Title: PSI 2017/2018 - Quantum Information - Lecture 9

Date: Mar 02, 2018 11:30 AM

URL: http://pirsa.org/18030022

Abstract:

Grover's algorithm:

$\Theta: \mathbb{Z}_N \to \mathbb{Z}_2$, Does there exist x_0 s.t. $\Theta(x_0)=1$?

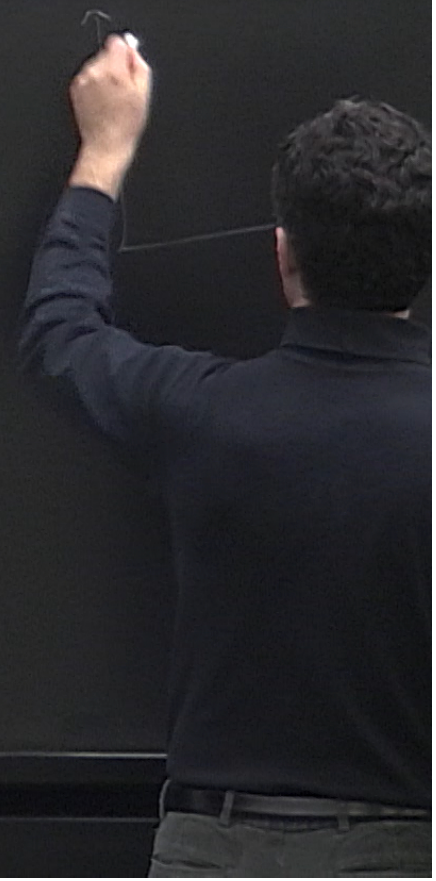
(Assume: if x_0 exists, it is unique)

1. Initialize to $|\psi_0\rangle = |U\rangle = H^{\otimes n}|0\ 0\rangle$

2. Apply $G = (-1)^U (-1)^0$ M times

$(-1)^U = H^{\otimes n} F_0 H^{\otimes n}$, $F_0|0\ 0\rangle = |0\ 0\rangle$, $F_0|a\rangle = -|a\rangle$

$(-1)^0 |x\rangle = (-1)^{\Theta(x)} |x\rangle$



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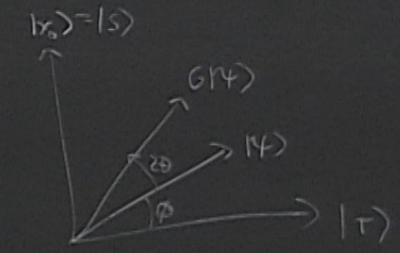
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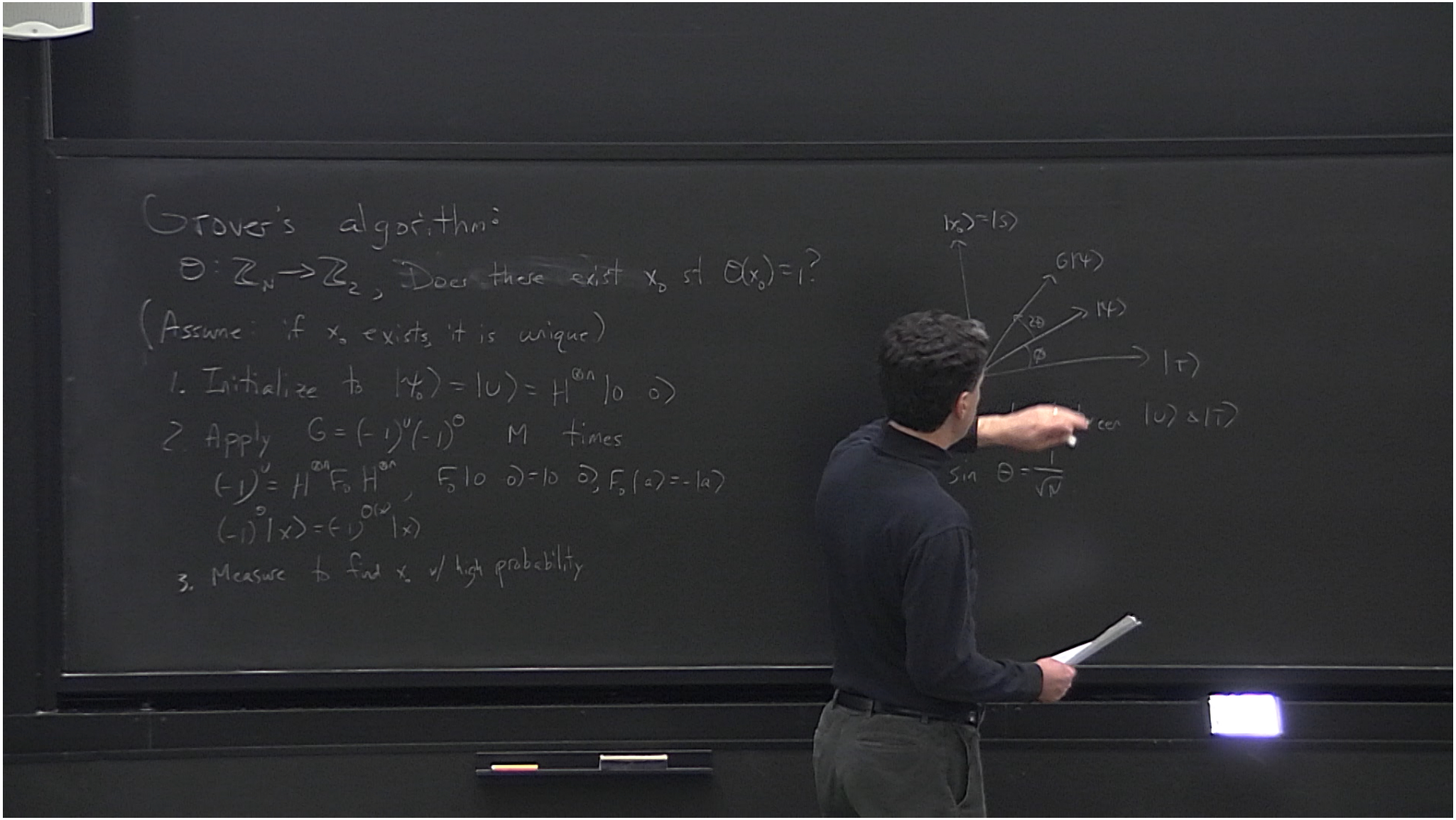
$(-1)^U = H^{\otimes n} F_0 H^{\otimes n}$, $F_0|0\ 0\rangle = |0\ 0\rangle$; $F_0|a\rangle = -|a\rangle$

$|x\rangle \to (-1)^{\Theta(x)}|x\rangle$



θ angle between $|U\rangle$ & $|\tau\rangle$

$\sin\theta = \frac{1}{\sqrt{N}}$


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$(-1)^U = H^{\otimes n} F_0 H^{\otimes n}$, $F_0|0\ 0\rangle = |0\ 0\rangle$, $F_0|a\rangle = -|a\rangle$
$(-1)^0|x\rangle = (-1)^{0(x)}|x\rangle$
3. Measure to find x_0 w/ high probability
$|x_0\rangle = |s\rangle$
$G|\psi\rangle$
$|\psi\rangle$
2θ
θ
$|T\rangle$
$|U\rangle$ & $|T\rangle$
$\sin\theta = \frac{1}{\sqrt{N}}$

Grover's algorithm:

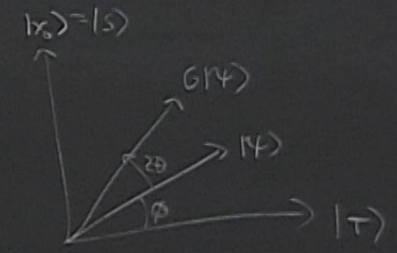
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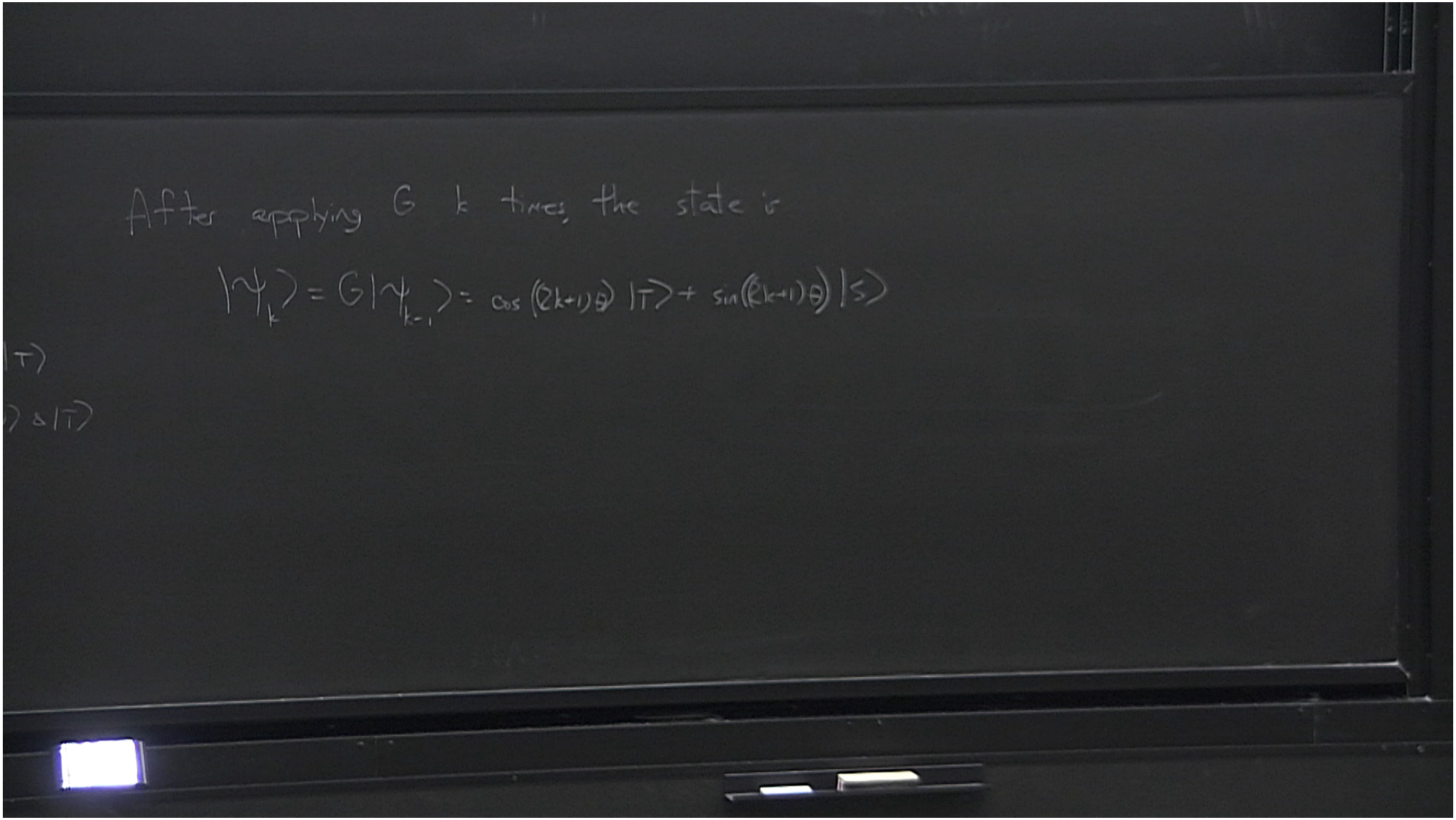
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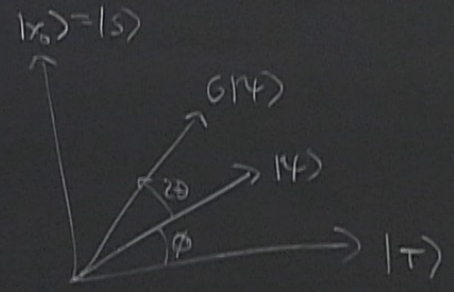


