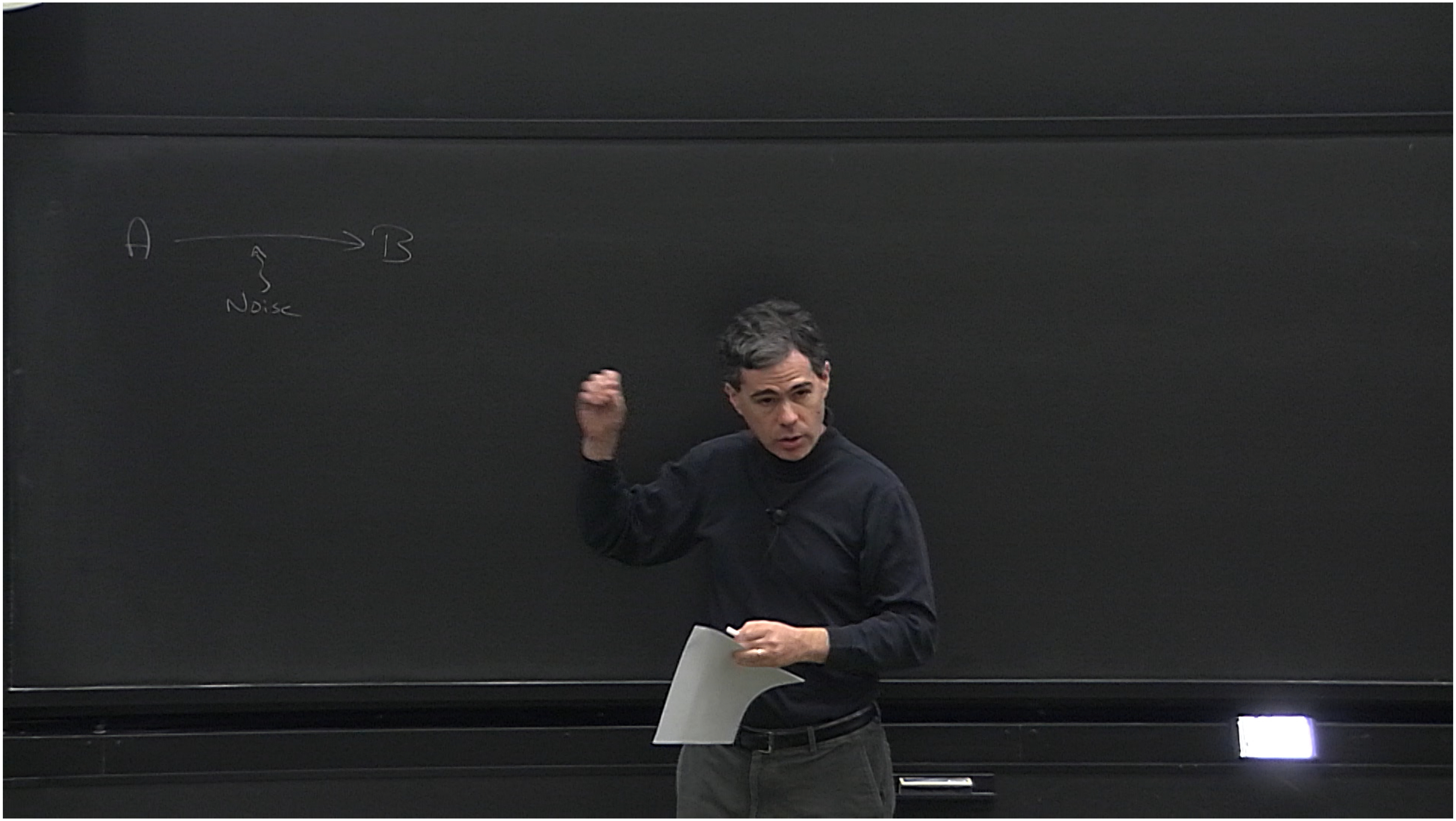
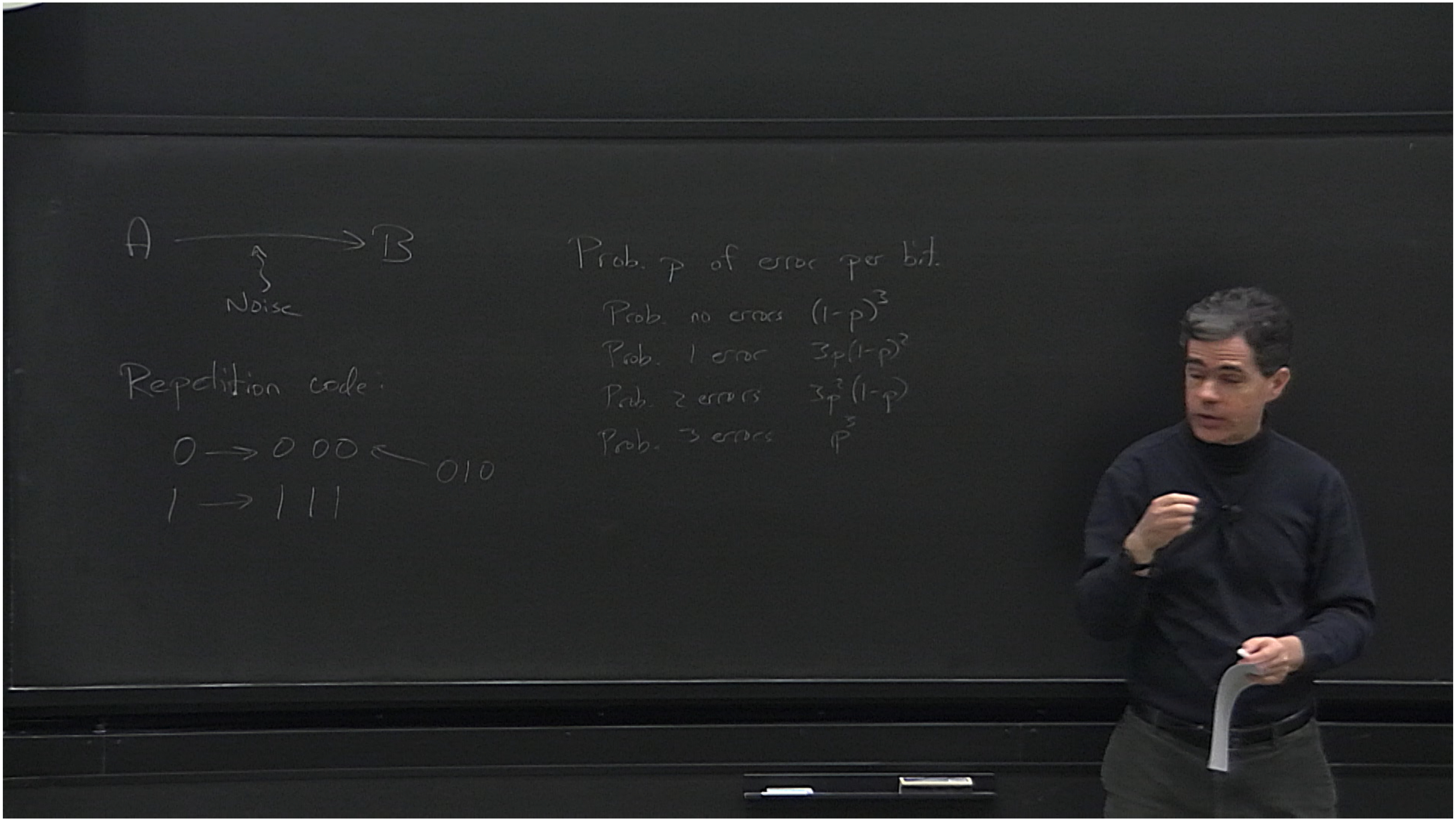
Title: PSI 2017/2018 - Quantum Information - Lecture 10

