Title: Quantum integrability, W-algebra from quiver gauge theory

Date: Feb 26, 2018 11:00 AM

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Abstract: <p>In this talk, I'd like to explain how quantum integrability and (q-deformed) W-algebraic structure arise from the moduli space of quiver gauge theory. It'll be also shown that our construction gives rise to a new family of W-algebras. </p>

Review : 1612.07590

5-90

Gauge theory / Integrable system.

4d $\mathcal{N}=2$ th.	Classical int. sys.
Coulomb branch.	Phase space. (base)
Seiberg-Witten curve.	Spectral curve.

$$\Sigma = \{ \det(y - \underbrace{L}_{\text{Lax}}(x)) = 0 \}.$$

$\Downarrow$ Quantization.

(Ω - background)

(ϵ_1, ϵ_2).

NS lim

$(\epsilon_1 = \hbar, \epsilon_2 \to 0)$

SUSY vacua

(Twisted F-term)

Quantum int. sys

Bethe equation

Review :

$\Downarrow$ Quantization.

(Ω-background)

(ϵ_1, ϵ_2).

NS lim

($\epsilon_1 = \hbar$, $\epsilon_2 \to 0$)

SUSY vacua

(Twisted F-term)

Quantum SW curve.

Quantum int. sys.

Bethe equation

(Transfer matrix

TQ-relation

$$[H_n, H_{n'}] = 0.$$

$$\text{sys}\left(\Theta \left[T(x), T(x') \right] = 0 \right)$$

Examples

4d. Pure $SU(N)$ SYM.

$SU(N)$ SQCD.
$(N_F = 2N)$

Affine Toda theory.

$SU(2)$ - XXX spin chain
(length N)

Examples

4d. Pure SU(N) SYM.

SU(N) SQCD. ($N_F = 2N$)

$\mathcal{N}=2^*$ SU(N) theory

Affine Toda theory.

SU(2) - XXX spin chain (length N)

Calogero - Moser sys.

8 SUSY.

4d $\mathcal{N}=2$ on $\mathbb{C}^2$	XXX (rat)
5d $\mathcal{N}=1$ on $\mathbb{C}^2 \times S^1$	XXZ (trig.)
6d $\mathcal{N}=(1,0)$ on $\mathbb{C}^2 \times T^2$	XYZ (elliptic)

g.

chain

N)

sys.

Examples

4d. Pure SU(N) SYM.

SU(N) SQCD.
($N_F = 2N$)

$\mathcal{N}=2^*$ SU(N) theory

A_{M-1} quiver theory.

Affine Toda theory.

SU(2) - XXX spin chain
(length N)

Calogero - Moser sys.