θ angle between $|U\rangle$ & $|T\rangle$

$\sin\theta = \frac{1}{\sqrt{N}}$

After appl


After applying G k times, the state is
$|\psi_k\rangle = G|\psi_{k-1}\rangle = \cos((2k+1)\theta)|T\rangle + \sin((2k+1)\theta)|S\rangle$



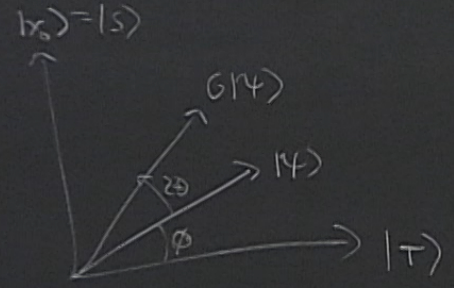
θ angle between $|U\rangle$ & $|\tau\rangle$

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After applying G k times, the state is

$$|\psi_k\rangle = G|\psi_{k-1}\rangle = \cos((2k+1)\theta)\,|\tau\rangle + \sin((2k+1)\theta)\,|s\rangle$$

$$(2k+1)\theta \approx \frac{\pi}{2} \Rightarrow k \approx \frac{\pi}{4\theta} = \frac{\pi}{4}\sqrt{N}$$



θ angle between $|U\rangle$ & $|\bar{T}\rangle$

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After applying G k times, the state is

$$|\psi_k\rangle = G|\psi_{k-1}\rangle = \cos((2k+1)\theta)\,|T\rangle + \sin((2k+1)\theta)\,|s\rangle$$

$$(2M+1)\theta \approx \frac{\pi}{2} \Rightarrow M \approx \frac{\pi}{4\theta} = \frac{\pi}{4}\sqrt{N}$$

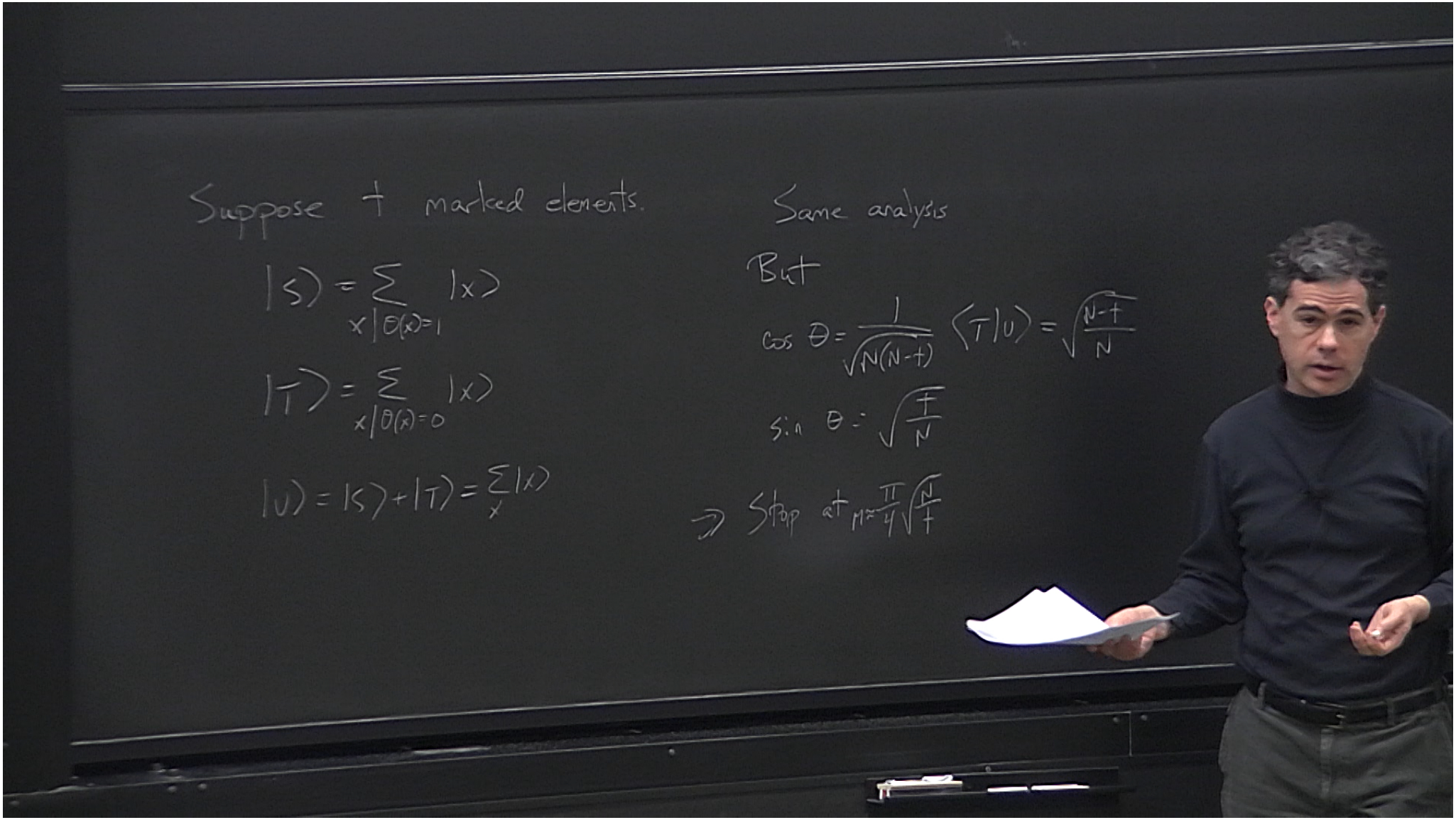
M is integer close to this

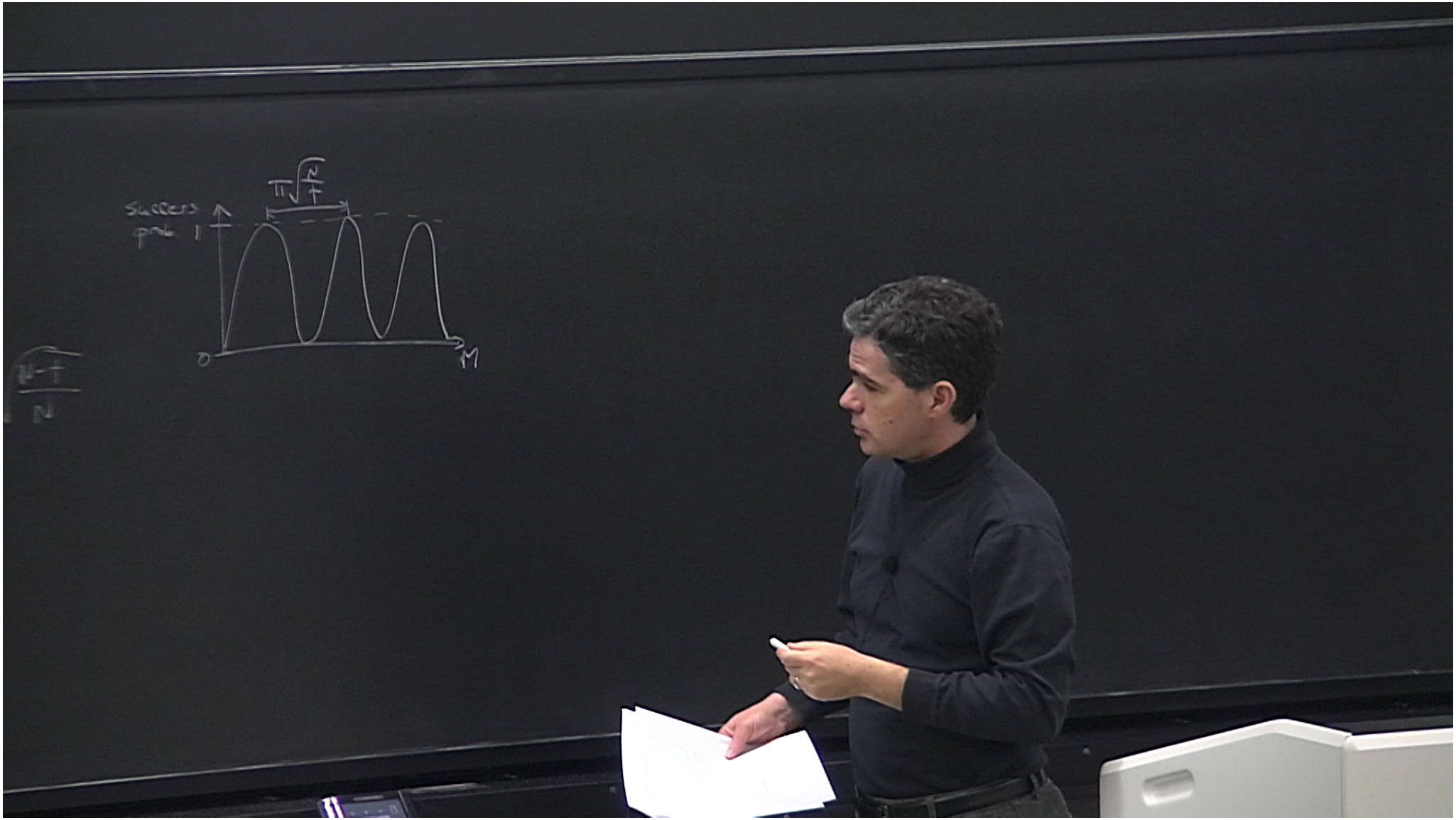
Suppose t marked elements.

