Title: PSI 2017/2018 - Quantum Information - Lecture 11

Date: Mar 06, 2018 11:30 AM

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Abstract:

Given subspace T, let

$S(T) = \{M \in \mathcal{P}_n \mid M|\psi\rangle = |\psi\rangle \ \forall |\psi\rangle \in T\}$

a) - If S

b) $S(T)$ is an Abelian group

Given $S \leq \mathcal{P}_n$ satisfying a & b,

let $T(S) = \{|\psi\rangle \mid M|\psi\rangle = |\psi\rangle \ \forall M \in S\}$

If S has r generators, then

$|S| =$

Given subspace T, let

$$S(T) = \{M \in \mathcal{P}_n \mid M|\psi\rangle = |\psi\rangle \ \forall |\psi\rangle \in T\}$$

a) $-I \notin S$

b) $S(T)$ is an Abelian group

Given $S \subseteq \mathcal{P}_n$ satisfying a & b,

let $T(S) = \{|\psi\rangle \mid M|\psi\rangle = |\psi\rangle \ \forall M \in S\}$

If S has r generators, then

$$|S| = 2^r.$$

To describe S, need $O(n)$ Paulis, each need $O(n)$ bits

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Thm: If S has r generators, then $T(S)$ has dimension 2^{n-r}, i.e. $k = n - r$ logical qubits.

