Title: PSI 2017/2018 - Relativistic Quantum Information - Lecture 3

Date: Mar 21, 2018 09:00 AM

URL: http://pirsa.org/18030030

Abstract:

DIPOLE COUPLING: $H_I = e\int d^3x\, \hat{\vec{d}}(t)\cdot\hat{\vec{E}}(t,\vec{x})$ $\qquad \hat{\vec{d}}(t) = \left(\vec{F}(\vec{x})\,\hat{\sigma}^{+}e^{i\Omega t} + \vec{F}^{*}(\vec{x})\,\hat{\sigma}^{-}e^{-i\Omega t}\right)$

Unruh-DeWitt:

Jaynes-Cummings (RWA + SMA): $H_I = \lambda\left(\hat{\sigma}^{+}\hat{a}_{\omega}e^{i(\Omega-\omega)t} + \hat{\sigma}^{-}\hat{a}^{\dagger}_{\omega}e^{i(\Omega+\omega)t}\right)$

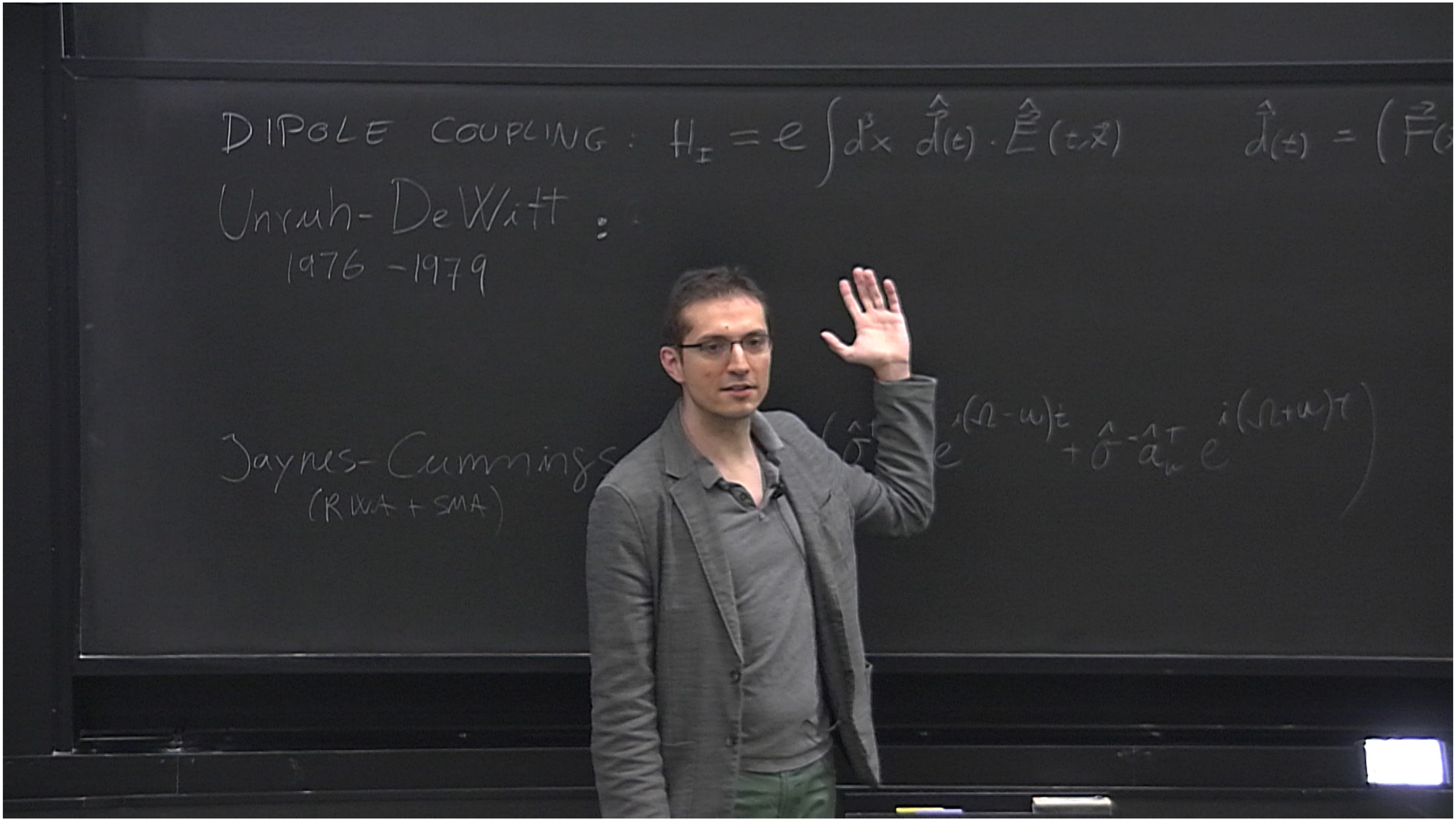
DIPOLE COUPLING: $H_I = e\int d^3x \; \hat{d}(t)\cdot\hat{\vec{E}}(t,\vec{x})$ $\qquad \hat{d}(t) = \big(\vec{F}($

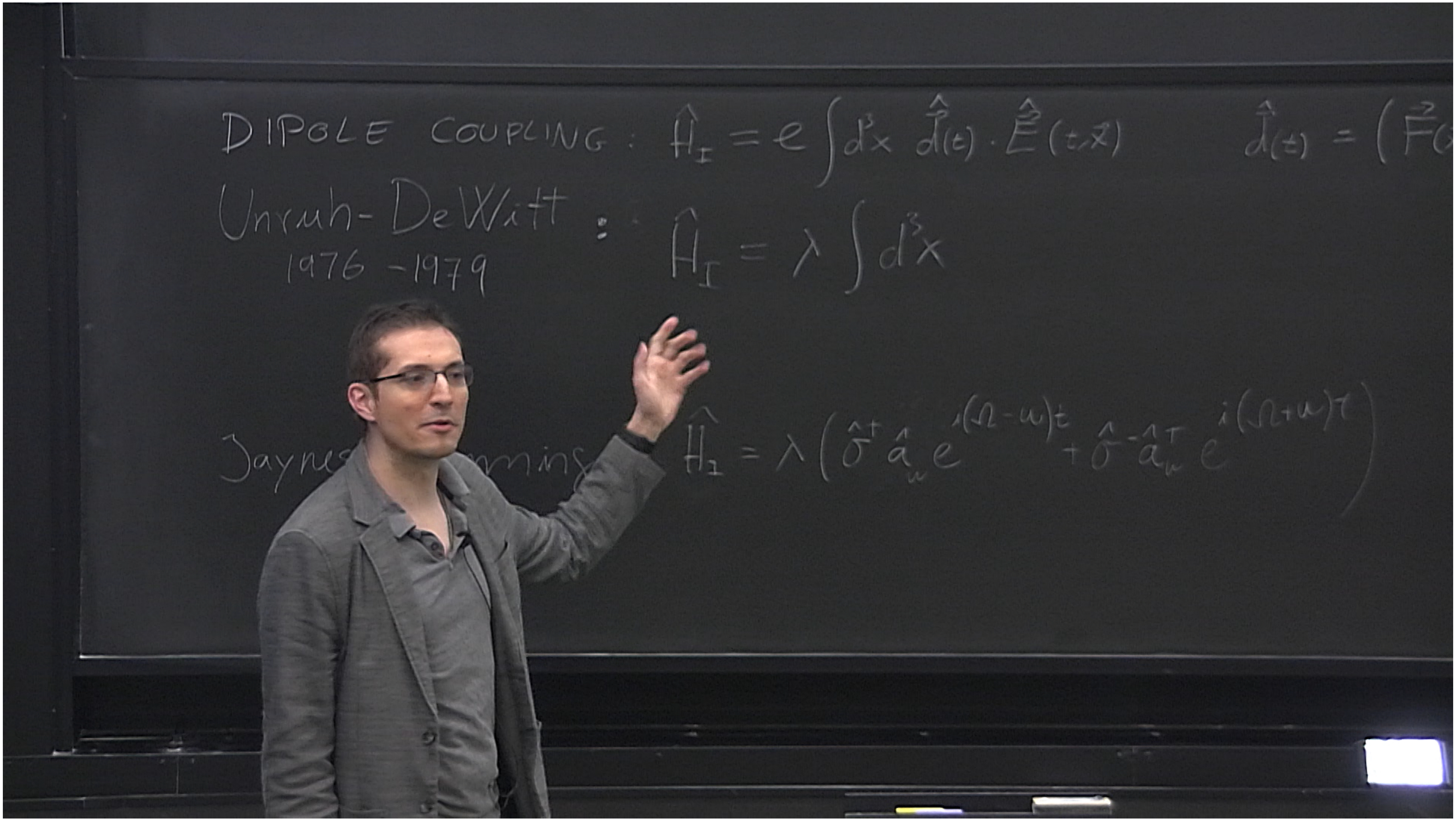
Unruh-DeWitt:

1976

Jaynes-Cummings: $H_I = \lambda\left(\hat{\sigma}^+ \hat{a}_\omega e^{i(\Omega-\omega)t} + \hat{\sigma}^- \hat{a}^\dagger_\omega e^{i(\Omega+\omega)t}\right)$

(RWA + SMA)


DIPOLE COUPLING :
Unruh-DeWitt :
1976 - 1979
Jaynes-Cummings
(RWA + SMA)


DIPOLE COUPLING:
Unruh-DeWitt:
1976-1979

DIPOLE COUPLING: $\hat{H}_I = e\int d^3x\; \hat{\vec{d}}(t)\cdot\hat{\vec{E}}(t,\vec{x})$ $\qquad \hat{\vec{d}}(t) = \left(\vec{F}(\vec{x})\,\hat{\sigma}^+ e^{i\Omega t} + \vec{F}^*(\vec{x})\,\hat{\sigma}^- e^{-i\Omega t}\right)$

Unruh-DeWitt: $\hat{H}_I = \lambda\int d^3x\; \hat{\mu}(t)\,\phi(t,\vec{x})$ $\qquad \hat{\mu}(t) = F(\vec{x})$

1976 - 1979

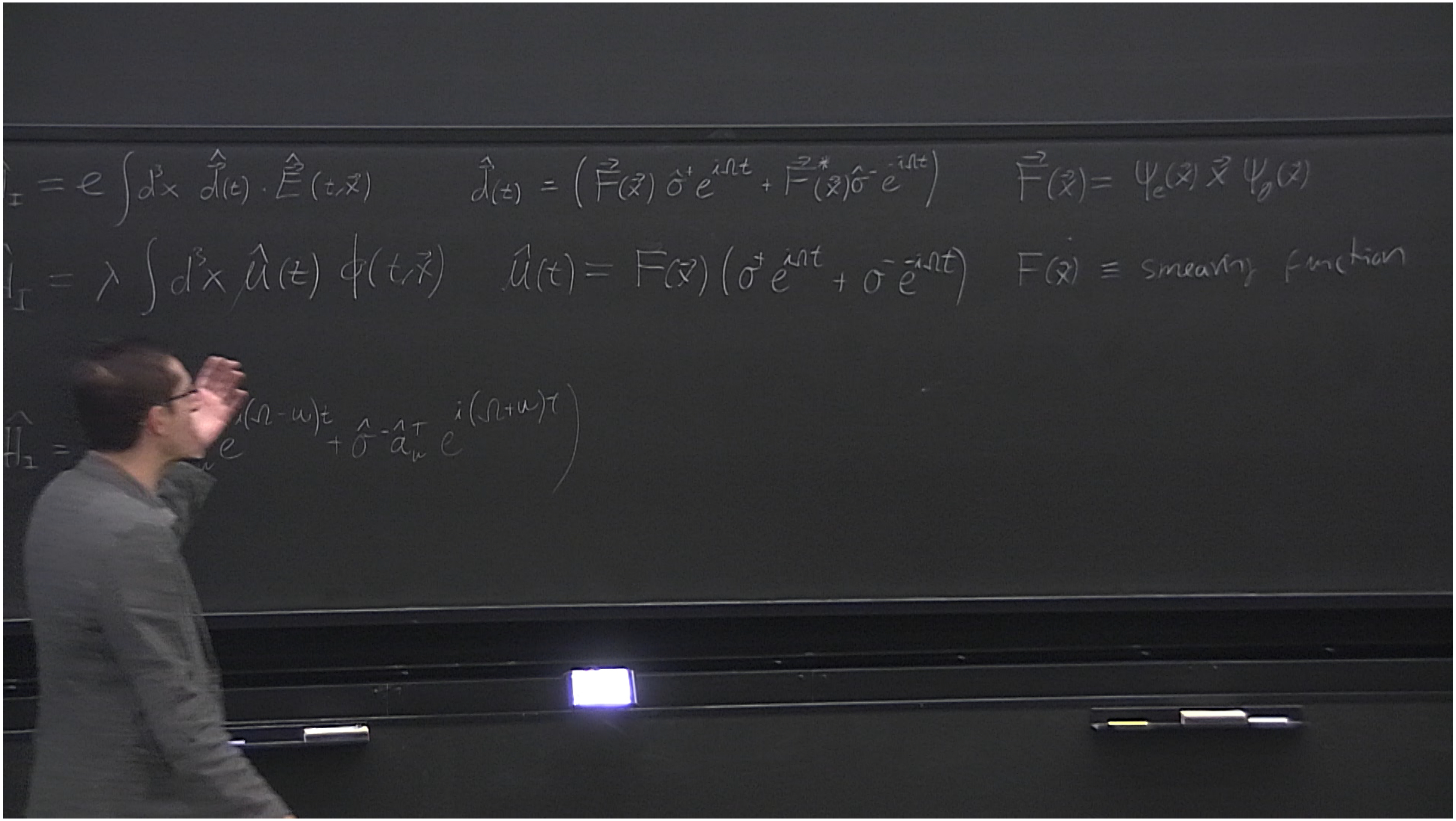
Jaynes-Cummings: $\hat{H}_I = \lambda\left(\hat{\sigma}^+\hat{a}_\omega e^{i(\Omega-\omega)t} + \hat{\sigma}^-\hat{a}^\dagger_\omega e^{i(\Omega+\omega)t}\right)$

(RWA + SMA)

