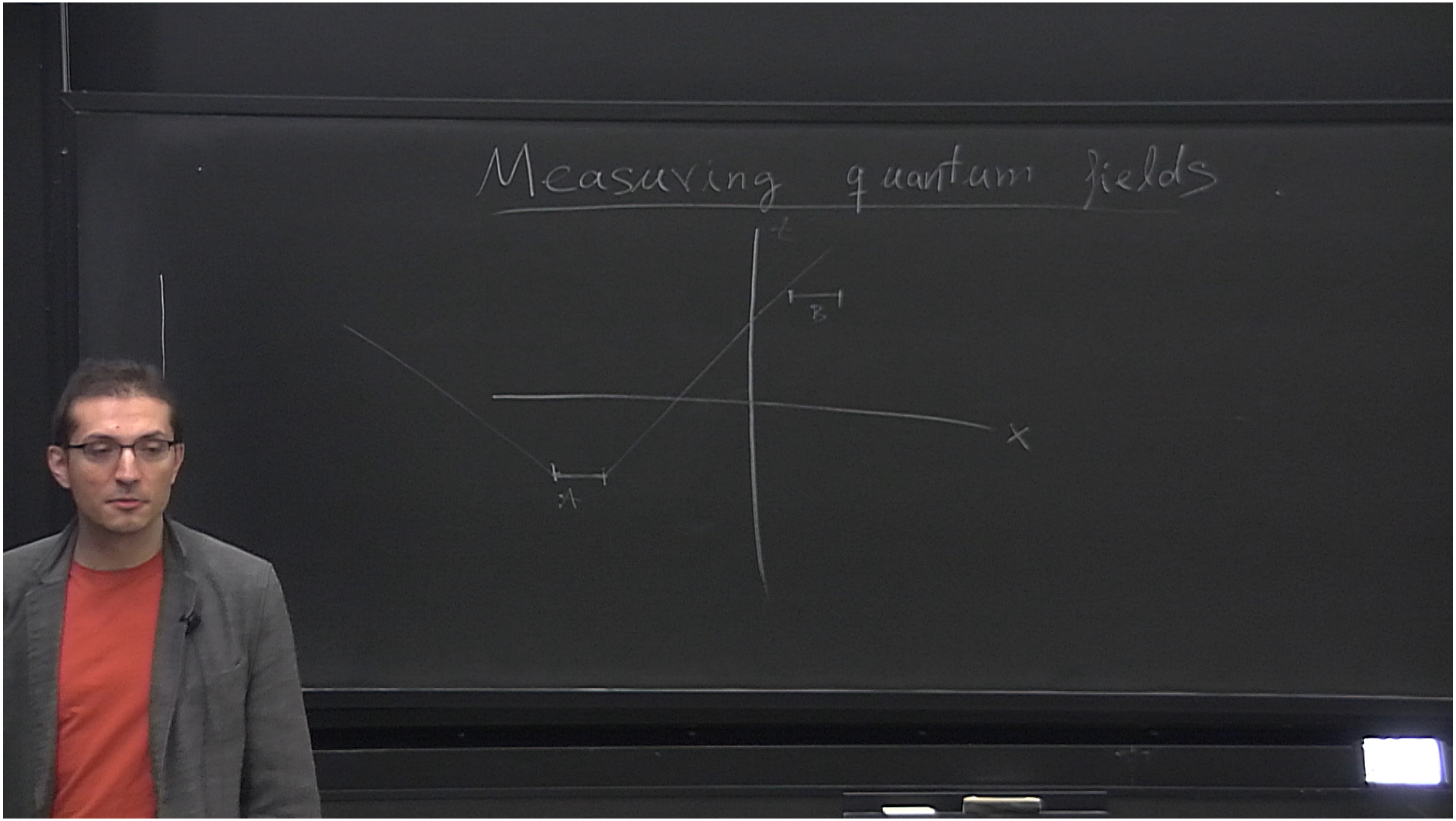
Title: PSI 2017/2018 - Relativistic Quantum Information - Lecture 2

Date: Mar 20, 2018 09:00 AM

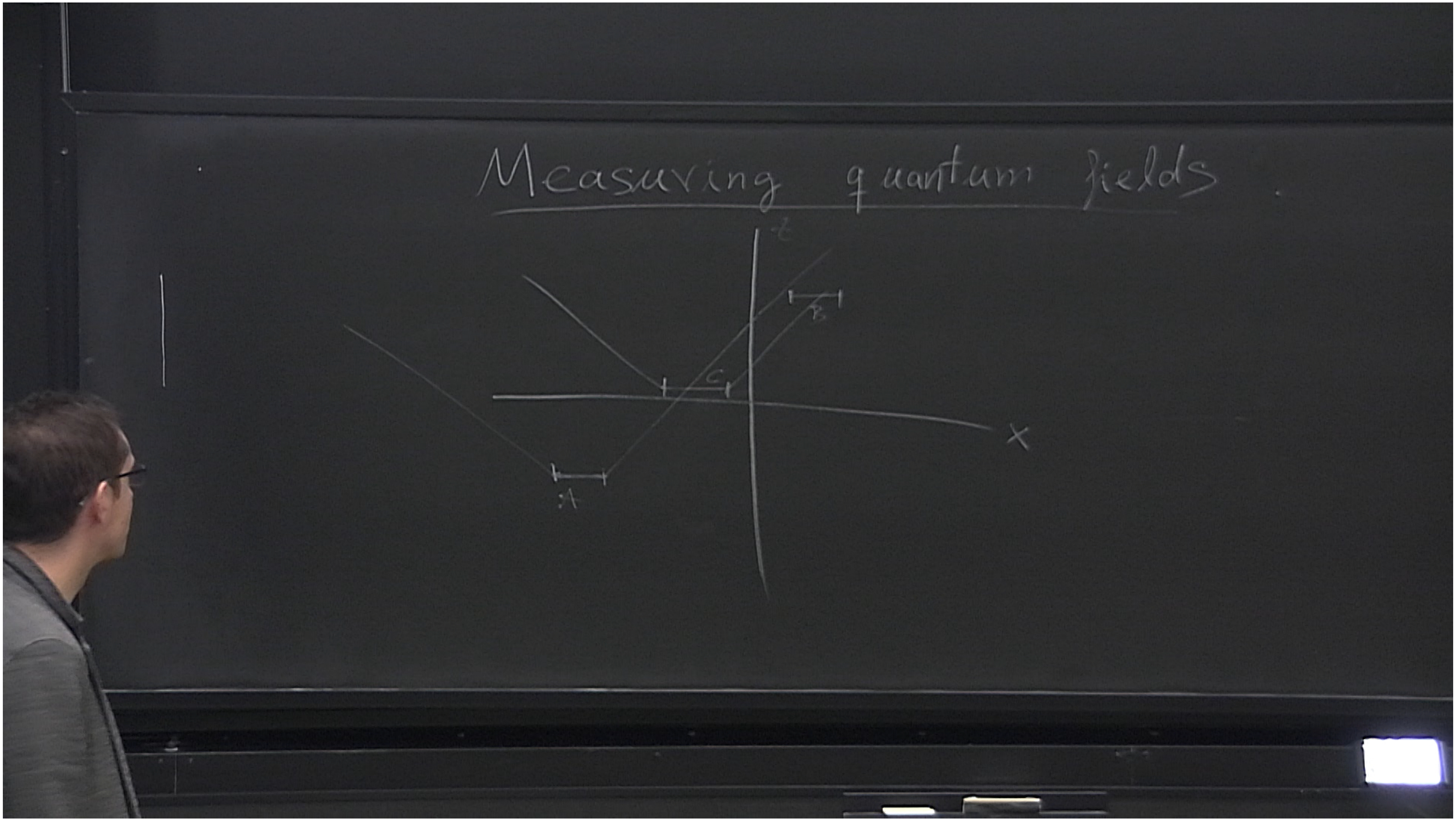
URL: http://pirsa.org/18030029

Abstract:

Measuring quantum fields



Measuring quantum fields
t
B
A
x


Measuring quantum fields
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x
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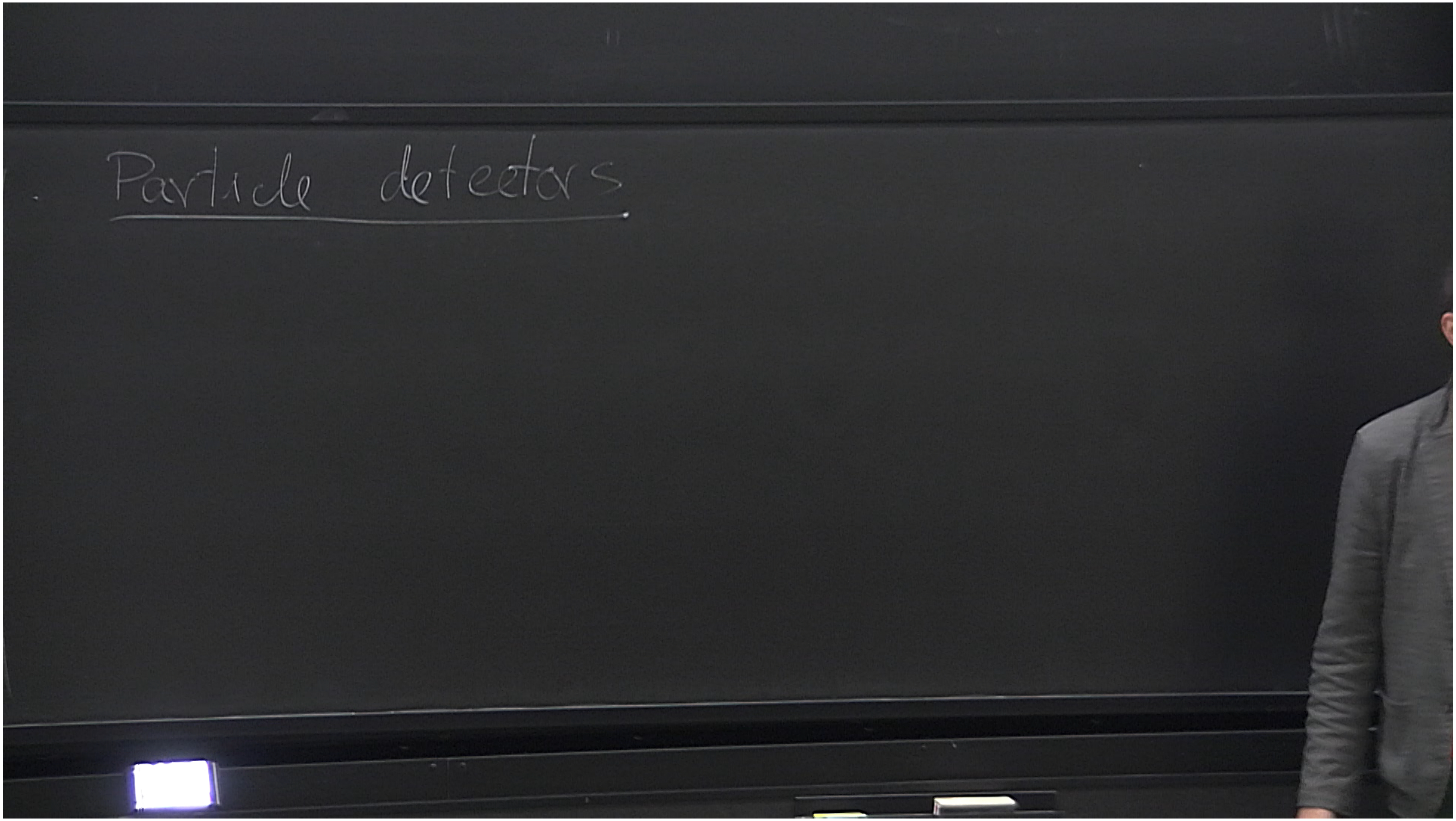