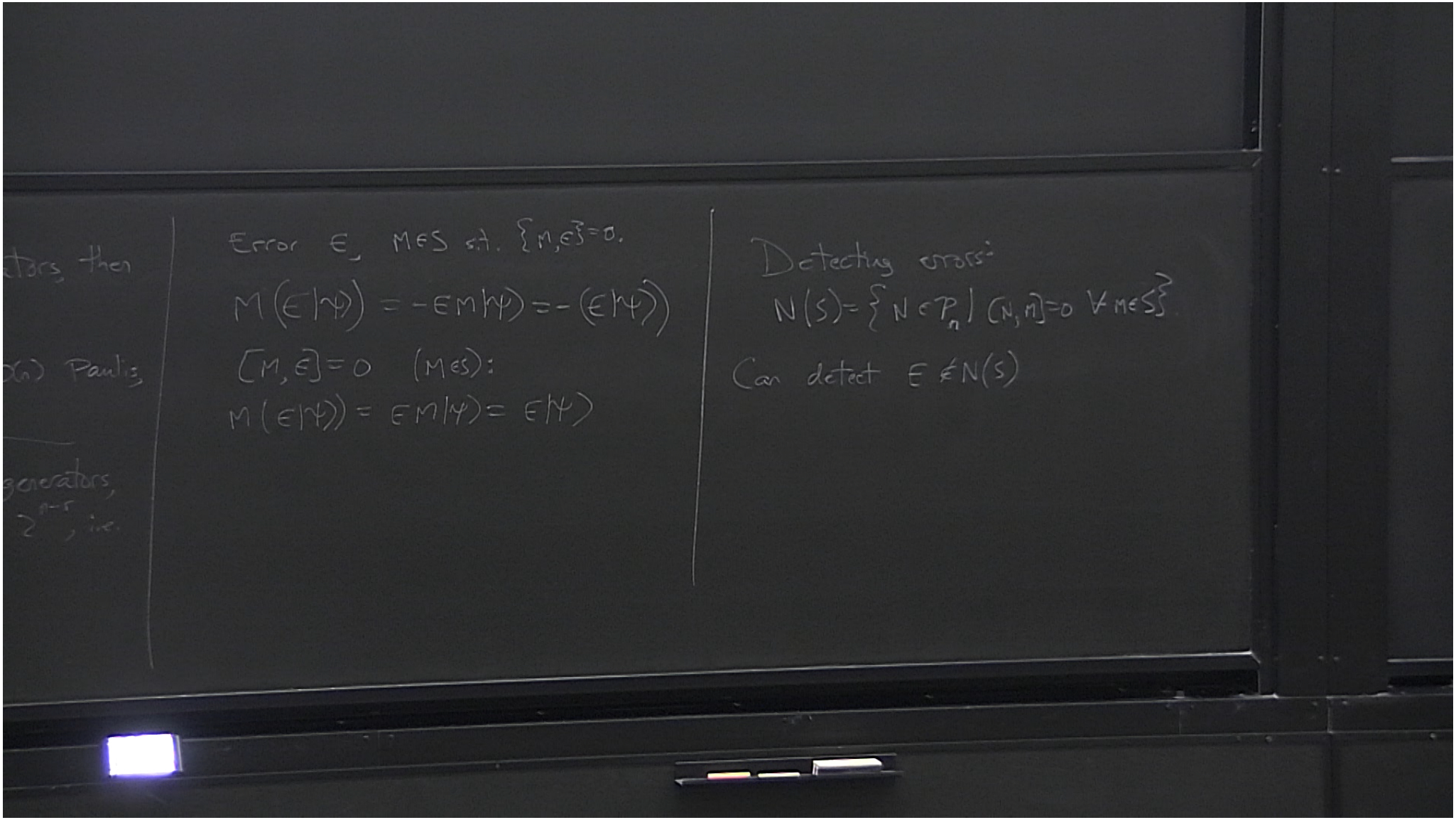
Error E,

Error ϵ, $M \in S$ s.t. $\{M, \epsilon\} = 0$.

$$M(\epsilon|\psi\rangle) = -\epsilon M|\psi\rangle = -(\epsilon|\psi\rangle)$$

$[M, \epsilon] = 0$ $(M \in S)$:

$$M(\epsilon|\psi\rangle) = \epsilon M|\psi\rangle = \epsilon|\psi\rangle$$


tors, then
(n) Pauli
generators,
2^{n-r}, i.e.
Error E, $M \in S$ s.t. $\{M, E\} = 0$.
