Title: PSI 2017/2018 - String Theory - Lecture 12

Date: Mar 07, 2018 10:15 AM

URL: http://pirsa.org/18030018

Abstract:

$$u' = F(u,\theta) = F_1(u) + \theta F_2(u)$$

$$\theta' = G(u,\theta) = G_1(u) + \theta G_2(u)$$

$$D_\theta = J\, D_{\theta'}$$

SUPER-DIFF

$$v_f = f(u)\left(\partial_\theta - \theta\partial_u\right)$$

$$\{v_f, D_\theta\} = \dot{f}\,\theta D_\theta$$

$$\{v_f, v_g\} = \boxed{-2fg\,\partial_u - \left(\dot{f} g + f\dot{g}\right)\theta\partial_\theta}$$

SDIFF-INVARIANT ACTIONS ?

NEED METRIC + GRAVITINO

$$S[x,\psi,e,\chi] \qquad \text{SDIFF-INVARIANT}$$

$$\Downarrow \text{ GAUGE-FIX}$$

$$S[x,\psi,\tau,\zeta]$$

$$y^\mu(v,\theta) = x^\mu(v) + \theta\psi^\mu(v)$$

$$[\hat{Q}, y^\mu] = \psi^\mu - \theta p^\mu = [\partial_\theta - \theta\partial_v]\, y^\mu$$

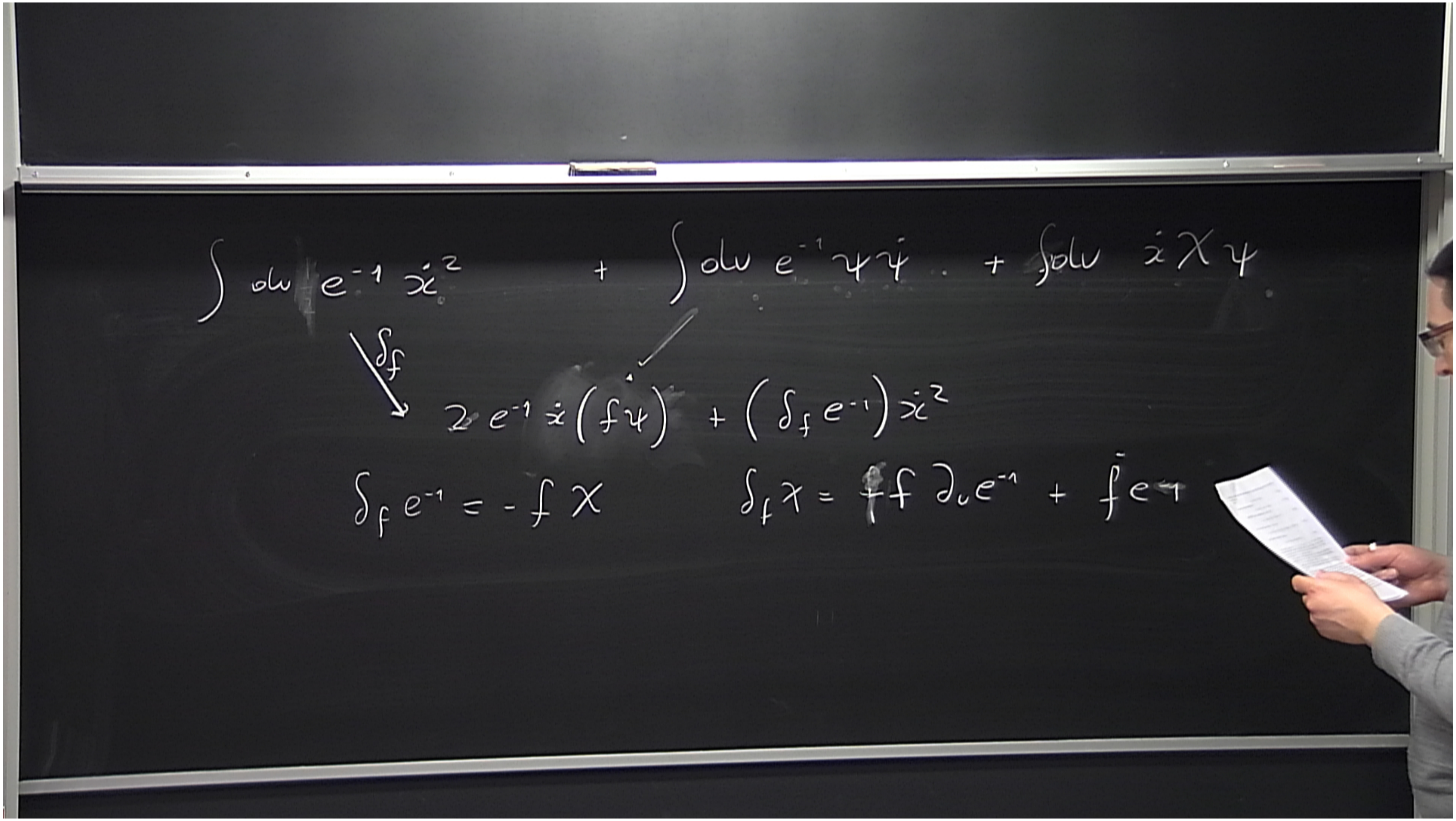
$$(\partial_\theta - \theta\partial_v)^2 \propto \partial_v$$