Particle detectors

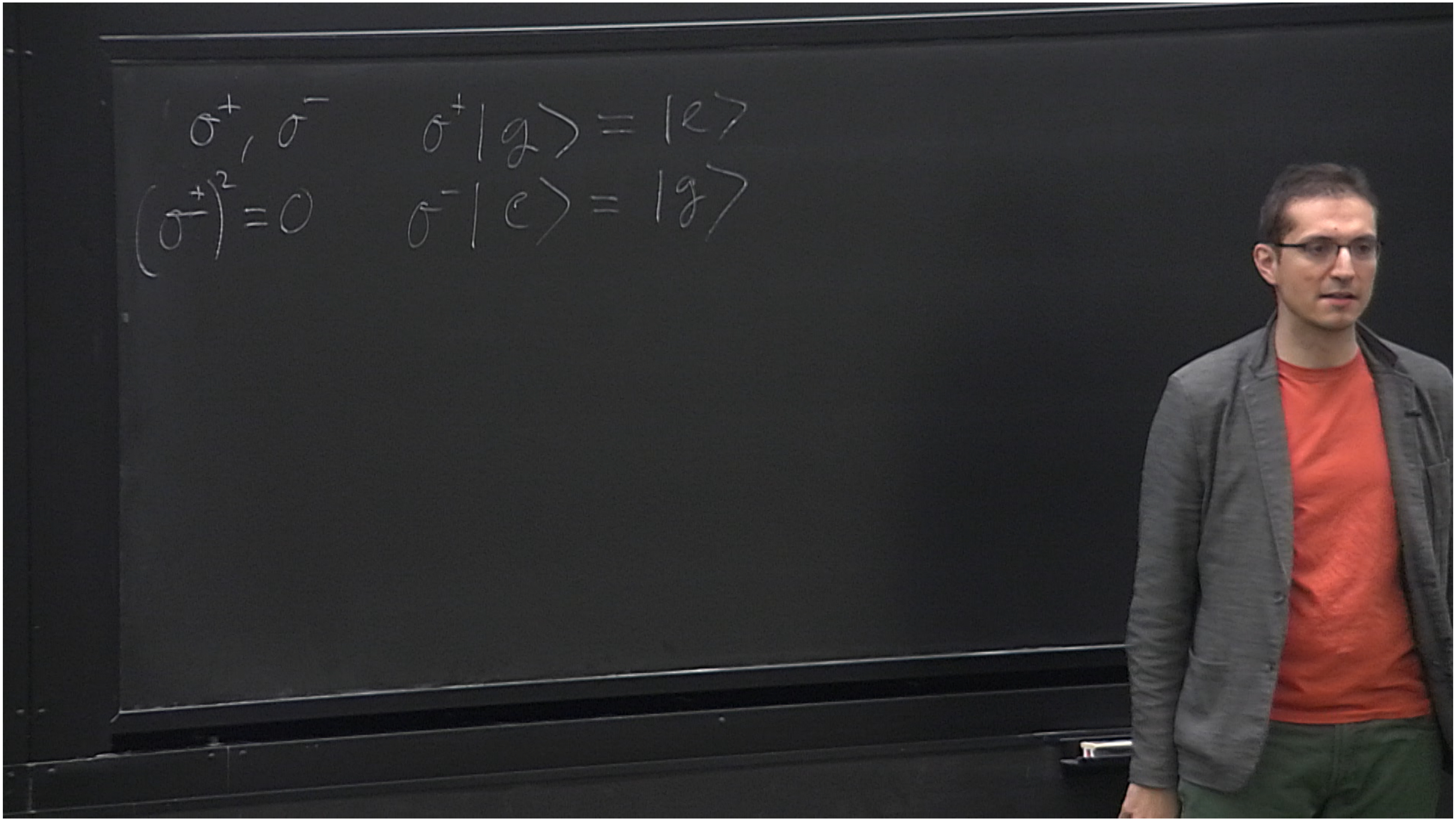
Particle detectors

What do people do in quantum optics?

Jaynes-Cummings model:

- Two-level "atom" $|e\rangle, |g\rangle$ (energy gap Ω)
- Coupling to a "mode" of the EM field (H. O.)

$$\hat{H}_I = \lambda\left(\hat{\sigma}^+ \hat{a}\, e^{i(\Omega-\omega)} + \hat{\sigma}^- \hat{a}^+ e^{-i(\Omega-\omega)}\right)$$


σ^+, σ^-
$\sigma^+|g\rangle = |e\rangle$
$(\sigma^\pm)^2 = 0$
$\sigma^-|e\rangle = |g\rangle$

Particle detectors

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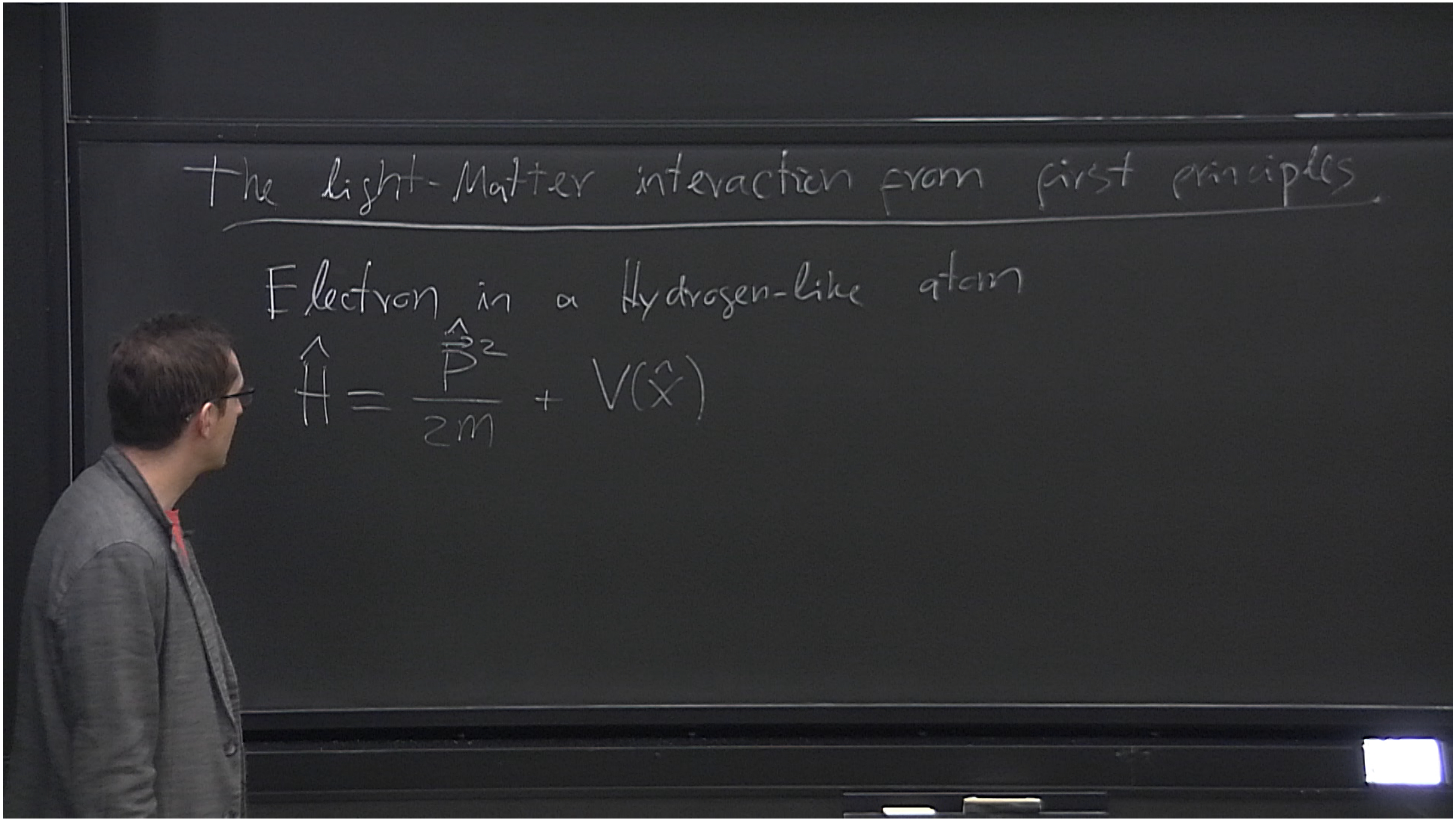
The light-Matter interaction from first principles
Electron in a Hydrogen-like atom
$\hat{H} = \frac{\hat{\vec{P}}^2}{2m} + V(\hat{\vec{x}})$

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The electron can couple to an EM field

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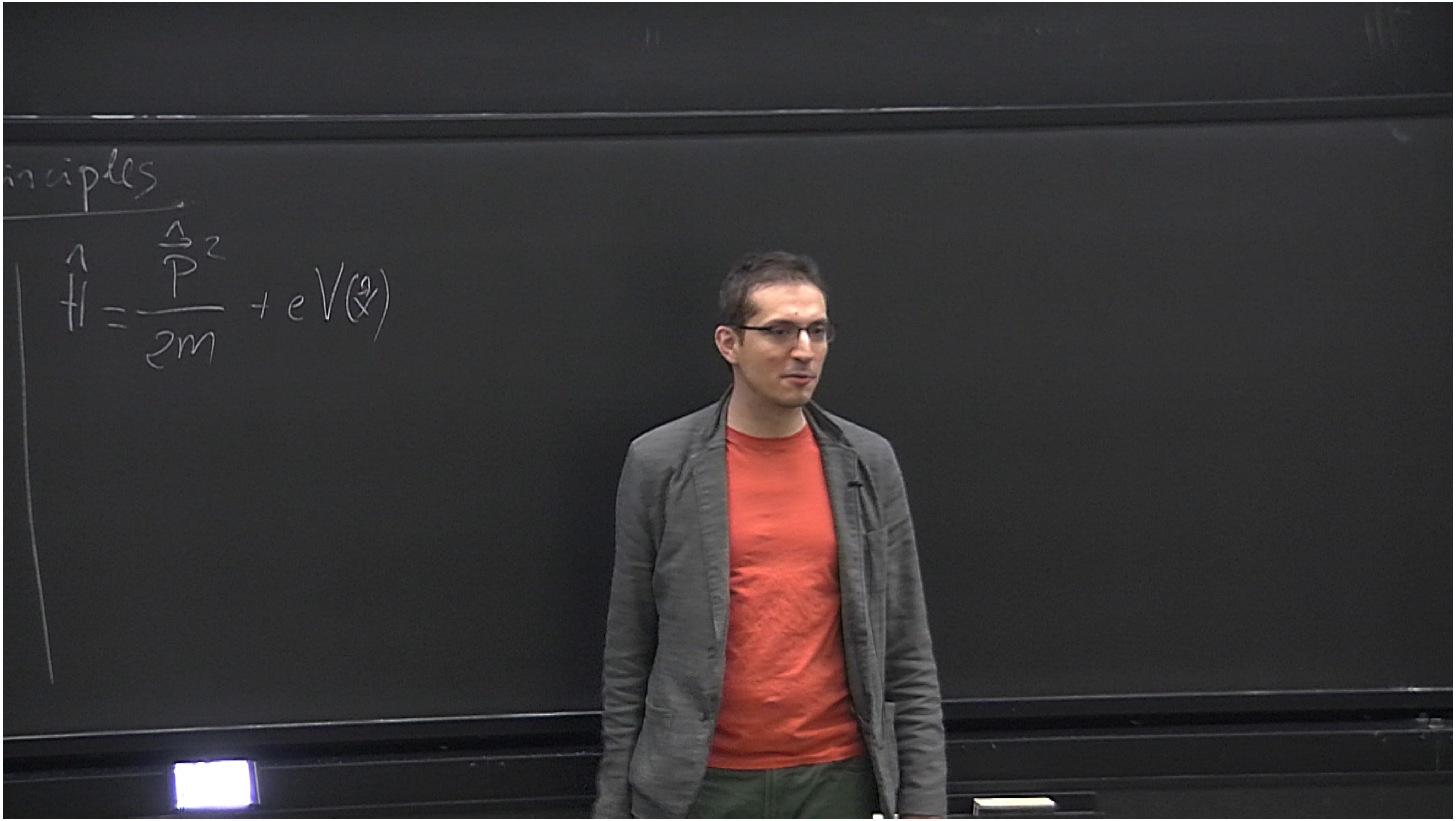
inciples
$\hat{H} = \frac{\hat{P}^2}{2m} + eV(\hat{x})$

inciples

$$\hat{H} = \underbrace{\frac{\hat{\vec{P}}^2}{2m} + eV(\hat{\vec{x}})}_{\hat{H}_0} - \frac{e}{2m}\left(\hat{\vec{p}}\cdot\hat{\vec{A}}(t,\hat{\vec{x}}) + \hat{\vec{A}}(t,\hat{\vec{x}})\cdot\hat{\vec{p}}\right) + eU(t,\hat{\vec{x}}) + \frac{e^2}{2m}\hat{\vec{A}}^2(t,\hat{\vec{x}})$$