SU(M) - XXX spin chain

$$[H_n, H_{n'}] = 0.$$

$$\Rightarrow \left([T(x), T(x')] = 0 \right)$$

$$\overbrace{\circ - \circ - \cdots - \circ}^{M-1}$$

Examples

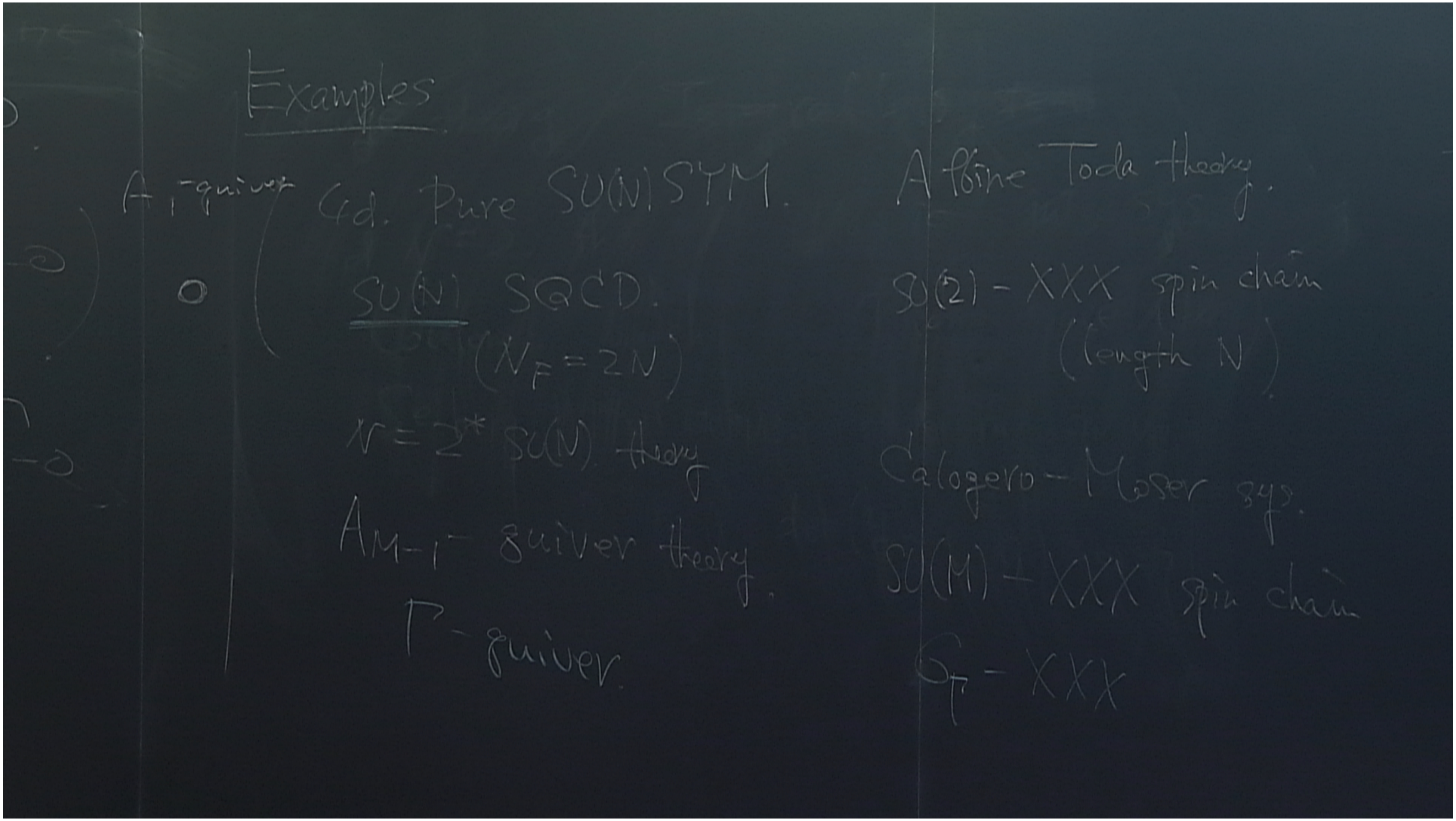
A_1-quiver $\circ$ (4d. Pure $SU(N)$ SYM

$SU(N)$ SQCD. $(N_F = 2N)$

$\mathcal{N} = 2^*$ $SU(N)$ theory

A_{M-1}- quiver theory.

Γ- quiver.


Examples
A1-quiver
4d. Pure SU(N) SYM.
SU(N) SQCD.
(NF = 2N)
N = 2* SU(N) theory
AM-1- quiver theory.
Γ- quiver.
Affine Toda theory.
SU(2) - XXX spin chain
(length N).
Calogero - Moser sys.
SU(M) - XXX spin chain
GΓ - XXX

$[H_n, H_{n'}] = 0$

$[T(x), T(x')] = 0$

$\overbrace{\circ - \circ - \cdots - \circ}^{M-1}$

A_1-quiver

$\circ$

$\hat{A}_0$

Examples

4d. Pure SU(N) SYM.

SU(N) SQCD. ($N_F = 2N$)

$\mathcal{N} = 2^*$ SU(N) theory

A_{M-1}-quiver theory.

Γ-quiver.

Affine

SU(2) – X

Calogero –

SU(M) – X

G_Γ – X

Generic $\dot{\mathcal{E}}_{1,2}$ $\Longleftrightarrow$ W-algebra.

