Title: PSI 2017/2018 - Relativistic Quantum Information - Lecture 5

Date: Mar 23, 2018 09:00 AM

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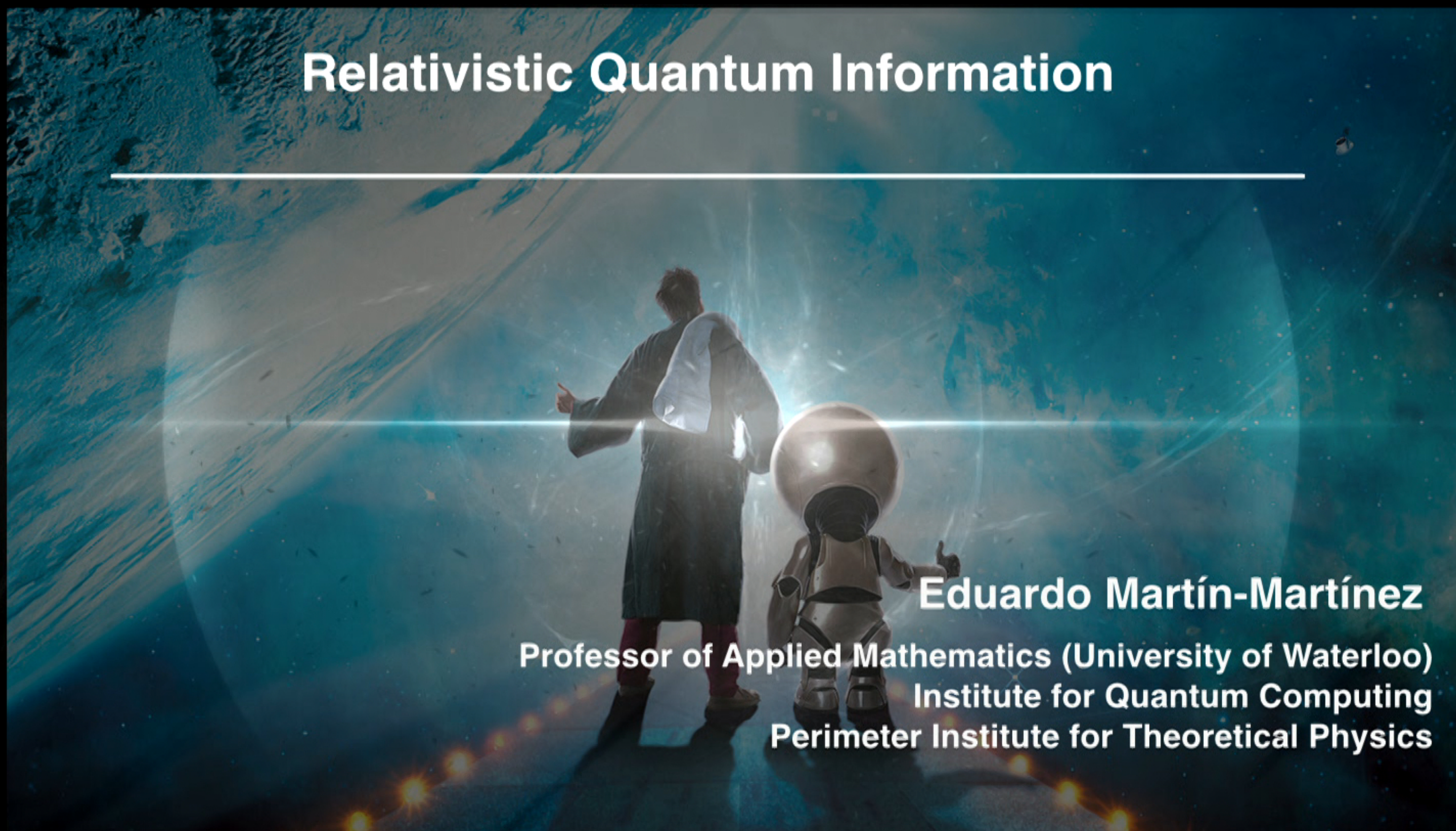
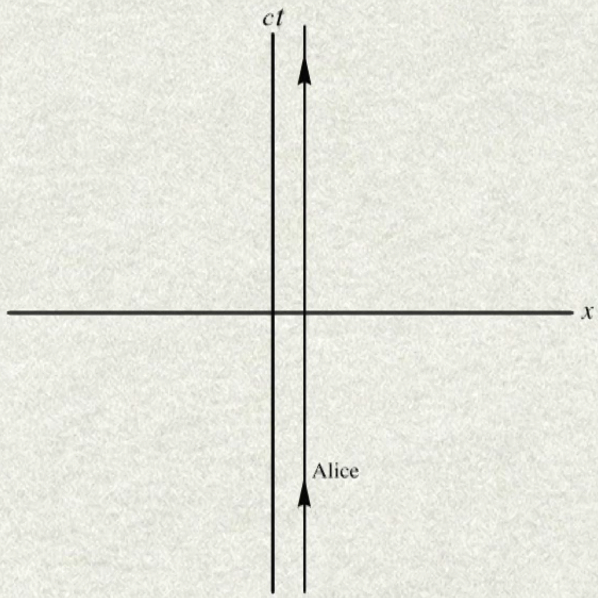
Abstract:

March 23rd 2018 PSI



Field QuantiZation Depends on the observer

- Example: different observers of flat spacetime



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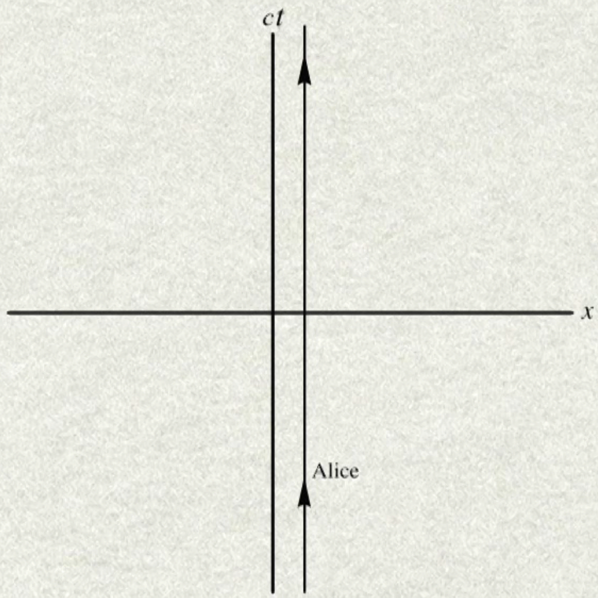

Positive freq. solutions $\nabla^\mu \nabla_\mu u = 0$

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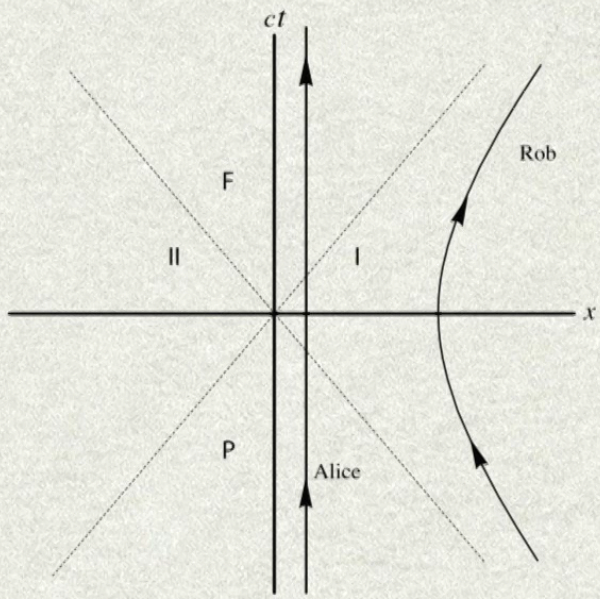

$$ct = \xi \sinh\left(\frac{a\tau}{c}\right), \qquad x = \xi \cosh\left(\frac{a\tau}{c}\right)$$