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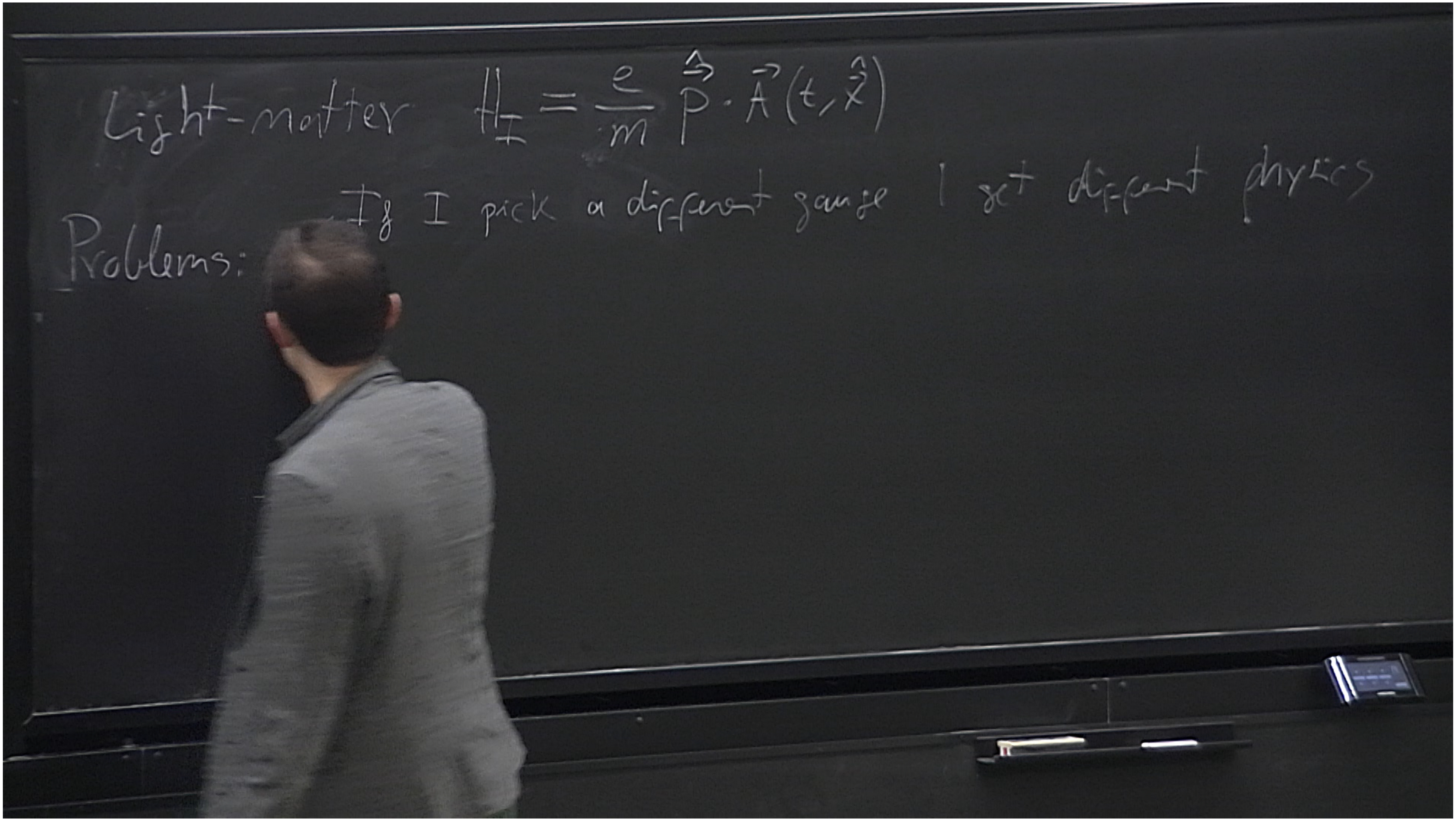
Path of Q.O.

1- Let's neglect $O(e^2)$

2- Let's work in the Coulomb gauge $[\hat{\vec{A}}(\hat{\vec{x}}), \hat{\vec{P}}] = 0$ $\quad U = 0$

Light-matter $H_I = \frac{e}{m} \hat{\vec{p}} \cdot \vec{A}(t, \hat{\vec{x}})$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Problems: If I pick a different gauge I get different physics

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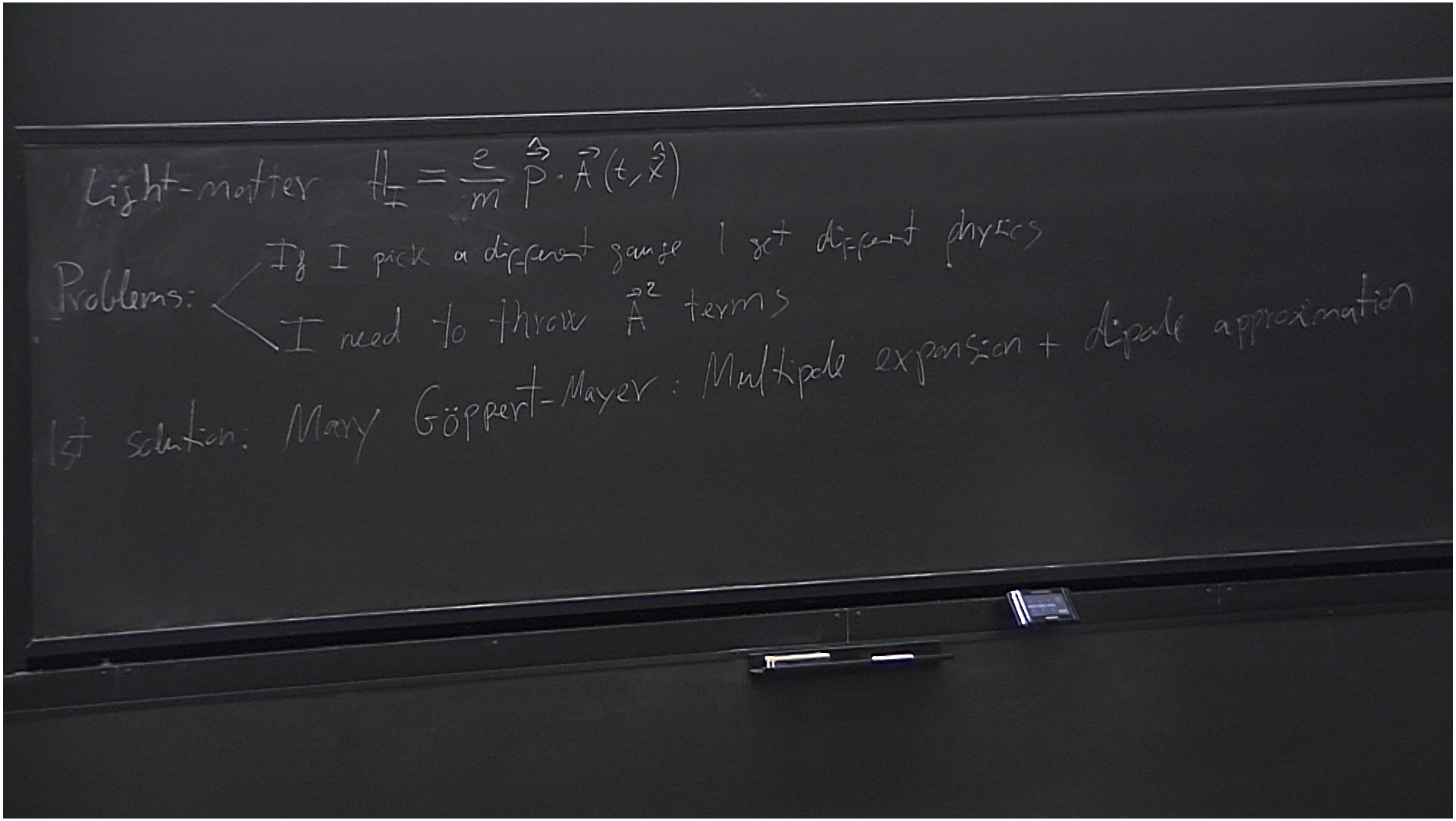
Problems:
- If I pick a different gauge I get different physics
- I need to throw $\vec{A}^2$ terms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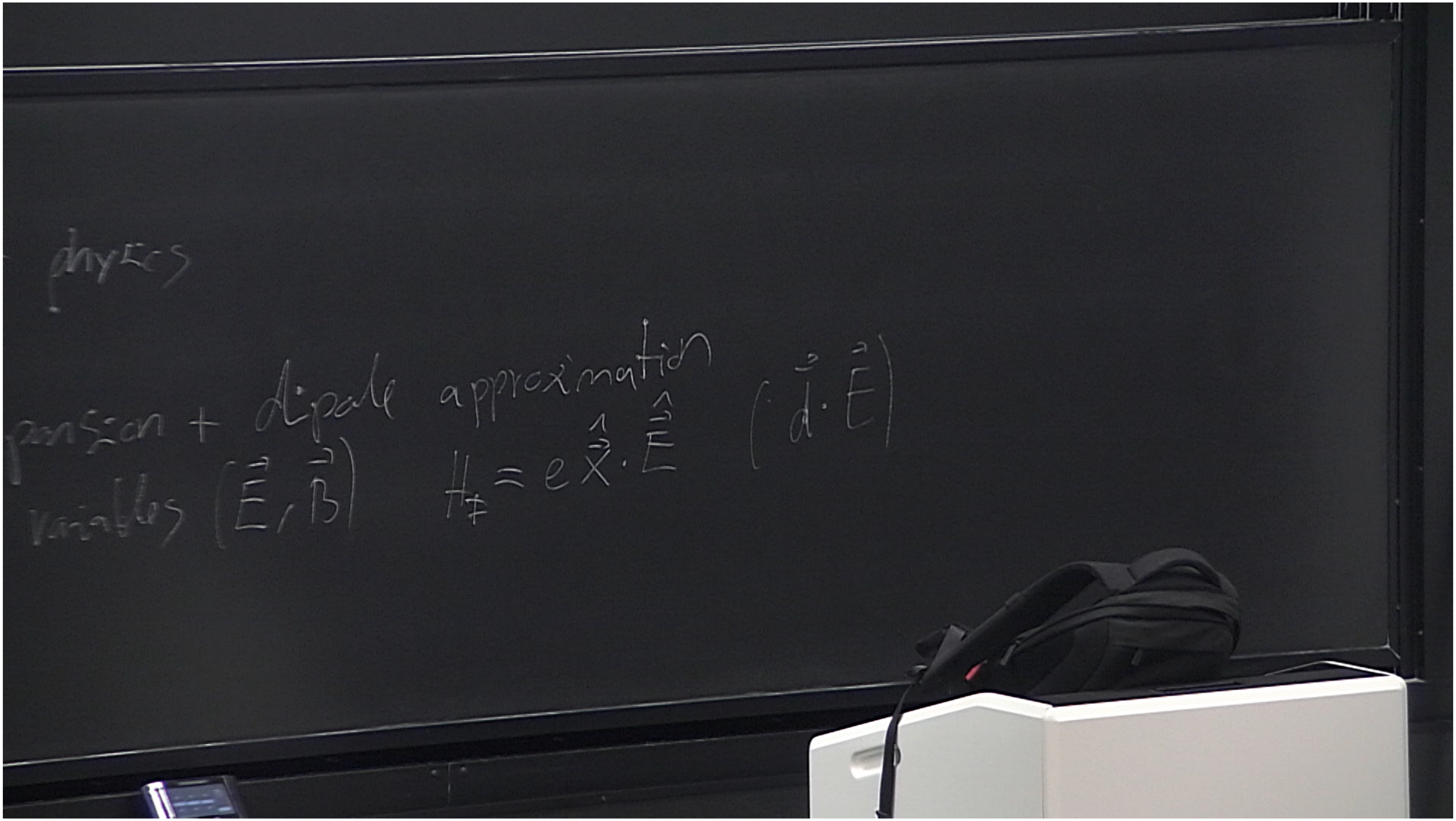
1st solution: Mary Göppert-Mayer: Multipole expansion + dipole approximation

(Write H_I as function of gauge independent variables $(\vec{E},\vec{B})$ $\quad H_I = e\,\hat{\vec{x}}\cdot\hat{\vec{E}}$


physics
pansion + dipole approximation
variables $(\vec{E}, \vec{B})$
$H_I = e\hat{\vec{x}} \cdot \hat{\vec{E}}$
$(\vec{d} \cdot \vec{E})$

Light-matter $H_I = \frac{e}{m}\hat{\vec{p}}\cdot\vec{A}(t,\hat{\vec{x}})$ $\quad (\vec{p}\cdot\vec{A})$

