if the field is stationary
w.r.t ∂z ⇒ W(z,z')=
Change of variables: μ = z - z'
ν = z + z'

$$= \frac{1}{\sigma} \int_{\mathbb{R}} dz \int_{\mathbb{R}} dz' \; \chi\left(\frac{z}{\sigma}\right) \chi\left(\frac{z'}{\sigma}\right) e^{i\Omega(z-z')} \; W(z,z')$$

$$\tilde{\chi}^*(w)\, \tilde{\chi}(w')\, e^{i\left(\frac{wz}{\sigma} - \frac{w'z'}{\sigma}\right)} \; W(z,z')\, e^{i(z-z')\Omega}$$

$$\tilde{\chi}(w')\, e^{\frac{i}{2\sigma}(w+w')u}\, e^{\frac{i}{2\sigma}(w-w')v}\, W(u)\, e^{i\Omega u}$$

↓ stationary

if the field is stationary w.r.t ∂_z $\Rightarrow W(z,z') = W(z-z')$

Change of variables: $u = z - z'$

$v = z + z'$

$$\int_{-\infty}^{\infty} dv\; e^{\frac{i}{2\sigma}(w-w')v} = 2\pi\delta\left(\frac{1}{2\sigma}(w-w')\right) =$$

$$= 4\pi\sigma\,\delta(w-w')$$

Recall, for a pointlike detector: $P(\Omega) = \lambda^2 \sigma \tilde{\mathcal{F}}(\Omega,\sigma)$, $\tilde{\mathcal{F}}(\Omega,\sigma) := \frac{1}{\sigma}\int_{\mathbb{R}} d\tau \int_{\mathbb{R}} d\tau'\ \chi\left(\frac{\tau}{\sigma}\right)\chi\left(\frac{\tau'}{\sigma}\right) e^{i\Omega(\tau-\tau')} W($

Assume a switching function $\|\chi(\frac{\tau}{\sigma})\|_2 = 1$ strongly supported in a timescale σ

$$\chi\left(\frac{\tau}{\sigma}\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty} du\ \tilde{\chi}(u)\, e^{-iu\frac{\tau}{\sigma}}$$

$$\chi\left(\frac{\tau}{\sigma}\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty} du\ \tilde{\chi}^*(u)\, e^{iu\frac{\tau}{\sigma}}$$

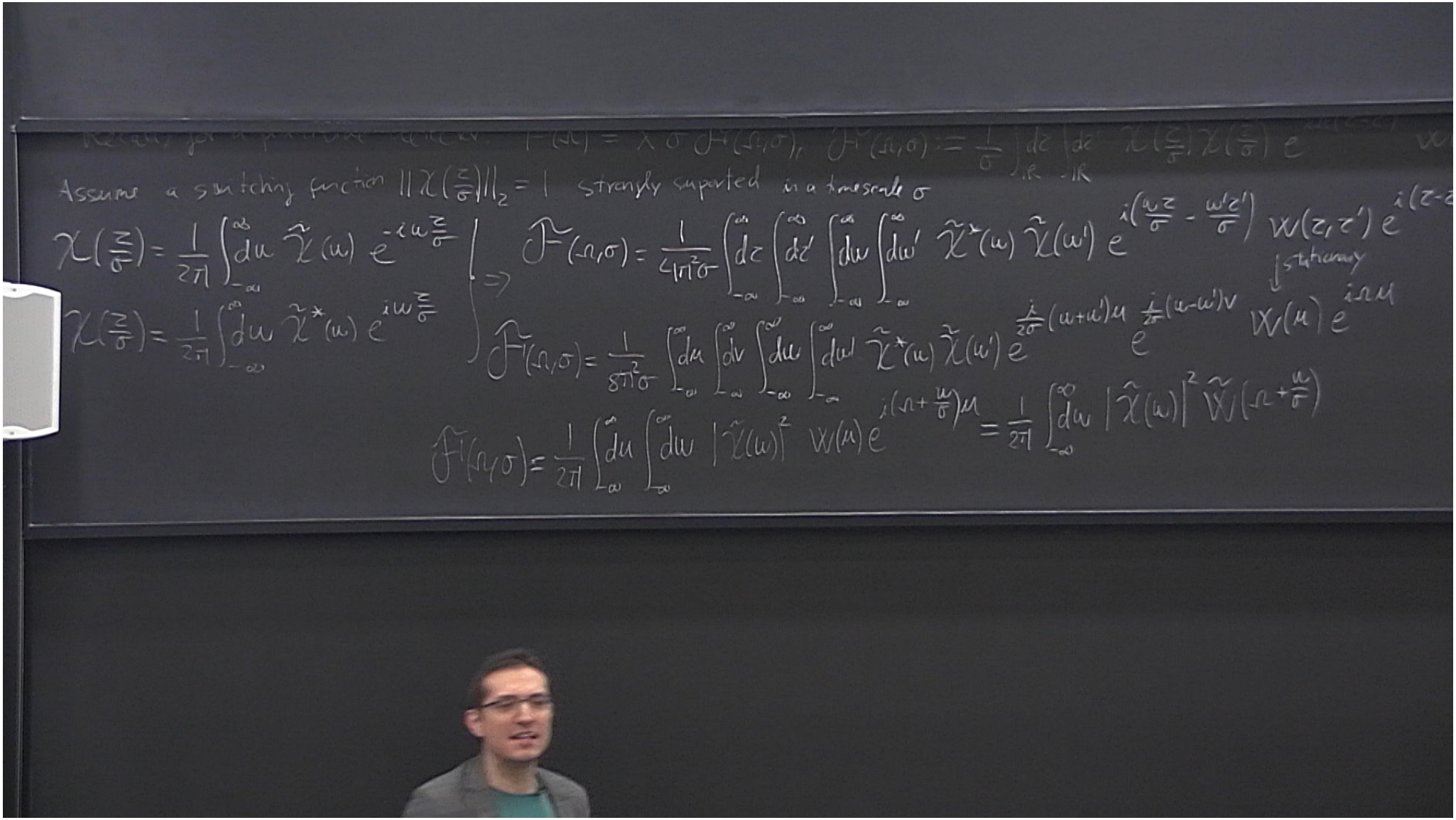
$$\Rightarrow \tilde{\mathcal{F}}(\Omega,\sigma) = \frac{1}{4\pi^2\sigma}\int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' \int_{-\infty}^{\infty} du \int_{-\infty}^{\infty} du'\ \tilde{\chi}^*(u)\,\tilde{\chi}(u')\, e^{i\left(\frac{u\tau}{\sigma} - \frac{u'\tau'}{\sigma}\right)} W(\tau,\tau')\, e^{i(\tau-$$

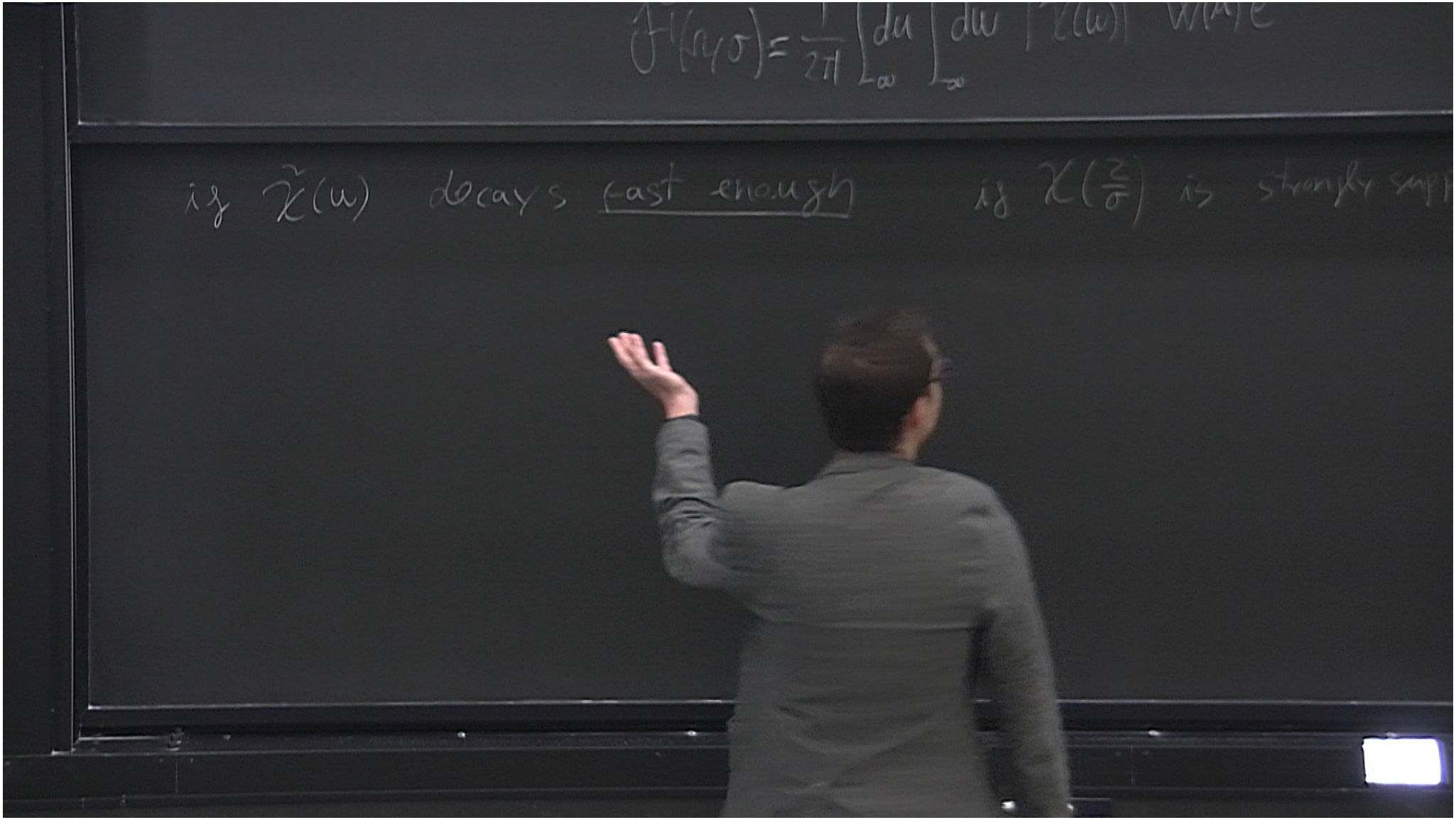
$$\tilde{\mathcal{F}}(\Omega,\sigma) = \frac{1}{8\pi^2\sigma}\int_{-\infty}^{\infty} du \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} du \int_{-\infty}^{\infty} du'\ \tilde{\chi}^*(u)\,\tilde{\chi}(u')\, e^{\frac{i}{2\sigma}(u+u')u}\, e^{\frac{i}{2\sigma}(u-u')v}$$

