Title: A web of dualities in many-body quantum physics

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Abstract: <p>Two seemingly different quantum field theories may secretly describe the same underlying physics â€” a phenomenon known as â€œdualityâ€•. Duality has been proved powerful in condensed matter physics, since many difficult questions can be drastically simplified in certain â€œdualâ€• pictures. This is especially valuable for strongly interacting many-body problems, for which traditional tools (such as perturbation theory) are often not applicable. </p>

<p>Recent developments on dualities have also revealed deep connections between several previously unrelated topics in modern condensed matter physics, including topological insulators, fractional quantum Hall effects and quantum phase transitions. Connections were also made between dualities in condensed matter physics and in high energy physics. I will give a brief review of some of these developments.</p>

A Web of Dualities in Many-body Quantum Physics

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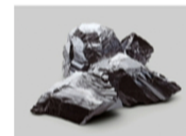
The grand challenge: interacting quantum many-body problems

Interacting electrons/atoms

Weakly interacting

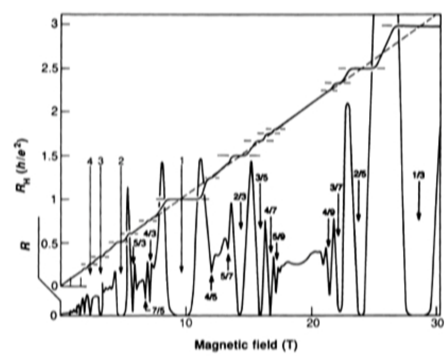


Li

Perturbation theory works, e.g. alkali metals

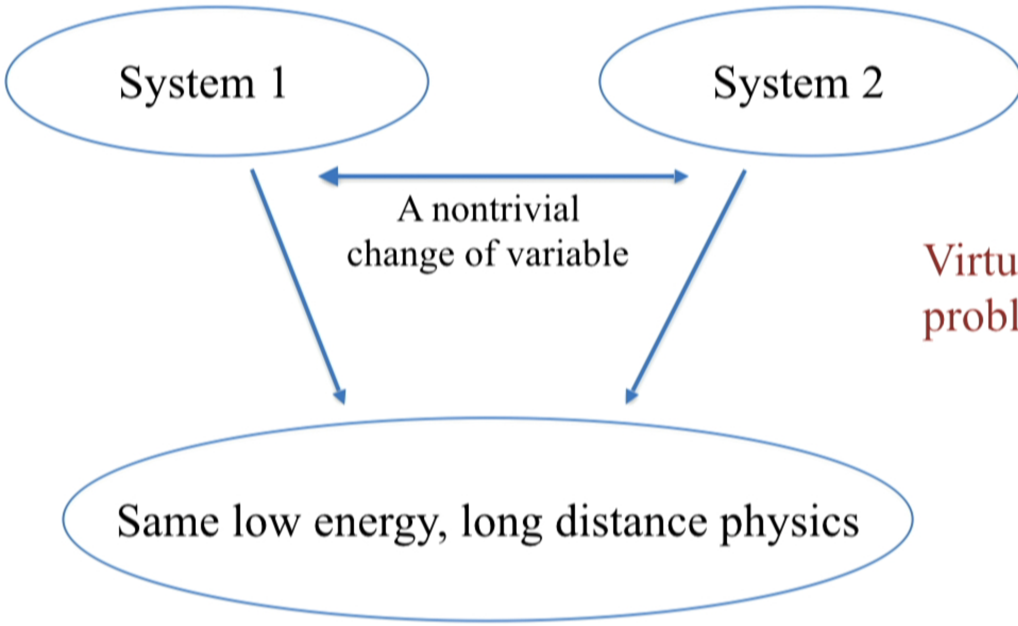
Strongly interacting

Perturbation theory may not work
— **need more tools!**
e.g. high temperature superconductors, fractional quantum Hall effects…



