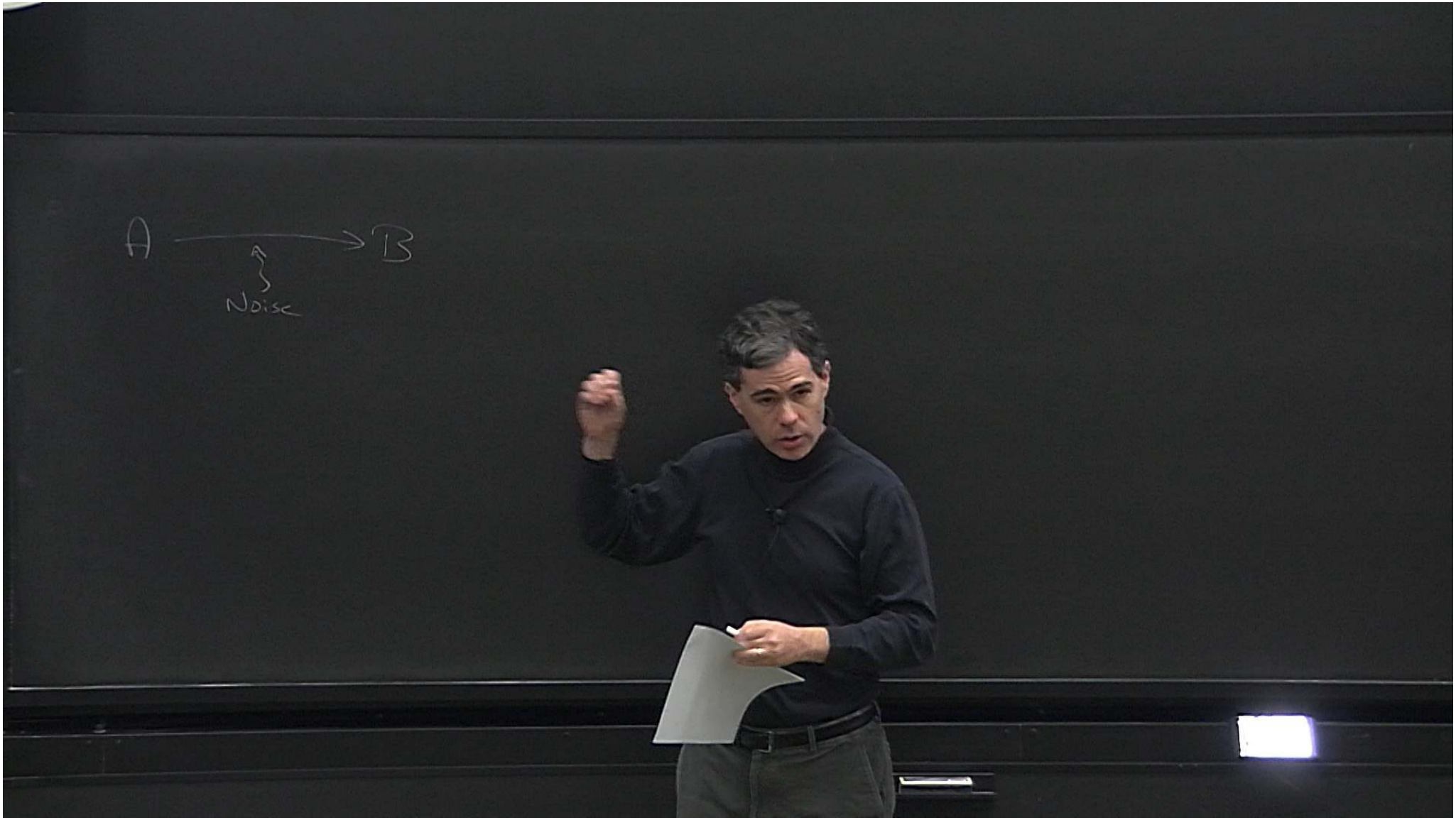
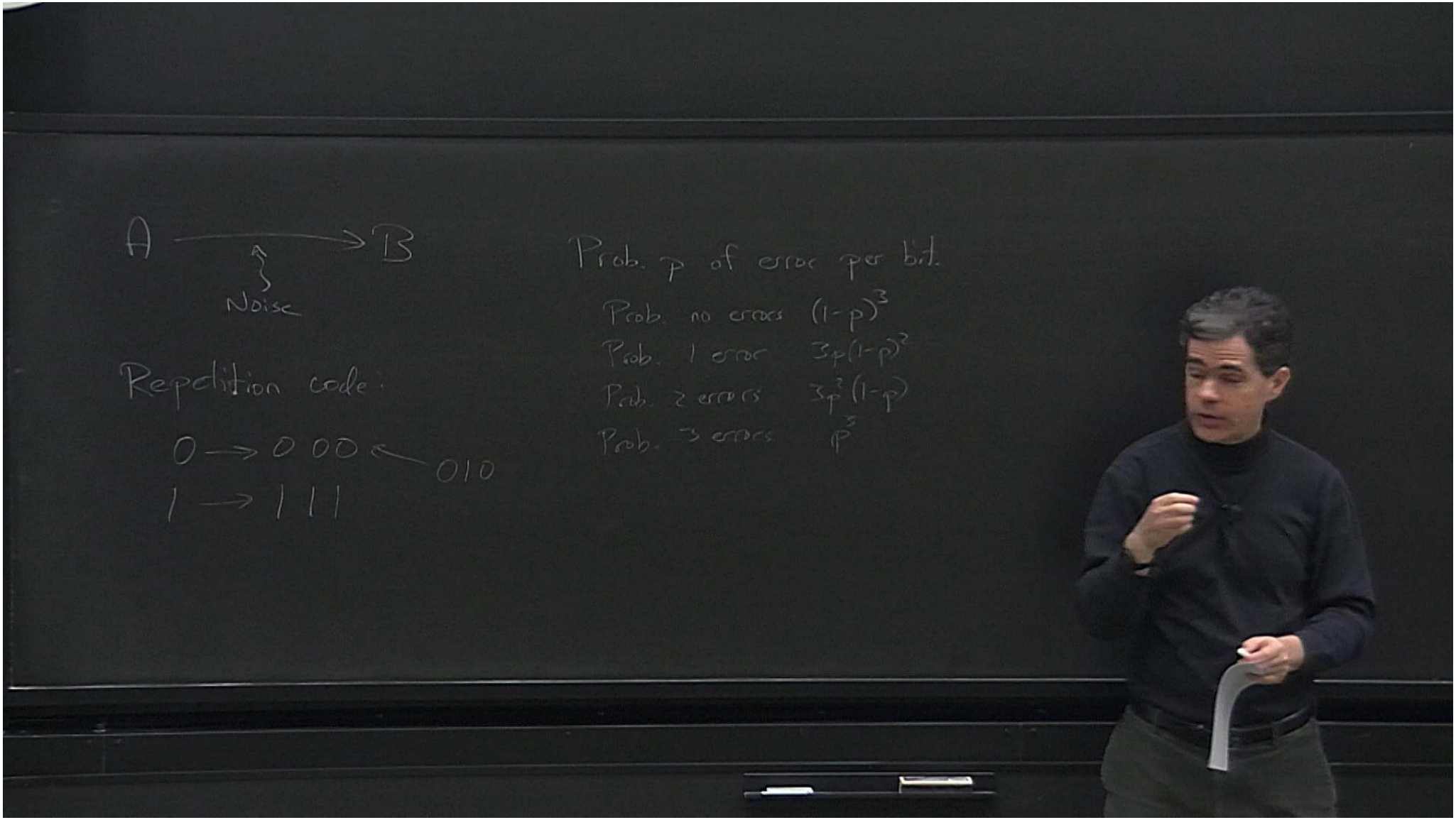
Title: PSI 2017/2018 - Quantum Information - Lecture 10