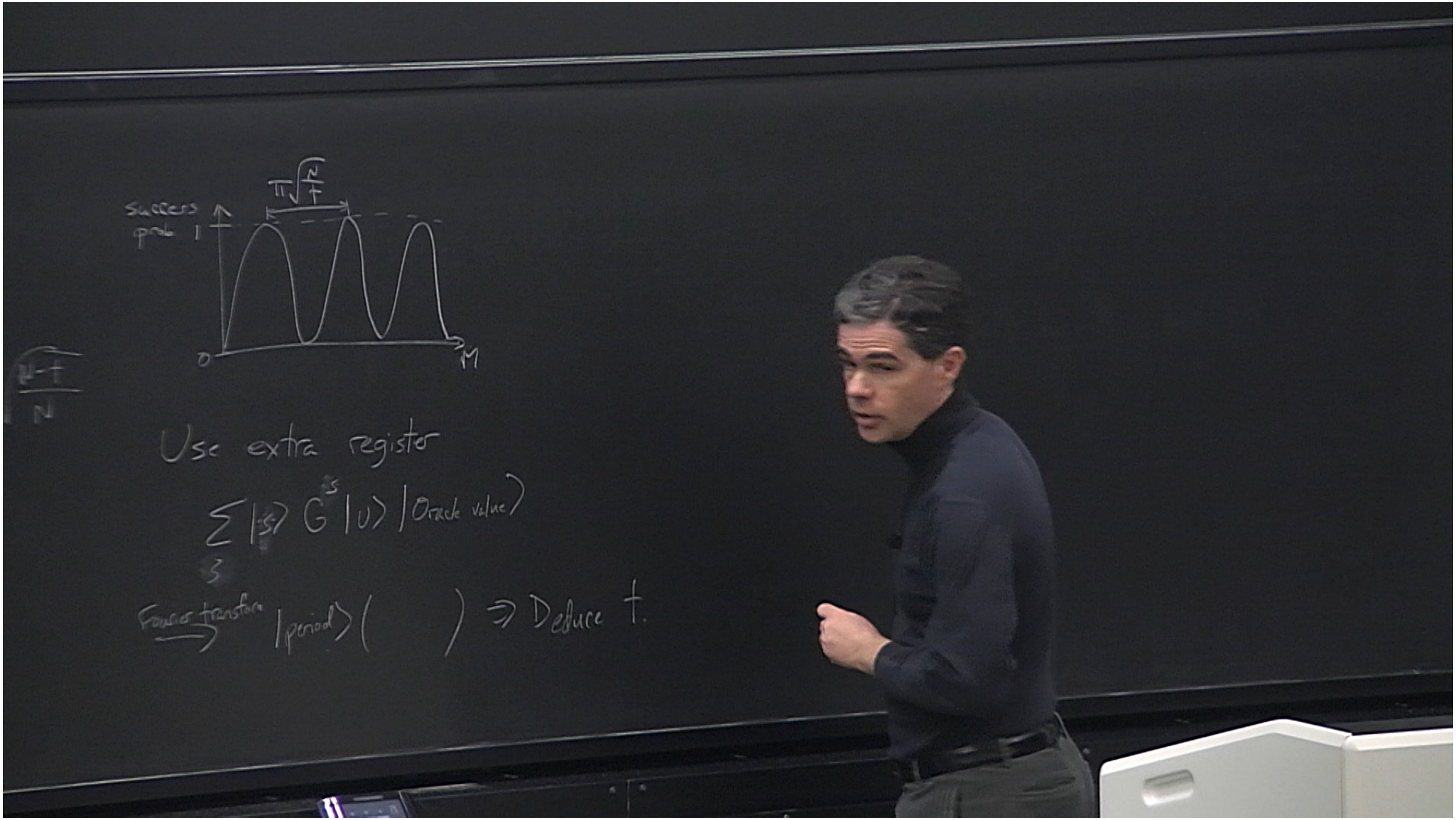
$$|S\rangle = \sum_{x \mid \theta(x)=1} |x\rangle$$

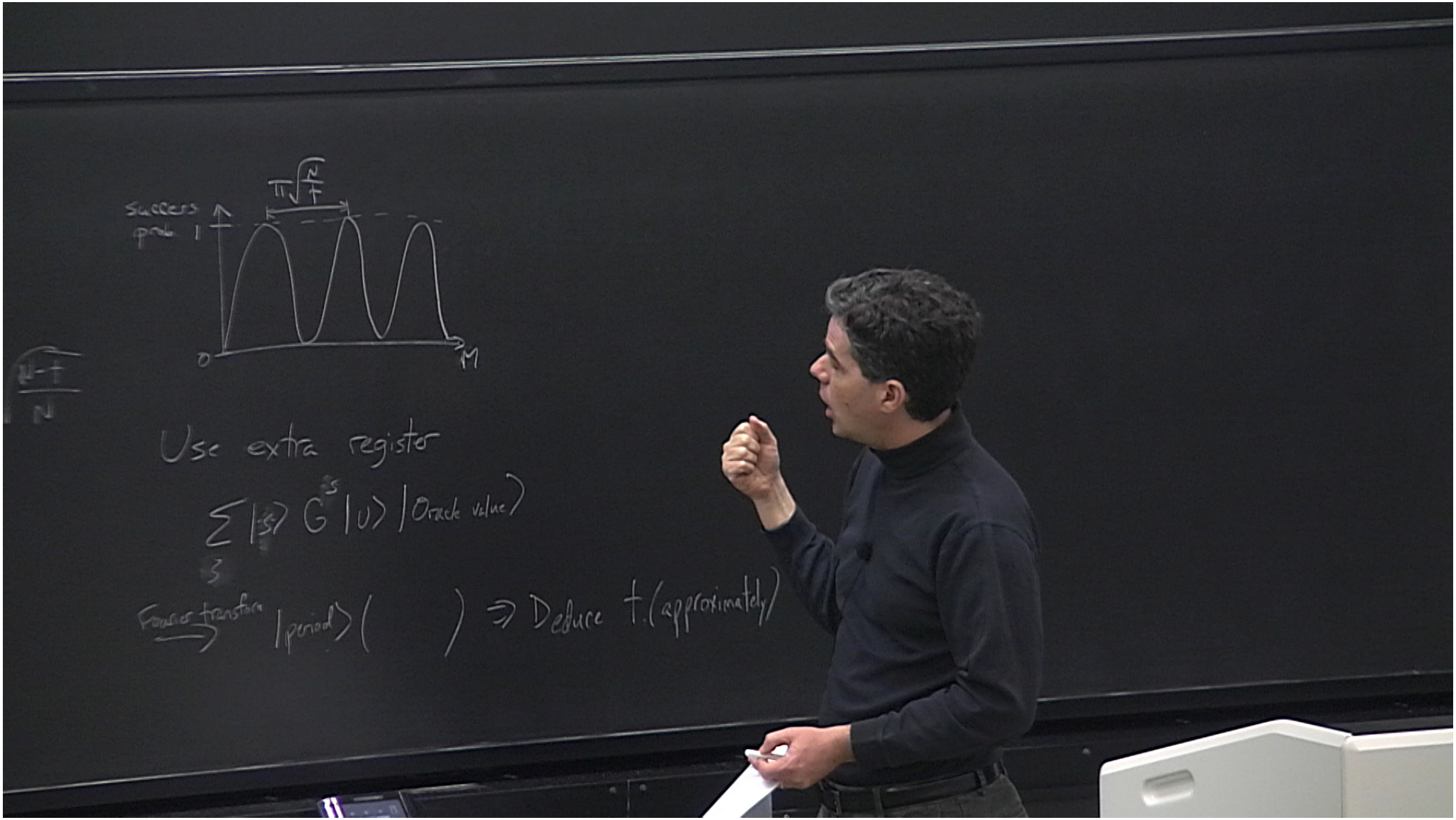
$$|T\rangle = \sum_{x \mid \theta(x)=0} |x\rangle$$

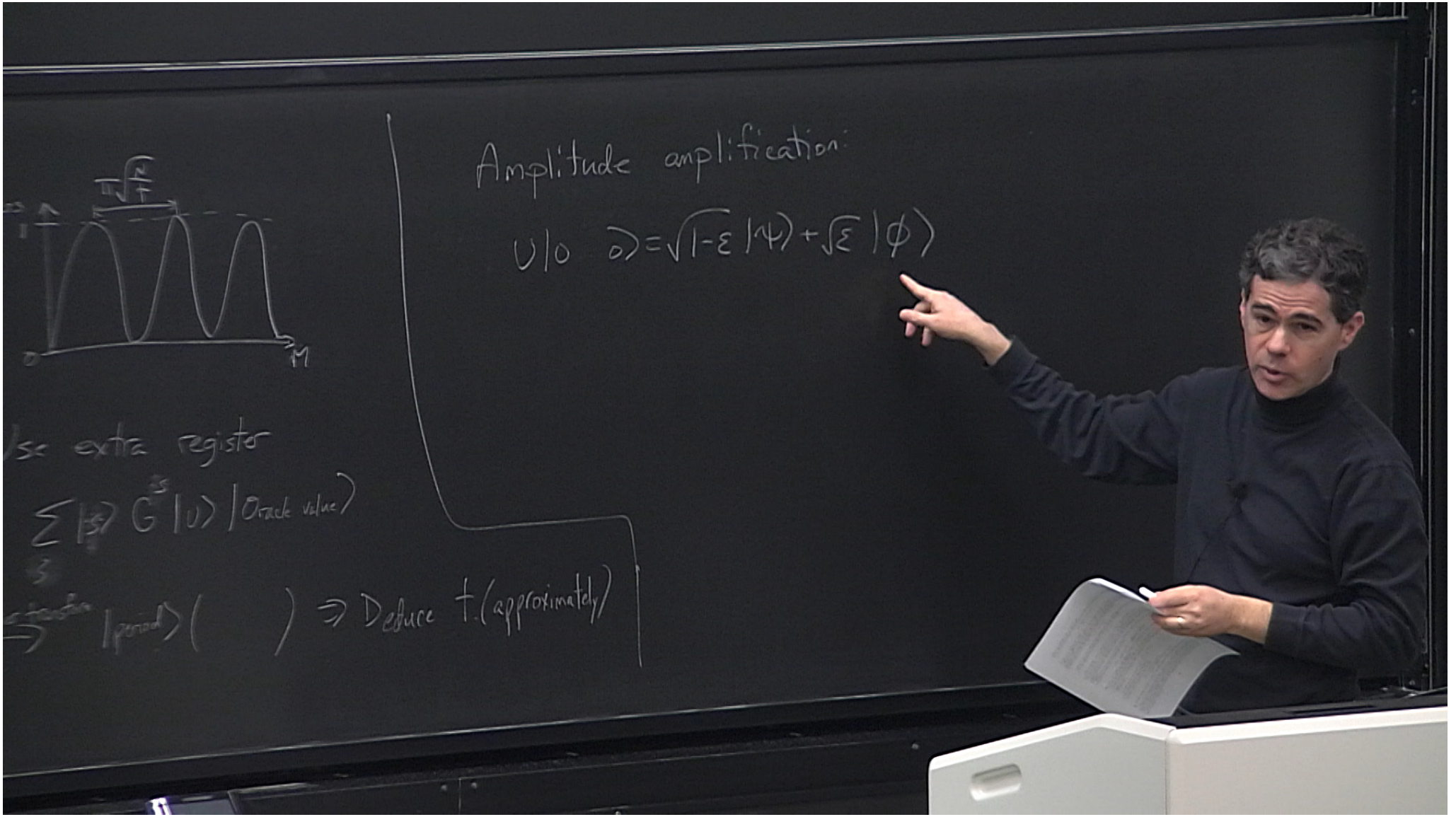
$$|U\rangle = |S\rangle + |T\rangle = \sum_{x} |x\rangle$$