<u>Quiver W-algebra.</u>

Γ-quiver G-gauge theory.

AGT-W $\swarrow$ $\searrow$ "quiver" AGT.

$W(G)$-algebra. $W(\Gamma)$-algebra.

Generic $G_{1,2}$ $\Longleftrightarrow$ W-algebra.

Quiver W-algebra.

$W(SU(N)) = W_N$

$= W(A_{N-1})$

Γ-quiver G-gauge theory.

AGT-W $\swarrow$ $\searrow$ "quiver" AGT.

$W(G)$-algebra $W(\Gamma)$-algebra.

Quiver W-algebra.

$Z[\Gamma\text{-quiver } G\text{-gauge theory}]$

AGT-W ↙ ↘ "quiver" AGT.

$W(G)$-algebra. $W(\Gamma)$-algebra.

$$\langle \cdots \rangle_{W(G)} = \langle \cdots \rangle_{W(\Gamma)}$$

Quiver W-algebra

$Z[\Gamma\text{-quiver } G\text{-gauge theory}]$

AGT-W ↙ ↘ "quiver" AGT

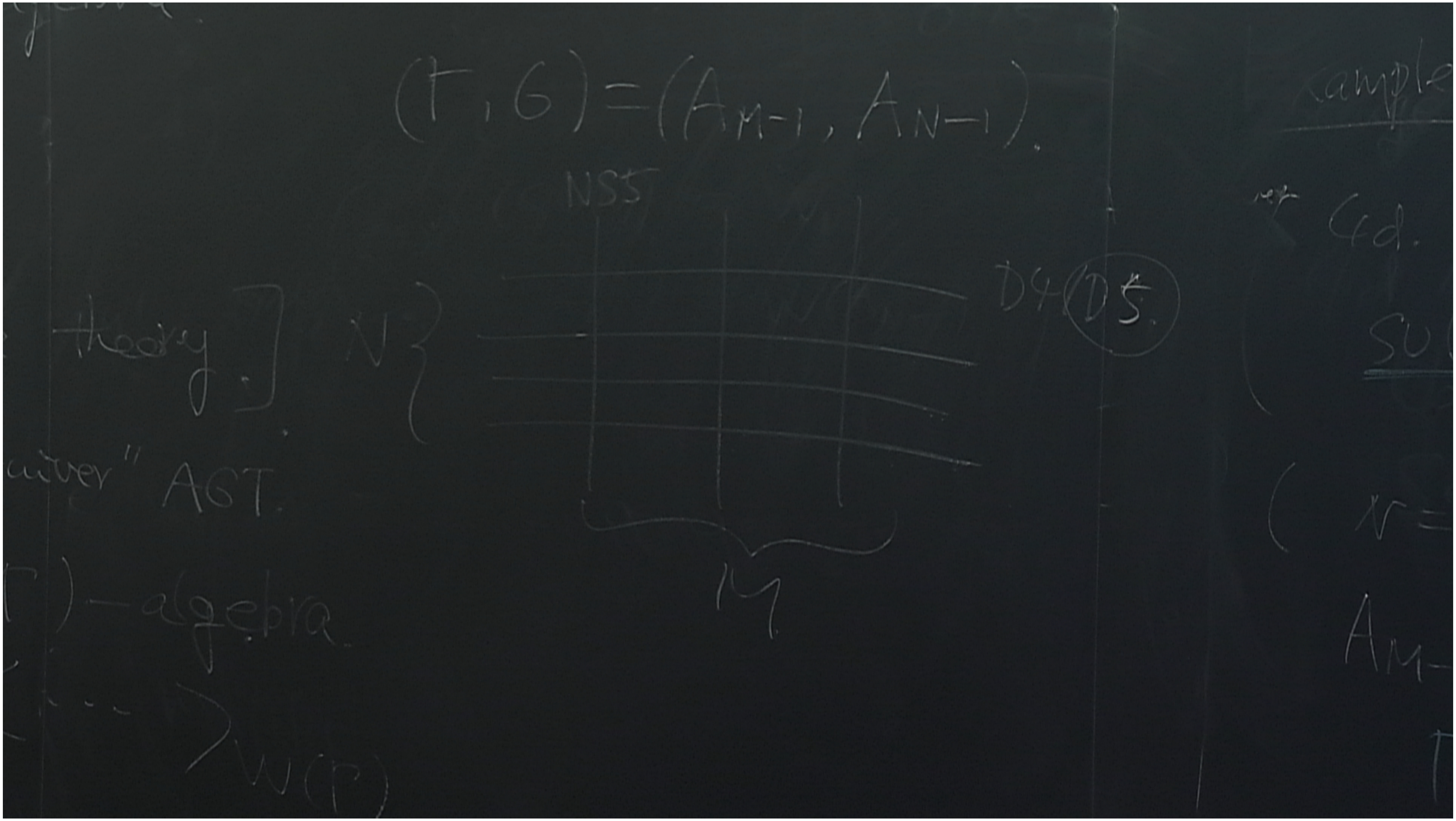
$W(G)$-algebra $\longleftrightarrow$ $W(\Gamma)$-algebra