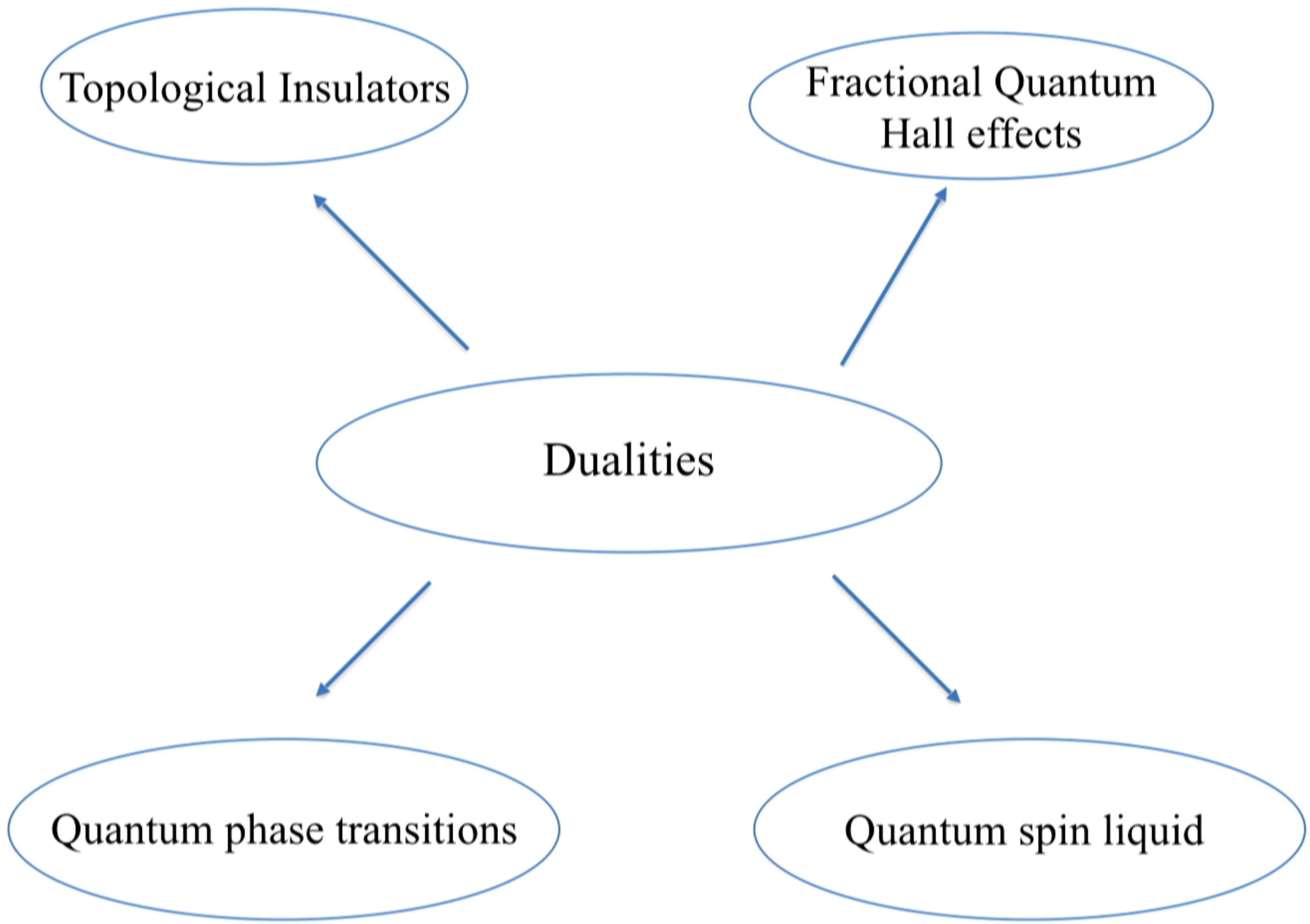
Quantum Hall effect

Duality



Virtue: sometimes a hard problem maps to a simpler one

Fruitful in condensed matter and high energy
e.g. 2d Ising model

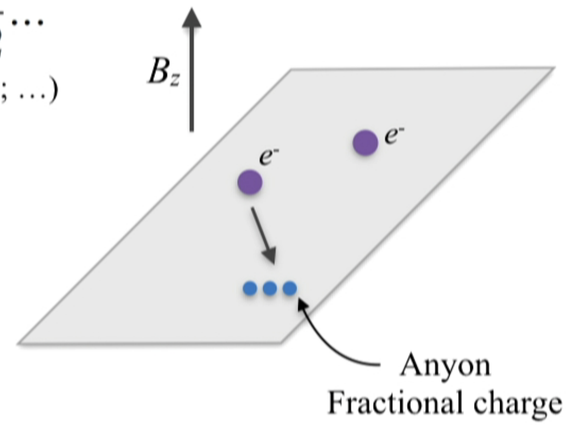

Topological Insulators
Fractional Quantum Hall effects
Dualities
Quantum phase transitions
Quantum spin liquid