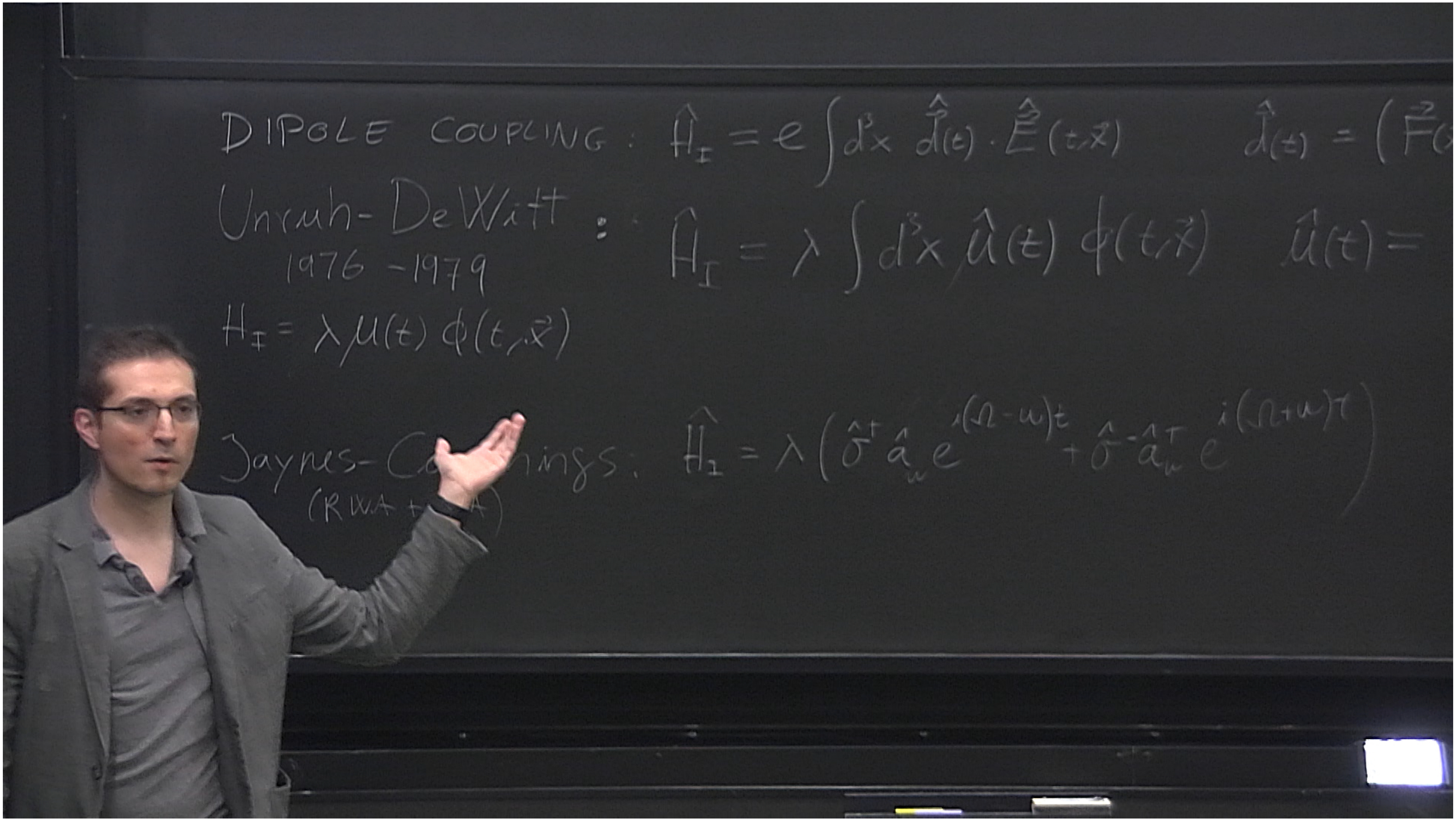
$$\hat{H}_I = e\int d^3x\ \hat{\vec{d}}(t)\cdot\hat{\vec{E}}(t,\vec{x}) \qquad \hat{\vec{d}}(t) = \left(\vec{F}(\vec{x})\,\hat{\sigma}^+ e^{i\Omega t} + \vec{F}^*(\vec{x})\,\hat{\sigma}^- e^{-i\Omega t}\right) \qquad \vec{F}(\vec{x}) = \psi_e(\vec{x})\,\vec{x}\,\psi_g(\vec{x})$$

$$\hat{H}_I = \lambda\int d^3x\ \hat{\mu}(t)\,\phi(t,\vec{x}) \qquad \hat{\mu}(t) = F(\vec{x})\left(\sigma^+ e^{i\Omega t} + \sigma^- e^{-i\Omega t}\right)$$

$$\hat{H}_I = \lambda\left(\hat{\sigma}^+\hat{a}_\omega e^{i(\Omega-\omega)t} + \hat{\sigma}^-\hat{a}^\dagger_\omega e^{i(\Omega+\omega)t}\right)$$



$\hat{H}_I = e\int d^3x\ \hat{\vec{d}}(t)\cdot\hat{\vec{E}}(t,\vec{x})$
$\hat{\vec{d}}(t) = \left(\vec{F}(\vec{x})\,\hat{\sigma}^+ e^{i\Omega t} + \vec{F}^*(\vec{x})\,\hat{\sigma}^- e^{-i\Omega t}\right)$
$\vec{F}(\vec{x}) = \psi_e(\vec{x})\,\vec{x}\,\psi_g(\vec{x})$
$\hat{H}_I = \lambda\int d^3x\ \hat{u}(t)\,\phi(t,\vec{x})$
$\hat{u}(t) = F(\vec{x})\left(\sigma^+ e^{i\Omega t} + \sigma^- e^{-i\Omega t}\right)$
$F(\vec{x}) \equiv$ smearing function


DIPOLE COUPLING: $\hat{H}_I = e\int d^3x\, \hat{d}(t)\cdot\hat{E}(t,\vec{x})$
$\hat{d}(t) = ($
Unruh-DeWitt: $\hat{H}_I = \lambda\int d^3x\, \hat{\mu}(t)\,\phi(t,\vec{x})$
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1976 - 1979
$H_I = \lambda\mu(t)\,\phi(t,\vec{x})$
Jaynes-C
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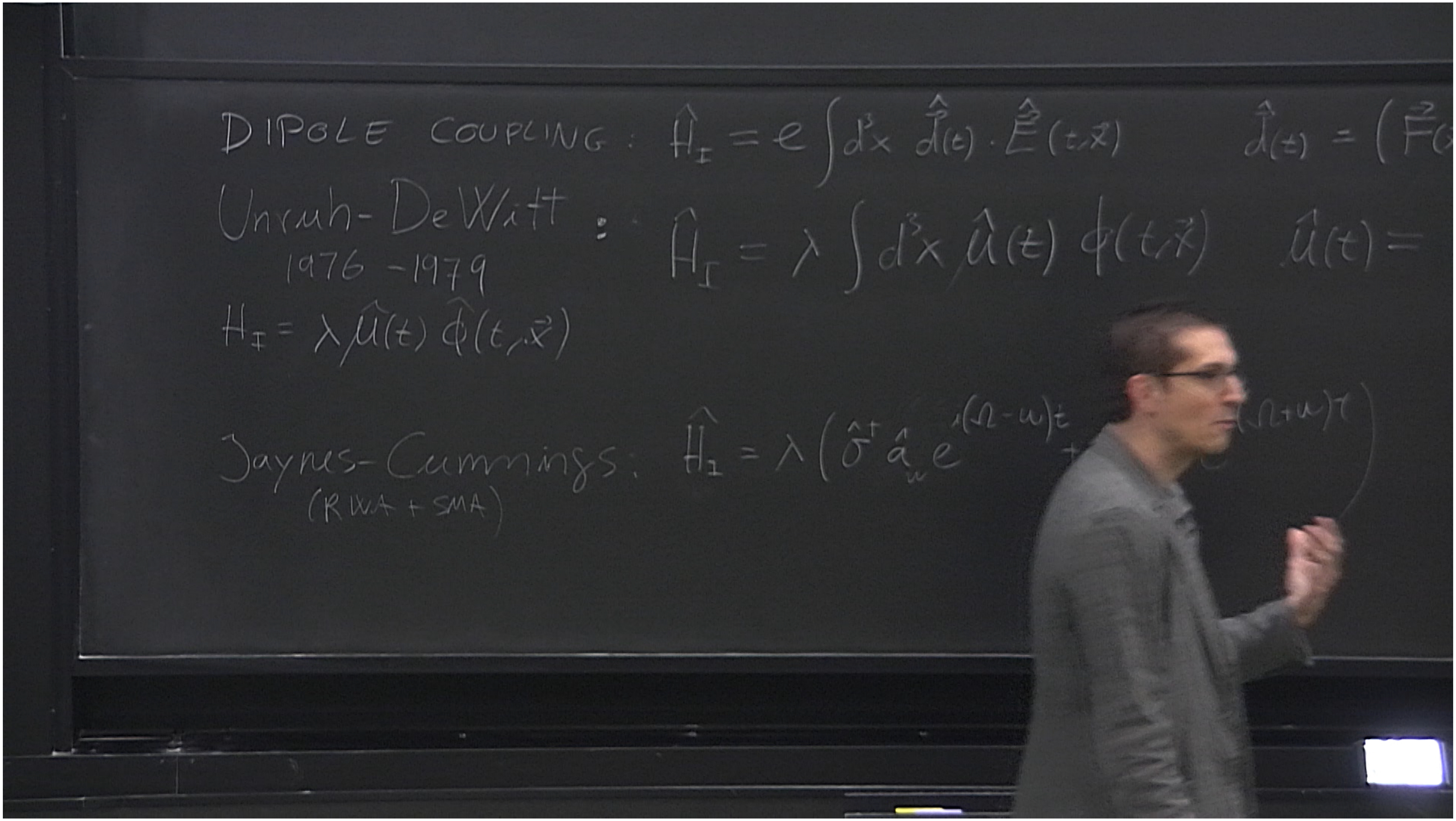
(RWA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(RWA + SMA)

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1976 - 1979

$\hat{H}_I = \lambda\,\hat{\mu}(t)\,\hat{\phi}(t,\vec{x})$

$\hat{H}_I = \lambda\,\chi(t)\,\hat{\mu}(t)\,\phi(t,\vec{x})$

Jaynes-Cummings: $\hat{H}_I = \lambda\left(\hat{\sigma}^+\hat{a}_\omega e^{i(\Omega-\omega)t} + \hat{\sigma}^-\hat{a}^\dagger_\omega e^{i(\Omega+\omega)t}\right)$

(RWA + SMA)

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1976 - 1979

$\hat{H}_I = \lambda\hat{\mu}(t)\,\hat{\phi}(t,\vec{x})$ $\qquad \hat{H}_I = \lambda\chi(t)\int d^3x\, \hat{\mu}(t)\, \hat{\phi}(t,\vec{x})$

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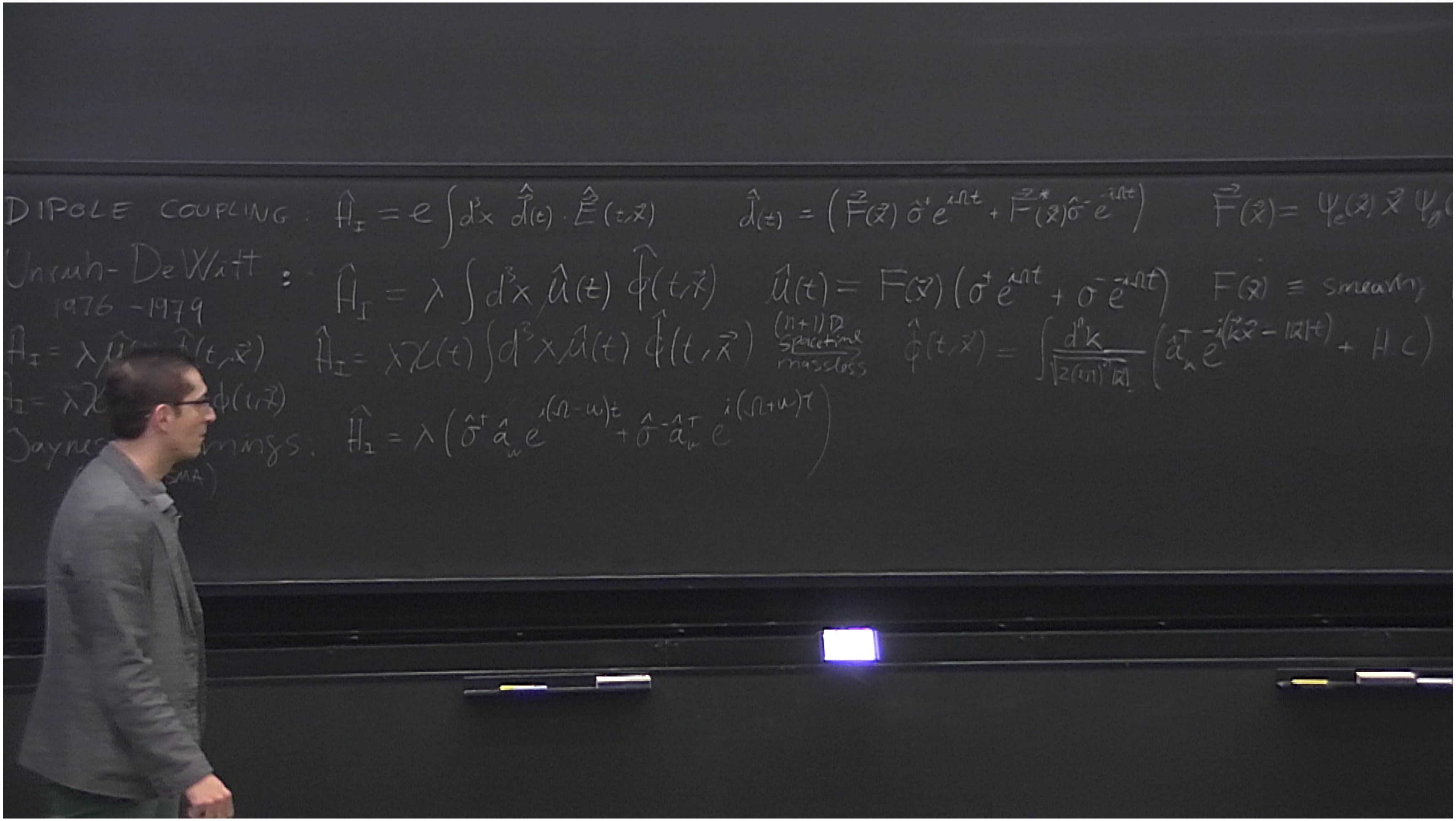
(RWA + SMA)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1976 - 1979
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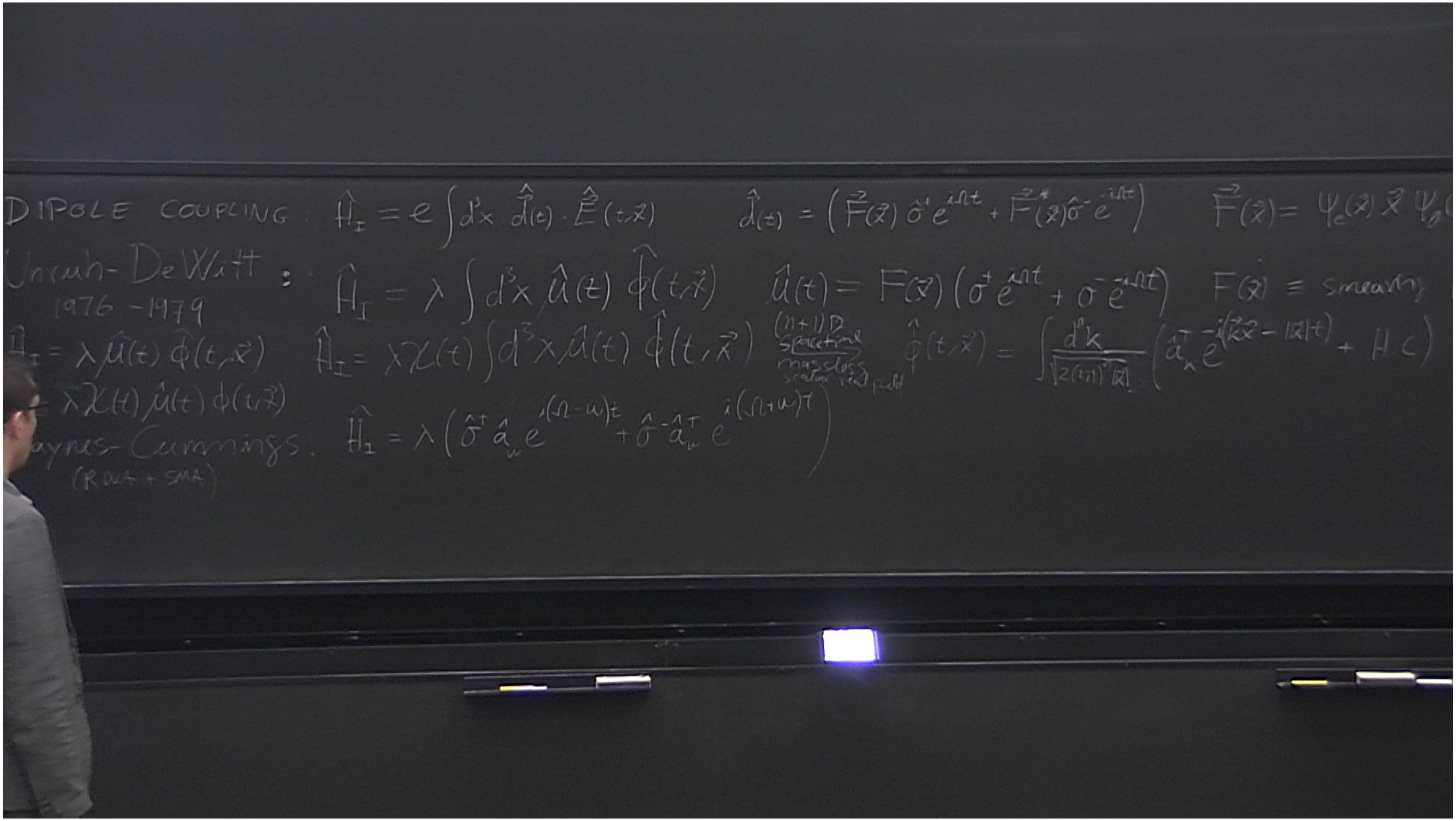
$F(\vec{x}) \equiv$ smearing
$\hat{H}_I = \lambda\chi(t)\int d^3x\, \hat{\mu}(t)\,\hat{\phi}(t,\vec{x})$
(n+1)D spacetime massless
Jaynes-Cummings: $\hat{H}_I = \lambda\left(\hat{\sigma}^+ \hat{a}_\omega e^{i(\Omega-\omega)t} + \hat{\sigma}^- \hat{a}^\dagger_\omega e^{i(\Omega+\omega)t}\right)$



