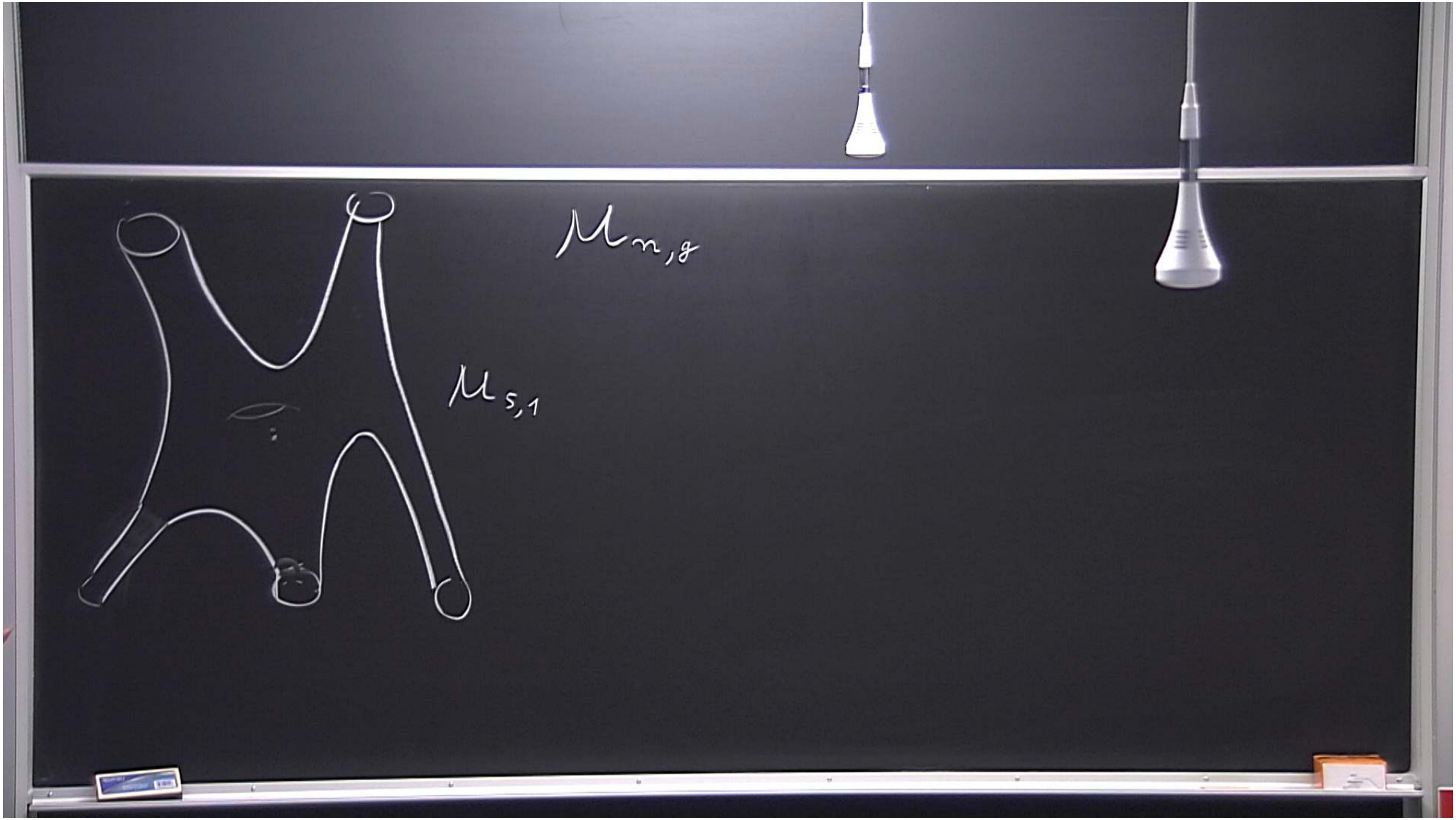
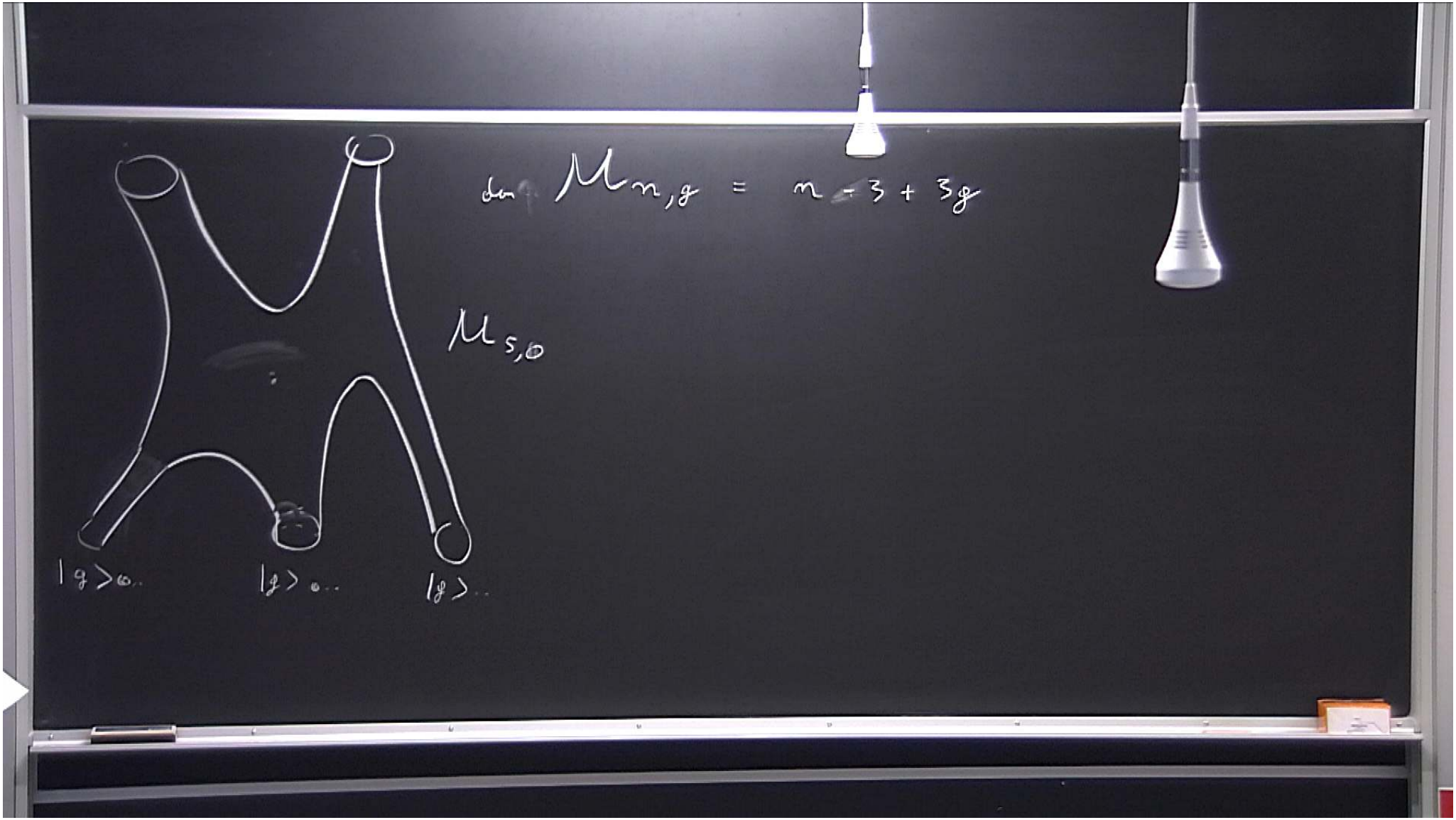
Title: PSI 2017/2018 - String Theory - Lecture 9

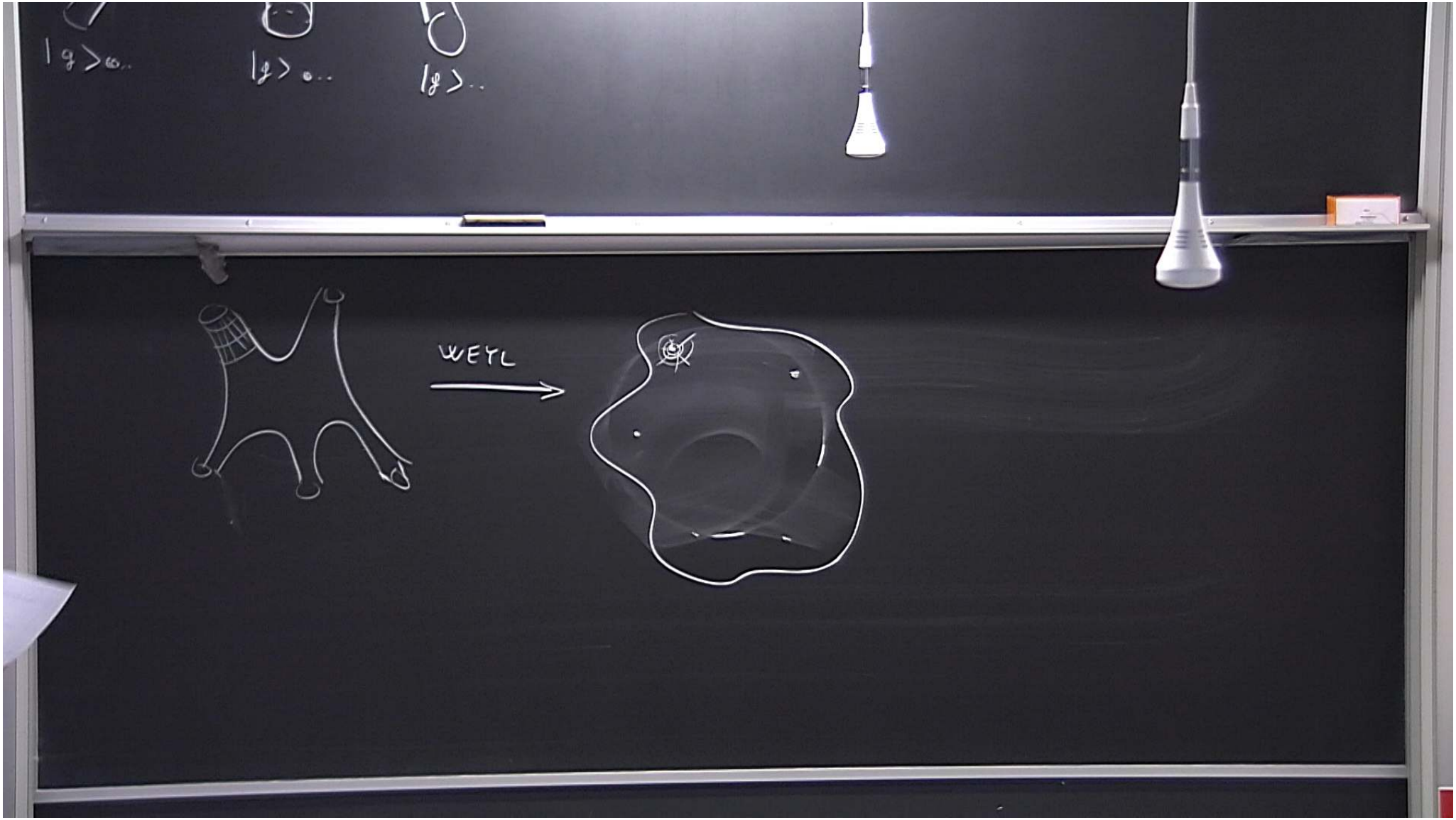
Date: Mar 02, 2018 10:15 AM

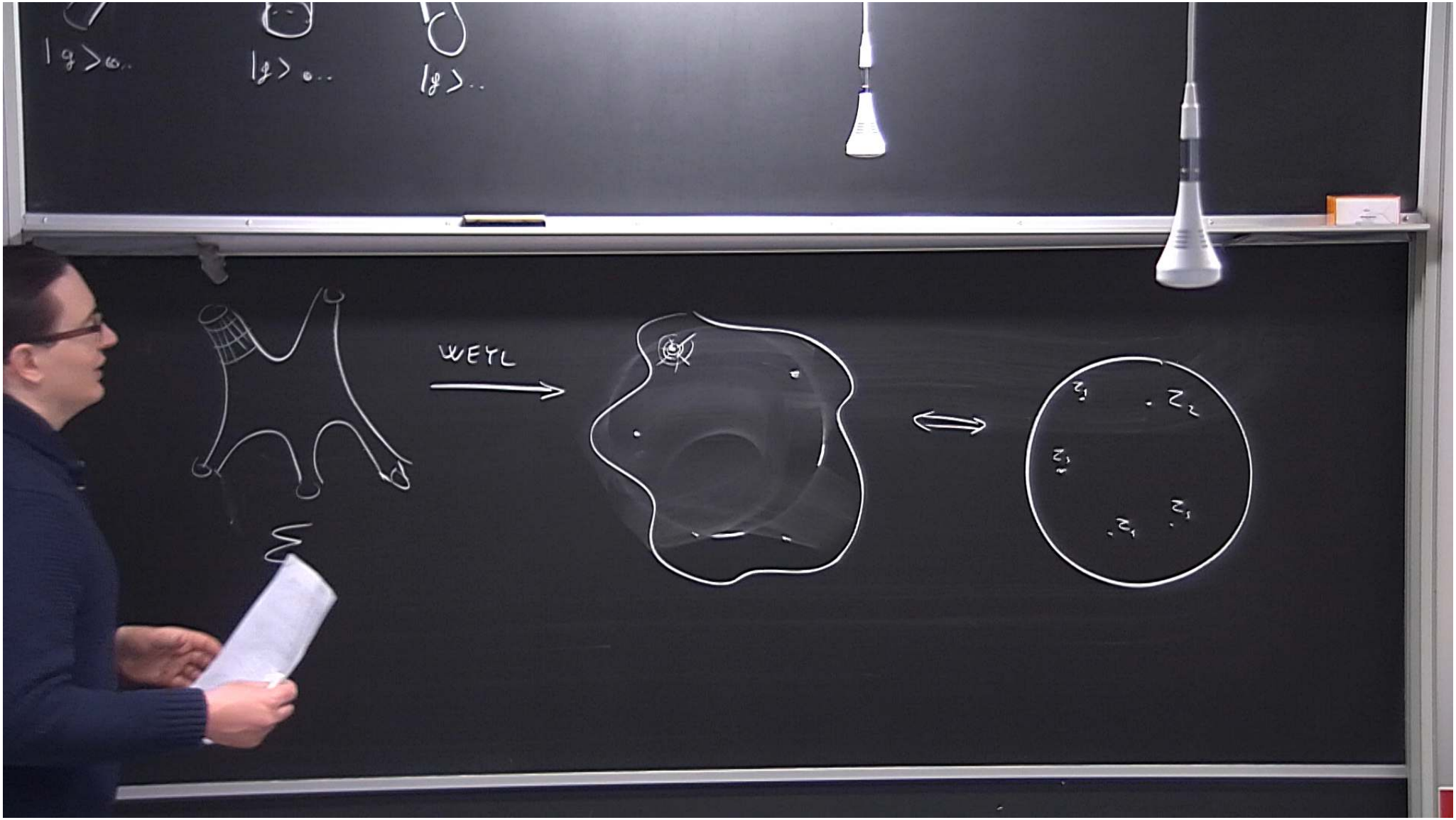
URL: http://pirsa.org/18030015

Abstract:


$\mathcal{M}_{n,g}$
$\mathcal{M}_{s,1}$


dim $\mathcal{M}_{n,g} = n - 3 + 3g$
$\mathcal{M}_{5,0}$
$|g\rangle_0$
$|g\rangle_0$
$|g\rangle_0$