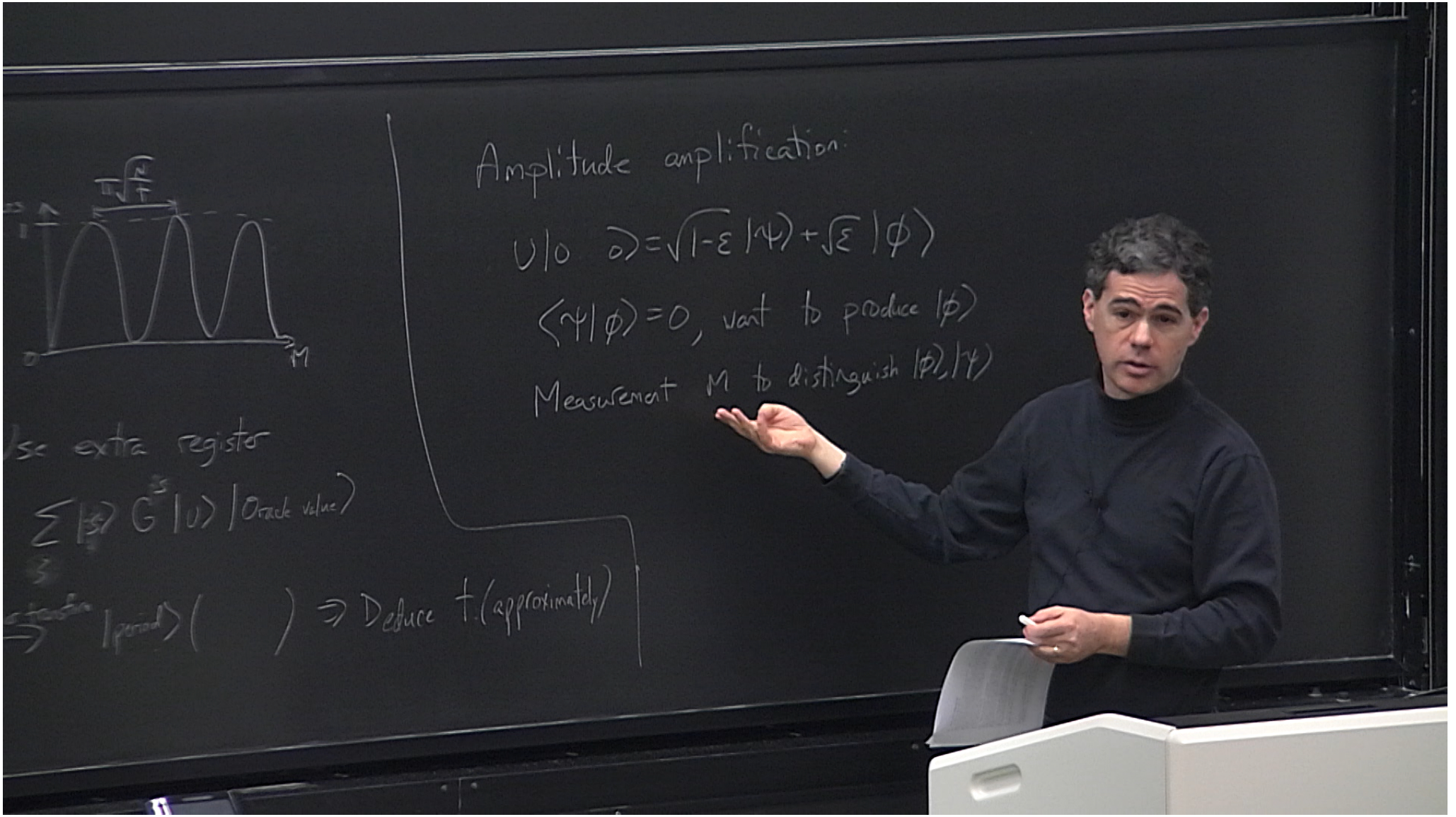

Suppose t marked elements.
$|S\rangle = \sum_{x|\theta(x)=1} |x\rangle$
$|T\rangle = \sum_{x|\theta(x)=0} |x\rangle$
$|U\rangle = |S\rangle + |T\rangle = \sum_x |x\rangle$
Same analysis
But
$\cos\theta = \frac{1}{\sqrt{N(N-t)}}$ $\langle T|U\rangle = \sqrt{\frac{N-t}{N}}$
$\sin\theta = \sqrt{\frac{t}{N}}$
$\Rightarrow$ Stop at $M \approx \frac{\pi}{4}\sqrt{\frac{N}{t}}$