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1976 - 1979
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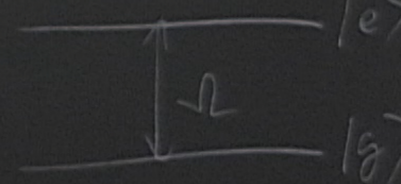
$= \lambda\chi(t)\,\hat{\mu}(t)\,\phi(t,\vec{x})$
$\hat{H}_I = \lambda\chi(t)\int d^3x\,\hat{\mu}(t)\,\hat{\phi}(t,\vec{x})$
(n+1)D spacetime
massless scalar real field
$\hat{\phi}(t,\vec{x}) = \int\frac{d^nk}{\sqrt{2(2\pi)^n|\vec{k}|}}\left(\hat{a}^\dagger_k e^{-i(\vec{k}\vec{x}-|\vec{k}|t)} + H.c\right)$
Jaynes-Cummings:
(RWA + SMA)
$\hat{H}_I = \lambda\left(\hat{\sigma}^+\hat{a}_\omega e^{i(\Omega-\omega)t} + \hat{\sigma}^-\hat{a}^\dagger_\omega e^{i(\Omega+\omega)t}\right)$

$$\left(\vec{F}(\vec{x})\,\hat{\sigma}^+ e^{i\Omega t} + \vec{F}^*(\vec{x})\,\hat{\sigma}^- e^{-i\Omega t}\right) \qquad \vec{F}(\vec{x}) = \psi_e(\vec{x})\,\vec{x}\,\psi_g(\vec{x})$$

$$t) = F(\vec{x})\left(\sigma^+ e^{i\Omega t} + \sigma^- e^{-i\Omega t}\right) \qquad F(\vec{x}) \equiv \text{smearing function}$$

1)D
cetime
ssless
lar real field

$$\hat{\phi}(t,\vec{x}) = \int \frac{d^n k}{\sqrt{2(2\pi)^n|\vec{k}|}}\left(\hat{a}_k^\dagger e^{-i(\vec{k}\vec{x} - |\vec{k}|t)} + H.C\right)$$



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1976 - 1979

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(n+1) D spacetime massless scalar field

$\hat{H}_I = \lambda\chi(t)\,\hat{\mu}(t)\,\phi(t,\vec{x})$

Jaynes-Cummings: $\hat{H}_I = \lambda\left(\hat{\sigma}^+\hat{a}_\omega e^{i(\Omega-\omega)t} + \hat{\sigma}^-\hat{a}^\dagger_\omega e^{-i(\Omega-\omega)t}\right)$

(RWA + SMA)

$(\sigma^+ + \sigma^-)$

J-C vs UDW

J-C: $\hat{\sigma}^{+}\hat{a} + \hat{\sigma}^{-}\hat{a}^{\dagger}$

UDW: $(\hat{\sigma}^{+} + \hat{\sigma}^{-})(\hat{a} + \hat{a}^{\dagger})$

$\propto \hat{\sigma}_x \cdot \hat{x}$

JC: $\hat{\sigma}^{+}\hat{a} + \hat{\sigma}^{-}\hat{a}^{\dagger}$

UDW: $(\hat{\sigma}^{+} + \hat{\sigma}^{-})(\hat{a} + \hat{a}^{\dagger}) = \hat{\sigma}^{+}\hat{a}^{\dagger} + \hat{\sigma}^{+}\hat{a} + \hat{\sigma}^{-}\hat{a} + \hat{\sigma}^{-}\hat{a}^{\dagger}$

$\propto \quad \hat{\sigma}_x \quad \cdot \quad \hat{x}$

Counter rotating

$$\text{JC}: \quad \hat{\sigma}^+ \hat{a} + \hat{\sigma}^- \hat{a}^\dagger$$

$$\text{UDW}: \quad (\hat{\sigma}^+ + \hat{\sigma}^-)(\hat{a} + \hat{a}^\dagger) = \hat{\sigma}^+ \hat{a}^\dagger + \hat{\sigma}^+ \hat{a} + \hat{\sigma}^- \hat{a} + \hat{\sigma}^- \hat{a}^\dagger$$

$$\propto \quad \hat{\sigma}_x \cdot \hat{x}$$

UDW

$$JC: \hat{\sigma}^+ \hat{a} + \hat{\sigma}^- \hat{a}^\dagger$$

Counter rotating

$$UDW: (\hat{\sigma}^+ + \hat{\sigma}^-)(\hat{a} + \hat{a}^\dagger) = \hat{\sigma}^+ \hat{a}^\dagger + \hat{\sigma}^+ \hat{a} + \hat{\sigma}^- \hat{a} + \hat{\sigma}^- \hat{a}^\dagger$$

$$\propto \hat{\sigma}_x \cdot \hat{x}$$

$$\underset{\hbar\Omega}{|e\rangle|0\rangle} \xrightarrow{\hat{\sigma}^- \hat{a}^\dagger} \underset{\hbar\omega}{|g\rangle|1\rangle}$$

$$\underset{0}{|g\rangle|0\rangle} \xrightarrow{\hat{\sigma}^+ \hat{a}^\dagger} \underset{\hbar(\Omega+\omega)}{|e\rangle|1\rangle}$$

Counter rotating   Do not preserve energy

JC: $\hat{\sigma}^+\hat{a} + \hat{\sigma}^-\hat{a}^\dagger$

UDW: $(\hat{\sigma}^+ + \hat{\sigma}^-)(\hat{a} + \hat{a}^\dagger) = \hat{\sigma}^+\hat{a}^\dagger + \hat{\sigma}^+\hat{a} + \hat{\sigma}^-\hat{a} + \hat{\sigma}^-\hat{a}^\dagger$

$\propto \hat{\sigma}_x \cdot \hat{x}$

$$\underset{\hbar\Omega}{|e\rangle|0\rangle} \xrightarrow{\hat{\sigma}^-\hat{a}^\dagger} \underset{\hbar\omega}{|g\rangle|1\rangle}$$

$$\underset{0}{|g\rangle|0\rangle} \xrightarrow{\hat{\sigma}^+\hat{a}^\dagger} \underset{\hbar(\Omega+\omega)}{|e\rangle|1\rangle}$$

JC: $\hat{\sigma}^{+}\hat{a} + \hat{\sigma}^{-}\hat{a}^{\dagger}$

Counter rotating

Some SAY: Do not preserve energy

UDW: $(\hat{\sigma}^{+} + \hat{\sigma}^{-})(\hat{a} + \hat{a}^{\dagger}) = \hat{\sigma}^{+}\hat{a}^{\dagger} + \hat{\sigma}^{+}\hat{a} + \hat{\sigma}^{-}\hat{a} + \hat{\sigma}^{-}\hat{a}^{\dagger}$

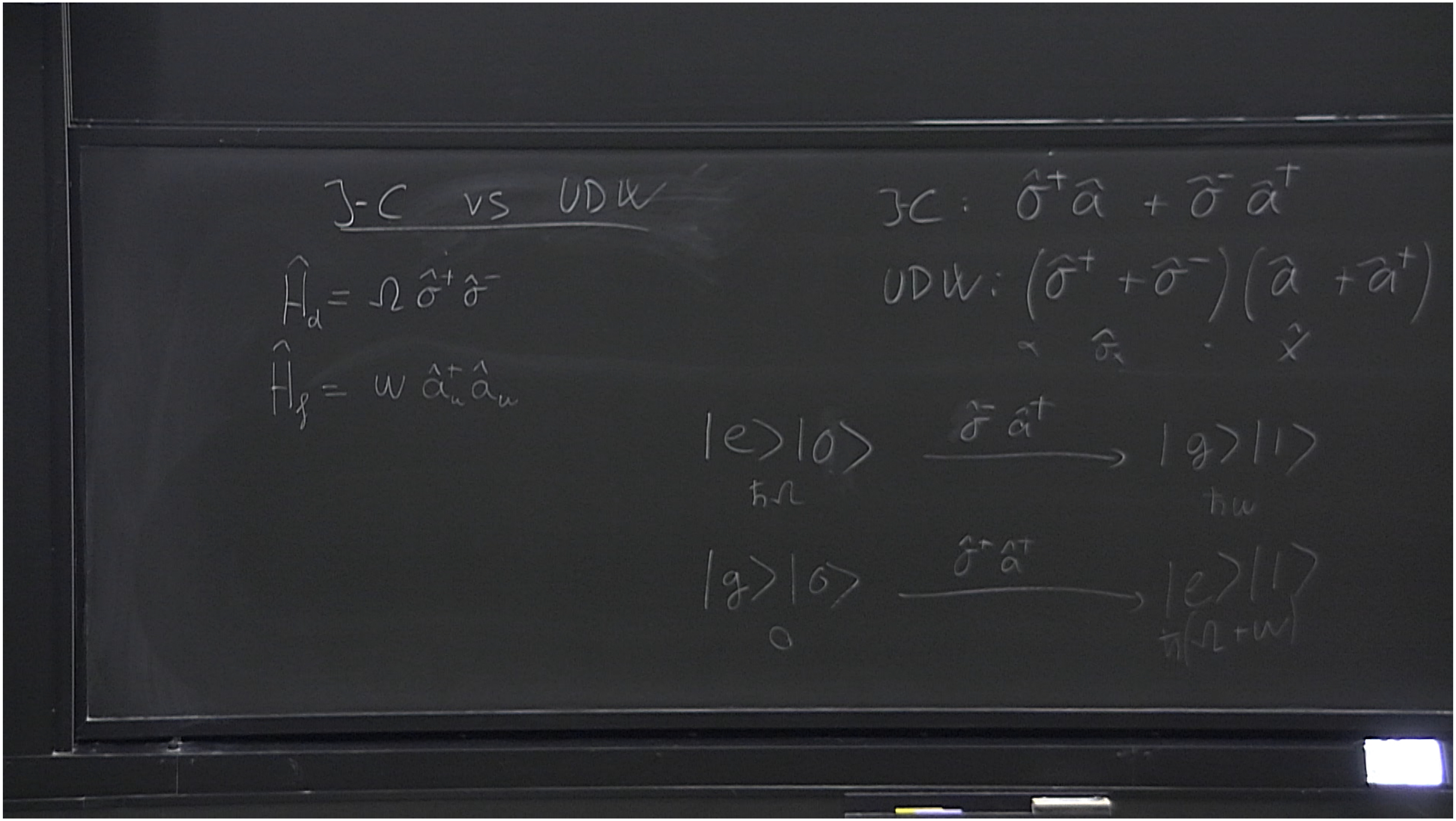
$\hat{\sigma}_x \qquad \hat{x}$

$|e\rangle|0\rangle \xrightarrow{\hat{\sigma}^{-}\hat{a}^{\dagger}} |g\rangle|1\rangle$

$\hbar\Omega \qquad \hbar\omega$

$|g\rangle|0\rangle \xrightarrow{\hat{\sigma}^{+}\hat{a}^{\dagger}} |e\rangle|1\rangle$

$0 \qquad \hbar(\Omega+\omega)$


J-C vs UDW
$\hat{H}_a = \Omega \hat{\sigma}^+ \hat{\sigma}^-$
$\hat{H}_f = \omega \hat{a}_n^\dagger \hat{a}_n$
JC: $\hat{\sigma}^+ \hat{a} + \hat{\sigma}^- \hat{a}^\dagger$
UDW: $(\hat{\sigma}^+ + \hat{\sigma}^-)(\hat{a} + \hat{a}^\dagger)$
$\propto \hat{\sigma}_x \cdot \hat{x}$
$|e\rangle|0\rangle \xrightarrow{\hat{\sigma}^- \hat{a}^\dagger} |g\rangle|1\rangle$
$\hbar\Omega$ $\hbar\omega$
$|g\rangle|0\rangle \xrightarrow{\hat{\sigma}^+ \hat{a}^\dagger} |e\rangle|1\rangle$
0 $\hbar(\Omega+\omega)$

