

Title: PSI 2017/2018 - String Theory - Lecture 9

Date: Mar 02, 2018 10:15 AM

URL: <http://pirsa.org/18030015>

Abstract:



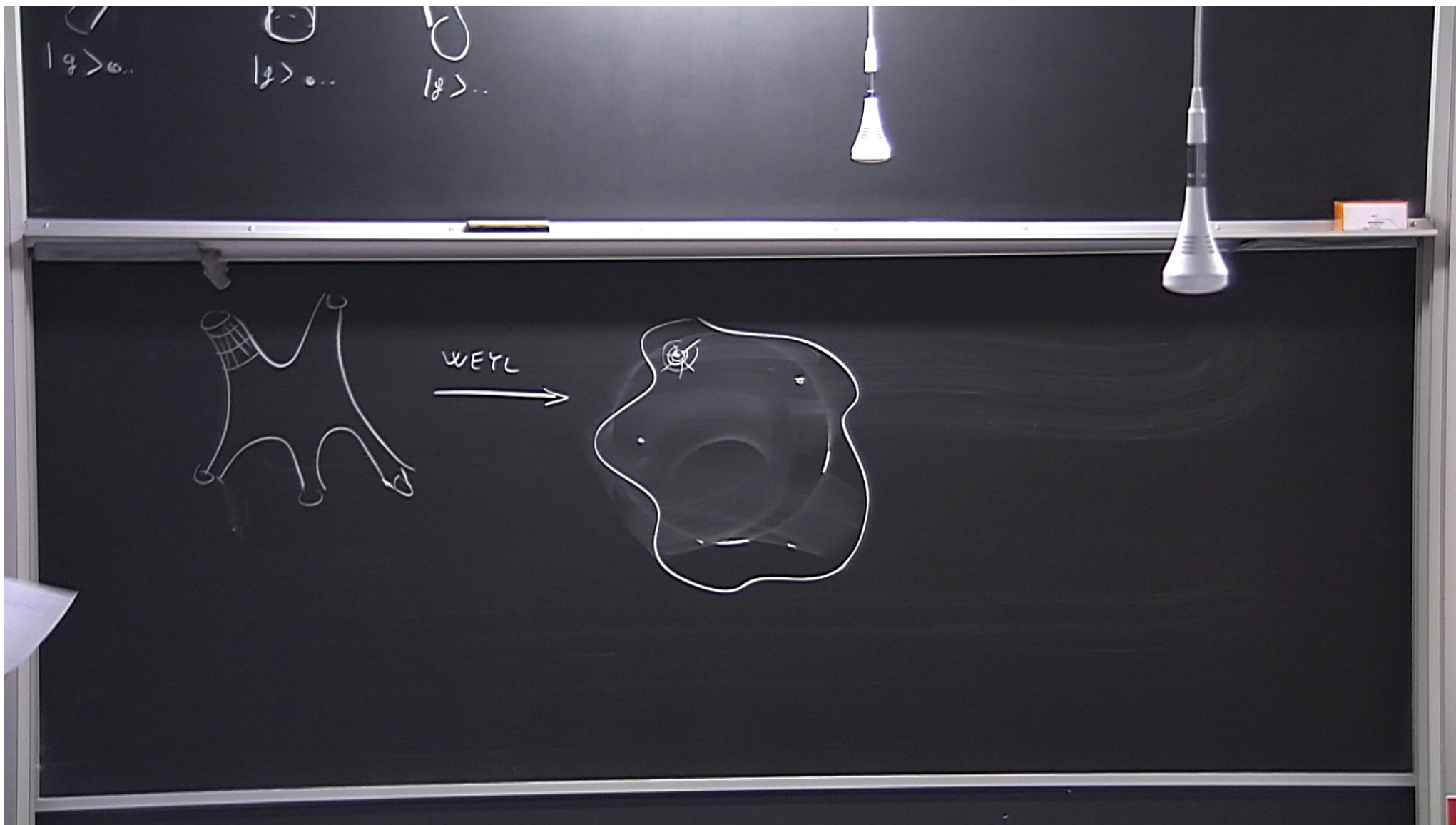
$\mathcal{M}_{n,g}$

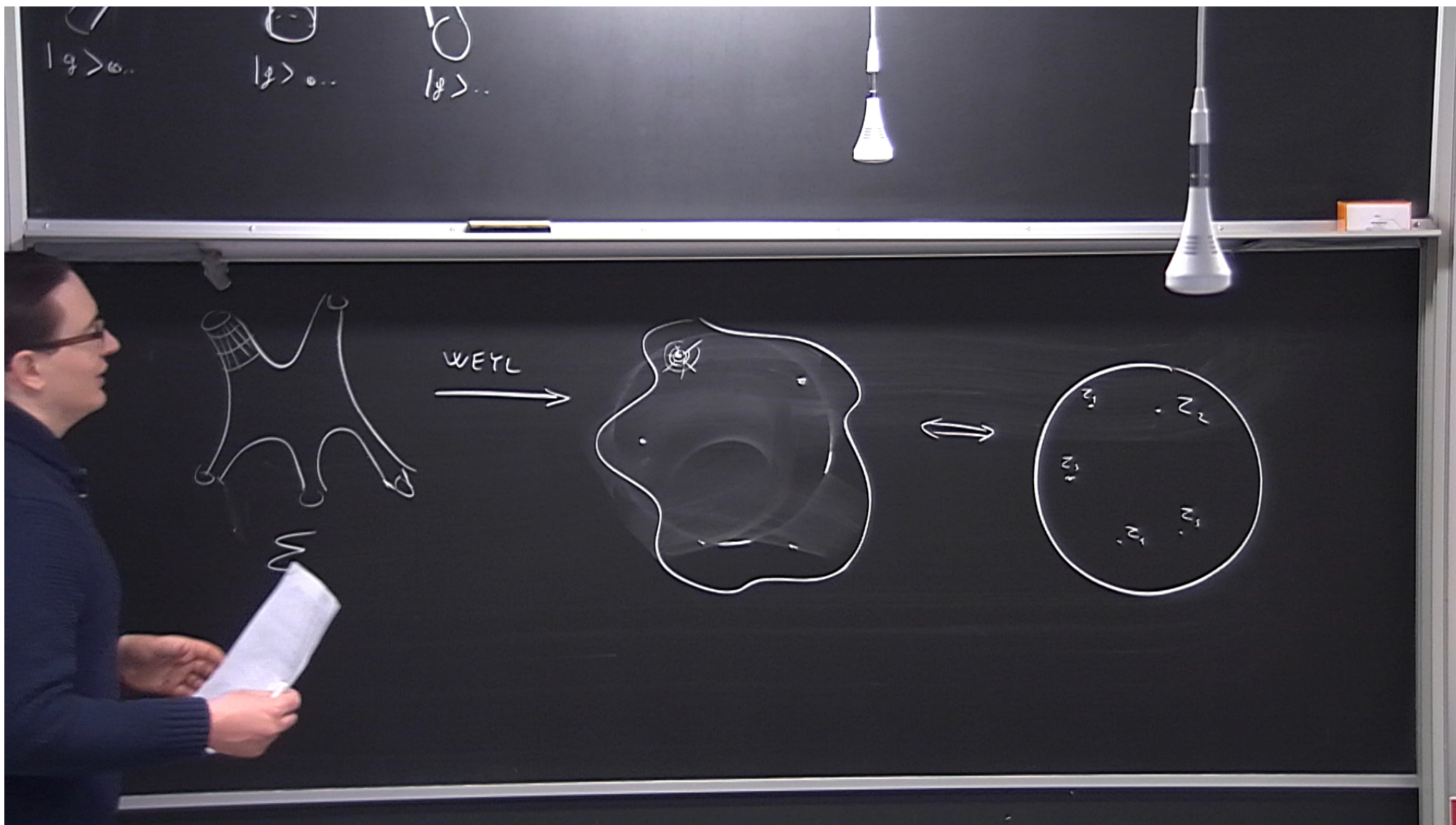
$\mathcal{M}_{5,1}$

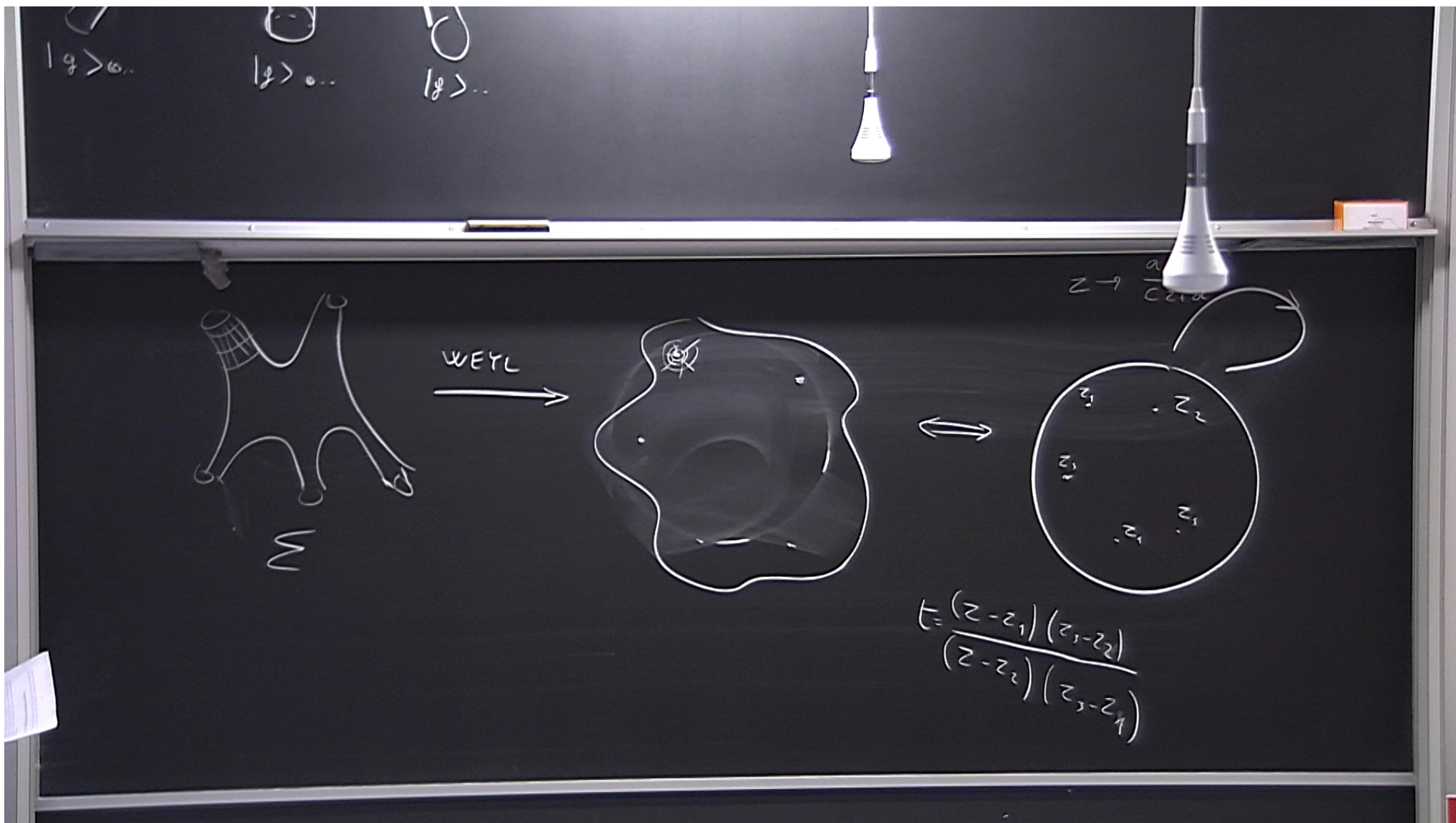


$$\dim \mathcal{M}_{n,g} = n - 3 + 3g$$

$\mathcal{M}_{5,0}$







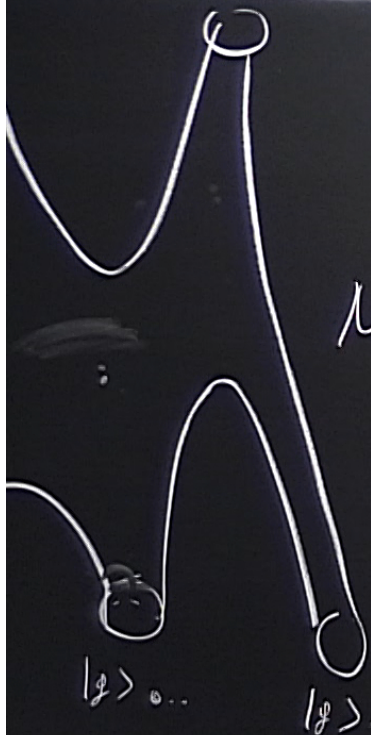
$$\dim \mathcal{M}_{n,g} = n - 3 + 3g$$