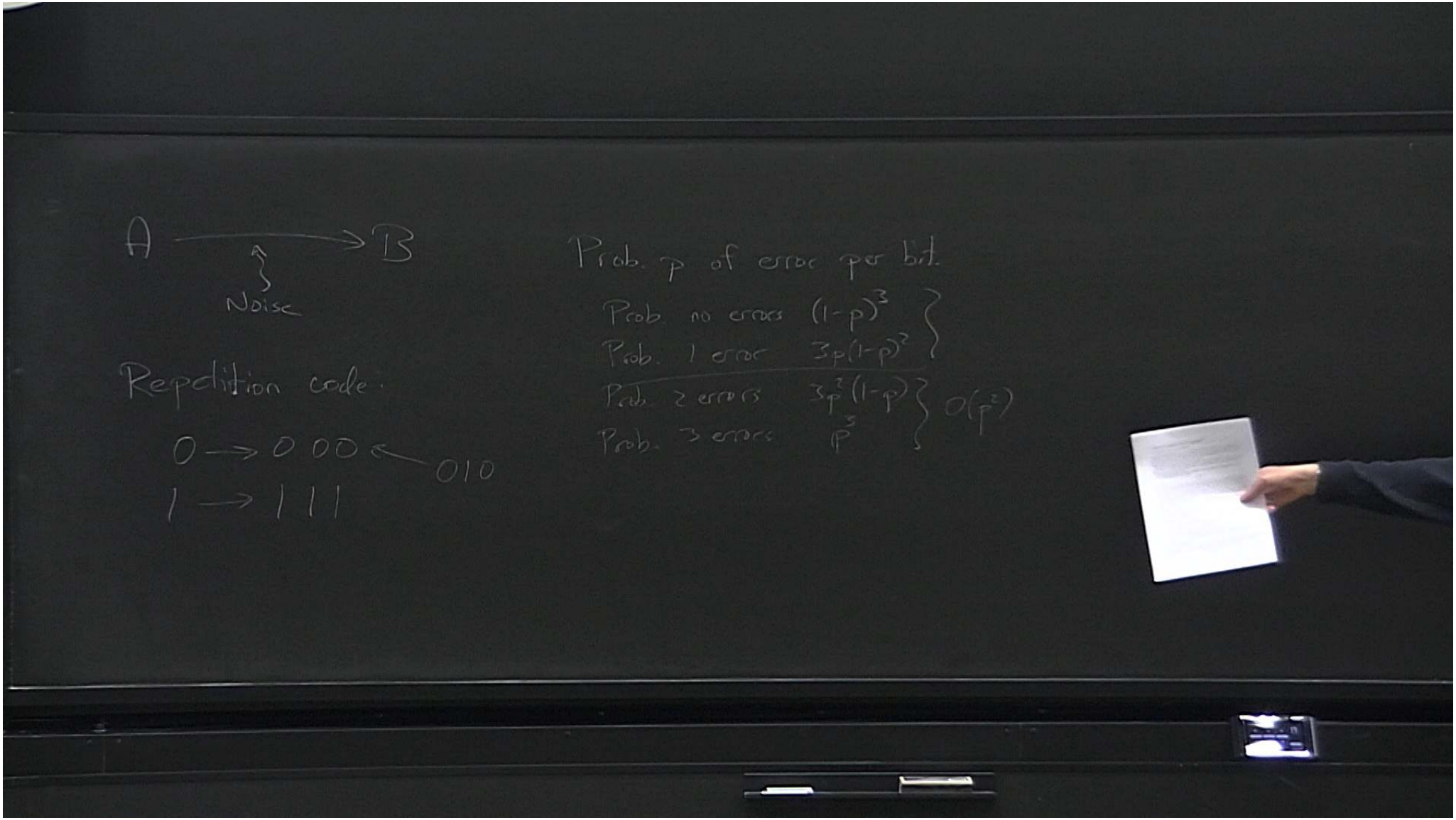
Date: Mar 05, 2018 11:30 AM

URL: http://pirsa.org/18030023

Abstract:


A
B
Noise


A
B
Noise
Repetition code:
0 → 000
010 ←
1 → 111
Prob. p of error per bit.
Prob. no errors $(1-p)^3$
Prob. 1 error $3p(1-p)^2$
Prob. 2 errors $3p^2(1-p)$
Prob. 3 errors p^3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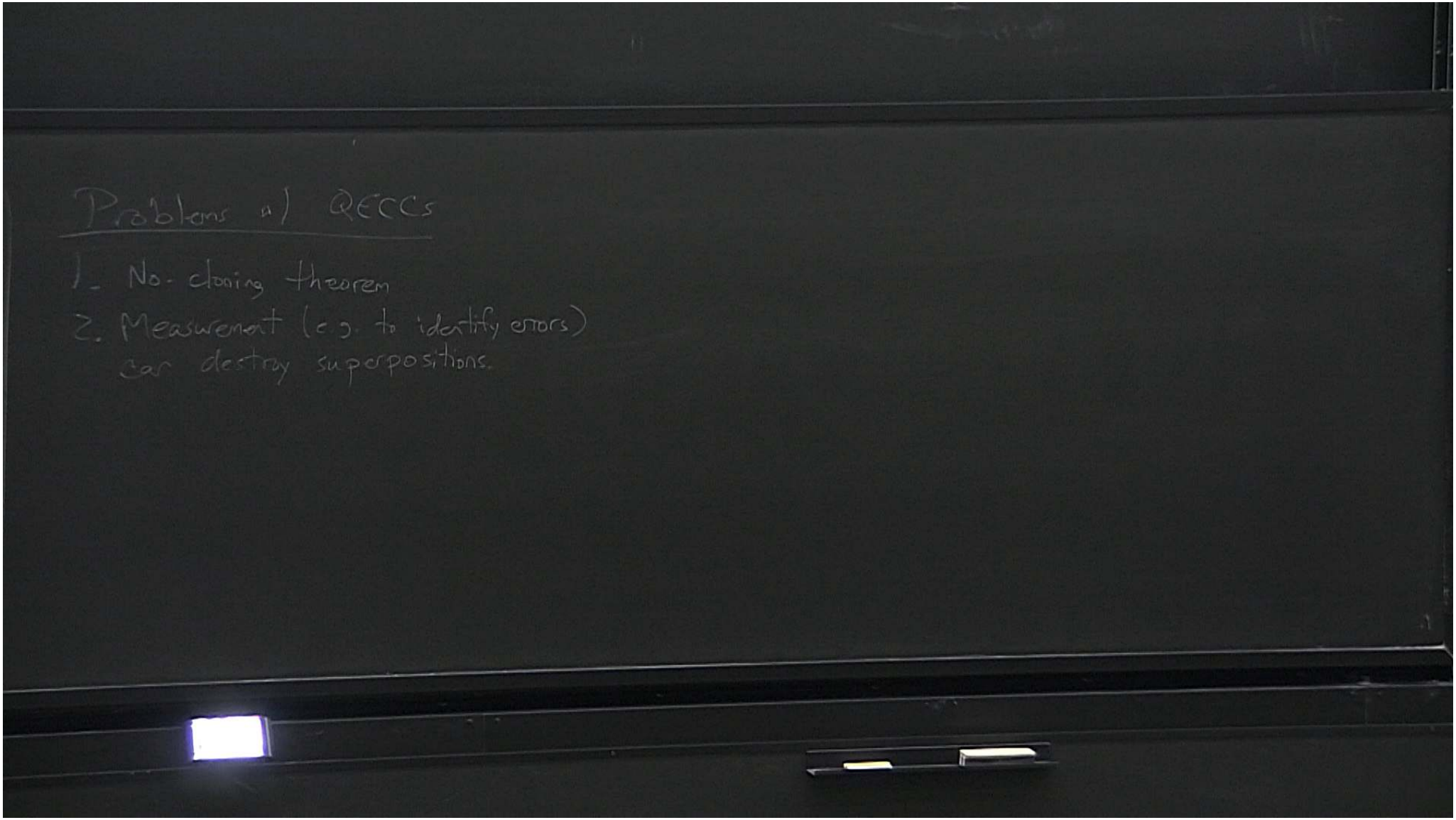
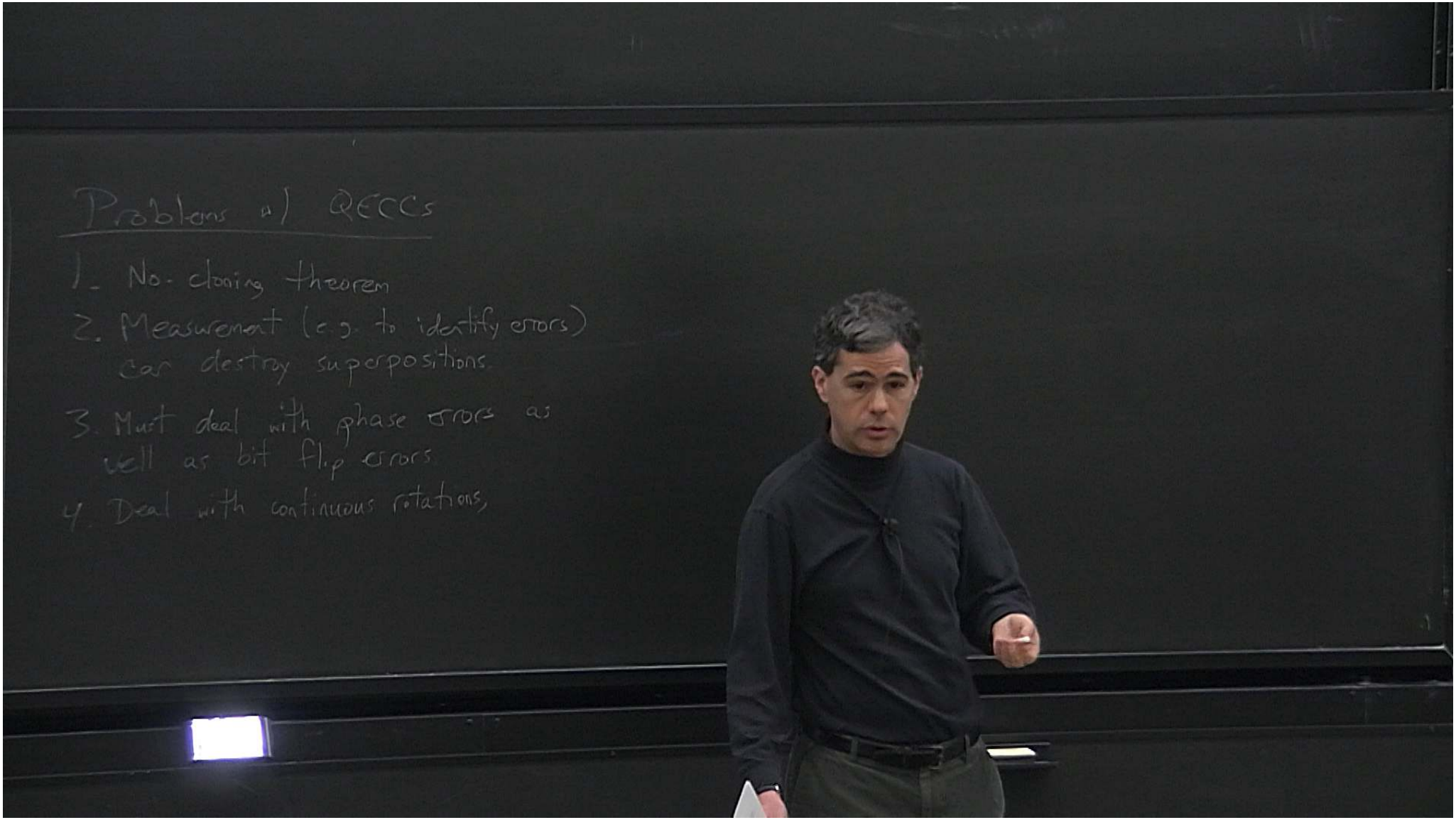
$O(p^2)$


Problems w/ QECCs
1. No-cloning theorem
2. Measurement (e.g. to identify errors)
can destroy superpositions.