J-C vs UDW

$$\hat{H}_a = \Omega \hat{\sigma}^+ \hat{\sigma}^-$$

$$\hat{H}_f = \omega \hat{a}^\dagger_\omega \hat{a}_\omega$$

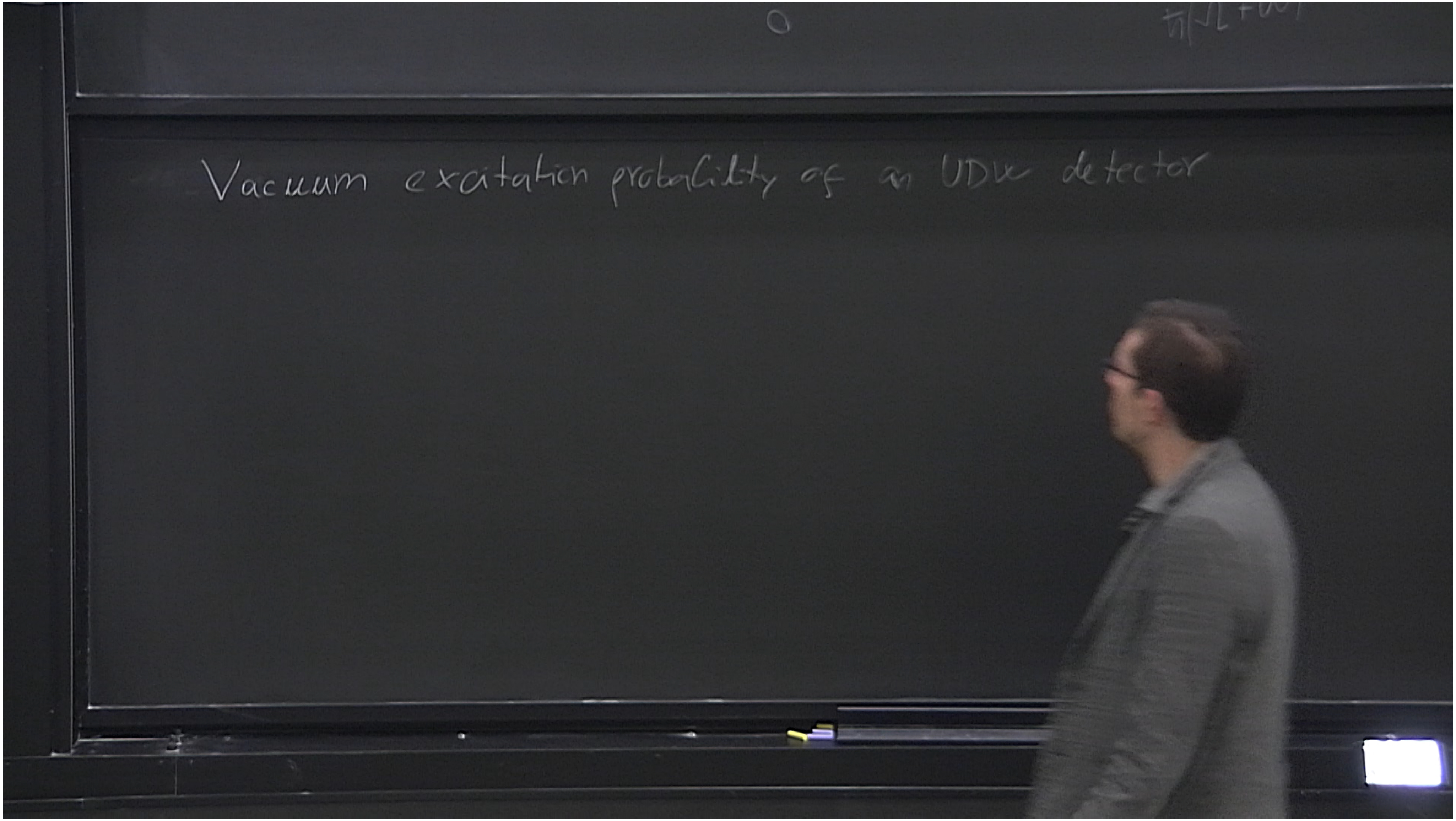
JC: $\hat{\sigma}^+ \hat{a} + \hat{\sigma}^- \hat{a}^\dagger$

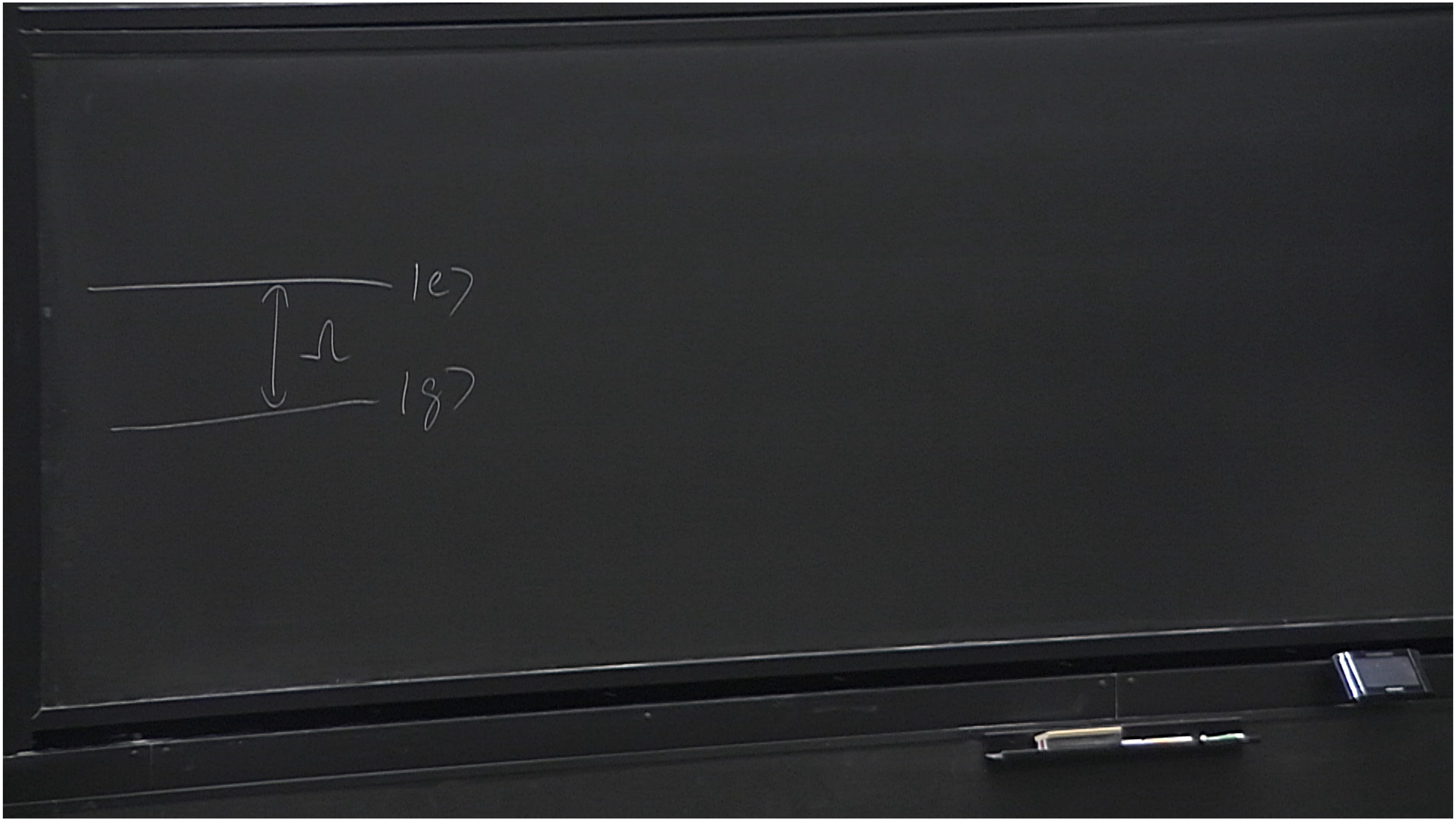
UDW: $(\hat{\sigma}^+ + \hat{\sigma}^-)(\hat{a} + \hat{a}^\dagger)$

$\propto \hat{\sigma}_x \cdot \hat{x}$

$$\underset{\hbar\Omega}{|e\rangle|0\rangle} \xrightarrow{\hat{\sigma}^- \hat{a}^\dagger} \underset{\hbar\omega}{|g\rangle|1\rangle}$$

$$\underset{0}{|g\rangle|0\rangle} \xrightarrow{\hat{\sigma}^+ \hat{a}^\dagger} \underset{\hbar(\Omega+\omega)}{|e\rangle|1\rangle}$$


Vacuum excitation probability of an UDW detector


|e⟩
Ω
|g⟩

Vacuum excitation probability of an UDW detector

$P^{|0\rangle}_{|g\rangle\to|e\rangle}(\Omega)$; $P^{|0\rangle}_{|e\rangle\to|g\rangle}(\Omega) = P^{|0\rangle}_{|g\rangle\to|e\rangle}(-\Omega)$; $P^{|0\rangle}_{|g\rangle\to|e\rangle} = \sum_{out} |\langle out, e|\hat{U}|g, 0\rangle|^2$

$$\rangle|^2 = \sum_{out} \langle 0,g|\,\hat{U}^\dagger\,|e,out\rangle\langle out,e|\,\hat{U}\,|g,0\rangle$$

$$\rangle|^2 = \sum_{out} \langle 0, g| \hat{U}^\dagger | e, out\rangle\langle out, e|\hat{U}|g, 0\rangle \qquad \hat{U} = T\exp\left(\int_{-\infty}^{\infty} dt\, H_I(t)\right)$$

$$|g\rangle|0\rangle \xrightarrow{\hat{\sigma}^+ \hat{a}^+} |e\rangle|1\rangle$$

$0 \qquad\qquad \hbar(\Omega+\omega)$

Vacuum excitation probability of an UDW detector

$$P^{|0\rangle}_{|g\rangle\to|e\rangle}(\Omega) \;;\quad P^{|0\rangle}_{|e\rangle\to|g\rangle}(\Omega) = P^{|0\rangle}_{|g\rangle\to|e\rangle}(-\Omega) \;;\quad P^{|0\rangle}_{|g\rangle\to|e\rangle} = \sum_{out} |\langle out, e|\hat{U}|g,0\rangle|^2 = \sum_{out} \langle 0,g|\hat{U}^\dagger|e,out^\dagger\rangle\langle out^\dagger, e$$

Perturbation theory (Dyson expansion) $\hat{U} = \mathbb{1} + \hat{U}^{(1)} + \hat{U}^{(2)} + O(\lambda)$

$$\rangle|^2 = \sum_{out} \langle 0,g|\,\hat{U}^\dagger\,|e,out\rangle\langle out,e|\,\hat{U}\,|g,0\rangle \qquad \hat{U} = T\exp\left(\int_{-\infty}^{\infty} dt\, H_I(t)\right)$$

$$1 + \hat{U}^{(1)} + \hat{U}^{(2)} + O(\lambda) \qquad \hat{U}^{(1)} = -i\int_{-\infty}^{\infty} dt\, H(t)$$

Vacuum excitation probability of an UDW detector

$P^{|0\rangle}_{|g\rangle\to|e\rangle}(\Omega)$; $P^{|0\rangle}_{|e\rangle\to|g\rangle}(\Omega) = P^{|0\rangle}_{|g\rangle\to|e\rangle}(-\Omega)$; $P^{|0\rangle}_{|g\rangle\to|e\rangle} = \sum_{out} |\langle out, e|\hat{U}|g,0\rangle|^2 = \sum_{out} \langle 0, g$

Perturbation theory (Dyson expansion) $\hat{U} = \mathbb{1} + \hat{U}^{(1)} + \hat{U}^{(}$

$$P^{|0\rangle}_{|g\rangle\to|e\rangle} = \sum_{out} \langle 0\, g| \mathbb{1} + U^{(1)\dagger} + O(\lambda^2) |e, out\rangle\langle e, out| \mathbb{1} + U^{(1)} + O(\lambda^2) | g, 0\rangle = \langle$$

$$\ldots\rangle|^2 = \sum_{out} \langle 0,g|\, \hat{U}^{\dagger}\, |e,out\rangle\langle out,e|\, \hat{U}\, |g,0\rangle \qquad \hat{U} = T\exp\left(\int_{-\infty}^{\infty} dt\, H_I(t)\right)$$

$$\mathbb{1} + \hat{U}^{(1)} + \hat{U}^{(2)} + O(\lambda) \qquad \hat{U}^{(1)} = -i\int_{-\infty}^{\infty} dt\, H(t)$$

$$O(\lambda^2)|\, g,0\rangle = \langle 0\, g|\, \hat{U}^{(1)\dagger}\, |e\, out\rangle\langle out,e|\, U^{(1)}\, |g,0\rangle + O(\lambda^3)$$

Vacuum excitation probability of an UDW detector

$$P^{|0\rangle}_{|g\rangle\to|e\rangle}(\Omega)\ ;\quad P^{|0\rangle}_{|e\rangle\to|g\rangle}(\Omega) = P^{|0\rangle}_{|g\rangle\to|e\rangle}(-\Omega)\ ;\quad P^{|0\rangle}_{|g\rangle\to|e\rangle} = \sum_{out} |\langle out, e|\hat{U}|g,0\rangle|^2 =$$

Perturbation theory (Dayson expansion) $\hat{U} = \mathbb{1} + \hat{U}$

$$P^{|0\rangle}_{|g\rangle\to|e\rangle} = \sum_{out} \langle 0\, g|\, \mathbb{1} + U^{(1)\dagger} + \mathcal{O}(\lambda^2)\, |e, out\rangle\langle e, out|\, \mathbb{1} + U^{(1)} + \mathcal{O}(\lambda^2)|\, g$$

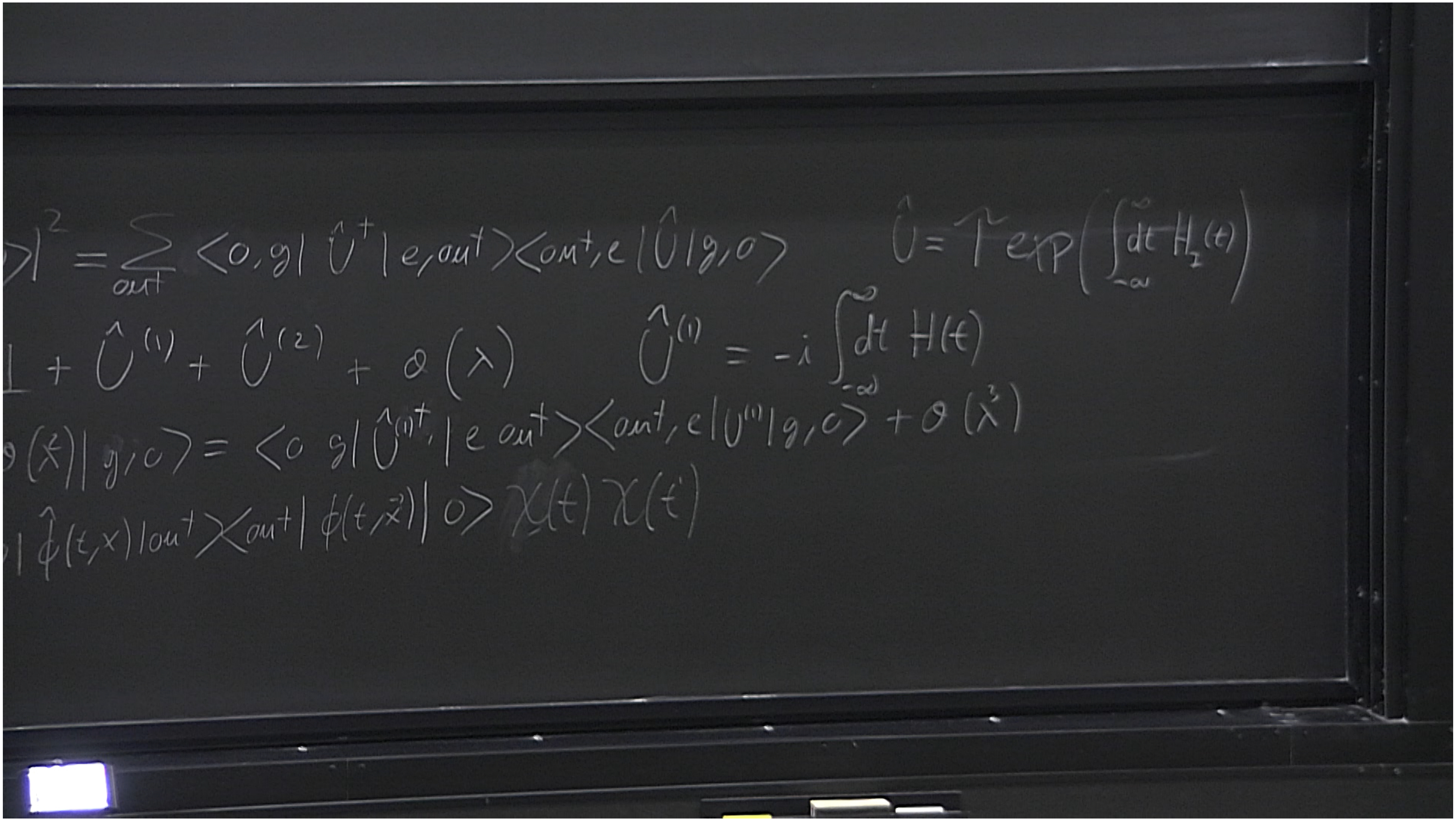
$$P_{|g\rangle\to|e\rangle}(\Omega) = \lambda^2 \sum_{out} \int dt \int dt' \int d^n x \int d^n x' \langle g|\hat{\mu}(t)|e\rangle\langle e|\hat{\mu}(t')|g\rangle F(x)F(x') \langle 0|\hat{\phi}(t, x$$