Motivation I: Fractional quantum Hall effect

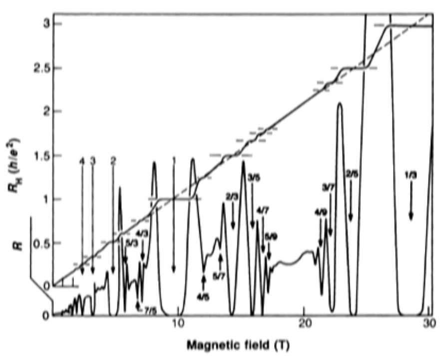
2D electron gas in magnetic field

$$\nu = \frac{2\pi n_e}{B_z} = \frac{1}{3}, \frac{2}{5}, \frac{5}{2} \ldots$$

(Tsui, Stormer, Gossard; Laughlin; …)



Collective motion:
Electrons "split" into fractional excitations
— fractional charge, anyon statistics…



Patterns of electron splitting: **topological order**

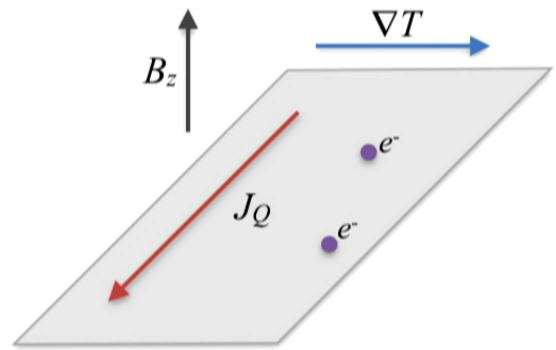
Low energy: topological quantum field theory

Simplest signature: quantized Hall conductance

An old puzzle

Nature of topological order at $\nu = \frac{2\pi n_e}{B_z} = \frac{5}{2}$

Promising host of non-abelian topological order



Popular candidates: Pfaffian or anti-Pfaffian

(Moore, Read; Levin, Halperin, Rosenow; Lee, Ryu, Nayak, Fisher)

Distinguished by thermal Hall conductance

$$\kappa_{xy} = \frac{7}{2}\frac{\pi^2 k_B^2 T}{3h} \quad \text{or} \quad \frac{3}{2}\frac{\pi^2 k_B^2 T}{3h}$$

Recently measured experimentally:
(Banerjee, et. al, arXiv:1710.00492)

$$\kappa_{xy} = \frac{5}{2}\frac{\pi^2 k_B^2 T}{3h}$$

What is going on?

Motivation II: Topological insulator

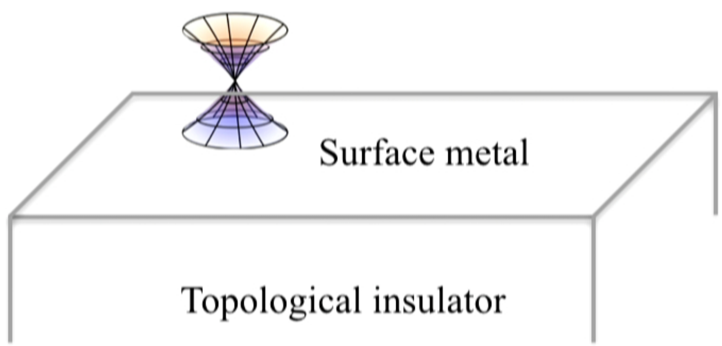
Insulating bulk, metallic surface

Surface electrons move as massless Dirac fermions

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial_\mu\psi$$

Free Dirac fermion: cannot be made insulating unless time-reversal symmetry is broken (Dirac mass)

Cannot happen in pure 2D metal: need a "topologically nontrivial" bulk



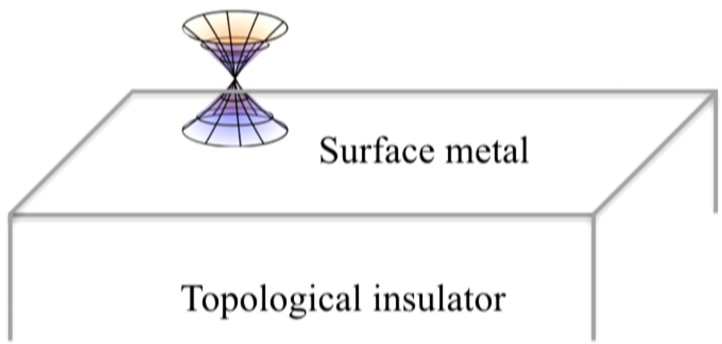
Q: what about with interaction?