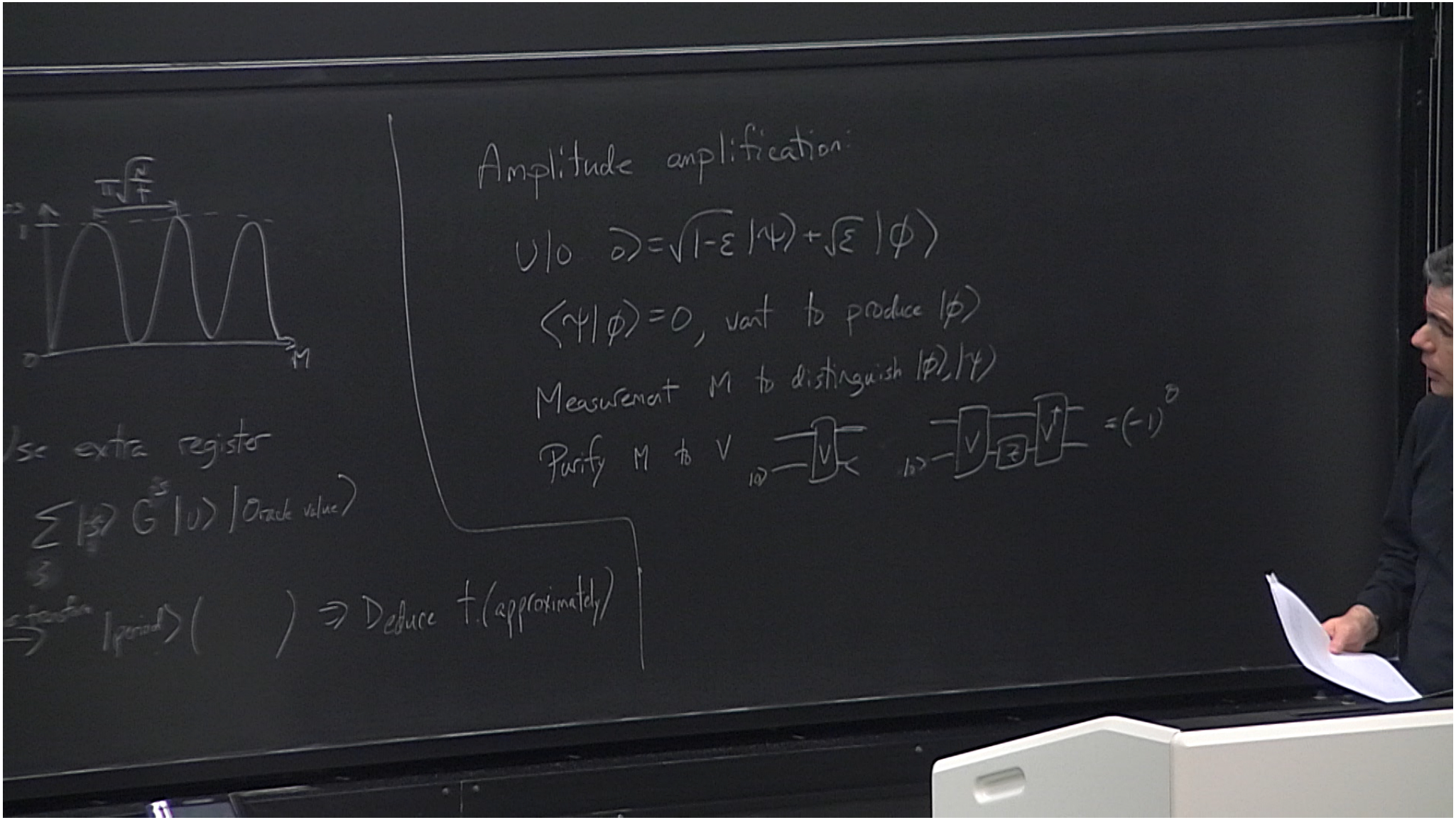

success prob
1
0
M

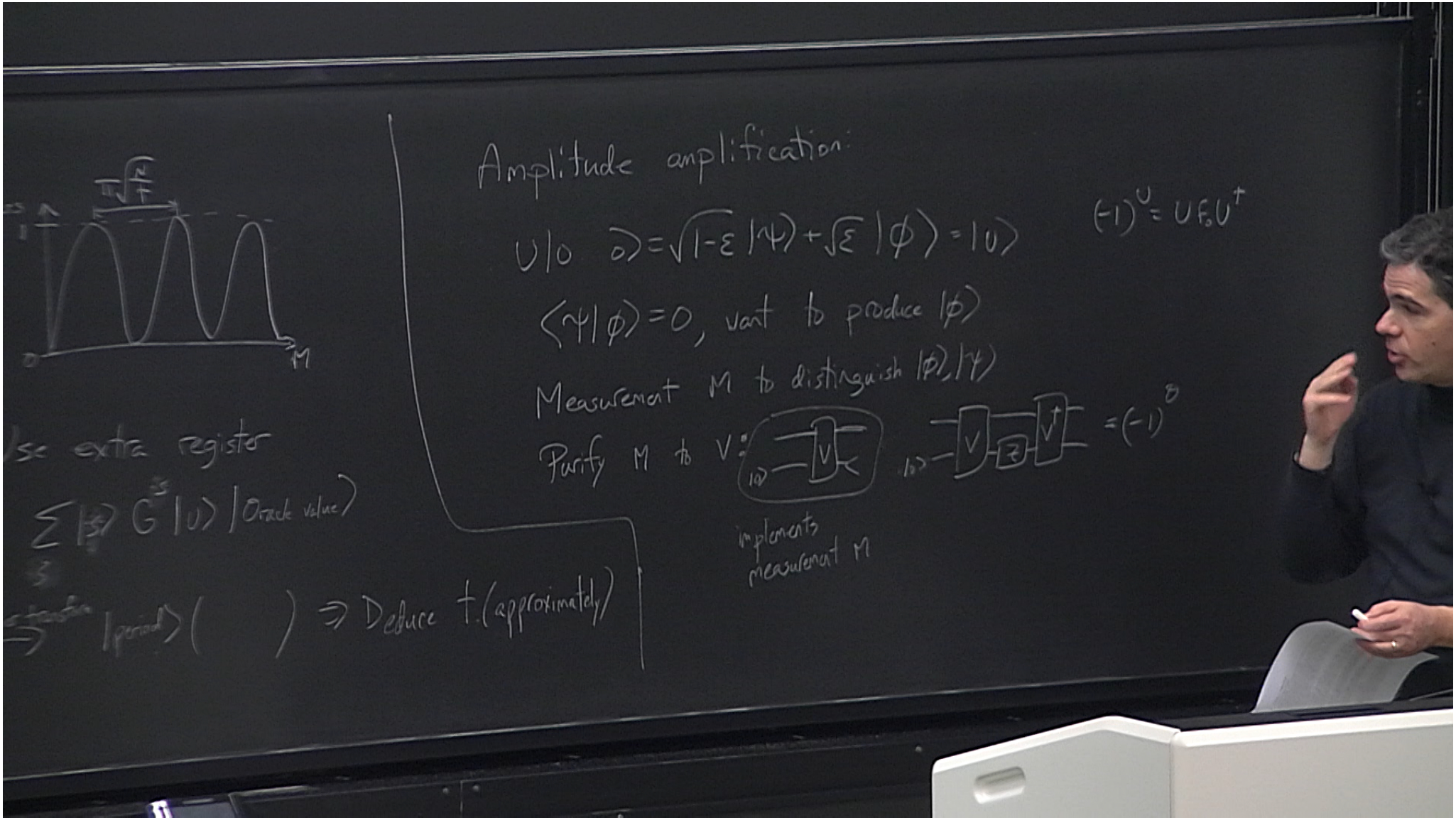

Success prob
1
0
M
Use extra registor
Fourier transform
Deduce t.

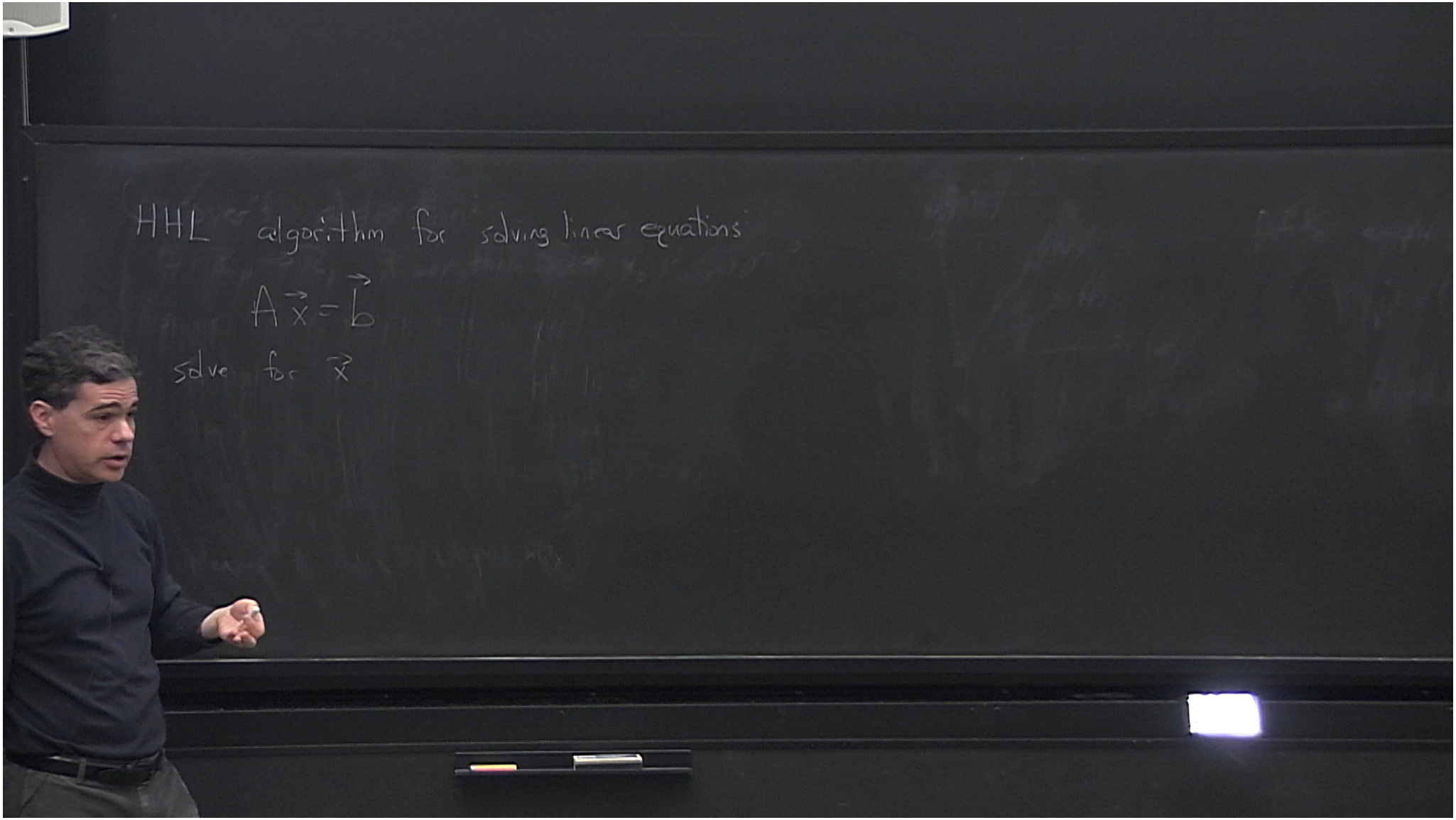

Success prob 1
0
M
Use extra register
Fourier transform
Deduce t (approximately)

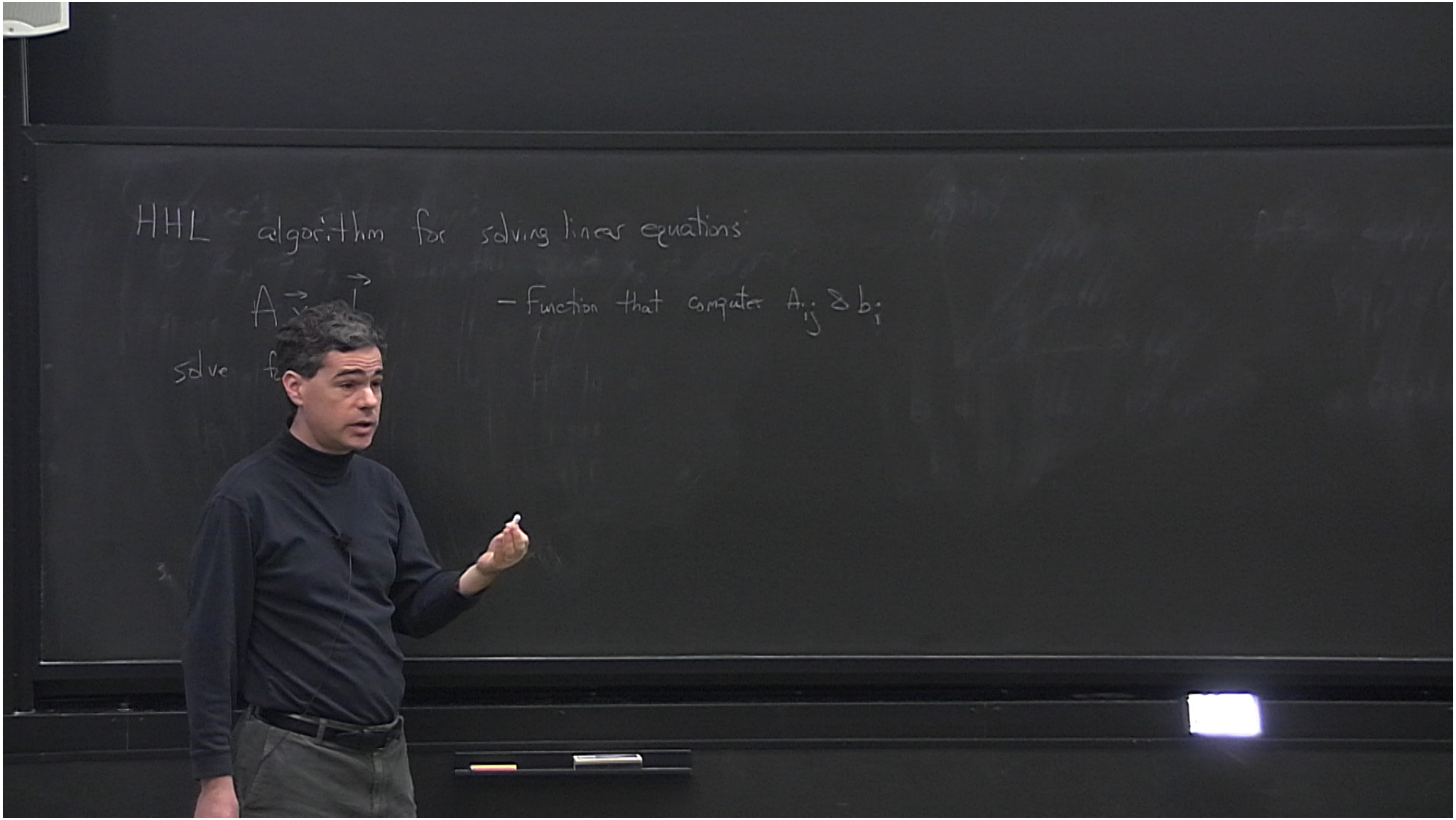

Amplitude amplification:
$U|0\rangle = \sqrt{1-\varepsilon}\,|\psi\rangle + \sqrt{\varepsilon}\,|\phi\rangle$
$\frac{\pi}{4}\sqrt{\frac{1}{t}}$
M
Use extra register
$\sum |s\rangle\, G^{2^j} |U\rangle\, |\text{Oracle value}\rangle$
$|\text{period}\rangle$ ( ) ⇒ Deduce t (approximately)