Date: Mar 05, 2018 11:30 AM

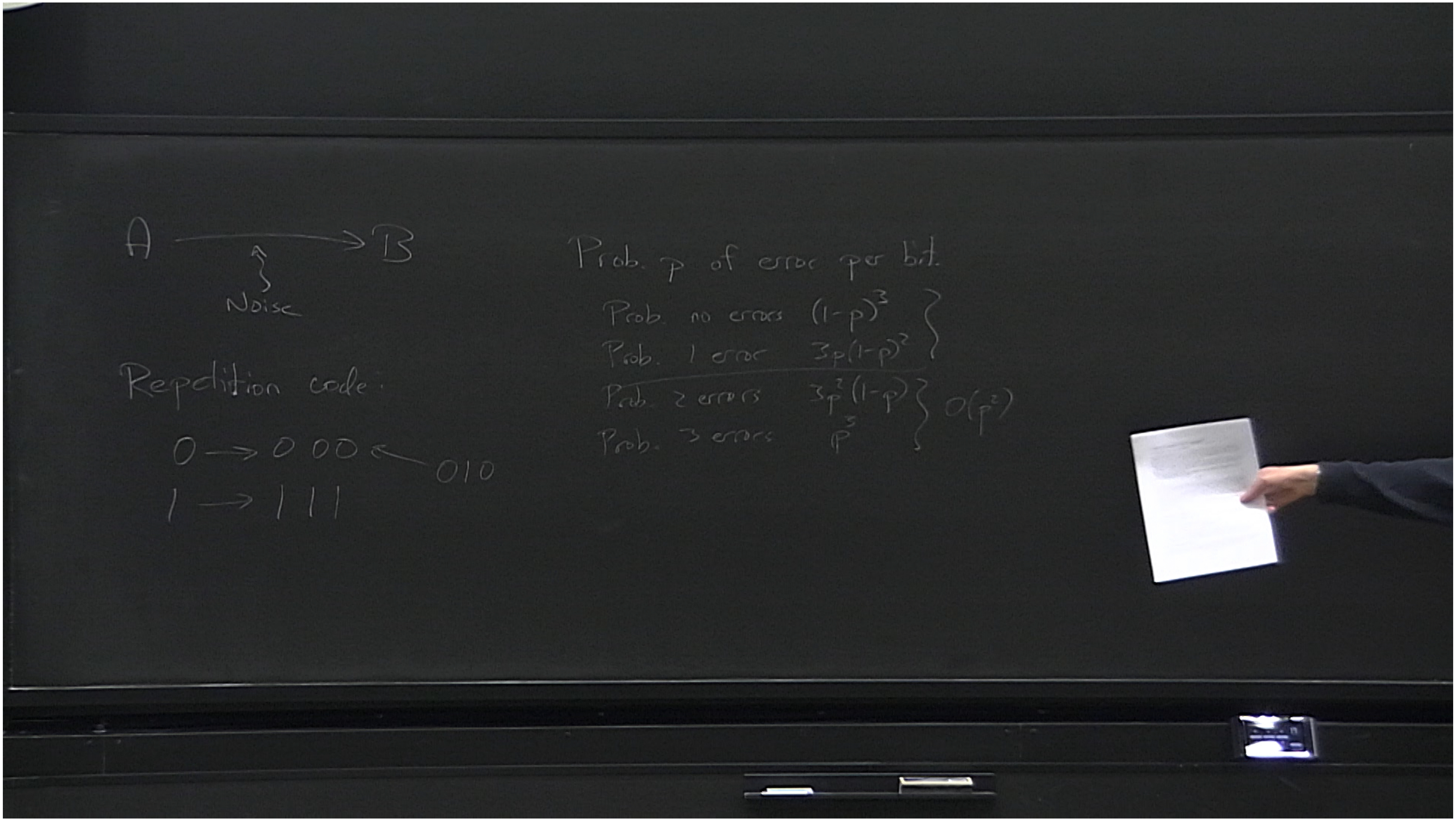
URL: http://pirsa.org/18030023

Abstract:


A
B
Noise



A → B
Noise
Repetition code:
0 → 000 ← 010
1 → 111
Prob. p of error per bit.
Prob. no errors $(1-p)^3$
Prob. 1 error $3p(1-p)^2$
Prob. 2 errors $3p^2(1-p)$
Prob. 3 errors p^3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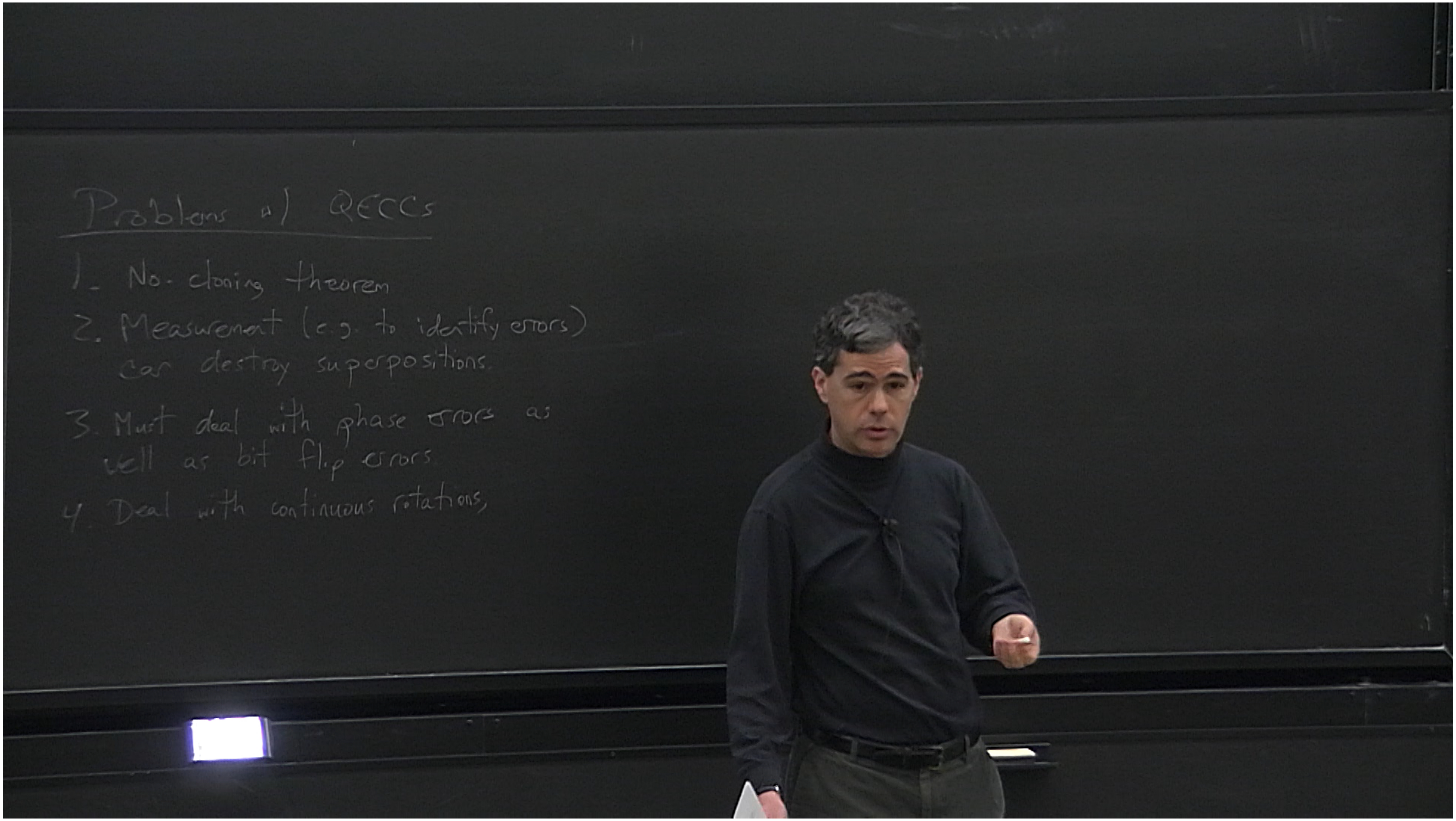
$O(p^2)$