With interactions, can we make the surface insulating?

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial_\mu\psi + \ldots$$

Weak interaction has no effect at low energy

Need strong interaction — but that's hard!



Outline

- Introduction: electric-magnetic duality and boson-vortex duality
- Particle-vortex duality for Dirac fermions:
  from topological insulator to quantum Hall effect
- A “web” of dualities and quantum criticality

Electric-magnetic (EM) duality

$$\nabla \cdot \mathbf{E} = 0, \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0, \qquad \nabla \times \mathbf{B} = +\frac{\partial \mathbf{E}}{\partial t}$$

EM duality: $(\mathbf{E}, \mathbf{B}) \to (\mathbf{B}, -\mathbf{E})$

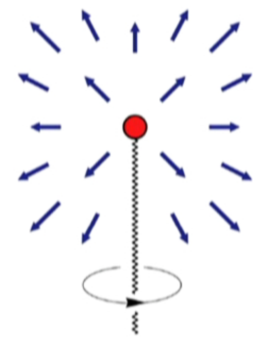


Image from Zyga, phys.org

Introduce magnetic monopoles (powerful conceptual tool)

Dirac: minimal magnetic flux $g_M = 2\pi/e$

weak interaction for charge ↔ strong for monopole

Merely a convention to name one "charge" and the other "monopole"

Define: $q_e = Q_e/e, \quad q_m = \Phi_B/g_M$

EM duality: $(q_e, q_m) \to (q_m, -q_e)$

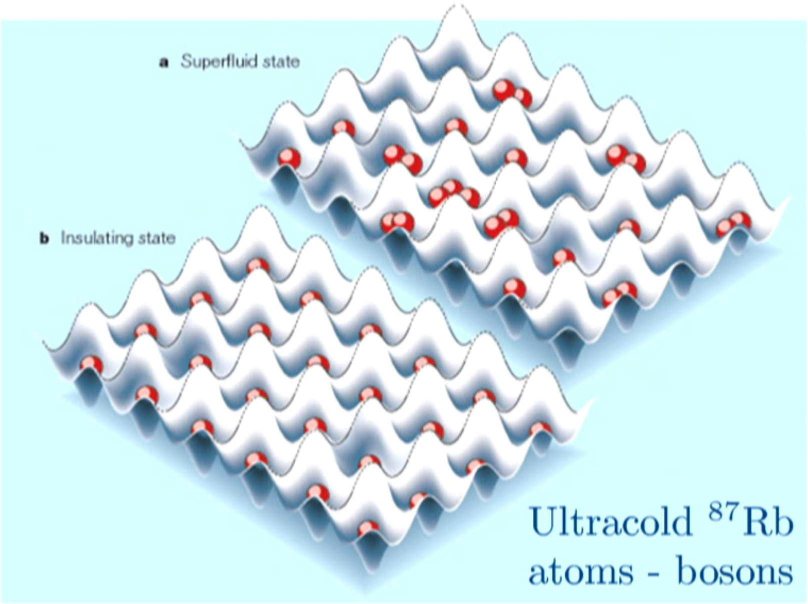
Superfluid and boson insulator

Bosons hopping on a 2D lattice, one particle per site on average

Weak interaction: superfluid, particles move freely

Strong on-site repulsion: Mott insulator, particles “frozen”

This is the “particle” picture



Realized in cold atoms (Greiner, et. al, Nature, 2002)

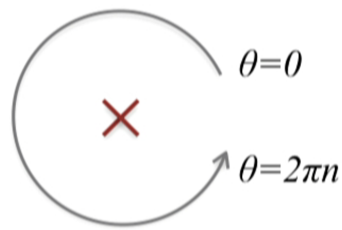
(Image from Sachdev, arXiv:1203.4565)

