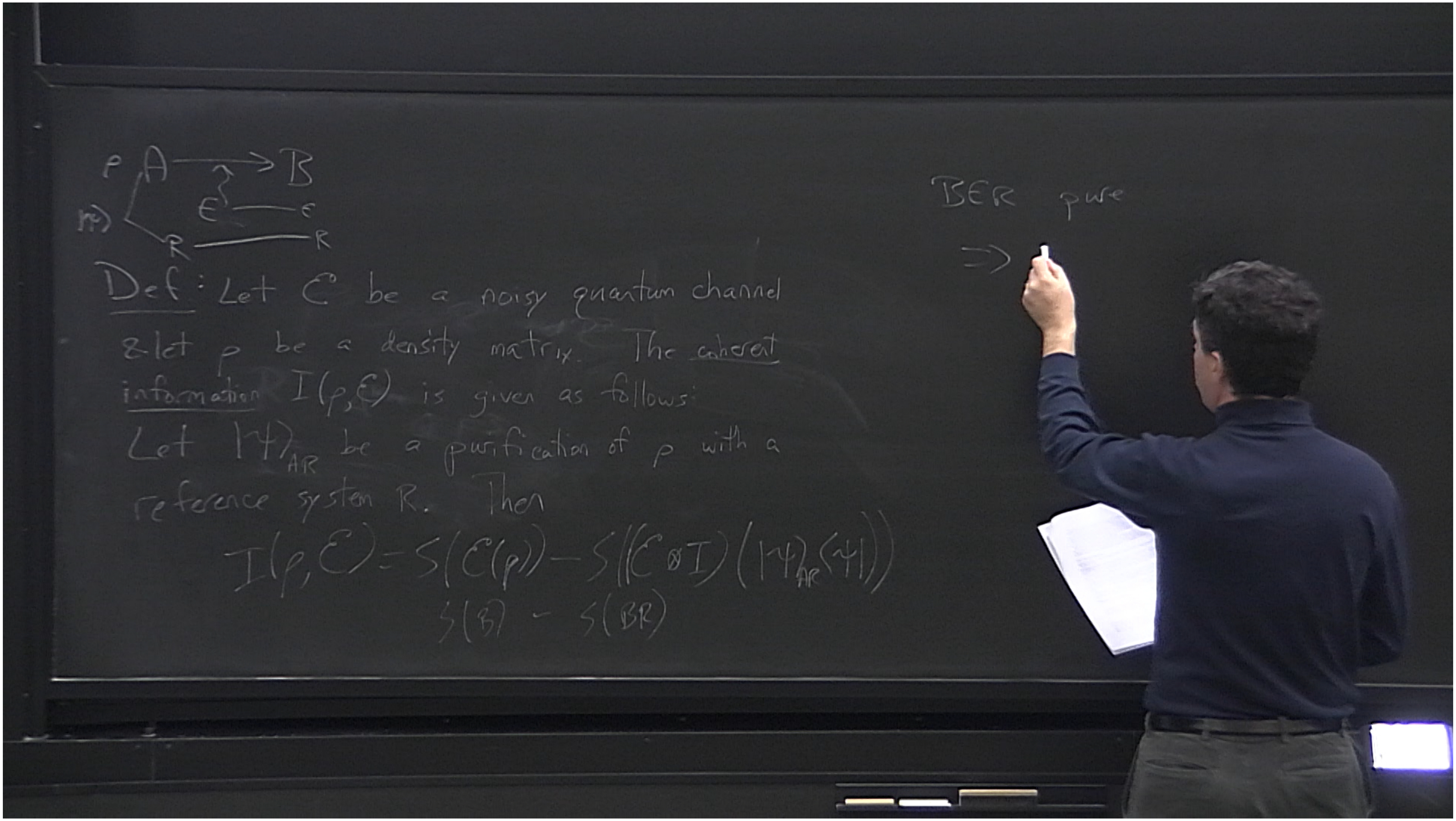
Title: PSI 2017/2018 - Quantum Information - Lecture 14