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Basis$\{$ $(x,t) \to u^{\mathrm{M}}_{\hat{\omega}} \propto e^{-i\hat{\omega}t}$

$$(\xi, \tau) \to u^{\mathrm{I}}_{\omega} \propto e^{-i\omega\tau}$$

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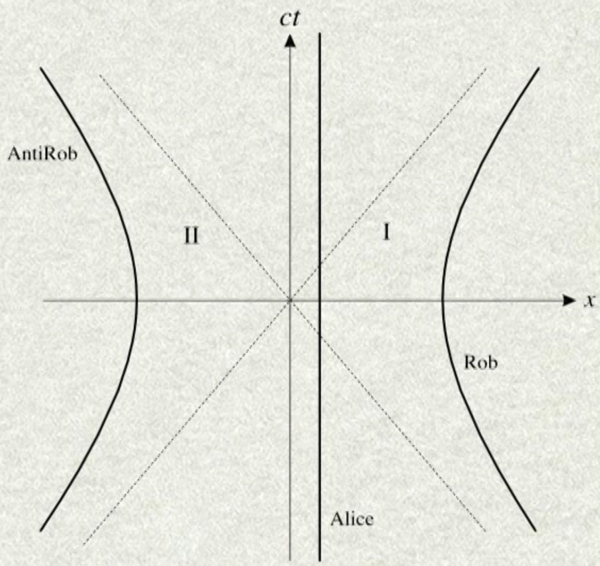

$$ct = \xi \sinh\left(\frac{a\tau}{c}\right), \qquad x = \xi \cosh\left(\frac{a\tau}{c}\right)$$

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$$(\xi,\tau) \rightarrow u^{\mathrm{II}}_{\omega} \propto e^{i\omega\tau'}$$

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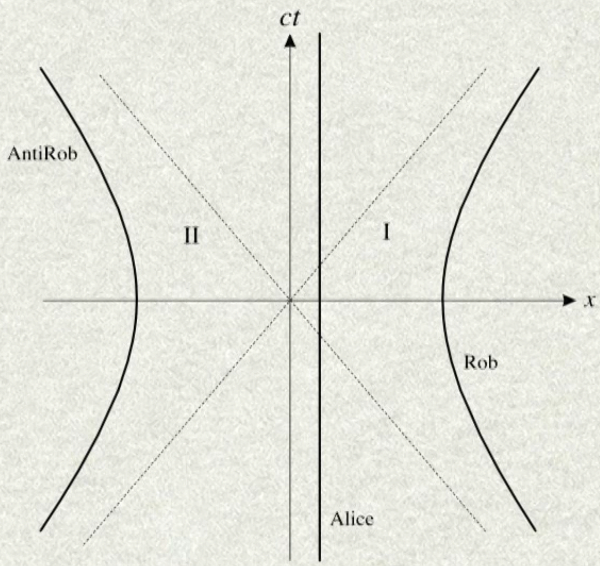

The field can be spanned in either basis

$$\phi = \sum_i \left(a_{\hat{\omega}_i,\mathrm{M}} u^{\mathrm{M}}_{\hat{\omega}_i} + a^{\dagger}_{\hat{\omega}_i,\mathrm{M}} u^{\mathrm{M}*}_{\hat{\omega}_i} \right)$$

or equivalently

$$\phi = \sum_i \left(a_{\omega_i,\mathrm{I}} u^{\mathrm{I}}_{\omega_i} + a^{\dagger}_{\omega_i,\mathrm{I}} u^{\mathrm{I}*}_{\omega_i} + a_{\omega_i,\mathrm{II}} u^{\mathrm{II}}_{\omega_i} + a^{\dagger}_{\omega_i,\mathrm{II}} u^{\mathrm{II}*}_{\omega_i} \right)$$

Basis $\left\{ (x,t) \rightarrow u^{\mathrm{M}}_{\hat{\omega}} \propto e^{-i\hat{\omega}t} \right.$