Quantum vortex

Superfluid order parameter: $\Phi = \sqrt{\rho_s} e^{i\theta}$

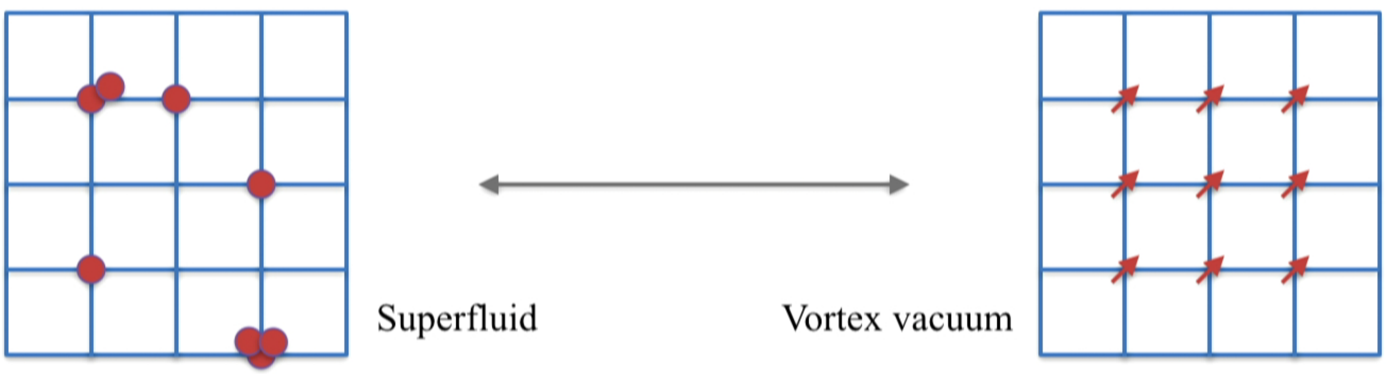
(≈ wave function of the Bose condensate)

Quantum vortex: a point defect around which the phase θ winds by $2\pi n$



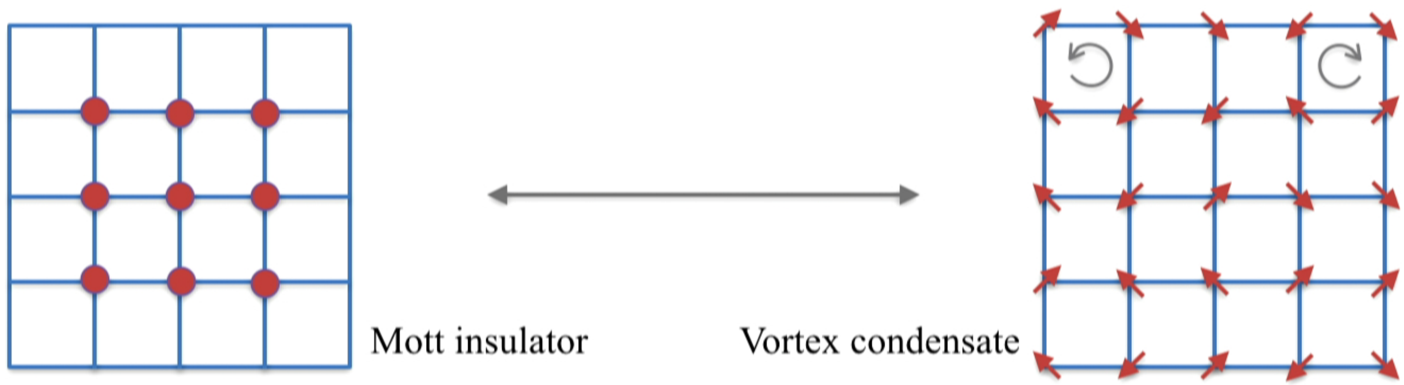
In a superfluid ground state, Φ=constant

→ a superfluid can be viewed as a "vacuum", or "insulator", of vortices



Vortex condensate = Mott insulator

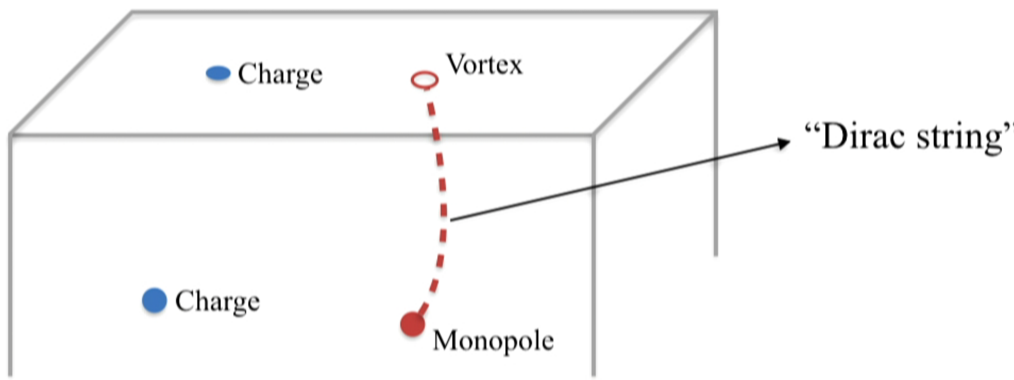
Vortices (bosons) can also Bose condense
→ wildly fluctuating phase θ
→ destroys the original (particle) condensate: Mott insulator!