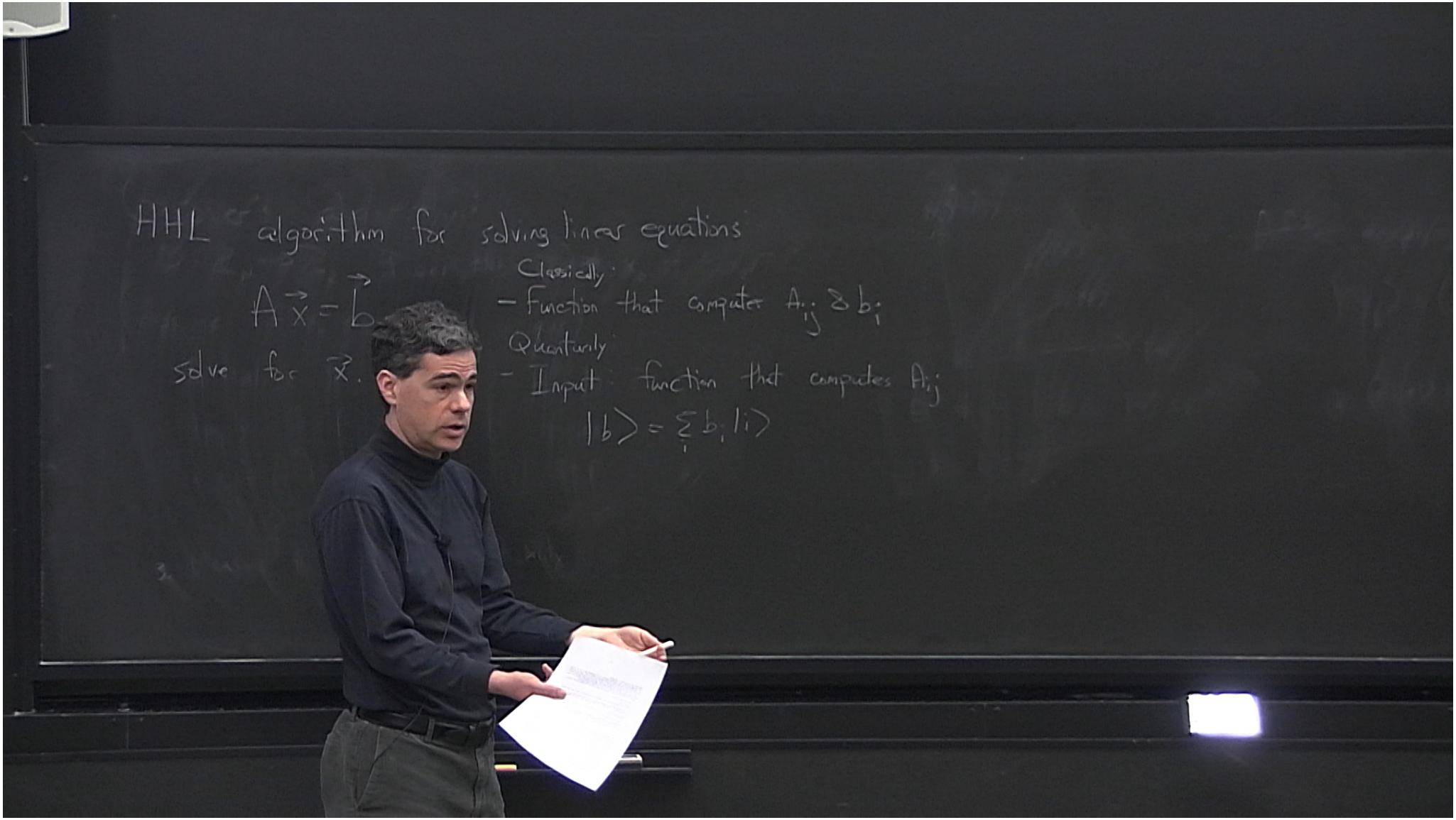

Amplitude amplification:
$U|0\rangle = \sqrt{1-\varepsilon}\,|\psi\rangle + \sqrt{\varepsilon}\,|\phi\rangle$
$\langle\psi|\phi\rangle = 0$, want to produce $|\phi\rangle$
Measurement M to distinguish $|\phi\rangle, |\psi\rangle$
$\sqrt{r}$
M
Use extra register
$\sum |s\rangle G^s |U\rangle |0_{\text{trial value}}\rangle$
$|\text{period}\rangle$ ( ) ⇒ Deduce r (approximately)

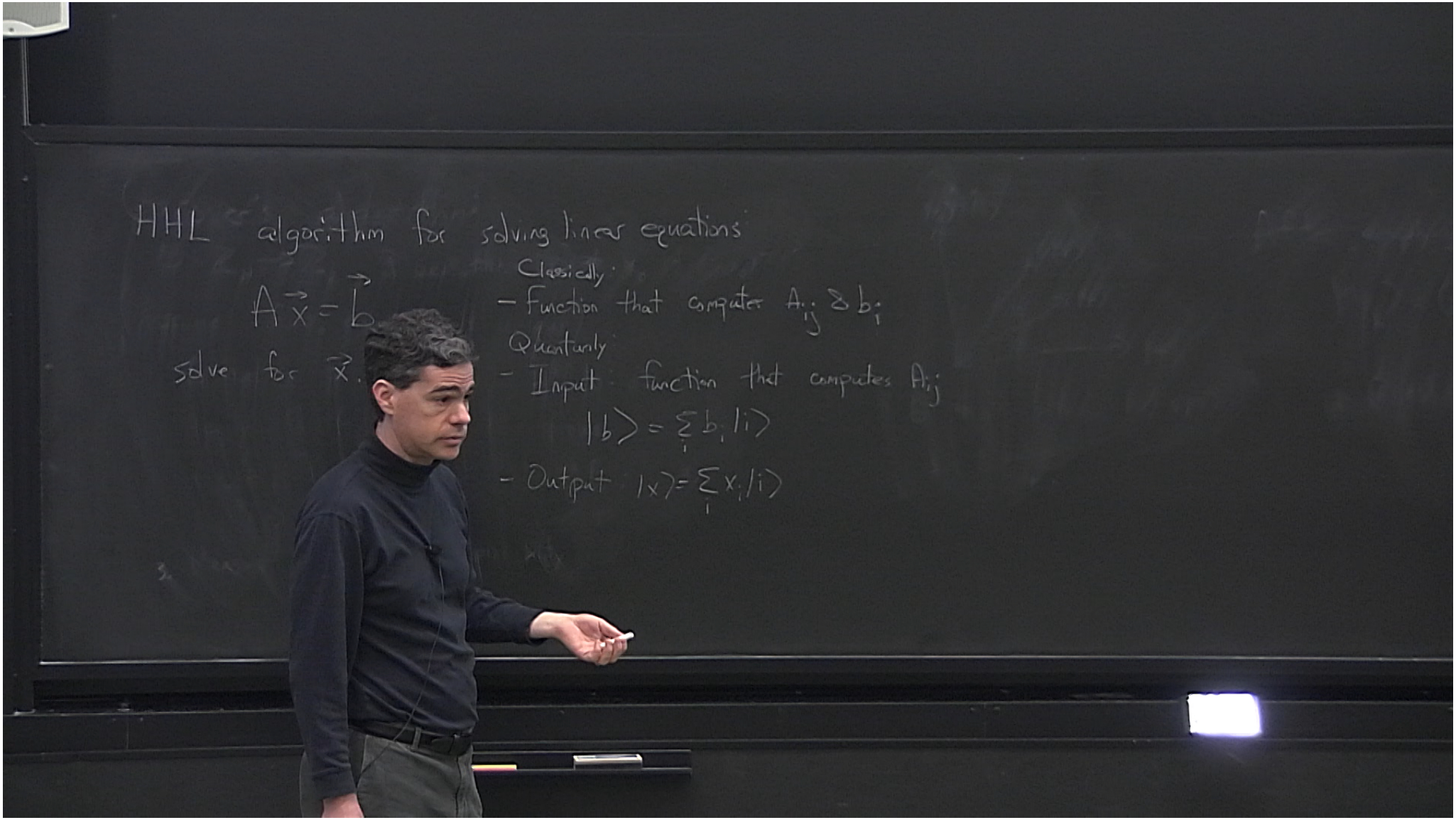

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Measurement M to distinguish $|\phi\rangle, |\psi\rangle$
Purify M to V
$= (-1)^{\delta}$
$\sum |s\rangle G^{s} |U\rangle |$ Oracle value $\rangle$
$|$ period $\rangle$ ( ) $\Rightarrow$ Deduce t (approximately)