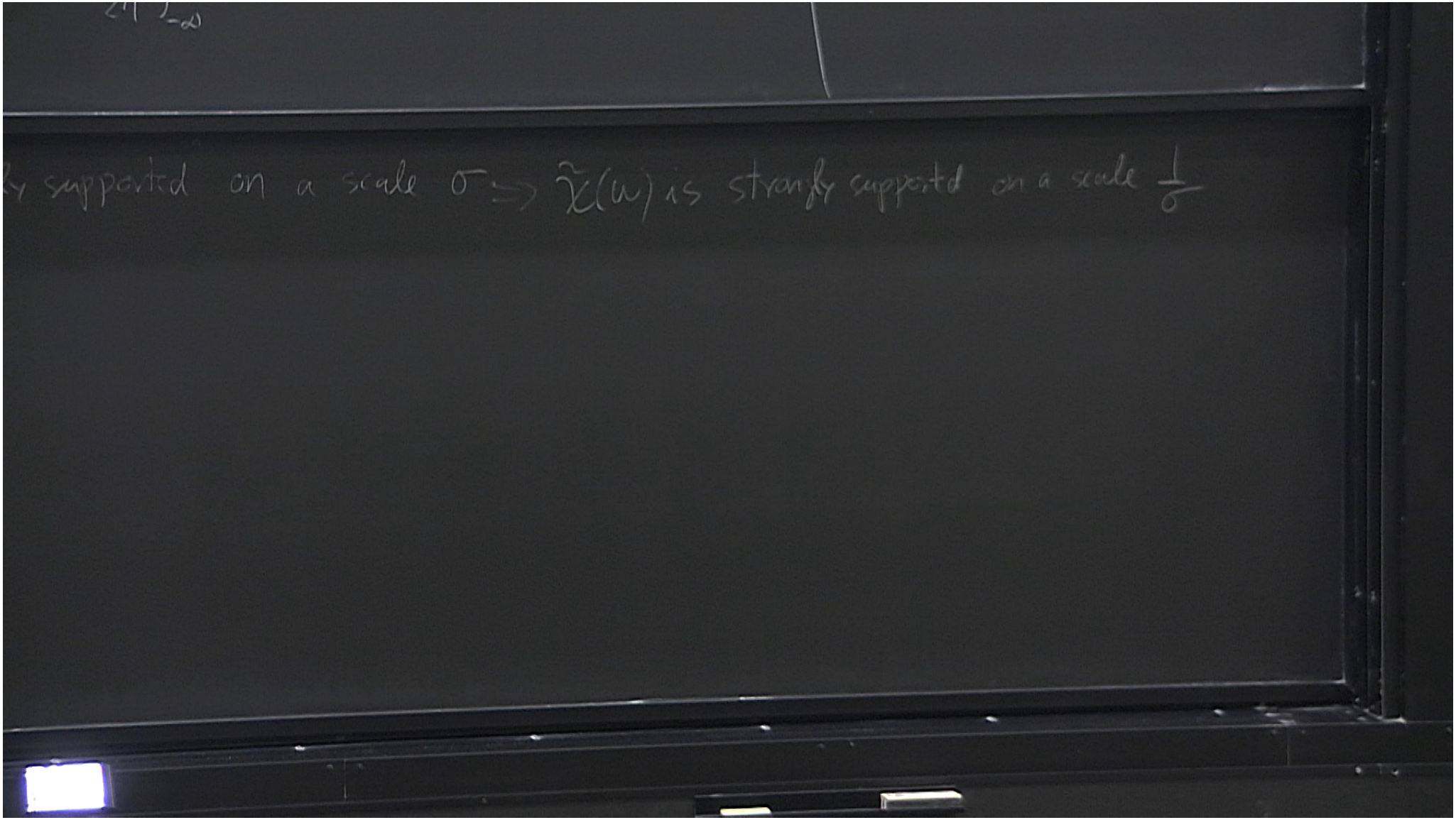
stationary

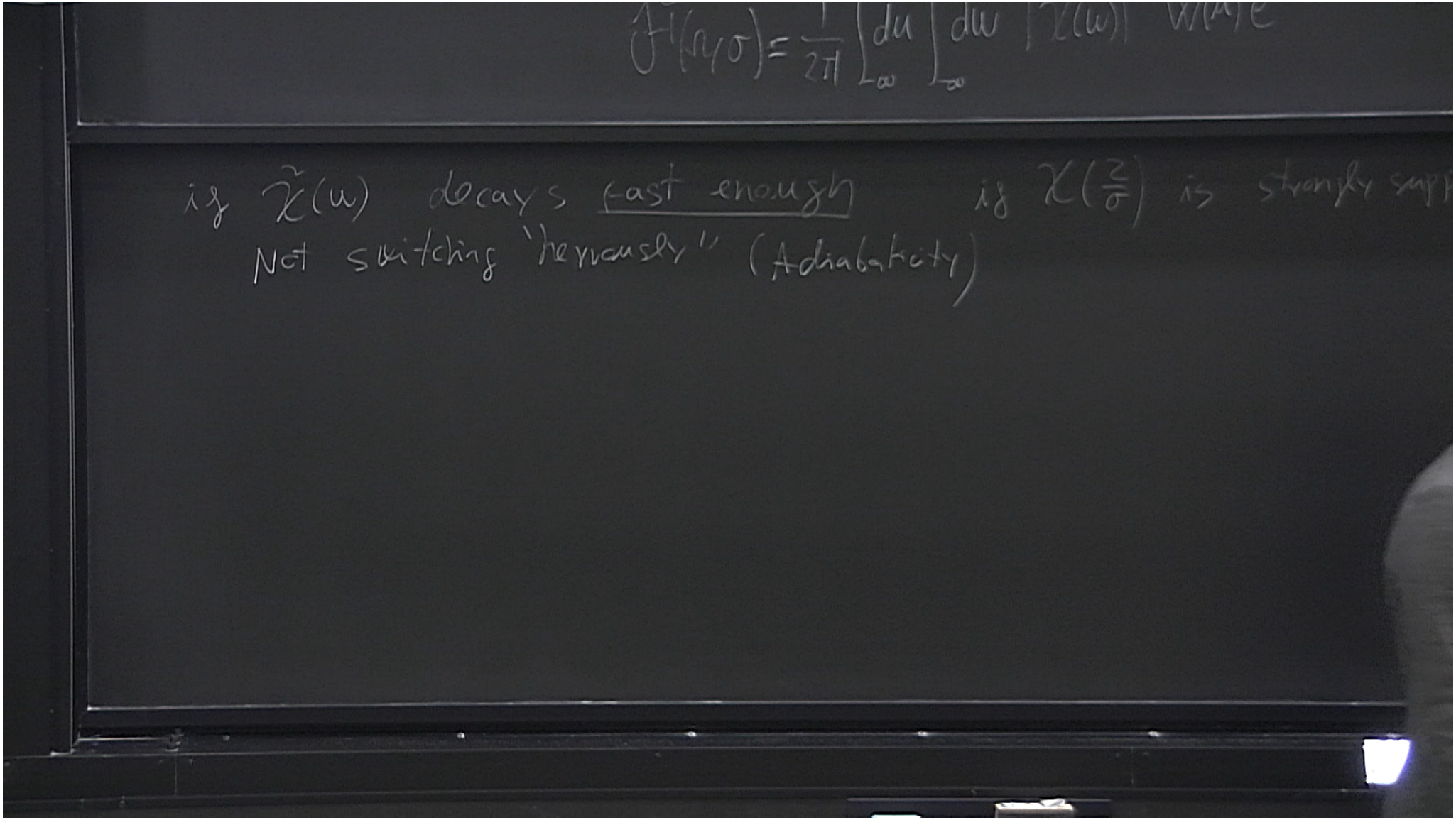
$$W(u)\, e^{i\Omega u}$$

$$\tilde{\mathcal{F}}(\Omega,\sigma) = \frac{1}{2\pi}\int_{-\infty}^{\infty} du \int_{-\infty}^{\infty} dw\ |\tilde{\chi}(w)|^2\, W(u)\, e^{i\left(\Omega + \frac{w}{2\sigma}\right)u}$$


Assume a switching function $\|\chi(\frac{z}{\sigma})\|_2 = 1$ strongly supported in a timescale σ
$\chi(\frac{z}{\sigma}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} du\, \tilde{\chi}(u)\, e^{-iu\frac{z}{\sigma}}$
$\chi(\frac{z}{\sigma}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} du\, \tilde{\chi}^*(u)\, e^{iu\frac{z}{\sigma}}$
$\Rightarrow \tilde{\mathcal{F}}(\Omega,\sigma) = \frac{1}{4\pi^2\sigma}\int dz \int dz' \int du \int du'\, \tilde{\chi}^*(u)\,\tilde{\chi}(u')\, e^{i(\frac{uz}{\sigma} - \frac{u'z'}{\sigma})}\, W(z,z')\, e^{i(z-z')\Omega}$
$\tilde{\mathcal{F}}(\Omega,\sigma) = \frac{1}{8\pi^2\sigma}\int du \int dv \int du' \int dw\, \tilde{\chi}^*(u)\,\tilde{\chi}(u')\, e^{\frac{i}{2\sigma}(u+u')w}\, e^{\frac{i}{2\sigma}(u-u')v}\, W(w)\, e^{i\Omega w}$
stationary
$\tilde{\mathcal{F}}(\Omega,\sigma) = \frac{1}{2\pi}\int du \int dw\, |\tilde{\chi}(u)|^2\, W(w)\, e^{i(\Omega+\frac{u}{\sigma})w} = \frac{1}{2\pi}\int_{-\infty}^{\infty} du\, |\tilde{\chi}(u)|^2\, \tilde{W}(\Omega+\frac{u}{\sigma})$


if $\tilde{\chi}(\omega)$ decays fast enough
if $\chi(\frac{z}{\sigma})$ is strongly


strongly supported on a scale $\sigma \Rightarrow \tilde{\chi}(\omega)$ is strongly supported on a scale $\frac{1}{\sigma}$


if $\tilde{\chi}(\omega)$ decays fast enough
Not switching "nervously" (Adiabaticity)
if $\chi(\frac{z}{v})$ is strongly

$$\mathcal{F}(\Omega,\sigma) = \frac{1}{2\pi}\int_{-\infty} du \int_{-\infty} d\omega \, |\tilde{\chi}(\omega)| \, W(u) e$$

if $\tilde{\chi}(\omega)$ decays fast enough if $\chi(\frac{z}{\sigma})$ is strongly supp

Not switching "nervously" (Adiabaticity)

Adiabatic

$$\lim_{\sigma\to\infty} \mathcal{F}(\Omega,\sigma) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \, |\tilde{\chi}(\omega)|^2 \, W(\Omega)$$

$$\mathcal{F}(\Omega,\sigma) = \frac{1}{2\pi}\int_{-\infty}^{\infty} du \left| \tilde{\chi}(u) \right|^2$$ where

if $\tilde{\chi}(u)$ decays fast enough if $\chi\left(\frac{t}{\sigma}\right)$ is strongly supp

Not switching "nervously" (Adiabaticity)

Adiabatic

$$\lim_{\sigma\to\infty} \mathcal{F}(\Omega,\sigma) = \frac{1}{2\pi}\int_{-\infty}^{\infty} du \left|\tilde{\chi}(u)\right|^2 \quad \tilde{W}(\Omega) = \frac{\tilde{W}(\Omega)}{2\pi}\int_{-\infty}^{\infty} du \left|\tilde{\chi}(u)\right|^2$$

$$\tilde{\mathcal{F}}(\Omega,\sigma) = \frac{1}{2\pi}\int_{-\infty}^{\infty} du \int_{-\infty}^{\infty} dw \; |\tilde{\chi}(w)|^2 \; W(u)\, e^{i(\Omega + \frac{w}{\sigma})u} = \frac{1}{2\pi}\int_{-\infty}^{\infty} du \; |$$

if $\tilde{\chi}(w)$ decays <u>fast enough</u> if $\chi(\frac{z}{\sigma})$ is strongly supported on

Not switching "nervously" (Adiabaticity)

Adiabatic

$$\lim_{\sigma\to\infty} \tilde{\mathcal{F}}(\Omega,\sigma) = \frac{1}{2\pi}\int_{-\infty}^{\infty} du\, |\tilde{\chi}(w)|^2 \; \tilde{W}(\Omega) = \frac{\tilde{W}(\Omega)}{2\pi}\int_{-\infty}^{\infty} du\, |\tilde{\chi}(w)|^2$$

$$\int_{-\infty}^{\infty} dz\, |\chi(\tfrac{z}{\sigma})|^2 = 1 \Rightarrow \left(\int_{-\infty}^{\infty} du\, |\tilde{\chi}(w)|^2 = \int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} dz' \int_{-\infty}^{\infty} du\; \chi(\tfrac{z}{\sigma})\, \chi^*(\tfrac{z'}{\sigma})\, e^{i(z-z')w} \right.$$

strongly supported on a scale $\sigma \Rightarrow \tilde{\chi}(w)$ is strongly supported on a scale $\frac{1}{\sigma}$

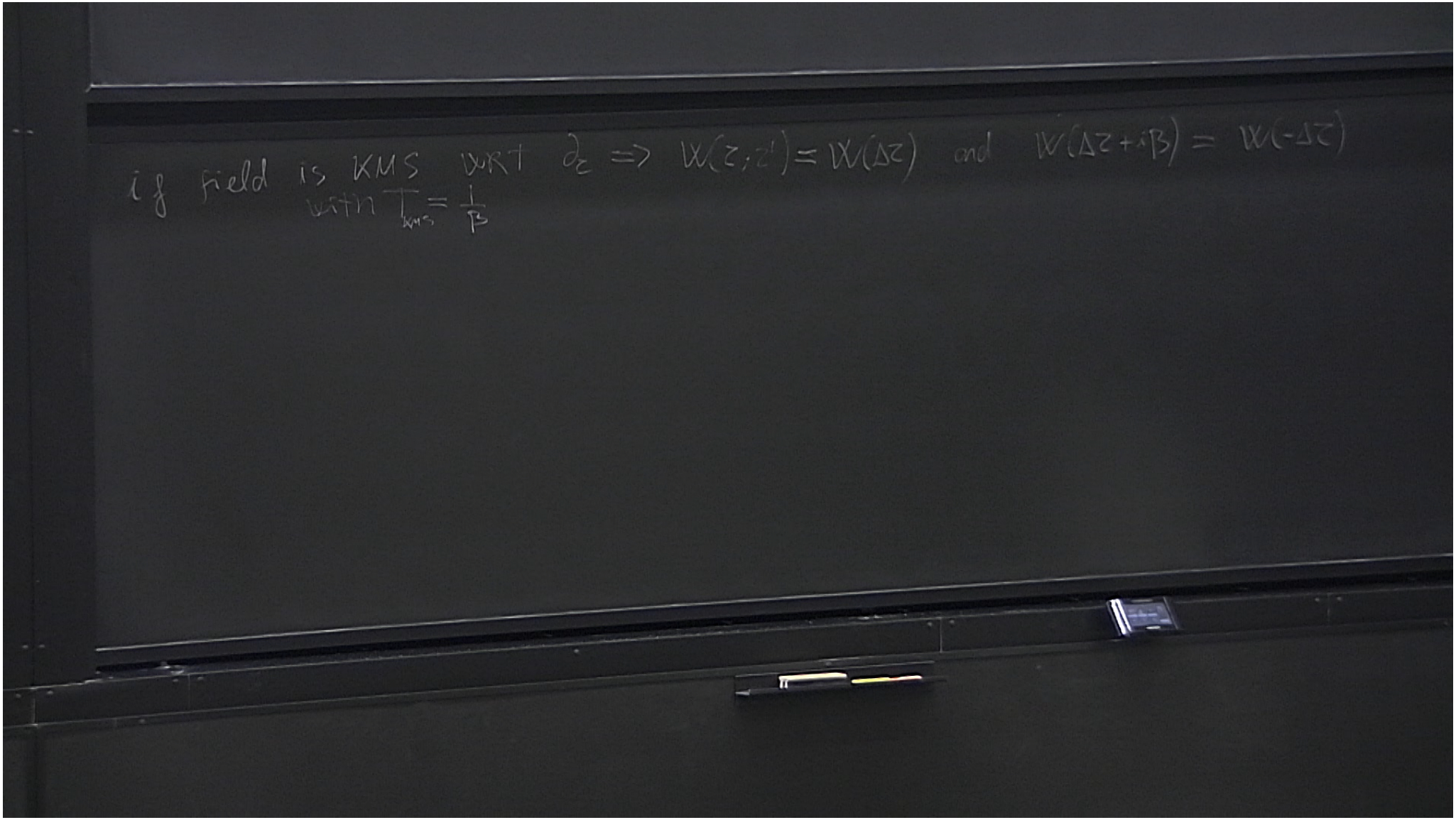
$$|\tilde{\chi}(w)|^2$$

$$\int \chi^*\left(\frac{z'}{\sigma}\right) e^{i(z-z')w} = 2\pi \int dz \int dz' \, \chi\left(\frac{z}{\sigma}\right) \chi^*\left(\frac{z'}{\sigma}\right) \delta(z-z') = 2\pi \underbrace{\int_{-\infty}^{\infty} dz \left|\chi\left(\frac{z}{\sigma}\right)\right|^2}_{1} = 2\pi$$

strongly supported on a scale $\sigma \Rightarrow \tilde{\chi}(\omega)$ is strongly supported on a scale $\frac{1}{\sigma}$