Problems w/ QECCs
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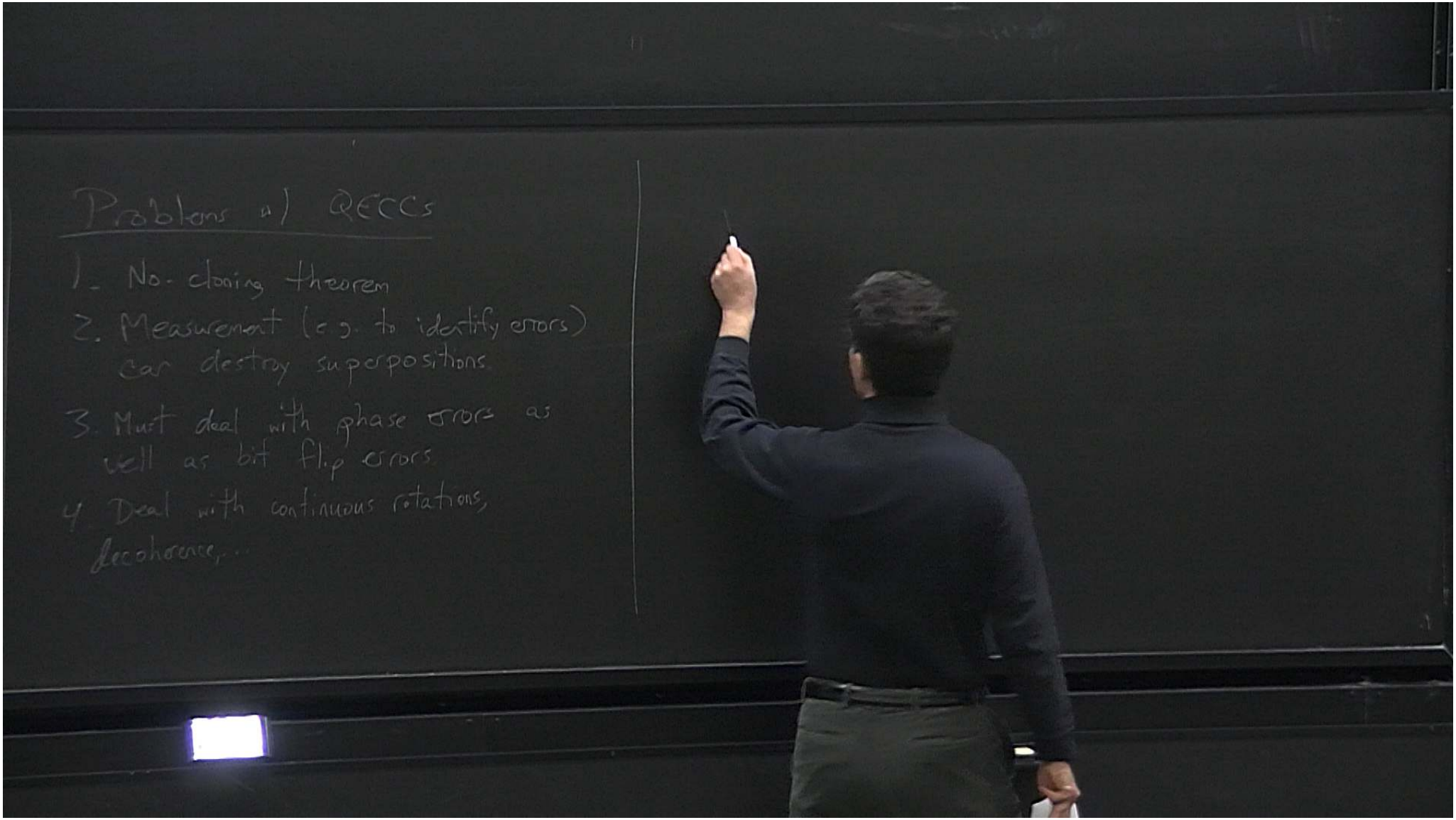
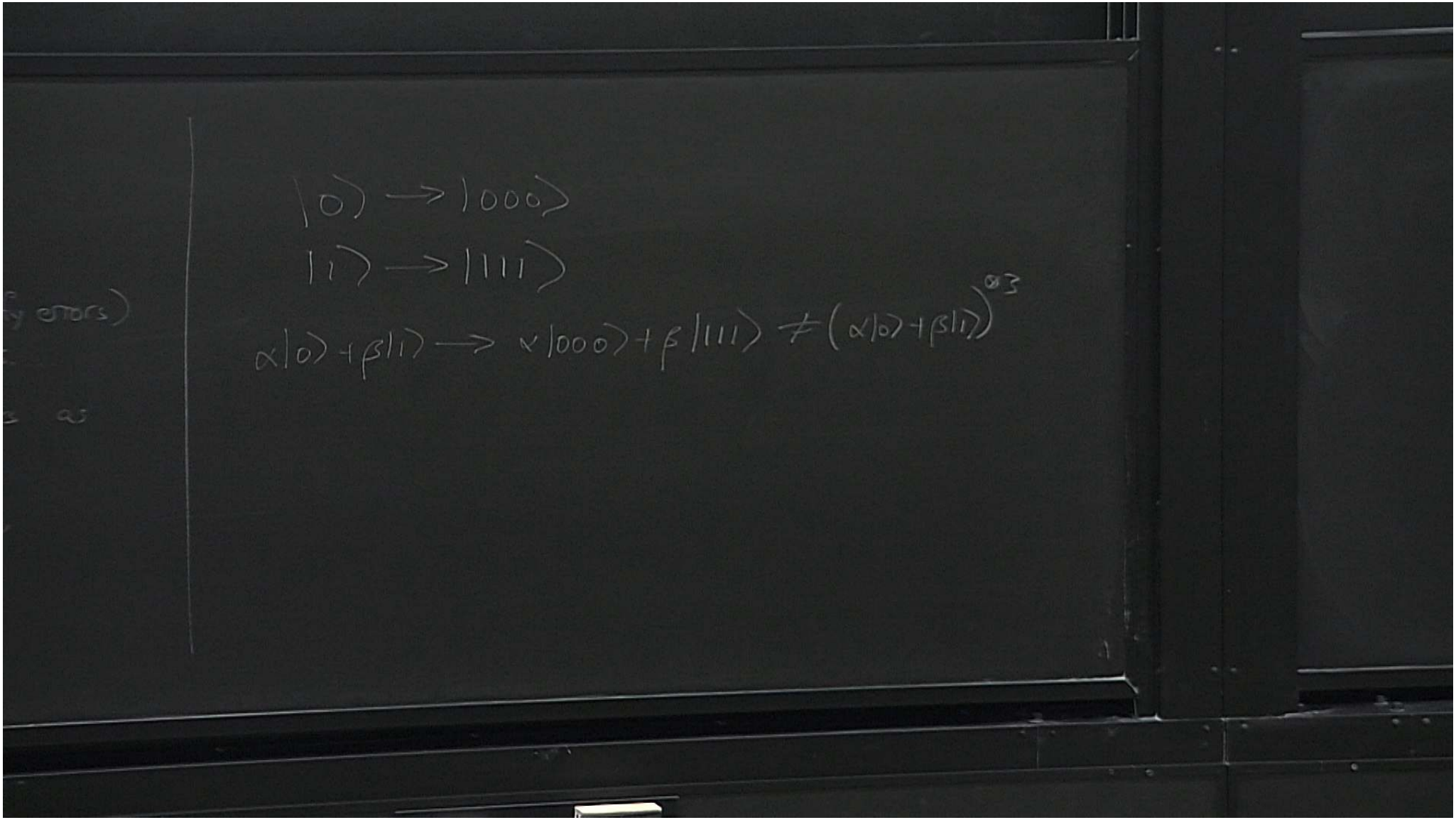
