Title: PSI 2017/2018 - Relativistic Quantum Information - Lecture 4

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Abstract:

$$P^{|0\rangle}_{|g\rangle\to|e\rangle} = \lambda^2 \int \frac{d^dk}{2(2\pi)^d|\vec{k}|} \left|\tilde{\chi}(\Omega+|\vec{k}|)\right|^2 \left|\tilde{F}(\vec{k})\right|^2$$

(contribution ONLY from co

$$P^{|0\rangle}_{|e\rangle\to|g\rangle} = \lambda^2 \int \frac{d^dk}{2(2\pi)^d|\vec{k}|} \left|\tilde{\chi}(\Omega-|\vec{k}|)\right|^2 \left|\tilde{F}(\vec{k})\right|^2$$

(contribution ONLY from c

if $\chi(t) = \chi_0 \Rightarrow \tilde{\chi}(\Omega \pm |\vec{k}|) \sim \delta(\Omega \pm |\vec{k}|)$ selection of a si

T support of $\chi(t)$, SMA: $T \gg (\Omega - |\vec{k}|)^{-1}$

RWA: $T \gg (\Omega + |\vec{k}|)^{-1}$

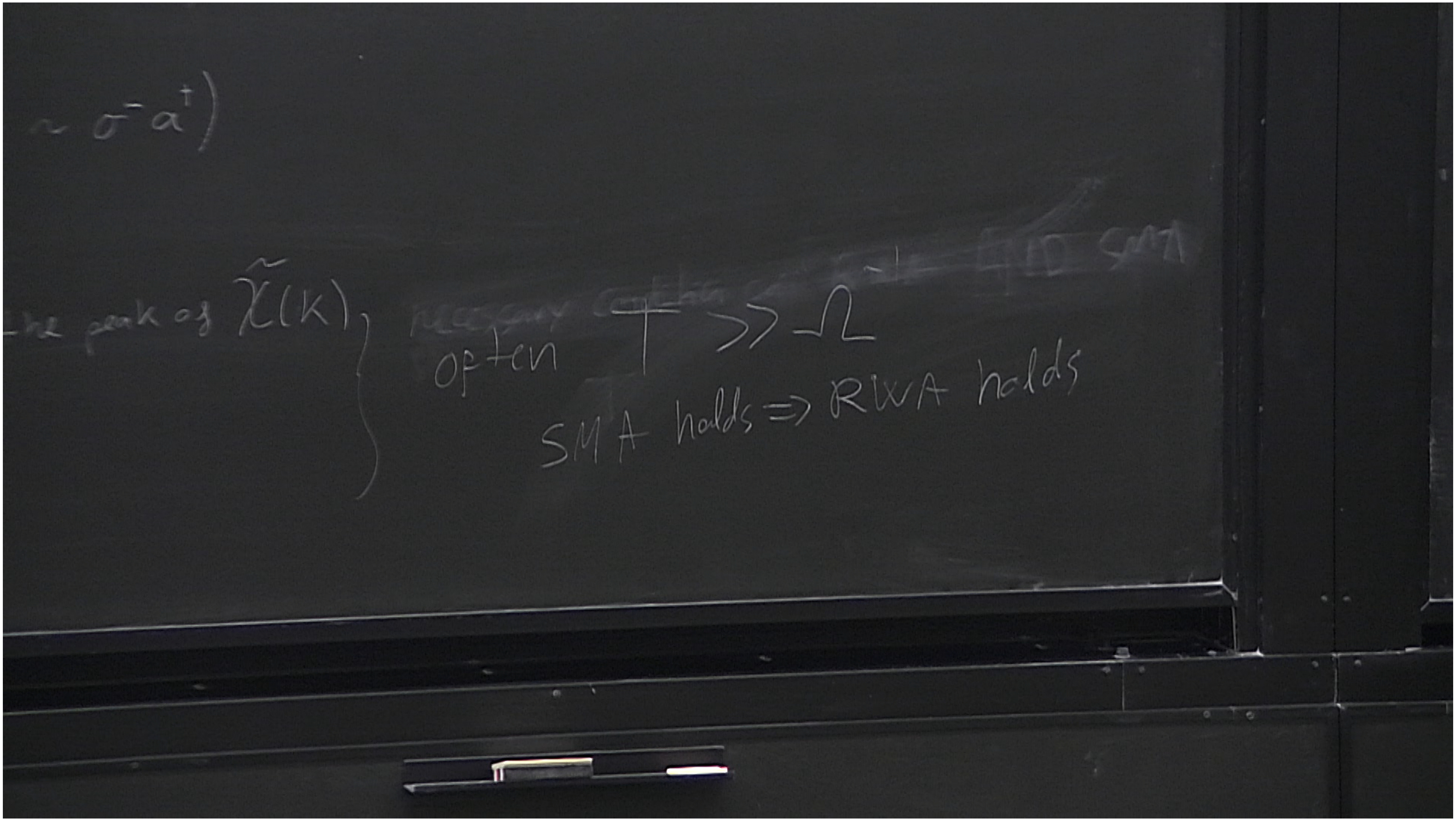
$$\Omega + |\vec{k}|)|^2 \, |\tilde{F}(\vec{k})|^2$$ (ONLY contribution from counter-rotating terms $\sim \sigma^+ a^\dagger$)