WEYL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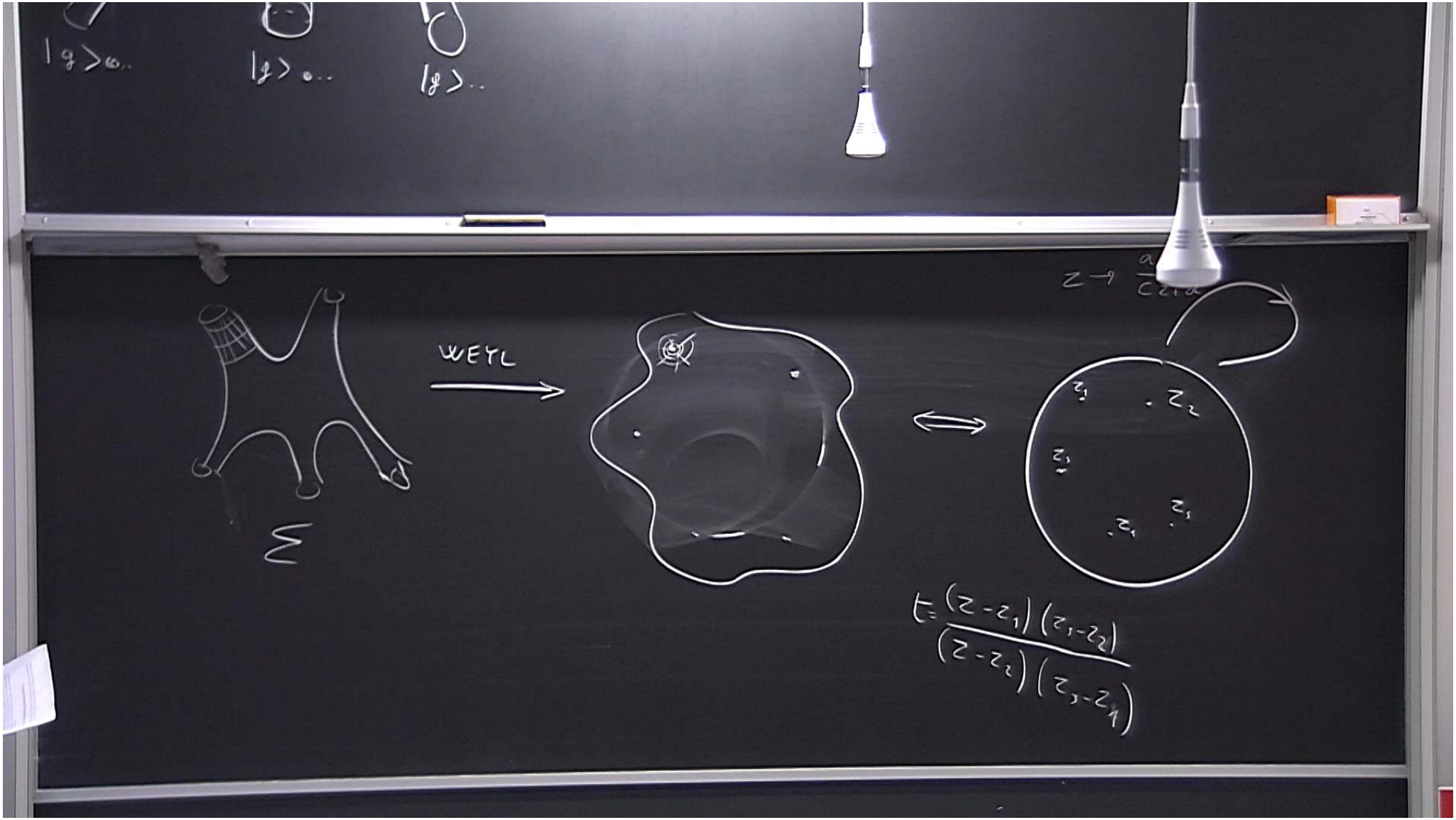
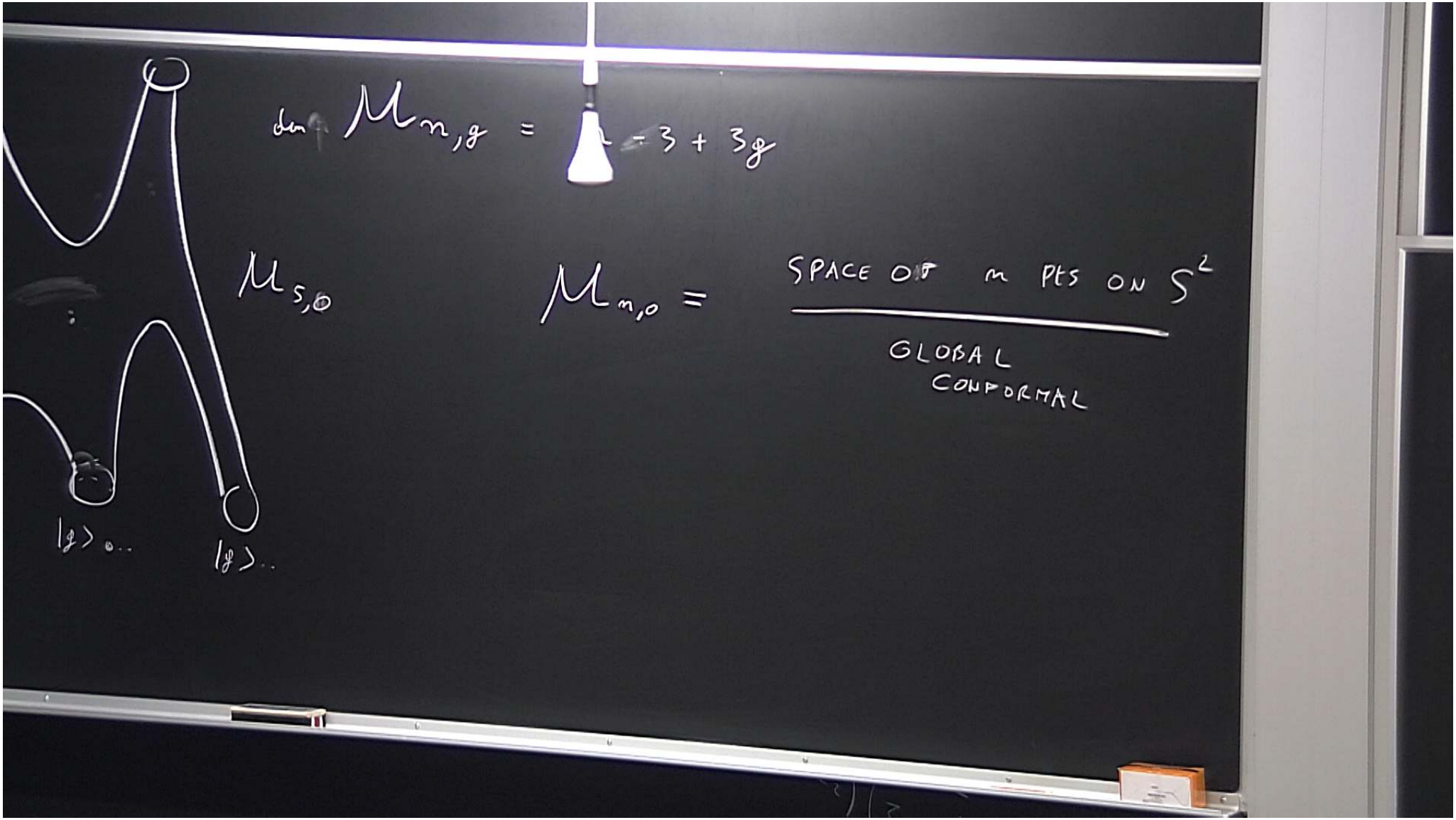
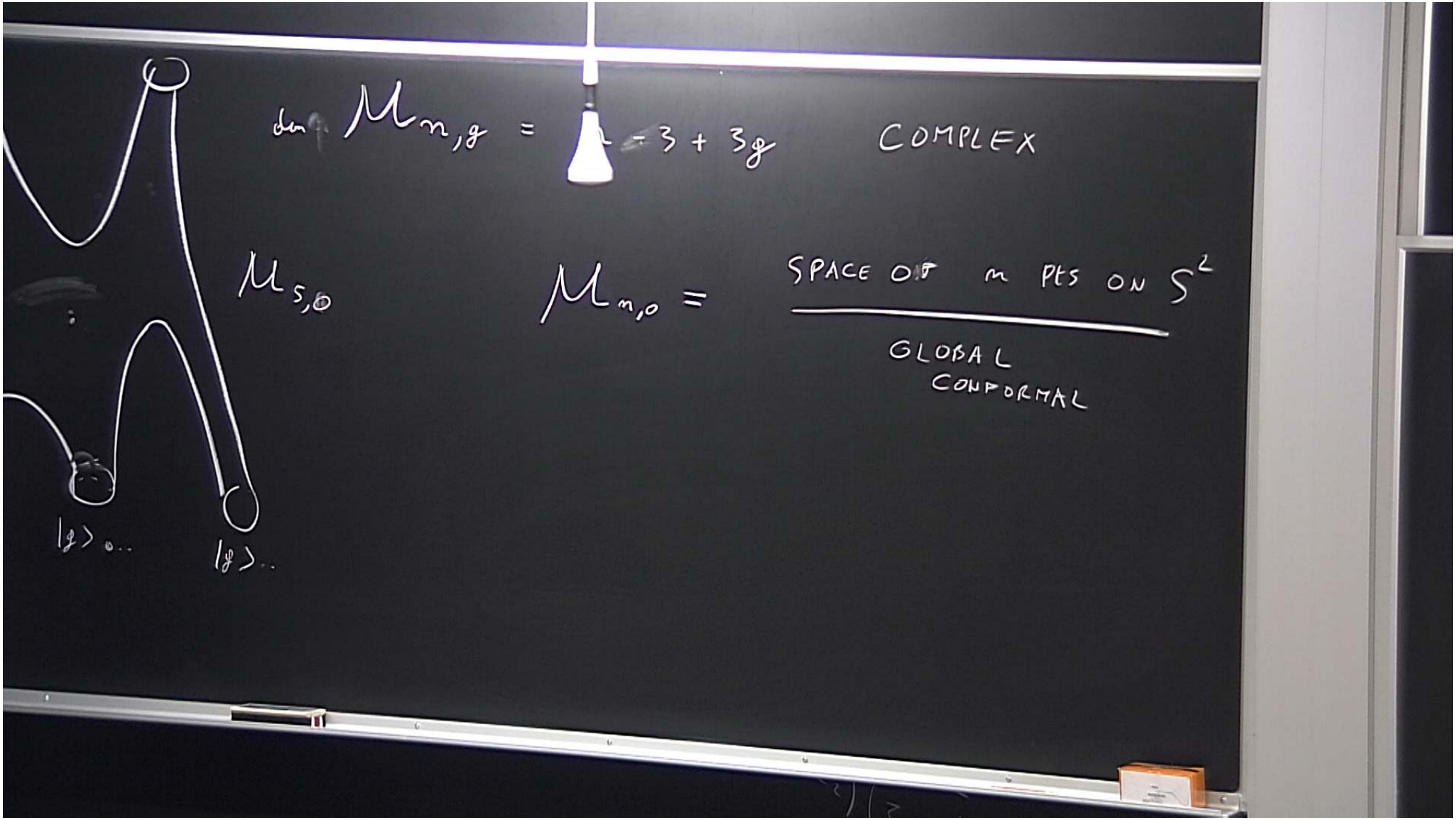
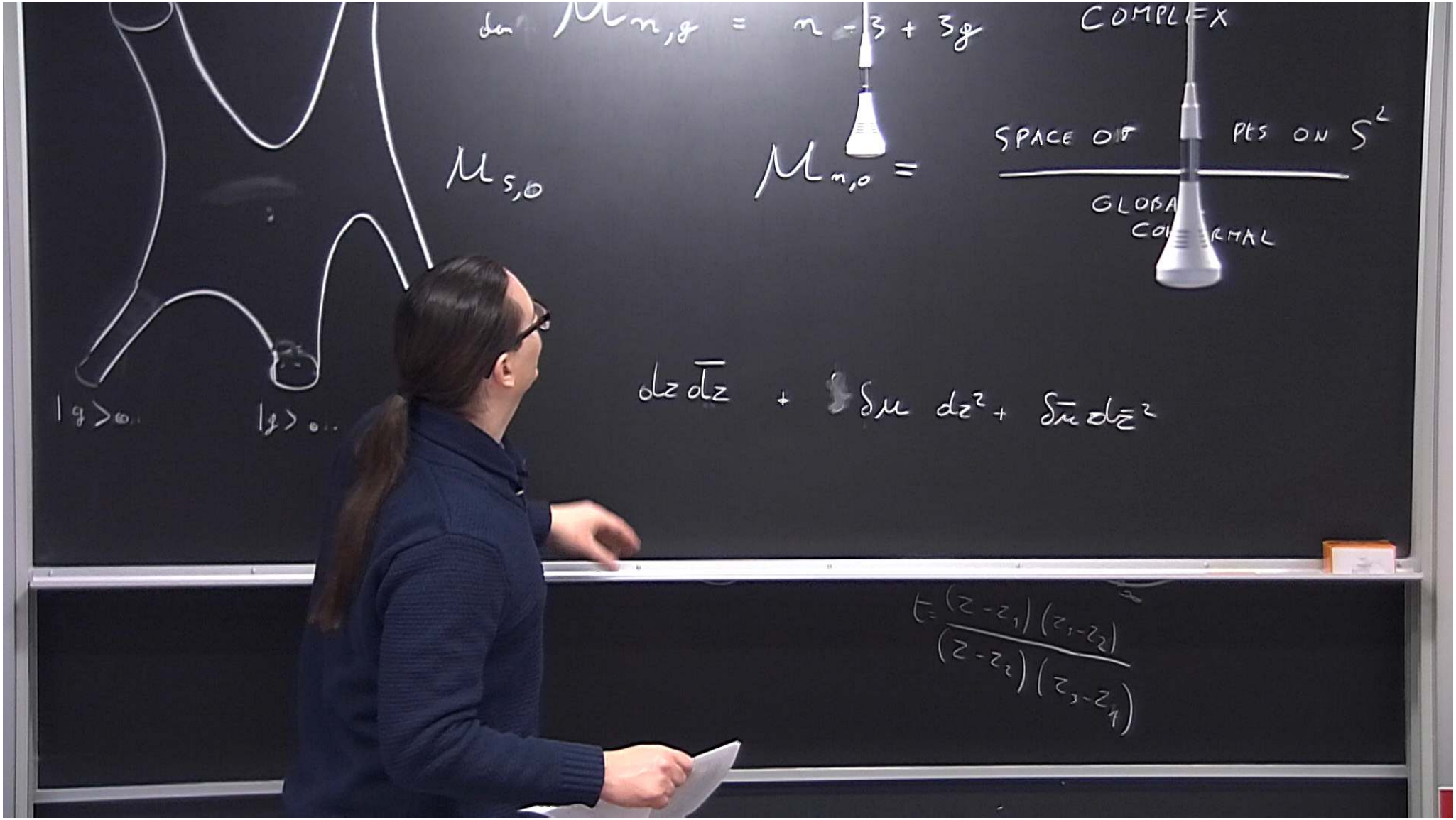
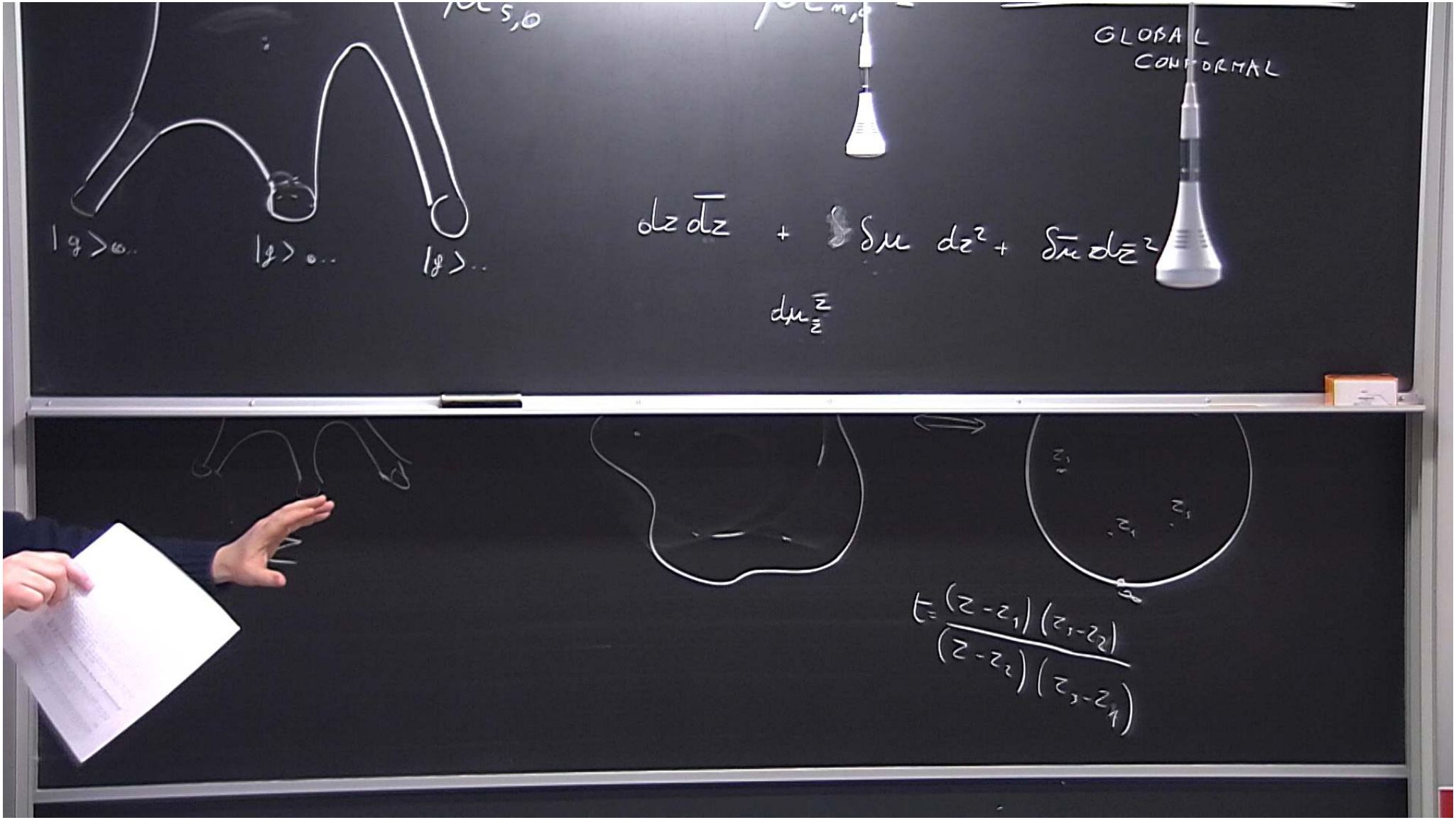
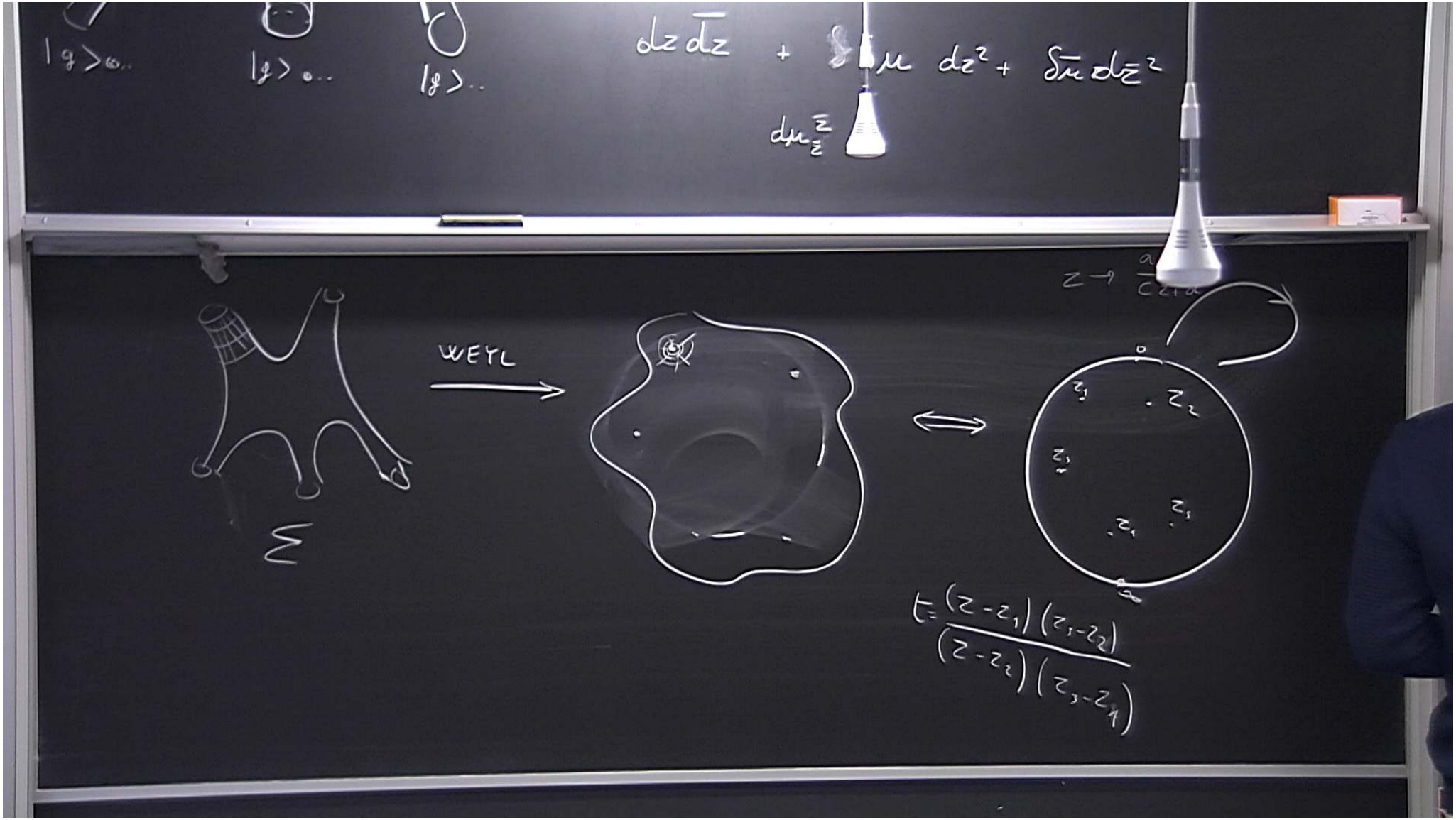

WEYL
Σ
$t = \frac{(z-z_1)(z_3-z_2)}{(z-z_2)(z_3-z_1)}$


$\dim \mathcal{M}_{n,g} = 3 + 3g$
$\mathcal{M}_{5,0}$
$\mathcal{M}_{n,0} =$ SPACE OF n PTS ON S^2 / GLOBAL CONFORMAL
$|g\rangle$ ...
$|g\rangle$ ..