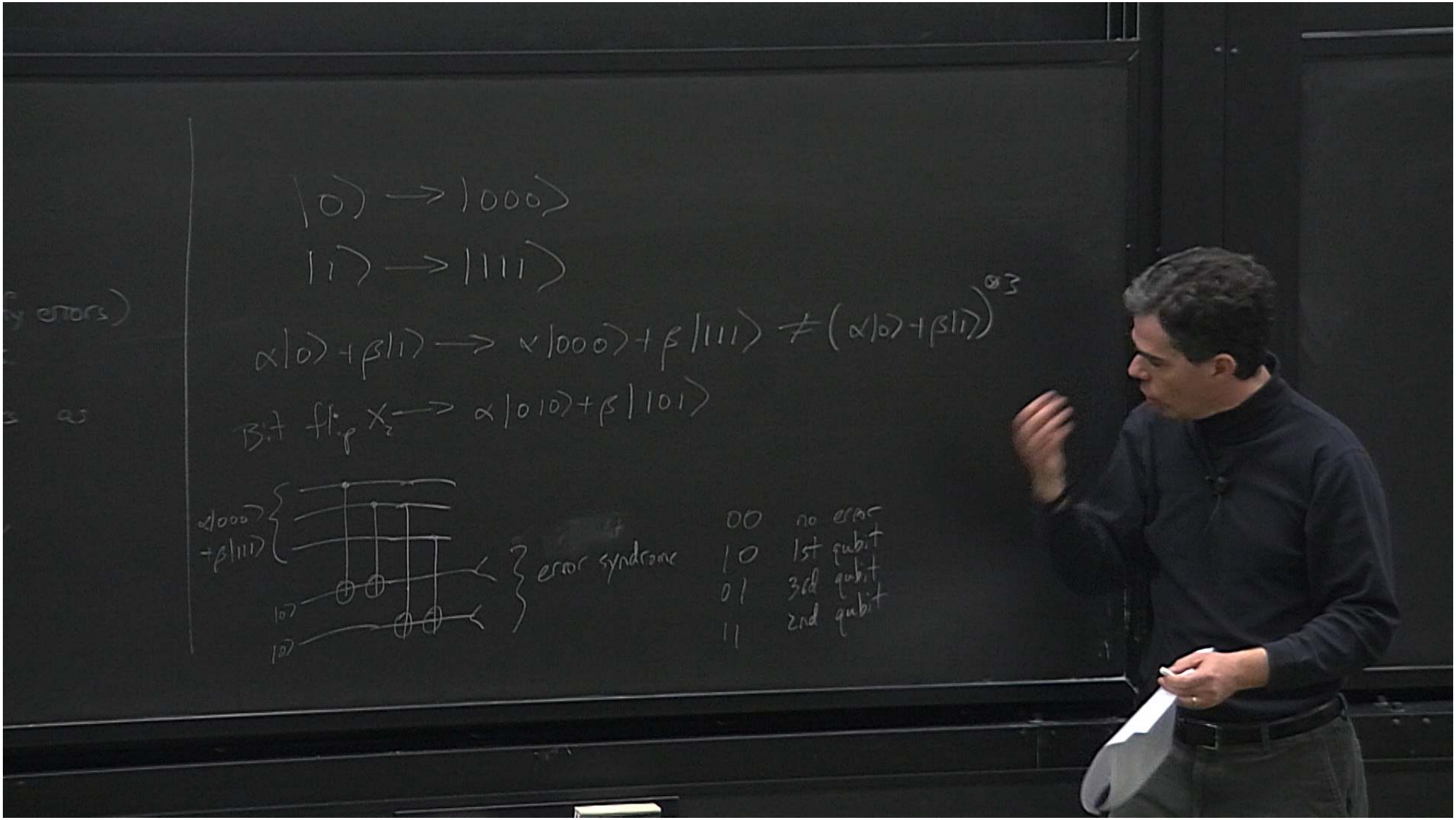
3. Must deal with phase errors as well as bit flip errors
4. Deal with continuous rotations,