Basis $\left\{ \begin{array}{l} (\xi,\tau) \rightarrow u^{\mathrm{I}}_{\omega} \propto e^{-i\omega\tau} \\ (\xi,\tau) \rightarrow u^{\mathrm{II}}_{\omega} \propto e^{i\omega\tau'} \end{array} \right.$

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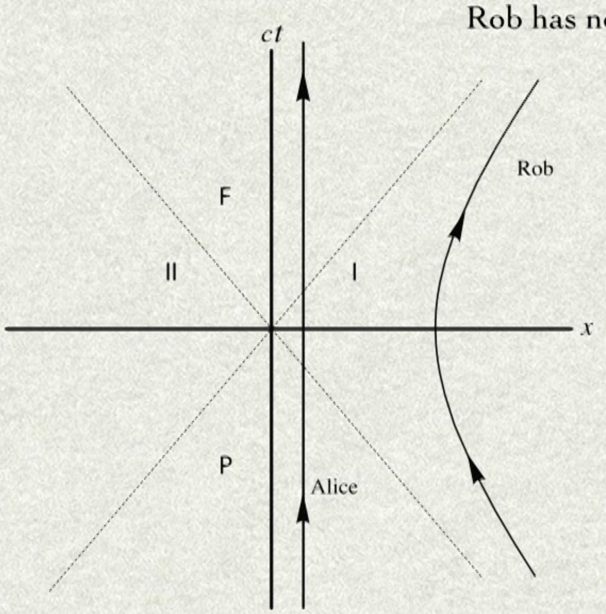
The Importance of The Horizon

Rob has no access to region II



$$\tanh r_\omega = \exp\left(-\frac{\pi c\,\omega}{a}\right)$$

Example: Minkowskian vacuum. Rob's perspective

$$|0\rangle_{\mathrm{M}}$$

First: change of Fock basis

$$|0\rangle_{\mathrm{M}} = \bigotimes_{\omega} \frac{1}{\cosh r_\omega} \sum_{n=0}^{\infty} \tanh^n r_\omega |n\rangle_{\mathrm{I}} |n\rangle_{\mathrm{II}}$$

Second: Trace out the disconnected region

$$\rho_{\mathrm{R},\omega} = \mathrm{Tr}_{\mathrm{II}}\left(|0_\omega\rangle\langle 0_\omega|\right) = \frac{1}{\cosh^2 r_\omega} \sum_n \tanh^{2n} r_\omega |n_\omega\rangle_{\mathrm{I}}\langle n_\omega|_{\mathrm{I}}$$

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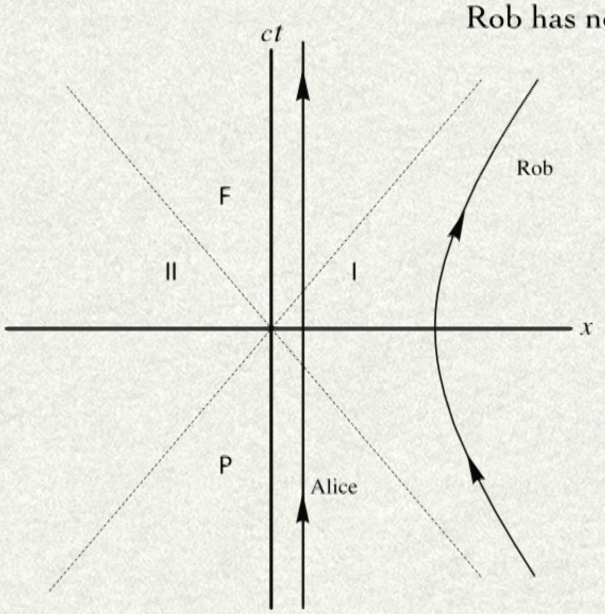