$$|\tilde{\chi}(\omega)|^2 = \tilde{W}(\Omega)$$

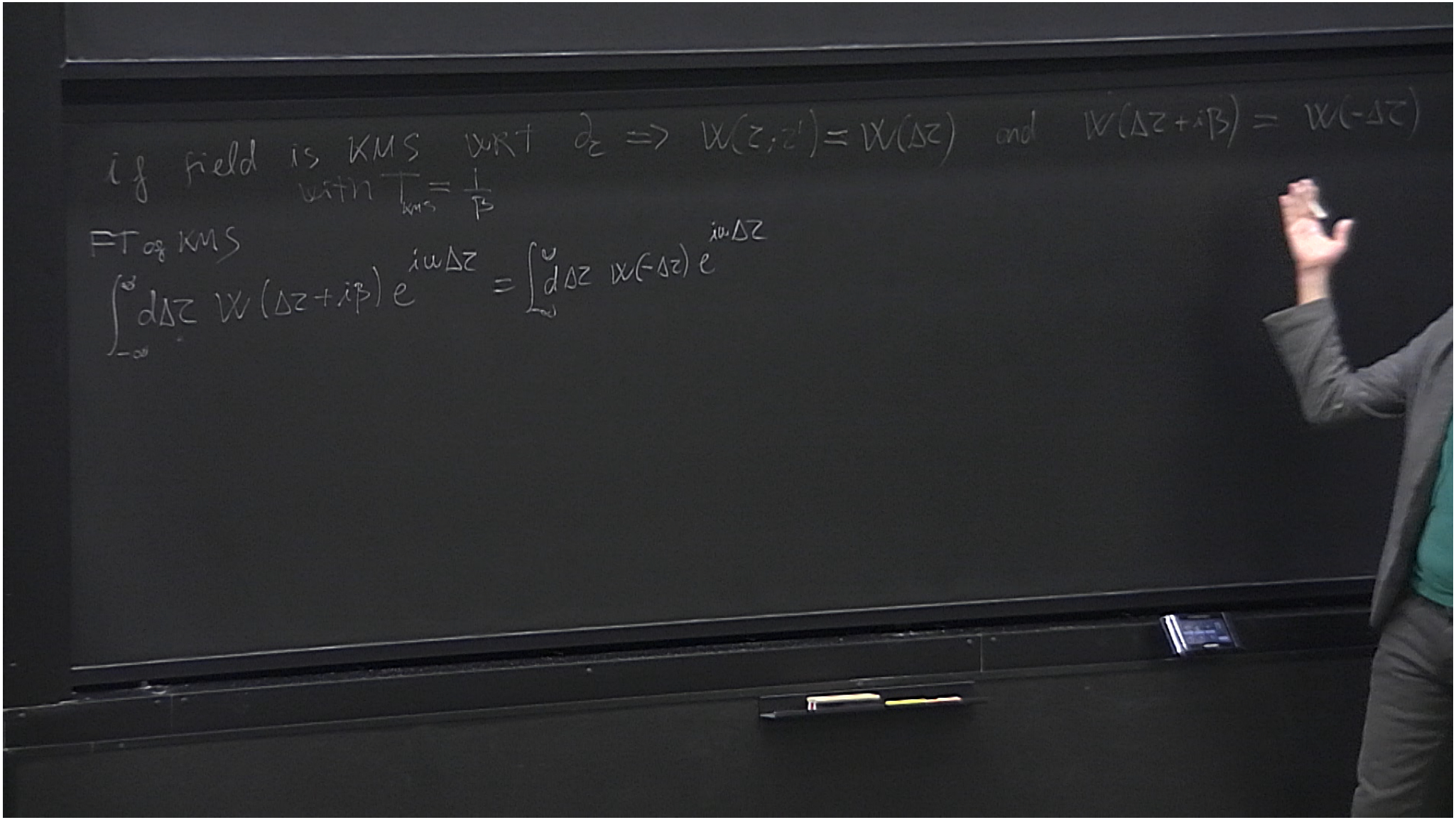
$$\int \chi^*\left(\frac{z'}{\sigma}\right) e^{i(z-z')w} = 2\pi \int dz \int dz' \, \chi\left(\frac{z}{\sigma}\right) \chi\left(\frac{z'}{\sigma}\right) \delta(z-z') = 2\pi \underbrace{\int_{-\infty}^{\infty} dz \left|\chi\left(\frac{z}{\sigma}\right)\right|^2}_{} = 2\pi$$

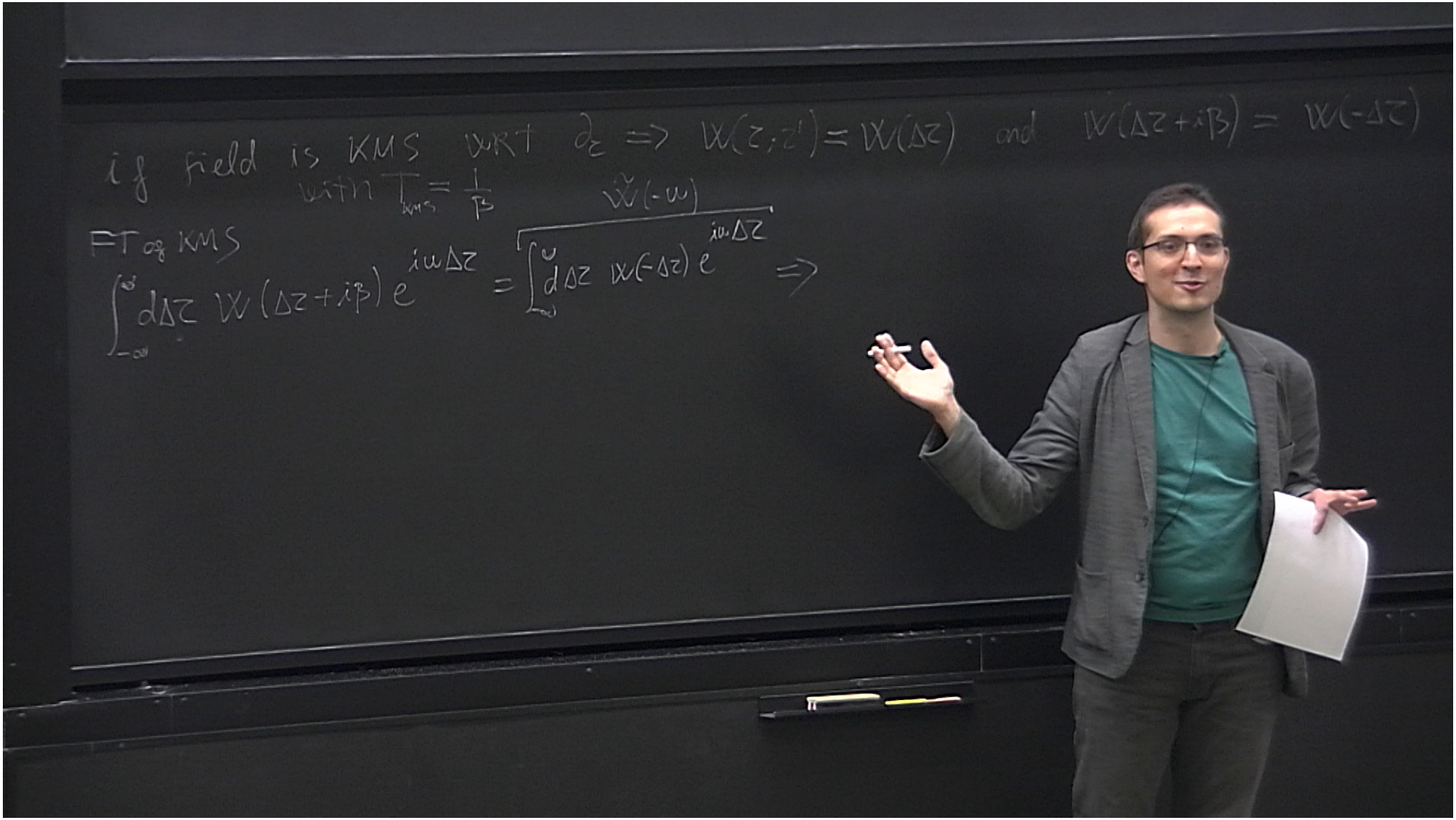

if field is KMS wrt ∂_τ => $W(\tau;\tau') = W(\Delta\tau)$ and $W(\Delta\tau + i\beta) = W(-\Delta\tau)$
with $T_{KMS} = \frac{1}{\beta}$

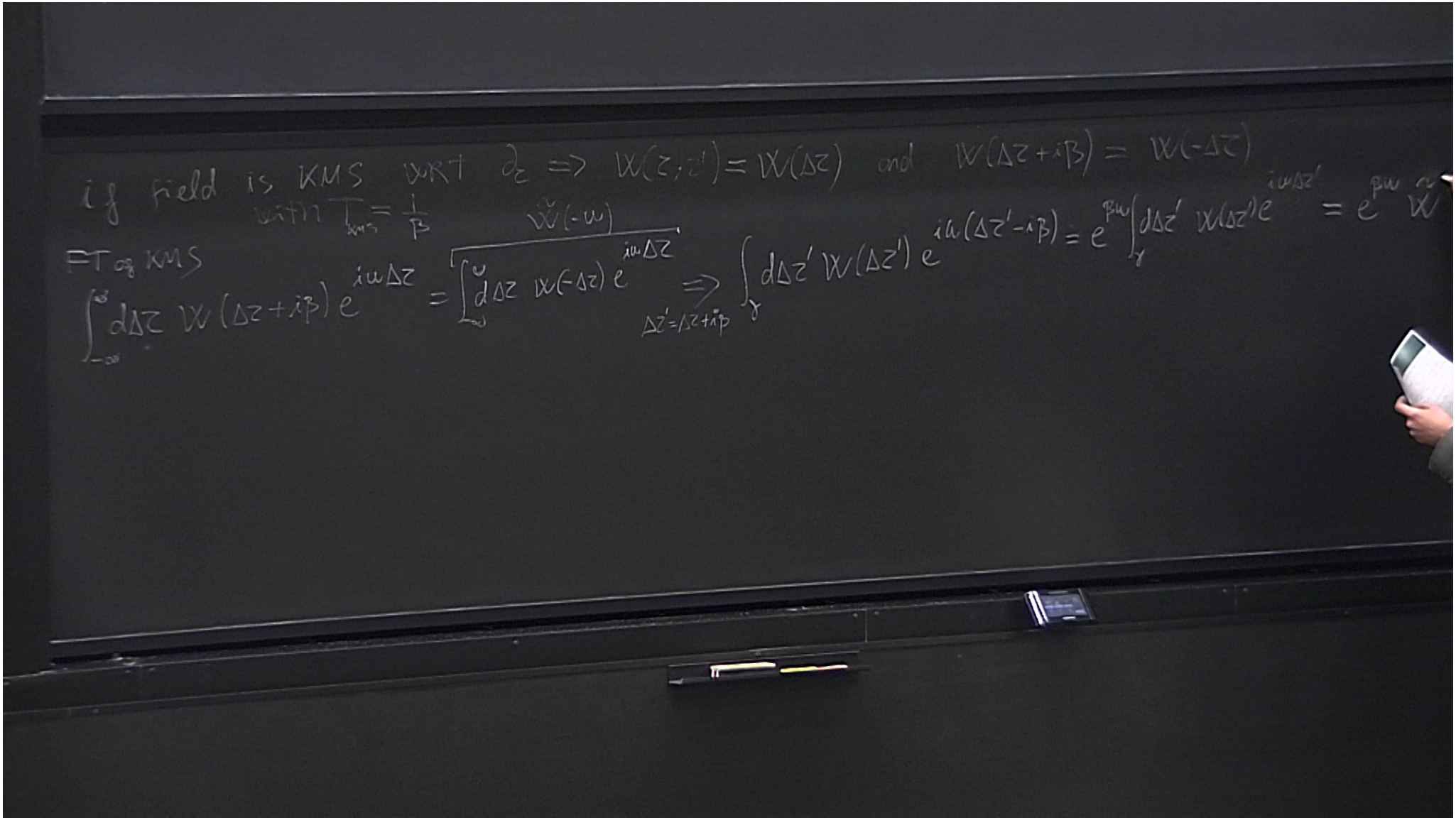
supported on a scale σ $\Rightarrow$ $\tilde{\chi}(w)$ is strongly supported on a scale $\frac{1}{\sigma}$

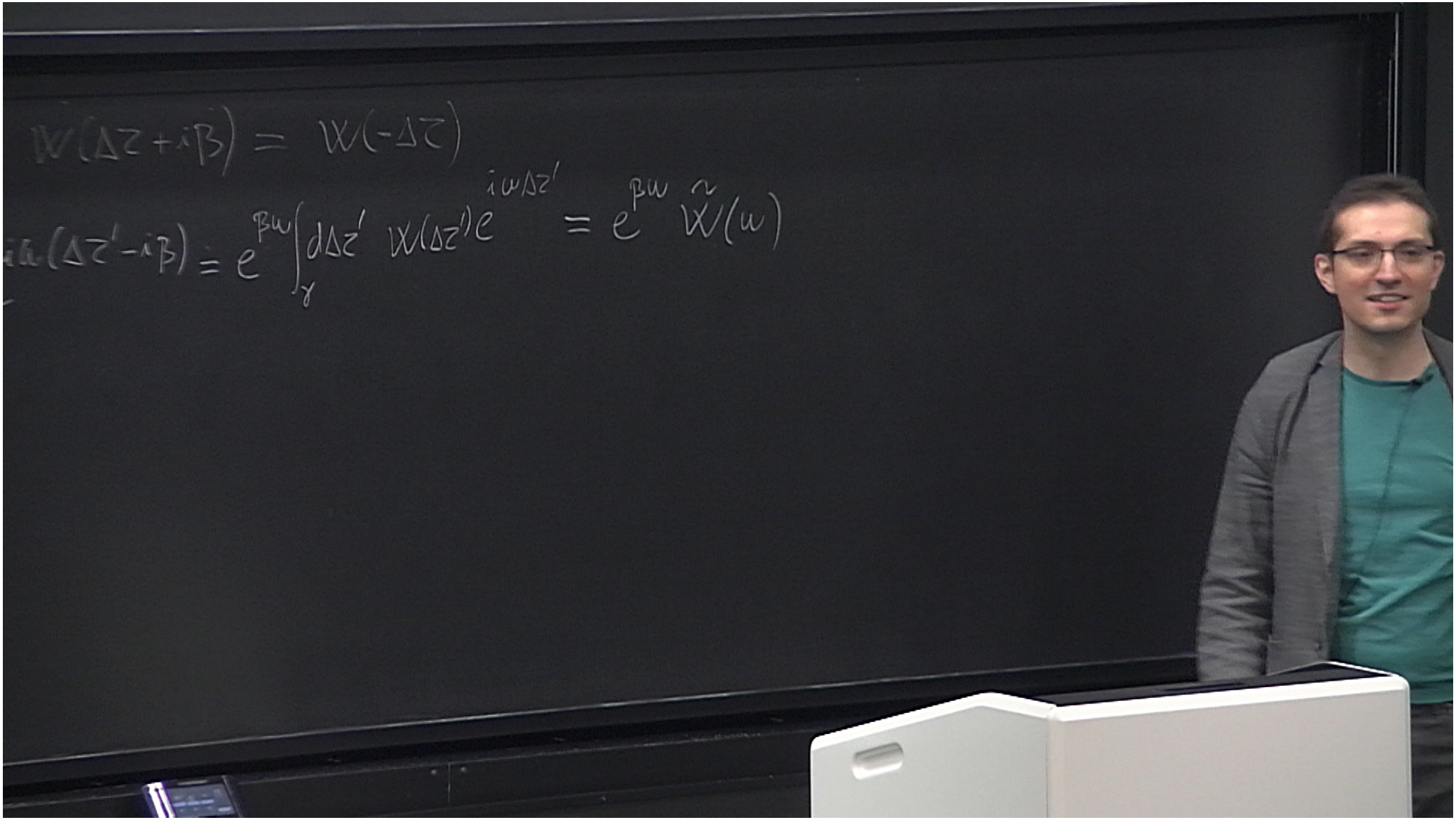
$= \tilde{\tilde{W}}(\Omega)$

$$e^{i(z-z')w} = 2\pi \int dz \int dz' \, \chi\left(\frac{z}{\sigma}\right) \chi\left(\frac{z'}{\sigma}\right) \delta(z-z') = 2\pi \underbrace{\int_{-\infty}^{\infty} dz \left|\chi\left(\frac{z}{\sigma}\right)\right|^2}_{1} = 2\pi$$