Problems w/ QECCs
1. No-cloning theorem
2. Measurement (e.g. to identify errors) can destroy superpositions
3. Must deal with phase errors as well as bit flip errors
4 Deal with continuous rotations, decoherence, ...


$|0\rangle \rightarrow |000\rangle$
$|1\rangle \rightarrow |111\rangle$
$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|000\rangle + \beta|111\rangle \neq (\alpha|0\rangle + \beta|1\rangle)^{\otimes 3}$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Bit flip $X_2 \rightarrow \alpha|010\rangle + \beta|101\rangle$
$\alpha|000\rangle + \beta|111\rangle$
$|0?\rangle$
$|?0\rangle$
error syndrome
00 no error
10 1st qubit
01 3rd qubit
11 2nd qubit

$|0\rangle \overset{H}{\longleftrightarrow} |+\rangle = |0\rangle + |1\rangle$

$|1\rangle \longleftrightarrow |-\rangle = |0\rangle - |1\rangle$

Z in $|+\rangle, |-\rangle$ basis acts like X

$|0\rangle \rightarrow |+++\rangle$

$|1\rangle \rightarrow |---\rangle$

$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|+++\rangle + \beta|---\rangle$

9-qubit code:

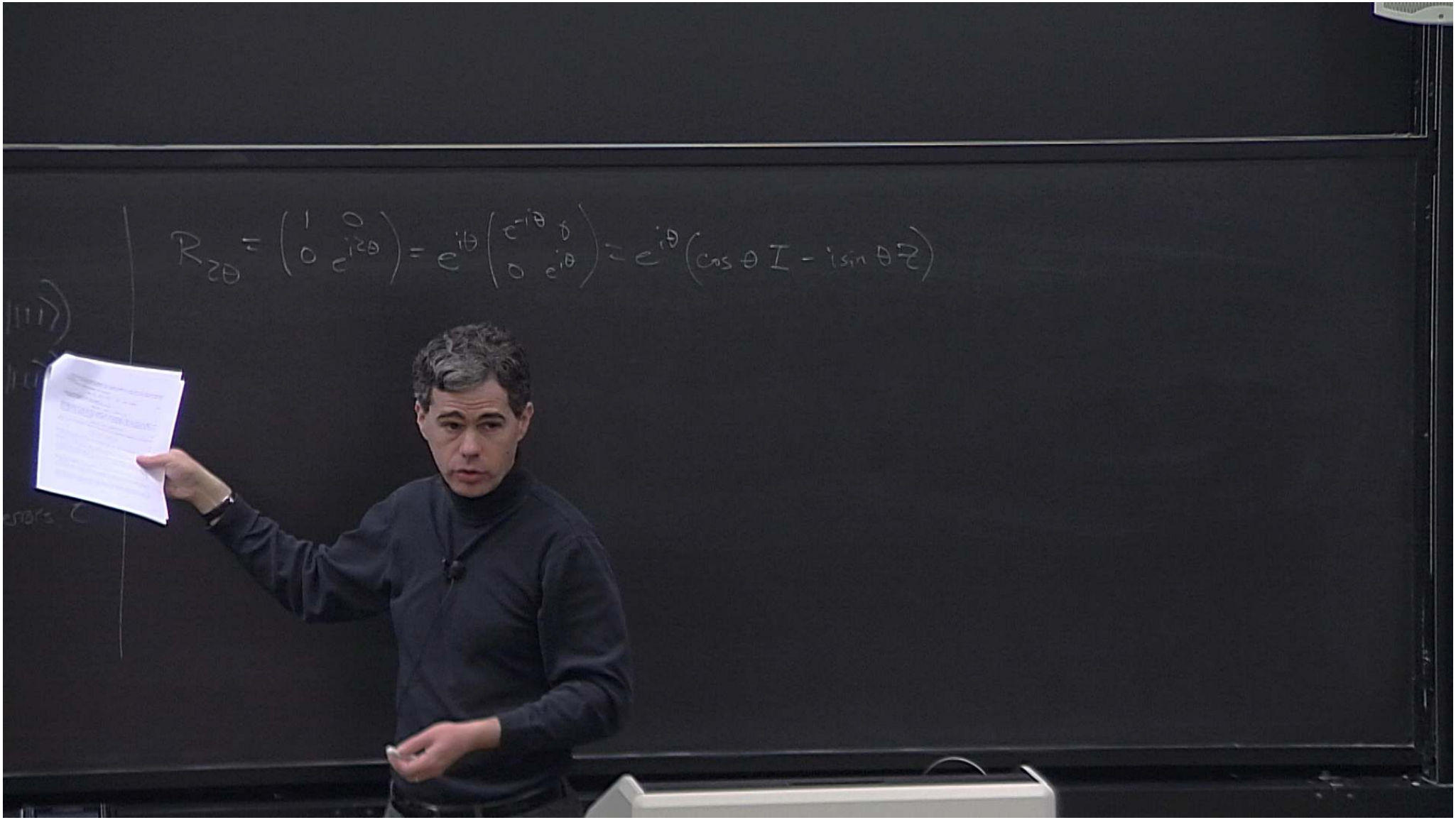
$|0\rangle \longrightarrow (|000\rangle + |111\rangle) \otimes (|000\rangle + |111\rangle) \otimes (|000\rangle + |111\rangle)$

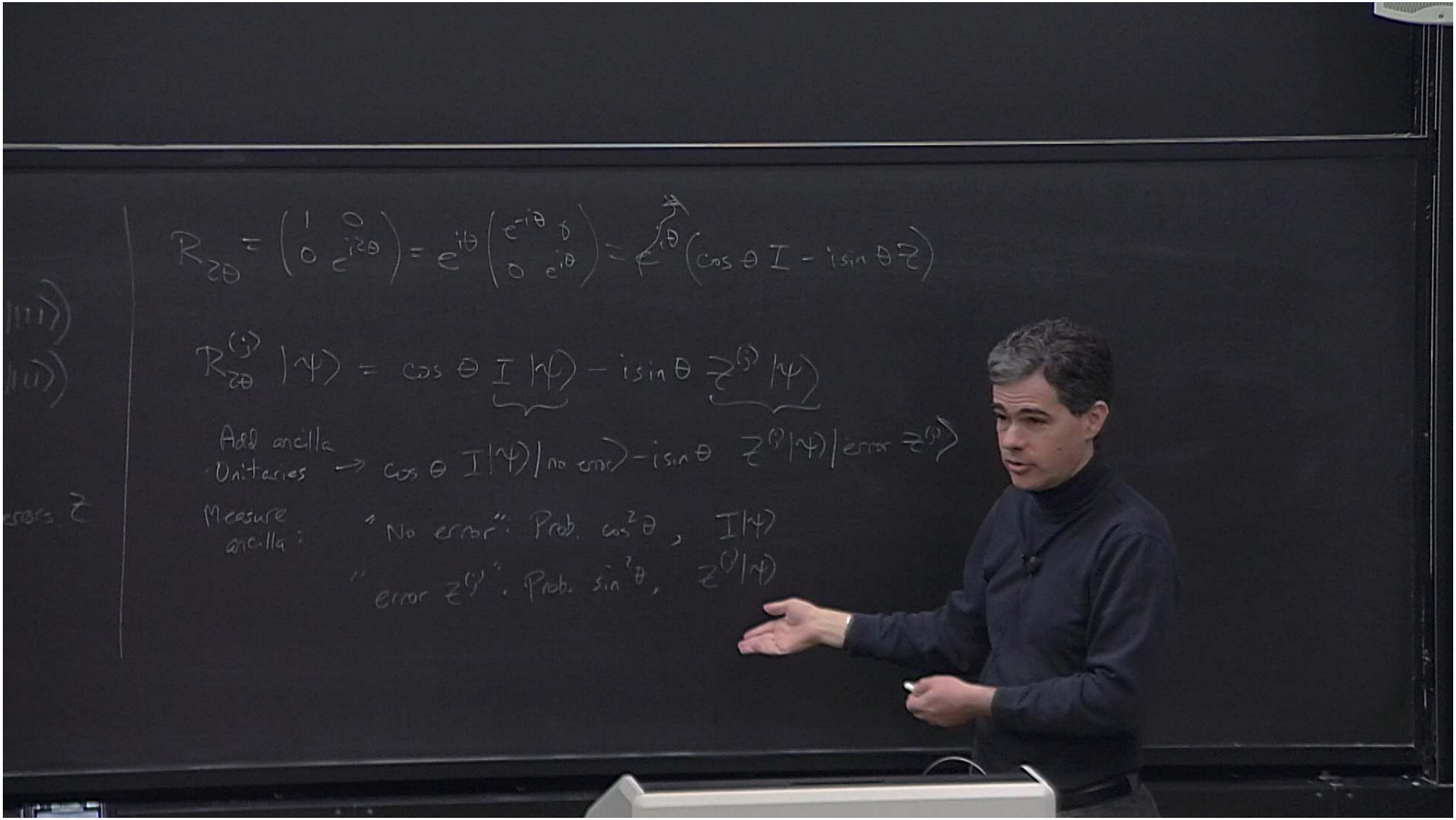
$|1\rangle \longrightarrow (|000\rangle - |111\rangle) \otimes (|000\rangle - |111\rangle) \otimes (|000\rangle - |111\rangle)$

Corrects bit flip (X) errors w/ in each set of 3

Compare phases between sets of 3 to correct phase errors (Z)

Also correct $Y = -iXZ = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$.