Problems w/ QECCs

1. No-cloning theorem
2. Measurement (e.g. to identify errors) can destroy superpositions.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3. Must deal with phase errors as
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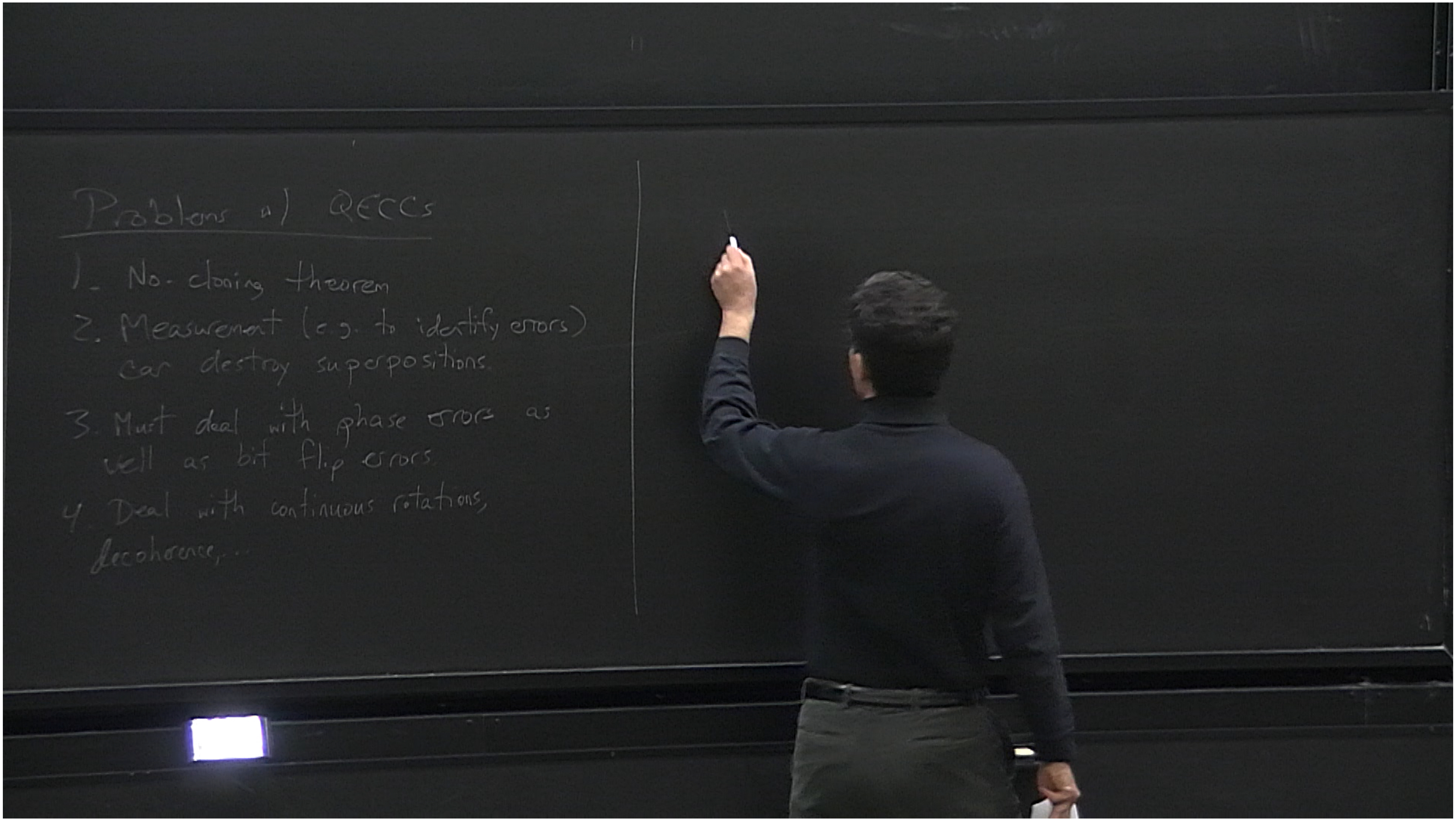
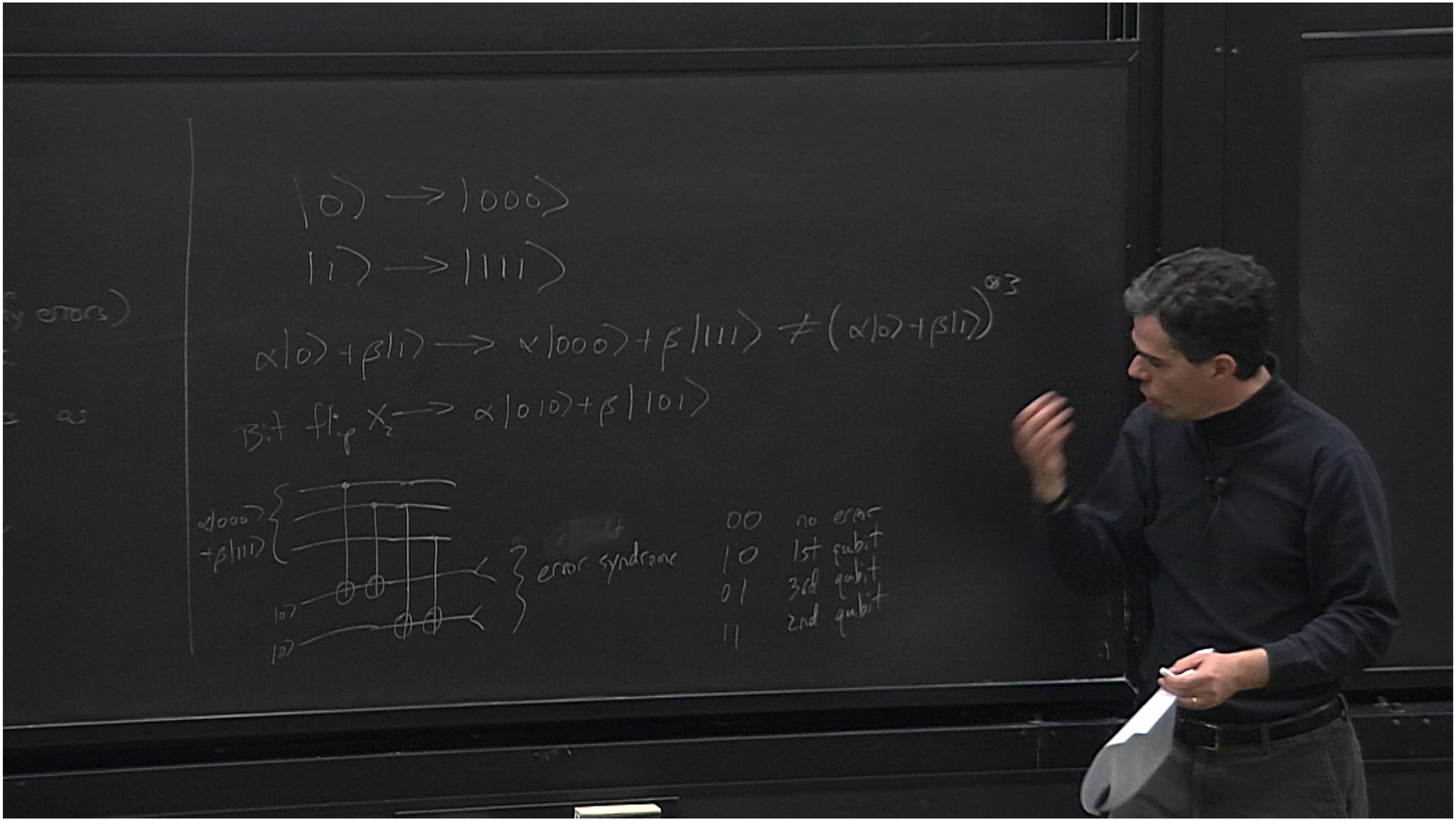
4. Deal with continuous rotations,


Problems w/ QECCs
1. No-cloning theorem
2. Measurement (e.g. to identify errors)
can destroy superpositions.
3. Must deal with phase errors as
well as bit flip errors.
4. Deal with continuous rotations,
decoherence, ...

$$|0\rangle \rightarrow |000\rangle$$

$$|1\rangle \rightarrow |111\rangle$$

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle \neq (\alpha|0\rangle + \beta|1\rangle)^{\otimes 3}$$


|0⟩ → |000⟩
|1⟩ → |111⟩
α|0⟩ + β|1⟩ → α|000⟩ + β|111⟩ ≠ (α|0⟩ + β|1⟩)⊗3
Bit flip X2 → α|010⟩ + β|101⟩
α|000⟩ + β|111⟩
|0⟩
|0⟩
error syndrome
00 no error
10 1st qubit
01 3rd qubit
11 2nd qubit

$$|0\rangle \overset{H}{\longleftrightarrow} |+\rangle = |0\rangle + |1\rangle$$

$$|1\rangle \longleftrightarrow |-\rangle = |0\rangle - |1\rangle$$

Z in $|+\rangle, |-\rangle$ basis acts like X

$$|0\rangle \rightarrow |+++\rangle$$

$$|1\rangle \rightarrow |---\rangle$$

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|+++\rangle + \beta|---\rangle$$

9-qubit code:

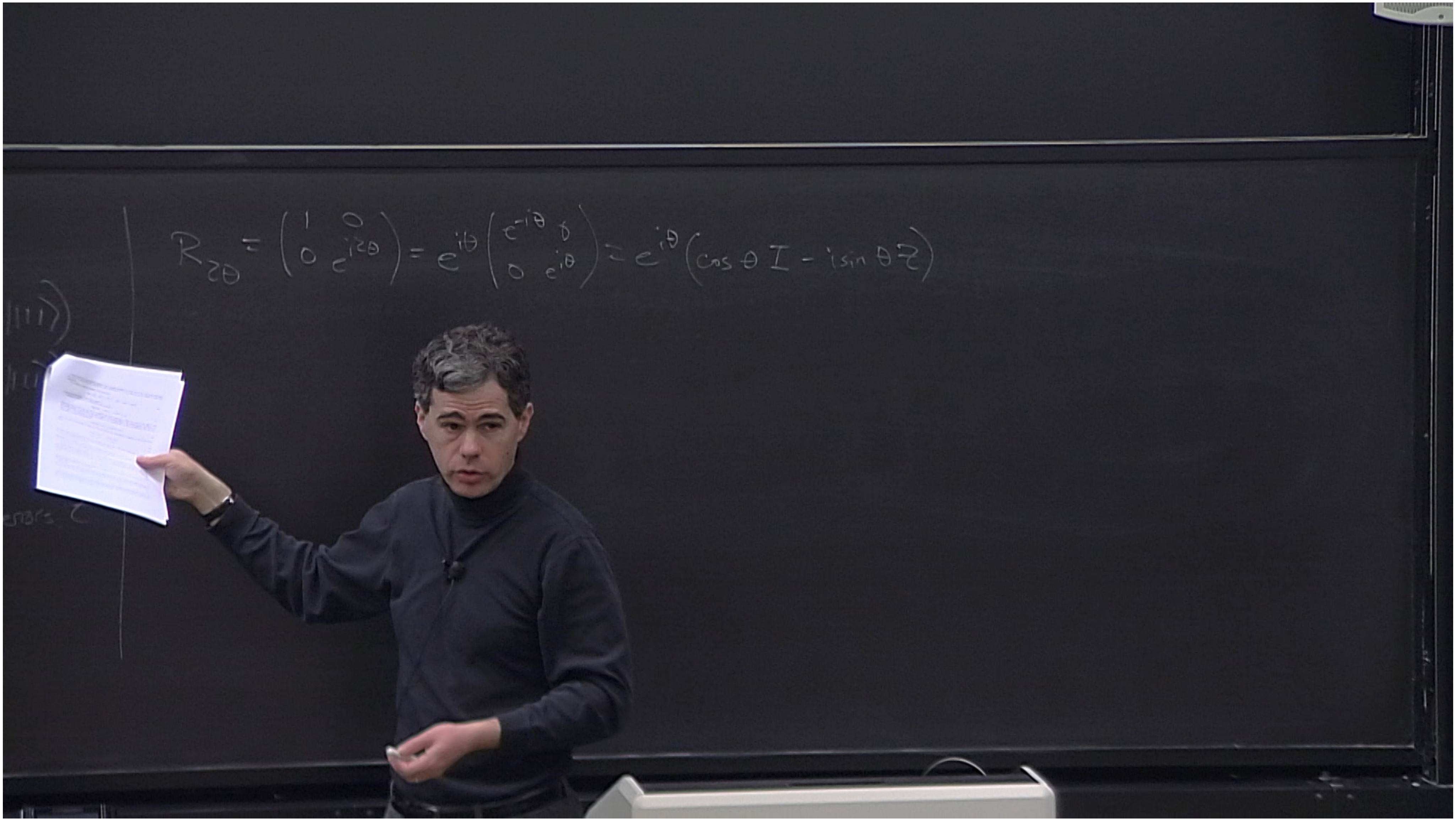
$$|0\rangle \longrightarrow \left(|000\rangle+|111\rangle\right)\otimes\left(|000\rangle+|111\rangle\right)\otimes\left(|000\rangle+|111\rangle\right)$$

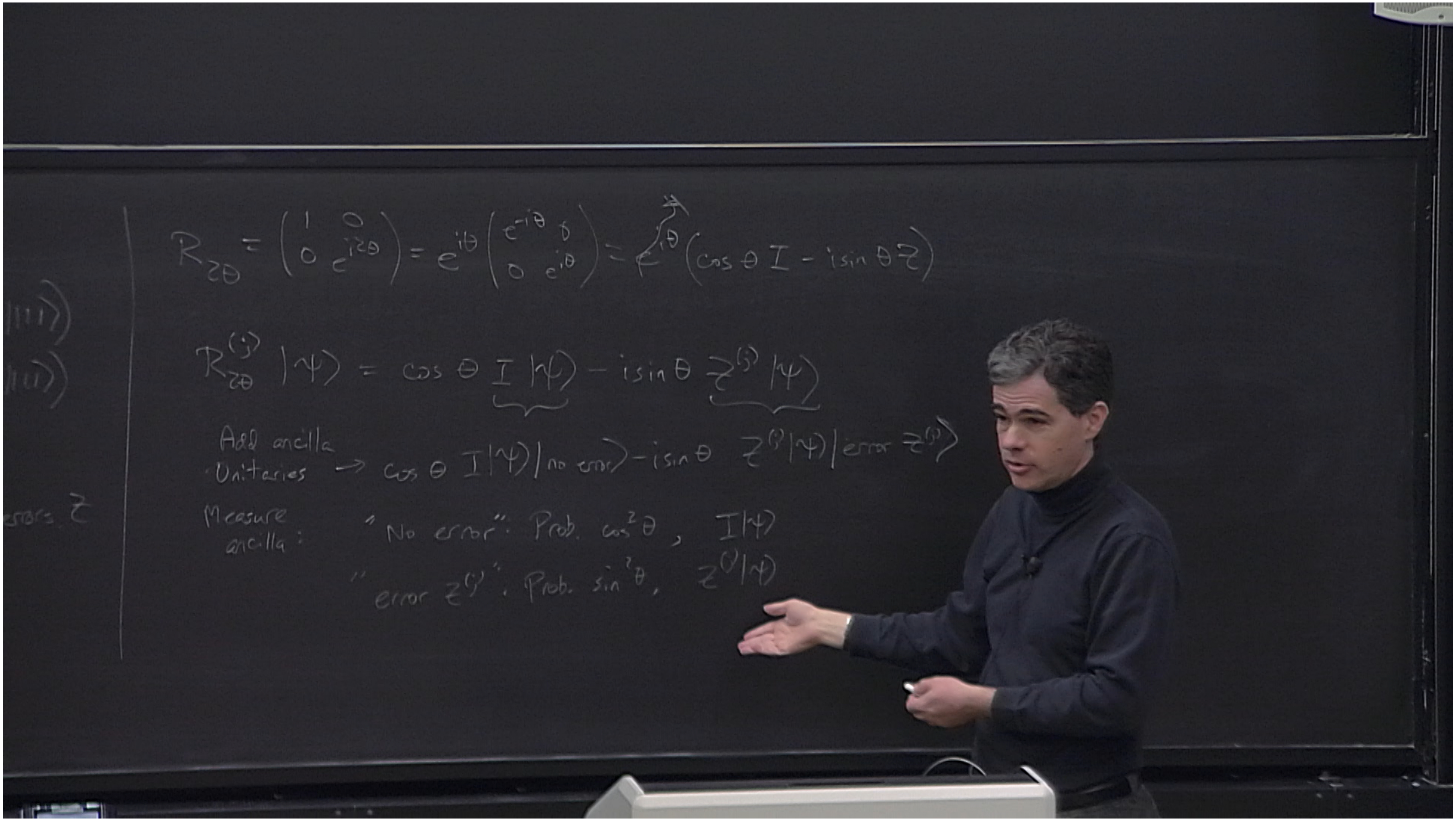
$$|1\rangle \longrightarrow \left(|000\rangle-|111\rangle\right)\otimes\left(|000\rangle-|111\rangle\right)\otimes\left(|000\rangle-|111\rangle\right)$$

Corrects bit flip $\overset{X}{}$ errors w/in each set of 3

Compare phases between sets of 3 to correct phase errors Z

Also correct $Y=-iXZ=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$.