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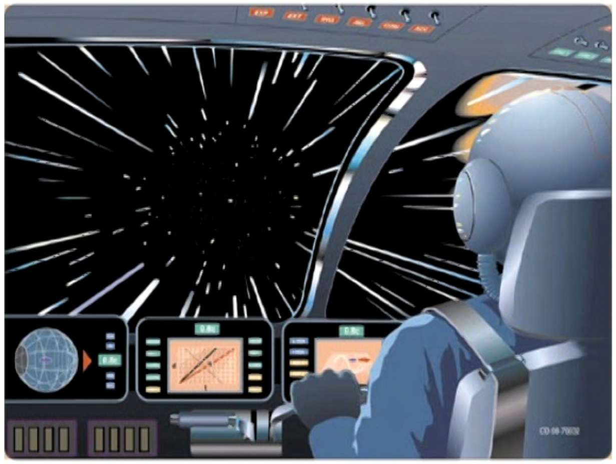
Result: thermal state

$$\langle N_{\omega,\mathrm{R}}\rangle = \frac{1}{e^{2\pi c/\omega a} - 1} \qquad T_{\mathrm{U}} = \frac{\hbar a}{2\pi K_{\mathrm{B}}}$$

THE UNRUH EFFECT

Inertial frame

Accelerated frame



- Alice Observes the field vacuum.
- Bob observes a thermal bath of temperature $T_{\mathrm{U}} \propto a$

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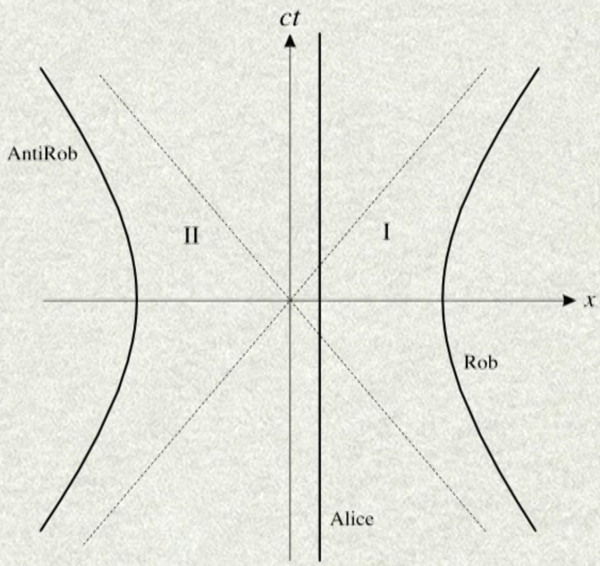

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...ibbs)

(Ill-defined for QFTs)

condition

$\hat{A}$

...relator of an observable of a quantum system in a thermal state (Gibbs)

$$\langle A(t) A(t') \rangle = \mathrm{tr}(\rho_\beta A(\Delta t) A(0))$$

stationarity

hat is a thermal state? (Gibbs)

Max S at const. Energy (Ill-defined for QFTs)

Kubo-Martin-Schwinger (KMS) condition

's compute the two-point correlator of an observable $\hat{A}$ of a quantum system

$$\hat{A}(t) = e^{-i\hat{H}t}\hat{A}(0)e^{i\hat{H}t}, \quad G(t,t') = tr(\hat{\rho}_\beta A(t)A(t')) \underset{\text{stationarity}}{=} tr(\hat{\rho}_\beta A(\Delta t)A(0)) = \frac{1}{Z} tr(e^{-\beta\hat{H}} e^{-iH\Delta t}$$

...ed for QFTs)

$\hat{A}$

...an observable of a quantum system in a thermal state (Gibbs)

$$= \operatorname{tr}\left(\hat{\rho}_\beta A(\Delta t) A(0)\right) = \frac{1}{Z} \operatorname{tr}\left(e^{-\beta \hat{H}} e^{-iH\Delta t} \hat{A}(0) e^{iH\Delta t} \hat{A}(0)\right)$$