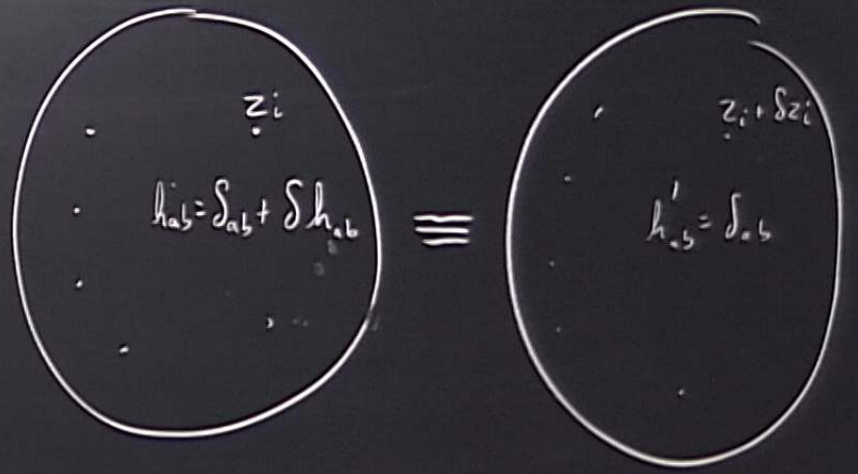

$\dim \mathcal{M}_{n,g} = 3 + 3g$
COMPLEX
$\mathcal{M}_{5,0}$
$\mathcal{M}_{n,0} =$
SPACE OF n PTS ON S^2
GLOBAL CONFORMAL
$|g\rangle$ ...
$|g\rangle$ ..

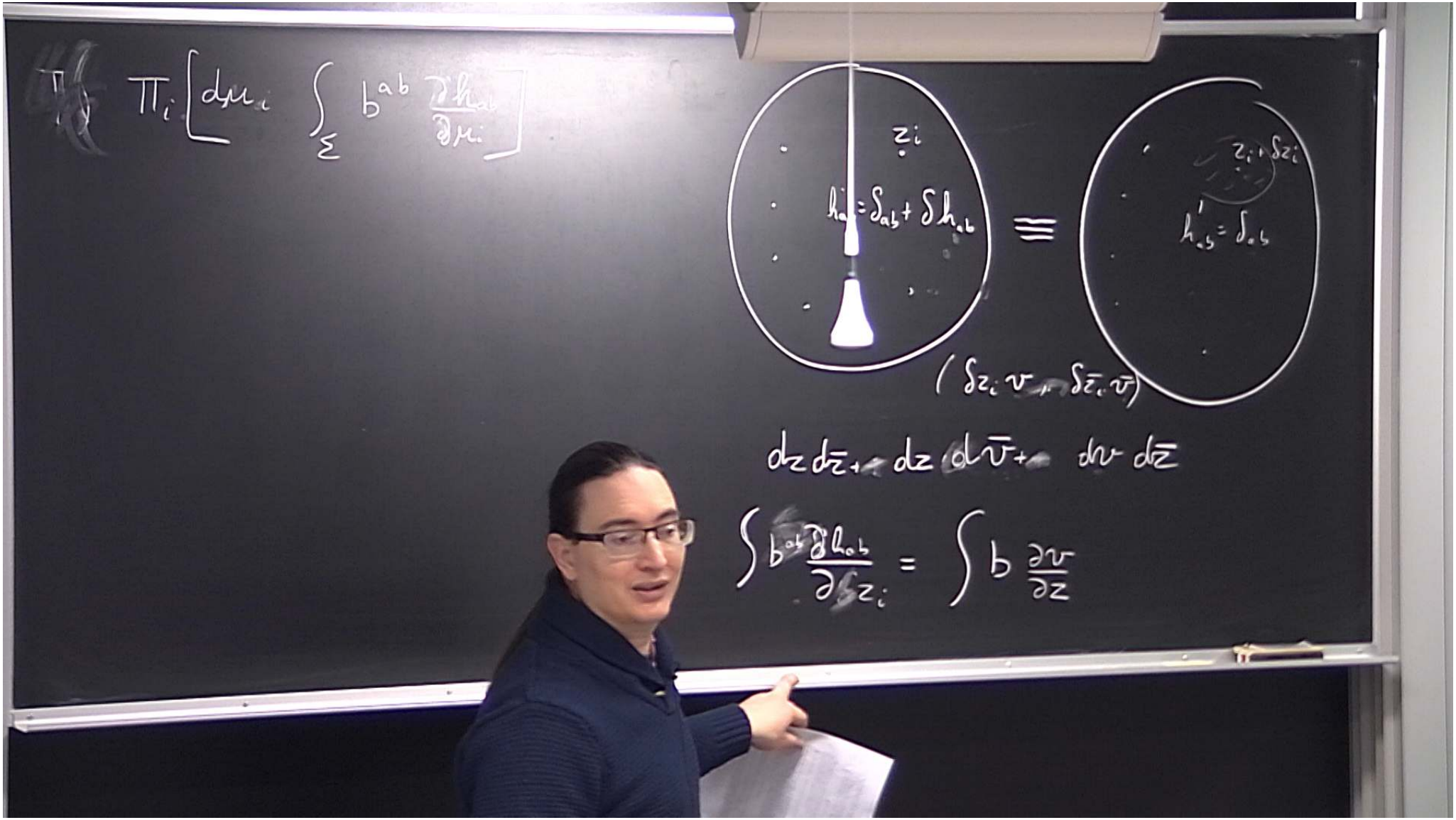

$\dim \mathcal{M}_{n,g} = n - 3 + 3g$
COMPLEX
$\mathcal{M}_{5,0}$
$\mathcal{M}_{n,0} =$
SPACE OF
PTS ON S^2
GLOBA
CONFORMAL
$|g\rangle_{\circ\circ}$
$|g\rangle_{\circ\circ}$
$dz\,d\bar{z} + \delta\mu\,dz^2 + \delta\bar{\mu}\,d\bar{z}^2$
$t = \frac{(z-z_1)(z_3-z_2)}{(z-z_2)(z_3-z_1)}$

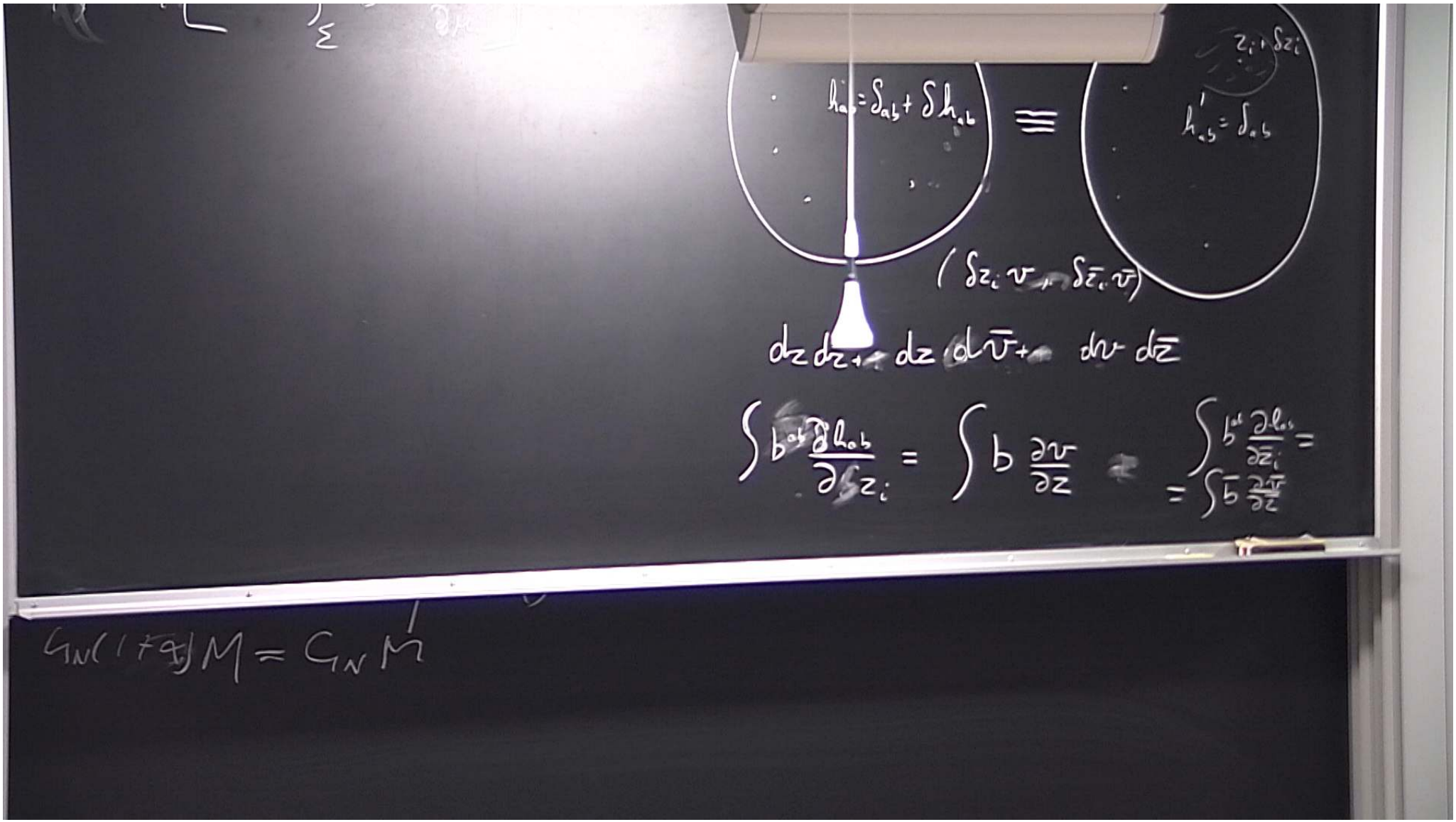

GLOBAL
CONFORMAL
$dz\,d\bar{z} + \delta\mu\, dz^2 + \delta\bar{\mu}\, d\bar{z}^2$
$d\mu_{\bar{z}}^{z}$
$|g\rangle$
$|g\rangle$
$|g\rangle$
$t = \frac{(z-z_1)(z_3-z_2)}{(z-z_2)(z_3-z_1)}$

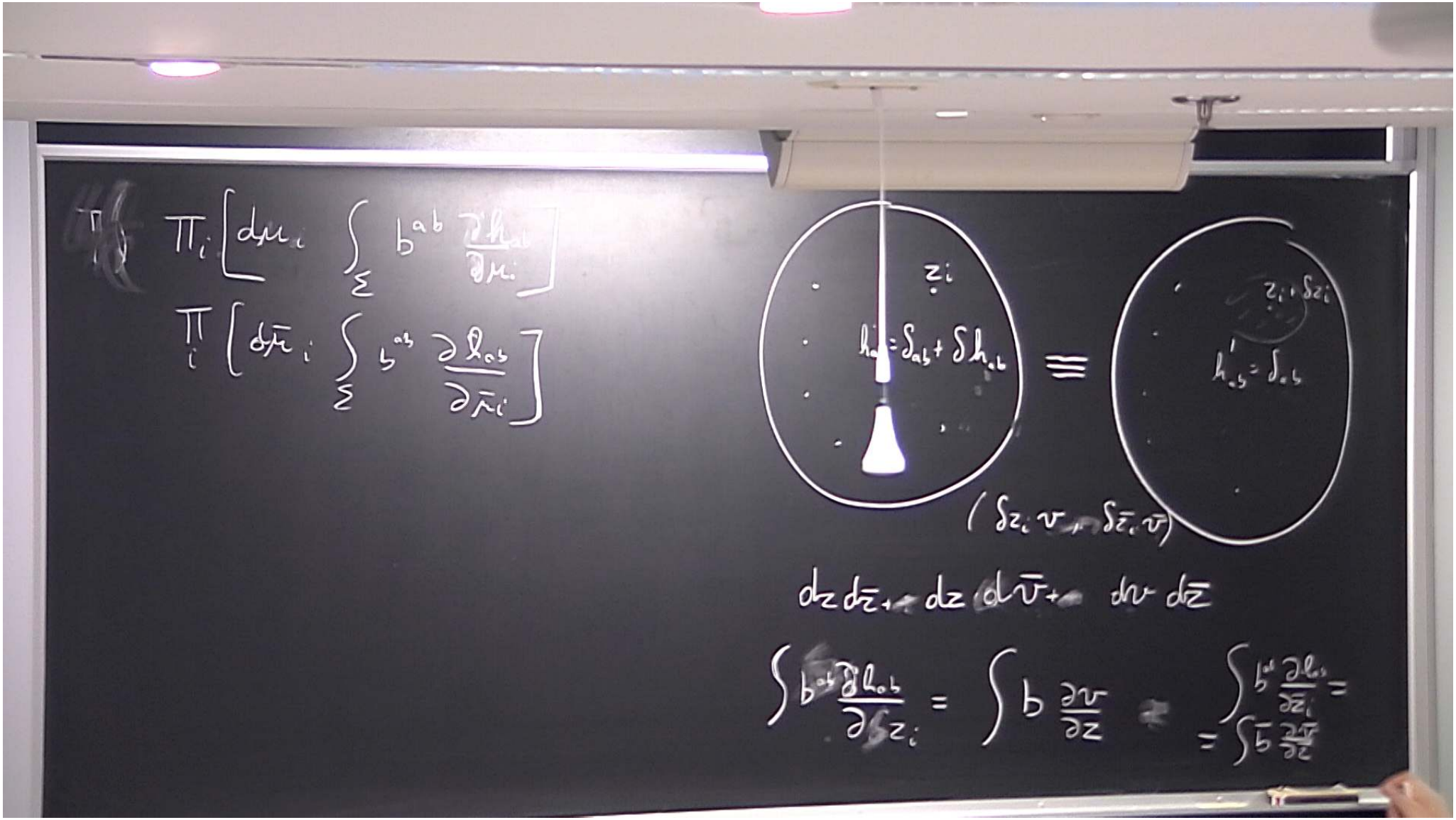

$|g\rangle_\omega$
$|g\rangle_\omega$
$|g\rangle_\omega$
$dz\,d\bar{z}$ + $\delta\mu\, dz^2$ + $\delta\bar{\mu}\, d\bar{z}^2$
$d\mu_{\bar{z}}^{z}$
WEYL
Σ
$z \to \frac{az+b}{cz+d}$
z_1
z_2
z_3
z_4
$t = \frac{(z-z_1)(z_3-z_2)}{(z-z_2)(z_3-z_1)}$

$$\Pi_i \left[d\mu_i \int_\Sigma b^{ab} \frac{\partial h_{ab}}{\partial \mu_i} \right]$$




$\Pi_i \left[d\mu_i \int_\Sigma b^{ab} \frac{\partial h_{ab}}{\partial \mu_i} \right]$
z_i
$h_{ab} = \delta_{ab} + \delta h_{ab}$
$\equiv$
$z_i + \delta z_i$
$h'_{ab} = \delta_{ab}$
$(\delta z_i v + \delta \bar{z}_i \bar{v})$
$dz\, d\bar{z} + dz\, d\bar{v} + dv\, d\bar{z}$
$\int b^{ab} \frac{\partial h_{ab}}{\partial z_i} = \int b \frac{\partial v}{\partial z}$