$R_{z\theta} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i2\theta} \end{pmatrix} = e^{i\theta} \begin{pmatrix} e^{-i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix} = e^{i\theta} (\cos\theta I - i \sin\theta Z)$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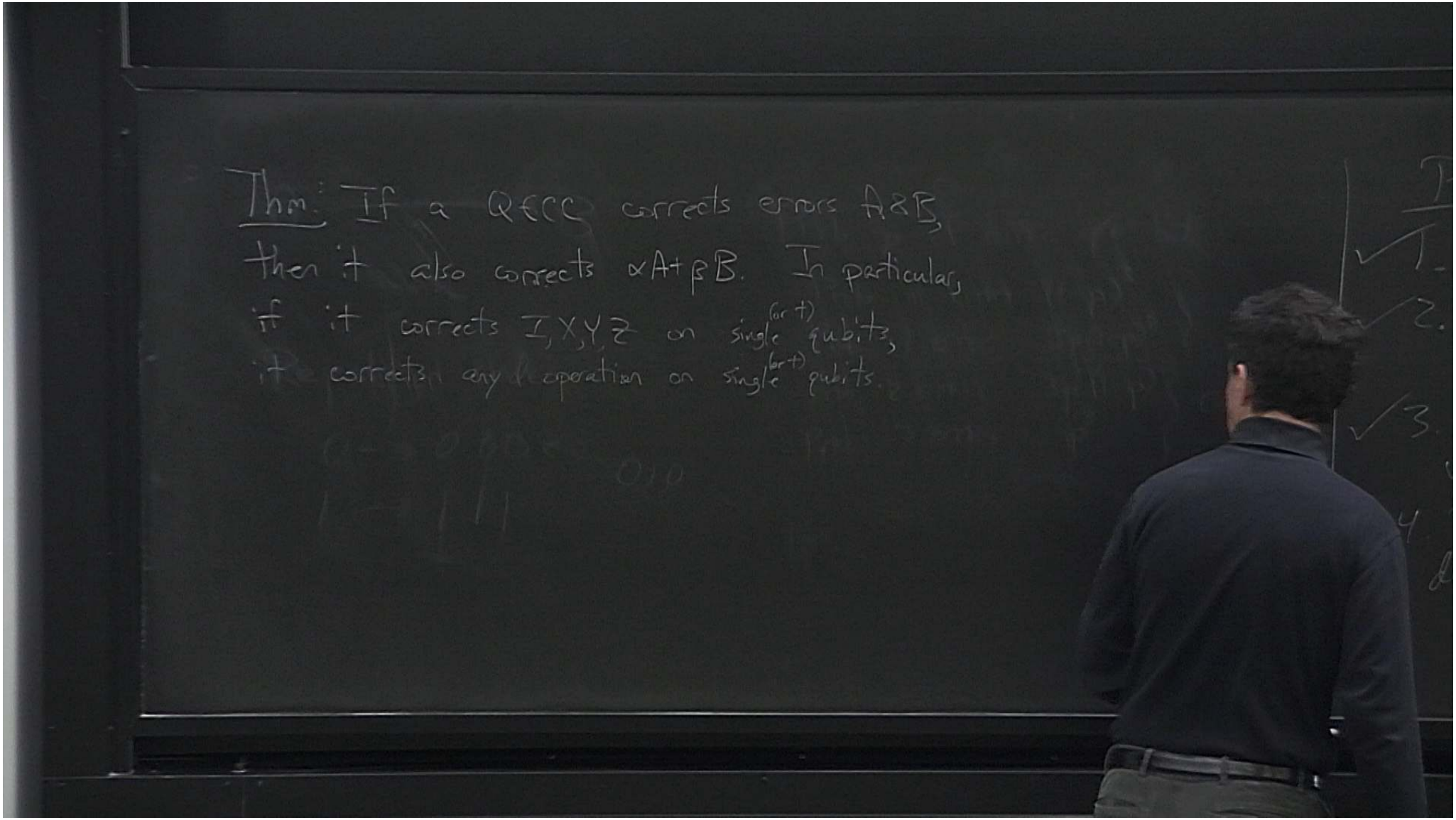
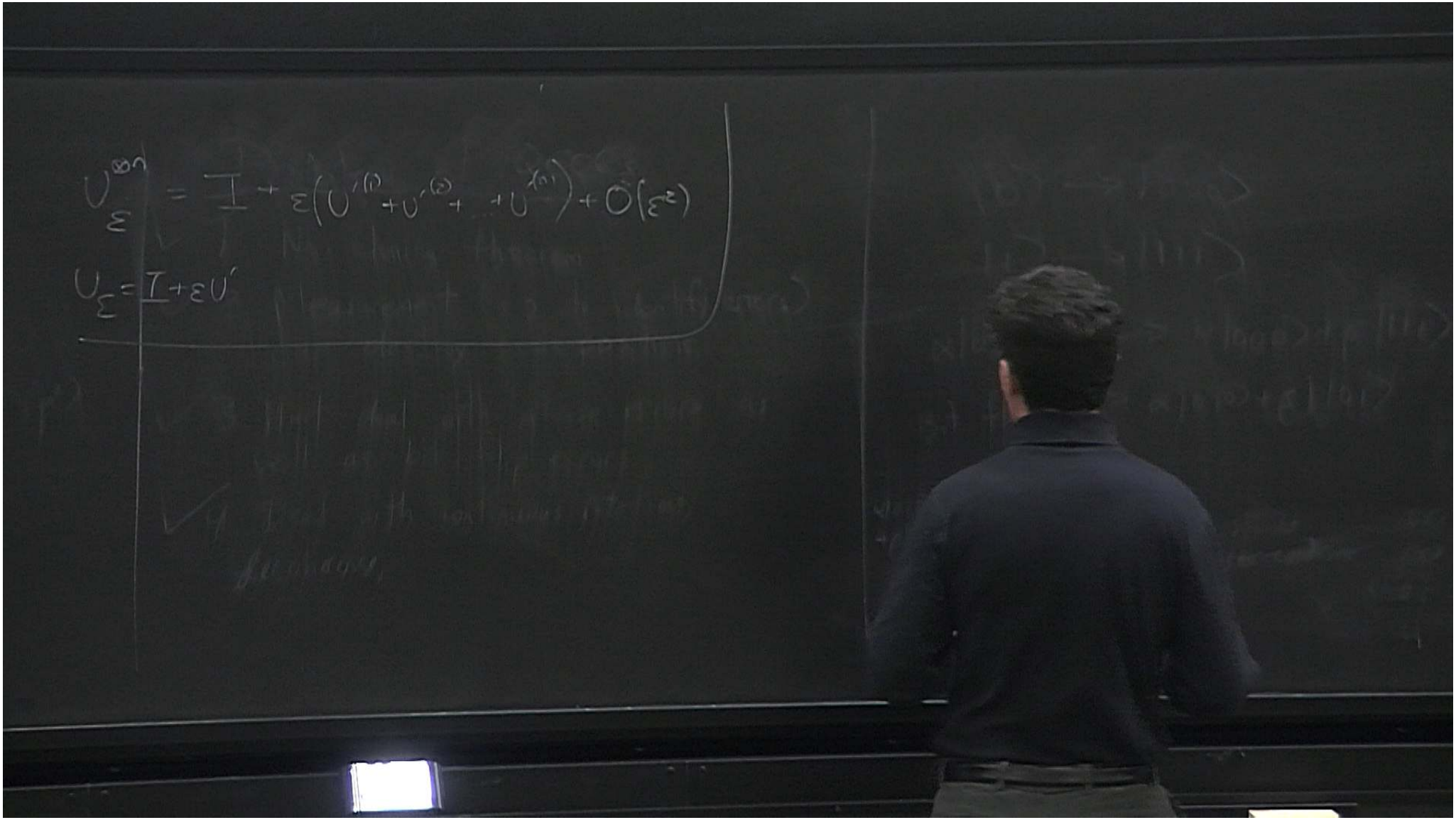
$R_{Z\theta}^{(j)} |\psi\rangle = \cos\theta I|\psi\rangle - i\sin\theta Z^{(j)}|\psi\rangle$
Add ancilla
Unitaries → $\cos\theta\, I|\psi\rangle|\text{no error}\rangle - i\sin\theta\, Z^{(j)}|\psi\rangle|\text{error } Z^{(j)}\rangle$
Measure ancilla:
"No error": Prob. $\cos^2\theta$, $I|\psi\rangle$
"error $Z^{(j)}$": Prob. $\sin^2\theta$, $Z^{(j)}|\psi\rangle$