$\mathcal{M}_{5,0}$

$\mathcal{M}_{n,0} =$

SPACE OF n PTS ON S^2

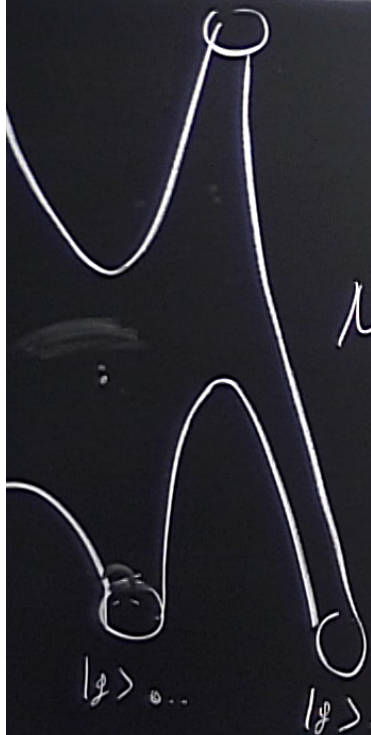
GLOBAL
CONFORMAL

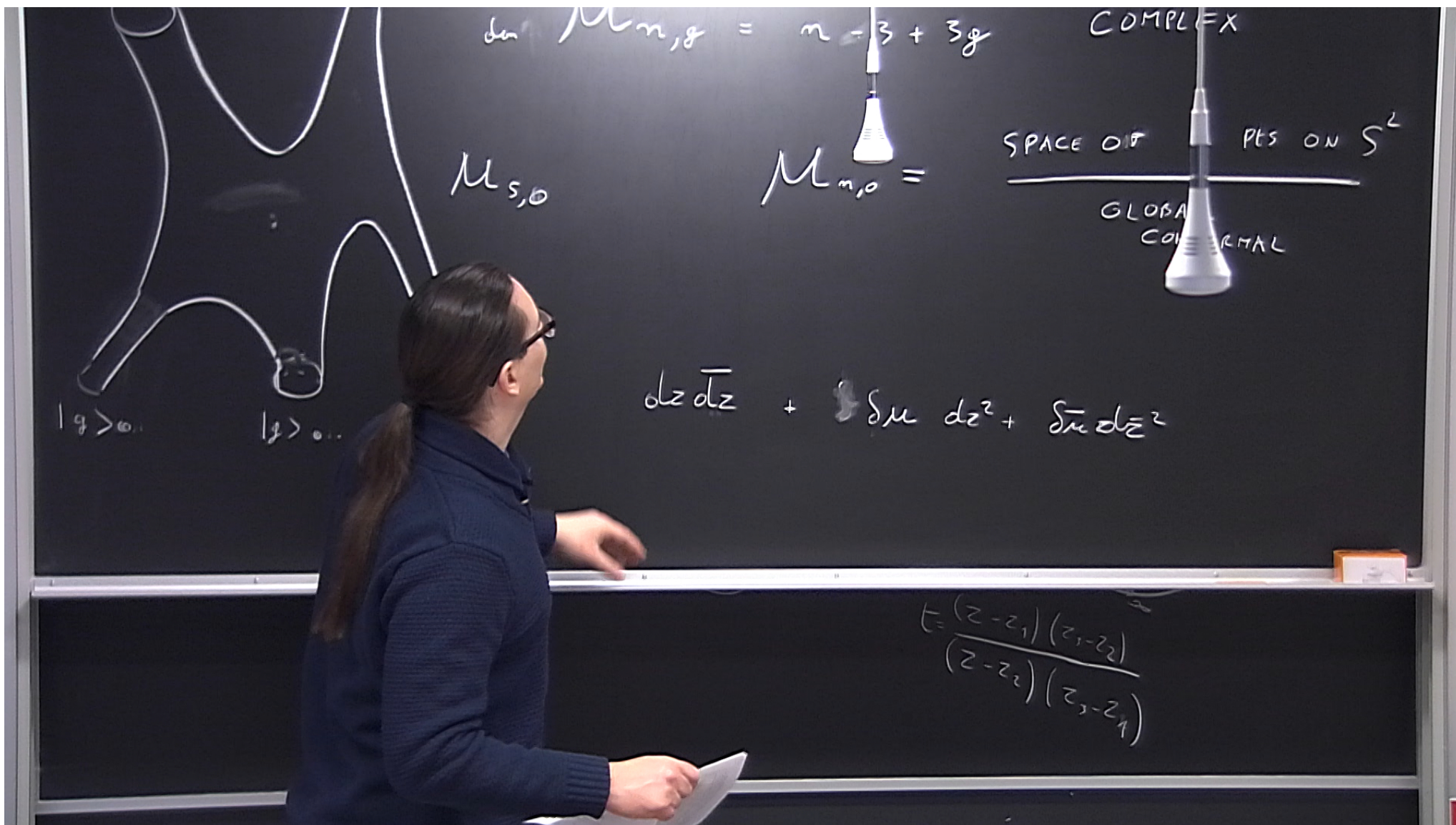


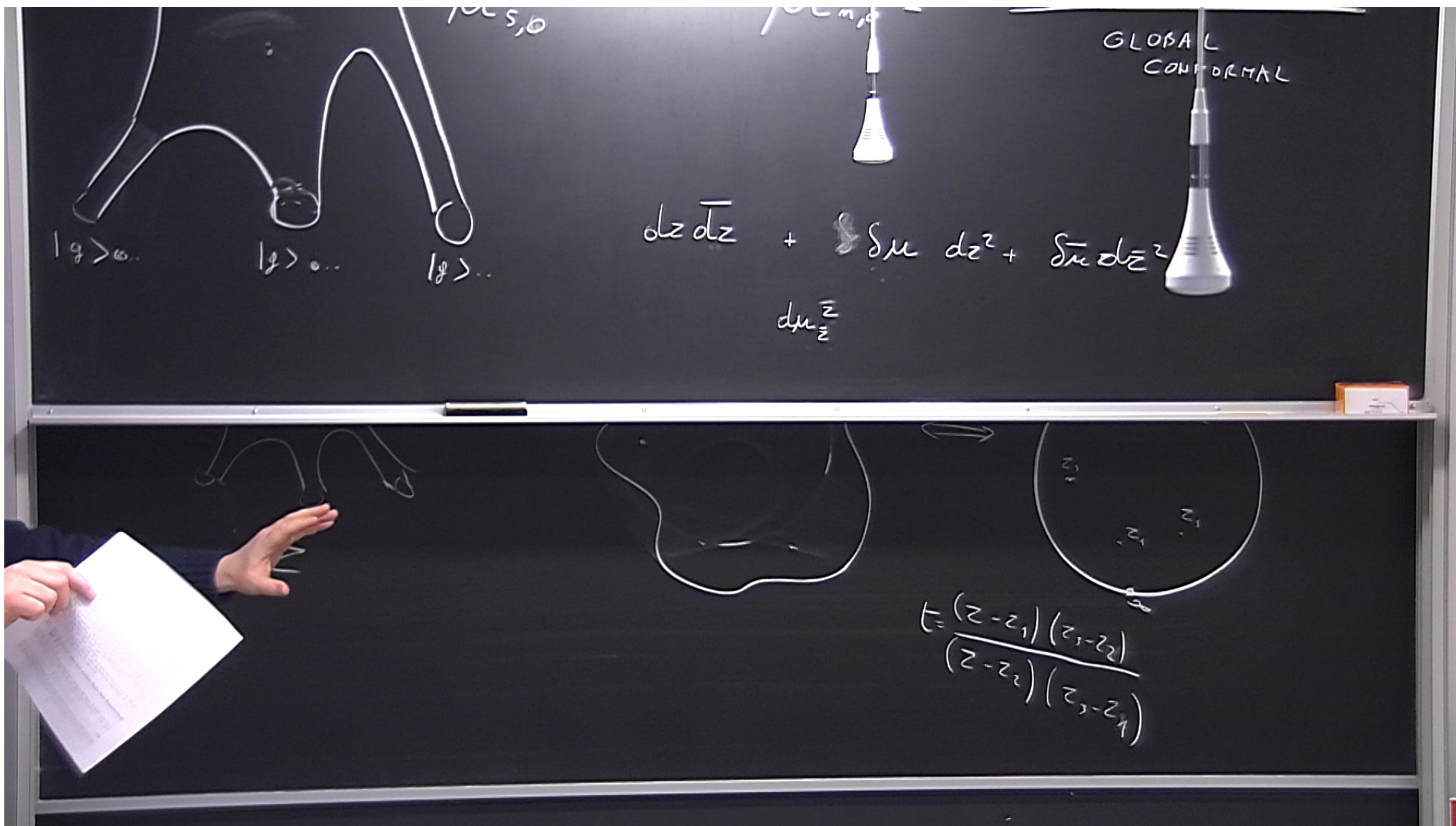
$$\dim \mathcal{M}_{n,g} = 6n - 3 + 3g \quad \text{COMPLEX}$$

$\mathcal{M}_{5,0}$

$\mathcal{M}_{n,0} = \frac{\text{SPACE OF } n \text{ PTS ON } S^2}{\text{GLOBAL CONFORMAL}}$







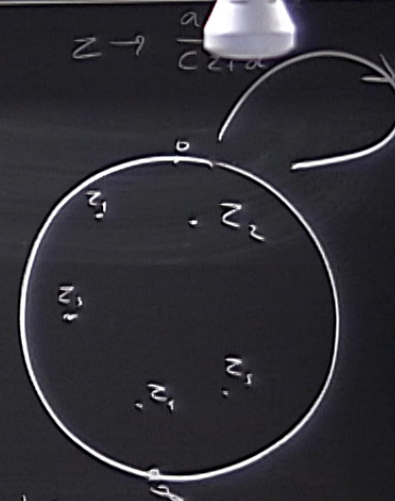
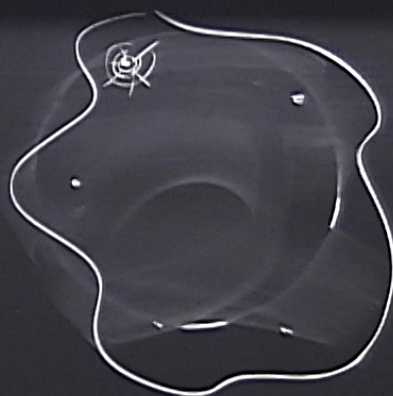
$|g\rangle \dots$ $|f\rangle \dots$ $|f\rangle \dots$

$$dz d\bar{z} + \int \mu dz^2 + \int \bar{\mu} d\bar{z}^2$$

$$d\mu_{\bar{z}}$$



WEYL



$$t = \frac{(z - z_1)(z_1 - z_2)}{(z - z_2)(z_3 - z_4)}$$

$$\pi_i \left[d\mu_i \int_{\Sigma} b^{ab} \frac{\delta h_{ab}}{\delta \mu_i} \right]$$

$$\begin{array}{c} z_i \\ h_{ab} = S_{ab} + \delta h_{ab} \end{array} \equiv \begin{array}{c} z_i + \delta z_i \\ h'_{ab} = S_{ab} \end{array}$$