$R_{2\theta} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i2\theta} \end{pmatrix} = e^{i\theta} \begin{pmatrix} e^{-i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix} = e^{i\theta} \left(\cos\theta \, I - i \sin\theta \, Z \right)$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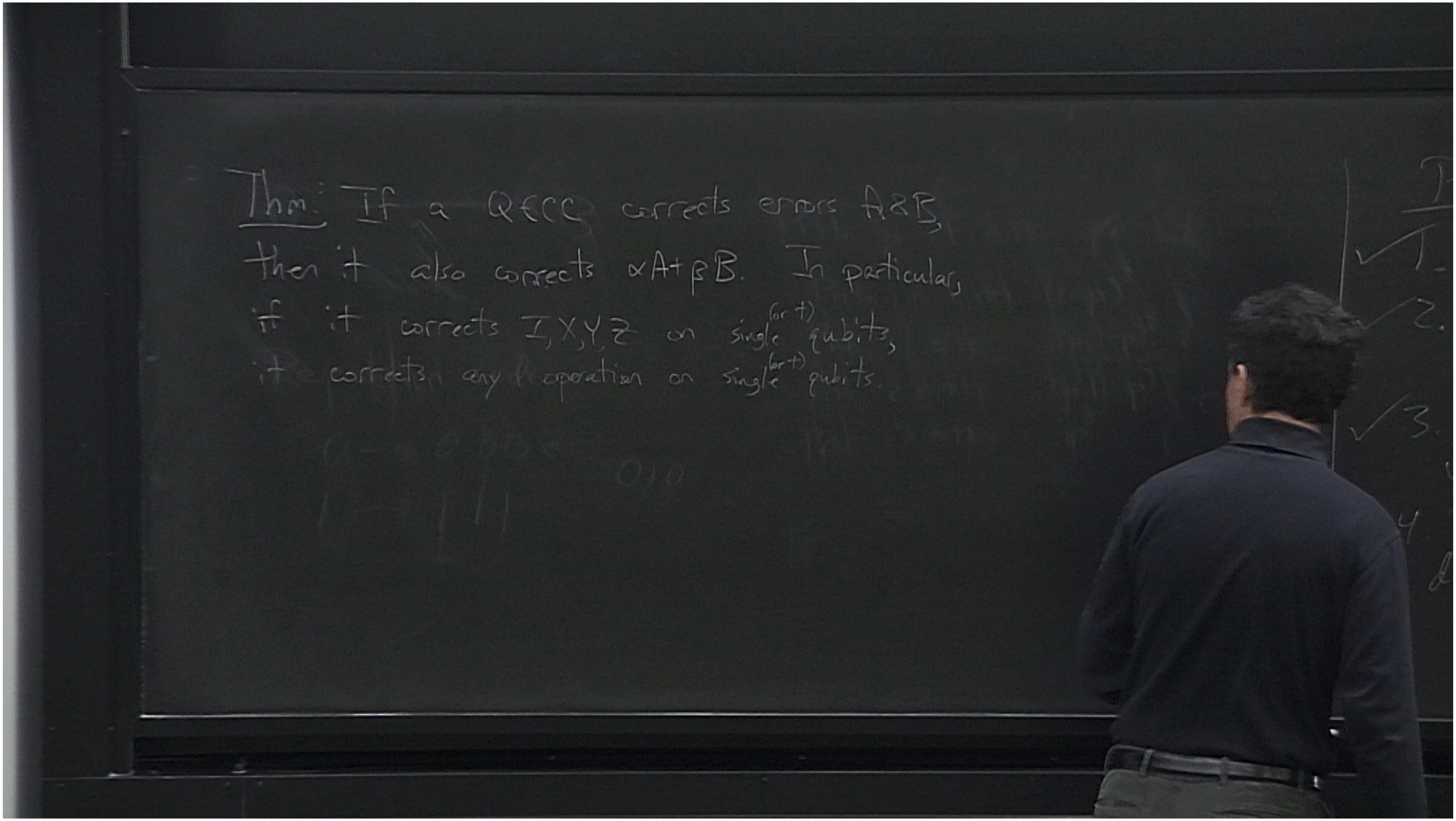
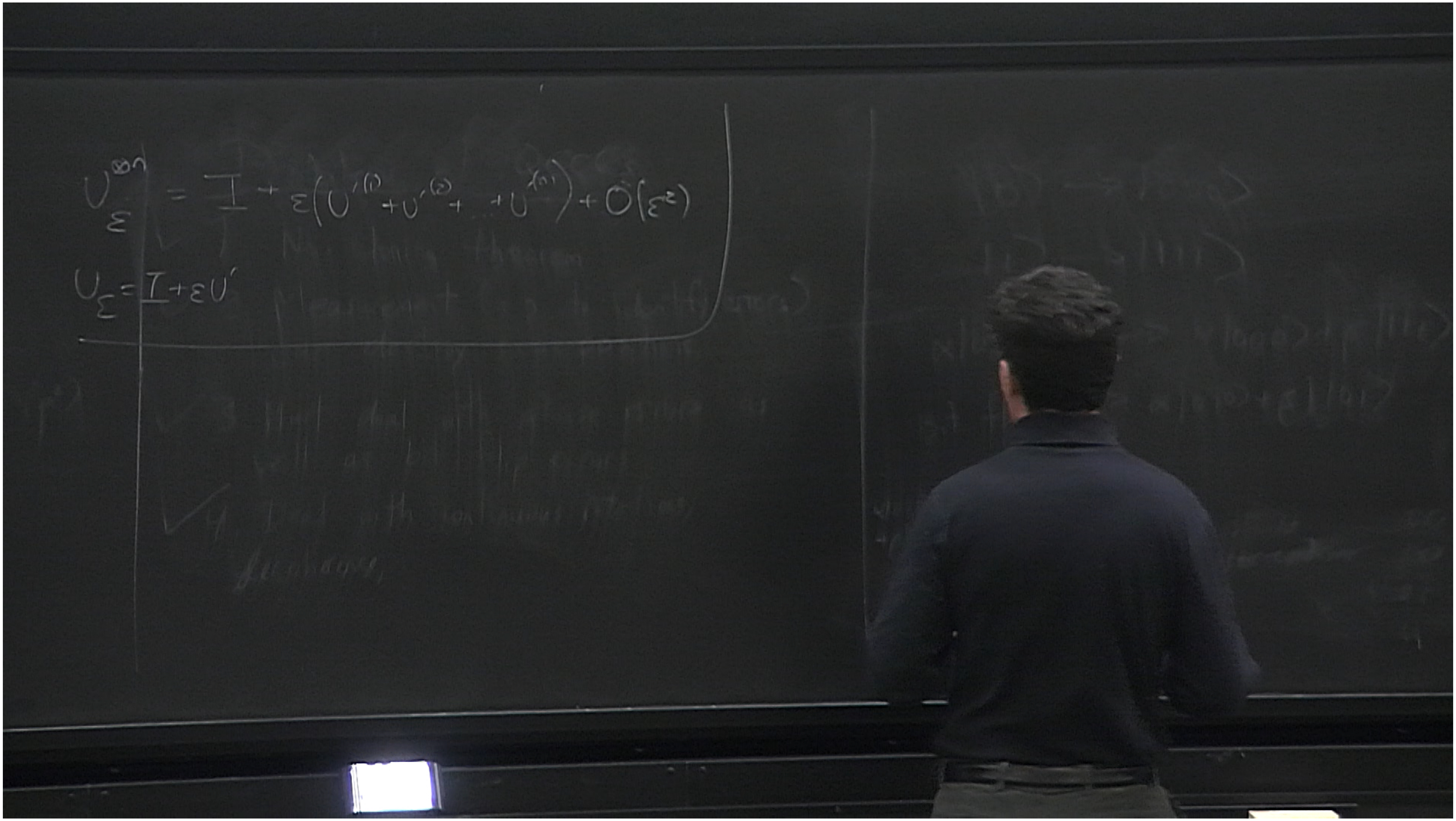
$R_{2\theta}^{(j)} |\psi\rangle = \cos\theta\, I |\psi\rangle - i\sin\theta\, Z^{(j)} |\psi\rangle$
Add ancilla
Unitaries
$\rightarrow \cos\theta\, I|\psi\rangle|\text{no error}\rangle - i\sin\theta\, Z^{(j)}|\psi\rangle|\text{error } Z^{(j)}\rangle$
Measure ancilla:
"No error": Prob. $\cos^2\theta$, $I|\psi\rangle$
"error $Z^{(j)}$": Prob. $\sin^2\theta$, $Z^{(j)}|\psi\rangle$
errors Z