$$-|\vec{k}|)|^2 \, |\tilde{F}(\vec{k})|^2$$ (ONLY contribution from co-rotating terms $\sim \sigma^- a^\dagger$)

$\tilde{\chi}(\Omega \pm |\vec{k}|) \sim \delta(\Omega \pm |\vec{k}|)$ selection of a single mode

T support of $\chi(t)$, SMA: $T \gg (\Omega - |\vec{k}|)^{-1}$ for $|\vec{k}|$ far from the peak of $\tilde{\chi}(k)$,

RWA: $T \gg (\Omega + |\vec{k}|)^{-1}$ $\forall k$


σ⁻a†)
the peak of χ̃(k),
often T >> Ω
SMA holds ⇒ RWA holds

ONLY
(contribution from co-rotating terms $\sim \sigma^- a^\dagger$)

$\delta(\Omega \pm |\vec{k}|)$ selection of a single mode

for $|\vec{k}|$ far from the peak of $\tilde{\chi}(k)$,

SMA: $T \gg |\Omega - |\vec{k}||^{-1}$ for $|\vec{k}|$ far from the peak of $\tilde{\chi}(k)$

RWA: $T \gg \|\Omega + |\vec{k}|\|^{-1}$ $\forall k$

Time reparametrizations $\tau(t)$

Let ${}^{t}\hat{H}(t)$ be the Hamiltonian of a quantum system generating tran

What is ${}^{\tau}\hat{H}(\tau)$ (generating translations WRT τ)? Nai

$$i\frac{d}{dt}|\psi\rangle = {}^{t}H(t)|\psi\rangle$$

under reparametrization $t \to t(\tau)$

$$\frac{d}{dt} = \frac{d\tau}{dt}\frac{d}{d\tau}$$

${}^{t}H = at \qquad t = z^3 \qquad$ Maybe ${}^{z}H(z) = az^3$

$${}^{z}H(z) = \frac{dt}{dz} H(t(z))$$

generating translations WRT t

z ?   Naive try ${}^{z}\hat{H}(z) \overset{?}{=} {}^{t}\hat{H}(t(z))$?

$$\frac{d}{dz} \Rightarrow i\frac{dz}{dt}\frac{d}{dz}|\psi\rangle = {}^{t}H(t(z))\,|\psi\rangle \Rightarrow i\frac{d}{dz}|\psi\rangle = \frac{dt}{dz}\,{}^{t}H(t(z))\,|\psi\rangle$$

Detector's free Hamiltonian $\quad {}^{c}\hat{H}_{0,d} = \Omega \hat{\sigma}^{+} \hat{\sigma}^{-}$ $\quad$ z is th

Field free Hamiltonian: $\quad {}^{t}\hat{H}_{0,\phi} = \int d^{d}k \, |\vec{k}| \, \hat{a}^{\dagger}_{k} \hat{a}_{k}$

Interaction Hamiltonian $\quad {}^{z}\hat{H}_{I} \propto$ (Obs of detector) $\otimes$

Is prescribed by the detector

Time evol : $\hat{U} = \int dt\, {}^{t}\hat{H}(t) = \int d\tau \frac{dt}{d\tau}\, {}^{\tau}\hat{H}(\tau)$

is the proper time of the detector

$\hat{a}_n$ t is "quantization frame" (Lab frame)

ector) $\otimes$ (Observable of ϕ) $\left({}^{\tau}\hat{H}_I = e\hat{X}\cdot\vec{E}(t,\hat{X}) \right)$

translations WRT τ

$$\boxed{{}^{\tau}H(\tau) = \frac{dt}{d\tau}{}^{t}H(t(\tau))}$$

Naive try ${}^{\tau}\hat{H}(\tau) \overset{?}{=} {}^{t}\hat{H}(t(\tau))$?