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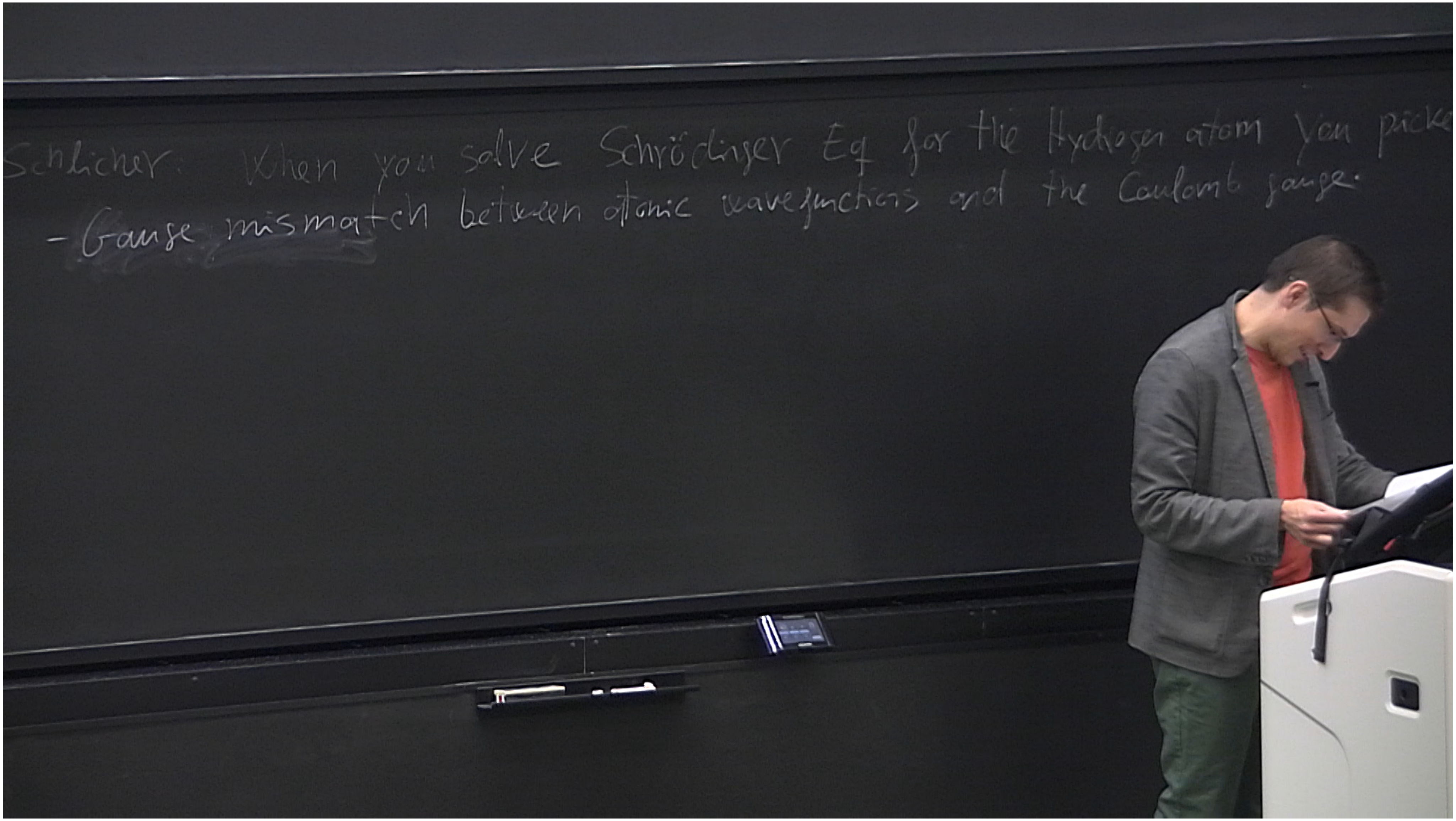
1st solution: [illegible]

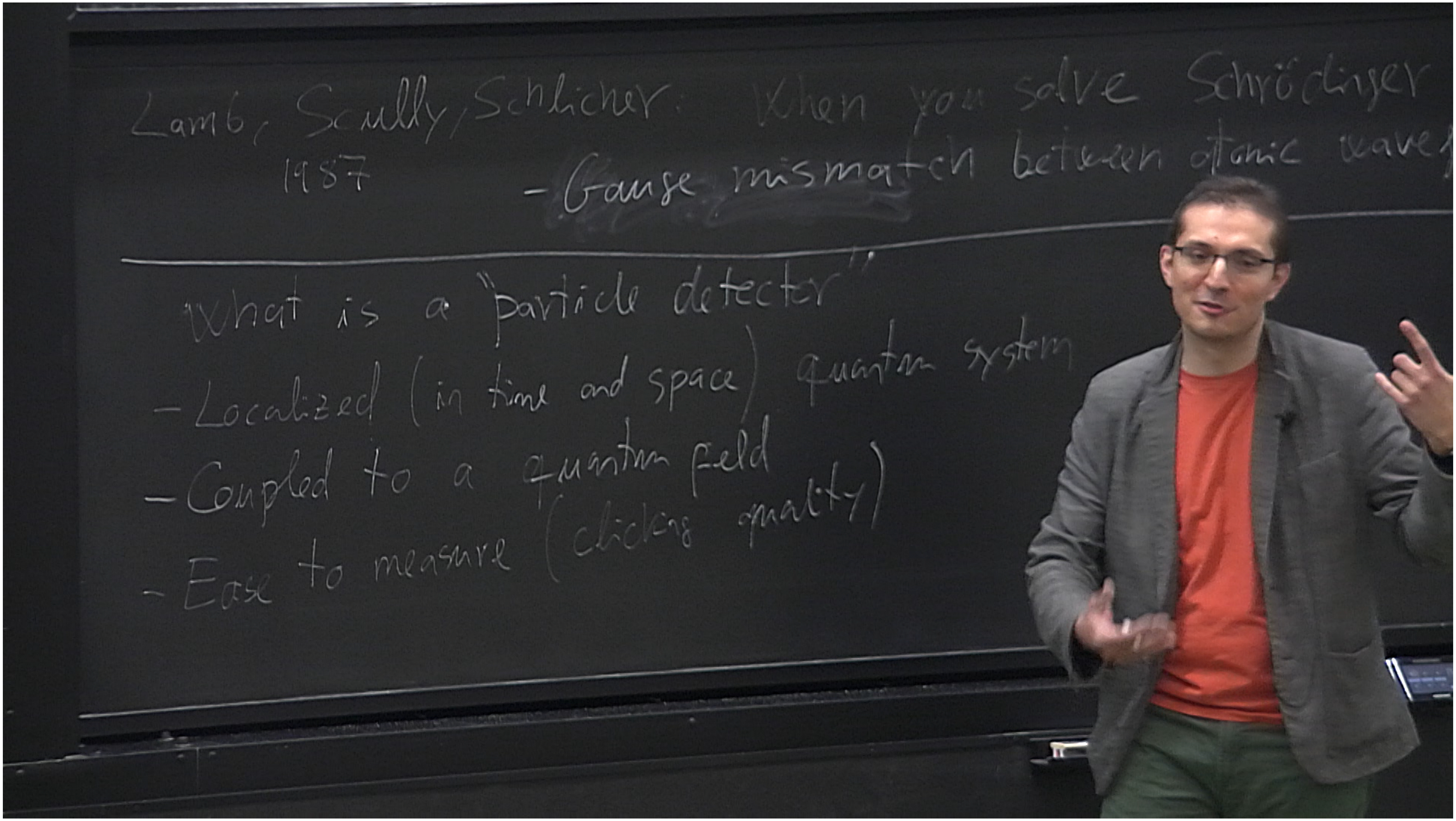
(Write H_I as function of gauge independent variables (E, B) H_I

Lamb, Scully, Schlicher: When you solve Schrödinger Eq for the Hydrogen atom you picked a gauge!!



Schlicher: When you solve Schrödinger Eq for the Hydrogen atom you picke
- Gauge mismatch between atomic wavefunctions and the Coulomb gauge.

Lamb, Scully, Schlicher: When you solve Schrödinger
1987
- Gauge mismatch between atomic wavef


Lamb, Scully, Schlicher: When you solve Schrödinger
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What is a "particle detector"
- Localized (in time and space) quantum system
- Coupled to a quantum field
- Ease to measure (clicking quality)

Lamb, Scully, Schlicher: When you solve Schrödinger Eq for

1987 — Gauge mismatch between atomic wavefunctions a

"What is a "particle detector""

- Localized (in time and space) quantum system
- Coupled to a quantum field
- Ease to measure (clicking quality) → Non relativistic (first quantized

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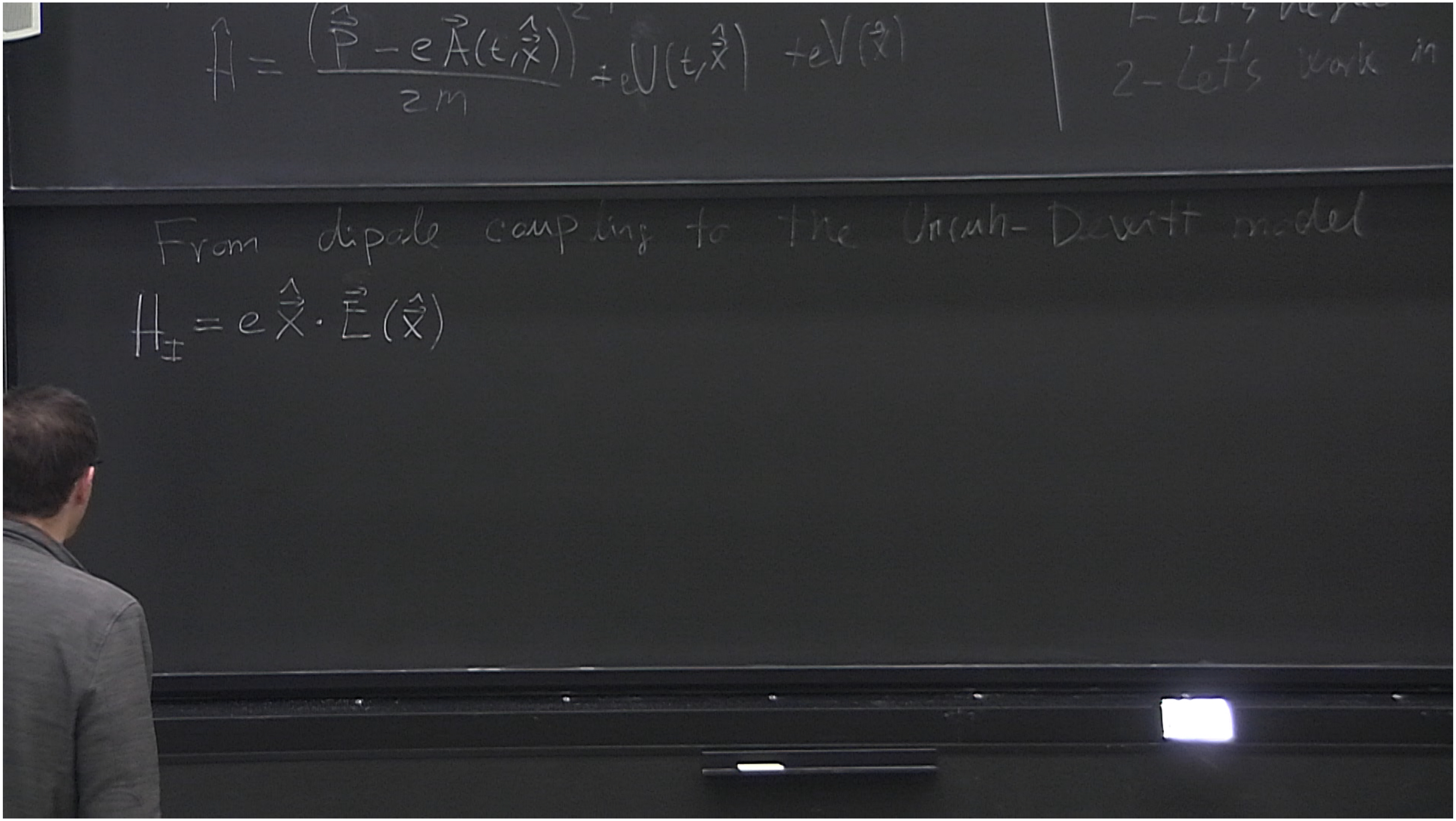
⇒ Hydrogen Atom


$\hat{H} = \frac{\left(\hat{\vec{P}} - e\vec{A}(t,\hat{\vec{x}})\right)^2}{2m} + eU(t,\hat{\vec{x}}) + eV(\hat{\vec{x}})$
2- Let's work in
From dipole coupling to the Unruh-DeWitt model
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From dipole coupling to the Unruh-DeWitt model

$$H_I = e\hat{\vec{x}}\cdot\vec{E}(\hat{\vec{x}}) = e\sum_i\sum_j \langle j|\hat{\vec{x}}\cdot\vec{E}(\hat{\vec{x}})|i\rangle e^{i\Omega_{ij}t}|j\rangle\langle i|$$

can couple to an EM f...