$$D_\theta = (\partial_\theta + \theta\partial_v) \qquad D_\theta^2 = \partial_v$$

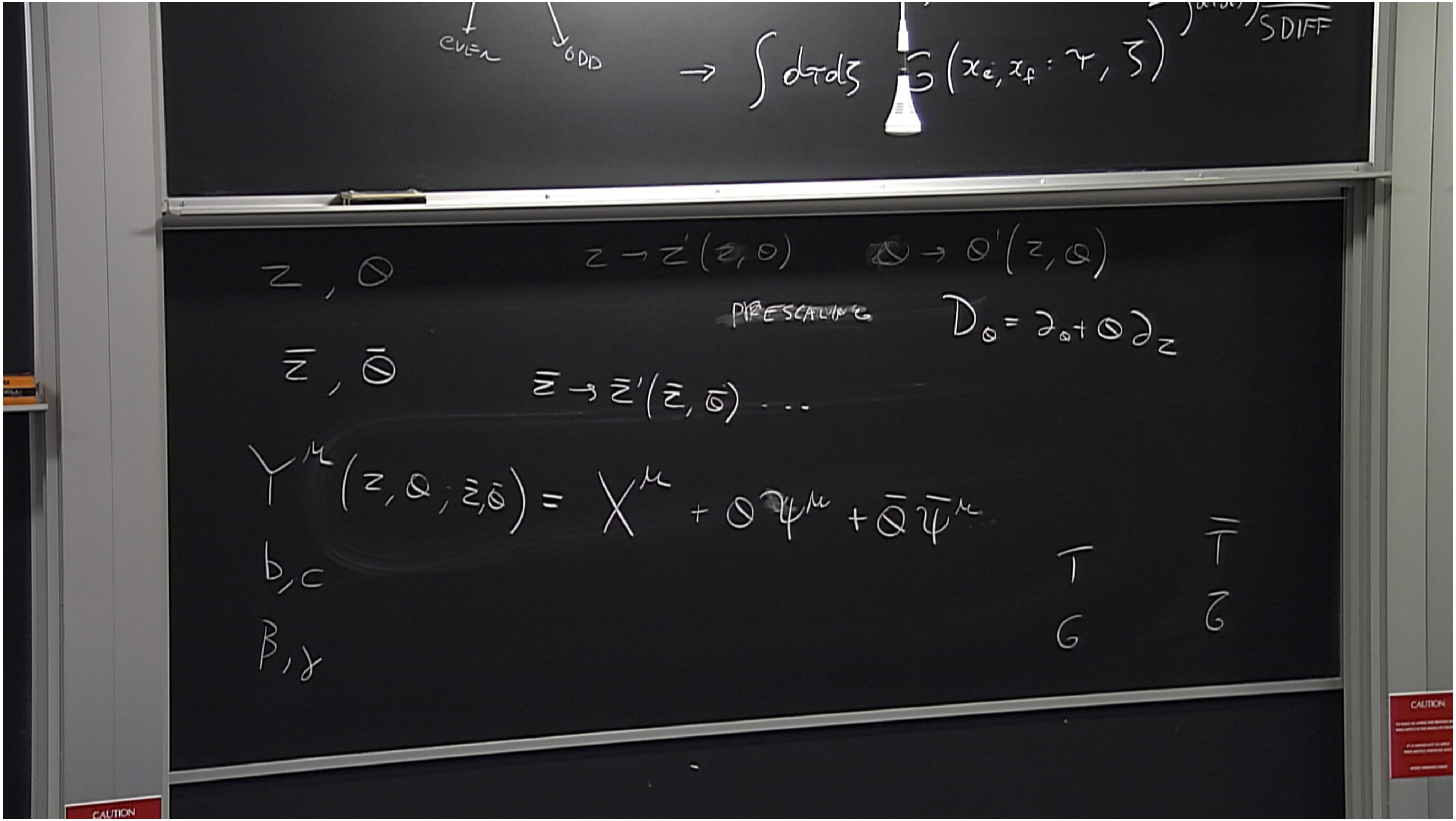
$$v_f(y(v,\theta)) = f(v)\psi - f(v)\dot{x}\theta$$