$$i\frac{d\tau}{dt}\frac{d}{d\tau}|\psi\rangle = {}^{t}H(t(\tau))|\psi\rangle \Rightarrow i\frac{d}{d\tau}|\psi\rangle = \frac{dt}{d\tau}{}^{t}H(t(\tau))|\psi\rangle$$

Time evol : $\hat{U} = \int dt\, {}^{t}H(t) = \int d\tau \frac{dt}{d\tau}{}^{\tau}H(t(\tau))$

τ the proper time of the detector

t is "quantization frame" (Lab frame)

(Observable of ϕ) $\quad \left({}^{\tau}H_I = e\hat{x}\cdot\vec{E}(t,\hat{x}) \right)$

Translations WRT τ $\qquad$ $\boxed{{}^{\tau}H(\tau) = \frac{dt}{d\tau}H(t(\tau))}$

Naive try ${}^{\tau}\hat{H}(\tau) \stackrel{?}{=} {}^{t}\hat{H}(t(\tau))$?

$$i\frac{d\tau}{dt}\frac{d}{d\tau}|\psi\rangle = {}^{t}H(t(\tau))\,|\psi\rangle \Rightarrow i\frac{d}{d\tau}|\psi\rangle = \overline{\frac{dt}{d\tau}{}^{t}H(t(\tau))}\,|\psi\rangle$$

Time evol : $\hat{U} = \int dt\, {}^{t}H(t) = \int d\tau\, \overbrace{\frac{dt}{d\tau}{}^{t}H(t(\tau))}^{{}^{\tau}H(\tau)}$

τ the proper time of the detector

t is "quantization frame" (Lab frame)

$\otimes$ (Observable of ϕ) $\qquad$ $\left({}^{\tau}H_I = e\hat{x}\cdot\vec{E}(t,\hat{x}) \right)$

For a detector in arbitrary motion

$(\tau, \vec{\xi})$ proper frame of the

$(t, \vec{x})$ Inertial "lab frame"

$$^{\tau}H_I = \lambda\, \chi(\tau) \int d^d\xi \; F(\vec{\xi})\, \overbrace{\hat{m}(\tau)}^{\left(\sigma^+ e^{i\Omega\tau} + \sigma^- e^{-i\Omega\tau}\right)}\, \phi\big(t(\tau,\vec{\xi}),\, \vec{x}(\tau,\vec{\xi})\big)$$

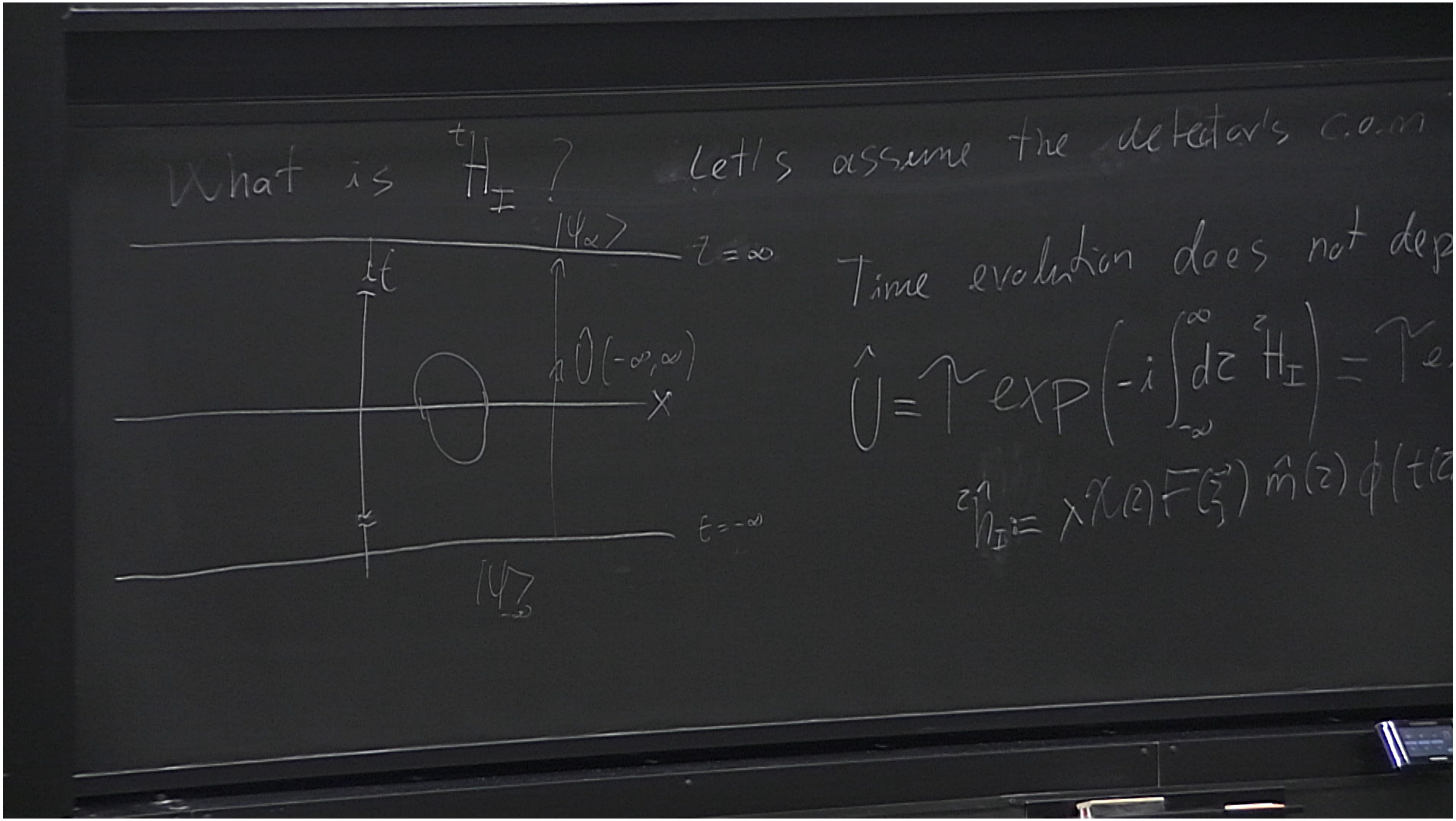
Time evol : $\hat{U} = -i\int dt\, {}^{t}H(t) = -i\int d\tau \overbrace{\frac{dt}{d\tau}\, {}^{t}H(t(\tau))}^{{}^{\tau}H(\tau)}$