$$e\vec{A}(t,\hat{\vec{x}}))^2 + eU(t,\hat{\vec{x}}) + eV(\hat{\vec{x}})$$

2m

1- Let's neglect $O(e^2)$

2- Let's work in the Coulomb gauge $[\hat{\vec{A}}(\hat{x}), \hat{P}]$

...le coupling to the Unruh-Dewitt model

$$\hat{\vec{x}}) = e\sum_i\sum_j \langle j|\hat{\vec{x}}\cdot\vec{E}(\hat{\vec{x}})|i\rangle e^{i\Omega_{ij}t}|j\rangle\langle i| = \sum_i\sum_j\int d^3x\int d^3x' \langle j|x\rangle\langle x|\hat{\vec{x}}\cdot\vec{E}(\hat{\vec{x}})|x'\rangle\langle x'$$

Let's work in the Coulomb gauge

…tt model

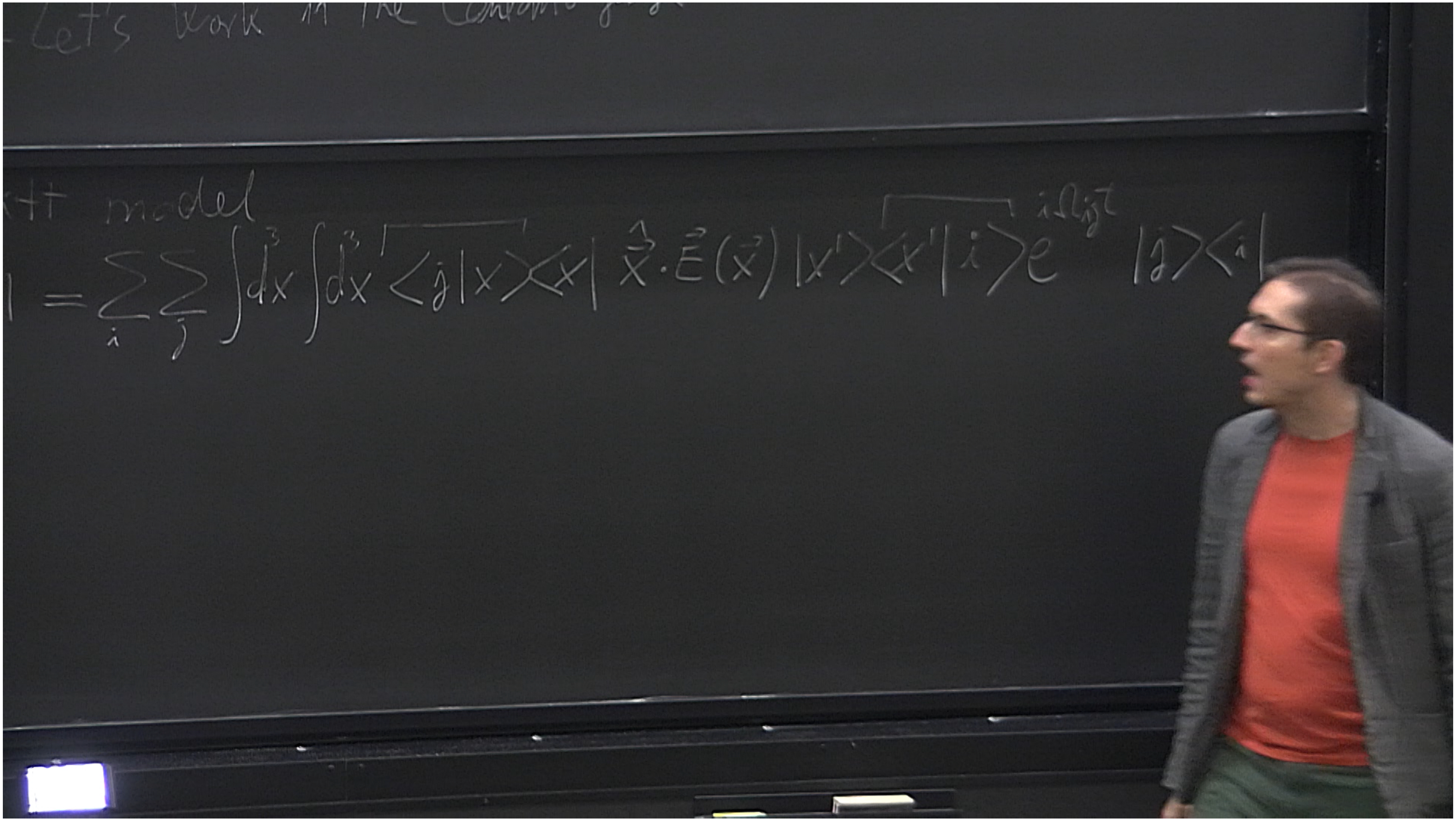
$$\ldots = \sum_i \sum_j \int d^3x \int d^3x' \langle j | x \rangle \langle x | \hat{\vec{x}} \cdot \vec{E}(\vec{x}) | x' \rangle \langle x' | i \rangle e^{i\Omega_{ij} t} | j \rangle \langle i |$$

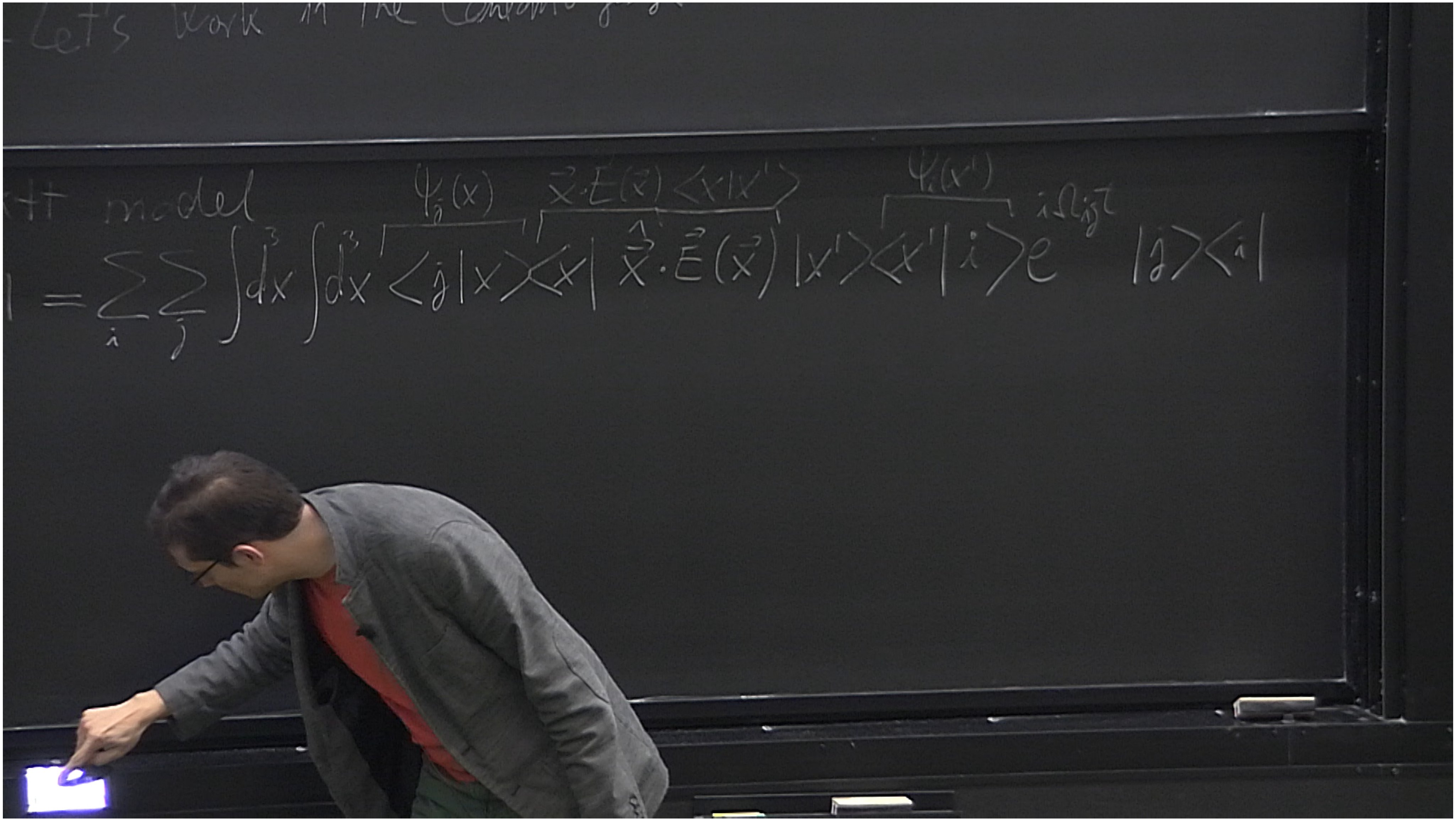
$$H = \frac{(\vec{P} - e\vec{A}(t,x))^2}{2m} + eU(t,x) + eV(x)$$

2-Let

Interaction From dipole coupling to the Unruh-Dewitt

picture $\quad {}^{D}H_I = e\hat{\vec{X}}\cdot\vec{E}(\hat{\vec{X}}) = e\sum_i\sum_j \langle j|\hat{\vec{X}}\cdot\vec{E}(\hat{\vec{X}})|i\rangle e^{i\Omega_{ij}t}\,|j\rangle\langle i| =$