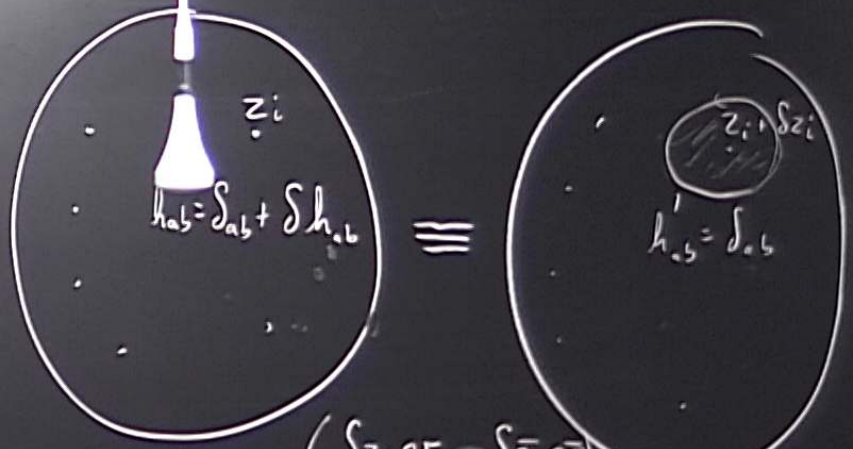

$h_{ab} = \delta_{ab} + \delta h_{ab}$
$\equiv$
$z_i + \delta z_i$
$h'_{ab} = \delta_{ab}$
$(\delta z_i v + \delta \bar{z}_i \bar{v})$
$dz\,dz + dz\,d\bar{v} + dv\,d\bar{z}$
$\int b^{ab} \frac{\partial h_{ab}}{\partial z_i} = \int b \frac{\partial v}{\partial z}$
$= \int \bar{b} \frac{\partial \bar{v}}{\partial \bar{z}}$
$G_N(1+\delta)M = G_N M'$


$\prod_i \left[d\mu_i \int_\Sigma b^{ab} \frac{\partial h_{ab}}{\partial \mu_i} \right]$
$\prod_i \left[d\bar{\mu}_i \int_\Sigma b^{ab} \frac{\partial h_{ab}}{\partial \bar{\mu}_i} \right]$
z_i
$h_{ab} = \delta_{ab} + \delta h_{ab}$
$\equiv$
$z_i + \delta z_i$
$h'_{ab} = \delta_{ab}$
$(\delta z_i v, \delta \bar{z}_i \bar{v})$
$dz\, d\bar{z} + dz\, d\bar{v} + dv\, d\bar{z}$
$\int b^{ab} \frac{\partial h_{ab}}{\partial z_i} = \int b \frac{\partial v}{\partial z} = \int b^{ab} \frac{\partial h_{ab}}{\partial \bar{z}_i} =$
$= \int \bar{b} \frac{\partial \bar{v}}{\partial \bar{z}}$

$$\prod_i \left[d\mu_i \int_\Sigma b^{ab} \frac{\partial h_{ab}}{\partial \mu_i} \right]$$

$$\prod_i \left[d\bar{\mu}_i \int_\Sigma b^{ab} \frac{\partial h_{ab}}{\partial \bar{\mu}_i} \right]$$

$$\int b \frac{\partial v}{\partial z} \, dz \, d\bar{z} = \oint_{l_i} b(z) V(z) \, dz$$



$(\delta z_i v, \delta \bar{z}_i \bar{v})$

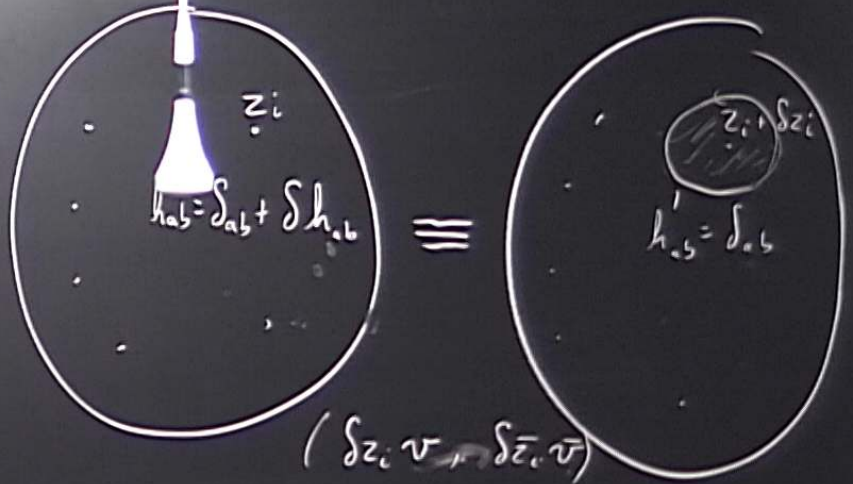
$dz\,d\bar{z} + dz\,d\bar{v} + dv\,d\bar{z}$

$$\int b^{ab} \frac{\partial h_{ab}}{\partial z_i} = \int b \frac{\partial v}{\partial z} = \int b^{ab} \frac{\partial h_{ab}}{\partial \bar{z}_i} = \int \bar{b} \frac{\partial \bar{v}}{\partial \bar{z}}$$

$$\prod_i \left[d\mu_i \int_\Sigma b^{ab} \frac{\partial h_{ab}}{\partial \mu_i} \right]$$

$$\prod_i \left[d\bar{\mu}_i \int_\Sigma b^{ab} \frac{\partial h_{ab}}{\partial \bar{\mu}_i} \right]$$

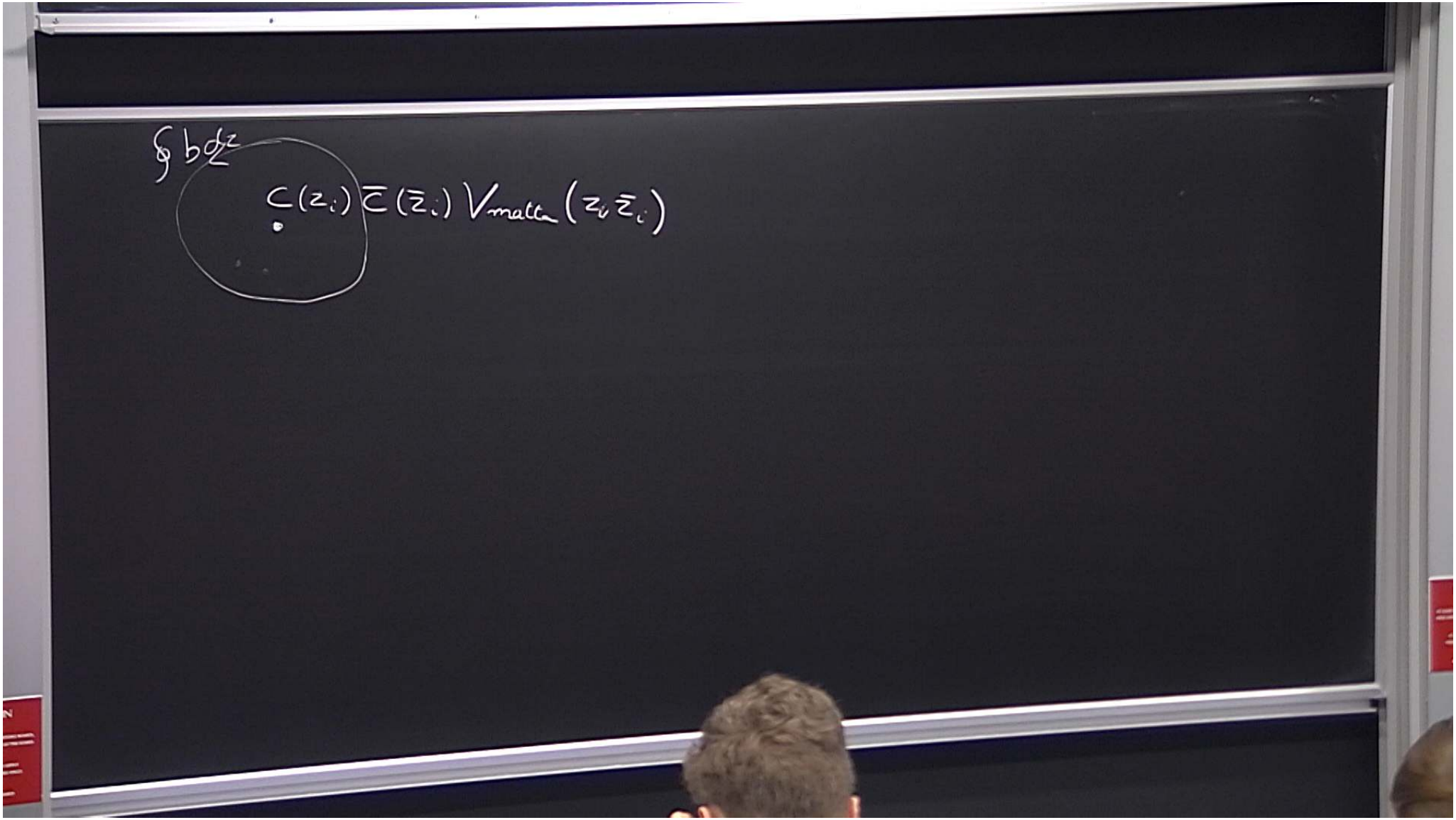
$$\int b \frac{\partial v}{\partial z} \, dz \, d\bar{z} = \oint_{l_i} b(z) \, v \, dz$$

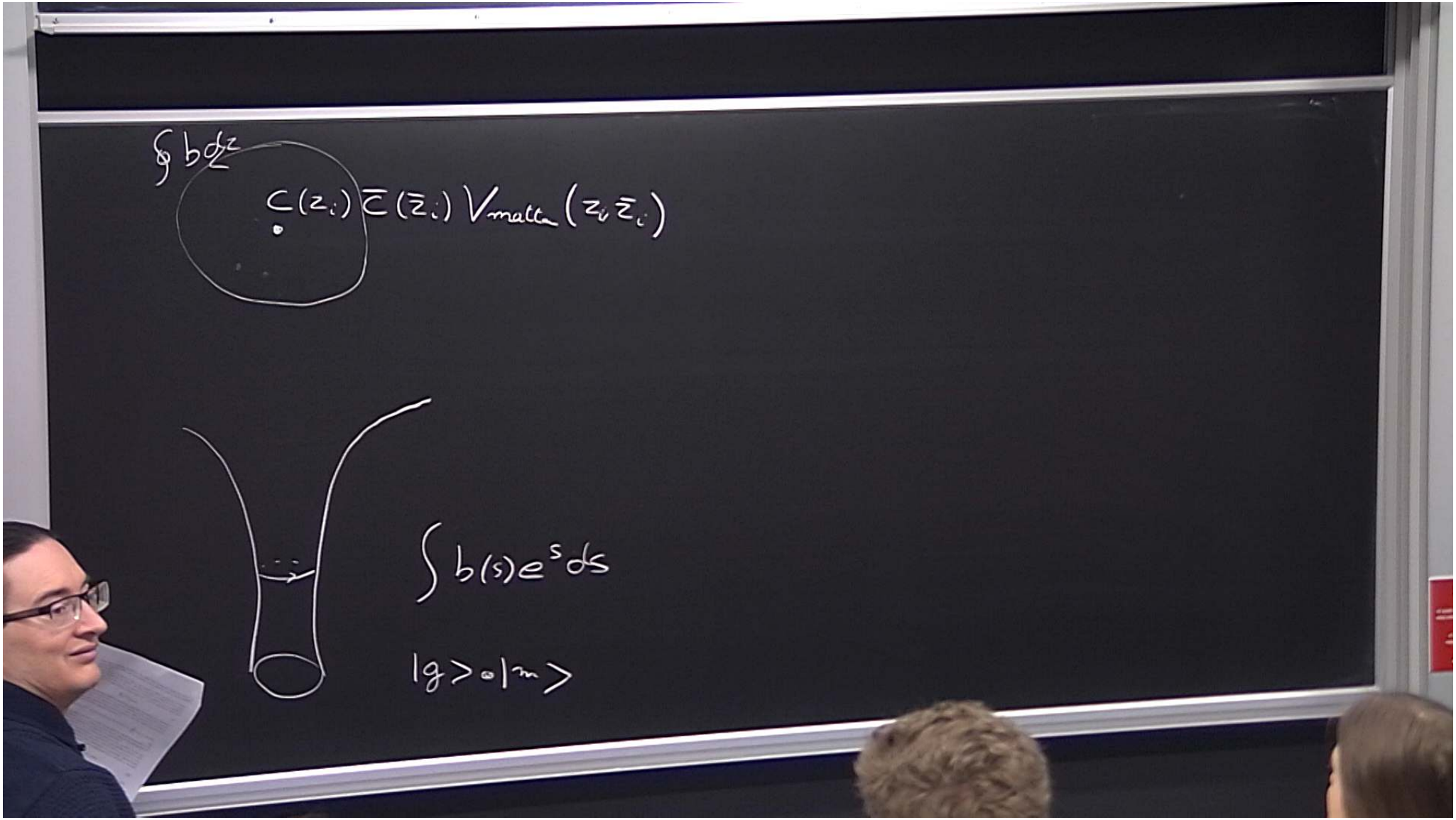


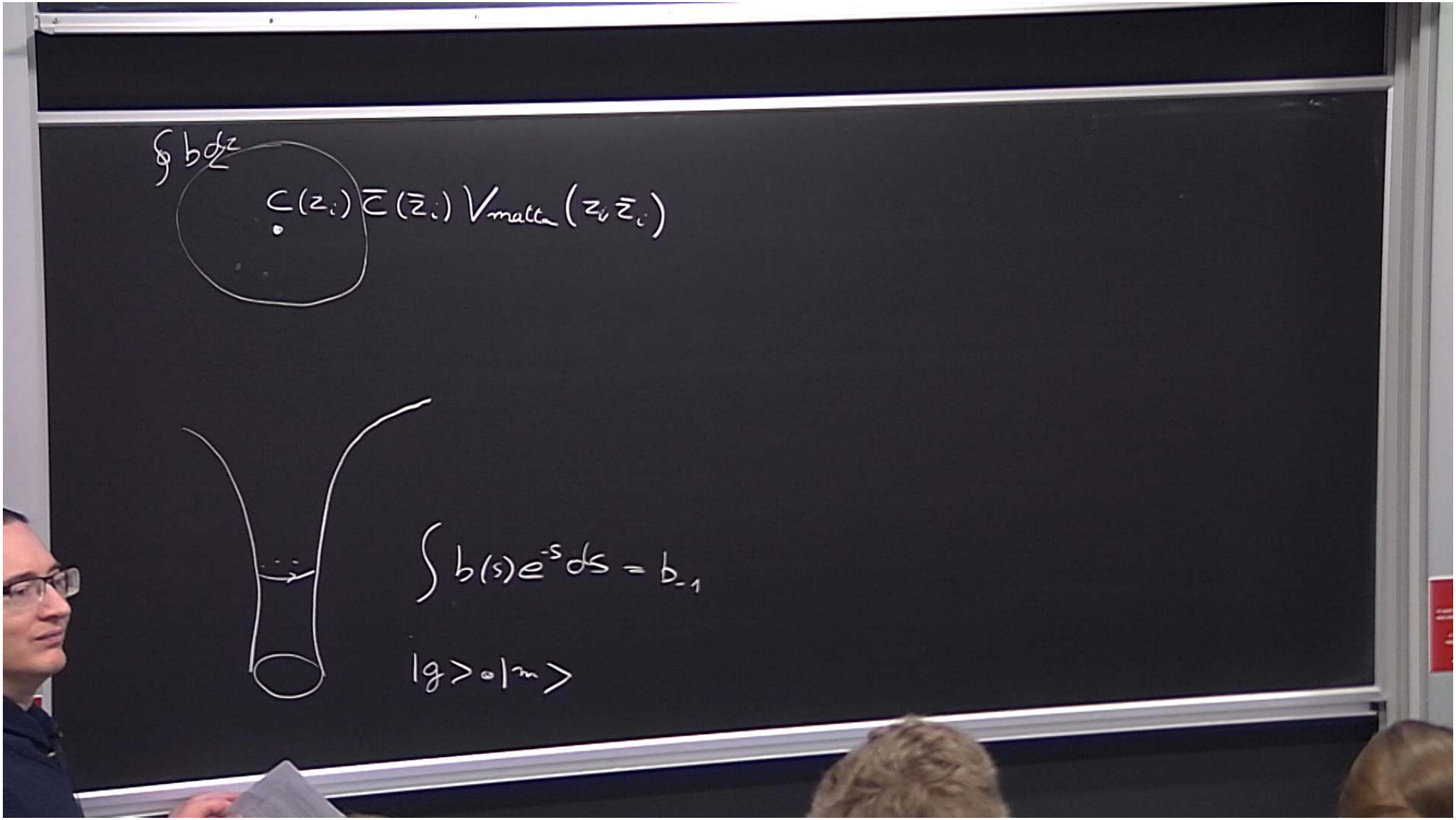
$(\delta z_i v, \delta \bar{z}_i \bar{v})$