$$\delta_f x = f\psi$$

$$\delta_f \psi = -f\dot{x}$$



$\int dw\, e^{-1} \dot{x}^2 + \int dw\, e^{-1} \psi \dot{\psi} + \int dw\, \dot{x} \chi \psi$
δ_f
$2 e^{-1} \dot{x} (\dot{f\psi}) + (\delta_f e^{-1}) \dot{x}^2$
$\delta_f e^{-1} = - f \chi$
$\delta_f \chi = f \partial_u e^{-1} + \dot{f} e^{-1}$


even
odd
$\rightarrow \int d\tau d\zeta \, G(x_i, x_f : \tau, \zeta)$
SDIFF
z, θ
$z \to z'(z,\theta)$
$\theta \to \theta'(z,\theta)$
RESCALING
$D_\theta = \partial_\theta + \theta \partial_z$
$\bar{z}, \bar{\theta}$
$\bar{z} \to \bar{z}'(\bar{z},\bar{\theta}) \ldots$
$Y^\mu(z,\theta;\bar{z},\bar{\theta}) = X^\mu + \theta\psi^\mu + \bar{\theta}\bar{\psi}^\mu$
b, c
β, γ
T
$\bar{T}$
G
$\bar{G}$
CAUTION

$$L = \frac{1}{2}\dot{x}^\mu \dot{x}_\mu + \psi^\mu \dot{\psi}_\mu$$

$$[\hat{x}^\mu, \hat{p}_\nu] = i\,\delta^\mu_\nu$$

$$\{\hat{\psi}^\mu, \hat{\psi}^\nu\} = \eta^{\mu\nu}$$

$$\gamma^\mu \equiv \hat{\psi}^\mu$$

$$\hat{H} = p^2$$

$$\hat{Q} = p_\mu \psi^\mu$$

SUPER-TRANSLATIONS

$$[\hat{H}, \hat{Q}] = 0 \qquad \hat{Q}^2 = \hat{H}$$

$\mathcal{H} \ni |\alpha\rangle \qquad \hat{\psi}^\mu |\alpha\rangle = (\gamma^\mu)^\alpha{}_\beta |\beta\rangle$

$$[\hat{Q}, x] = \psi$$

$$[\hat{Q}, \psi] = p$$

2

FERMIONS WORLDLINES

- $\frac{p_\mu \gamma^\mu}{p^2}$ FROM INTEGRAL OVER PROPER TIME

- $p^2 |\text{PHYSICAL}\rangle = 0 \Rightarrow p^\mu \gamma_\mu |\text{PHYSICAL}\rangle = 0$

$$p^\mu \psi_\mu | \quad \rangle = 0$$

- HOW TO GET γ^μ ?