if field is KMS wrt ∂_τ with $T_{KMS} = \frac{1}{\beta}$
$\Rightarrow W(\tau;\tau') = W(\Delta\tau)$ and $W(\Delta\tau + i\beta) = W(-\Delta\tau)$
FT of KMS
$\int_{-\infty}^{\infty} d\Delta\tau\, W(\Delta\tau + i\beta)\, e^{i\omega\Delta\tau} = \int_{-\infty}^{\infty} d\Delta\tau\, W(-\Delta\tau)\, e^{i\omega\Delta\tau}$


if field is KMS WRT ∂_t => $W(t,t') = W(\Delta t)$ and $W(\Delta t + i\beta) = W(-\Delta t)$
with $T_{KMS} = \frac{1}{\beta}$
FT of KMS
$\int_{-\infty}^{\infty} d\Delta t\, W(\Delta t + i\beta)\, e^{i\omega\Delta t} = \int_{-\infty}^{\infty} d\Delta t\, W(-\Delta t)\, e^{i\omega\Delta t}$ =>
$\tilde{W}(-\omega)$


if field is KMS wrt $\partial_t \Rightarrow W(t,t') = W(\Delta t)$ and $W(\Delta t + i\beta) = W(-\Delta t)$
with $T_{KMS} = \frac{1}{\beta}$
FT of KMS
$\int_{-\infty}^{\infty} d\Delta t\, W(\Delta t + i\beta)\, e^{i\omega \Delta t} = \int_{-\infty}^{\infty} d\Delta t\, W(-\Delta t)\, e^{i\omega \Delta t} = \tilde{W}(-\omega)$
$\Delta t' = \Delta t + i\beta \Rightarrow \int_\gamma d\Delta t'\, W(\Delta t')\, e^{i\omega(\Delta t' - i\beta)} = e^{\beta\omega} \int_\gamma d\Delta t'\, W(\Delta t') e^{i\omega \Delta t'} = e^{\beta\omega} \tilde{W}$


$W(\Delta\tau + i\beta) = W(-\Delta\tau)$
$W(\Delta\tau' - i\beta) = e^{\beta w} \int_\gamma d\Delta\tau' \, W(\Delta\tau') e^{i w \Delta\tau'} = e^{\beta w} \tilde{W}(w)$

if field is KMS wrt $\partial_\tau \Rightarrow W(\tau;\tau') = W(\Delta\tau)$ and $W(\Delta\tau + i\beta) =$

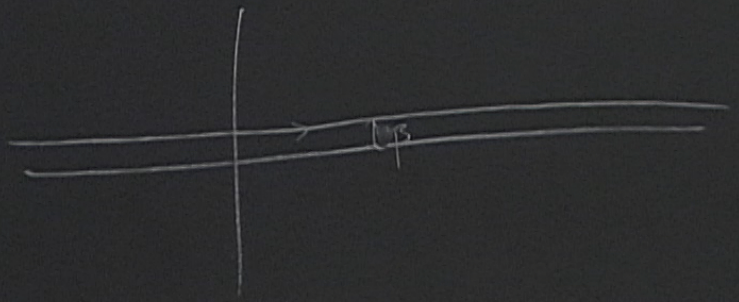
with $T_{KMS} = \frac{1}{\beta}$

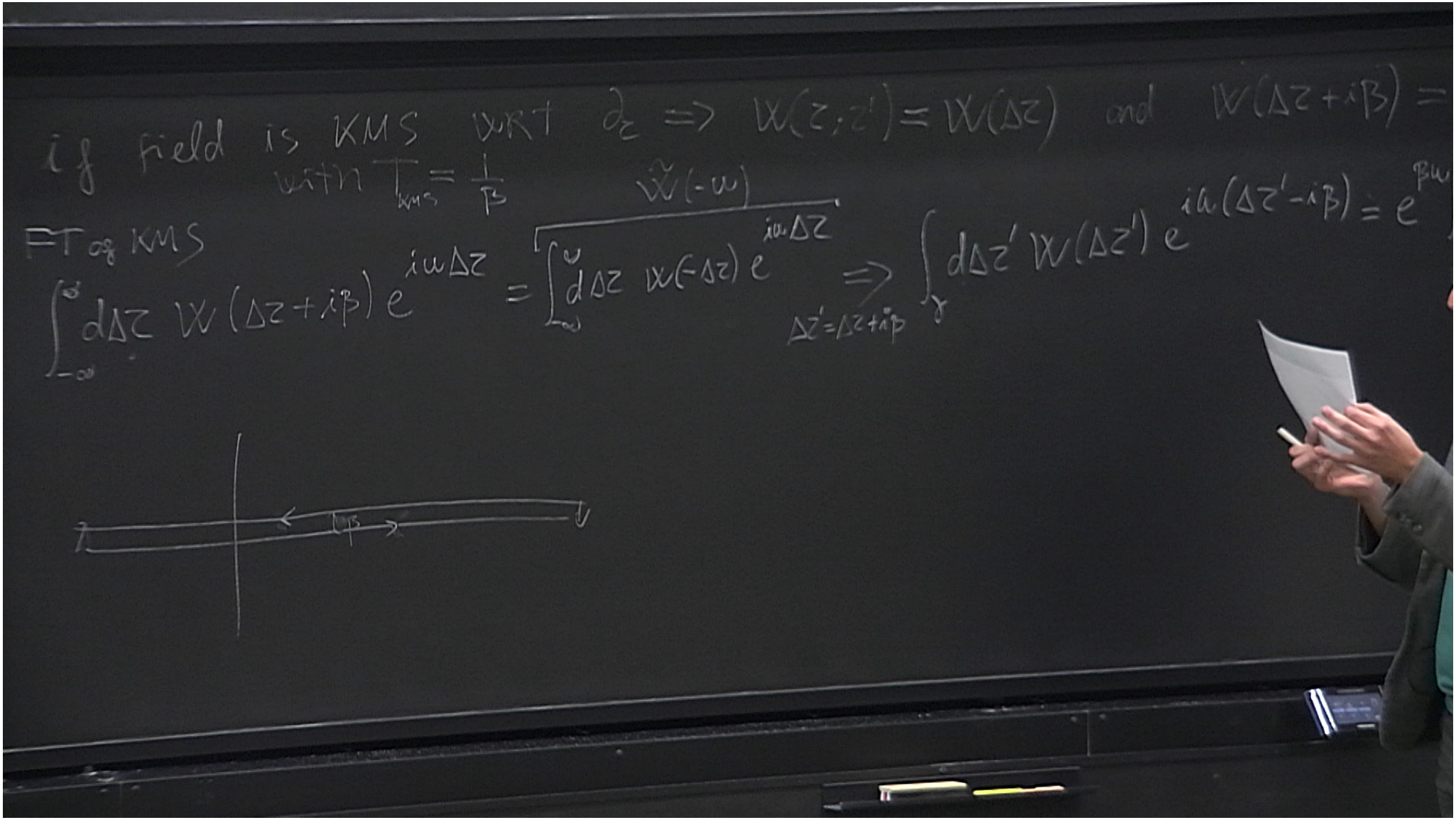
FT of KMS

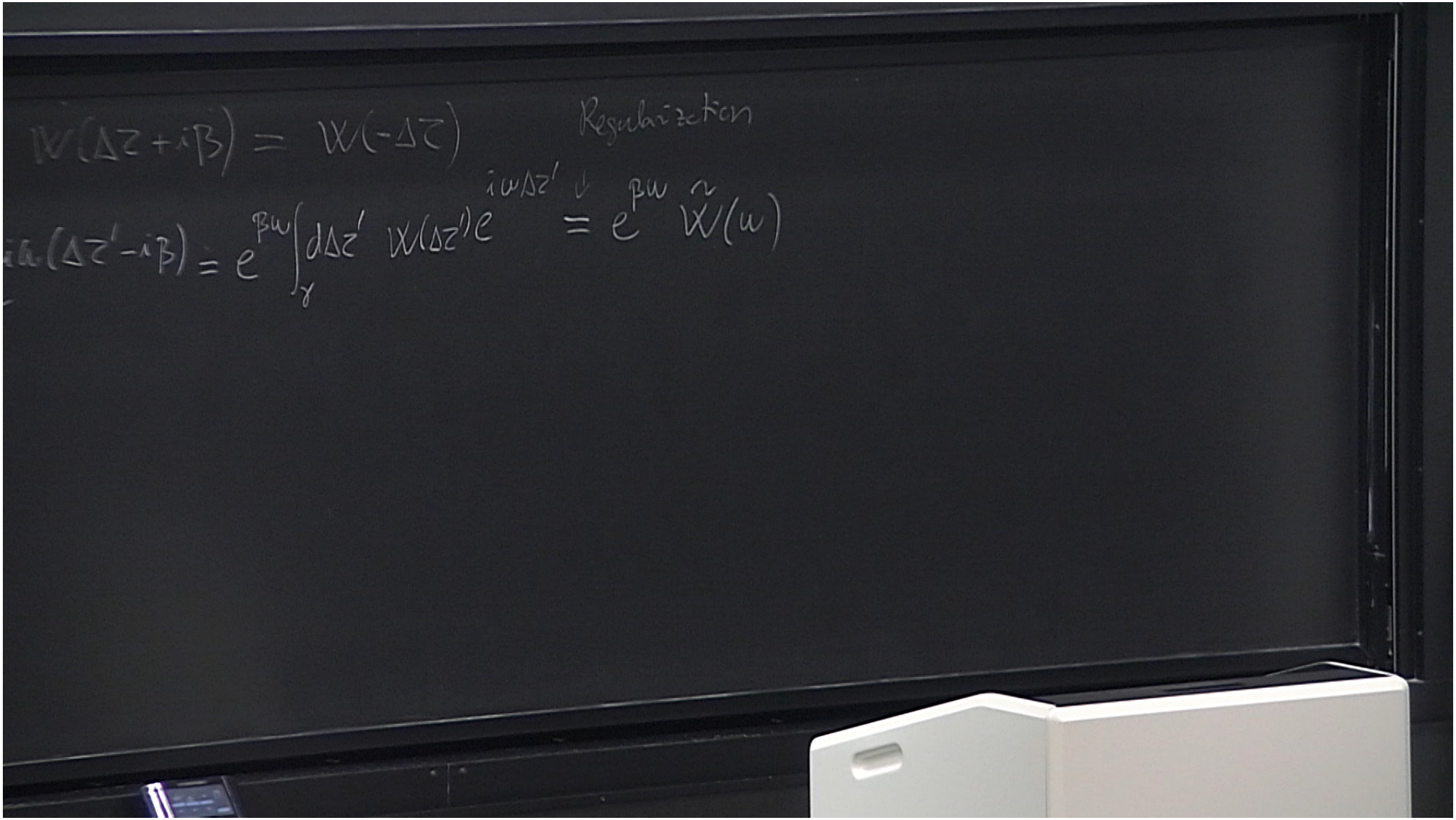
$$\int_{-\infty}^{\infty} d\Delta\tau\, W(\Delta\tau + i\beta)\, e^{i\omega\Delta\tau} = \int_{-\infty}^{\infty} d\Delta\tau\, W(-\Delta\tau)\, e^{i\omega\Delta\tau} = \tilde{W}(-\omega)$$

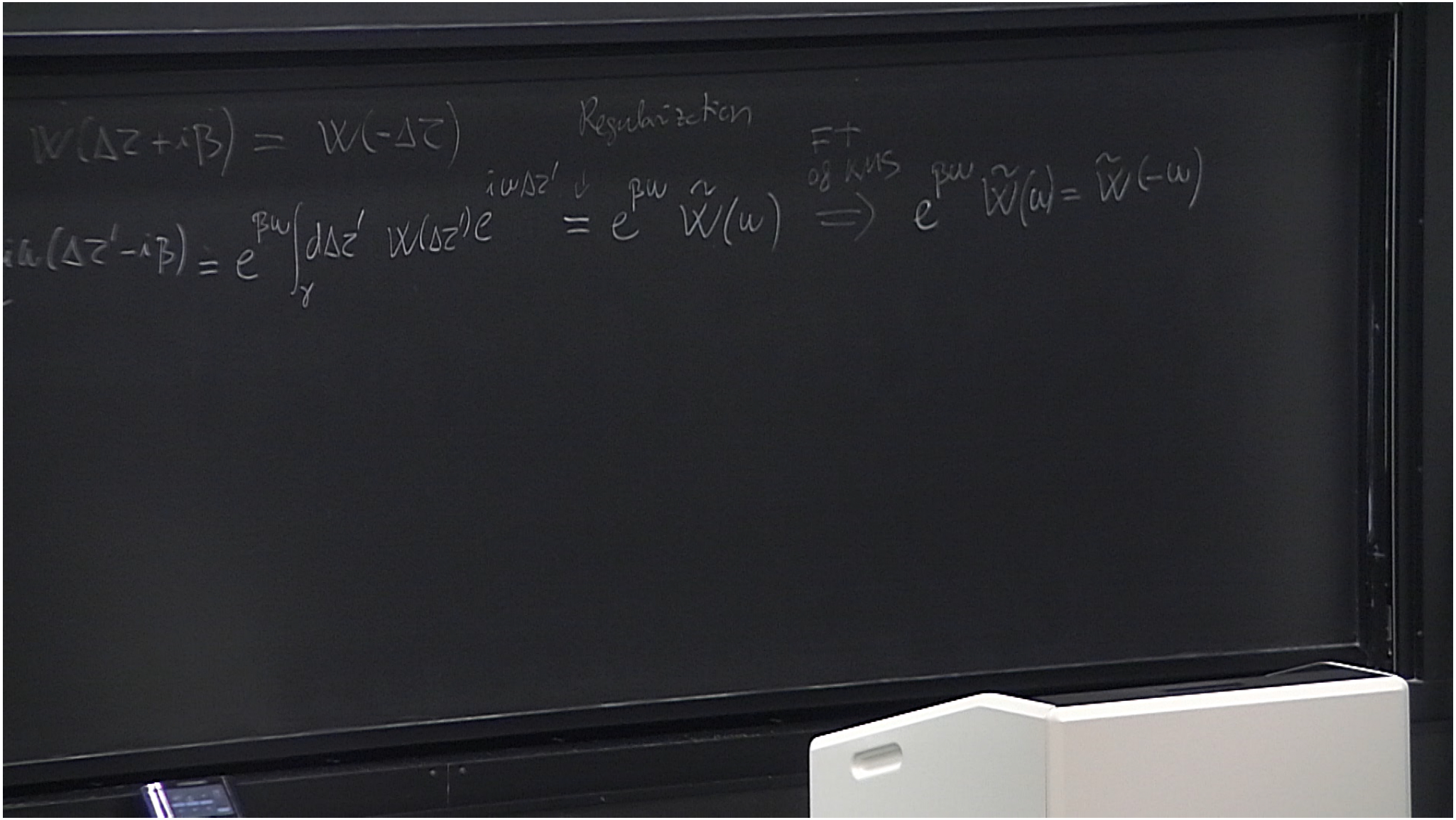
$$\Rightarrow \int_{\gamma} d\Delta\tau'\, W(\Delta\tau')\, e^{i\omega(\Delta\tau' - i\beta)} = e^{\beta\omega}$$