Let's work in the
model
$= \sum_i \sum_j \int d^3x \int d^3x' \langle j|x\rangle\langle x| \hat{\vec{x}}\cdot\vec{E}(\vec{x}) |x'\rangle\langle x'|i\rangle e^{i\Omega_{ij}t} |j\rangle\langle i|$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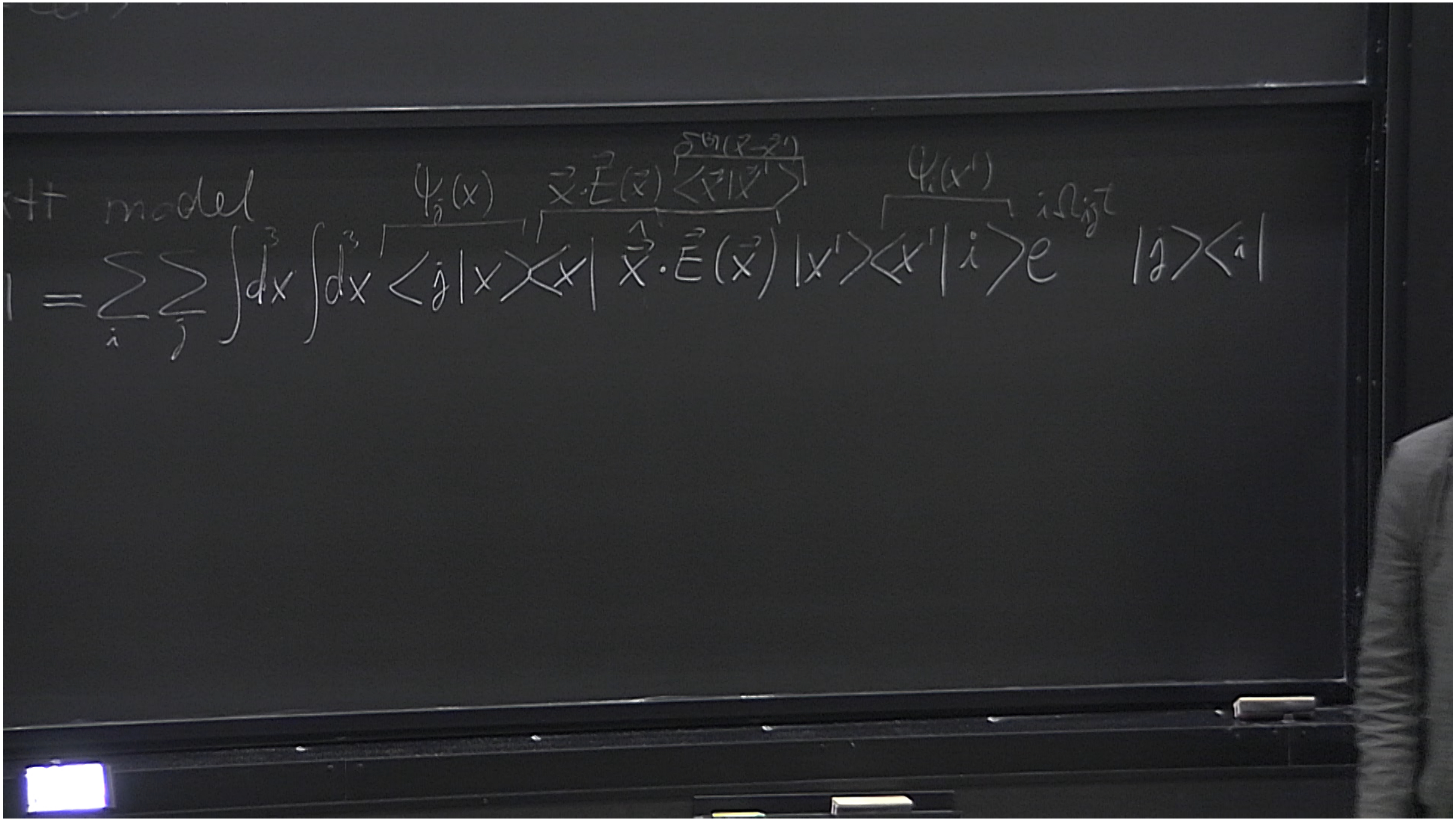
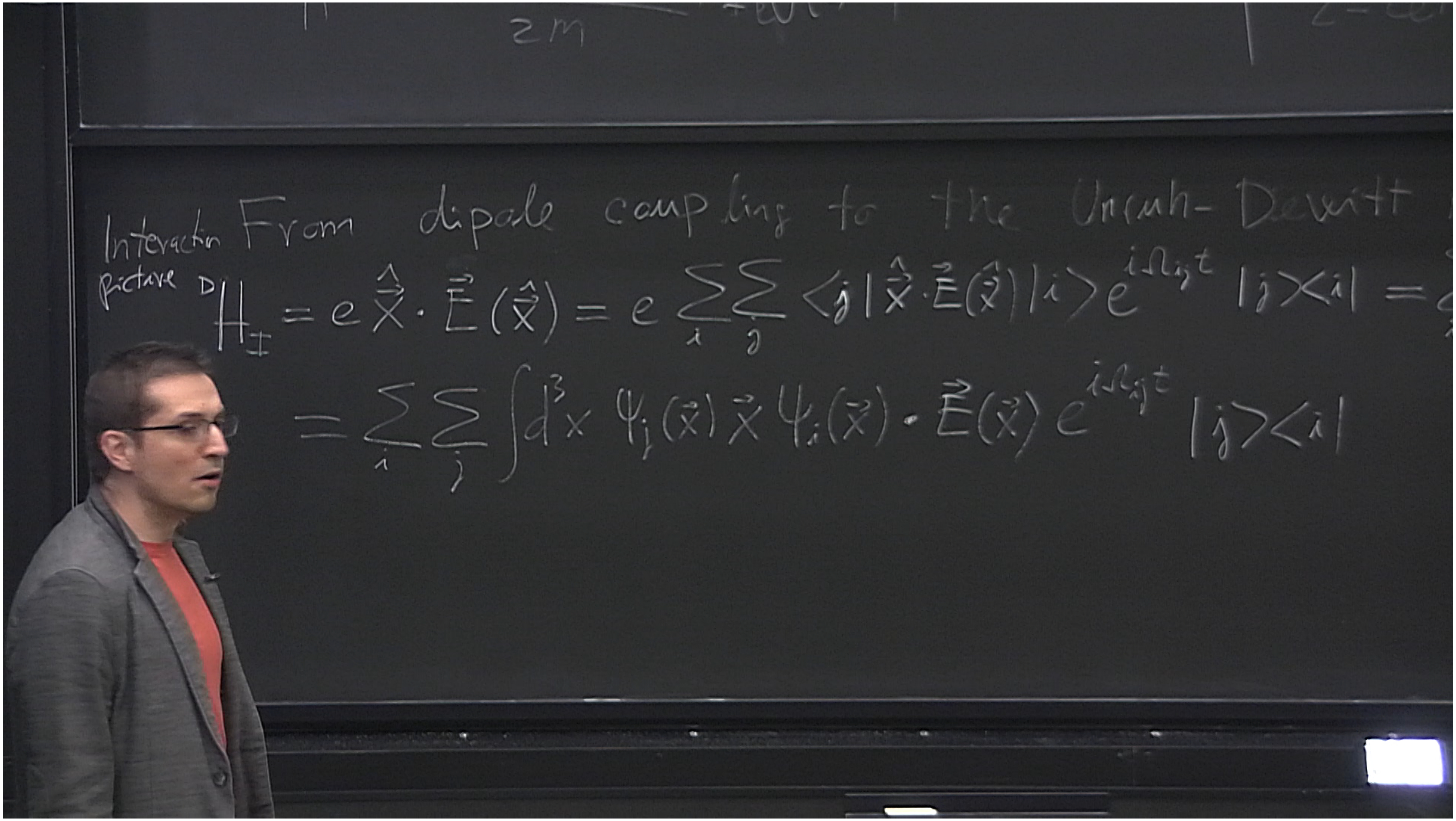


...tt model

$$\ldots = \sum_i \sum_j \int d^3x \int d^3x' \overbrace{\langle j|x\rangle}^{\psi_j(x)} \overbrace{\langle x| \hat{\vec{x}}\cdot\vec{E}(\vec{x})|x'\rangle}^{\vec{x}\cdot\vec{E}(\vec{x})\,\overbrace{\langle \vec{x}|\vec{x}'\rangle}^{\delta^{(3)}(\vec{x}-\vec{x}')}} \overbrace{\langle x'|i\rangle}^{\psi_i(x')} e^{i\Omega_{ji}t} |j\rangle\langle i|$$



Interaction picture
From dipole coupling to the Unruh-Dewitt
$H_I = e\hat{\vec{x}}\cdot\vec{E}(\hat{\vec{x}}) = e\sum_i\sum_j \langle j|\hat{\vec{x}}\cdot\vec{E}(\hat{\vec{x}})|i\rangle e^{i\Omega_{ij}t}|j\rangle\langle i| =$
$= \sum_i\sum_j\int d^3x\, \psi_j(\vec{x})\,\vec{x}\,\psi_i(\vec{x})\cdot\vec{E}(\vec{x})\, e^{i\Omega_{ij}t}|j\rangle\langle i|$

Interaction picture From dipole coupling to the Unruh-Dewitt

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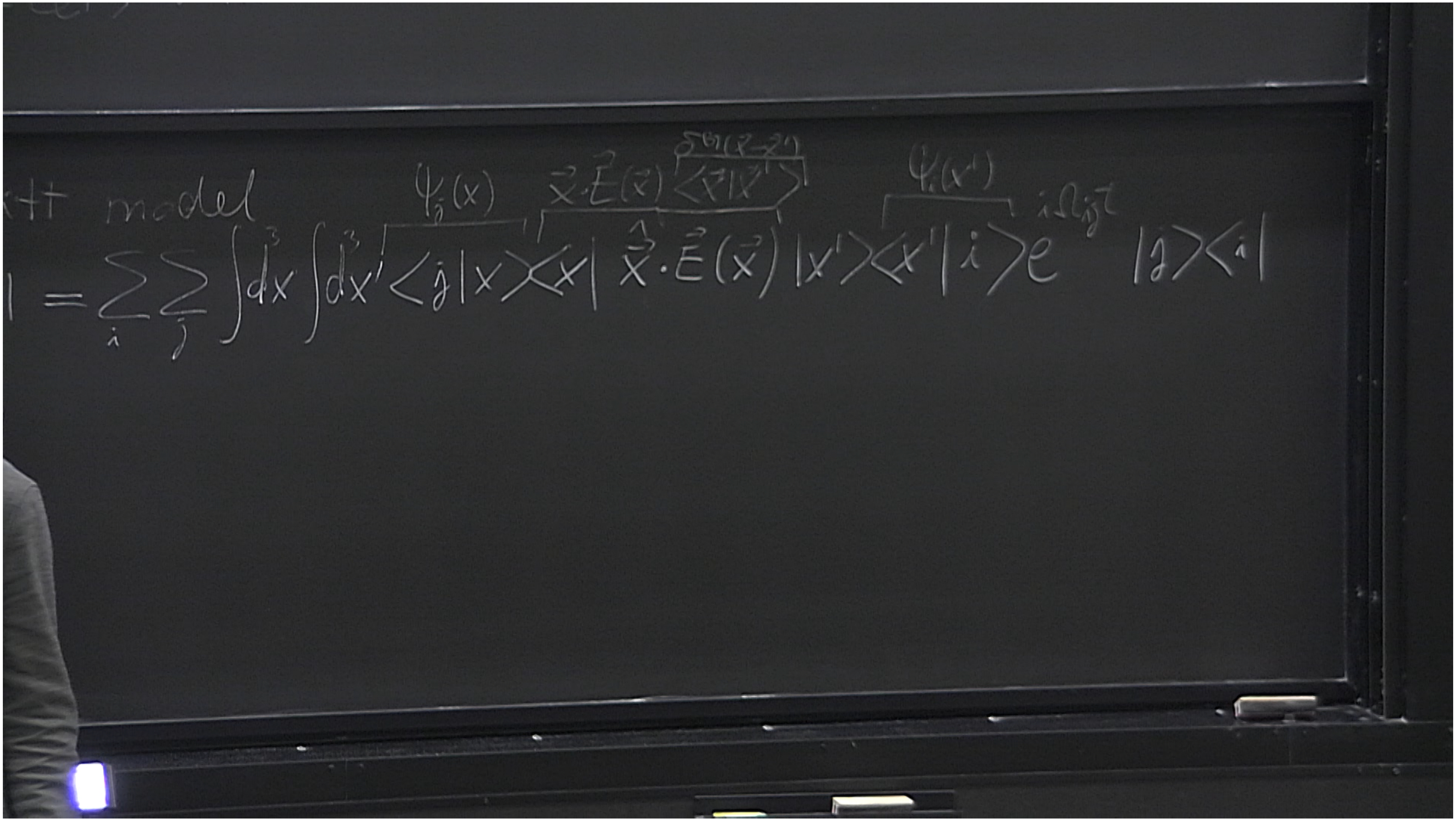
$$\vec{E}(\vec{x}) \downarrow \propto e^{i\vec{k}\cdot\vec{x}}$$


model
$$= \sum_i \sum_j \int d^3x \int d^3x' \overbrace{\langle j|x\rangle}^{\psi_j^*(x)} \overbrace{\langle x| \hat{\vec{x}}\cdot\vec{E}(\vec{x}) |x'\rangle}^{\vec{x}\cdot\vec{E}(\vec{x})\overbrace{\langle \vec{x}|\vec{x}'\rangle}^{\delta^{(3)}(\vec{x}-\vec{x}')}} \overbrace{\langle x'|i\rangle}^{\psi_i(x')} e^{i\Omega_{ji}t} |j\rangle\langle i|$$

...t model

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many of these matrix elements are zero

$i \not\to i$ selection rules

...tt model

$$| = \sum_i \sum_j \int d^3x \int d^3x' \, \overbrace{\langle j|x\rangle}^{\psi_j^*(x)} \overbrace{\langle x| \hat{\vec{x}}\cdot\vec{E}(\hat{\vec{x}}) |x'\rangle}^{\vec{x}\cdot\vec{E}(\vec{x}) \overbrace{\langle \vec{x}|\vec{x}'\rangle}^{\delta^{(3)}(\vec{x}-\vec{x}')}} \overbrace{\langle x'|i\rangle}^{\psi_i(x')} e^{i\Omega_{ij}t} \, |j\rangle\langle i|$$

many of these matrix elements are zero

$i \not\to i$

Selection rules $\Delta l = 0, \pm 1$

$l=0 \not\to 0$

$2m$

Interaction picture $\Rightarrow$ From dipole coupling to the Unruh-Dewitt

$$H_I = e\,\hat{\vec{x}}\cdot\vec{E}(\hat{\vec{x}}) = e\sum_i\sum_j \langle j|\hat{\vec{x}}\cdot\vec{E}(\hat{\vec{x}})|i\rangle\, e^{i\Omega_{ij}t}\,|j\rangle\langle i| =$$

$$= \underbrace{\sum_i\sum_j}_{\sum_{i\neq j}} \int d^3x\, \psi_j(\vec{x})\,\vec{x}\,\psi_i(\vec{x})\cdot \underbrace{\vec{E}(\vec{x})}_{\propto e^{i\vec{k}\cdot\vec{x}}}\, e^{i\Omega_{ij}t}\,|j\rangle\langle i|$$

2m

Interaction picture From dipole coupling to the Unruh-Dewitt

$$H_I = e\hat{\vec{X}}\cdot\vec{E}(\hat{\vec{X}}) = e\sum_i\sum_j \langle j|\hat{\vec{x}}\cdot\vec{E}(\hat{\vec{x}})|i\rangle e^{i\Omega_{ij}t}\,|j\rangle\langle i| =$$