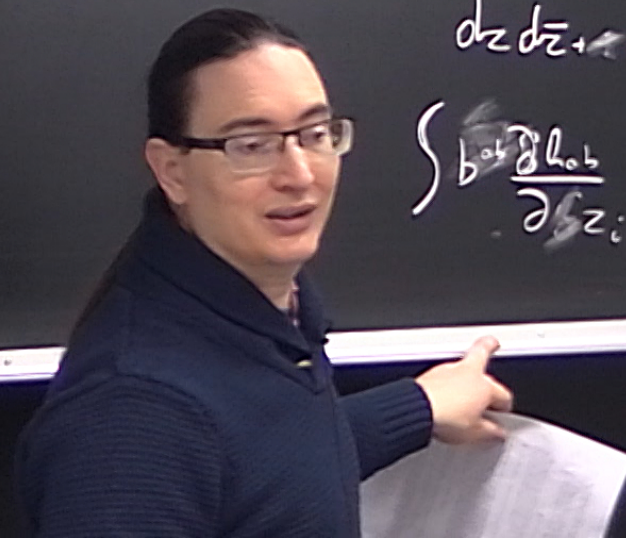
$$\Pi_i \left[d\mu_i \int_{\Sigma} b^{ab} \frac{\delta h_{ab}}{\delta \mu_i} \right]$$

$$\begin{array}{c} \text{Left circle: } z_i \\ h_{ab} = \delta_{ab} + \delta h_{ab} \\ \text{Right circle: } z_i, \delta z_i \\ h'_{ab} = \delta_{ab} \end{array} \equiv$$

($\delta z_i v, \delta \bar{z}_i \bar{v}$)

$$dz d\bar{z} + dz d\bar{v} + dv d\bar{z}$$

$$\int b^{ab} \frac{\delta h_{ab}}{\delta z_i} = \int b \frac{\partial v}{\partial z}$$



$$h_{ab} = \delta_{ab} + \delta h_{ab} \equiv \left(\begin{array}{c} z_i + \delta z_i \\ h'_{ab} = \delta_{ab} \end{array} \right)$$

$$(\delta z_i, \bar{v}, \delta \bar{z}_i, \bar{v})$$

$$dz dz + dz d\bar{v} + d\bar{v} d\bar{z}$$

$$\int b \frac{\delta h_{ab}}{\delta z_i} = \int b \frac{\partial v}{\partial z} = \int b \frac{\partial h_{ab}}{\partial \bar{z}_i} = \int \bar{b} \frac{\partial \bar{v}}{\partial \bar{z}}$$

$$G_N(1+\epsilon)M = G_N M$$

$$\prod_i \left[d\mu_i \int_{\Sigma} b^{ab} \frac{\partial h_{ab}}{\partial \mu_i} \right]$$

$$\prod_i \left[d\bar{\mu}_i \int_{\Sigma} b^{ab} \frac{\partial h_{ab}}{\partial \bar{\mu}_i} \right]$$

z_i
 $h_{ab} = S_{ab} + \delta h_{ab}$
 $\equiv$
 $z_i + \delta z_i$
 $h'_{ab} = S_{ab}$
 $(\delta z_i, v, \delta \bar{z}_i, \bar{v})$

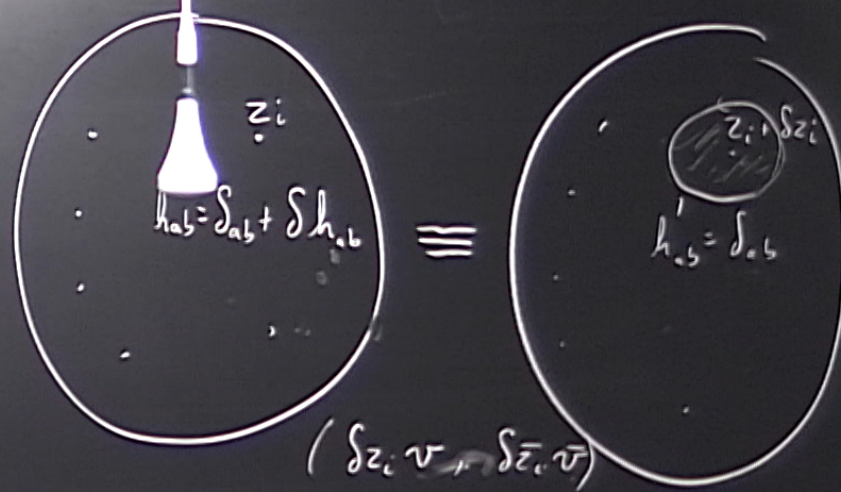
$$dz d\bar{z} + dz d\bar{v} + dv d\bar{z}$$

$$\int b^{ab} \frac{\partial h_{ab}}{\partial z_i} = \int b \frac{\partial v}{\partial z} = \int b \frac{\partial v}{\partial \bar{z}} = \int \bar{b} \frac{\partial \bar{v}}{\partial \bar{z}}$$

$$\prod_i \left[d\mu_i \int_{\Sigma} b^{ab} \frac{\partial h_{ab}}{\partial \mu_i} \right]$$

$$\prod_i \left[d\bar{\mu}_i \int_{\Sigma} b^{ab} \frac{\partial h_{ab}}{\partial \bar{\mu}_i} \right]$$

$$\int b \frac{\partial v}{\partial z} dz d\bar{z} = \oint_{l_i} b(z) V(z) dz$$



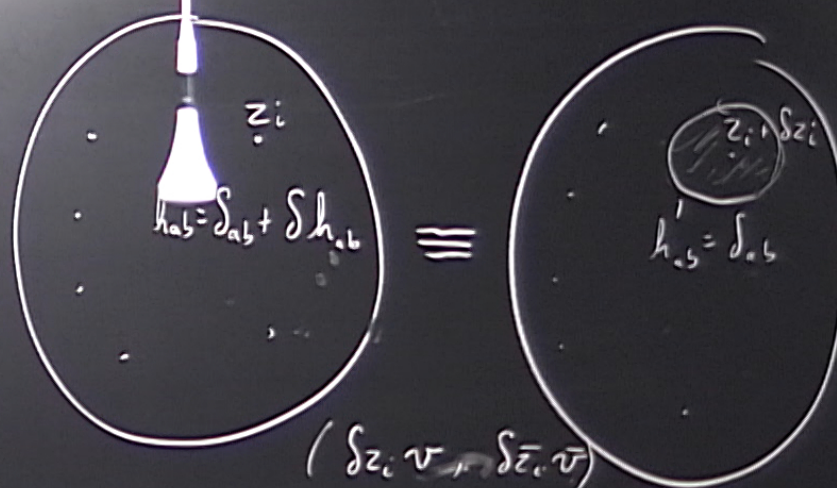
$$dz d\bar{z}, dz d\bar{v}, dv d\bar{z}$$

$$\int b^{ab} \frac{\partial h_{ab}}{\partial z_i} = \int b \frac{\partial v}{\partial z} = \int b^{ab} \frac{\partial h_{ab}}{\partial z_i} = \int b \frac{\partial v}{\partial z}$$

$$\prod_i \left[d\mu_i \int_{\Sigma} b^{ab} \frac{\partial h_{ab}}{\partial \mu_i} \right]$$

$$\prod_i \left[d\bar{\mu}_i \int_{\Sigma} b^{ab} \frac{\partial h_{ab}}{\partial \bar{\mu}_i} \right]$$

$$\int b \frac{\partial v}{\partial z} dz d\bar{z} = \oint_{l_i} b^{ab} \frac{\partial h_{ab}}{\partial z_i} dz d\bar{z}$$



$$dz d\bar{z} + dz d\bar{v} + dv d\bar{z}$$

$$\int b^{ab} \frac{\partial h_{ab}}{\partial z_i} dz d\bar{z} = \int b \frac{\partial v}{\partial z} dz d\bar{z} = \int b^{ab} \frac{\partial h_{ab}}{\partial z_i} dz d\bar{z} = \int b \frac{\partial v}{\partial z} dz d\bar{z}$$