$\Delta\tau' = \Delta\tau + i\beta$




if field is KMS wrt $\partial_z \Rightarrow W(z,z') = W(\Delta z)$ and $W(\Delta z + i\beta) =$
with $T_{KMS} = \frac{1}{\beta}$
FT of KMS
$\int_{-\infty}^{\infty} d\Delta z\, W(\Delta z + i\beta)\, e^{i\omega\Delta z} = \int_{-\infty}^{\infty} d\Delta z\, W(-\Delta z)\, e^{i\omega\Delta z}$
$\tilde{W}(-\omega)$
$\Rightarrow$
$\Delta z' = \Delta z + i\beta$
$\int_{\gamma} d\Delta z'\, W(\Delta z')\, e^{i\omega(\Delta z' - i\beta)} = e^{\beta\omega}$
$i\beta$


$W(\Delta\tau + i\beta) = W(-\Delta\tau)$
Regularization
$\tilde{W}(\Delta\tau' - i\beta) = e^{\beta w} \int_\gamma d\Delta\tau' \, W(\Delta\tau') e^{i w \Delta\tau'} = e^{\beta w} \tilde{W}(w)$


$W(\Delta\tau + i\beta) = W(-\Delta\tau)$
Regularization
$\tilde{W}(\omega) = e^{\beta\omega} \int_\gamma d\Delta\tau' \, W(\Delta\tau') e^{i\omega\Delta\tau'} = e^{\beta\omega} \tilde{W}(\omega)$
FT of KMS
$\Rightarrow e^{\beta\omega} \tilde{W}(\omega) = \tilde{W}(-\omega)$

if field is KMS wrt $\partial_t \Rightarrow W(t,t') = W(\Delta t)$ and $W(\Delta t + i\beta) =$

with $T_{KMS} = \frac{1}{\beta}$

FT of KMS

$$\int_{-\infty}^{\infty} d\Delta t\, W(\Delta t + i\beta)\, e^{i\omega \Delta t} = \int_{-\infty}^{\infty} d\Delta t\, W(-\Delta t)\, e^{i\omega \Delta t} = \tilde{W}(-\omega)$$

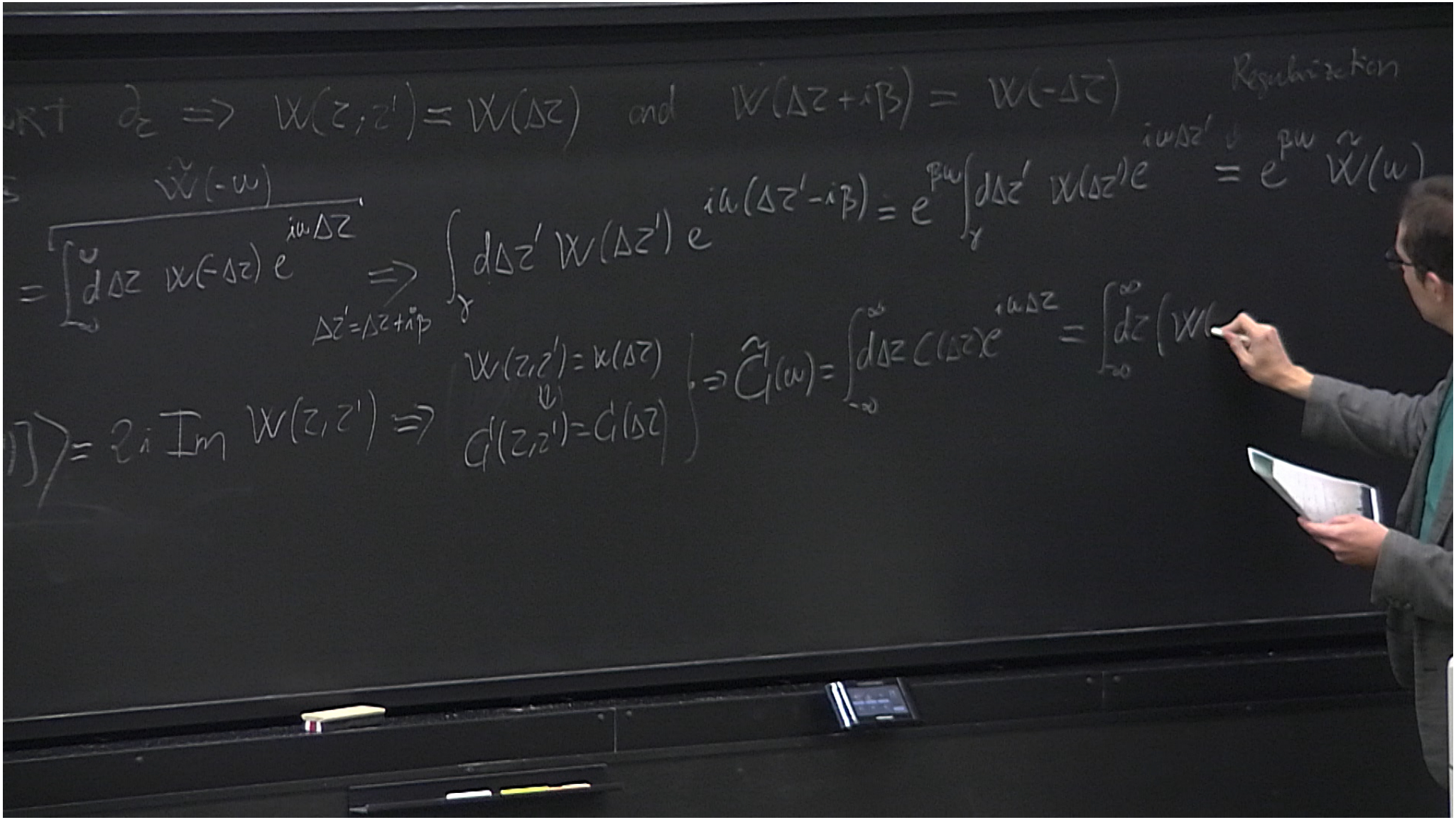
$$\Delta t' = \Delta t + i\beta \Rightarrow \int_{\gamma} d\Delta t'\, W(\Delta t')\, e^{i\omega(\Delta t' - i\beta)} = e^{\beta\omega}$$

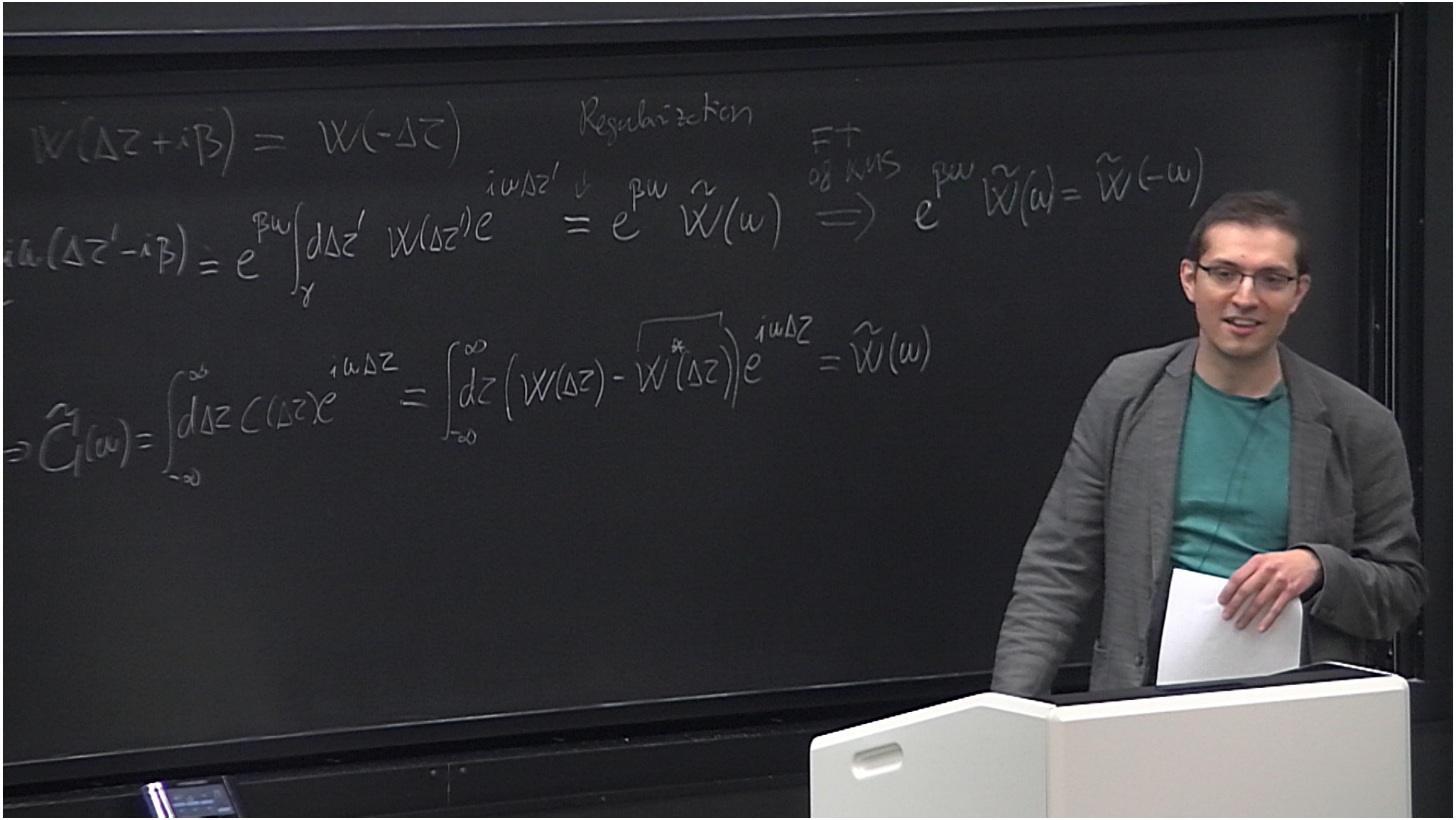
$$G(t,t') := \langle [\hat{\phi}(t), \hat{\phi}(t')] \rangle = 2i\, \mathrm{Im}\, W(t,t') \Rightarrow$$