Thm: If a QECC corrects errors A & B,
then it also corrects αA+βB. In particular,
if it corrects I, X, Y, Z on single (or t) qubits,
it corrects any operation on single (or t) qubits.

Thm: If a QECC corrects errors A & B, then it also corrects $\alpha A + \beta B$. In particular, if it corrects I, X, Y, Z on single (or t) qubits, it corrects any operation on single (or t) qubits.

CPTP map $\rho \to \sum_k A_k \rho A_k^\dagger$

$|\psi\rangle\langle\psi| \to \sum_k A_k |\psi\rangle\langle\psi| A_k^\dagger$


$U_\varepsilon^{\text{app}} = I + \varepsilon(U^{(1)} + U^{(2)} + \dots + U^{(n)}) + O(\varepsilon^2)$
$U_\varepsilon = I + \varepsilon U'$

Pauli group on n qubits

$P_n = \{$ tensor products of I, X, Y, Z on n qubits w/ phase $\pm 1, \pm i\}$.

- Any element squares to $+I$ or $-I$.
- Any two $P, Q \in P_n$ either commute $[P, Q] = 0$ or anticommute $\{P, Q\} = 0$.

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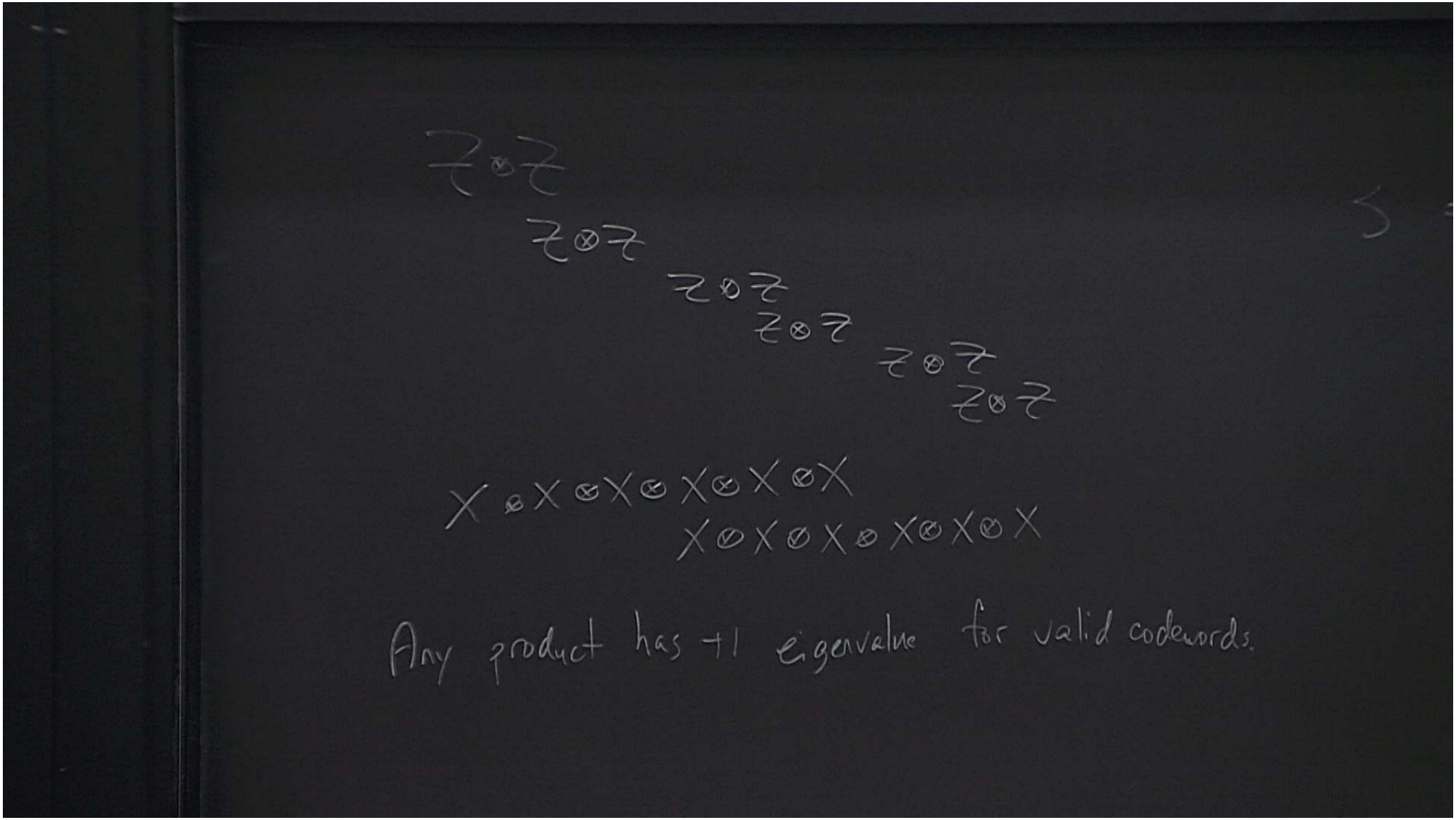
Def: Weight of $P \in P_n$ is # qubits where P is not I.


Z⊗Z
Z⊗Z
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Z⊗Z
Z⊗Z
X⊗X⊗X⊗X⊗X⊗X
X⊗X⊗X⊗X⊗X⊗X
Any product has +1 eigenvalue for valid codewords.

$$S = \{ M \mid M \in P_n,\ M|\psi\rangle = |\psi\rangle \ \forall |\psi\rangle \in \text{code} \}$$

Has following properties:

- Does not contain $-I$.
- Is a group: $M, N \in S \Rightarrow MN|\psi\rangle = M|\psi\rangle = |\psi\rangle$ ✓
- Is an Abelian group: $M, N \in S \rightarrow [M,N]|\psi\rangle = (MN - NM)|\psi\rangle = 0$. ✓

Paulis commute or anticommute $\Rightarrow (M,N) = 0$.

codewords.