...ty

$A(t) = e \; A(0) e \quad , \quad G(t,t') = tr(\rho_\beta A(t) A(t'))$

stationarity

$$G(\Delta\tau) = \frac{1}{Z} tr\left(e^{-\beta H} e^{iH\Delta\tau} A(0) e^{-iH\Delta\tau} A(0)\right) = \frac{1}{Z} tr\left(e^{-iH(\Delta t}\right.$$

$$G(\Delta\tau + i\beta) = \frac{1}{Z} tr\left(e\right.$$

$$A(t) = e^{iHt} A(0) e^{-iHt}, \quad G(t,t') = \mathrm{tr}\left(\rho_\beta A(t) A(t')\right) = \mathrm{tr}\left(\rho_\beta A(\Delta t \ldots\right.$$

stationarity

$$G(\Delta t) = \frac{1}{Z}\mathrm{tr}\left(e^{-\beta H} e^{-iH\Delta t} A(0) e^{+iH\Delta t} A(0)\right) = \frac{1}{Z}\mathrm{tr}\left(e^{-iH(\Delta t - i\beta)} A(0)\, e^{i H \ldots}\right.$$

$$G(\Delta t + i\beta) = \frac{1}{Z}\mathrm{tr}\left(e^{-iH\Delta t} A(0)\, e^{iH(\Delta t + i\beta)} A(0)\right) = \frac{1}{Z}\mathrm{tr}\left(A(0) e^{-iH\Delta t} A(0 \ldots\right.$$

(anti)-
Complex periodicity

$$G(\Delta\tau) = \frac{1}{Z} tr\left(e^{-\beta H} e^{-iH\Delta\tau} A(0) e^{+iH\Delta\tau} A(0)\right) = \frac{1}{Z} tr\left(e^{-iH(\Delta t - i\beta)} A(0)\, e\right.$$

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(anti)-
Complex periodicity

KMS condition "defines" thermality in QFT stationarity

$$G(\Delta\tau) = \frac{1}{Z} tr\left(e^{-\beta H} e^{-iH\Delta\tau} A(0) e^{+iH\Delta\tau} A(0)\right) = \frac{1}{Z} tr\left(e^{-iH(\Delta t - i\beta)} A(0 \ldots\right.$$

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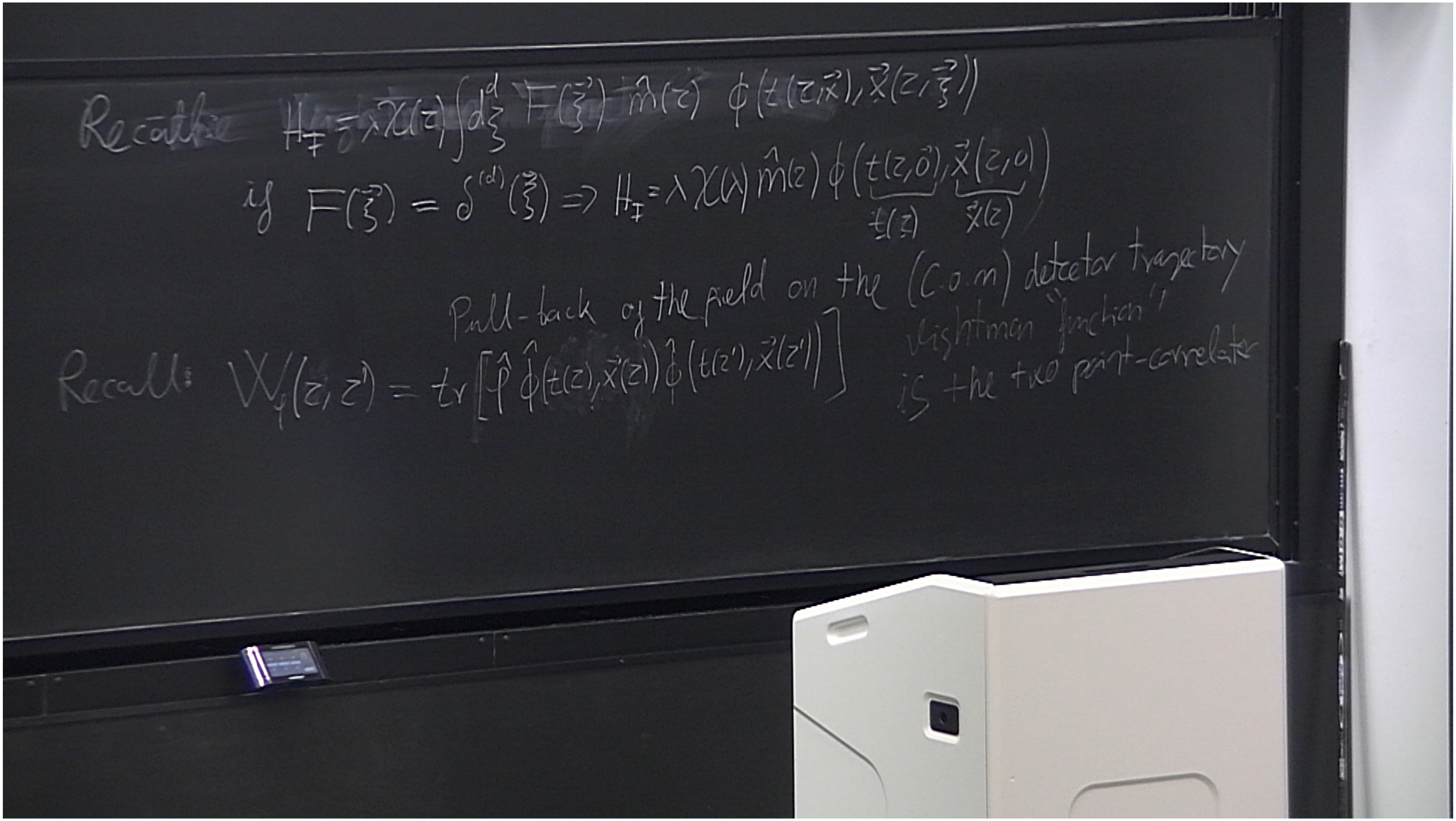
(anti)-
Complex periodicity