$\langle \cdots \rangle_{W(G)} = \langle \cdots \rangle_{W(\Gamma)}$

(base/fibre

S-duality

Spectral duality


(T, G) = (A_{M-1}, A_{N-1}).
NS5
D4/D5
N
M
theory.
"quiver" AGT.
)-algebra
W(p)
4d.
SU

5d notation (K-th; multiplicative) $\mathbb{C}^*$)

Rad. of S^1

$$(q_1, q_2) = (e^{\epsilon_1}, e^{\epsilon_2})$$

4d $\mathcal{N}=2$ (D5)

$$q := q_1 q_2 = e^{\epsilon_1+\epsilon_2}$$

quiver

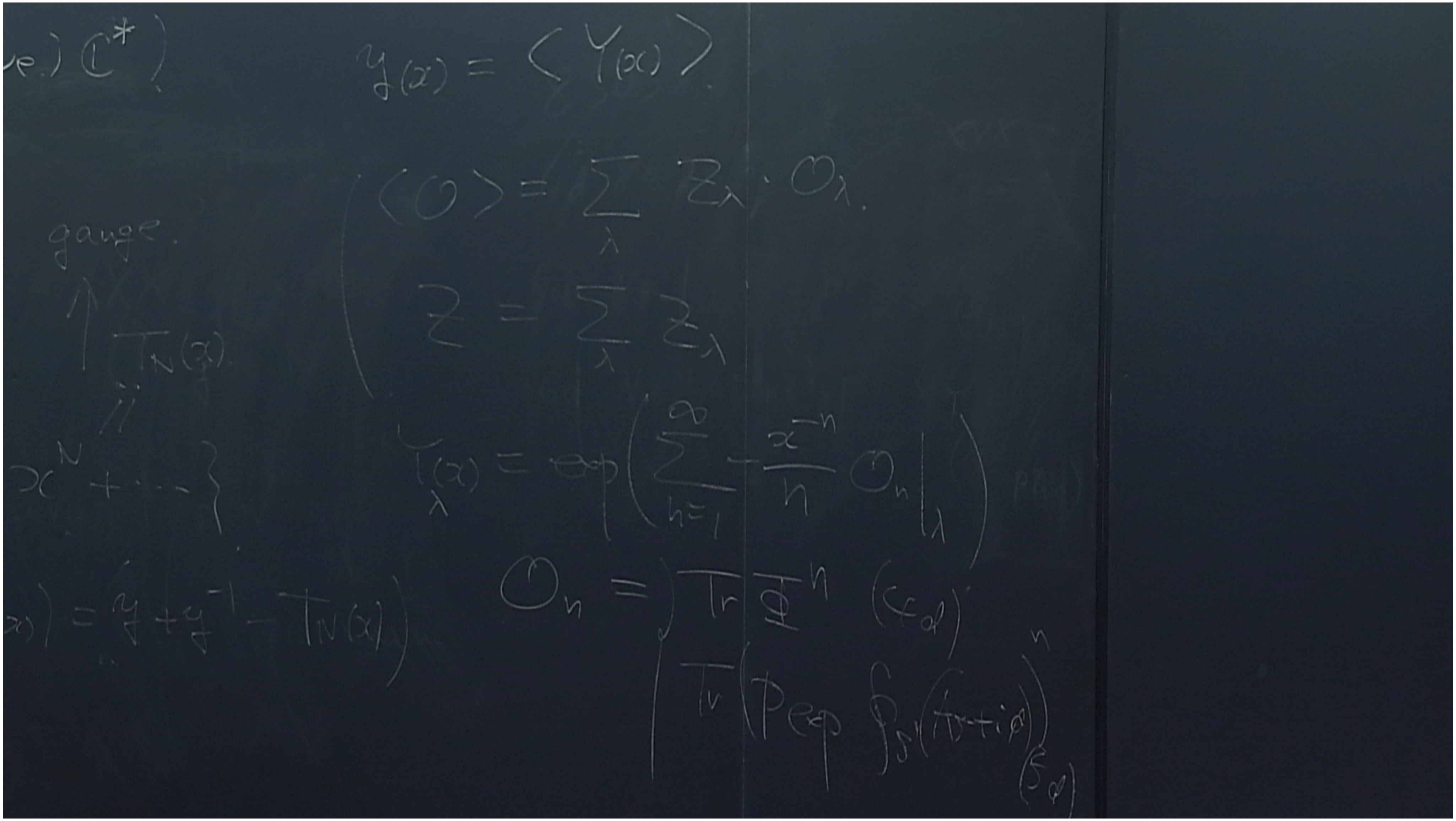
gauge

$T_N(x)$

SW curve: $\Sigma = \{ y_{(x)} + y^{-1}_{(x)} = x^N + \cdots \}$

$(\Gamma = A_1, SU(N))$

$$y^{\#} \times \det(y - L(x)) = y + y^{-1} - T_N(x)$$



$y(x) = \langle Y(x) \rangle$
$\langle \mathcal{O} \rangle = \sum_\lambda Z_\lambda \cdot \mathcal{O}_\lambda$
$Z = \sum_\lambda Z_\lambda$
