Title: PSI 2017/2018 - Quantum Information - Lecture 11

Date: Mar 06, 2018 11:30 AM

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Abstract:

Given subspace T, let

$S(T) = \{M \in \mathcal{P}_n \mid M|\psi\rangle = |\psi\rangle \ \forall |\psi\rangle \in T\}$

a) - If S

b) $S(T)$ is an Abelian group

Given $S \leq \mathcal{P}_n$ satisfying a & b,

let $T(S) = \{|\psi\rangle \mid M|\psi\rangle = |\psi\rangle \ \forall M \in S\}$

If S has r generators, then

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Thm: If S has r generators, then $T(S)$ has dimension 2^{n-r}, i.e. $k = n - r$ logical qubits.

Error E,

Error ϵ, $M \in S$ s.t. $\{M, \epsilon\} = 0$.

$$M(\epsilon|\psi\rangle) = -\epsilon M|\psi\rangle = -(\epsilon|\psi\rangle)$$

$[M, \epsilon] = 0$ $(M \in S)$:

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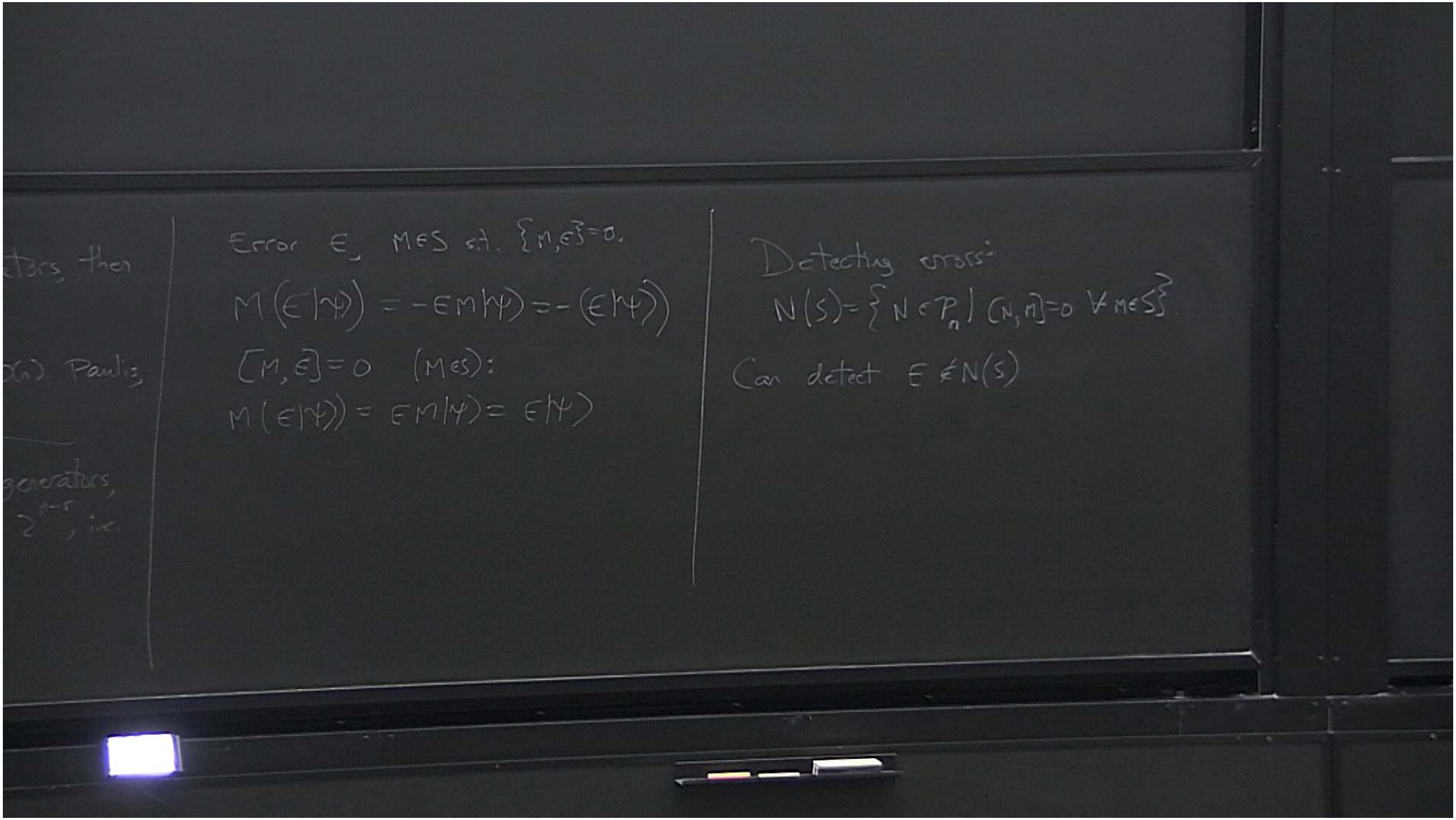

tors, then
(n) Pauli
generators,
Error E, $M \in S$ s.t. $\{M, E\} = 0$.
$M(E|\psi\rangle) = -EM|\psi\rangle = -(E|\psi\rangle)$
$[M, E] = 0$ $(M \in S)$:
$M(E|\psi\rangle) = EM|\psi\rangle = E|\psi\rangle$
Detecting errors:
$N(S) = \{N \in \mathcal{P}_n \mid [N, M] = 0 \ \forall M \in S\}$
Can detect $E \notin N(S)$