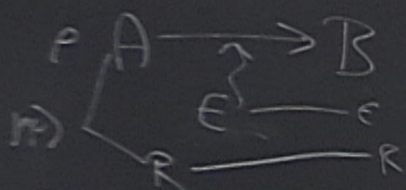
Date: Mar 09, 2018 11:30 AM

URL: http://pirsa.org/18030027

Abstract:



ρ $A \longrightarrow B$
$\mathcal{E}$ — E
$|\psi\rangle$ R — R
Def: Let $\mathcal{E}$ be a noisy quantum channel
& let ρ be a density matrix. The coherent
information $I(\rho,\mathcal{E})$ is given as follows:
Let $|\psi\rangle_{AR}$ be a purification of ρ with a
reference system R. Then
$I(\rho,\mathcal{E}) = S(\mathcal{E}(\rho)) - S((\mathcal{E}\otimes I)(|\psi\rangle_{AR}\langle\psi|))$
$S(B) - S(BR)$
BER pure
$\Rightarrow$



Def: Let $\mathcal{E}$ be a noisy quantum channel & let ρ be a density matrix. The coherent information $I(\rho,\mathcal{E})$ is given as follows: Let $|\psi\rangle_{AR}$ be a purification of ρ with a reference system R. Then

$$I(\rho,\mathcal{E}) = S(\mathcal{E}(\rho)) - S\big((\mathcal{E}\otimes I)(|\psi\rangle_{AR}\langle\psi|)\big)$$

$$S(B) - S(BR) = S(B) - S(E)$$

BER pure

$\Rightarrow S(BR) = S(E)$

Thm (LSD):

BER pure

$$\Rightarrow S(BR) = S(E)$$

Thm (LSD): Quantum channel capacity is

$$\lim_{n\to\infty} \sup_{\rho} \frac{1}{n} I(\rho, \mathcal{E}^{\otimes n})$$

state on $\mathcal{H}^{\otimes n}$ (Alice's Hilbert space of n messages)