$$\psi \to \psi' \left(\frac{d\tau'}{d\tau}\right)^{\frac{1}{2}}$$

1)

2

$$\frac{1}{p^2} = \int d\tau\, e^{-\tau p^2} \to \int d\tau\, G(x_a, x_b, \tau) \to \int d\tau \int Dx\, e^{S[x,\tau]}$$

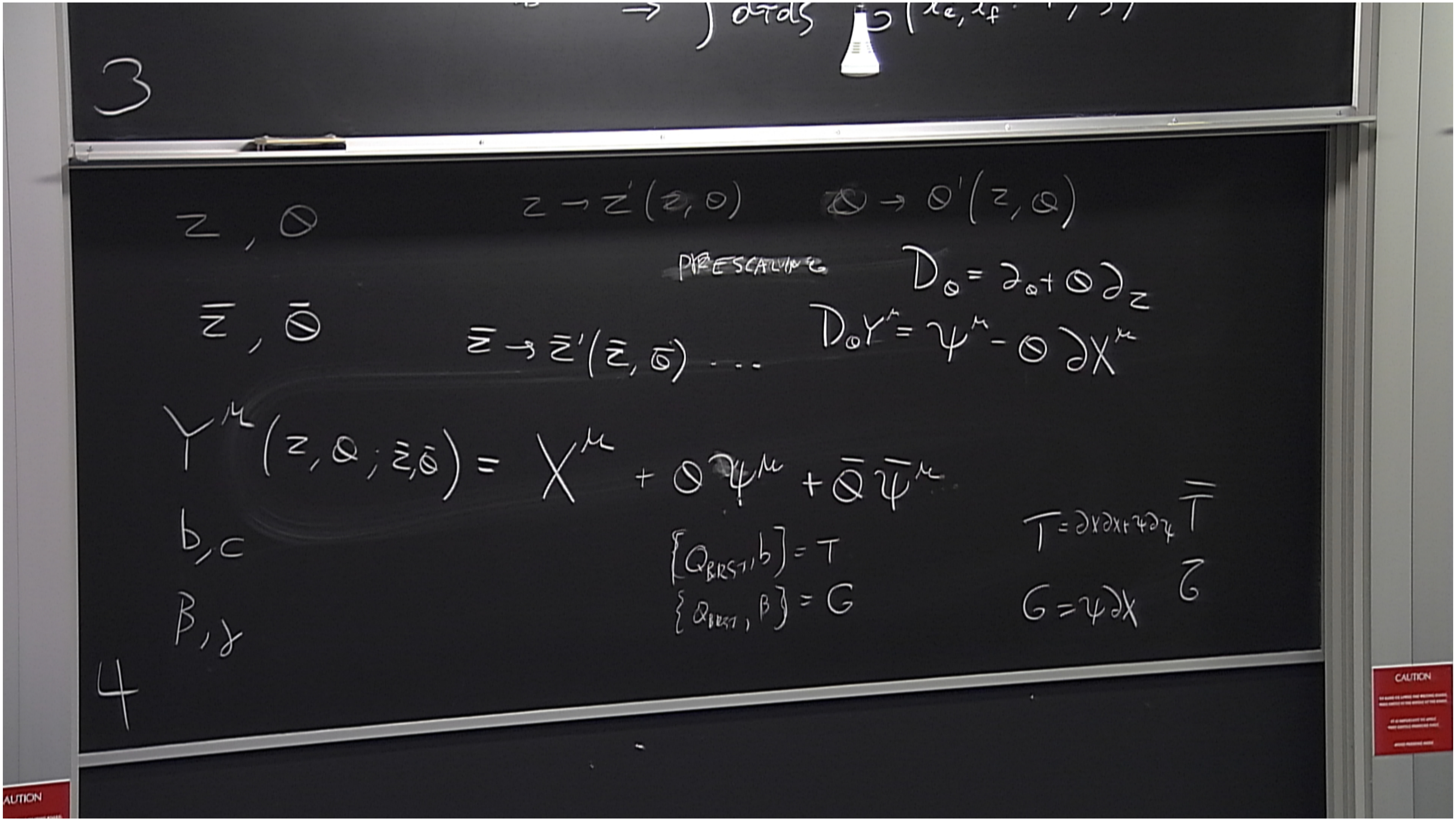
$$\downarrow$$

$$\int d\tau \int \frac{Dx\, De}{\mathrm{DIFF}}\, e^{S[x,e]}$$

$$\frac{p_\mu \psi^\mu}{p^2} = \int \underbrace{d\tau}_{\text{even}}\, \underbrace{d\zeta}_{\text{odd}}\, e^{-\tau p^2 - \zeta p_\mu \psi^\mu} = \int d\tau\, d\zeta\, e^{-\tau \hat{H} - \zeta \hat{Q}} \to \int d\tau\, d\zeta \int \frac{D \cdots}{\mathrm{SDIFF}}$$

$$\to \int d\tau\, d\zeta\; G(x_i, x_f : \tau, \zeta)$$

3


3
4
z , θ
$z \to z'(z,\theta) \quad \theta \to \theta'(z,\theta)$
PRESCALING
$D_\theta = \partial_\theta + \theta \partial_z$