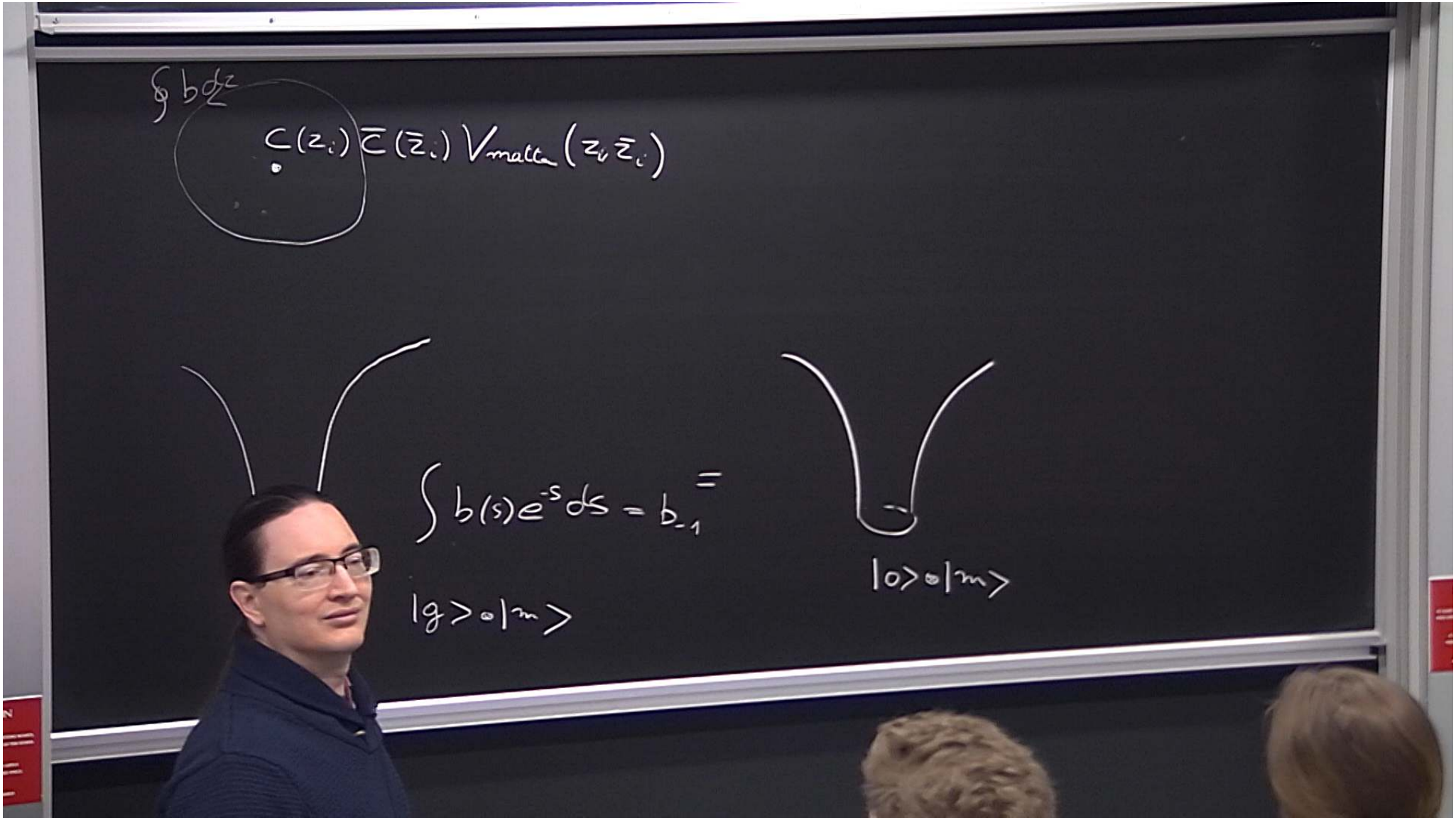
$dz\,d\bar{z} + dz\,d\bar{v} + dv\,d\bar{z}$

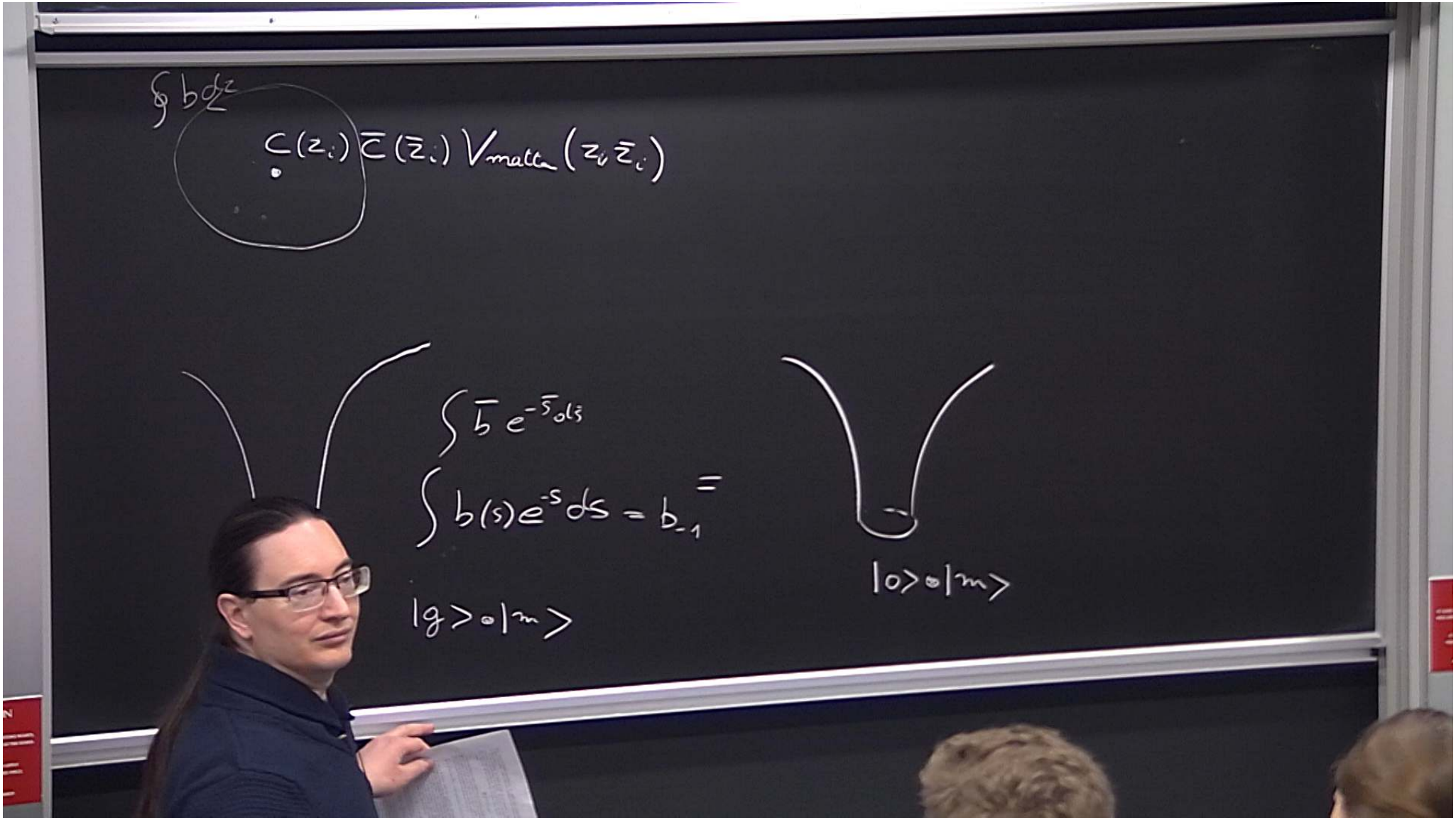
$$\int b^{ab} \frac{\partial h_{ab}}{\partial z_i} = \int b \frac{\partial v}{\partial z} = \int b^{ab} \frac{\partial h_{ab}}{\partial \bar{z}_i} = \int \bar{b} \frac{\partial \bar{v}}{\partial \bar{z}}$$

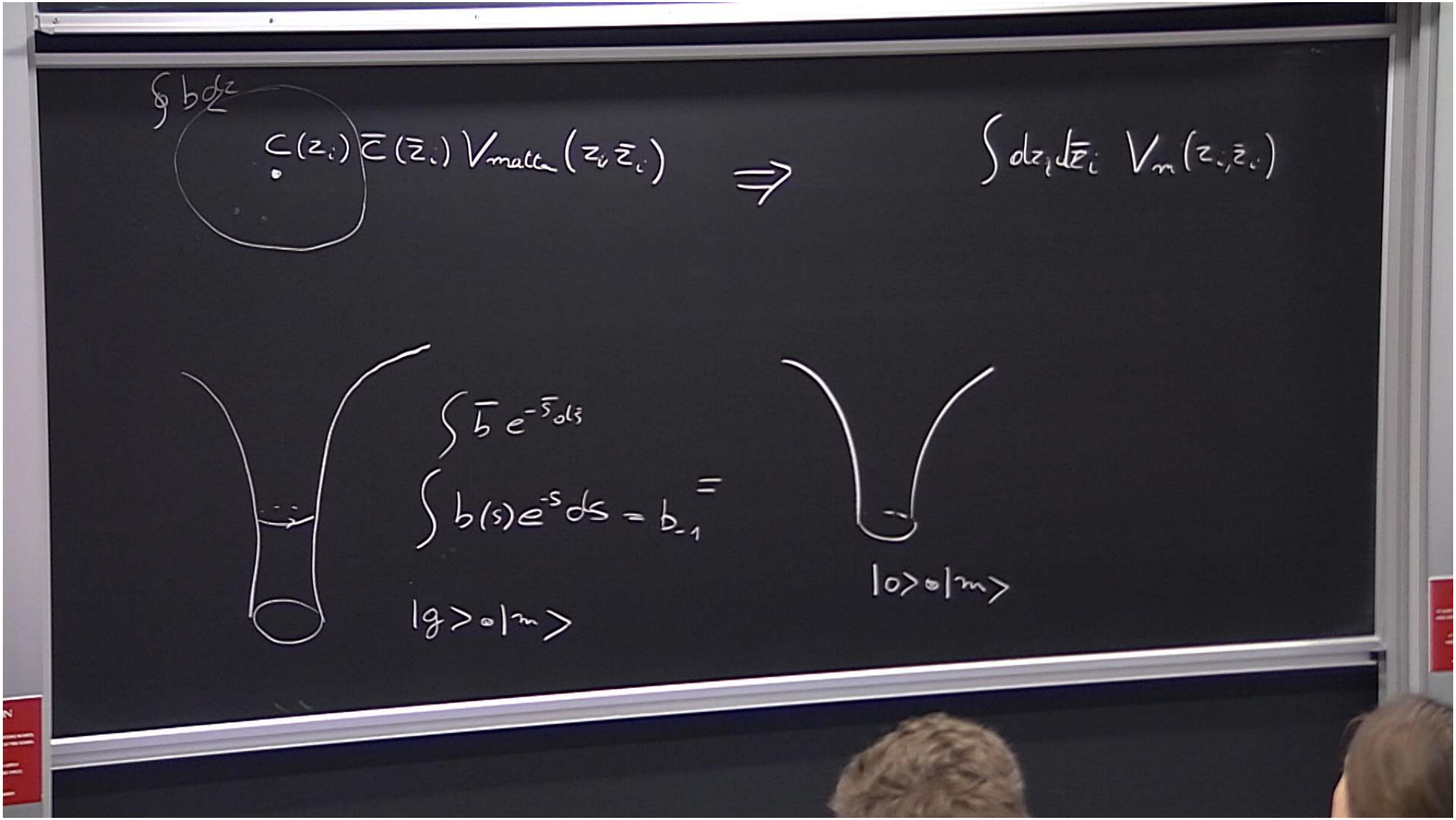

$\oint b\,dz$
$c(z_i)\,\bar{c}(\bar{z}_i)\,V_{matter}(z_i,\bar{z}_i)$

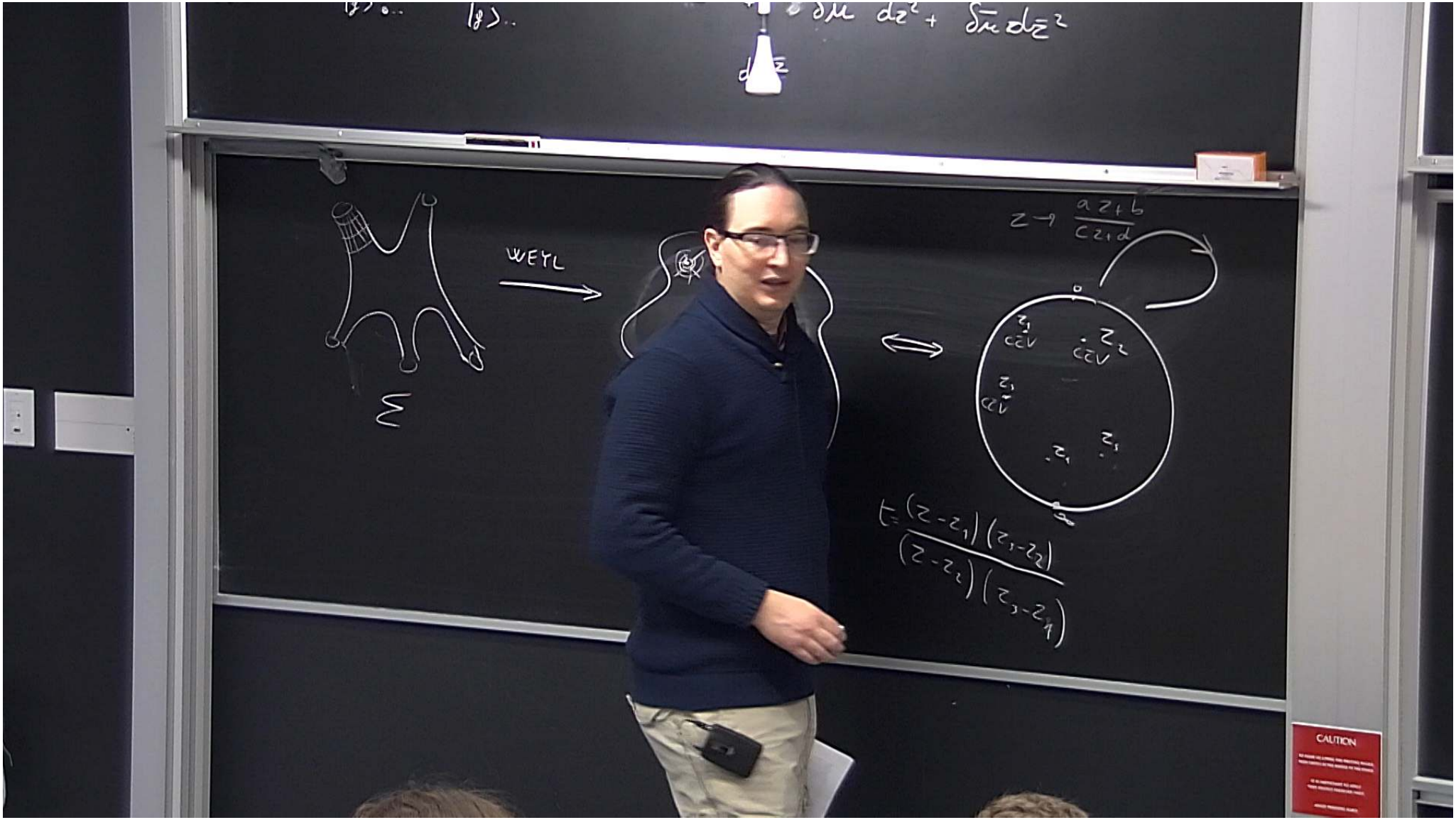

$\oint b dz$
$c(z_i)\,\bar{c}(\bar{z}_i)\,V_{matter}(z_i, \bar{z}_i)$
$\int b(s) e^s ds$
$|g\rangle \otimes |m\rangle$

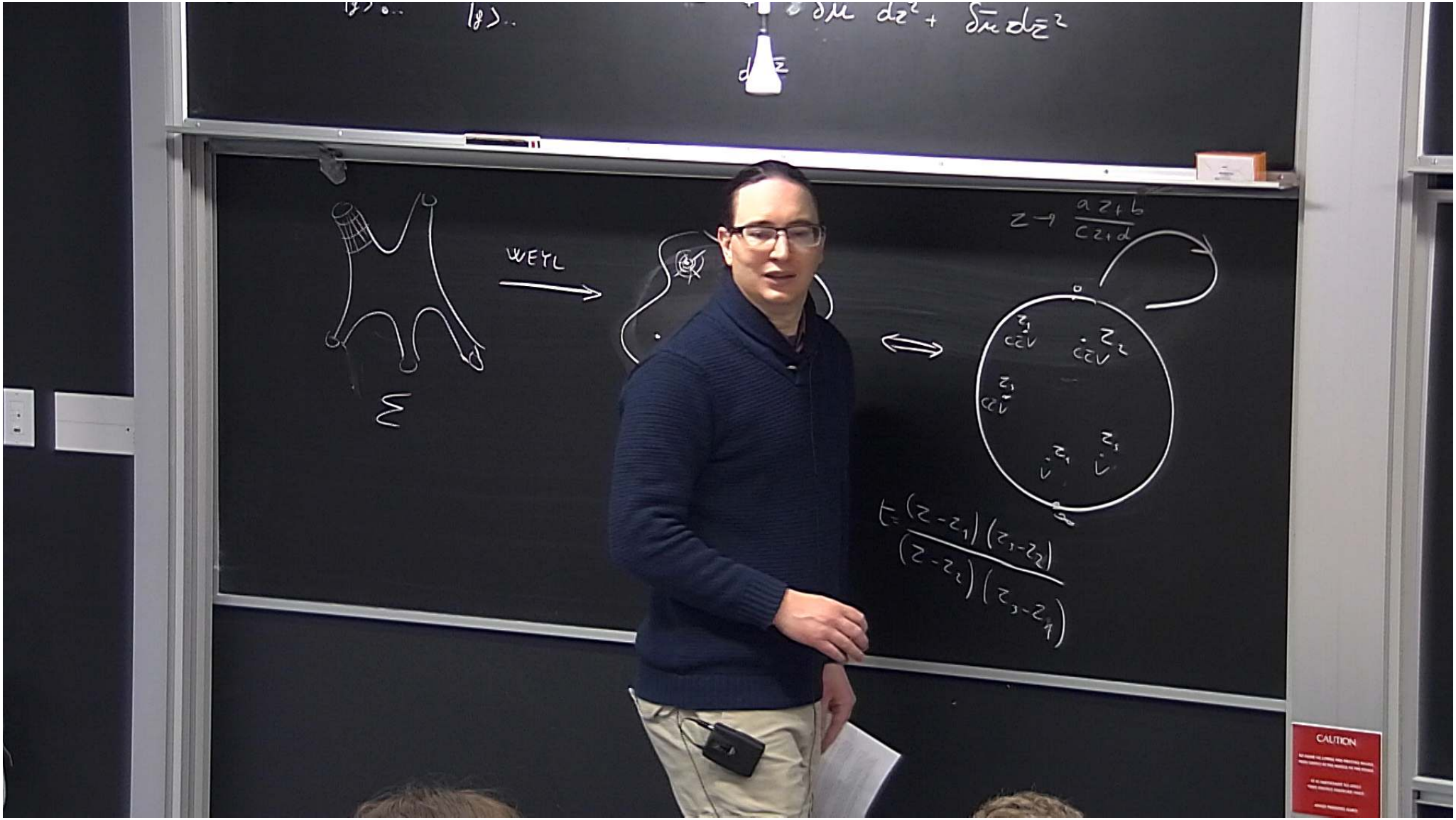

$\oint b dz$
$c(z_i)\,\bar{c}(\bar{z}_i)\,V_{matter}(z_i, \bar{z}_i)$
$\int b(s) e^{-s} ds = b_{-1}$
$|g\rangle \otimes |m\rangle$