Classical capacity $\max_X I(X:Y)$

BER pure $\quad I(X:Y) = H(X) + H(Y) - H(X,Y)$

$\Rightarrow S(BR) = S(E)$

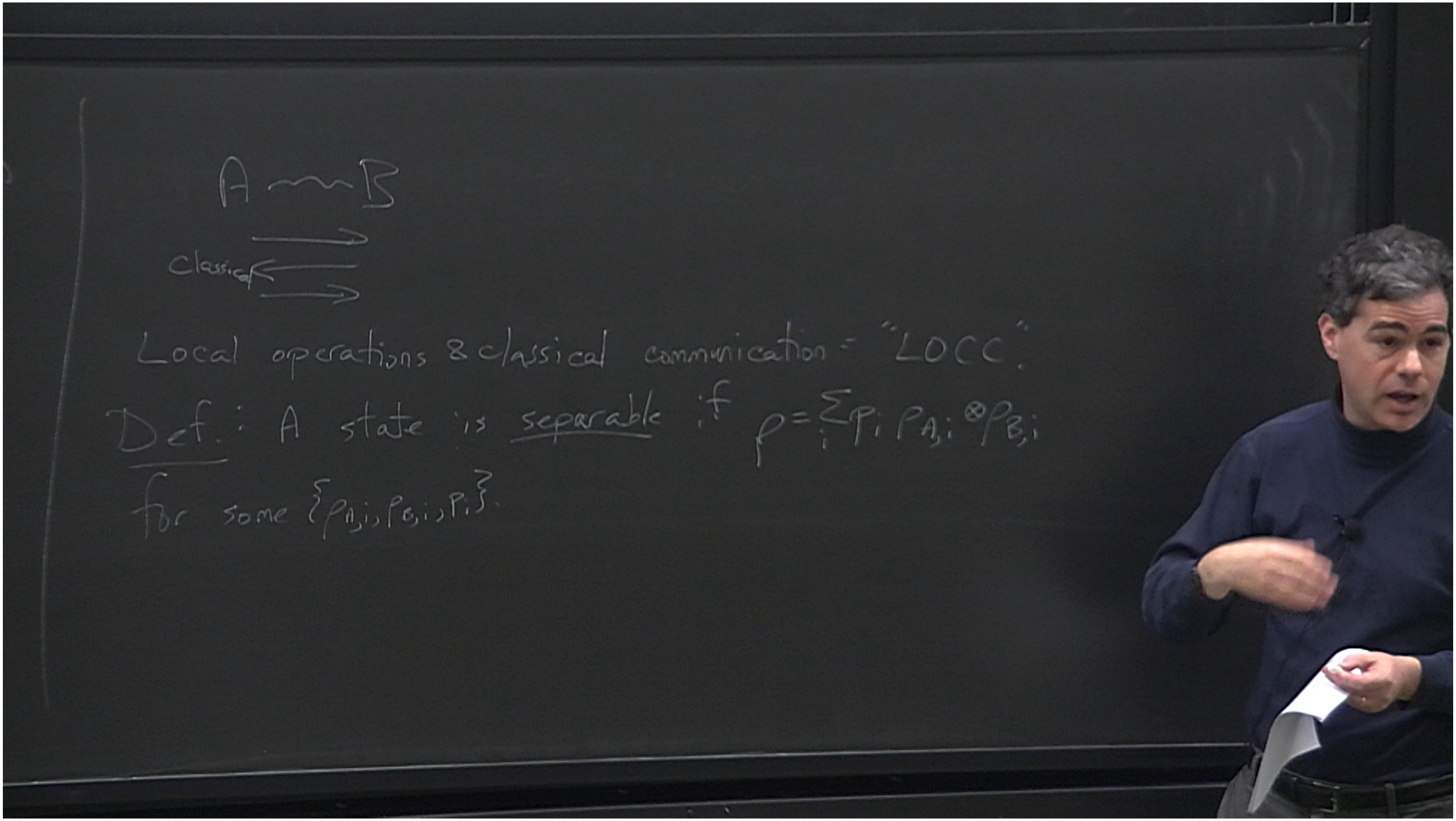
Thm (LSD): Quantum channel capacity is

$$\lim_{n\to\infty} \sup_{\rho} \frac{1}{n} I(\rho, \mathcal{E}^{\otimes n})$$

state on $\mathcal{H}^{\otimes n}$ (Alice's Hilbert space of n messages)