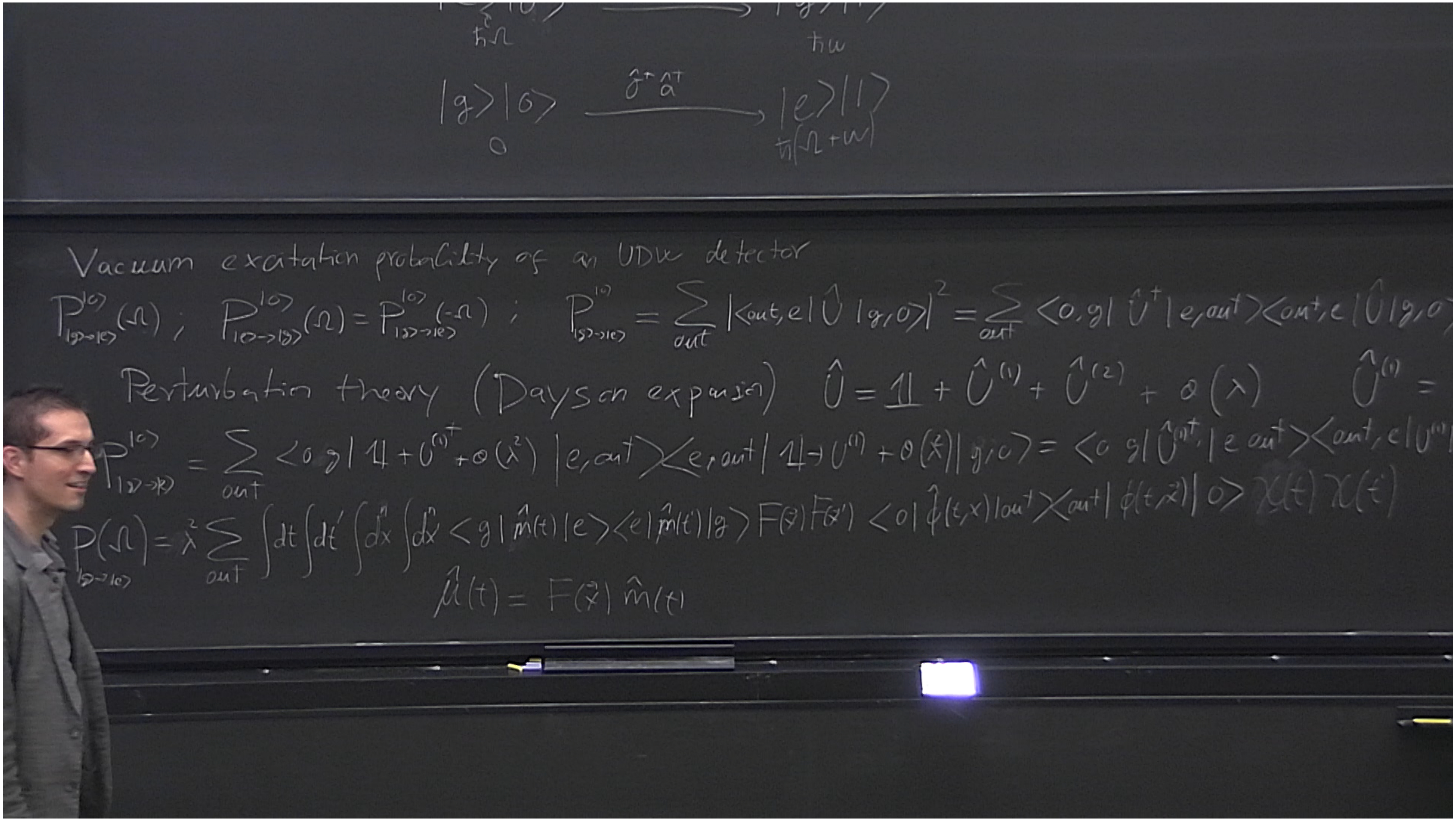
$$\rangle|^2 = \sum_{out} \langle 0,g|\hat{U}^{\dagger}|e,out\rangle\langle out,e|\hat{U}|g,0\rangle \qquad \hat{U} = T\exp\left(\int_{-\infty}^{\infty} dt\, H_I(t)\right)$$

$$1 + \hat{U}^{(1)} + \hat{U}^{(2)} + O(\lambda) \qquad \hat{U}^{(1)} = -i\int_{-\infty}^{\infty} dt\, H(t)$$

$$O(\lambda^2)|\,g,0\rangle = \langle 0\; g|\hat{U}^{(1)\dagger}|e\; out\rangle\langle out,e|U^{(1)}|g,0\rangle + O(\lambda^3)$$

$$|\,\hat{\phi}(t,x)|out\rangle\langle out|\,\phi(t,\vec{x})|\,0\rangle$$


$\rangle|^2 = \sum_{out} \langle 0,g| \hat{U}^\dagger |e,out\rangle\langle out,e| \hat{U} |g,0\rangle$
$\hat{U} = T\exp\left(\int_{-\infty}^{\infty} dt\, H_I(t)\right)$
$1 + \hat{U}^{(1)} + \hat{U}^{(2)} + O(\lambda)$
$\hat{U}^{(1)} = -i \int_{-\infty}^{\infty} dt\, H(t)$
$O(\lambda^2)| g,0\rangle = \langle 0\ g| \hat{U}^{(1)\dagger} |e\ out\rangle\langle out,e| U^{(1)} |g,0\rangle + O(\lambda^3)$
$| \hat{\phi}(t,x) |out\rangle\langle out| \phi(t,\vec{x}')| 0\rangle\, \chi(t)\, \chi(t')$


|g>|0> → |e>|1>
ħ(Ω+ω)
Vacuum excitation probability of an UDW detector
Perturbation theory (Dyson expansion)
μ̂(t) = F(x) m̂(t)

$$|e\rangle|0\rangle \longrightarrow |g\rangle|1\rangle$$
$$\hbar\Omega \qquad \hbar\omega$$

$$|g\rangle|0\rangle \xrightarrow{\hat{\sigma}^+ \hat{a}^\dagger} |e\rangle|1\rangle$$
$$0 \qquad \hbar(\Omega+\omega)$$

Vacuum excitation probability of an UDW detector

$$P^{|0\rangle}_{|g\rangle\to|e\rangle}(\Omega) \; ; \; P^{|0\rangle}_{|e\rangle\to|g\rangle}(\Omega) = P^{|0\rangle}_{|g\rangle\to|e\rangle}(-\Omega) \; ; \; P^{|0\rangle}_{|g\rangle\to|e\rangle} = \sum_{out} |\langle out, e| \hat{U} |g, 0\rangle|^2 = \sum_{out} \langle 0, g| \hat{U}^\dagger |e, out\rangle\langle out, e| \hat{U} |g, 0\rangle \qquad \hat{U} =$$

Perturbation theory (Dyson expansion) $\hat{U} = \mathbb{1} + \hat{U}^{(1)} + \hat{U}^{(2)} + \mathcal{O}(\lambda) \qquad \hat{U}^{(1)} = -i\int_{-\infty}^{\infty} dt\, H$

$$P^{|0\rangle}_{|g\rangle\to|e\rangle} = \sum_{out} \langle 0\, g| \mathbb{1} + \hat{U}^{(1)\dagger} + \mathcal{O}(\lambda^2) |e, out\rangle\langle e, out| \mathbb{1} + \hat{U}^{(1)} + \mathcal{O}(\lambda^2) | g, 0\rangle = \langle 0\, g| \hat{U}^{(1)\dagger} |e\, out\rangle\langle out, e| \hat{U}^{(1)} |g, 0\rangle + \mathcal{O}$$

$$P_{|g\rangle\to|e\rangle}(\Omega) = \lambda^2 \sum_{out} \int dt \int dt' \int d^n x \int d^n x' \langle g| \hat{m}(t) |e\rangle\langle e| \hat{m}(t') |g\rangle F(\vec{x}) F(\vec{x}') \langle 0| \hat{\phi}(t,x) |out\rangle\langle out| \hat{\phi}(t',\vec{x}') |0\rangle\, \chi(t)\, \chi(t')$$

$$\hat{\mu}(t) = F(\vec{x})\, \hat{m}(t)$$

$$P(\Omega) = \lambda^2 \int dt \int dt' \, \chi(t)\chi(t') \langle g|\hat{m}(t)|e\rangle\langle e|\hat{m}(t')|g\rangle \int d^n\vec{x} \int d^n\vec{x}' \, F(\vec{x})F(\vec{x}') \langle 0|\hat{\phi}(t,\vec{x})\hat{\phi}(t',\vec{x}')|0\rangle$$

$\hat{H}_g = \omega\, \hat{a}^\dagger_\omega \hat{a}_\omega$

$$\underset{\hbar\Omega}{|e\rangle|0\rangle} \xrightarrow{\hat{\sigma}^-\hat{a}^\dagger} \underset{\hbar\omega}{|g\rangle|1\rangle}$$

$$\underset{0}{|g\rangle|0\rangle} \xrightarrow{\hat{\sigma}^+\hat{a}^\dagger} \underset{\hbar(\Omega+\omega)}{|e\rangle|1\rangle}$$

Vacuum excitation probability of an UDW detector

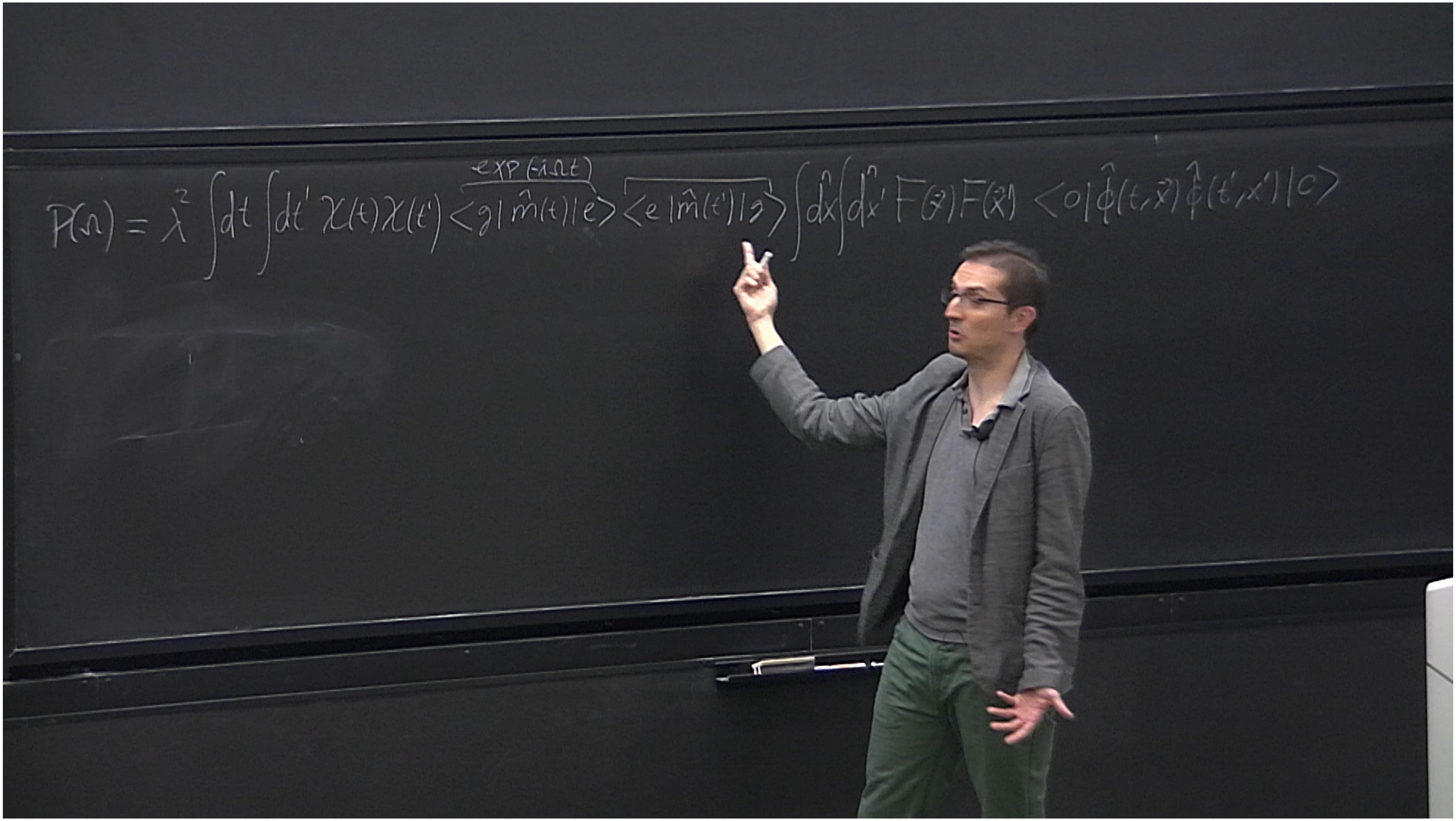
$$P^{|0\rangle}_{|g\rangle\to|e\rangle}(\Omega)\,;\quad P^{|0\rangle}_{|e\rangle\to|g\rangle}(\Omega) = P^{|0\rangle}_{|g\rangle\to|e\rangle}(-\Omega)\,;\quad P^{|0\rangle}_{|g\rangle\to|e\rangle} = \sum_{out} |\langle out, e|\hat{U}|g,0\rangle|^2 = \sum_{out} \langle 0,g|\hat{U}^\dagger|e,out\rangle\langle out,e|\hat{U}|g,0\rangle \qquad \hat{U} = \mathcal{T}\exp\left(\int_{-\infty}^{\infty} dt\, \hat{H}_I(t)\right)$$