$\oint b dz$
$c(z_i)\,\bar{c}(\bar{z}_i)\,V_{matter}(z_i, \bar{z}_i)$
$\int b(s) e^{-s} ds = b_{-1}$
$|g\rangle \otimes |m\rangle$
$|0\rangle \otimes |m\rangle$


$\oint b dz$
$c(z_i)\bar{c}(\bar{z}_i)V_{matter}(z_i,\bar{z}_i)$
$\int \bar{b} e^{-\bar{S}_{ol}}$
$\int b(s)e^{-s}ds = b_{-1} =$
$|g\rangle \otimes |m\rangle$
$|0\rangle \otimes |m\rangle$


$\oint b\,dz$
$c(z_i)\,\bar{c}(\bar{z}_i)\,V_{matter}(z_i,\bar{z}_i)$
$\Rightarrow$
$\int dz_i\,d\bar{z}_i\;V_m(z_i,\bar{z}_i)$
$\int \bar{b}\,e^{-\bar{S}}\,d\xi$
$\int b(s)\,e^{-s}\,ds = b_{-1}$
$|g\rangle \otimes |m\rangle$
$|0\rangle \otimes |m\rangle$


WEYL
Σ
CAUTION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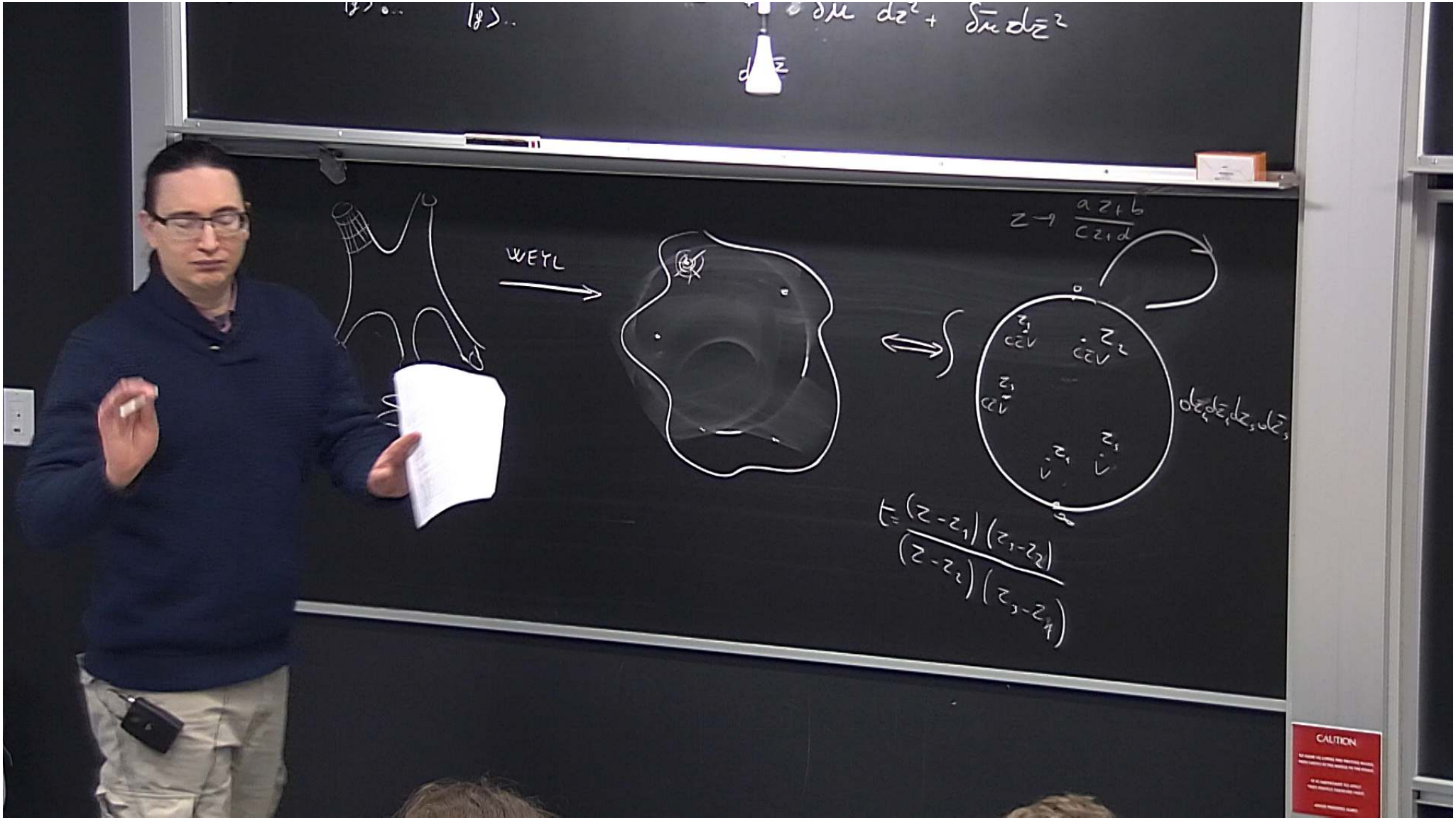
CAUTION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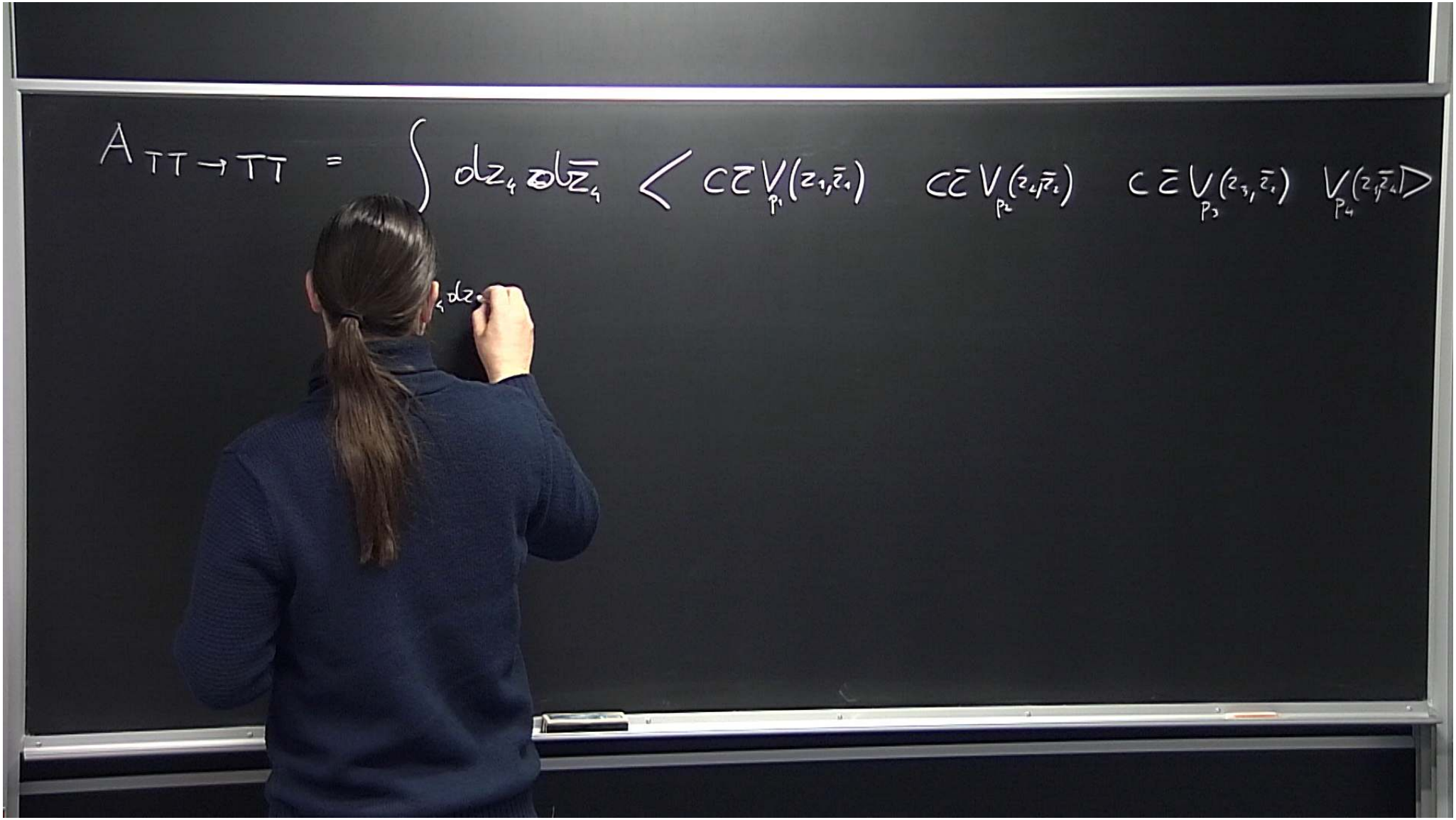
CAUTION


$A_{TT\to TT} = \int dz_4 d\bar{z}_4 \langle c\bar{c}V_{p_1}(z_1,\bar{z}_1)\ c\bar{c}V_{p_2}(z_2,\bar{z}_2)\ c\bar{c}V_{p_3}(z_3,\bar{z}_3)\ V_{p_4}(z_4,\bar{z}_4)\rangle$

$$A_{TT\to TT} = \int dz_4 d\bar{z}_4 \; \langle c\bar{c} V_{p_1}(z_1,\bar{z}_1) \; c\bar{c} V_{p_2}(z_2,\bar{z}_2) \; c\bar{c} V_{p_3}(z_3,\bar{z}_3) \; V_{p_4}(z_4,\bar{z}_4) \rangle$$

$$e^{ip_1 X} \quad e^{ip_2 X} \quad e^{ip_3 X} \quad e^{ip_4 X}$$

$$= \prod_{1\le i<j\le 3} |z_i - z_j|^{2p_i\cdot p_j + 2} \int dz_4 d\bar{z}_4 \prod_{1\le i\le 3} |z_4 - z_i|^{2p_i\cdot p_4}$$

$$A_{TT\to TT} = \int dz_4 d\bar{z}_4 \; \langle c\bar{c} V_{p_1}(z_1,\bar{z}_1) \; c\bar{c} V_{p_2}(z_2,\bar{z}_2) \; c\bar{c} V_{p_3}(z_3,\bar{z}_3) \; V_{p_4}(z_4,\bar{z}_4) \rangle$$

$$e^{ip_1 X} \qquad e^{ip_2 X} \qquad e^{ip_3 X} \qquad e^{ip_4 X}$$

$$= \prod_{1\le i<j\le 3} |z_i - z_j|^{2p_i\cdot p_j + 2} \int dz_4 d\bar{z}_4 \prod_{1\le i\le 3} |z_4 - z_i|^{2p_i\cdot p_4}$$

$$z_4 \to \infty \quad |z_4|^{-2p_4^2}$$

$$A_{TT\to TT} = \int dz_4 d\bar{z}_4 \; \langle c\bar{c} V_{p_1}(z_1,\bar{z}_1) \; c\bar{c} V_{p_2}(z_2,\bar{z}_2) \; c\bar{c} V_{p_3}(z_3,\bar{z}_3) \; V_{p_4}(z_4,\bar{z}_4) \rangle$$

$$e^{ip_1 X} \quad e^{ip_2 X} \quad e^{ip_3 X} \quad e^{ip_4 X}$$

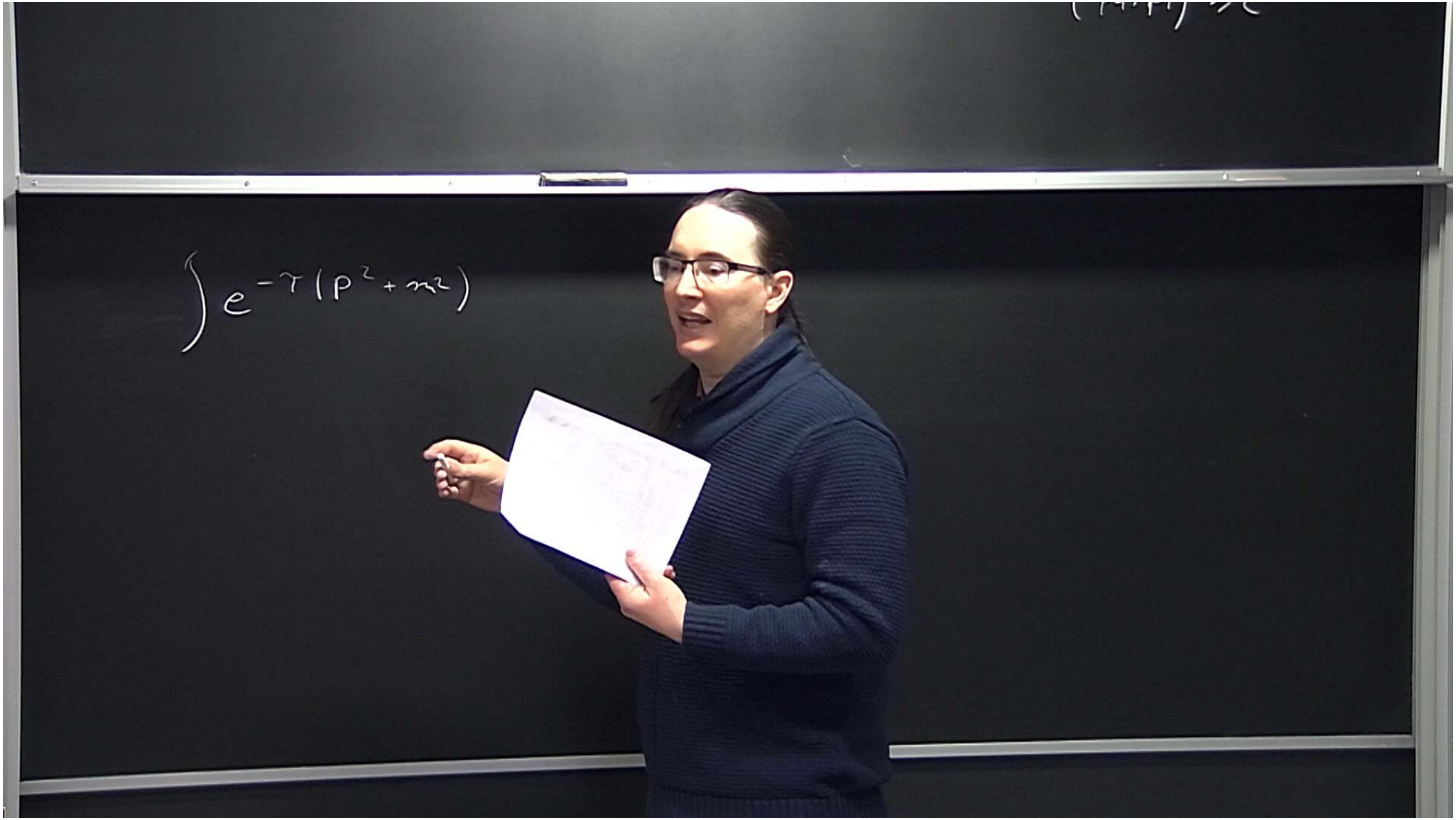
$$= \prod_{1\le i<j\le 3} |z_i - z_j|^{2p_i\cdot p_j + 2} \int dz_4 d\bar{z}_4 \prod_{1\le i\le 3} |z_4 - z_i|^{2p_i\cdot p_4}$$