$\oint b dz$

$$C(z_i) \bar{C}(\bar{z}_i) V_{\text{matter}}(z_i, \bar{z}_i)$$

$\oint b dz$

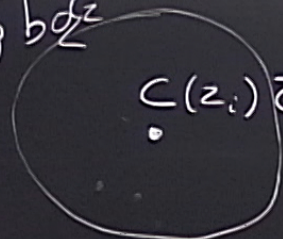
$$C(z_i) \bar{C}(\bar{z}_i) V_{\text{matter}}(z_i, \bar{z}_i)$$



$$\int b(s) e^s ds$$

$$|g\rangle \otimes |m\rangle$$

$\oint b dz$



$$C(z_i) \bar{C}(\bar{z}_i) V_{\text{matter}}(z_i, \bar{z}_i)$$



$$\int b(s) e^{-s} ds = b_{-1}$$

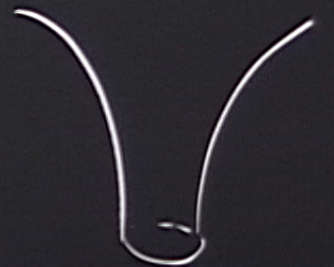
$$|g\rangle \otimes |m\rangle$$

$\oint b dz$

$$C(z_i) \bar{C}(\bar{z}_i) V_{\text{matter}}(z_i, \bar{z}_i)$$

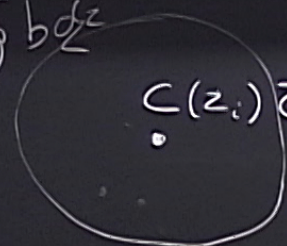
$$\int b(s) e^{-s} ds = b_{-1}$$

$$|g\rangle \otimes |m\rangle$$



$$|0\rangle \otimes |m\rangle$$

$\oint b dz$



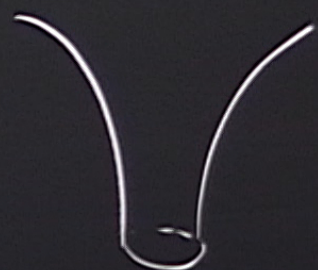
$$C(z_i) \bar{C}(\bar{z}_i) V_{\text{matter}}(z_i, \bar{z}_i)$$



$$\int \bar{b} e^{-\bar{s} \partial \bar{s}}$$

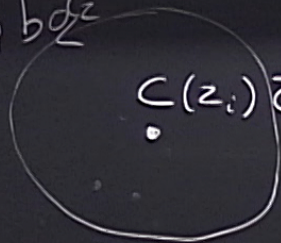
$$\int b(s) e^{-s \partial s} = b_{-1}$$

$$|g\rangle \otimes |m\rangle$$



$$|0\rangle \otimes |m\rangle$$

$\oint b dz$



$$C(z_i) \bar{C}(\bar{z}_i) V_{\text{matter}}(z_i, \bar{z}_i)$$

$\Rightarrow$

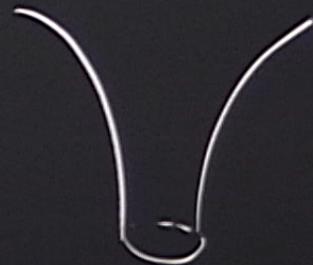
$$\int dz_i d\bar{z}_i V_m(z_i, \bar{z}_i)$$



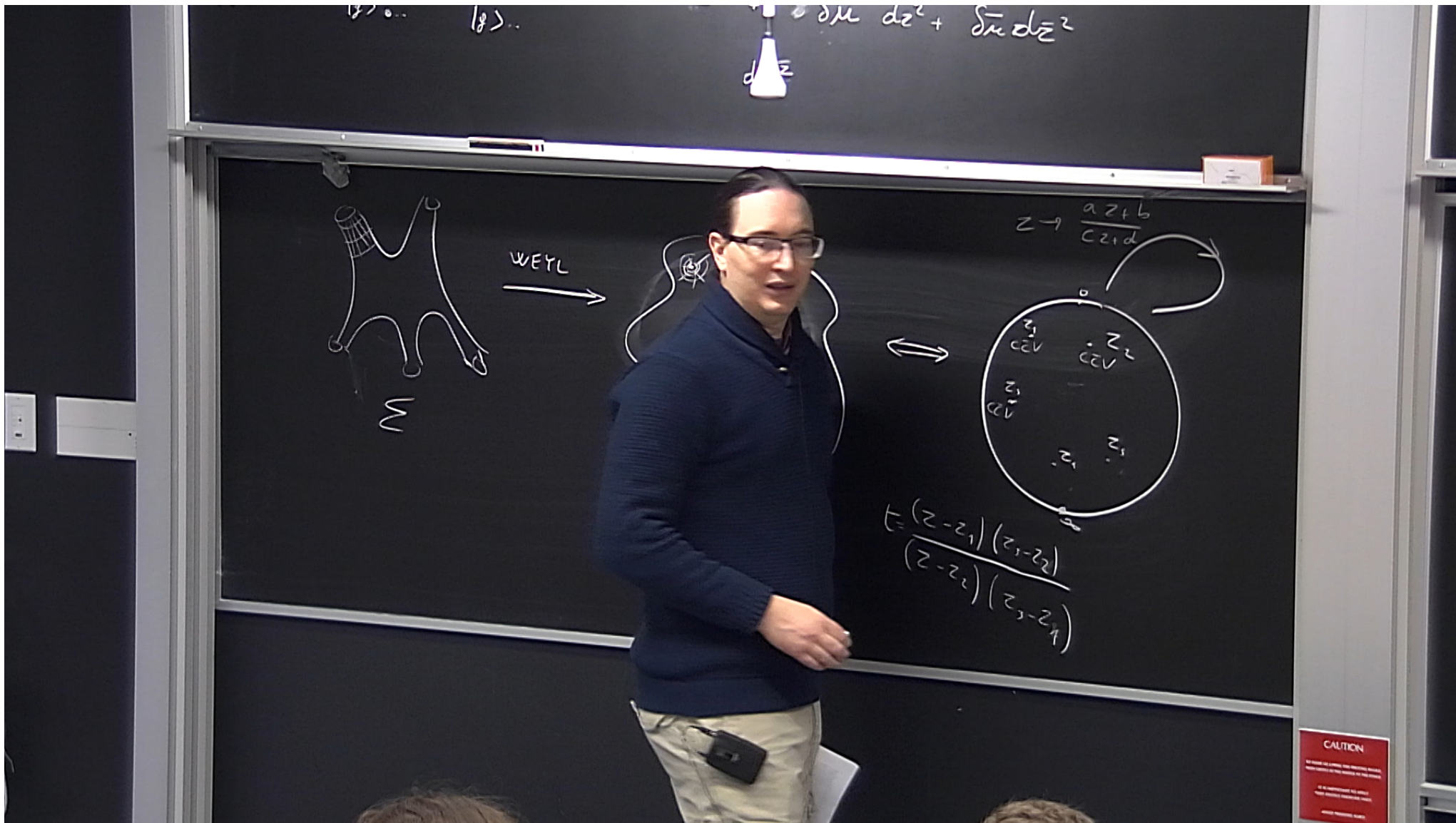
$$\int \bar{b} e^{-\bar{s} \alpha \bar{\lambda}}$$