$Y_\lambda(x) = \exp\left(\sum_{n=1}^{\infty} \frac{x^{-n}}{n} \mathcal{O}_n \Big|_\lambda \right)$
$\mathcal{O}_n = \mathrm{Tr}\, \Phi^n$
gauge
$T_N(x)$
$x^N + \cdots$
$= y + y^{-1} - T_N(x)$

$$F_{UV} \longrightarrow F_{UV} + \sum_{n=1}^{\infty} t_n \cdot \mathcal{O}_n$$

$$Z \longrightarrow Z(t) = \sum_{\lambda} Z_\lambda \cdot Z_\lambda^{pot}(t)$$

$$Z_\lambda^{pot}(t) = \exp\left(\sum_{n=1}^{\infty} t_n \cdot \mathcal{O}_n \Big|_\lambda \right)$$

$$F_{UV} \longrightarrow F_{UV} + \sum_{n=1}^{\infty} t_n \cdot \mathcal{O}_n$$

$$Z \longrightarrow Z(t) = \sum_{\lambda} Z_\lambda \cdot Z_\lambda^{pot}(t)$$

$$Z_\lambda^{pot}(t) = \exp\left(\sum_{n=1}^{\infty} t_n \cdot \mathcal{O}_n \Big|_\lambda \right)$$