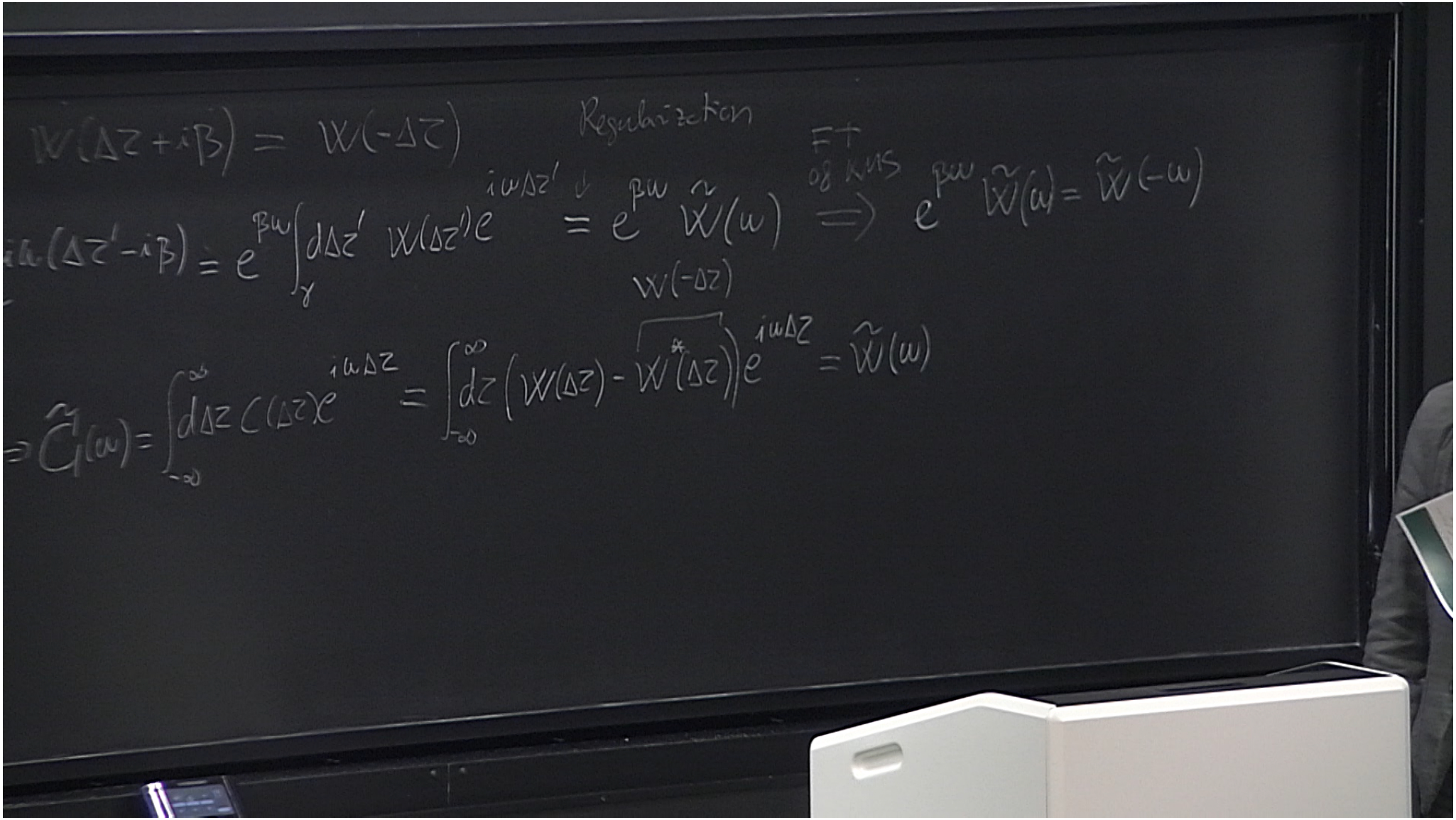
$$W(t,t') = W(\Delta t)$$

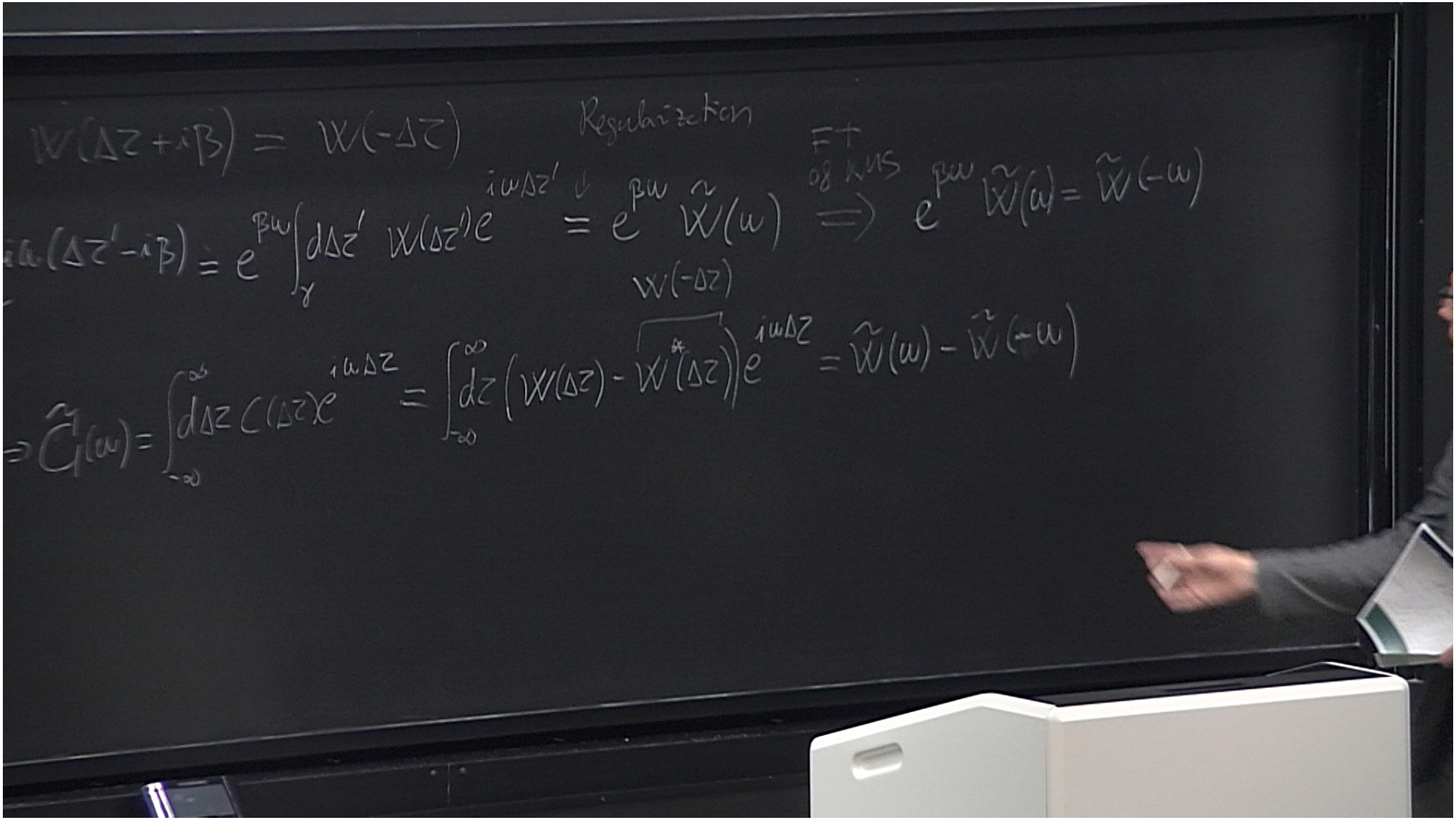
$$\Downarrow$$

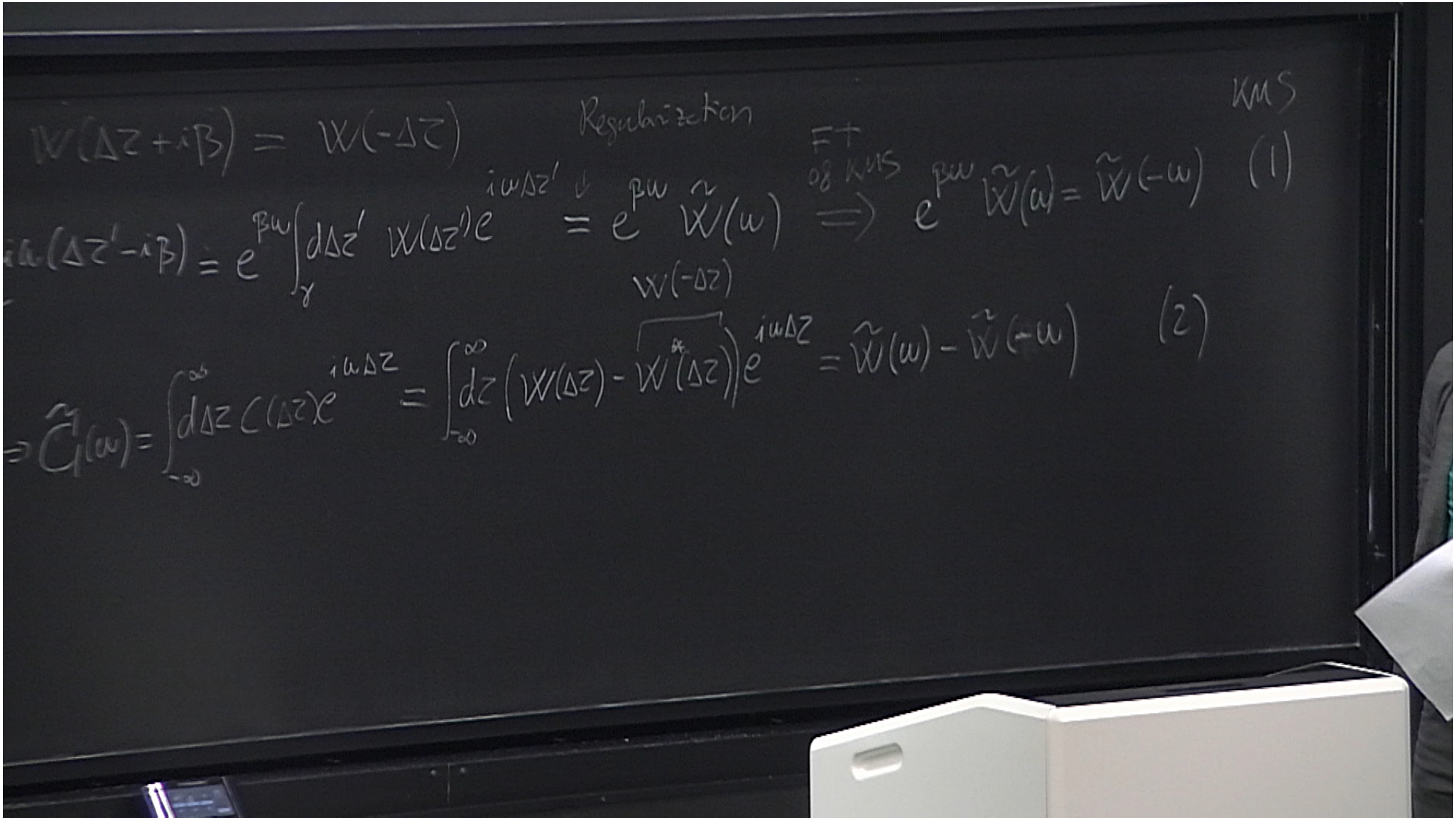
$$G(t,t') = G(\Delta t)$$


Regularization


Regularization
FT
of KMS


$W(\Delta z + i\beta) = W(-\Delta z)$
Regularization
$\tilde{W}(\Delta z' - i\beta) = e^{\beta w} \int_\gamma d\Delta z' \, W(\Delta z') e^{i w \Delta z'} = e^{\beta w} \tilde{W}(w)$
FT of KMS
$\Rightarrow e^{\beta w} \tilde{W}(w) = \tilde{W}(-w)$
$W(-\Delta z)$
$\Rightarrow \tilde{C}(w) = \int_{-\infty}^{\infty} d\Delta z \, C(\Delta z) e^{i w \Delta z} = \int_{-\infty}^{\infty} dz \left(W(\Delta z) - \overline{W^*(\Delta z)} \right) e^{i w \Delta z} = \tilde{W}(w)$


$W(\Delta z + i\beta) = W(-\Delta z)$
Regularization
$\tilde{G}(\Delta z' - i\beta) = e^{\beta\omega} \int_\gamma d\Delta z' \, W(\Delta z') e^{i\omega\Delta z'} = e^{\beta\omega} \tilde{W}(\omega)$
FT of KMS
$\Rightarrow e^{\beta\omega} \tilde{W}(\omega) = \tilde{W}(-\omega)$
$W(-\Delta z)$
$\Rightarrow \tilde{C}(\omega) = \int_{-\infty}^{\infty} d\Delta z \, C(\Delta z) e^{i\omega\Delta z} = \int_{-\infty}^{\infty} dz \left(W(\Delta z) - \overline{W^*(\Delta z)} \right) e^{i\omega\Delta z} = \tilde{W}(\omega) - \tilde{W}(-\omega)$



$$W(\Delta\tau + i\beta) = W(-\Delta\tau)$$

Regularization

KMS

$$i\omega(\Delta\tau' - i\beta) = e^{\beta\omega}\int_\gamma d\Delta\tau'\, W(\Delta\tau')\, e^{i\omega\Delta\tau'} = e^{\beta\omega}\,\tilde{W}(\omega) \overset{\text{FT of KMS}}{\Longrightarrow} e^{\beta\omega}\,\tilde{W}(\omega) = \tilde{W}(-\omega) \qquad (1)$$

$$\Rightarrow \tilde{C}(\omega) = \int_{-\infty}^{\infty} d\Delta\tau\, C(\Delta\tau)\, e^{i\omega\Delta\tau} = \int_{-\infty}^{\infty} d\tau \left(W(\Delta\tau) - \overbrace{W^*(\Delta\tau)}^{W(-\Delta\tau)}\right) e^{i\omega\Delta\tau} = \tilde{W}(\omega) - \tilde{W}(-\omega) \qquad (2)$$

if field is KMS wrt $\partial_t \Rightarrow W(t,t') = W(\Delta t)$ and $W(\Delta t + i\beta) = W(-\Delta t)$

with $T_{KMS} = \frac{1}{\beta}$

$\tilde{W}(-\omega)$

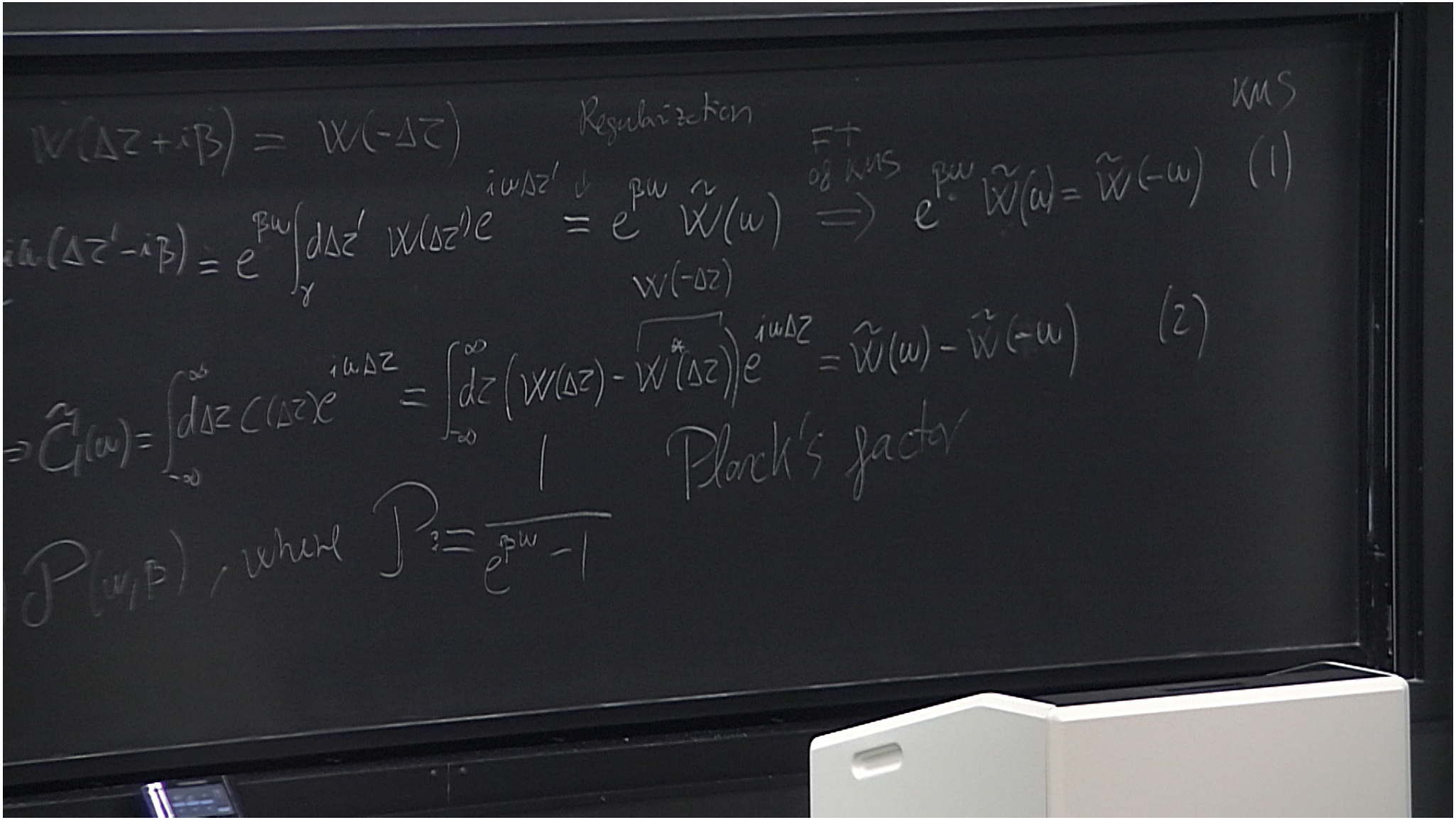
$$\int_{-\infty}^{\infty} d\Delta t\, W(\Delta t + i\beta)\, e^{i\omega\Delta t} = \int_{\infty}^{\infty} d\Delta t\, W(-\Delta t)\, e^{i\omega\Delta t} \Rightarrow \int_{\gamma} d\Delta t'\, W(\Delta t')\, e^{i\omega(\Delta t' - i\beta)} = e^{\beta\omega} \int_{\gamma} d\Delta t'\, W(\Delta t')\, e^{i\omega\Delta t'} = e^{\beta\omega}\, \tilde{W}(\omega)$$