Thm: If a QECC corrects errors A & B,
then it also corrects $\alpha A + \beta B$. In particular,
if it corrects I, X, Y, Z on single (or t) qubits,
it corrects any operation on single (or t) qubits.

Thm: If a QECC corrects errors A & B, then it also corrects $\alpha A + \beta B$. In particular, if it corrects I, X, Y, Z on single$^{(\text{or } t)}$ qubits, it corrects any operation on single$^{(\text{or } t)}$ qubits.

CPTP map $\rho \longrightarrow \sum_k A_k \rho A_k^\dagger$

$$|\psi\rangle\langle\psi| \rightarrow \sum_k A_k |\psi\rangle\langle\psi| A_k^\dagger$$


$U_\varepsilon^{\otimes n} = I + \varepsilon(U'^{(1)} + U'^{(2)} + \dots + U'^{(n)}) + O(\varepsilon^2)$
$U_\varepsilon = I + \varepsilon U'$

(ε^2)

Pauli group on n qubits

$\mathcal{P}_n = \{$ tensor products of I, X, Y, Z on n qubits w/ phase $\pm 1, \pm i \}$.

- Any element squares to $+I$ or $-I$.
- Any two $P, Q \in \mathcal{P}_n$ either commute $[P, Q] = 0$
  or anticommute $\{P, Q\} = 0$.

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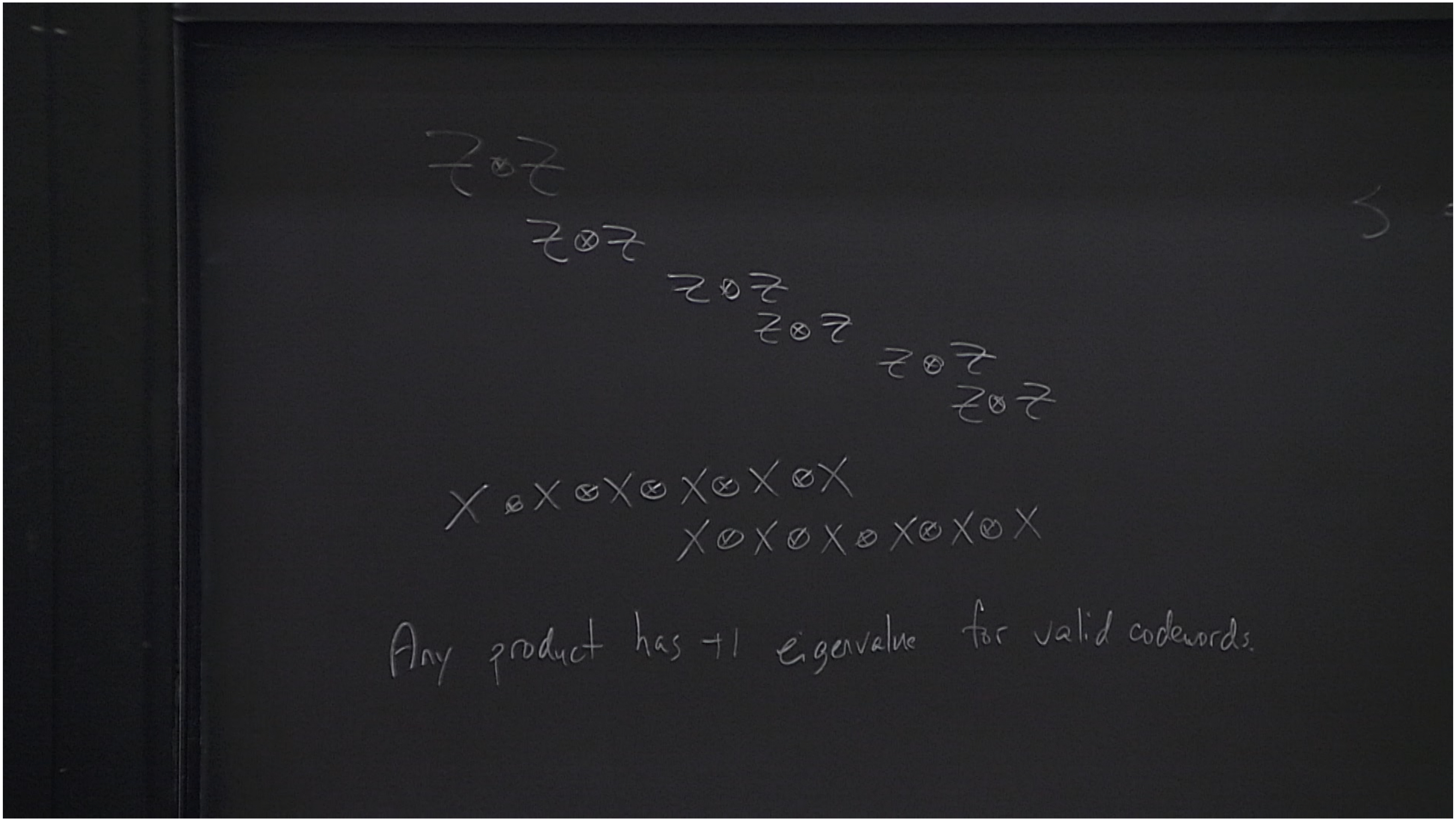
<u>Def.</u>: <u>Weight</u> of $P \in \mathcal{P}_n$ is # qubits where P is not I.