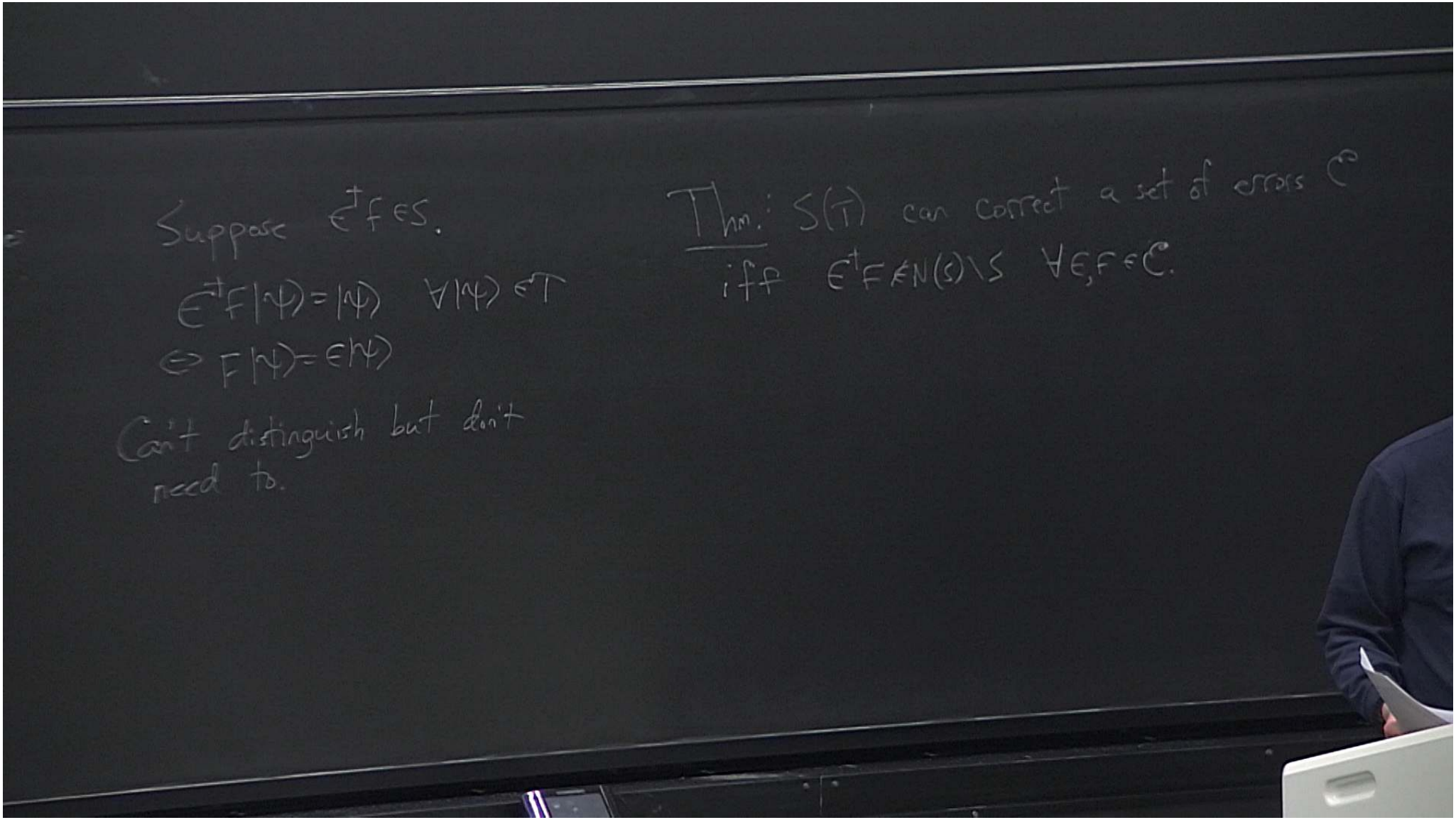
Sufficient if E, F have different error syndromes