$(\psi|))$

$-S(E)$

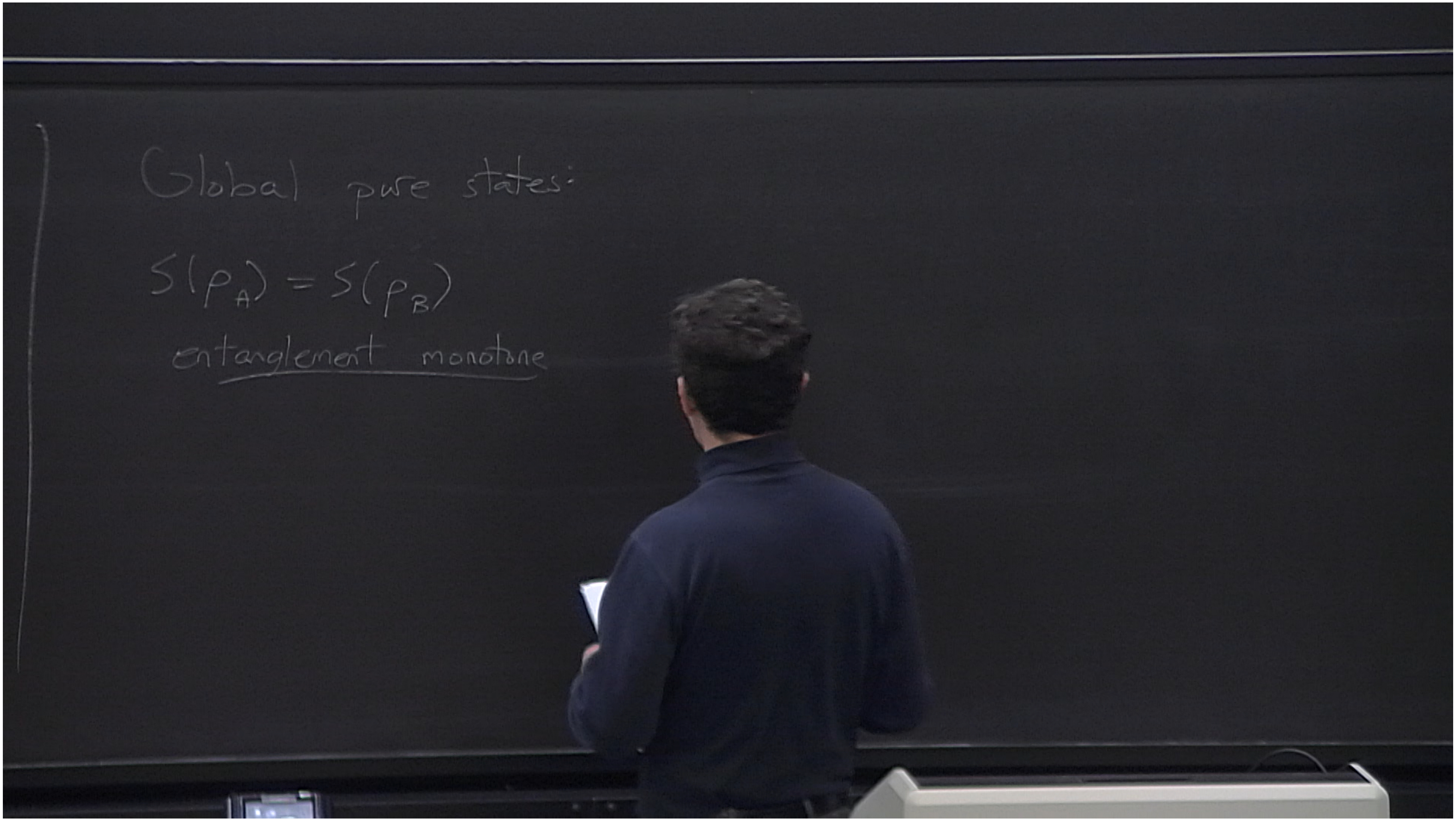

A B
Classical
Local operations & classical communication = "LOCC"
Def.: A state is separable if $\rho = \sum_i p_i \rho_{A,i} \otimes \rho_{B,i}$
for some $\{\rho_{A,i}, \rho_{B,i}, p_i\}$.