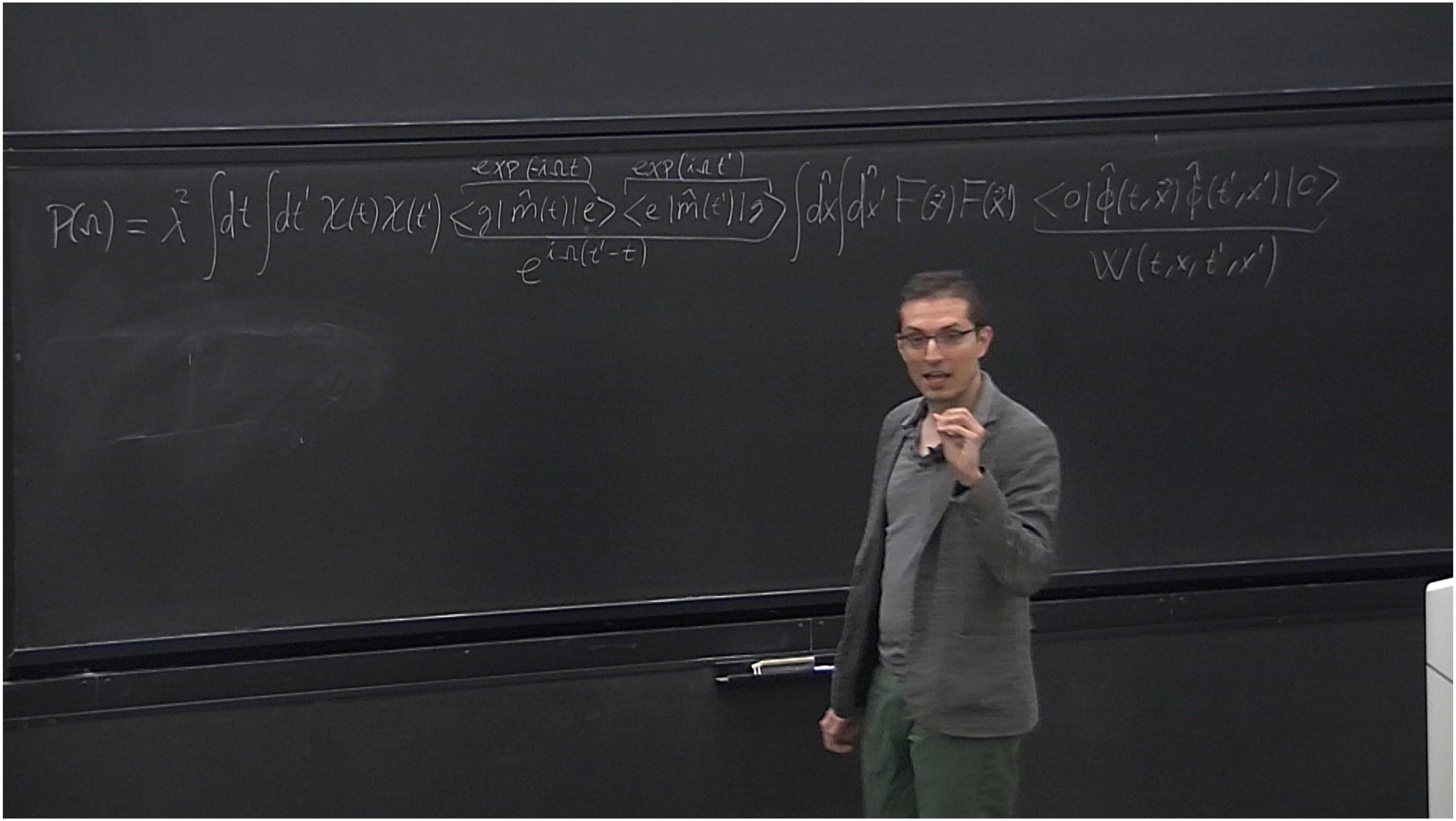
Perturbation theory (Dayson expansion) $\hat{U} = \mathbb{1} + \hat{U}^{(1)} + \hat{U}^{(2)} + \mathcal{O}(\lambda)$ $\qquad \hat{U}^{(1)} = -i\int_{-\infty}^{\infty} dt\, H(t)$

$$P^{|0\rangle}_{|g\rangle\to|e\rangle} = \sum_{out} \langle 0\, g|\mathbb{1} + \hat{U}^{(1)\dagger} + \mathcal{O}(\lambda^2)\,|e,out\rangle\langle e,out|\,\mathbb{1} + \hat{U}^{(1)} + \mathcal{O}(\lambda)|g,0\rangle = \langle 0\, g|\hat{U}^{(1)\dagger}|e\, out\rangle\langle out,e|\hat{U}^{(1)}|g,0\rangle + \mathcal{O}(\lambda^2)$$

$$P_{|g\rangle\to|e\rangle}(\Omega) = \lambda^2 \sum_{out} \int dt \int dt' \int d^n x \int d^n x' \langle g|\hat{m}(t)|e\rangle\langle e|\hat{m}(t')|g\rangle F(x)F(x')\langle 0|\phi(t,x)|out\rangle\langle out|\phi(t,x')|0\rangle\, \chi(t)\,\chi(t')$$

$$\hat{\mu}(t) = F(x)\,\hat{m}(t) \qquad \hat{m}(t) = \sigma^+ e^{i\Omega t} + \sigma^- e^{-i\Omega t}$$


$\exp(-i\Omega t)$
$P(\Omega) = \lambda^2 \int dt \int dt' \, \chi(t)\chi(t') \langle g|\hat{m}(t)|e\rangle \langle e|\hat{m}(t')|g\rangle \int d^n\vec{x} \int d^n\vec{x}' \, F(\vec{x})F(\vec{x}') \, \langle 0|\hat{\phi}(t,\vec{x})\hat{\phi}(t',\vec{x}')|0\rangle$



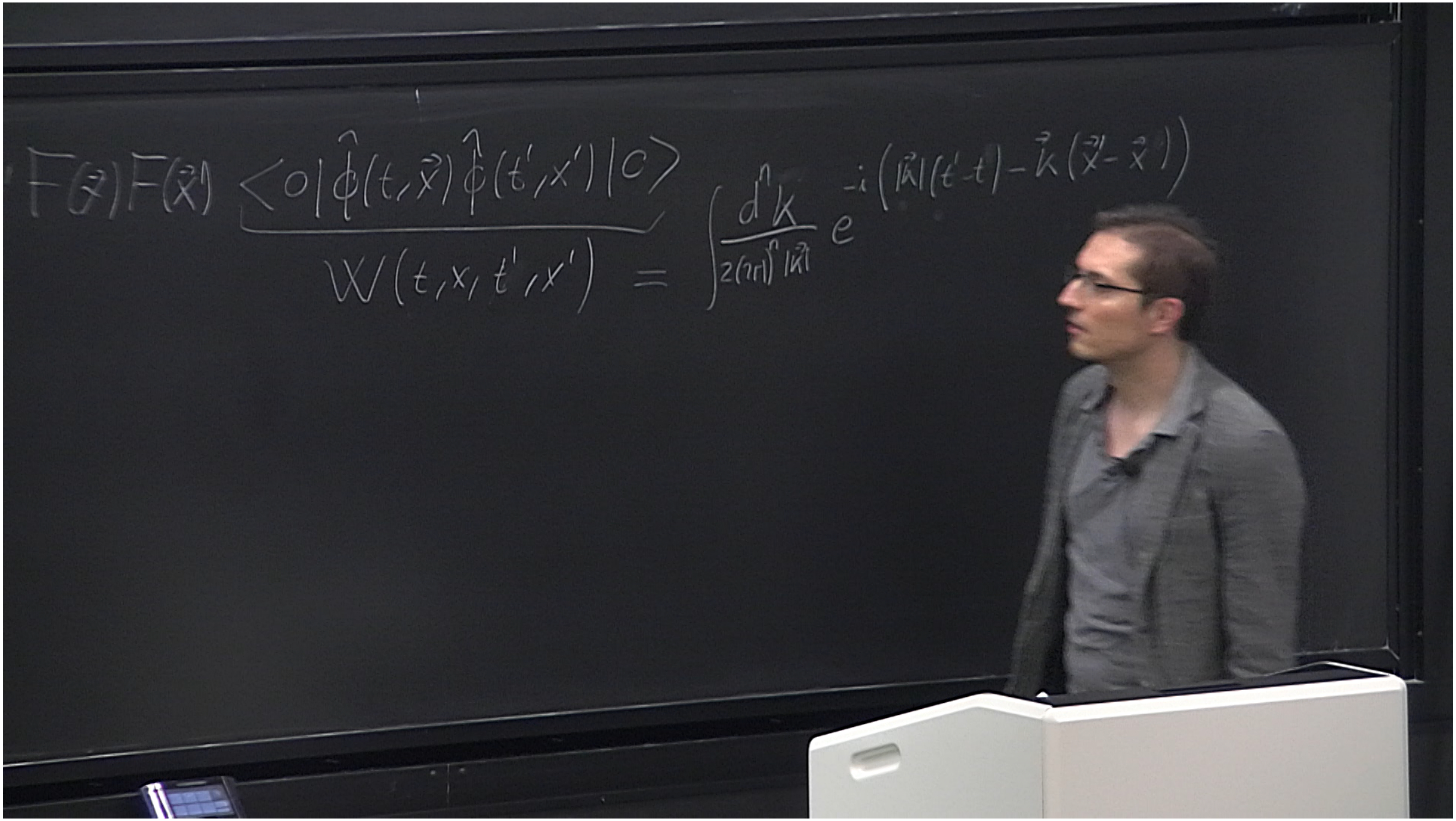
$$P(\Omega) = \lambda^2 \int dt \int dt' \, \chi(t)\chi(t') \underbrace{\overbrace{\langle g|\hat{m}(t)|e\rangle}^{\exp(-i\Omega t)} \overbrace{\langle e|\hat{m}(t')|g\rangle}^{\exp(i\Omega t')}}_{e^{i\Omega(t'-t)}} \int d^n x \int d^n x' \, F(\vec{x}) F(\vec{x}') \underbrace{\langle 0|\hat{\phi}(t,\vec{x})\hat{\phi}(t',\vec{x}')|0\rangle}_{W(t,x,t',x')}$$

$$
\left. t' \right) \underbrace{\overbrace{\langle g | \hat{m}(t) | e \rangle}^{\exp(-i\Omega t)} \overbrace{\langle e | \hat{m}(t') | g \rangle}^{\exp(i\Omega t')}}_{e^{i\Omega(t'-t)}} \int d^n x \int d^n x' \, F(\vec{x}) F(\vec{x}') \underbrace{\langle 0 | \hat{\phi}(t,\vec{x}) \hat{\phi}(t',\vec{x}') | 0 \rangle}_{W(t,x,t',x') \; = \; \int \frac{d^n k}{2(2\pi)^n |\vec{k}|}}
$$

$$F(\vec{x})F(\vec{x}')\ \underbrace{\langle 0|\hat{\phi}(t,\vec{x})\hat{\phi}(t',x')|0\rangle}_{W(t,x,t',x')} = \int \frac{d^n k}{2(2\pi)^n|\vec{k}|}\, e^{-i\left(|\vec{k}|(t-t')-\vec{k}(\vec{x}-\vec{x}')\right)}$$

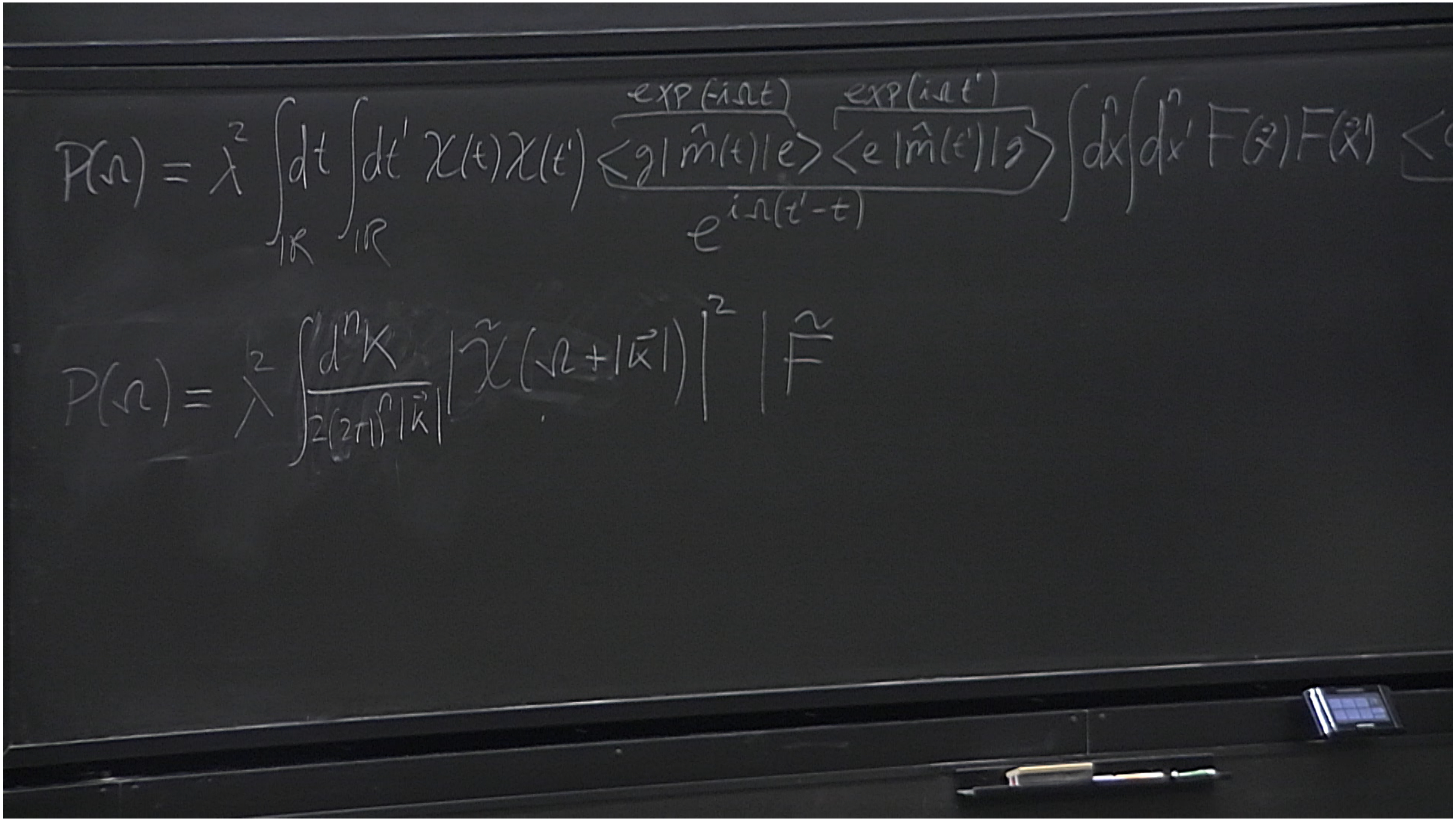
$$P(\Omega) = \lambda^2 \int_{\mathbb{R}} dt \int_{\mathbb{R}} dt' \, \chi(t)\chi(t') \underbrace{\overbrace{\langle g|\hat{m}(t)|e\rangle}^{\exp(-i\Omega t)} \overbrace{\langle e|\hat{m}(t')|g\rangle}^{\exp(i\Omega t')}}_{e^{i\Omega(t'-t)}} \int d^n x \int d^n x' \, F(\vec{x}) F(\vec{x}')$$

$$P(\Omega) = \lambda^2$$


$F(\vec{x})F(\vec{x}') \underbrace{\langle 0|\hat{\phi}(t,\vec{x})\hat{\phi}(t',x')|0\rangle}_{W(t,x,t',x')} = \int \frac{d^n k}{2(2\pi)^n|\vec{k}|} e^{-i(|\vec{k}|(t'-t)-\vec{k}(\vec{x}'-\vec{x}))}$

$$P(\Omega) = \lambda^2 \int_{\mathbb{R}} dt \int_{\mathbb{R}} dt' \, \chi(t)\chi(t') \underbrace{\overbrace{\langle g|\hat{m}(t)|e\rangle}^{\exp(-i\Omega t)} \overbrace{\langle e|\hat{m}(t')|g\rangle}^{\exp(i\Omega t')}}_{e^{i\Omega(t'-t)}} \int d^n x \int d^n x' \, F(\vec{x}) F(\vec{x}') \, \langle$$