Vortex picture can describe both superfluid and Mott insulators
Vortex insulator/superfluid = Particle superfluid/insulator

Also useful in superconductor-insulator transitions (Dasgupta, Halperin; Fisher, Lee)
and some exotic phases hard to describe in particle picture (Senthil, Fisher)

From EM to particle-vortex duality



Charge-vortex duality ≈ "surface version" of EM duality

$$|\nabla\Phi|^2 + m|\Phi|^2 + \lambda|\Phi|^4 + \ldots$$

Dual ⟷

$$|(\nabla - i\mathbf{a})\phi|^2 - m|\phi|^2 + \lambda|\phi|^4 + \frac{1}{4e^2}f_{\mu\nu}^2 + \ldots$$

Particle-vortex duality for Dirac fermions

Topological insulator

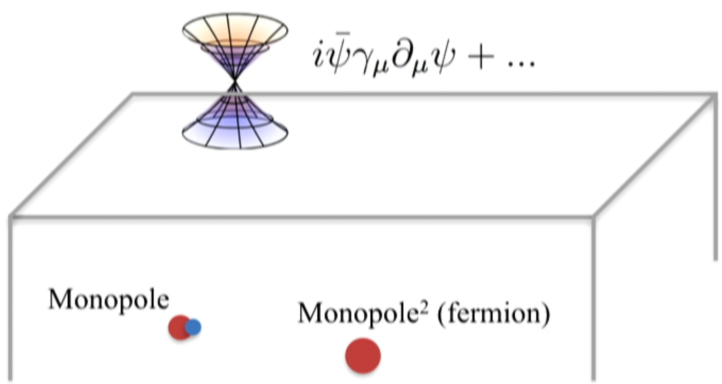
Surface: single Dirac cone (Fu, Kane, Mele)

Bulk: Witten effect — monopole traps electric charge (Qi, Hughes, Zhang; Witten)