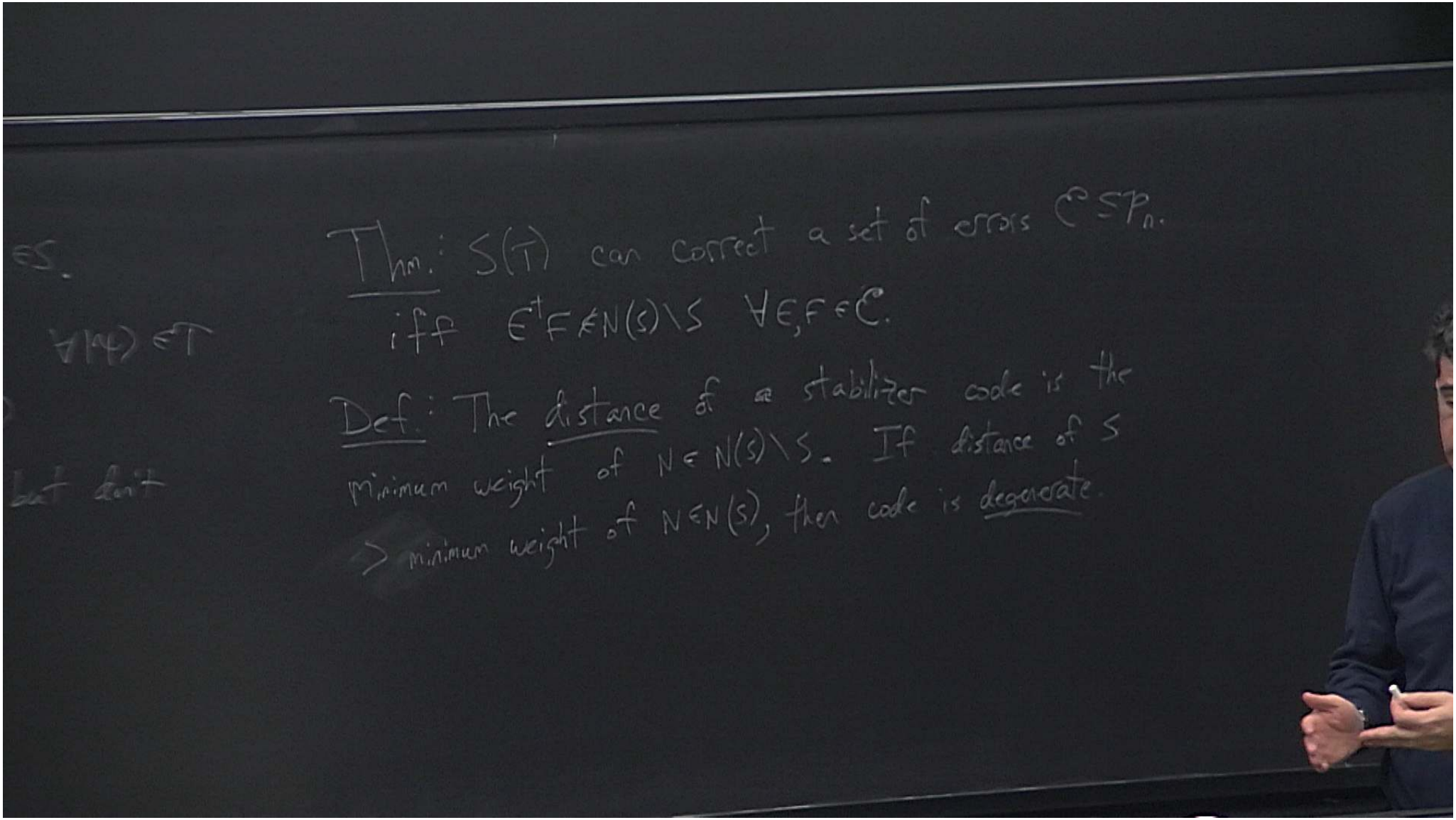
$(\forall E, F \in \mathcal{E})$

$\Leftrightarrow$ E & F have different sets of $M \in S$ with which they commute.

$\Leftrightarrow \{E^\dagger F, M\} = 0$ for at least one $M \in S$.

$\Leftrightarrow E^\dagger F \notin N(S)$.


Suppose $E_i^\dagger f \in S$.
$E_i^\dagger f |\psi\rangle = |\psi\rangle \quad \forall |\psi\rangle \in T$
$\Leftrightarrow F|\psi\rangle = E|\psi\rangle$
Can't distinguish but don't need to.
Thm: $S(T)$ can correct a set of errors $\mathcal{E}$
iff $E_i^\dagger F \notin N(S) \setminus S \quad \forall E, F \in \mathcal{E}$.