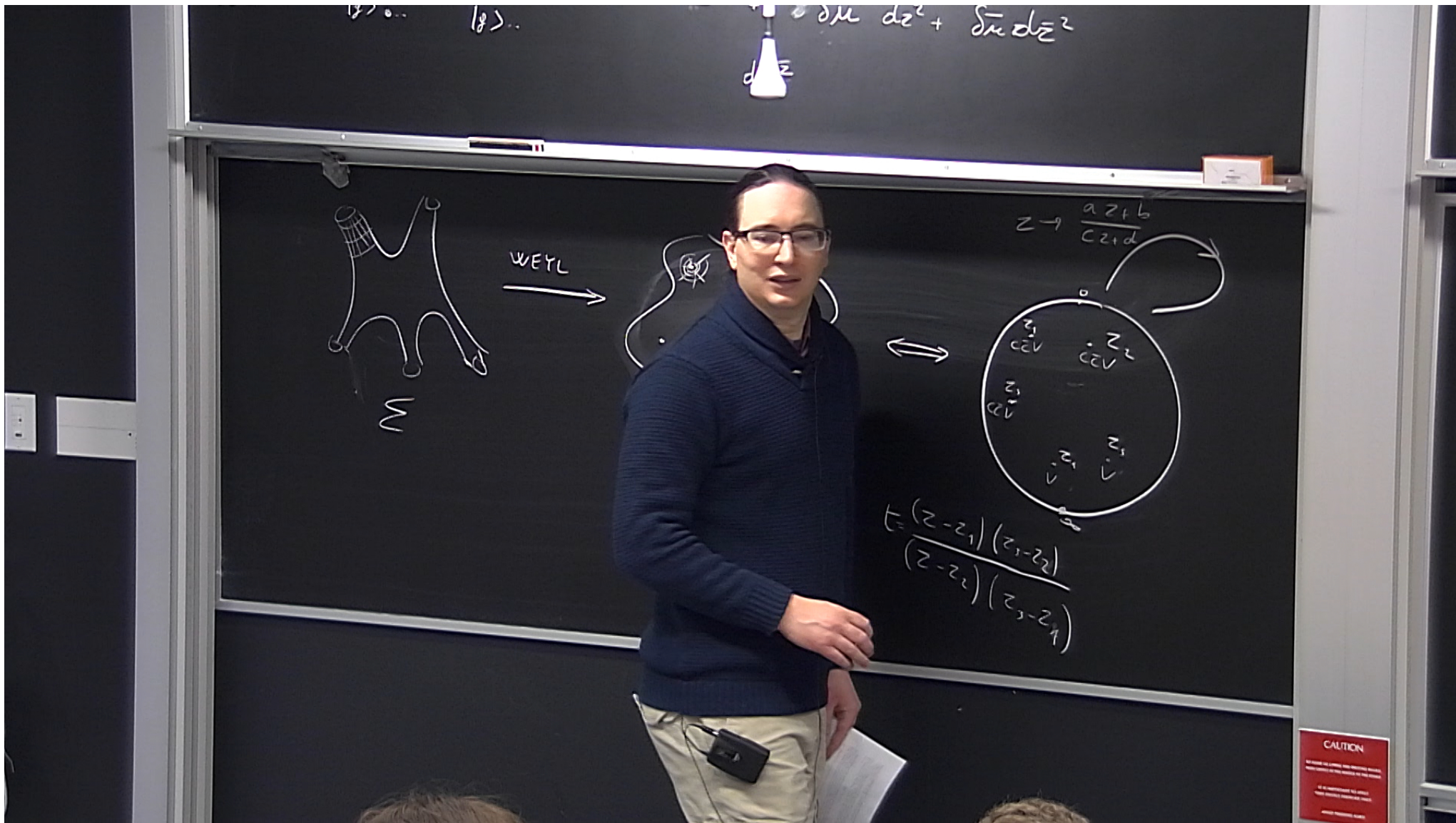
$$\int b(s) e^{-s} ds = b_{-1}$$

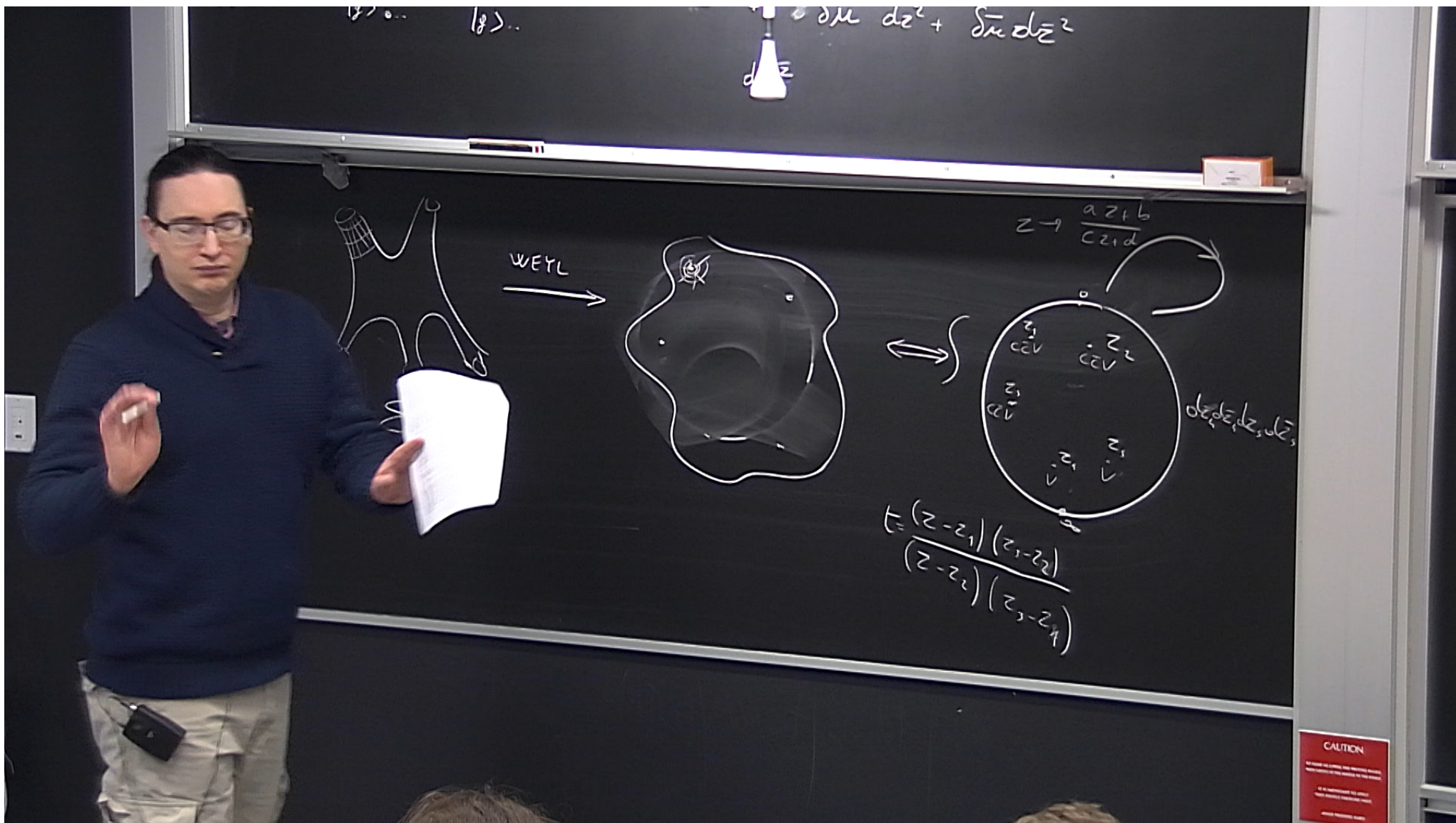
$$|g\rangle \otimes |m\rangle$$



$$|0\rangle \otimes |m\rangle$$







$$A_{TT \rightarrow TT} = \int dz_4 d\bar{z}_4 \langle c\bar{c}_{p_1} V(z_1, \bar{z}_1) c\bar{c}_{p_2} V(z_2, \bar{z}_2) c\bar{c}_{p_3} V(z_3, \bar{z}_3) V(z_4, \bar{z}_4) \rangle$$