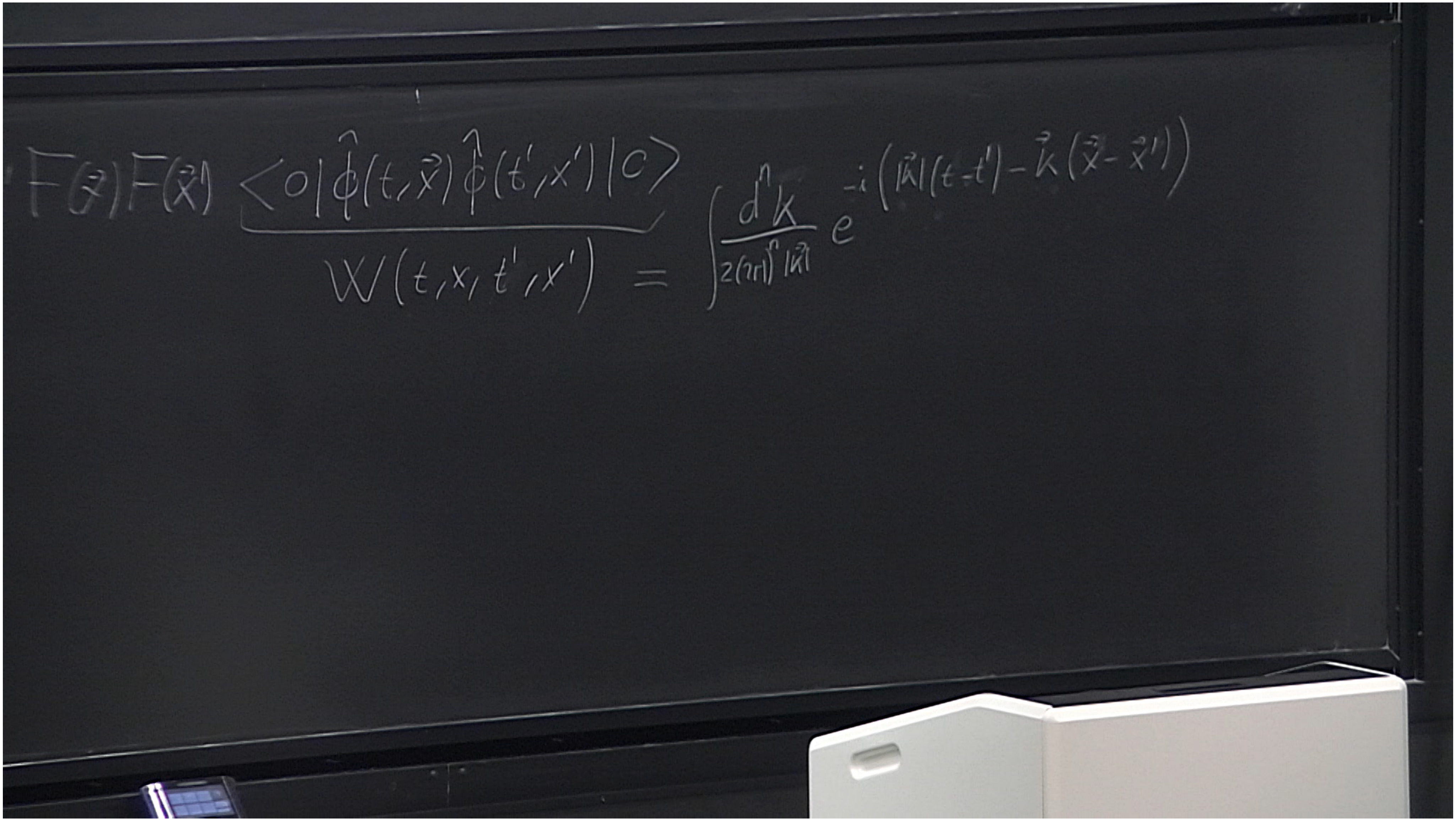
$$P(\Omega) = \lambda^2 \left| \tilde{\chi}(\Omega + |\vec{k}|) \right|^2$$



$P(\Omega) = \lambda^2 \int_{\mathbb{R}} dt \int_{\mathbb{R}} dt' \, \chi(t)\chi(t') \underbrace{\overbrace{\langle g|\hat{m}(t)|e\rangle}^{\exp(-i\Omega t)} \overbrace{\langle e|\hat{m}(t')|g\rangle}^{\exp(i\Omega t')}}_{e^{i\Omega(t'-t)}} \int d^n x \int d^n x' \, F(\vec{x}) F(\vec{x}')$
$P(\Omega) = \lambda^2 \int \frac{d^n k}{2(2\pi)^n |\vec{k}|} \left|\tilde{\chi}(\Omega + |\vec{k}|)\right|^2 |\tilde{F}$

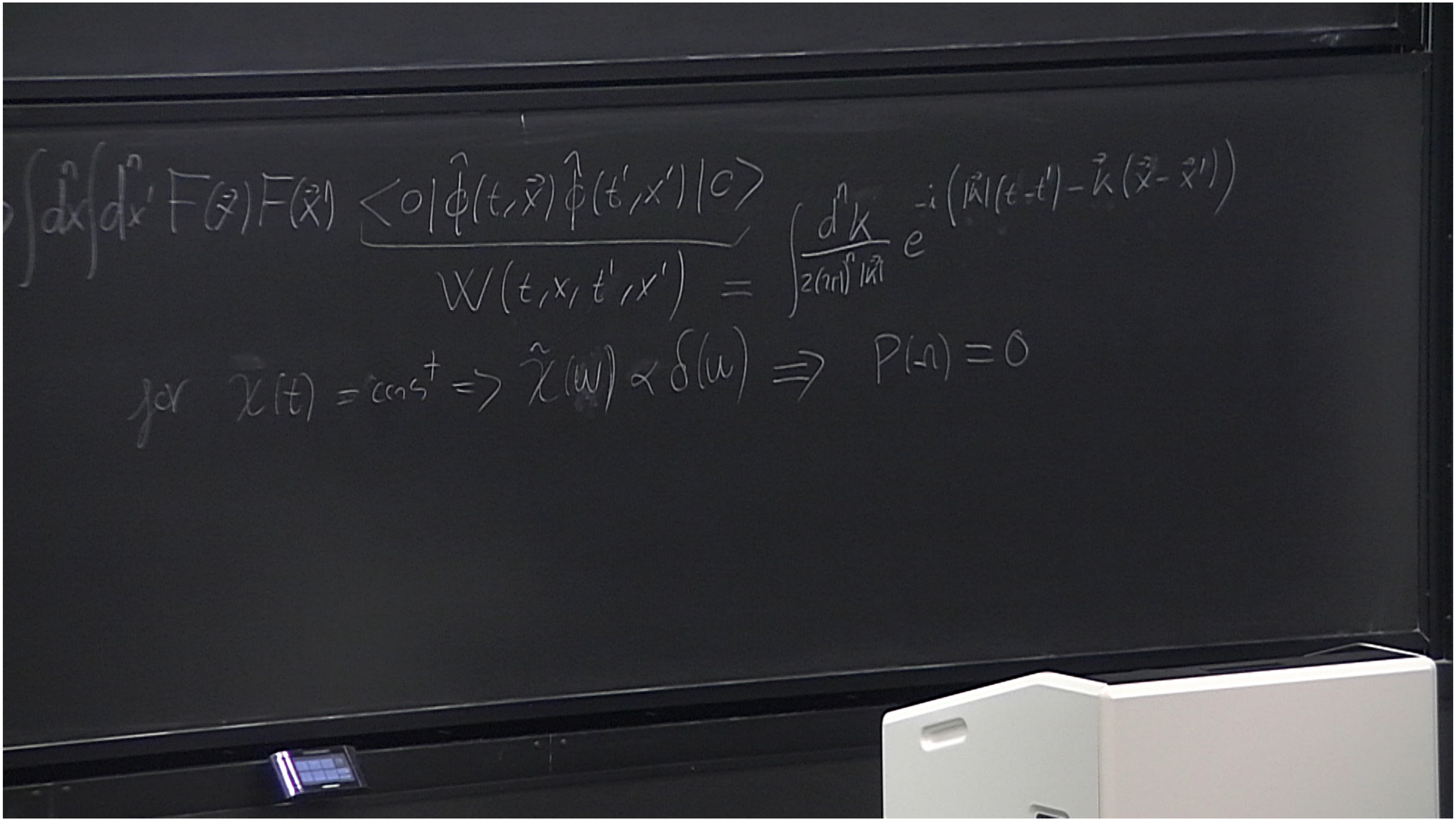
$$P(\Omega) = \lambda^2 \int_{\mathbb{R}} dt \int_{\mathbb{R}} dt' \, \chi(t)\chi(t') \underbrace{\overbrace{\langle g|\hat{m}(t)|e\rangle}^{\exp(-i\Omega t)} \overbrace{\langle e|\hat{m}(t')|g\rangle}^{\exp(i\Omega t')}}_{e^{i\Omega(t'-t)}} \int d^n x \int d^n x' \, F(\vec{x}) F(\vec{x}') \langle$$

$$P(\Omega) = \lambda^2 \int \frac{d^n k}{2(2\pi)^n |\vec{k}|} \left|\tilde{\chi}(\Omega + |\vec{k}|)\right|^2 \left|\tilde{F}(\vec{k})\right|^2$$


$F(\vec{x})F(\vec{x}')\underbrace{\langle 0|\hat{\phi}(t,\vec{x})\hat{\phi}(t',x')|0\rangle}_{W(t,x,t',x')} = \int \frac{d^n k}{2(2\pi)^n |\vec{k}|} e^{-i\left(|\vec{k}|(t-t') - \vec{k}(\vec{x}-\vec{x}')\right)}$

$$\int d^n x \int d^n x' \, F(\vec{x}) F(\vec{x}') \underbrace{\langle 0 | \hat{\phi}(t,\vec{x}) \hat{\phi}(t',x') | 0 \rangle}_{W(t,x,t',x')} = \int \frac{d^n k}{2(2\pi)^n |\vec{k}|} e^{-i\left(|\vec{k}|(t-t') - \vec{k}(\vec{x}-\vec{x}')\right)}$$

for $\chi(t) = \text{const} \Rightarrow \tilde{\chi}(\omega) \propto \delta(\omega)$

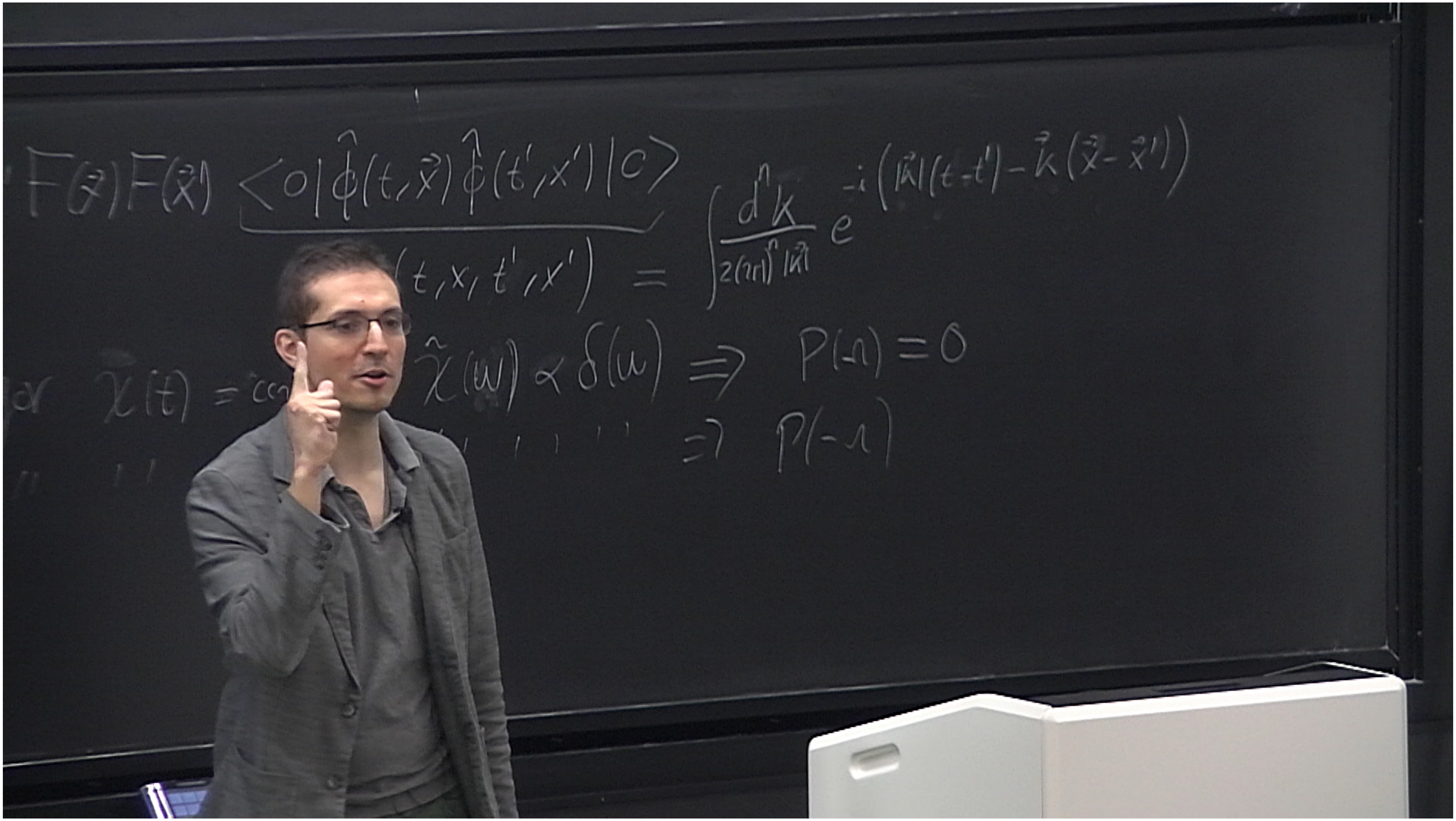

$\int d^n\vec{x} \int d^n\vec{x}' \, F(\vec{x}) F(\vec{x}') \underbrace{\langle 0|\hat{\phi}(t,\vec{x})\hat{\phi}(t',x')|0\rangle}_{W(t,x,t',x')} = \int \frac{d^n k}{2(2\pi)^n |\vec{k}|} e^{-i(|\vec{k}|(t-t') - \vec{k}(\vec{x}-\vec{x}'))}$
for $\chi(t) = \text{const} \Rightarrow \tilde{\chi}(\omega) \propto \delta(\omega) \Rightarrow P(\Omega) = 0$