$Z \otimes Z$
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$Z \otimes Z$
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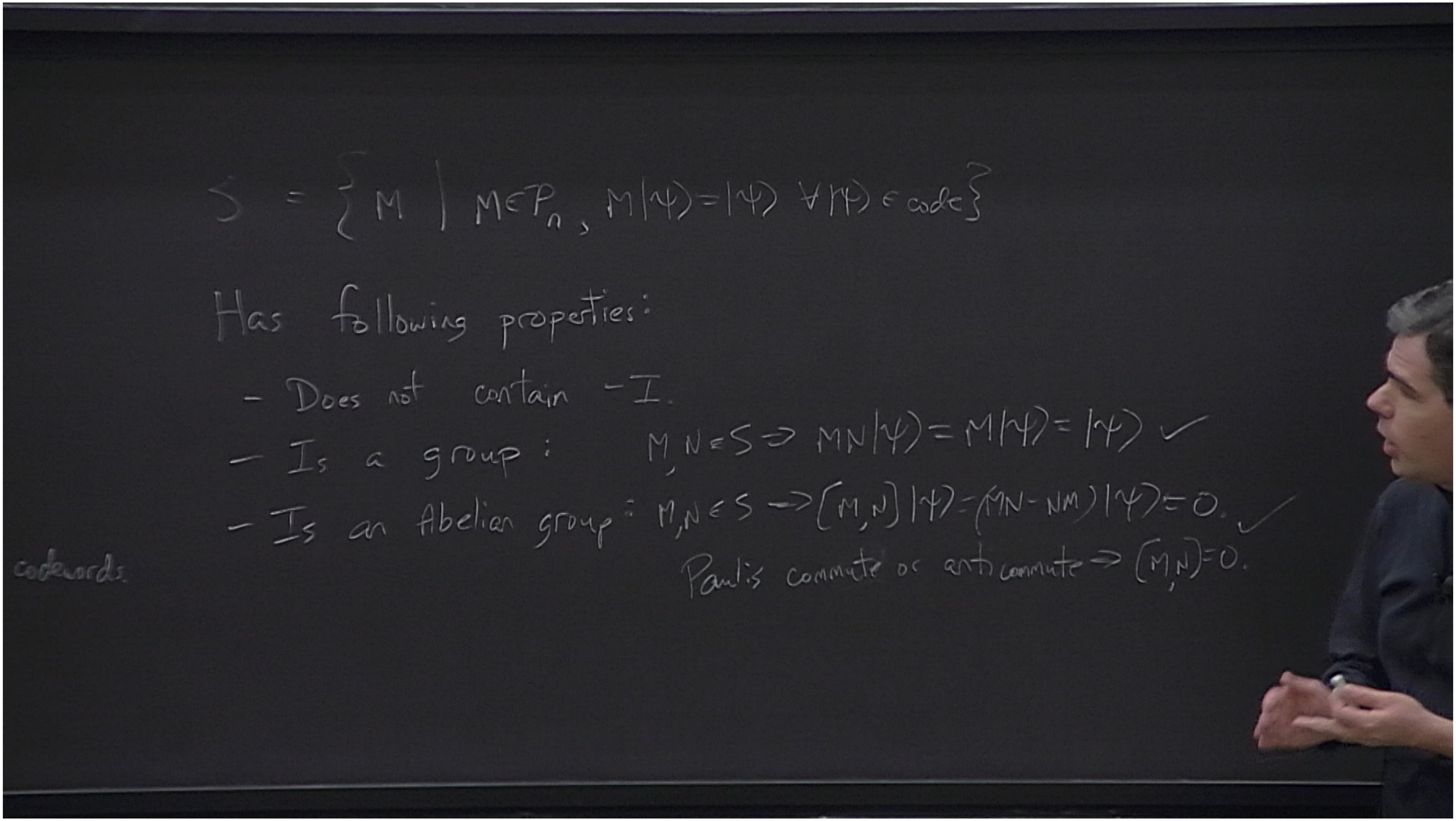
$X \otimes X \otimes X \otimes X \otimes X \otimes X$
$X \otimes X \otimes X \otimes X \otimes X \otimes X$
Any product has +1 eigenvalue for valid codewords.


$S = \{ M \mid M \in P_n, M|\psi\rangle = |\psi\rangle \ \forall |\psi\rangle \in \text{code} \}$
Has following properties:
- Does not contain $-I$.
- Is a group: $M, N \in S \Rightarrow MN|\psi\rangle = M|\psi\rangle = |\psi\rangle$ ✓
- Is an Abelian group: $M, N \in S \rightarrow [M,N]|\psi\rangle = (MN - NM)|\psi\rangle = 0$. ✓
Paulis commute or anticommute $\Rightarrow [M,N] = 0$.
codewords.