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...tt model

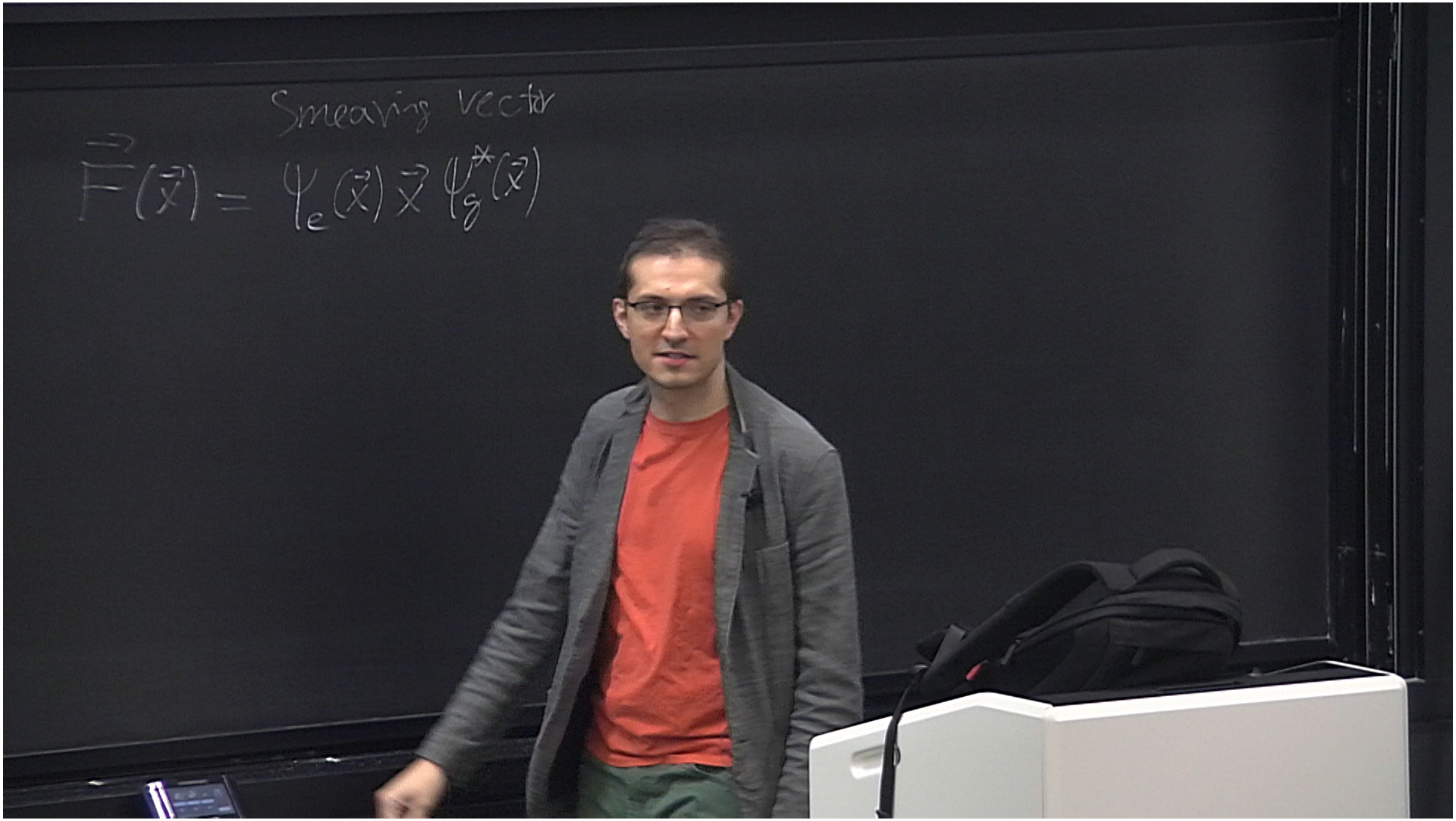
$$| = e\sum_i \sum_j \int d^3x \int d^3x' \overbrace{\langle j|x\rangle}^{\psi_j^*(x)} \overbrace{\langle x|\,\hat{\vec{x}}\cdot\vec{E}(\vec{x})\,|x'\rangle}^{\vec{x}\cdot\vec{E}(\vec{x})\,\overbrace{\langle\vec{x}|\vec{x}'\rangle}^{\delta^{(3)}(\vec{x}-\vec{x}')}} \overbrace{\langle x'|i\rangle}^{\psi_i(x')} e^{i\Omega_{ji}t}\, |j\rangle\langle i|$$

many of these matrix elements are zero

$i \not\to i$ Selection rules $\Delta l = 0, \pm 1$

$l=0 \not\to 0$

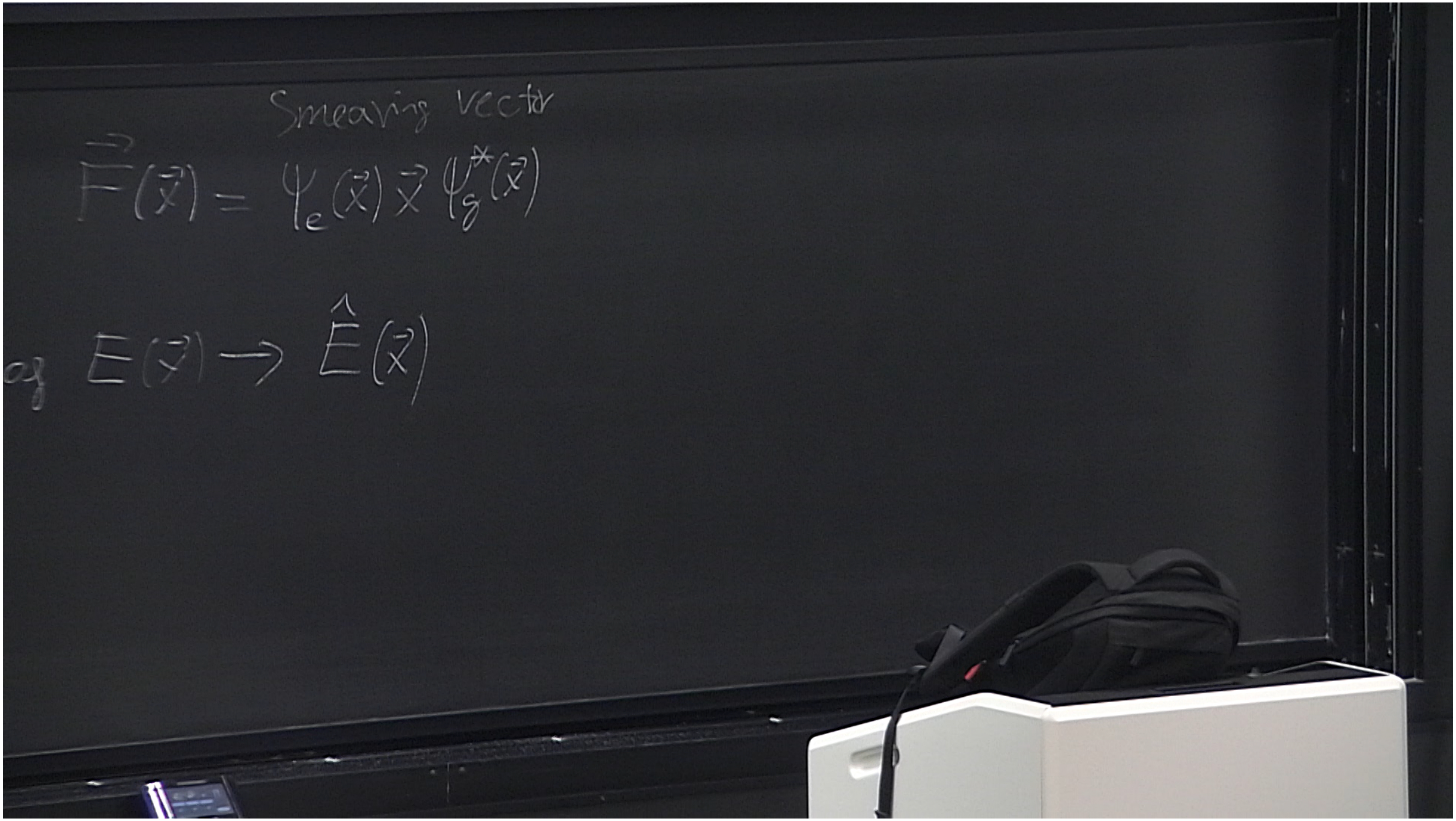
$$H_I = e\int dx \left(\vec{F}(\vec{x})\, e^{i\Omega t} \underset{|e\rangle\langle g|}{\underset{\shortparallel}{\sigma^+}} + \vec{F}^*(\vec{x})\, e^{-i\Omega t} \underset{|g\rangle\langle e|}{\underset{\shortparallel}{\sigma^-}} \right) \cdot \vec{E}(\vec{x})$$

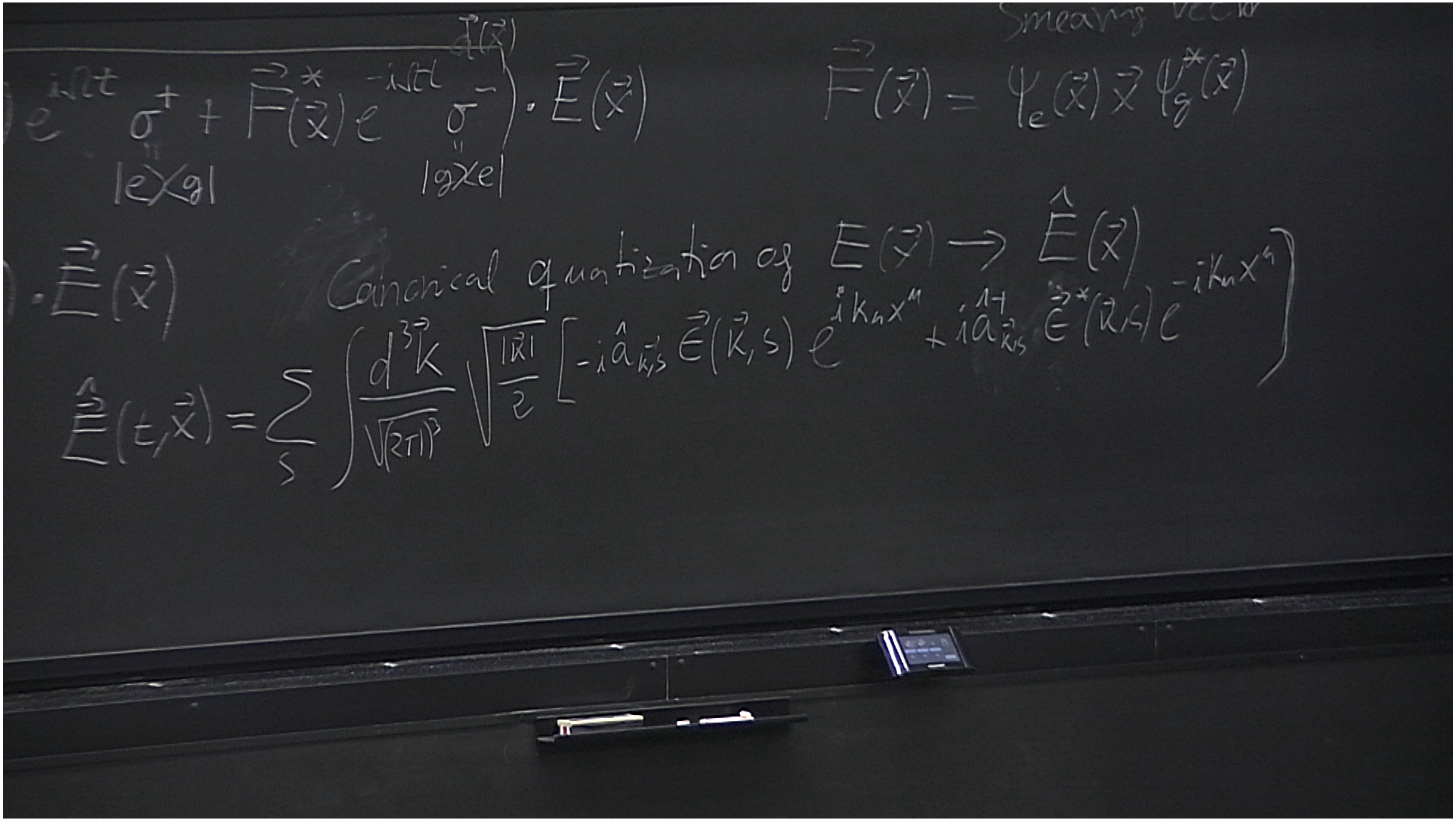

Smearing vector
$\vec{F}(\vec{x}) = \psi_e(\vec{x})\,\vec{x}\,\psi_g^*(\vec{x})$

$$H_I = e\int dx \, \overbrace{\left(\vec{F}(\vec{x})\, e^{i\Omega t}\, \underset{|e\rangle\langle g|}{\sigma^+} + \vec{F}^*(\vec{x})\, e^{-i\Omega t}\, \underset{|g\rangle\langle e|}{\sigma^-}\right)}^{\hat{\vec{d}}(\vec{x})} \cdot \vec{E}(\vec{x}) \qquad \vec{F}(\vec{x})$$

$$H_I = e\int dx \; \hat{\vec{d}}(\vec{x}) \cdot \vec{E}(\vec{x})$$

Canonical quantization of $E(\vec{x})$


Smearing vector
$\vec{F}(\vec{x}) = \psi_e(\vec{x})\,\vec{x}\,\psi_g^*(\vec{x})$
$E(\vec{x}) \rightarrow \hat{E}(\vec{x})$