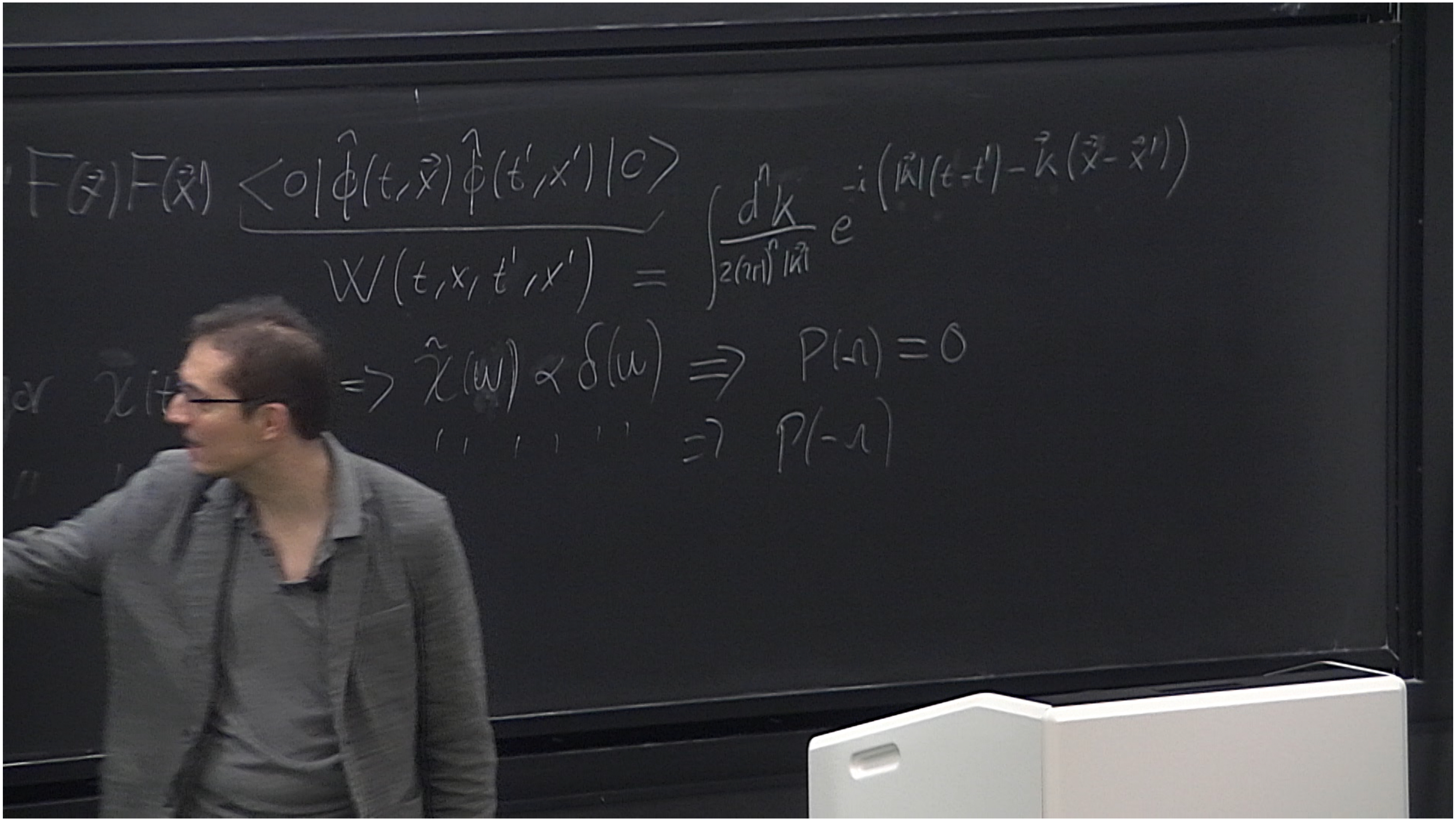
$$P(\Omega) = \lambda^2 \int_{\mathbb{R}} dt \int_{\mathbb{R}} dt'\, \chi(t)\chi(t') \underbrace{\overbrace{\langle g|\hat{m}(t)|e\rangle}^{\exp(-i\Omega t)} \overbrace{\langle e|\hat{m}(t')|g\rangle}^{\exp(i\Omega t')}}_{e^{i\Omega(t'-t)}} \int d^n x \int d^n x'\, F(\vec{x}) F(\vec{x}')\, \langle \ldots$$

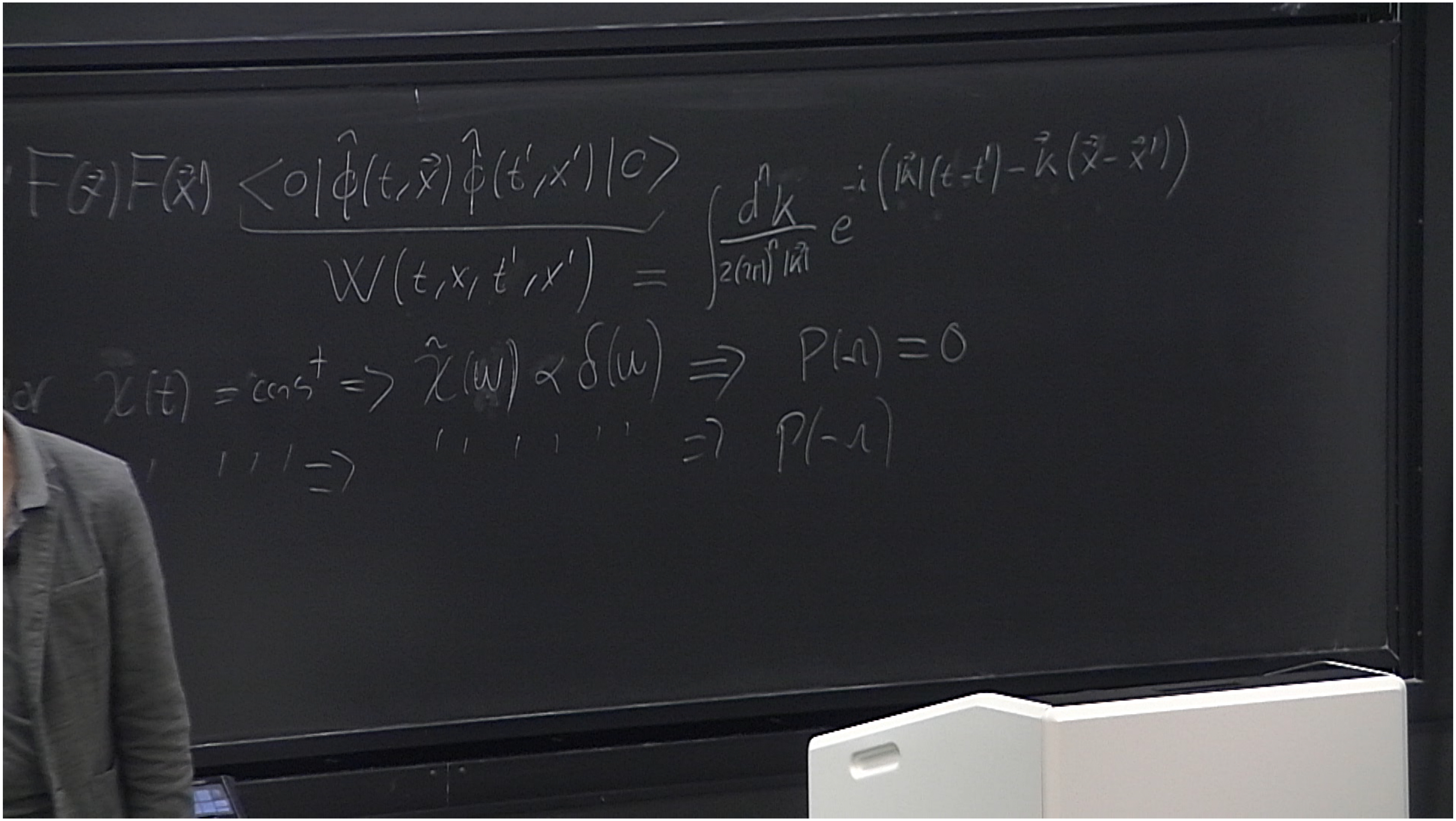
$$P(\Omega) = \lambda^2 \int \frac{d^n k}{2(2\pi)^n |\vec{k}|} \left|\tilde{\chi}(\Omega + |\vec{k}|)\right|^2 \left|\tilde{F}(\vec{k})\right|^2 \qquad \text{for } \chi(t) = \cos \ldots$$

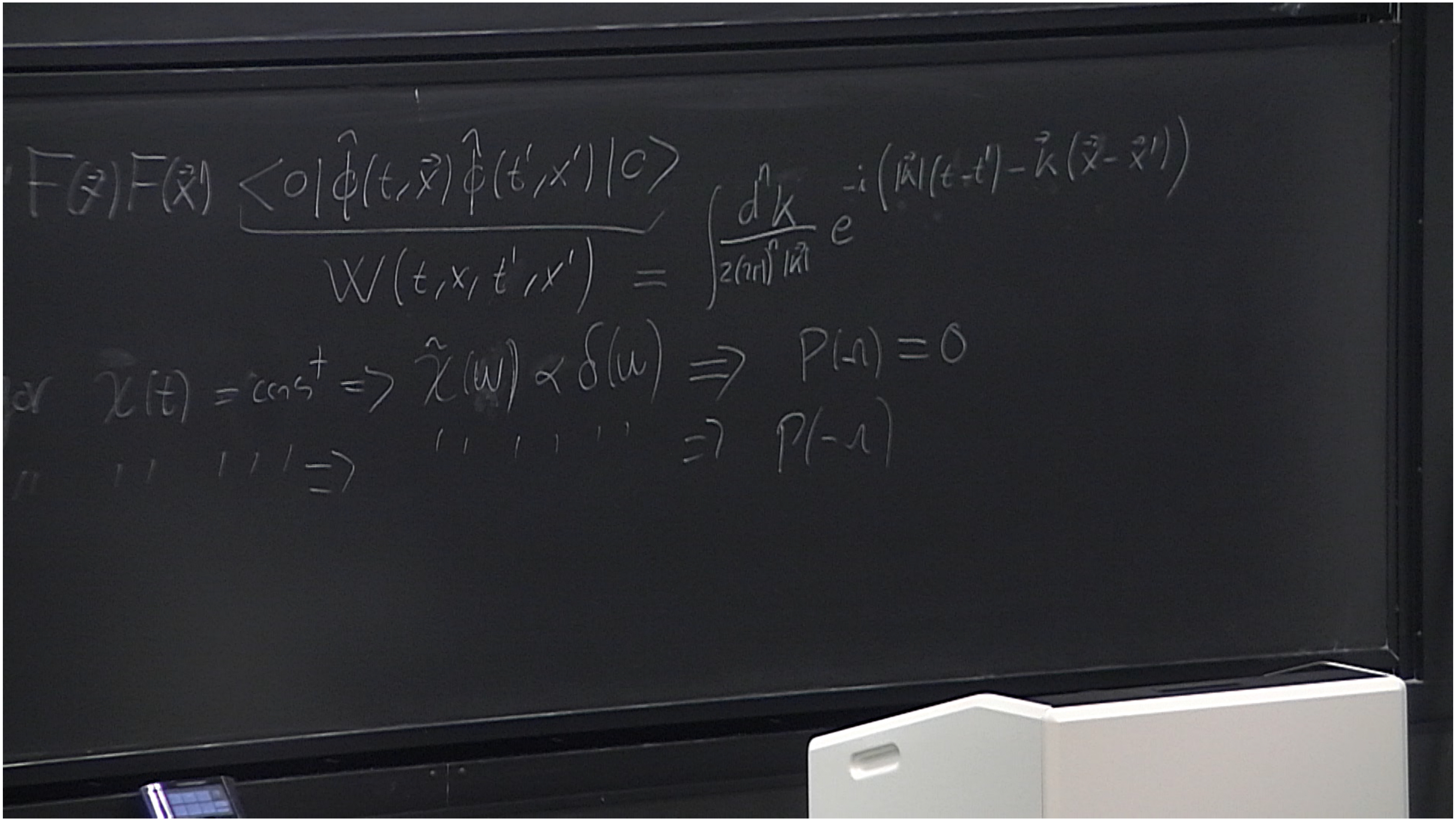
$$\tilde{\chi}(|\vec{k}| - \Omega)$$

$$P(-\Omega) = \quad '' \quad '' \quad '' \quad ''$$


$F(\vec{x})F(\vec{x}') \langle 0|\hat{\phi}(t,\vec{x})\hat{\phi}(t',x')|0\rangle$
$(t,x,t',x') = \int \frac{d^nk}{2(2\pi)^n|\vec{k}|} e^{-i(|\vec{k}|(t-t')-\vec{k}(\vec{x}-\vec{x}'))}$
$\tilde{\chi}(\omega) \propto \delta(\omega) \Rightarrow P(-\Omega)=0$
$\Rightarrow P(-\Omega)$


$F(\vec{x})F(\vec{x}') \langle 0|\hat{\phi}(t,\vec{x})\hat{\phi}(t',x')|0\rangle$
$W(t,x,t',x') = \int \frac{d^n k}{2(2\pi)^n|\vec{k}|} e^{-i(|\vec{k}|(t-t') - \vec{k}(\vec{x}-\vec{x}'))}$
$\Rightarrow \tilde{\chi}(\omega) \propto \delta(\omega) \Rightarrow P(\Omega) = 0$
$\Rightarrow P(-\Omega)$


$F(\vec{x})F(\vec{x}') \underbrace{\langle 0|\hat{\phi}(t,\vec{x})\hat{\phi}(t',x')|0\rangle}_{W(t,x,t',x')} = \int \frac{d^n k}{2(2\pi)^n|\vec{k}|} e^{-i(|\vec{k}|(t-t') - \vec{k}(\vec{x}-\vec{x}'))}$
$\chi(t) = \text{const} \Rightarrow \tilde{\chi}(\omega) \propto \delta(\omega) \Rightarrow P(\Omega) = 0$
$\Rightarrow P(-\Omega)$


$F(\vec{x})F(\vec{x}')\,\underbrace{\langle 0|\hat{\phi}(t,\vec{x})\hat{\phi}(t',x')|0\rangle}_{W(t,x,t',x')} = \int \frac{d^n k}{2(2\pi)^n|\vec{k}|} e^{-i\left(|\vec{k}|(t-t')-\vec{k}(\vec{x}-\vec{x}')\right)}$
or $\chi(t) = \text{const} \Rightarrow \hat{\chi}(\omega) \propto \delta(\omega) \Rightarrow P(-\Omega) = 0$
$\Rightarrow P(-\Omega)$