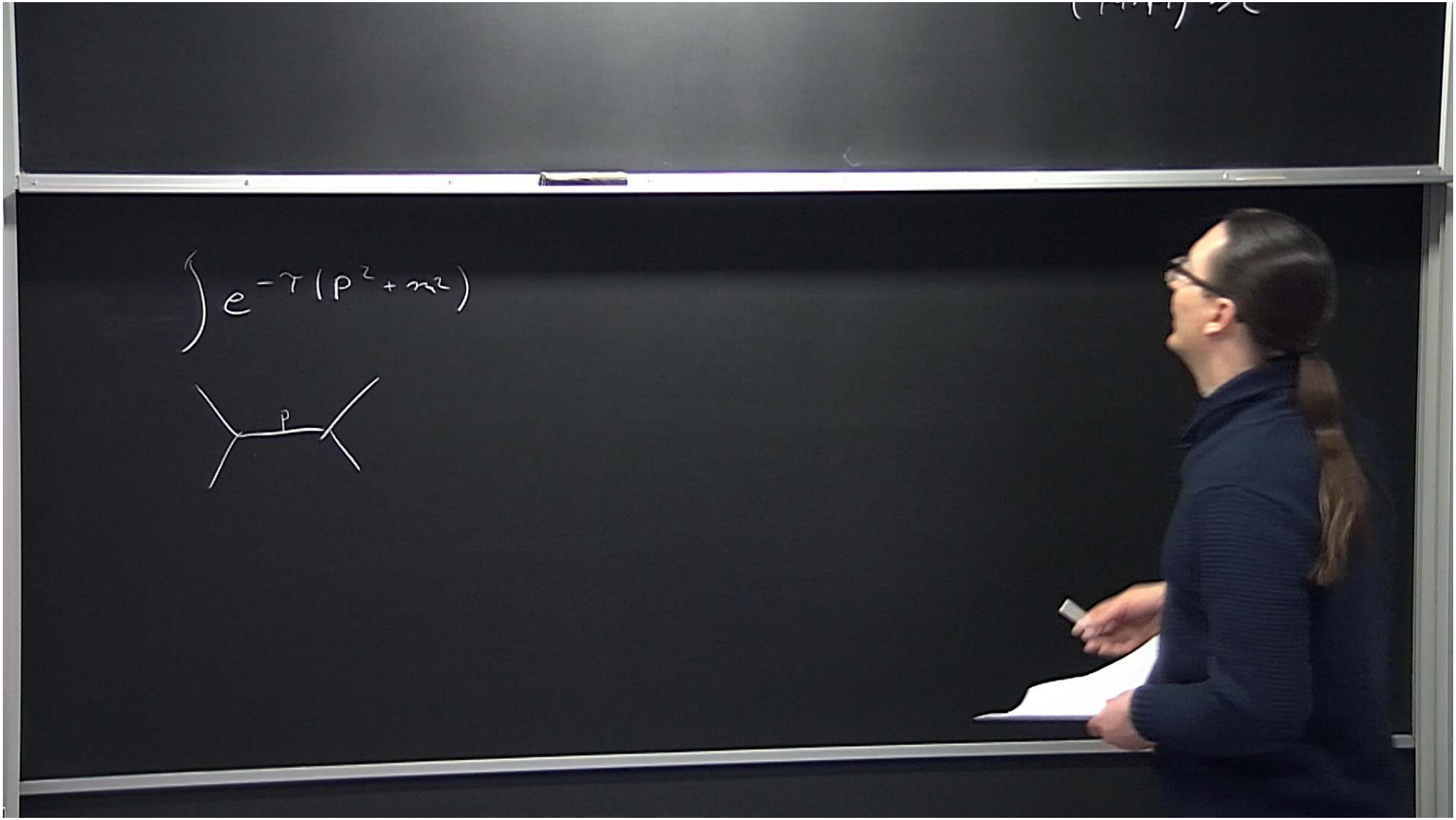
$z_4 \to \infty \quad |z_4|^{-2p_4^2}$

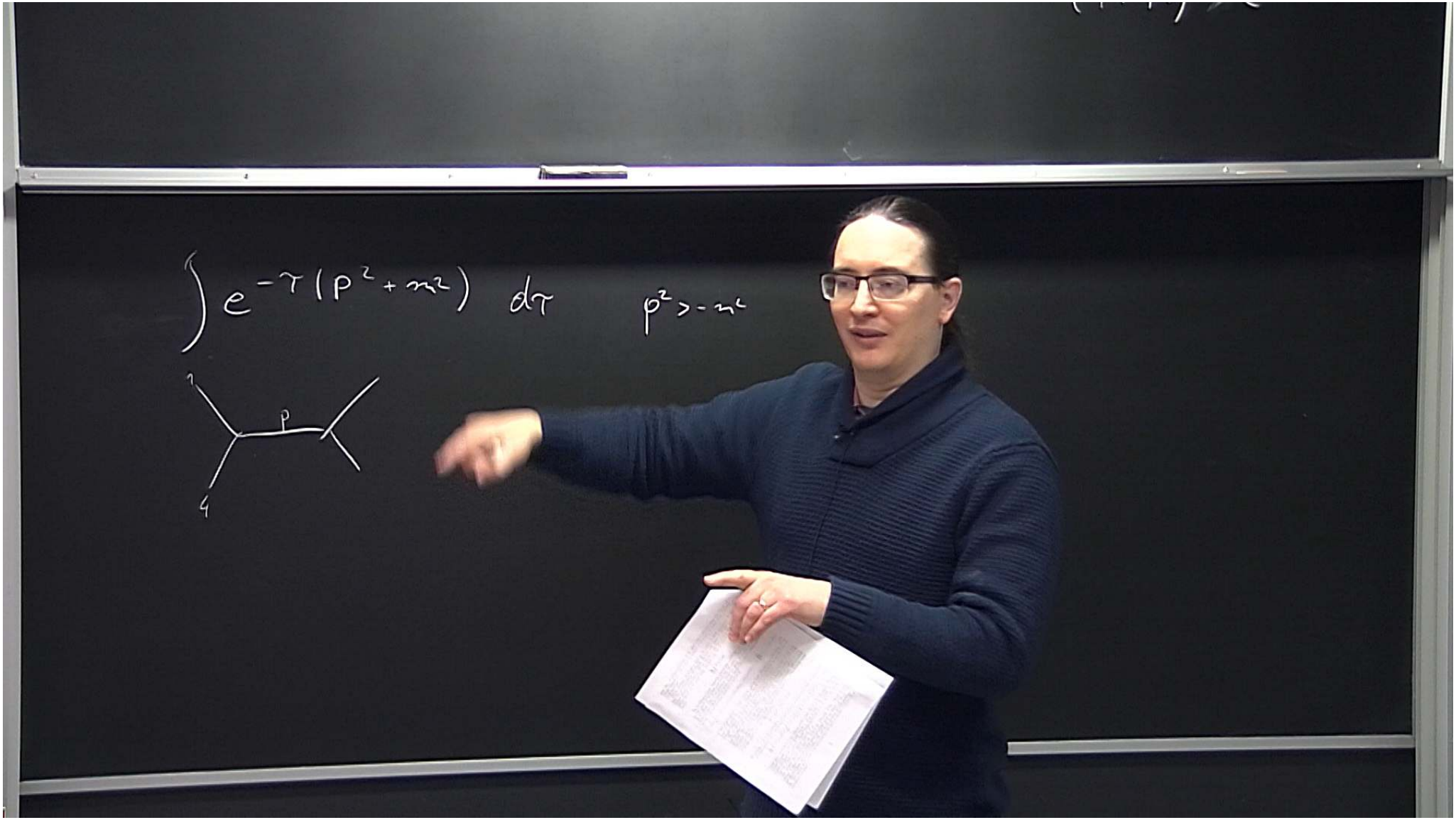
$z_4 \to z_i \quad |z_4 - z_i|^{2p_4\cdot p_i}$