$\Delta t' = \Delta t + i\beta$

$W(-\Delta t)$

$$G(t,t') := \langle [\hat{\phi}(t), \hat{\phi}(t')] \rangle = 2i\, \mathrm{Im}\, W(t,t') \Rightarrow \begin{matrix} W(t,t') = W(\Delta t) \\ \Downarrow \\ G(t,t') = G(\Delta t) \end{matrix} \Rightarrow \tilde{G}(\omega) = \int_{-\infty}^{\infty} d\Delta t\, G(\Delta t) e^{i\omega\Delta t} = \int_{-\infty}^{\infty} dt \left(W(\Delta t) - W^*(\Delta t) \right) e^{i\omega}$$

$$(2) \to (1) \Rightarrow \tilde{G}(\omega) = \tilde{W}(\omega) - e^{\beta\omega}\, \tilde{W}(\omega) \Rightarrow \tilde{W}(\omega,\beta) = -\tilde{G}(\omega,\beta)\, \mathcal{P}(\omega,\beta), \text{ where } \mathcal{P} := \frac{1}{e^{\beta\omega} - 1}$$



$$W(\Delta\tau + i\beta) = W(-\Delta\tau)$$

Regularization

KMS

$$e^{i\omega(\Delta\tau' - i\beta)} = e^{\beta\omega}\int_\gamma d\Delta\tau'\, W(\Delta\tau')\, e^{i\omega\Delta\tau'} = e^{\beta\omega}\,\tilde{W}(\omega) \overset{\text{FT of KMS}}{\Longrightarrow} e^{\beta\omega}\,\tilde{W}(\omega) = \tilde{W}(-\omega) \qquad (1)$$

$$\Rightarrow \tilde{G}(\omega) = \int_{-\infty}^{\infty} d\Delta\tau\, C(\Delta\tau)\, e^{i\omega\Delta\tau} = \int_{-\infty}^{\infty} d\tau \left(W(\Delta\tau) - \overbrace{W^*(\Delta\tau)}^{W(-\Delta\tau)}\right) e^{i\omega\Delta\tau} = \tilde{W}(\omega) - \tilde{W}(-\omega) \qquad (2)$$

Planck's factor

$$\mathcal{P}(\omega,\beta), \text{ where } \mathcal{P} := \frac{1}{e^{\beta\omega} - 1}$$

$$\chi\left(\frac{z}{\sigma}\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty} du\, \tilde{\chi}^*(u)\, e^{iu\frac{z}{\sigma}}$$

$$\tilde{F}(\Omega,\sigma) = \frac{1}{8\pi^2\sigma}\int_{-\infty}^{\infty}du\int_{-\infty}^{\infty}dv\int_{-\infty}^{\infty}du\int_{-\infty}^{\infty}du'\,\tilde{\chi}^*(u)\,\tilde{\chi}(u')\,e^{\frac{i}{2\sigma}(u+u')u}\,e^{\frac{i}{2\sigma}(u-u')v}\,W(v)\,e^{i(\Omega+\frac{u}{\sigma})v}$$

$$\tilde{F}(\Omega,\sigma) = \frac{1}{2\pi}\int_{-\infty}^{\infty}du\int_{-\infty}^{\infty}du\,|\tilde{\chi}(u)|^2\,\tilde{W}(\Omega+\tfrac{u}{\sigma}) = \frac{1}{2\pi}\int_{-\infty}^{\infty}du\,|\tilde{\chi}(u)|^2\,\tilde{W}(\Omega+\tfrac{u}{\sigma})$$

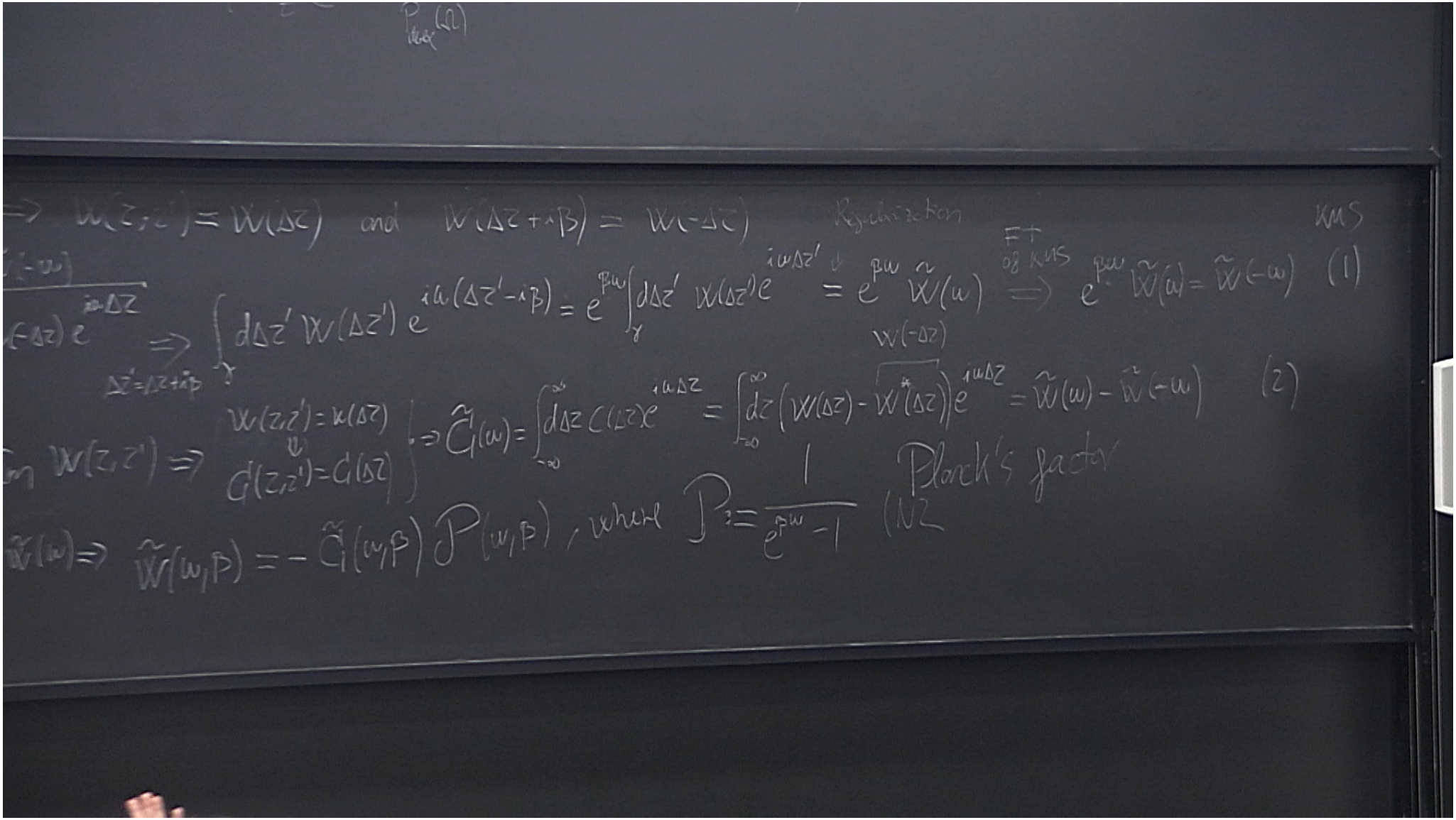
if $\tilde{\chi}(u)$ decays fast enough — if $\chi\left(\frac{z}{\sigma}\right)$ is strongly supported on a scale $\sigma \Rightarrow \tilde{\chi}(u)$

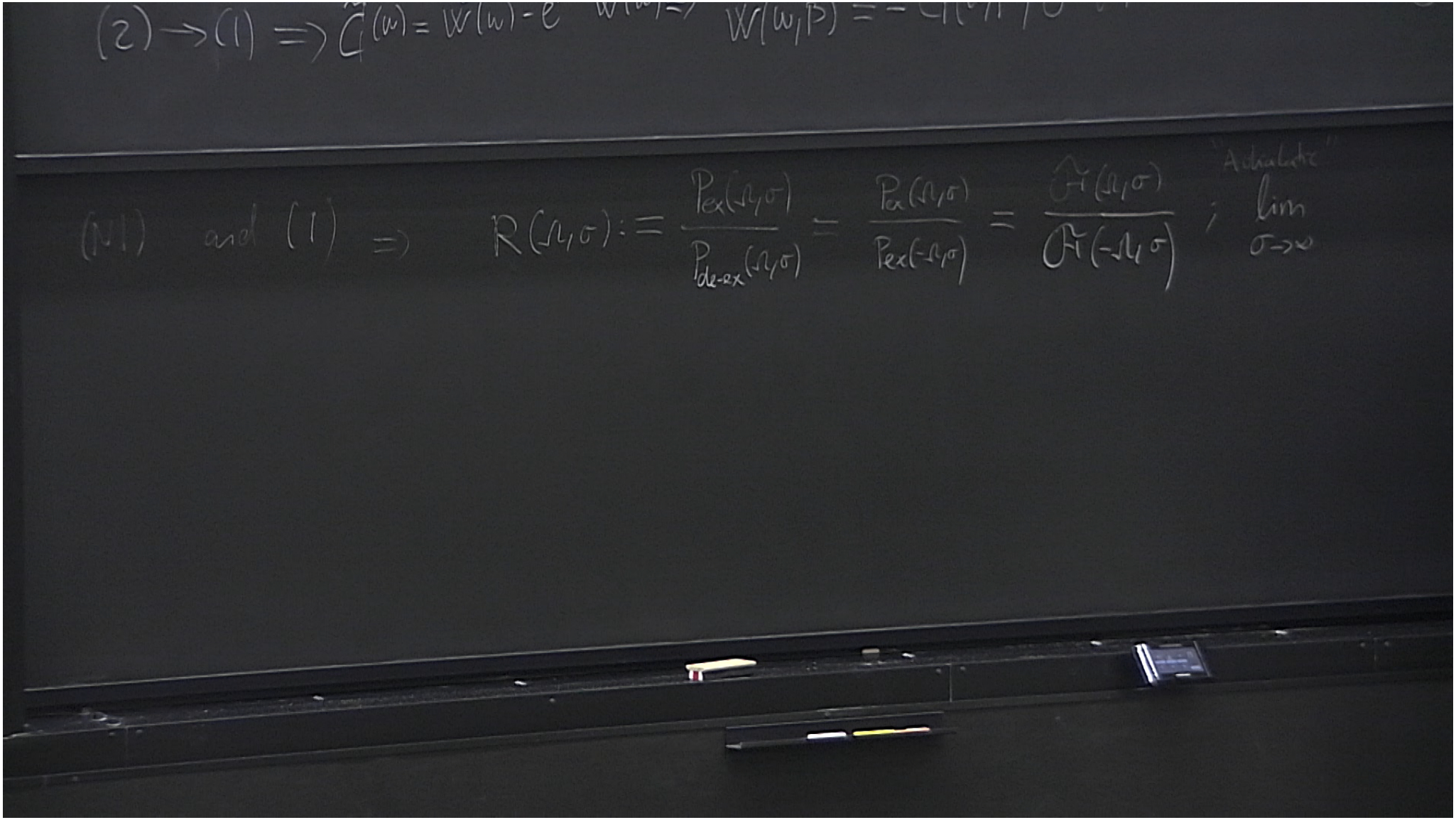
Not switching "nervously" (Adiabaticity)