$\bar{z} , \bar{\theta}$
$\bar{z} \to \bar{z}'(\bar{z},\bar{\theta}) \ldots$
$D_\theta Y^\mu = \psi^\mu - \theta \partial X^\mu$
$Y^\mu(z,\theta;\bar{z},\bar{\theta}) = X^\mu + \theta \psi^\mu + \bar{\theta}\bar{\psi}^\mu$
b, c
β, γ
$[Q_{BRST}, b] = T$
$\{Q_{BRST}, \beta\} = G$
$T = \partial X \partial X + \psi \partial \psi$
$\bar{T}$
$G = \psi \partial X$
$\bar{G}$

$$\rightarrow \int d\tau\, ds \ldots (x_e, x_f \ldots)$$

3

$$G = \int Dx\, D\psi\, e^{-\int_0^\tau du\ \frac{1}{2}\dot{x}^2 + \frac{1}{2}\psi\dot{\psi} + \tau^{-1}\zeta\psi\dot{x}}$$

$$(\partial_\theta - \theta\partial_u)^2 \propto \partial_u$$

$$y^\mu(u,\theta) = x^\mu(u) + \theta\psi^\mu(u)$$

$$[\hat{Q}, y^\mu] = \psi^\mu - \theta p^\mu = [\partial_\theta - \theta\partial_u]\, y^\mu$$