$M(E|\psi\rangle) = -EM|\psi\rangle = -(E|\psi\rangle)$
$[M, E] = 0 \quad (M \in S)$:
$M(E|\psi\rangle) = EM|\psi\rangle = E|\psi\rangle$
Detecting errors:
$N(S) = \{N \in \mathcal{P}_n \mid [N, M] = 0 \ \forall M \in S\}$
Can detect $E \notin N(S)$

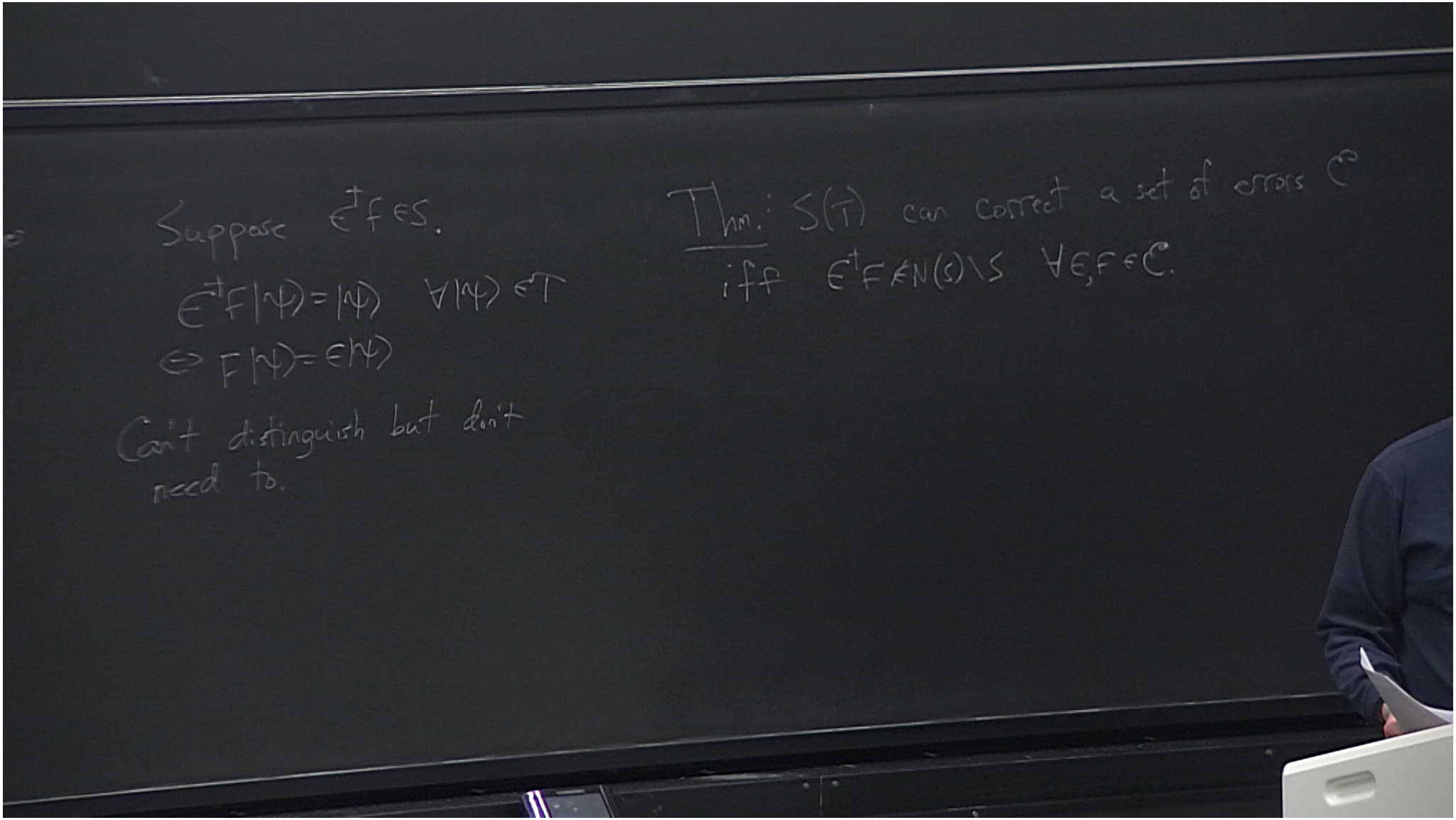
Sufficient if E, F have different error syndromes