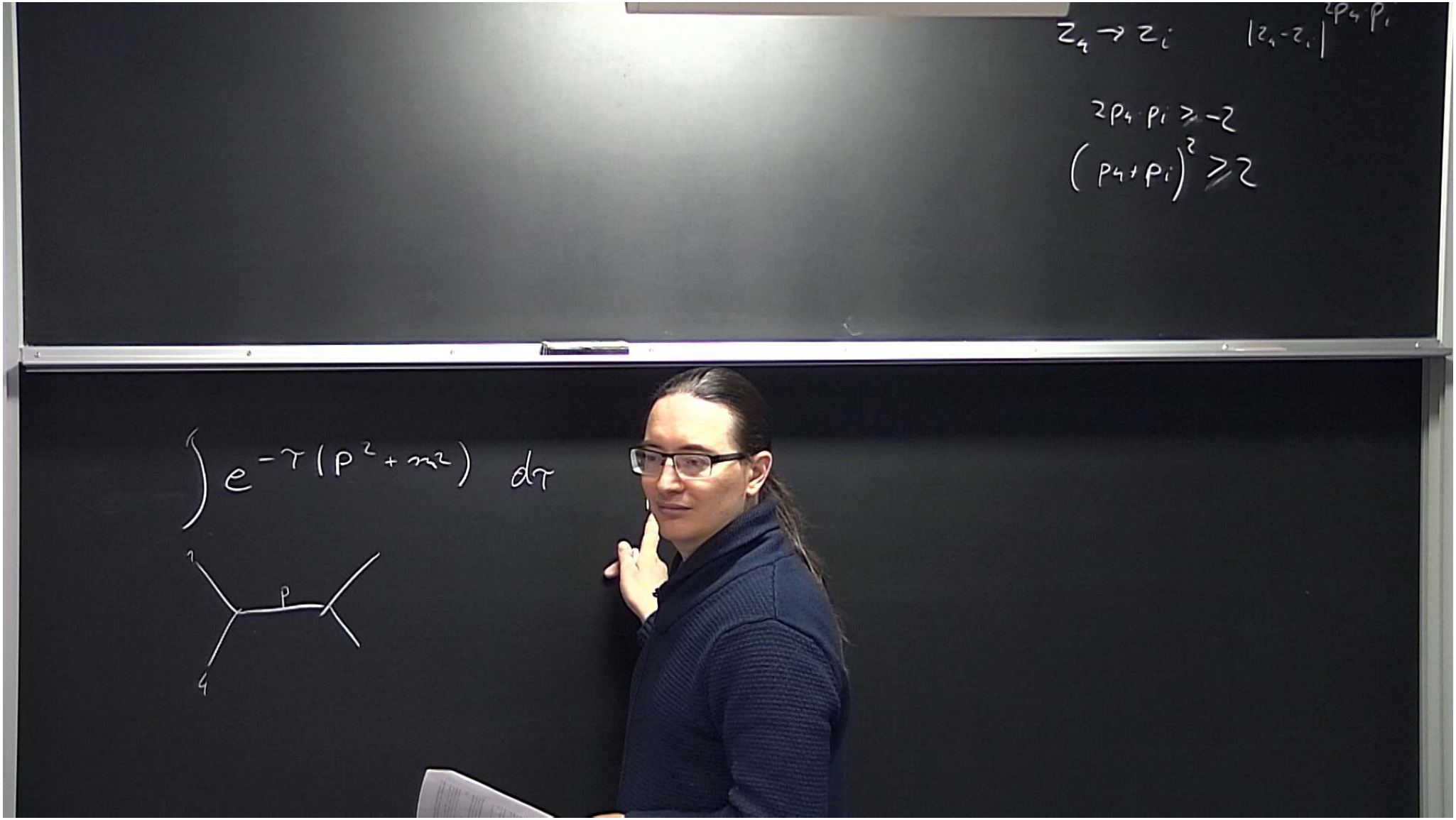
$2p_4\cdot p_i \geq -2$

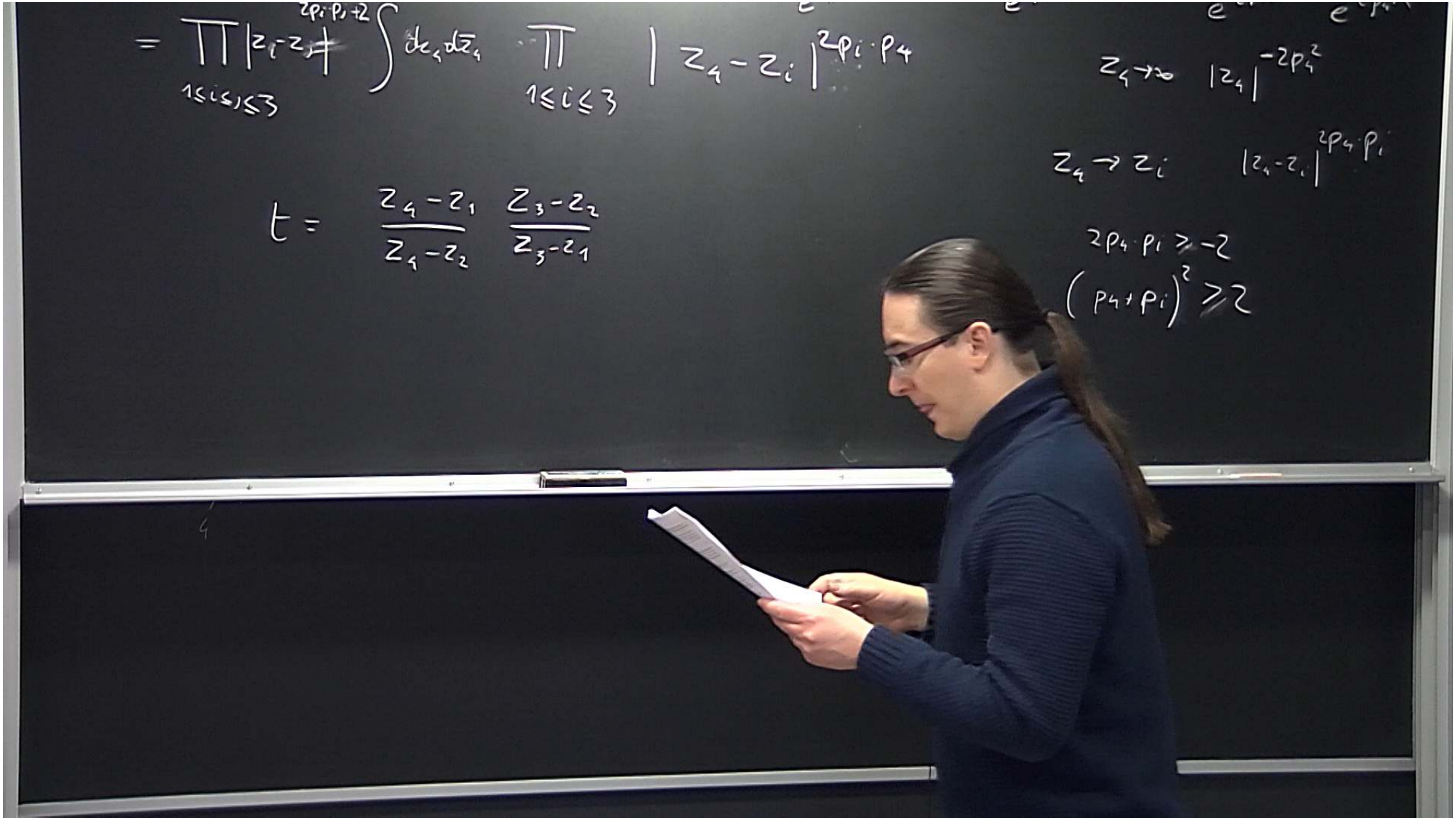
$(p_4 + p_i)^2 \geq 2$

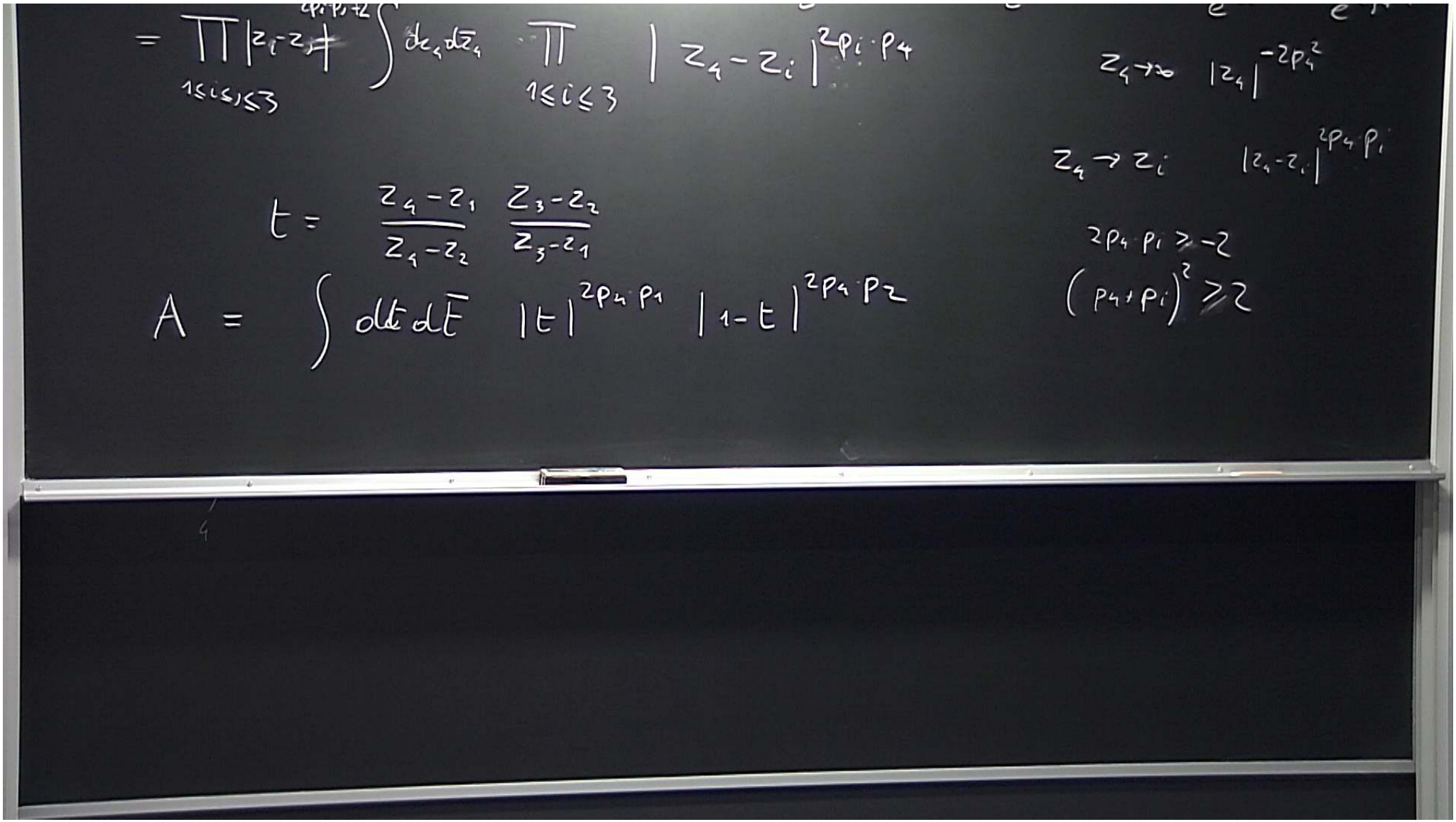

$\int e^{-\tau(p^2+m^2)}$

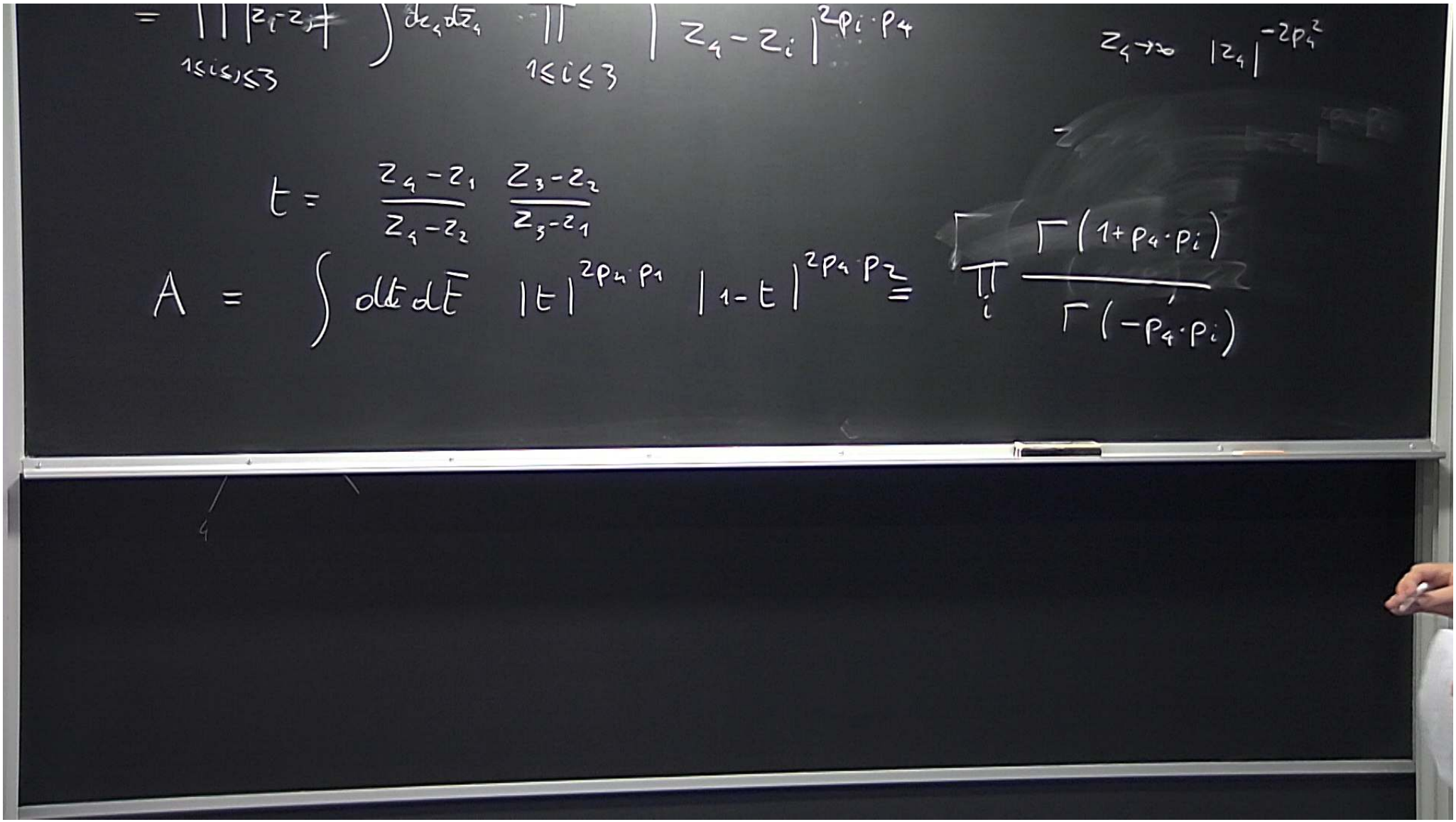

$\int e^{-\tau(p^2+m^2)}$
p


$\int e^{-\tau(p^2+m^2)}\, d\tau \qquad p^2 > -m^2$
p


$z_4 \to z_i \quad |z_4 - z_i|^{2p_4 p_i}$
$2p_4 p_i \geq -2$
$(p_4 + p_i)^2 \geq 2$
$\int e^{-\tau(p^2 + m^2)} \, d\tau$
1
p
4

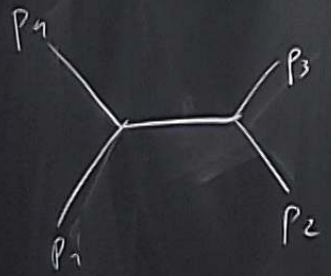

$= \prod_{1\le i<j\le 3} |z_i - z_j|^{2p_i\cdot p_j+2} \int d^2z_4 \prod_{1\le i\le 3} |z_4 - z_i|^{2p_i\cdot p_4}$
$z_4 \to \infty \quad |z_4|^{-2p_4^2}$
$z_4 \to z_i \quad |z_4 - z_i|^{2p_4\cdot p_i}$
$t = \frac{z_4 - z_1}{z_4 - z_2} \frac{z_3 - z_2}{z_3 - z_1}$
$2p_4\cdot p_i \geq -2$
$(p_4 + p_i)^2 \geq 2$


$= \prod_{1\le i<j\le 3} |z_i - z_j|^{2p_i\cdot p_4} \int dz_4 d\bar{z}_4 \prod_{1\le i\le 3} |z_4 - z_i|^{2p_i\cdot p_4}$
$z_4 \to \infty \quad |z_4|^{-2p_4^2}$
$z_4 \to z_i \quad |z_4 - z_i|^{2p_4\cdot p_i}$
$t = \frac{z_4 - z_1}{z_4 - z_2} \frac{z_3 - z_2}{z_3 - z_1}$
$2p_4\cdot p_i \ge -2$
$A = \int dt\, d\bar{t}\, |t|^{2p_4\cdot p_1} |1-t|^{2p_4\cdot p_2}$
$(p_4 + p_i)^2 \ge 2$


$= \prod_{1\le i<j\le 3} |z_i - z_j|^{\dots} \int d^2 z_4 \prod_{1\le i\le 3} |z_4 - z_i|^{2p_i\cdot p_4}$
$z_4 \to \infty \quad |z_4|^{-2p_4^2}$
$t = \frac{z_4 - z_1}{z_4 - z_2} \frac{z_3 - z_2}{z_3 - z_1}$
$A = \int d^2 t \, |t|^{2p_4\cdot p_1} |1-t|^{2p_4\cdot p_2} = \prod_i \frac{\Gamma(1+p_4\cdot p_i)}{\Gamma(-p_4\cdot p_i)}$

$$A = \int d|t|\, d\bar{t}\; |t|^{2p_4\cdot p_1}\, |1-t|^{2p_4\cdot p_2} \equiv \prod_i \frac{\Gamma(\quad)}{\Gamma(-p_4\cdot p_i)}$$

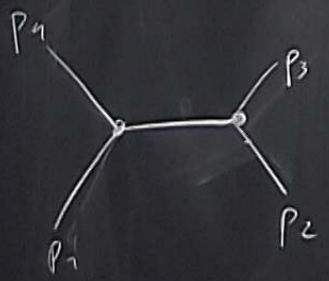
POLES $\quad p_4\cdot p_i = -1, -2, -3, \ldots \quad (p_4+p_i)^2 = 2, 0, -2, -4 \ldots$



$$t = \frac{z_4 - z_1}{z_4 - z_2} \frac{z_3 - z_2}{z_3 - z_1}$$

$$A = \int d\mathrm{Re}\, t\, d\mathrm{Im}\, t\; |t|^{2p_4\cdot p_1} |1-t|^{2p_4\cdot p_2} = \prod_i \frac{\Gamma(1+p_4\cdot p_i)}{\Gamma(-p_4\cdot p_i)}$$

POLES $\quad p_4 p_i = -1, -2, -3, \ldots \quad (p_4+p_i)^2 = 2, 0, -2, -4 \ldots$



$$\mathrm{Res}_{p_4 p_1 = -n} A = \prod_{k=0}^{n-2} (p_4\cdot p_2 - k)(p_4\cdot p_3 - k)$$

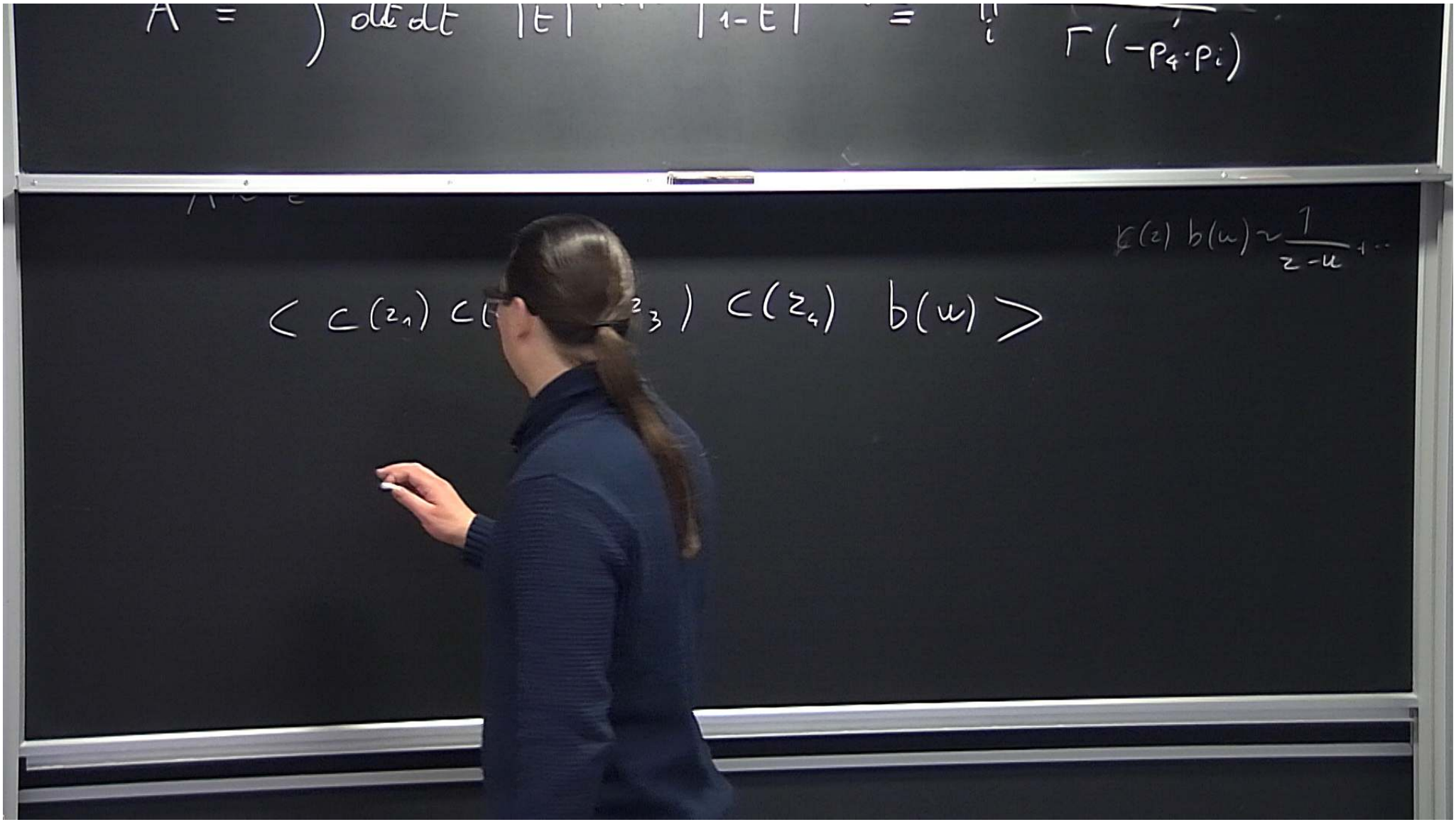
$$A = \int d|t|\, d\bar{t}\; |t|^{2p_4\cdot p_1}\, |1-t|^{2p_4\cdot p_2} = \prod_i \frac{\Gamma(\quad)}{\Gamma(-p_4\cdot p_i)}$$

$$A \sim e^{-s\ln s - t\ln t - u\ln u}$$

$A = \int d\ell\, dt\, |t|^{\cdots} |1-t|^{\cdots} = \frac{1}{i}\, \Gamma(-p_4 \cdot p_i)$

$A \sim e^{-s h_s - t h_t - u h_u}$

$\langle c(z_1)\, c(z_2)\, c(z_3)\, c(z_4)\, b(w) \rangle$

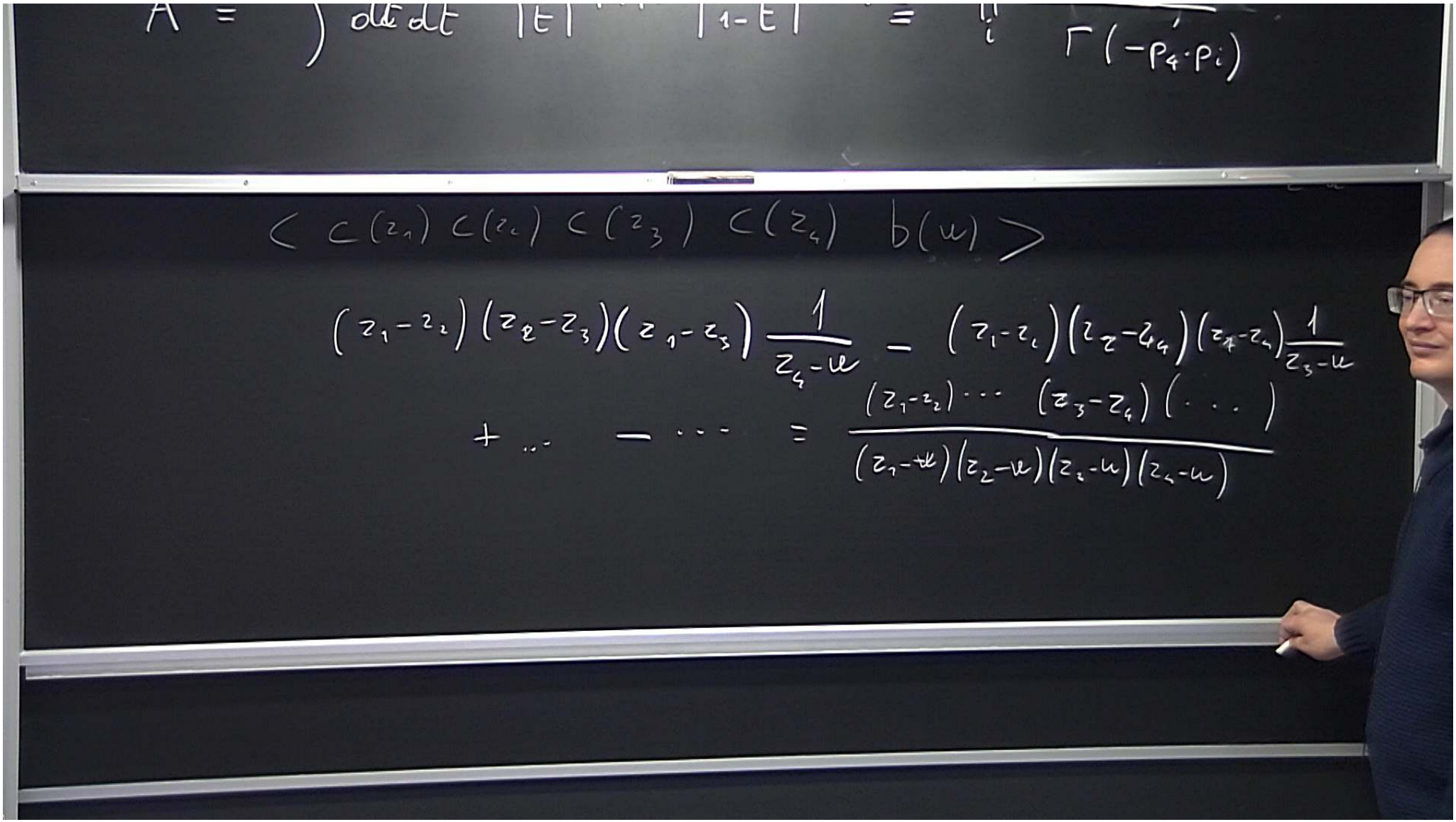

$\Gamma(-p_4 \cdot p_i)$
$c(z) b(w) \sim \frac{1}{z-w} + \ldots$
$\langle c(z_1) c(\; z_3) c(z_4) \; b(w) \rangle$

$$A = \int d\omega\, dt\, |t|^{\cdots} |1-t|^{\cdots} = \frac{\prod_i}{} \Gamma(-p_4 \cdot p_i)$$

$$\langle c(z_1)\, c(z_2)\, c(z_3)\, c(z_4)\, b(w) \rangle$$

$$(z_1 - z_2)(z_2 - z_3)(z_1 - z_3)\frac{1}{z_4 - w} - (z_1 - z_2)(z_2 - z_4)(z_4 - z_4)\frac{1}{z_3 - w}$$

$$+ \cdots \quad - \cdots \quad = \frac{}{(z_1 - w)(z_2 - w)(z_3 - w)(z_4 - w)}$$


$A = \int du\, dt\, |t|^{\cdots} |1-t|^{\cdots} = \prod_i \Gamma(-p_4 \cdot p_i)$
$\langle c(z_1)\, c(z_2)\, c(z_3)\, c(z_4)\, b(w) \rangle$
$(z_1-z_2)(z_2-z_3)(z_1-z_3)\frac{1}{z_4-w} - (z_1-z_4)(z_2-z_4)(z_4-z_4)\frac{1}{z_3-w}$
$+ \cdots \quad - \cdots = \frac{(z_1-z_2)\cdots (z_3-z_4)(\cdots)}{(z_1-w)(z_2-w)(z_3-w)(z_4-w)}$

$$t = \frac{z_4 - z_1}{z_4 - z_2} \, \frac{z_3 - z_2}{z_3 - z_1}$$

$$A = \int d^2t \, |t|^{2p_4 \cdot p_1} \, |1-t|^{2p_4 \cdot p_2} = \prod_i \frac{\Gamma(1 + p_4 \cdot p_i)}{\Gamma(-p_4 \cdot p_i)}$$

$$A \sim e^{-s \ln s - t \ln t - u \ln u}$$

$$b(z)\, b(w) \sim \frac{1}{z-w} + \dots$$

$$\langle c(z_1)\, c(z_2)\, c(z_3)\, c(z_4)\, b(w) \rangle$$

$$(z_1 - z_2)(z_2 - z_3)(z_1 - z_3) \frac{1}{z_4 - w} - (z_1 - z_2)(z_2 - z_4)(z_4 - z_3) \frac{1}{z_3 - w}$$

$$+ \dots \quad - \dots = \frac{(z_1 - z_2) \cdots (z_3 - z_4)(\dots)}{(z_1 - w)(z_2 - w)(z_3 - w)(z_4 - w)}$$