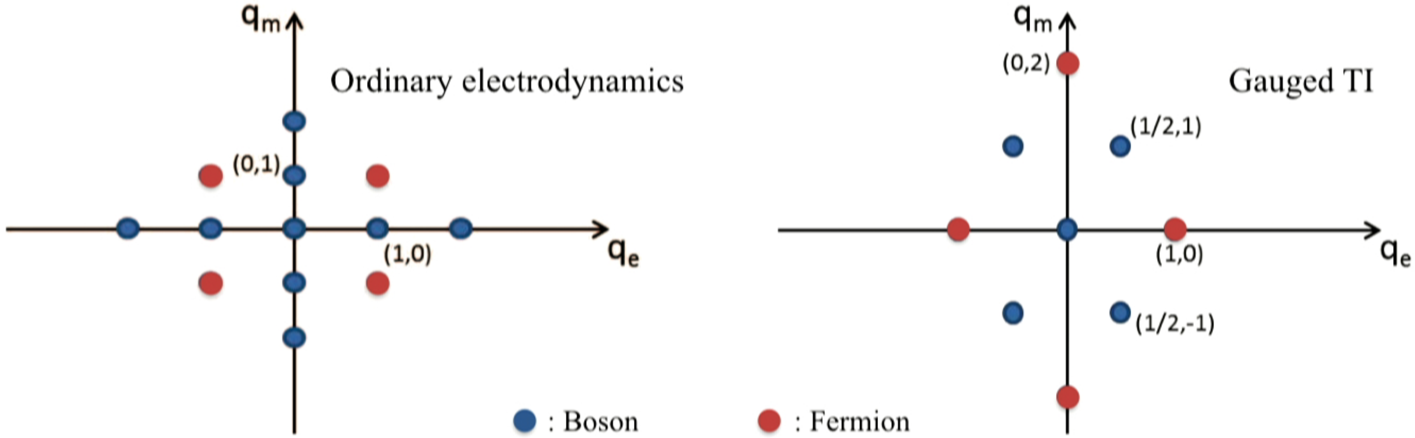
$$q_e = \frac{q_m}{2}$$

→ If q_m=2: q_e=1, can be neutralized by attaching an electron

→ A "pure" strength-2 monopole is a fermion



EM duality in TI



Ordinary EM duality: $(q_e, q_m) \rightarrow (q_m, -q_e)$

A 90° rotation of charge-monopole lattice

EM duality in gauged TI: $(q_e, q_m) \rightarrow (2q_m, -q_e/2)$

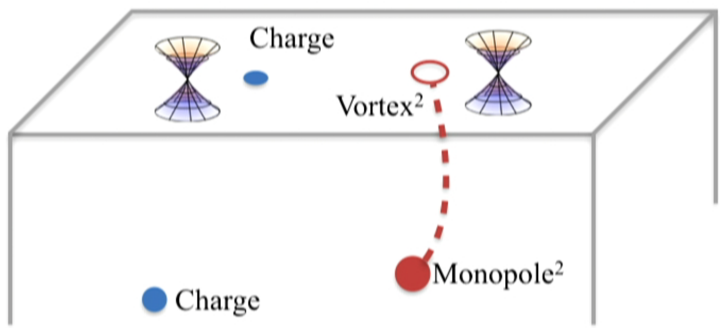
A 90° rotation, followed by rescaling of axes, lattice remains the same*

*: Time-reversal symmetry acts differently

CW, Senthil; Metlitski, Vishwanath; Metltiski

EM duality in TI

"Surface version" of this special EM duality ≈ Dirac particle-vortex duality



Dirac fermion: $i\bar{\psi}\gamma_\mu\partial_\mu\psi + \ldots$

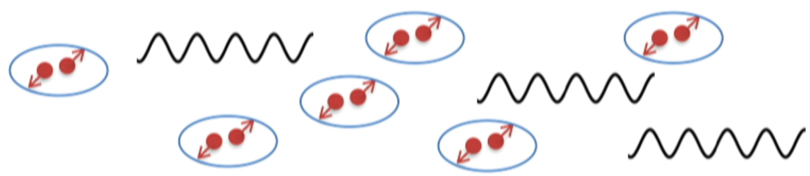
Dual

Dirac fermion as 4π vortex: $i\bar{\chi}\gamma_\mu(\partial_\mu - ia_\mu)\chi + \frac{1}{4e^2}f_{\mu\nu}^2 + \ldots$

Application: interacting TI surface

Can we make TI surface insulating with strong interaction?

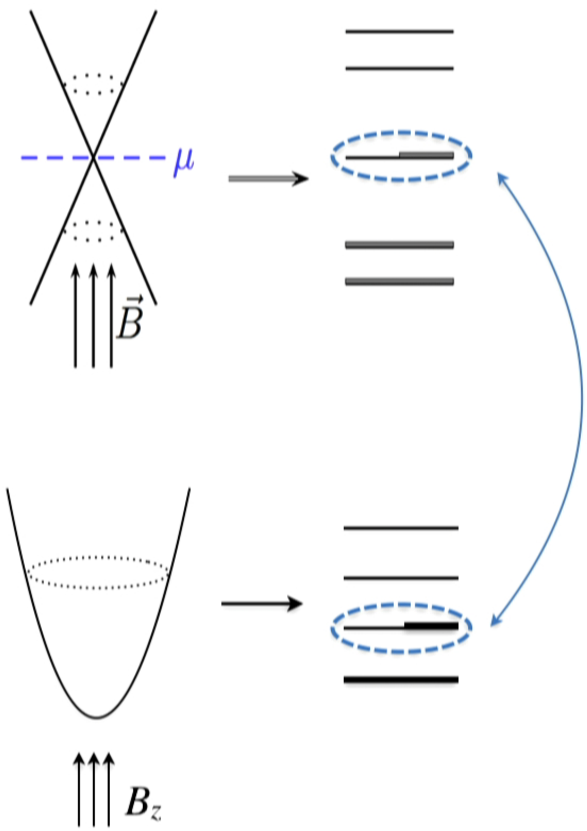
Go to vortex picture:
Insulator = vortex condensate (vortex superconductor)



Resulting insulator: "T-Pfaffian" ("PH-Pfaffian") topological order, a highly entangled topological quantum phase
(Fidkowski, Chen, Vishwanath; Son)

Similar results obtained in other ways, but much more complicated
(CW, Potter, Senthil; Metlitski, Kane, Fisher; Chen, Fidkowski, Vishwanath; Bonderson, Nayak, Qi, 2013; Mross, Essin, Alicea, 2014)

From Dirac to quantum Hall



Under magnetic filled:
same Landau level problem

Topological orders on TI surface can also be realized in quantum Hall systems!