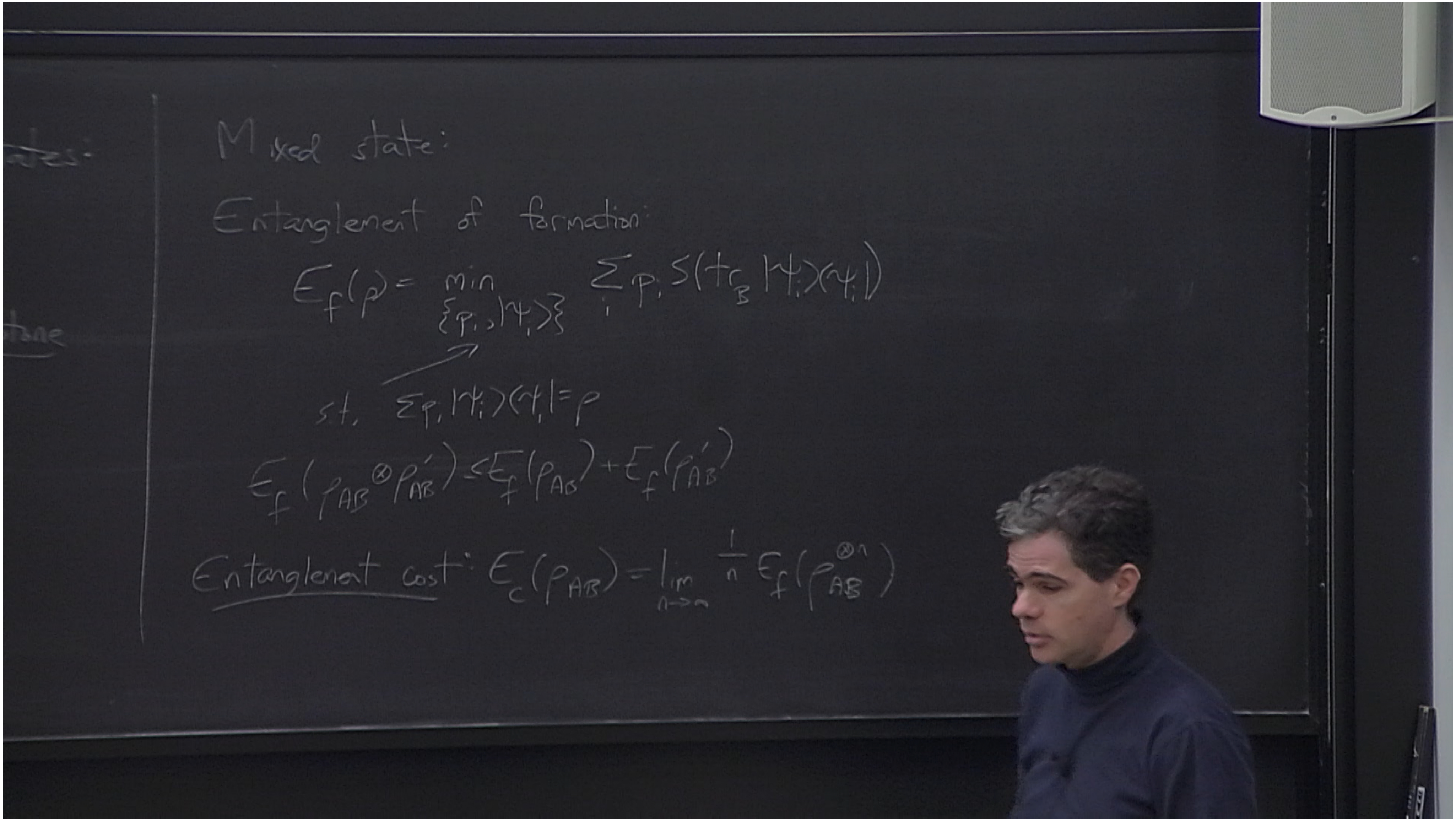
Def.: An entanglement monotone is a function $f(\rho)$ from quantum states to non-negative real number s.t.