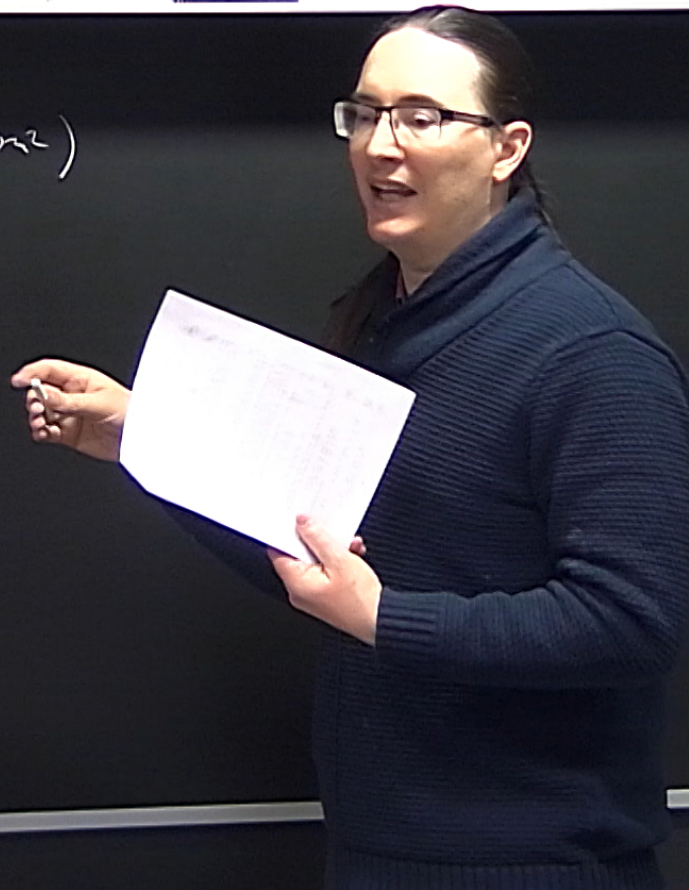
dz_4

$$\begin{aligned}
 A_{TT \rightarrow TT} &= \int dz_4 d\bar{z}_4 \left\langle c z_1 V_{p_1}(z_1, \bar{z}_1) e^{ip_1 X} c \bar{z}_2 V_{p_2}(z_2, \bar{z}_2) e^{ip_2 X} c \bar{z}_3 V_{p_3}(z_3, \bar{z}_3) e^{ip_3 X} V_{p_4}(z_4, \bar{z}_4) e^{ip_4 X} \right\rangle \\
 &= \prod_{1 \leq i < j \leq 3} |z_i - z_j|^{2p_i \cdot p_j + 2} \int dz_4 d\bar{z}_4 \prod_{1 \leq i \leq 3} |z_4 - z_i|^{2p_i \cdot p_4}
 \end{aligned}$$

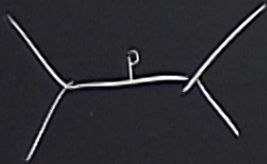
$$\begin{aligned}
 A_{TT \rightarrow TT} &= \int dz_4 d\bar{z}_4 \left\langle c z_1 V_{p_1}(z_1, \bar{z}_1) e^{ip_1 X} c \bar{z}_2 V_{p_2}(z_2, \bar{z}_2) e^{ip_2 X} c \bar{z}_3 V_{p_3}(z_3, \bar{z}_3) e^{ip_3 X} V_{p_4}(z_4, \bar{z}_4) e^{ip_4 X} \right\rangle \\
 &= \prod_{1 \leq i < j \leq 3} |z_i - z_j|^{2p_i \cdot p_j + 2} \int dz_4 d\bar{z}_4 \prod_{1 \leq i \leq 3} |z_4 - z_i|^{2p_i \cdot p_4} |z_4|^{-2p_4^2}
 \end{aligned}$$

$$\begin{aligned}
 A_{TT \rightarrow TT} &= \int dz_4 d\bar{z}_4 \left\langle c z_{p_1} V(z_1, \bar{z}_1) e^{i p_1 X} c \bar{z}_{p_2} V(z_2, \bar{z}_2) e^{i p_2 X} c \bar{z}_{p_3} V(z_3, \bar{z}_3) e^{i p_3 X} V(z_4, \bar{z}_4) e^{i p_4 X} \right\rangle \\
 &= \prod_{1 \leq i < j \leq 3} |z_i - z_j|^{2 p_i \cdot p_j + 2} \int dz_4 d\bar{z}_4 \prod_{1 \leq i \leq 3} |z_4 - z_i|^{2 p_i \cdot p_4} \\
 &\quad z_4 \rightarrow \infty \quad |z_4|^{-2 p_4^2} \\
 &\quad z_4 \rightarrow z_i \quad |z_4 - z_i|^{2 p_4 \cdot p_i} \\
 &\quad 2 p_4 \cdot p_i \geq -2 \\
 &\quad (p_4 + p_i)^2 \geq 2
 \end{aligned}$$

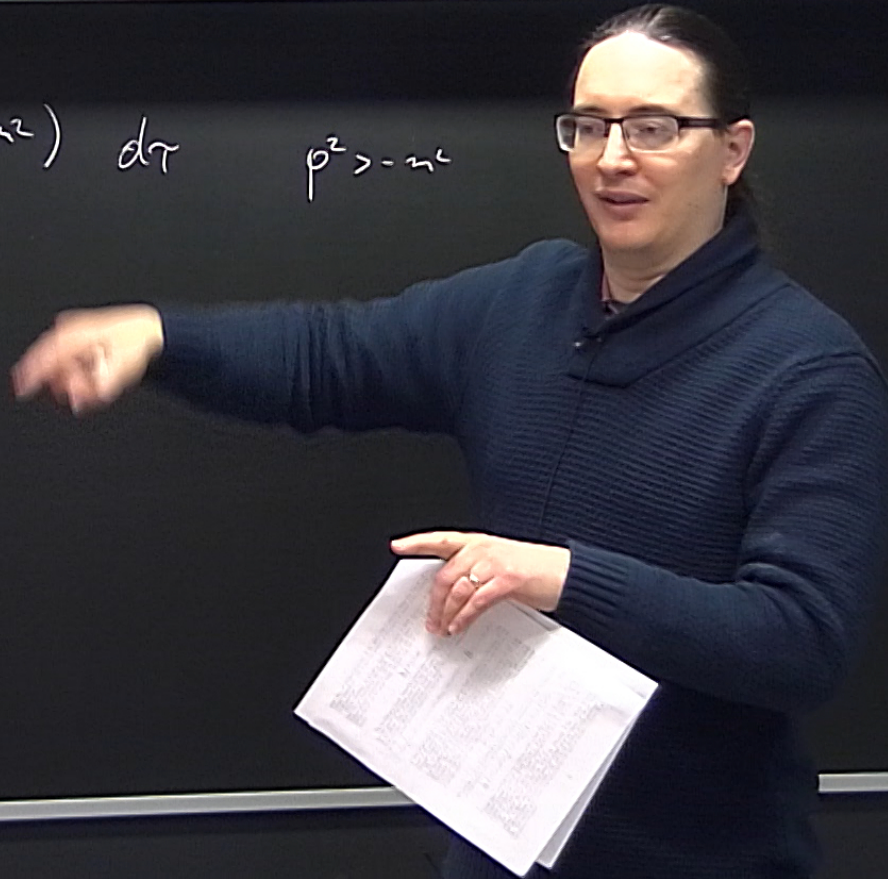
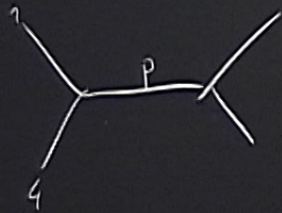
$$\int e^{-\tau(p^2 + m^2)}$$