KMS condition "defines" thermality in QFT

$$A(0)\, e^{iH\Delta t} A(0) \Big)$$

$$\Big)\, e^{-iH\Delta t} A(0)\, e^{iH(\Delta t + i\beta)} \Big) = \frac{1}{Z} \mathrm{tr}\Big(e^{iH(\Delta t + i\beta)} A(0)\, e^{-iH\Delta t} A(0) \Big) =$$

$$= \frac{1}{Z} \mathrm{tr}\Big(e^{-\beta H} e^{iH\Delta t} A(0)\, e^{-iH\Delta t} A(0) \Big) = G(-\Delta\tau)$$


Recall $H_I = \lambda \chi(\tau) \int d^d\xi \, F(\vec{\xi}) \, \hat{m}(\tau) \, \phi(t(\tau, \vec{x}), \vec{x}(\tau, \vec{\xi}))$
if $F(\vec{\xi}) = \delta^{(d)}(\vec{\xi}) \Rightarrow H_I = \lambda \chi(\tau) \hat{m}(\tau) \phi(\underbrace{t(\tau, \vec{0})}_{t(\tau)}, \underbrace{\vec{x}(\tau, 0)}_{\vec{x}(\tau)})$
Pull-back of the field on the (c.o.m) detector trajectory
Recall: $W_\rho(\tau, \tau') = tr[\hat{\rho} \, \hat{\phi}(t(\tau), \vec{x}(\tau)) \hat{\phi}(t(\tau'), \vec{x}(\tau'))]$
Wightman "function" is the two point-correlator

KMS states in QFT

- a timlike vector ∂_τ
- a Hamiltonian
- field state

$\hat{\rho}$ is a KMS state of KMS temperature $T_{KMS} = \frac{1}{\beta}$ with respect to ∂_τ

Recall the

if

Recall: $W_\varphi($

KMS states in QFT

- a timlike vector ∂_τ
- a Hamiltonian
- field state

$\hat{\rho}$ is a KMS state of KMS-temperature $T_{KMS} = \frac{1}{\beta}$ with respect to ∂_τ iff it is stationary and

$$\underbrace{W_\rho(\tau,\tau') = W_\rho(\Delta\tau)}_{\text{stationarity}}$$

$$W_\rho(\Delta\tau + i\beta) = W_\rho(-\Delta\tau)$$

Rec

Recal