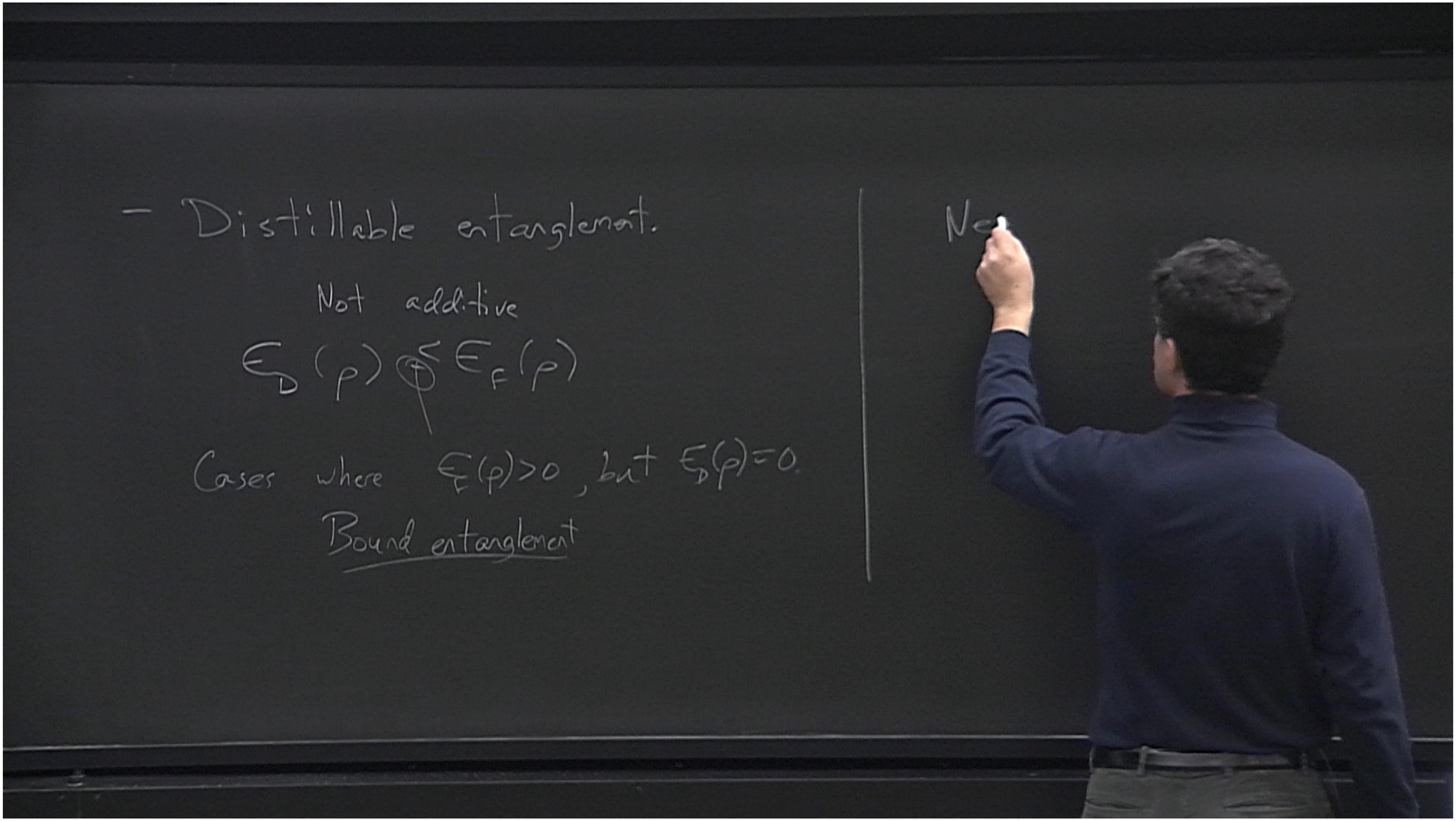
1. $f(\rho) = 0$ for separable states, $f(\rho) \neq 0$ for at least some non-separable states.

2. $f\left((U_A \otimes U_B)\rho(U_A^\dagger \otimes U_B^\dagger)\right) = f(\rho)$

3. $f(\rho)$ is non-increasing on average under LOCC.


Global pure states:
$S(\rho_A) = S(\rho_B)$
entanglement monotone


Mixed state:
Entanglement of formation:
$E_f(\rho) = \min_{\{p_i, |\psi_i\rangle\}} \sum_i p_i S(\mathrm{tr}_B |\psi_i\rangle\langle\psi_i|)$
s.t. $\sum p_i |\psi_i\rangle\langle\psi_i| = \rho$
$E_f(\rho_{AB} \otimes \rho'_{AB}) \leq E_f(\rho_{AB}) + E_f(\rho'_{AB})$
Entanglement cost: $E_c(\rho_{AB}) = \lim_{n \to \infty} \frac{1}{n} E_f(\rho_{AB}^{\otimes n})$



- Distillable entanglement.
Not additive
$E_D(\rho) \leq E_F(\rho)$
Cases where $E_F(\rho)>0$, but $E_D(\rho)=0$.
Bound entanglement

Negativity:

ρ^{T_A} partial transpose

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ρ^{T_A} partial transpose

positive, but not completely positive

Negativity = sum − eigenvalues under partial transpose

Additive

Negativity:

ρ^{T_A} partial transpose

positive, but not completely positive

Negativity = sum (absolute value of) − eigenvalues under partial transpose.

Additive