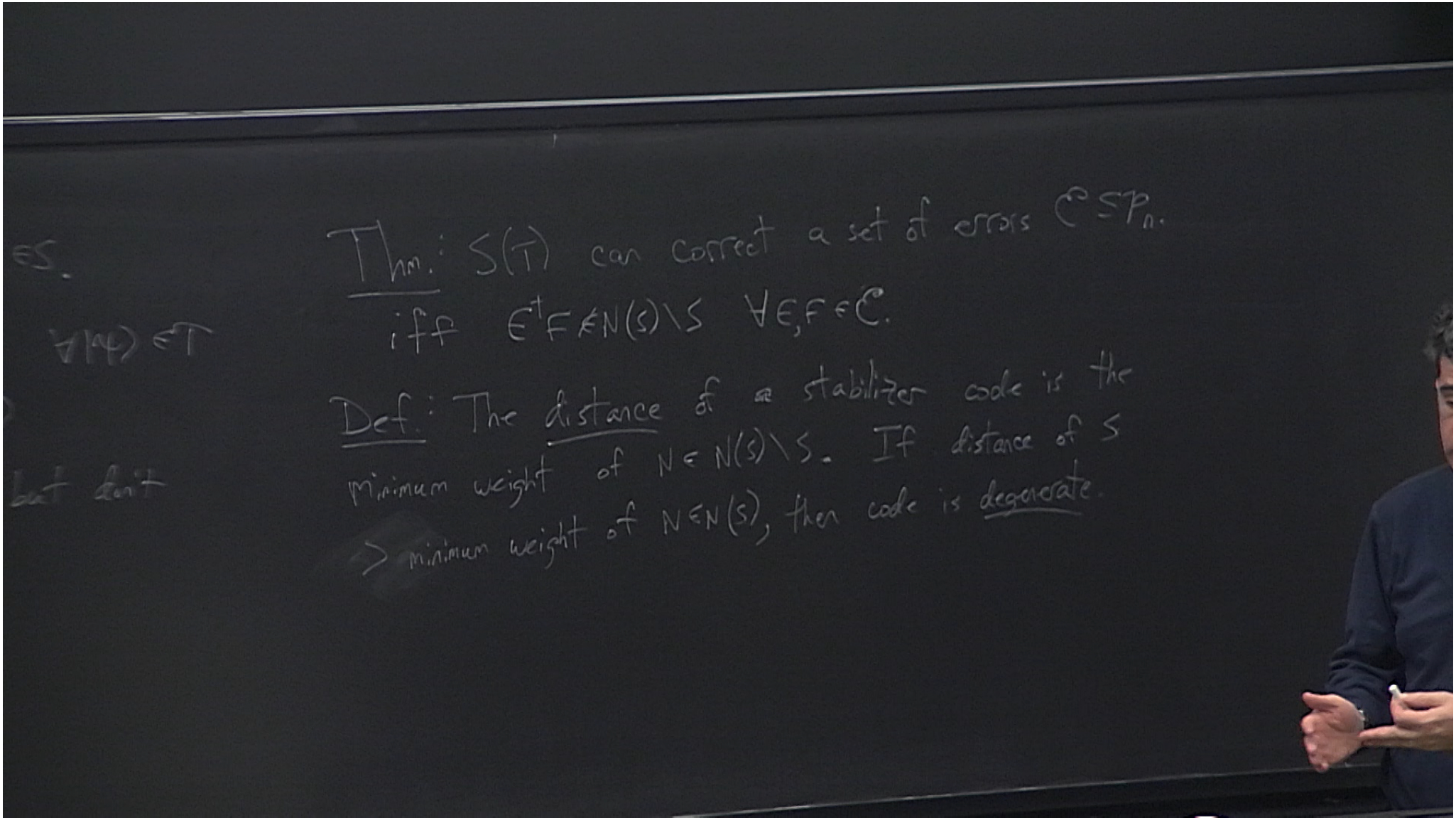
$(\forall E, F \in \mathcal{E})$

$\Leftrightarrow$ E & F have different sets of $M \in S$ with which they commute.

$\Leftrightarrow \{E^\dagger F, M\} = 0$ for at least one $M \in S$.

$\Leftrightarrow E^\dagger F \notin N(S)$.


Suppose $E_f^\dagger \in S$.
$E_f^\dagger F|\psi\rangle = |\psi\rangle \quad \forall |\psi\rangle \in T$
$\Leftrightarrow F|\psi\rangle = E|\psi\rangle$
Can't distinguish but don't need to.
Thm.: $S(T)$ can correct a set of errors $\mathcal{C}$
iff $E^\dagger F \notin N(S) \setminus S \quad \forall E, F \in \mathcal{C}$.