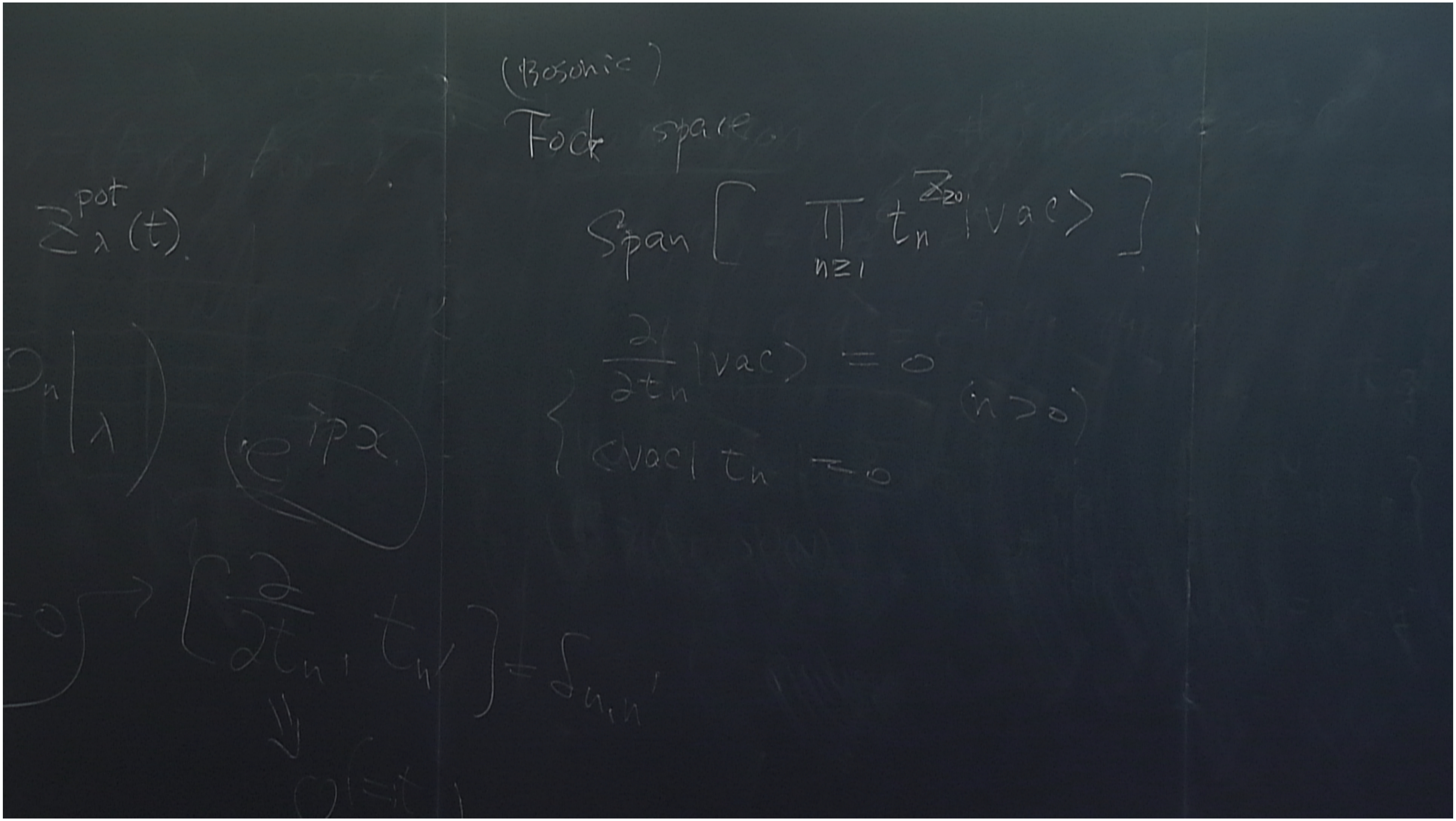
$$e^{ipx}$$

$$\langle \mathcal{O}_n \rangle = \frac{\partial}{\partial t_n} Z(t) \Big|_{t=0} \longrightarrow \left[\frac{\partial}{\partial t_n}, t_{n'} \right] = \delta_{n,n'}$$

$$\boxed{\mathcal{O}_n \Longleftrightarrow \frac{\partial}{\partial t_n}}$$

$$\Downarrow$$

$$\mathcal{O}_s (= t_n)$$


(Bosonic)
Fock space
Span[$\prod_{n\geq 1} t_n^{\mathbb{Z}_{\geq 0}} |vac\rangle$]
$\frac{\partial}{\partial t_n}|vac\rangle = 0$ $(n>0)$
$\langle vac| t_n = 0$
$Z^{pot}_{\lambda}(t)$
e^{ipx}
$[\frac{\partial}{\partial t_n}, t_{n'}] = \delta_{n,n'}$