∈S.
∀|ψ⟩ ∈T
but don't
Thm: S(T) can correct a set of errors $\mathcal{E} \subseteq \mathcal{P}_n$.
iff $E^\dagger F \notin N(S) \setminus S$ $\forall E, F \in \mathcal{E}$.
Def: The distance of a stabilizer code is the
minimum weight of $N \in N(S) \setminus S$. If distance of S
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Thm: $S(T)$ can correct a set of errors $\mathcal{E} \subseteq P_n$.
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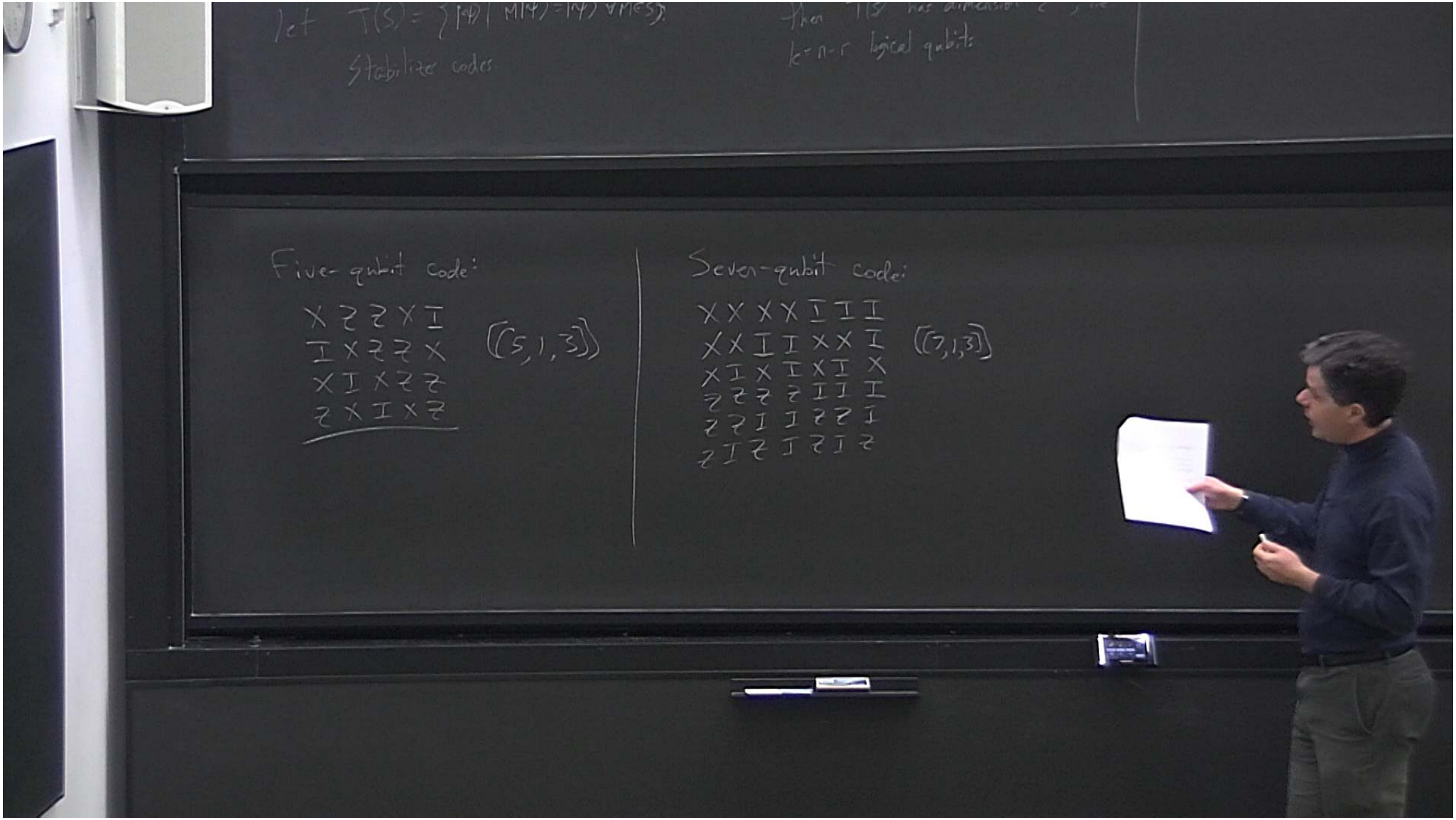
Notation: $[[n, k, d]]$ for code with n physical qubits, k logical qubits, & distance d.

To correct t-qubit errors; need $d \geq 2t+1$.


Stabilizer codes
Five-qubit code:
([5,1,3])
Seven-qubit code:
([7,1,3])

1,3]]

Fault Tolerance:

Threshold theorem:

∃ threshold error rate p_T s.t. if error p per gate or time step is below p_T, then ∃ fault-tolerant protocols for arbitrarily long quantum computations. The overhead is polylog (T), where T is the length of the logical computation