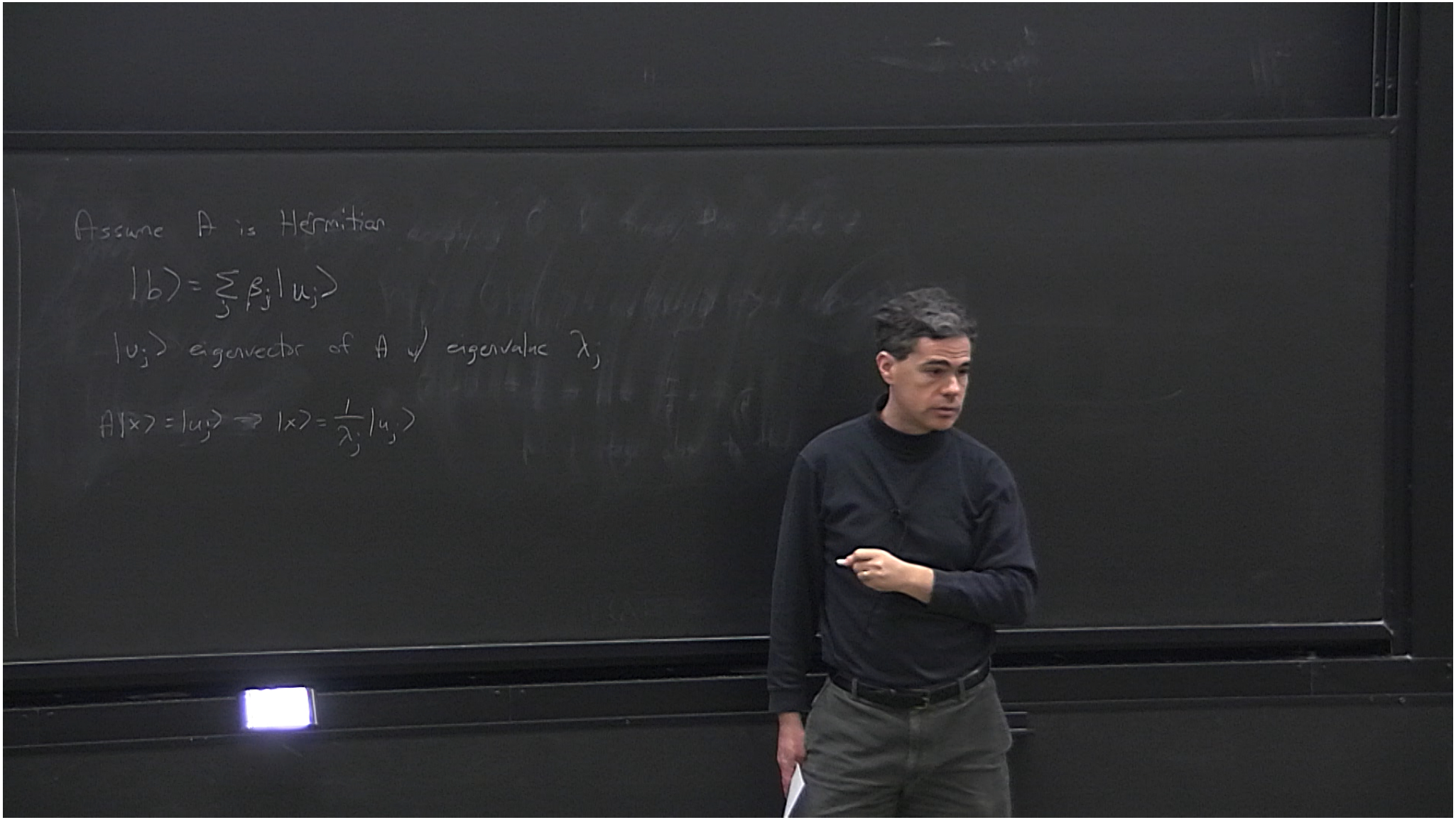

Amplitude amplification
$U|0\rangle = \sqrt{1-\varepsilon}\,|\psi\rangle + \sqrt{\varepsilon}\,|\phi\rangle = |\psi\rangle$
$\langle\psi|\phi\rangle = 0$, want to produce $|\phi\rangle$
Measurement M to distinguish $|\phi\rangle, |\psi\rangle$
Purify M to V:
$|0\rangle$
V
$|0\rangle$
V
V†
$= (-1)^B$
implements measurement M
$(-1)^U = U f_0 U^\dagger$
$\pi\sqrt{\frac{N}{t}}$
M
Use extra register
$\sum_x |x\rangle G^x |U\rangle |0\text{racle value}\rangle$
$|\text{period}\rangle$ ( ) ⇒ Deduce t (approximately)

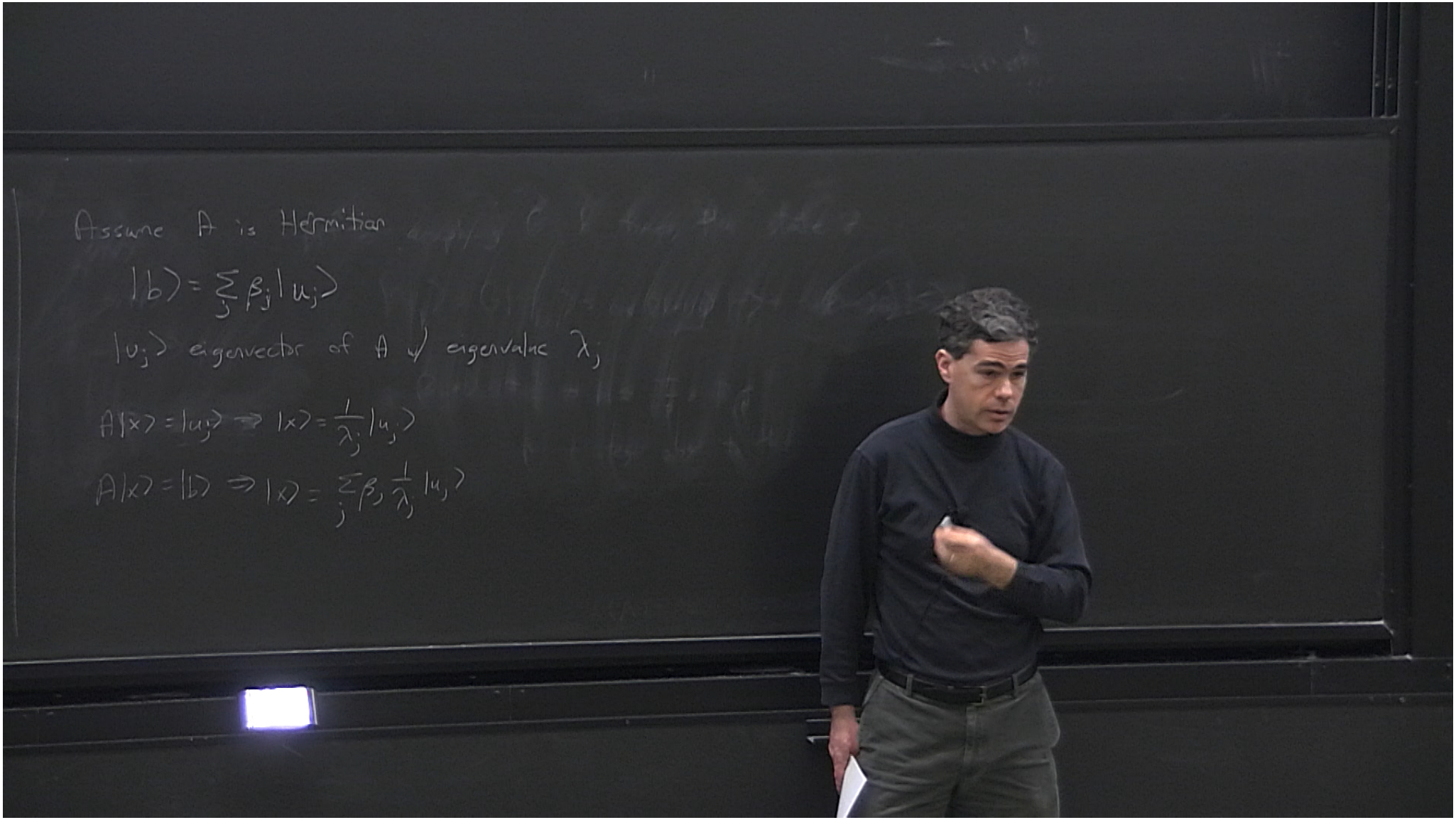

HHL algorithm for solving linear equations
$A\vec{x} = \vec{b}$
solve for $\vec{x}$


HHL algorithm for solving linear equations
- function that computes A_{ij} & b_i
solve


HHL algorithm for solving linear equations
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Classically:
- Function that computes A_{ij} & b_i
Quantumly
- Input function that computes A_{ij}
$|b\rangle = \sum_i b_i |i\rangle$


HHL algorithm for solving linear equations
$A\vec{x} = \vec{b}$
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Classically
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Quantumly
- Input function that computes A_{ij}
$|b\rangle = \sum_i b_i |i\rangle$
- Output $|x\rangle = \sum_i x_i |i\rangle$


Assume A is Hermitian
$|b\rangle = \sum_j \beta_j |u_j\rangle$
$|u_j\rangle$ eigenvector of A w/ eigenvalue λ_j
$A|x\rangle = |u_j\rangle \rightarrow |x\rangle = \frac{1}{\lambda_j}|u_j\rangle$


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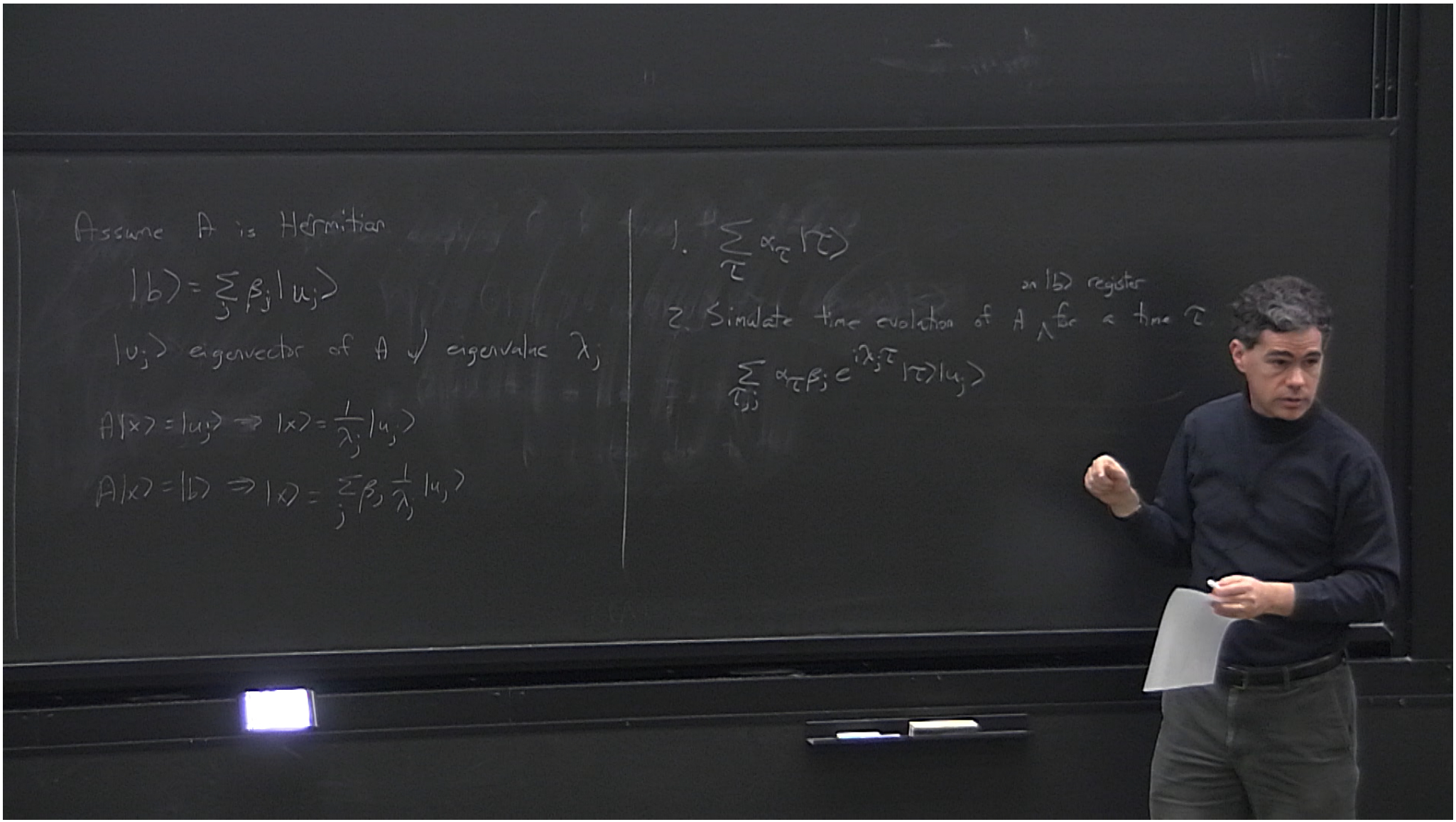
$A|x\rangle = |u_j\rangle \Rightarrow |x\rangle = \frac{1}{\lambda_j}|u_j\rangle$

$A|x\rangle = |b\rangle \Rightarrow |x\rangle = \sum_j \beta_j \frac{1}{\lambda_j}|u_j\rangle$