Z - state

Operator $Z(t) \Longleftrightarrow$ State $|Z\rangle = Z(t)|vac\rangle$

$$Z(t=0) = \langle vac|Z\rangle$$

$$= \langle vac|Z(t)|vac\rangle$$

Z - state

Operator $Z(t) \Longleftrightarrow$ State $|Z\rangle = Z(t)|vac\rangle$

$$Z(t=0) = \langle vac|Z\rangle$$

$$= \langle vac|\underbrace{Z(t)}|vac\rangle$$

Screening of $W(r)$ charges

alg. rel ; $y(x) + y^{-1}(x) = T_N(x)$

$$\Downarrow$$

op. rel ; $Y(x) + Y^{-1}(q^{-1}x) = T(x)$

$$= \sum_{n \in \mathbb{Z}} T_n \cdot x^{-n}$$

$$Y(x) = \exp\left(\sum_{n \neq 0} y_n \cdot x^{-n} \right)$$

$$[y_n, y_{n'}] = \frac{1}{n} \frac{(1-q_1^n)(1-q_2^n)}{1+q^n} \delta_{n+n',0}$$

alg. rel.: $y(x) + y^{-1}(x) = N(x)$

$$\Downarrow$$

op. rel.: $Y(x) + Y^{-1}(q^{-1}x) = T(x)$

$$= \sum_{n \in \mathbb{Z}} T_n \cdot x^{-n}$$

A_1

$$Y(x) = \exp\left(\sum_{n \neq 0} y_n \cdot x^{-n} \right)$$

$$[y_n, y_{n'}] = \frac{1}{n} \frac{(1-q_1^n)(1-q_2^n)}{1+q^n} \delta_{n+n',0}$$

$$[T_n, T_{n'}] = \cdots$$

OPE of $T(x)$'s

q-deformed Virasoro.

(q-$W(A_1)$-alg.)

$$f\left(\frac{x'}{x}\right) T(x) T(x') - f\left(\frac{x}{x'}\right) T(x') T(x)$$

$$= -\frac{(1-q_1)(1-q_2)}{1-q}\left(\delta\left(q\frac{x'}{x}\right) - \delta\left(q^{-1}\frac{x'}{x}\right)\right)$$

$$f(z) = \exp\left(\sum_{n=1}^{\infty} \frac{(1-q_1^n)(1-q_2^n)}{n(1+q^n)} z^n\right)$$

q-deformed W...

(q-$W(A_1)$-alg.)

OPE of $T(x)$'s

$$f\left(\frac{x'}{x}\right) T(x)\, T(x') - f\left(\frac{x}{x'}\right) T(x')\, T(x) = -\frac{(1-q_1)(1-q_2)}{1-q}\left(\delta\left(q\frac{x'}{x}\right) - \delta\left(q^{-1}\frac{x'}{x}\right)\right)$$