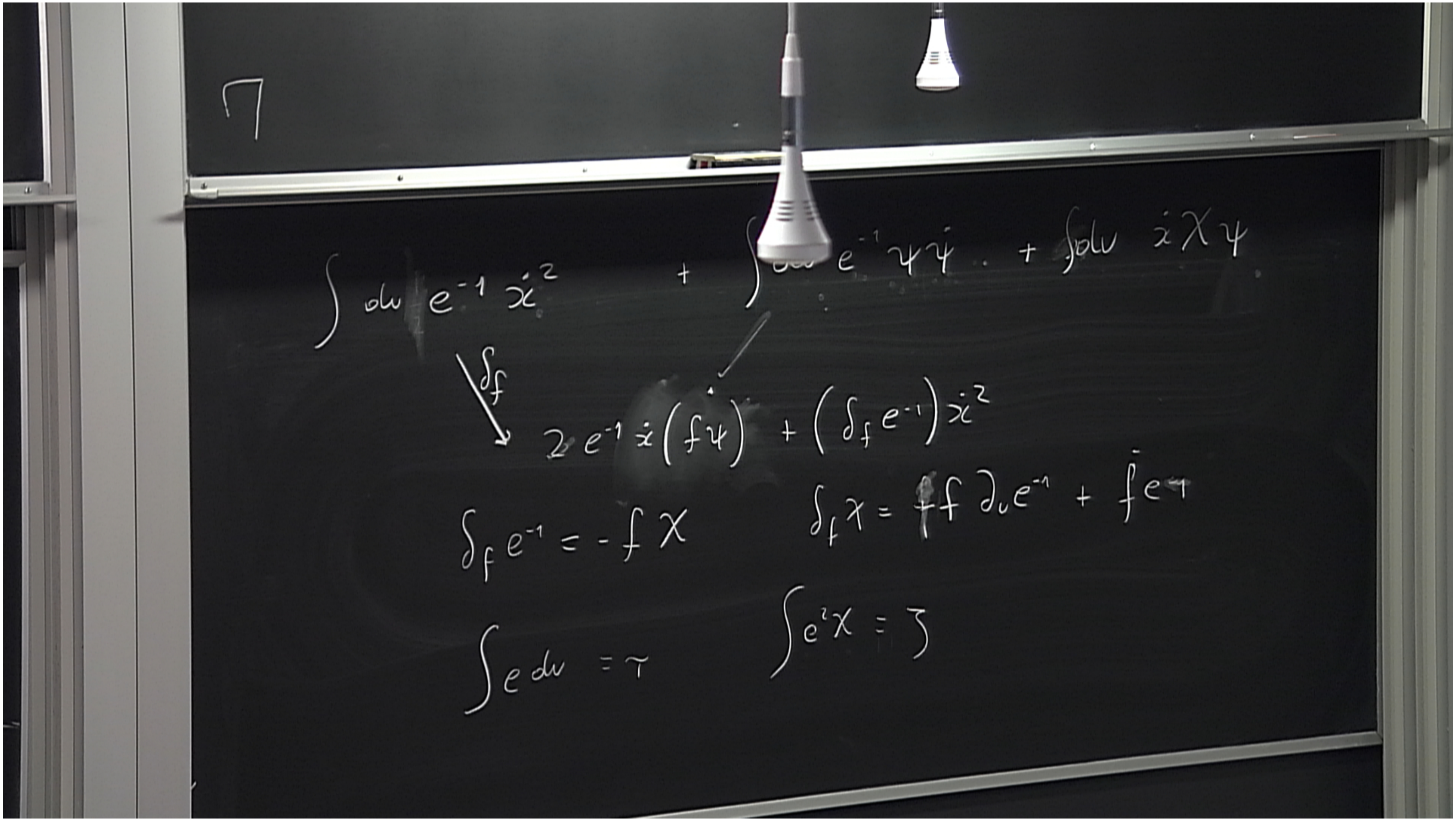
$$D_\theta = (\partial_\theta + \theta\partial_u) \qquad D_\theta^2 = \partial_u$$

$$v_f(y(u,\theta)) = f(u)\psi - f(u)\dot{x}\theta$$

$$\delta_f x = f\psi$$

$$\delta_f \psi = -f\dot{x}$$

5


7
$\int d\upsilon \, e^{-1} \dot{x}^2 + \int d\upsilon \, e^{-1} \psi \dot{\psi} + \int d\upsilon \, \dot{x} \chi \psi$
$\xrightarrow{\delta_f} 2 e^{-1} \dot{x} \dot{(f\psi)} + (\delta_f e^{-1}) \dot{x}^2$
$\delta_f e^{-1} = - f \chi$
$\delta_f \chi = f \partial_\upsilon e^{-1} + \dot{f} e^{-1}$
$\int e \, d\upsilon = \tau$
$\int e^2 \chi = \zeta$

$$\psi' = F(u,\theta) = F_1(u) + \theta F_2(u)$$

$$\theta' = G(u,\theta) = G_1(u) + \theta G_2(u)$$

$$D_\theta = J\, D_{\theta'}$$

SUPER-DIFF

$$v_f = f(u)\left(\partial_\theta - \theta\,\partial_u\right)$$

$$\{v_f, D_\theta\} = \dot{f}\,\theta\, D_\theta$$

$$\{v_f, v_g\} = \boxed{-2fg\,\partial_u - \left(\dot{f}g + f\dot{g}\right)\theta\,\partial_\theta}$$

SDIFF-INVARIANT ACTIONS ?

NEED METRIC + GRAVITINO

$$S[x, \psi, e, \chi]$$ SDIFF-INVARIANT

$$\Big\Downarrow \text{ GAUGE-FIX}$$

$$S[x, \psi, \tau, \zeta]$$

8