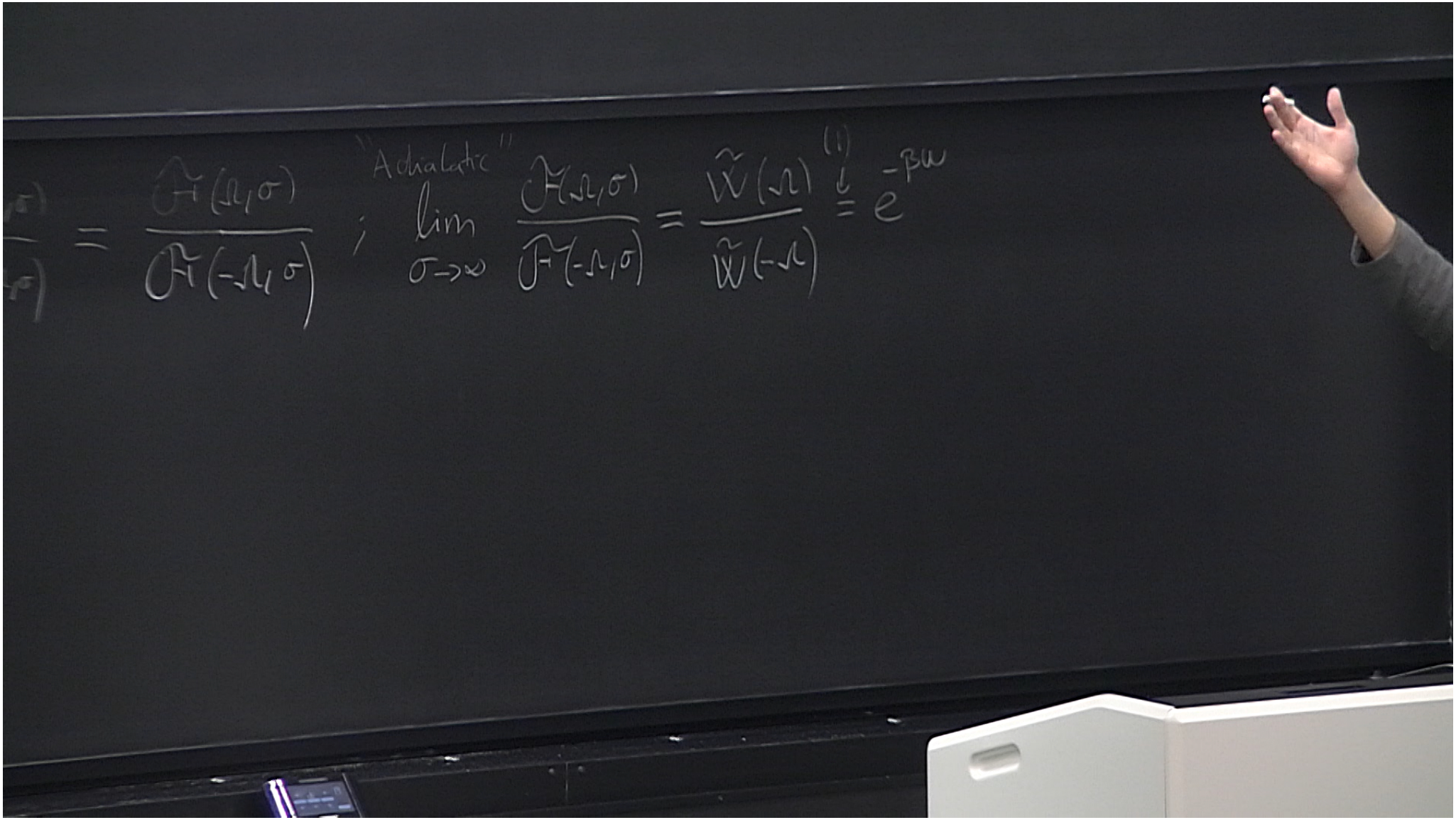
Adiabatic

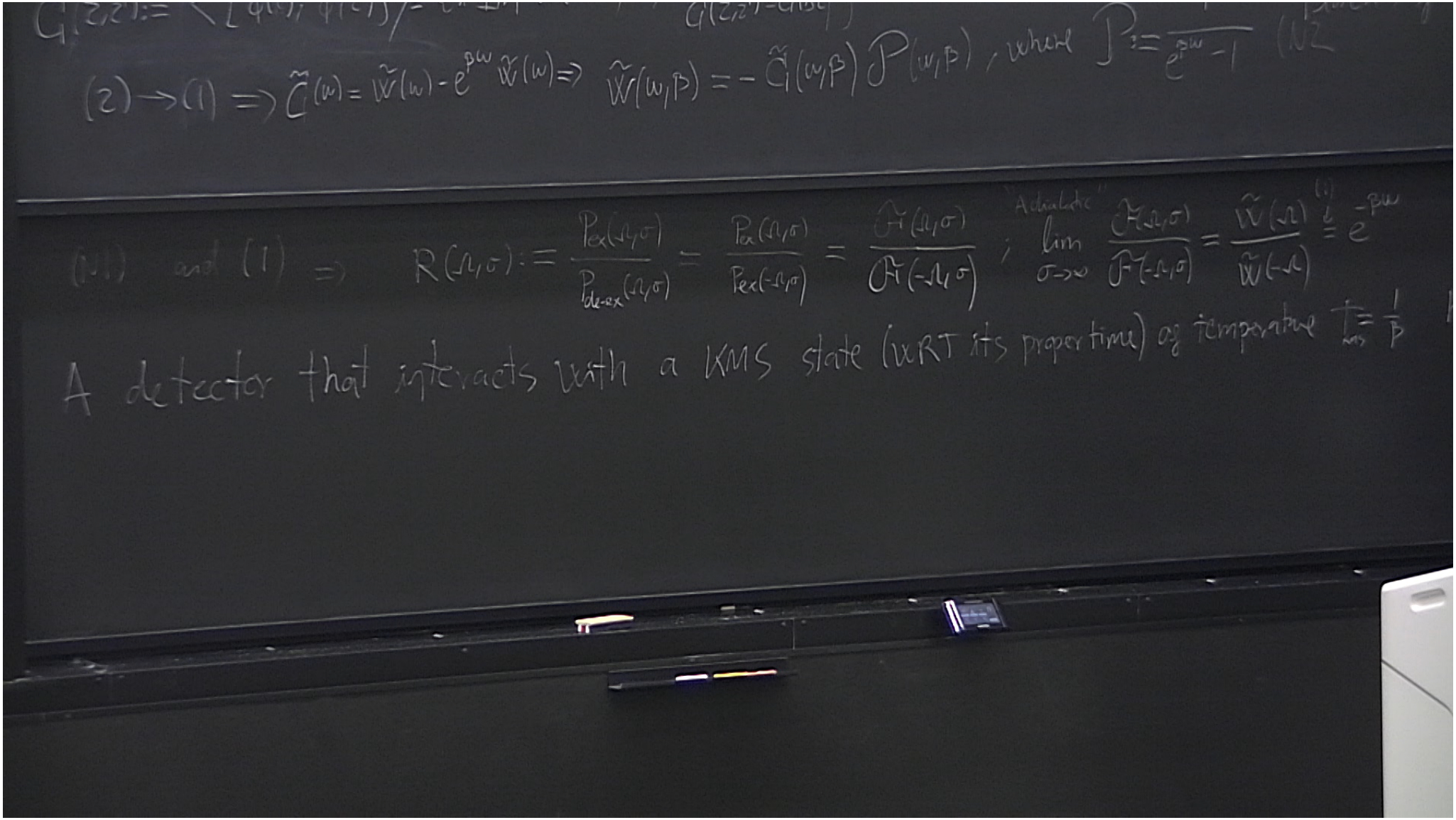
$$\lim_{\sigma\to\infty}\tilde{F}(\Omega,\sigma) = \frac{1}{2\pi}\int_{-\infty}^{\infty}du\,|\tilde{\chi}(u)|^2\,\tilde{W}(\Omega) = \frac{\tilde{W}(\Omega)}{2\pi}\int_{-\infty}^{\infty}du\,|\tilde{\chi}(u)|^2 = \tilde{W}(\Omega) \quad (N1)$$

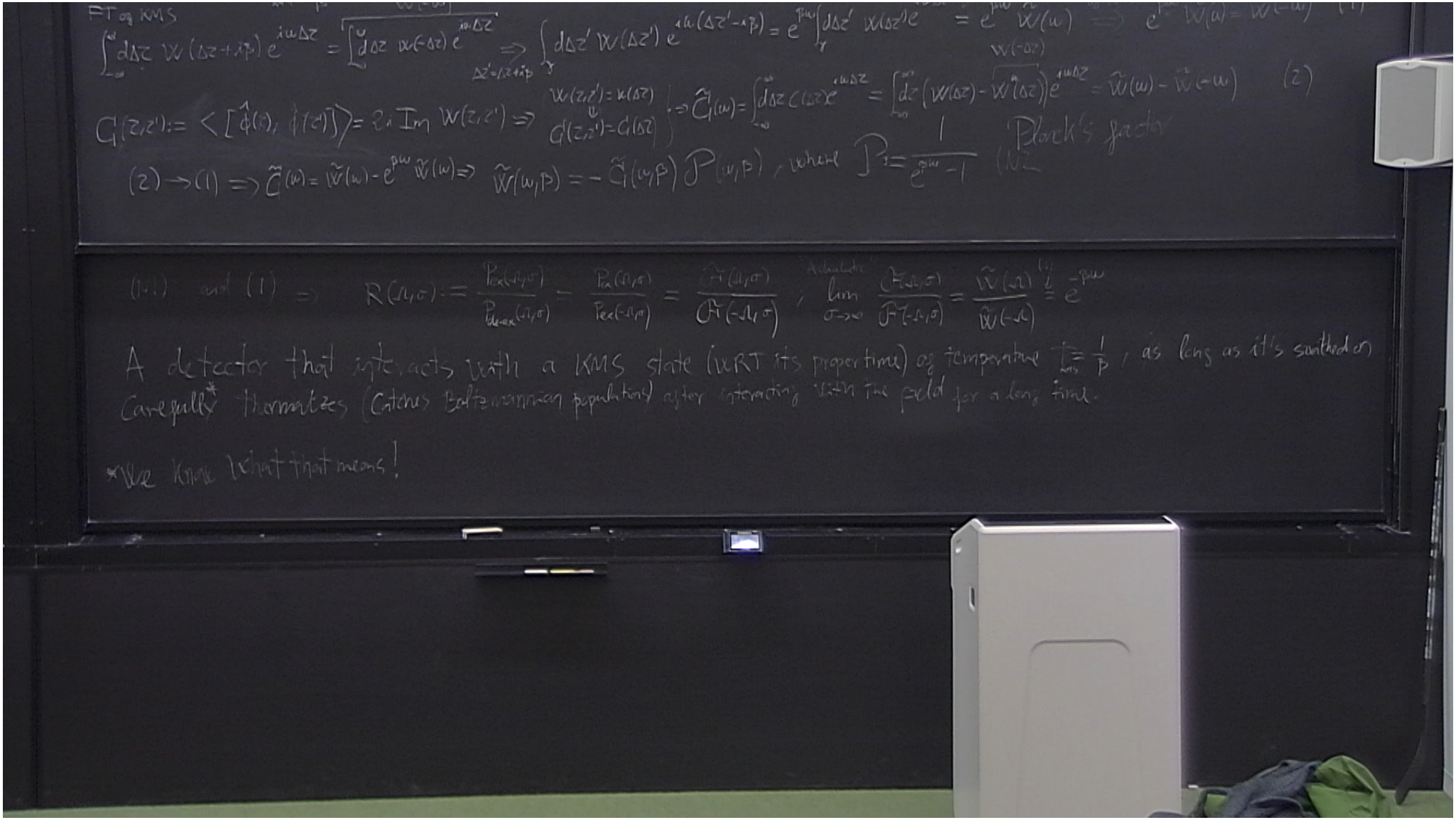
$$\int_{-\infty}^{\infty}dz\,\left|\chi\left(\frac{z}{\sigma}\right)\right|^2 = 1 \Rightarrow \left(\int_{-\infty}^{\infty}du\,|\tilde{\chi}(u)|^2 = \int_{-\infty}^{\infty}dz\int_{-\infty}^{\infty}dz'\int_{-\infty}^{\infty}du\,\chi\left(\frac{z}{\sigma}\right)\chi^*\left(\frac{z'}{\sigma}\right)e^{i(z-z')u} = 2\pi\int dz\int dz'\,\chi\left(\frac{z}{\sigma}\right)\chi\left(\frac{z'}{\sigma}\right)\right.$$


$\Rightarrow W(z,z') = W(\Delta z)$ and $W(\Delta z + i\beta) = W(-\Delta z)$
Regularization
KMS
FT of KMS
$\int_\gamma d\Delta z' W(\Delta z') e^{i\omega(\Delta z' - i\beta)} = e^{\beta\omega} \int_\gamma d\Delta z' W(\Delta z') e^{i\omega\Delta z'} = e^{\beta\omega} \tilde{W}(\omega) \Rightarrow e^{\beta\omega} \tilde{W}(\omega) = \tilde{W}(-\omega)$ (1)
$W(-\Delta z)$
$\Delta z' = \Delta z + i\beta$
$W(z,z') = W(\Delta z)$
$G(z,z') = G(\Delta z)$
$\Rightarrow \tilde{G}(\omega) = \int_{-\infty}^{\infty} d\Delta z\, G(\Delta z) e^{i\omega\Delta z} = \int_{-\infty}^{\infty} dz \left(W(\Delta z) - W^*(\Delta z)\right) e^{i\omega\Delta z} = \tilde{W}(\omega) - \tilde{W}(-\omega)$ (2)
$W(z,z') \Rightarrow$
$\tilde{W}(\omega) \Rightarrow \tilde{W}(\omega,\beta) = -\tilde{G}(\omega,\beta)\, \mathcal{P}(\omega,\beta)$, where $\mathcal{P} = \frac{1}{e^{\beta\omega} - 1}$
Planck's factor


(2) → (1) ⇒
(N.I) and (1) ⇒ $R(\lambda,\sigma) := \frac{P_{ex}(\lambda,\sigma)}{P_{de-ex}(\lambda,\sigma)} = \frac{P_{ex}(\lambda,\sigma)}{P_{ex}(-\lambda,\sigma)} = \frac{\hat{G}(\lambda,\sigma)}{\hat{G}(-\lambda,\sigma)}$; "Achtung" $\lim_{\sigma\to\infty}$


"Adiabatic"


(2) → (1) ⇒ $\tilde{C}(w) = \tilde{W}(w) - e^{\beta w}\tilde{W}(w) \Rightarrow \tilde{W}(w,\beta) = -\tilde{G}_1(w,\beta)\mathcal{P}(w,\beta)$, where $\mathcal{P} := \frac{1}{e^{\beta w}-1}$
(1.1) and (1) ⇒ $R(\Omega,\sigma) := \frac{P_{ex}(\Omega,\sigma)}{P_{de-ex}(\Omega,\sigma)} = \frac{P_{ex}(\Omega,\sigma)}{P_{ex}(-\Omega,\sigma)} = \frac{\tilde{G}^+(\Omega,\sigma)}{\tilde{G}^+(-\Omega,\sigma)}$; "Adiabatic" $\lim_{\sigma\to\infty} \frac{\tilde{G}^+(\Omega,\sigma)}{\tilde{G}^+(-\Omega,\sigma)} = \frac{\tilde{W}(\Omega)}{\tilde{W}(-\Omega)} \overset{(1)}{=} e^{-\beta\Omega}$
A detector that interacts with a KMS state (wRT its proper time) as temperature $T_{KMS} = \frac{1}{\beta}$


FT of KMS
A detector that interacts with a KMS state (wrt its proper time) at temperature $T_{KMS} = \frac{1}{\beta}$, as long as it's switched on
Carefully* thermalizes (Catches Boltzmannian population) after interacting with the field for a long time.
*We know what that means!