$$\int e^{-\tau(p^2 + m^2)}$$



$$\int e^{-\tau(p^2 + m^2)} d\tau \quad p^2 > -m^2$$

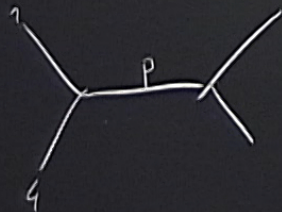


$$z_4 \rightarrow z_i \quad |z_4 - z_i| \quad p_4 \cdot p_i$$

$$2p_4 \cdot p_i \geq -2$$

$$(p_4 + p_i)^2 \geq 2$$

$$\int e^{-\tau(p^2 + m^2)} d\tau$$



$$= \prod_{1 \leq i \leq j \leq 3} |z_i - z_j|^{2p_i \cdot p_j + 2} \int dz_4 d\bar{z}_4 \prod_{1 \leq i \leq 3} |z_4 - z_i|^{2p_i \cdot p_4}$$

$$t = \frac{z_4 - z_1}{z_4 - z_2} \frac{z_3 - z_2}{z_3 - z_1}$$

$$z_4 \rightarrow \infty \quad |z_4|^{-2p_4^2}$$

$$z_4 \rightarrow z_i \quad |z_4 - z_i|^{2p_4 \cdot p_i}$$

$$2p_4 \cdot p_i \geq -2$$

$$(p_4 + p_i)^2 \geq 2$$

$$= \prod_{1 \leq i \leq 3} |z_i - z_4|^{\frac{p_i + p_4}{2}} \int dz_4 d\bar{z}_4 \prod_{1 \leq i \leq 3} |z_4 - z_i|^{2p_i \cdot p_4}$$

$$z_4 \rightarrow \infty \quad |z_4|^{-2p_4^2}$$

$$z_4 \rightarrow z_i \quad |z_4 - z_i|^{2p_4 p_i}$$

$$t = \frac{z_4 - z_1}{z_4 - z_2} \frac{z_3 - z_2}{z_3 - z_1}$$

$$A = \int dt d\bar{t} |t|^{2p_4 p_1} |1-t|^{2p_4 p_2}$$

$$2p_4 p_i \geq -2$$

$$(p_4 + p_i)^2 \geq 2$$

$$= \prod_{1 \leq i < j \leq 3} |z_i - z_j|^{2p_i \cdot p_j} \prod_{1 \leq i \leq 3} |z_4 - z_i|^{2p_i \cdot p_4}$$

$$z_4 \rightarrow \infty \quad |z_4|^{-2p_4^2}$$

$$t = \frac{z_4 - z_1}{z_4 - z_2} \frac{z_3 - z_2}{z_3 - z_1}$$

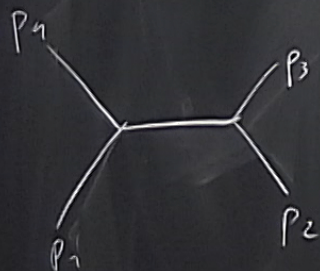
$$A = \int dt d\bar{t} |t|^{2p_4 \cdot p_1} |1-t|^{2p_4 \cdot p_2} = \prod_i \frac{\Gamma(1+p_4 \cdot p_i)}{\Gamma(-p_4 \cdot p_i)}$$

$$A = \int dt d\bar{t} |t|^{2p_4 \cdot p_1} |1-t|^{2p_4 \cdot p_2} = \prod_i \frac{1}{\Gamma(-p_4 \cdot p_i)}$$

POLES

$$p_4 \cdot p_i = -1, -2, -3, \dots$$

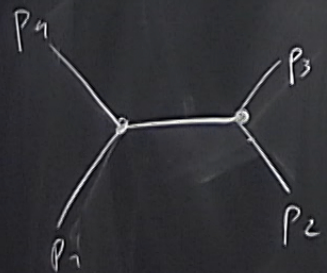
$$(p_4 + p_i)^2 = 2, 0, -2, -4, \dots$$



$$t = \frac{z_4 - z_1}{z_4 - z_2} \frac{z_3 - z_2}{z_3 - z_1}$$

$$A = \int dt d\bar{t} |t|^{2p_4 \cdot p_1} |1-t|^{2p_4 \cdot p_2} \prod_i \frac{\Gamma(1+p_4 \cdot p_i)}{\Gamma(-p_4 \cdot p_i)}$$

POLES $p_4 \cdot p_i = -1, -2, -3, \dots$ $(p_4 + p_i)^2 = 2, 0, -2, -4, \dots$



$$\text{Res } A_{p_4 \cdot p_i = -n} = \prod_{k=0}^{n-2} (p_4 \cdot p_2 - k) (p_4 \cdot p_3 - k)$$

$$A = \int dt d\bar{t} |t|^{2p_4 \cdot p_1} |1-t|^{2p_4 \cdot p_2} = \prod_i \frac{1}{\Gamma(-p_4 \cdot p_i)}$$

$$A \sim e^{-s h_5 - t h_t - u h_u}$$

$$A = \int_0^1 dt \int_0^1 dt \frac{|t|^{p_1-1} |1-t|^{p_2-1}}{\Gamma(-p_1-p_2)} = \frac{1}{i} \Gamma(-p_1-p_2)$$

$$A \sim e^{-s \ln s - t \ln t - u \ln u}$$

$$\langle c(z_1) c(z_2) c(z_3) c(z_4) b(w) \rangle$$

$$A = \int dt dt |t|^{1-\epsilon} |1-t| = \frac{1}{\epsilon} \Gamma(-p_4 \cdot p_i)$$

$$\langle c(z_1) c(z_2) c(z_3) \rangle \langle c(z_4) b(w) \rangle$$

$$k(z) b(w) \sim \frac{1}{z-w} + \dots$$

$$A = \int_0^1 dt \int_0^1 dt \frac{|t|^{p_1-1} |1-t|^{p_2-1}}{\Gamma(-p_1-p_2)} = \dots$$

$$\langle c(z_1) c(z_2) c(z_3) c(z_4) b(w) \rangle$$

$$(z_1 - z_2)(z_2 - z_3)(z_1 - z_3) \frac{1}{z_4 - w} - (z_1 - z_2)(z_2 - z_4)(z_4 - z_3) \frac{1}{z_3 - w} + \dots = \frac{\dots}{(z_1 - w)(z_2 - w)(z_3 - w)(z_4 - w)}$$

$$A = \int_0^1 dt \int_0^1 dt \frac{|t|^{p_1-1} |1-t|^{p_2-1}}{\Gamma(-p_1-p_2)} = \dots$$

$$\langle c(z_1) c(z_2) c(z_3) c(z_4) b(u) \rangle$$

$$\begin{aligned} & (z_1 - z_2)(z_2 - z_3)(z_1 - z_3) \frac{1}{z_4 - u} - (z_1 - z_2)(z_2 - z_4)(z_4 - z_3) \frac{1}{z_3 - u} \\ & + \dots - \dots = \frac{(z_1 - z_2) \dots (z_3 - z_4) (\dots)}{(z_1 - u)(z_2 - u)(z_3 - u)(z_4 - u)} \end{aligned}$$

$$t = \frac{z_4 - z_1}{z_4 - z_2} \frac{z_3 - z_2}{z_3 - z_1}$$

$$A = \int dt d\bar{t} |t|^{2p_4 \cdot p_1} |1-t|^{2p_4 \cdot p_2} = \prod_i \frac{\Gamma(1 + p_4 \cdot p_i)}{\Gamma(-p_4 \cdot p_i)}$$

$$A \sim e^{-s \ln s - t \ln t - u \ln u}$$

$$c(z) b(w) \sim \frac{1}{z-w} + \dots$$

$$\langle c(z_1) c(z_2) c(z_3) c(z_4) b(w) \rangle$$

$$\begin{aligned} & (z_1 - z_2)(z_2 - z_3)(z_1 - z_3) \frac{1}{z_4 - w} - (z_1 - z_2)(z_2 - z_4)(z_4 - z_3) \frac{1}{z_3 - w} \\ & + \dots = \frac{(z_1 - z_2) \dots (z_3 - z_4) (\dots)}{(z_1 - w)(z_2 - w)(z_3 - w)(z_4 - w)} \end{aligned}$$