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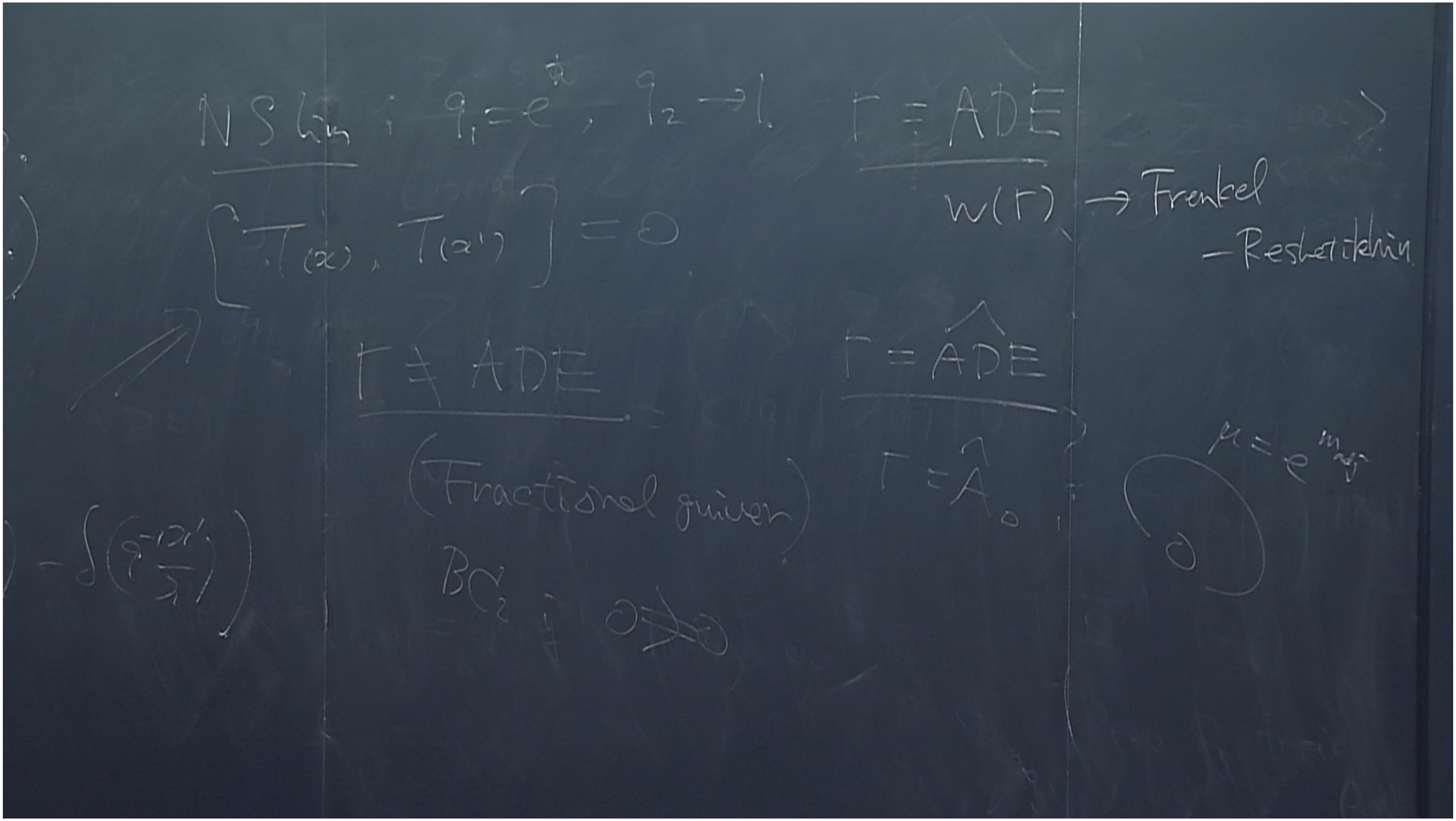
NS lim : q1 = e , q2 → 1.
[T(x), T(x')] = 0
Γ ≠ ADE
(Fractional quiver)
BC2 :
Γ = ADE
W(Γ) → Frenkel
– Reshetikhin
Γ = ÂDE
Γ = Â0

q-deformed Virasoro.

(q-$W(A_1)$-alg.)

OPE of $T(x)$'s

$$f\left(\frac{x'}{x}\right) T(x)\, T(x') - f\left(\frac{x}{x'}\right) T(x')\, T(x) = -\frac{(1-q_1)(1-q_2)}{1-q}\left(\delta\left(q\frac{x'}{x}\right) - \delta\left(q^{-1}\frac{x'}{x}\right)\right)$$

$\theta(q_2 ; p)$

$$f(z) = \exp\left(\sum_{n=1}^{\infty} \frac{(1-q_1^n)(1-q_2^n)}{n(1+q^n)} z^n\right)$$