1. $\sum_\tau \alpha_\tau |\tau\rangle$ on $|b\rangle$ register

2. Simulate time evolution of A for a time τ

$\sum_{\tau,j} \alpha_\tau \beta_j e^{i\lambda_j \tau} |\tau\rangle|u_j\rangle$



Assume A is Hermitian

$|b\rangle = \sum_j \beta_j |u_j\rangle$

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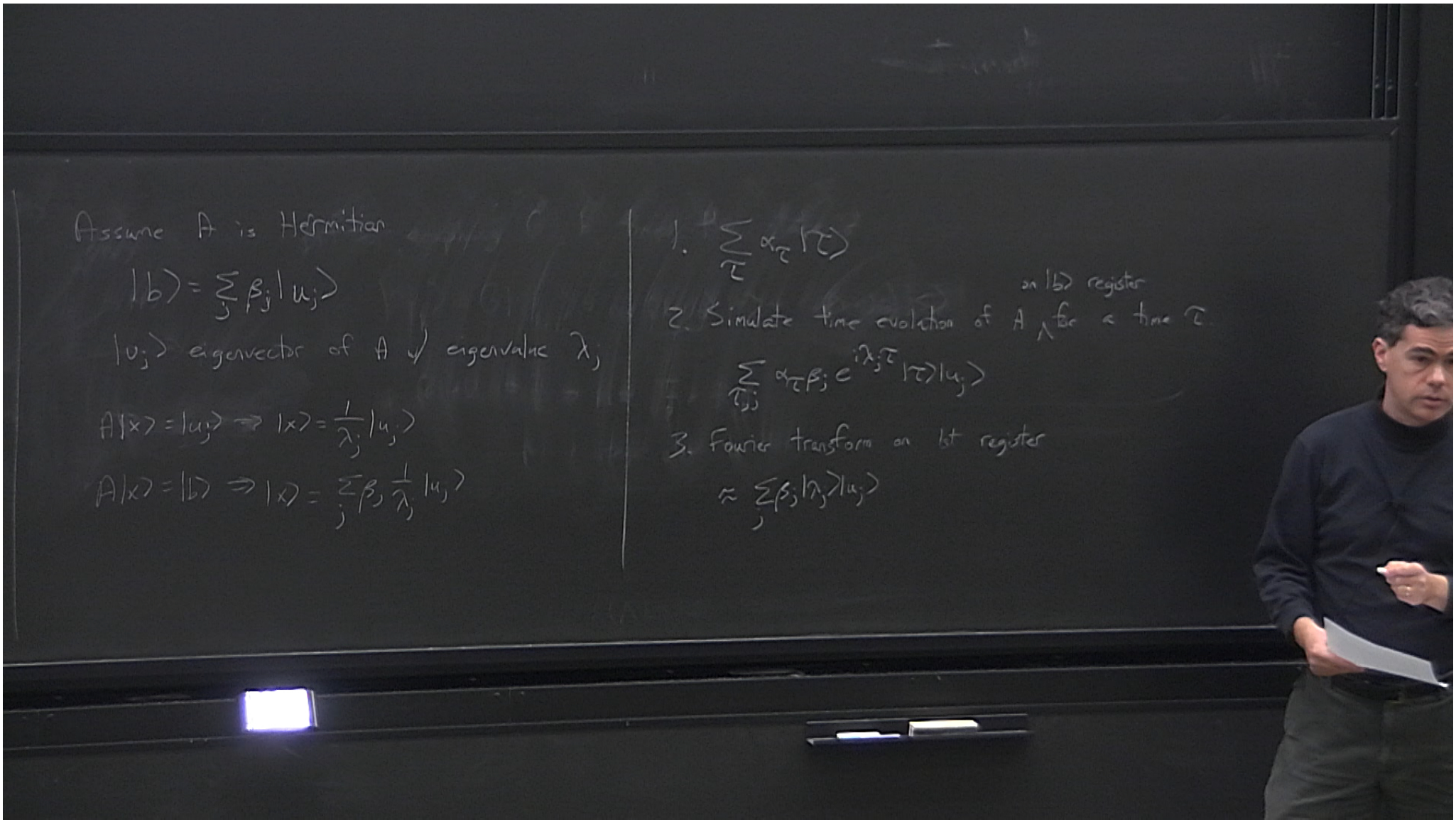
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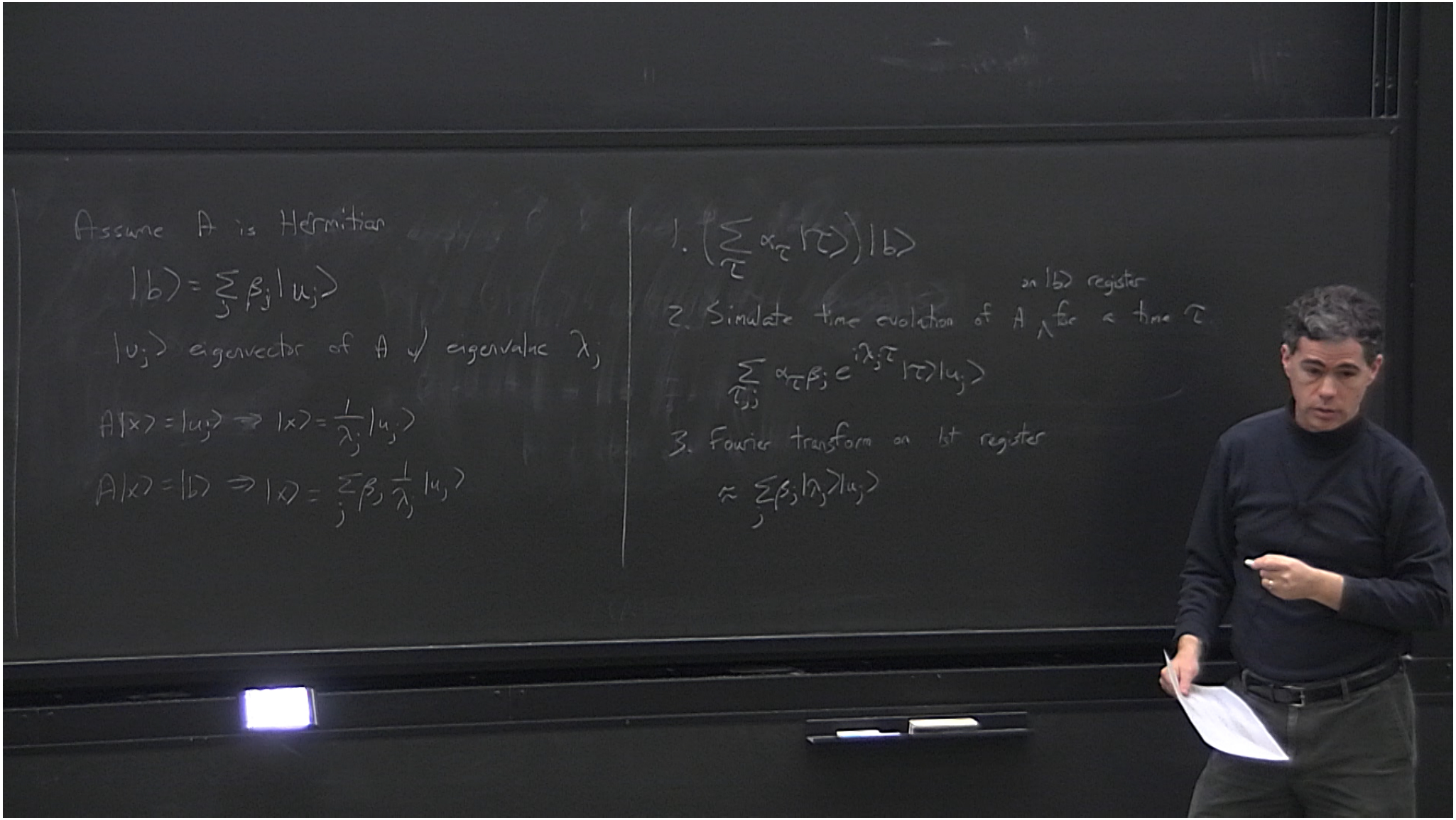
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3. Fourier transform on 1st register

$\approx \sum_j \beta_j |\lambda_j\rangle|u_j\rangle$




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$A|x\rangle = |b\rangle \Rightarrow |x\rangle = \sum_j \beta_j \frac{1}{\lambda_j}|u_j\rangle$
1. $\left(\sum_\tau \alpha_\tau |\tau\rangle\right)|b\rangle$
2. Simulate time evolution of A on $|b\rangle$ register for a time τ
$\sum_{\tau,j} \alpha_\tau \beta_j e^{i\lambda_j \tau} |\tau\rangle|u_j\rangle$
3. Fourier transform on 1st register
$\approx \sum_j \beta_j |\lambda_j\rangle|u_j\rangle$

is Hermitian

$\sum_j \beta_j |u_j\rangle$

eigenvector of A w/ eigenvalue λ_j

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3. Fourier transform on 1st register

$$\approx \sum_j \beta_j |\lambda_j\rangle|u_j\rangle$$

4. Probabilistically $\rightarrow \sum_j \frac{\beta_j}{\lambda_j}|\lambda_j\rangle|u_j\rangle$

5. Undo steps 2&3 $\rightarrow \propto \sum_j \frac{\beta_j}{\lambda_j}|u_j\rangle$

is Hermitian

$\sum_j \beta_j |u_j\rangle$

eigenvector of A w/ eigenvalue λ_j

$\Rightarrow |x\rangle = \frac{1}{\lambda_j}|u_j\rangle$

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2. Simulate time evolution of A for a time τ on $|b\rangle$ register

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3. Fourier transform on 1st register

$\approx \sum_j \beta_j |\lambda_j\rangle|u_j\rangle$

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Ratio biggest eigenvalue to smallest = "condition #"

is Hermitian

$\sum_j \beta_j |u_j\rangle$

eigenvector of A w/ eigenvalue λ_j

$\Rightarrow |x\rangle = \frac{1}{\lambda_j}|u_j\rangle$

$\Rightarrow |x\rangle = \sum_j \beta_j \frac{1}{\lambda_j}|u_j\rangle$

1. $\left(\sum_\tau \alpha_\tau |\tau\rangle\right)|b\rangle$

2. Simulate time evolution of A for a time τ. on $|b\rangle$ register

$$\sum_{\tau,j} \alpha_\tau \beta_j e^{i\lambda_j \tau} |\tau\rangle|u_j\rangle$$

3. Fourier transform on 1st register

$$\approx \sum_j \beta_j |\lambda_j\rangle|u_j\rangle$$

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Ratio biggest eigenvalue to smallest = "condition #"

is Hermitian

$\sum_j \beta_j |u_j\rangle$

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$$\approx \sum_j \beta_j |\lambda_j\rangle|u_j\rangle$$

4. Probabilistically $\rightarrow \sum_j \frac{\beta_j}{\lambda_j}|\lambda_j\rangle|u_j\rangle$
5. Undo steps 2&3 $\rightarrow \approx \sum_j \frac{\beta_j}{\lambda_j}|u_j\rangle$

- Ratio biggest eigenvalue to smallest = "condition #"
- Sparsity of A