...e of the detector

...uantization frame" (Lab frame)

...of ϕ) $\left({}^{\tau}H_I = e\hat{x}\cdot\vec{E}(t,\hat{x}) \right)$

$$U^0 = T\exp\left(i\int dt\, {}^{t}H_0\right) = T\exp\left(i\int d\tau\, {}^{\tau}H_0\right)$$


What is $^{t}H_I$?
Let's assume the detector's c.o.m
$|\psi_\alpha\rangle$
$t=\infty$
t
$\hat{U}(-\infty,\infty)$
x
$t=-\infty$
$|\psi\rangle_{-\infty}$
Time evolution does not dep
$\hat{U}=\mathcal{T}\exp\left(-i\int_{-\infty}^{\infty}d\tau\,{}^{\tau}H_I\right)=\mathcal{T}e$
${}^{\tau}\hat{h}_I:=\lambda\chi(\tau)F(\vec{\xi})\,\hat{m}(\tau)\,\phi(t(\tau$

et's assume the detector's c.o.m. describes a trajectory $\vec{x}(t)$ in the lab fra

$=\infty$

Time evolution does not depend on the observer

$$\hat{U} = \mathcal{T}\exp\left(-i\int_{-\infty}^{\infty} d\tau\, {}^{\tau}\hat{H}_I\right) = \mathcal{T}\exp\left(-i\int d\tau\, d\vec{\xi}\, {}^{\tau}\hat{h}_I\right)$$

$=-\infty$

$${}^{\tau}\hat{h}_I := \lambda\chi(\tau)F(\vec{\xi})\,\hat{m}(\tau)\,\phi\big(t(\tau,\vec{\xi}),\vec{x}(\tau,\vec{\xi})\big)$$

$$\hat{U} = \mathcal{T}\exp\left(-i\int dz\, d^d\xi\, {}^z h_I\right) = \mathcal{T}\exp\left(-i\int dt\, d^dx\, {}^t h_I\right)$$

$${}^t h_I = \lambda\, \chi\big(z(t,\vec{x})\big)\, F\big(\vec{\xi}(t,\vec{x})\big)\, \hat{m}\big(z(t,\vec{x})\big)\, \phi(t,\vec{x}) \left|\frac{\partial(z,\vec{\xi})}{\partial(t,\vec{x})}\right|$$

$${}^t H_I = \lambda \int d^dx\, \chi\big(z(t,\vec{x})\big)\, F\big(\vec{\xi}(t,\vec{x})\big)\, \hat{m}\big(z(t,\vec{x})\big)\, \phi(t,\vec{x}) \left|\frac{\partial(z,\vec{\xi})}{\partial(t,\vec{x})}\right|$$