PH-Pfaffian: a candidate state at

$$\nu = \frac{2\pi n_e}{B_z} = \frac{5}{2}$$

different from previous proposals
(Son)

PH-Pfaffian in quantum Hall

Key signature: Thermal Hall conductance

$$\kappa_{xy} = \frac{5}{2}\frac{\pi^2 k_B^2 T}{3h}$$

as opposed to 7/2 (Pfaffian) or 3/2 (anti-Pfaffian)

Agrees with experiment! (Banerjee, et. al, arXiv:1710.00492)

Also supported by other, less direct, experimental evidences
(Zucker, Feldman)

A Web of Dualities
and quantum criticality

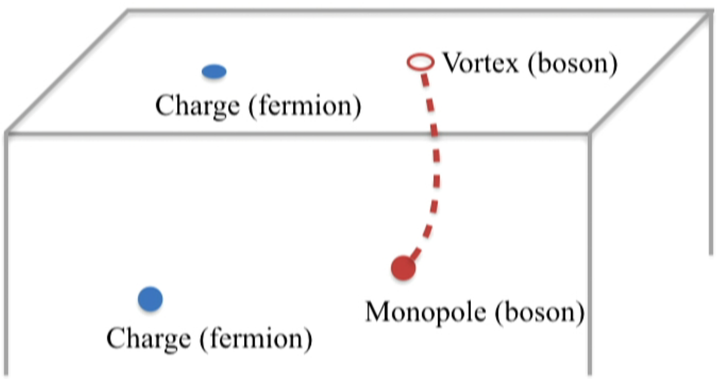
A web of dualities

Infinitely many different EM dualities

Each one motivates a duality in (2+1)d

(Seiberg, Senthil, CW, Witten; Karch, Tong)

Example: a fermion-boson duality, a.k.a. (2+1)d bosonization



$$i\bar{\psi}\gamma_\mu\partial_\mu\psi + \ldots$$

$$\updownarrow$$

$$|(\nabla - i\mathbf{a})\phi|^2 + |\phi|^4 + \frac{i}{4\pi}ada + \ldots$$

Also related to supersymmetric mirror dualities (Kachru, Mulligan, Torroba, Wang)

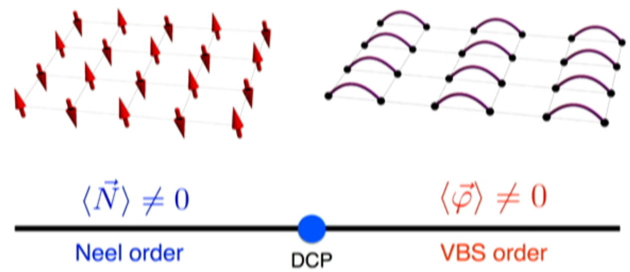
Duality between quantum phase transitions

Transition 1,
Symmetry G_1 easy to see

Transition 2,
Symmetry G_2 easy to see

$G_1 \neq G_2$

Same low energy physics
Symmetry $G \supset G1,G2$



A non-perturbative mechanism for emergent symmetries at critical points

CW, Nahum, Metlitski, Xu, Senthil

Quantum criticality: more to be understood

Tension between different numerics: Monte Carlo vs. Conformal Bootstrap
— unconventional finite-size scaling? (Sandvik)
— pseudo-criticality? (CW, Nahum, Metlitski, Xu, Senthil)

More relations among known quantum critical points/phases? (Jian, Thomson, Rasmussen, Bi, Xu)

Systematic relationship between lattice detail and allowed criticality? (Thorngren, Metlitski; ……)

What about effect of disorder? Could duality be helpful?
— Potentially relevant for superconductor-insulator transitions (Mulligan, Raghu), quantum Hall transitions (Hui, Kim, Mulligan) …

Summary

- Dualities: electric-magnetic dualities, particle-vortex duality for bosons and Dirac fermions, and beyond

- Applications: topological insulators, quantum Hall effects, quantum phase transitions……

Thank you!