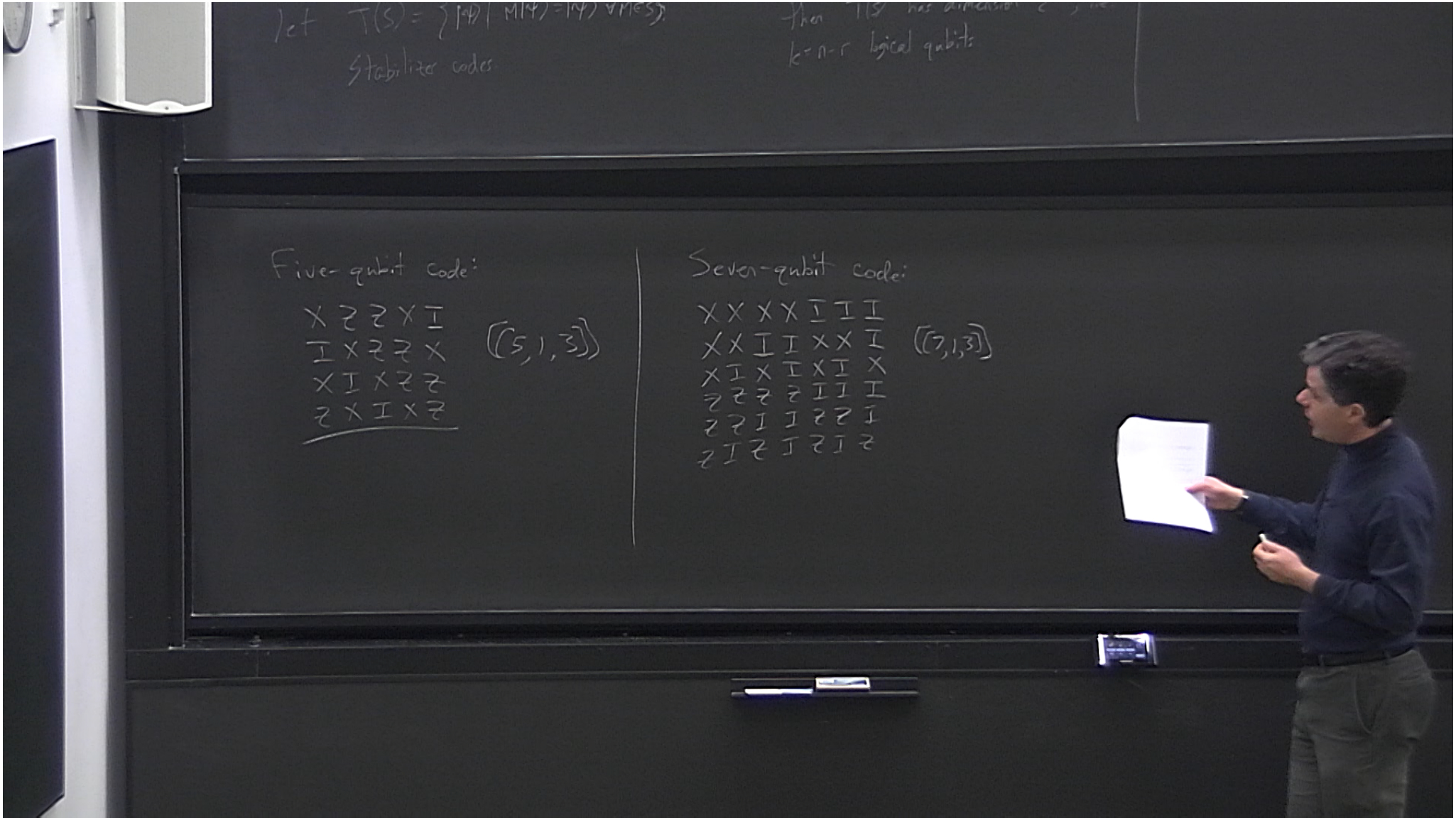

∈S.
∀|ψ⟩ ∈T
but don't
Thm: S(T) can correct a set of errors ℰ ⊆ P_n.
iff E†F ∉ N(S)\S ∀E,F ∈ ℰ.
Def: The distance of a stabilizer code is the
minimum weight of N ∈ N(S)\S. If distance of S
> minimum weight of N ∈ N(S), then code is degenerate.

Thm: $S(T)$ can correct a set of errors $\mathcal{E} \subseteq \mathcal{P}_n$.
iff $E^\dagger F \notin N(S) \setminus S \quad \forall E, F \in \mathcal{E}$.

Def: The distance of a stabilizer code is the minimum weight of $N \in N(S) \setminus S$. If distance of S > minimum weight of $N \in N(S)$, then code is degenerate.

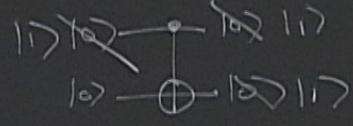
Notation: $[[n, k, d]]$ for code with n physical qubits, k logical qubits, & distance d.

To correct t-qubit errors, need $d \geq 2t+1$.


Let T(S) = {|ψ⟩ | M|ψ⟩ = |ψ⟩ ∀M∈S}
Stabilizer codes
then
k = n - r logical qubits
Five-qubit code:
X Z Z X I
I X Z Z X
X I X Z Z
Z X I X Z
([5,1,3])
Seven-qubit code:
X X X X I I I
X X I I X X I
X I X I X I X
Z Z Z Z I I I
Z Z I I Z Z I
Z I Z I Z I Z
([7,1,3])

,3]]

Fault Tolerance:



Threshold theorem:

∃ threshold error rate p_T s.t. if error p per gate or time step is below p_T, then ∃ fault-tolerant protocols for arbitrarily long quantum computations. The overhead is polylog (T), where T is the length of the logical computation