Smearing vector
$e^{i\Omega t}\sigma^+ + \vec F^*(\vec x)\, e^{-i\Omega t}\sigma^-)\cdot \vec E(\vec x)$
$|e\rangle\langle g|$
$|g\rangle\langle e|$
$\vec F(\vec x) = \psi_e(\vec x)\,\vec x\,\psi_g^*(\vec x)$
$\cdot \vec E(\vec x)$
Canonical quantization of $E(\vec x) \to \hat E(\vec x)$
$\hat{\vec E}(t,\vec x) = \sum_s \int \frac{d^3\vec k}{\sqrt{(2\pi)^3}} \sqrt{\frac{|\vec k|}{2}} \left[-i\hat a_{\vec k,s}\,\vec\epsilon(\vec k,s)\, e^{ik_\mu x^\mu} + i\hat a^\dagger_{\vec k,s}\,\vec\epsilon^*(\vec k,s)\, e^{-ik_\mu x^\mu} \right]$

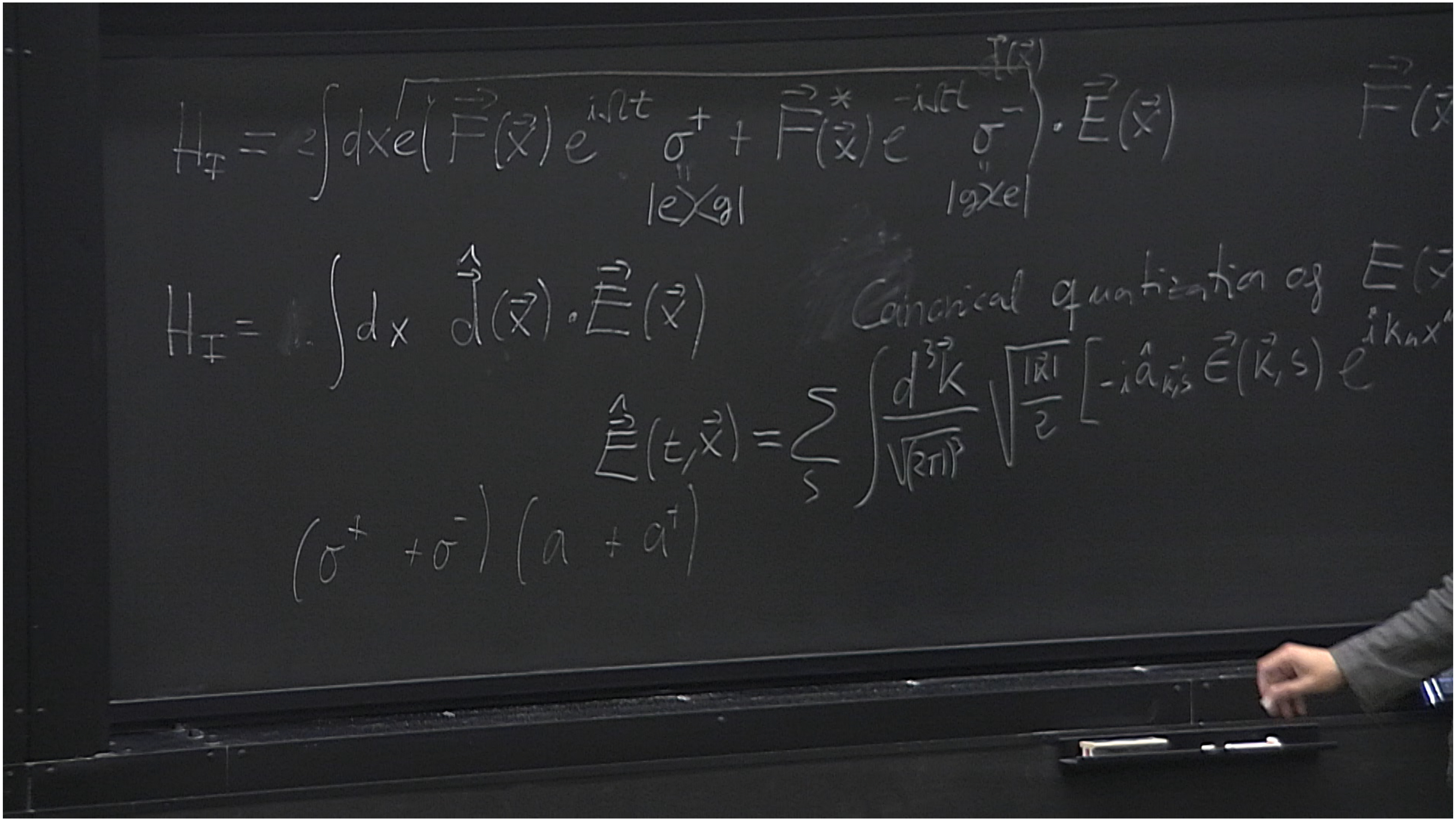
$$H_I = \int dx\, e\,\overbrace{\left(\vec{F}(\vec{x})\, e^{i\Omega t}\, \underset{|e\rangle\langle g|}{\sigma^+} + \vec{F}^*(\vec{x})\, e^{-i\Omega t}\, \underset{|g\rangle\langle e|}{\sigma^-}\right)}^{\vec{d}(\vec{x})} \cdot \vec{E}(\vec{x})$$

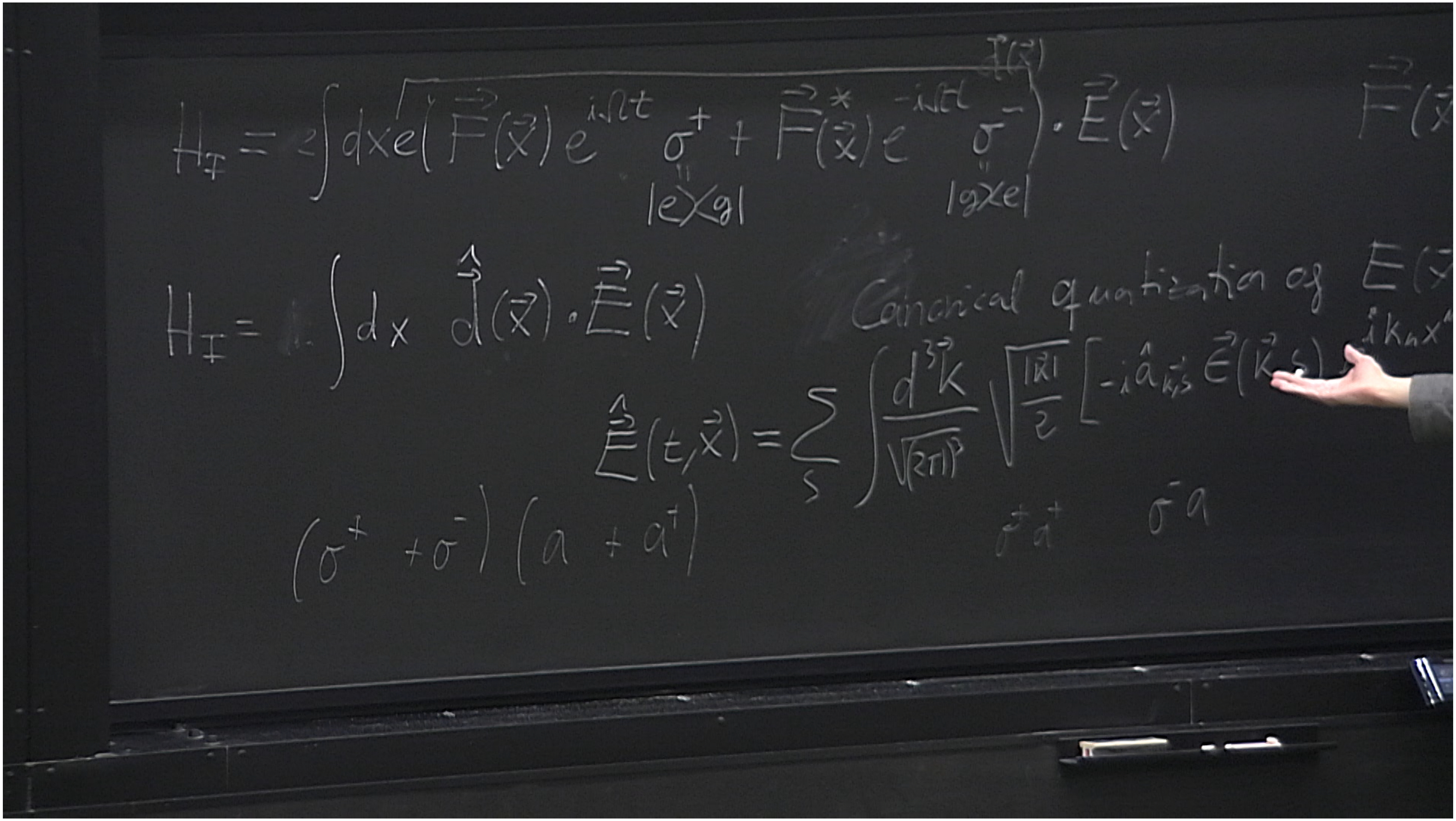
$\vec{F}(\vec{x}$

$$H_I = \int dx\; \hat{\vec{d}}(\vec{x}) \cdot \vec{E}(\vec{x})$$

Canonical quantization of $E(\vec{x}$

$$\hat{\vec{E}}(t,\vec{x}) = \sum_s \int \frac{d^3\vec{k}}{\sqrt{(2\pi)^3}} \sqrt{\frac{|\vec{k}|}{2}} \left[-i\, \hat{a}_{\vec{k},s}\, \vec{\epsilon}(\vec{k},s)\, e^{i k_\mu x^\mu} \right.$$


$H_I = \int dx\, e\left(\vec{F}(\vec{x})\, e^{i\Omega t}\, \sigma^+ + \vec{F}^*(\vec{x})\, e^{-i\Omega t}\, \sigma^-\right) \cdot \vec{E}(\vec{x})$
$|e\rangle\langle g|$
$|g\rangle\langle e|$
$\vec{I}(\vec{x})$
$\vec{F}(\vec{x}$
$H_I = \int dx\; \hat{\vec{d}}(\vec{x}) \cdot \vec{E}(\vec{x})$
Canonical quantization of $E(\vec{x}$
$\hat{\vec{E}}(t, \vec{x}) = \sum_s \int \frac{d^3\vec{k}}{\sqrt{(2\pi)^3}} \sqrt{\frac{|\vec{k}|}{2}} \left[-i\, \hat{a}_{\vec{k},s}\, \vec{\epsilon}(\vec{k}, s)\, e^{i k_\mu x^\mu} \right.$
$(\sigma^+ + \sigma^-)(a + a^\dagger)$


$H_I = \int dx\, e\left(\vec{F}(\vec{x}) e^{i\Omega t} \sigma^+ + \vec{F}^*(\vec{x}) e^{-i\Omega t} \sigma^-\right) \cdot \vec{E}(\vec{x})$
$|e\rangle\langle g|$
$|g\rangle\langle e|$
$H_I = \int dx\, \hat{\vec{d}}(\vec{x}) \cdot \vec{E}(\vec{x})$
Canonical quantization of E(
$\hat{\vec{E}}(t,\vec{x}) = \sum_s \int \frac{d^3k}{\sqrt{(2\pi)^3}} \sqrt{\frac{|\vec{k}|}{2}} \left[-i \hat{a}_{\vec{k},s} \vec{\epsilon}(\vec{k},s) \right.$
$(\sigma^+ + \sigma^-)(a + a^\dagger)$
$\sigma^+ a^\dagger$
$\sigma^- a$