Title: Is the Born Rule Unstable in Quantum Gravity?

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Abstract: <p>We argue that in quantum gravity there is no stable equilibrium state corresponding to the Born rule. Our main argument rests on the continued controversy over the physical meaning of the Wheeler-DeWitt equation. We suggest that attempts to interpret it are hampered by the conventional assumption that probabilities should be governed by a fixed Born rule. It is possible to abandon this assumption in a de Broglie-Bohm interpretation. We argue that a stable Born rule emerges only in the limiting SchrÃ¶dinger regime in which the system is described by an effective (time-dependent) SchrÃ¶dinger equation with an appropriate conserved current. We consider a regime in which quantum-gravitational corrections to the SchrÃ¶dinger equation induce a non-conservation of the standard current. Such a regime can be consistently described in de Broglie-Bohm theory. As a result, quantum-gravitational effects can generate non-equilibrium deviations from the Born rule even starting from an initial equilibrium state. We provide estimates for such effects in an inflationary de Sitter background.</p>

Is the Born rule unstable in quantum gravity?

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A. Valentini, Black Holes, information loss, and hidden variables, arXiv:hep-th/0407032

A. Valentini, Trans-Planckian fluctuations and the stability of quantum mechanics, arXiv:1409.7467 [hep-th]

A. Example of a theory in which the Born rule is stable
(pilot-wave theory of de Broglie and Bohm)

B. Three arguments for the instability of the Born rule in gravitational physics

-- non-globally hyperbolic spacetime, black hole evaporation
(warm-up, brief)

-- regularization of phase singularities in pilot-wave theory
(warm up, brief)

-- non-Schrödinger regime in quantum gravity, no conserved quantum current
(main focus)

C. A quantum-gravitational model for quantum instability, application to inflation, speculative laboratory experiment

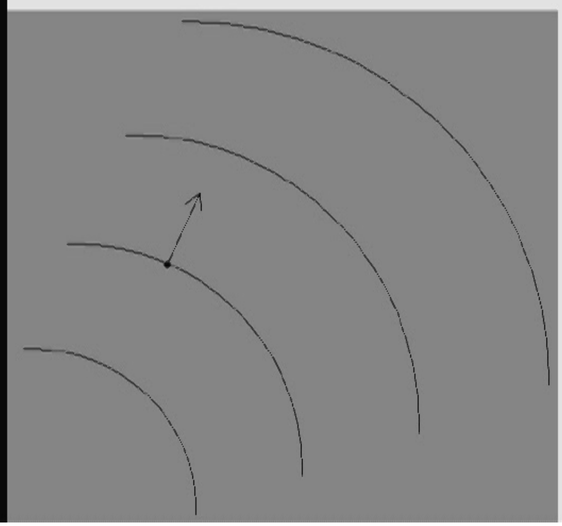
D. Discussion

De Broglie's Pilot-Wave Dynamics (1927)

$$i\frac{\partial\Psi}{\partial t} = \sum_{i=1}^{N} -\frac{1}{2m_i}\nabla_i^2\Psi + V\Psi$$

$$(\ \Psi = |\Psi|\, e^{iS}\ ) \qquad m_i\frac{d\mathbf{x}_i}{dt} = \nabla_i S$$

(cf. WKB, but for *any* wave function)



(Generalise: configuration $q(t)$)

Get QM if *assume* initial Born-rule distribution, $P = |\Psi|^2$ (preserved in time by the dynamics)

(shown fully by Bohm in 1952)

Illustration for one particle

$$i\frac{\partial\Psi}{\partial t} = -\frac{1}{2m}\nabla^2\Psi + V\Psi \quad \Longrightarrow \quad \frac{\partial|\Psi|^2}{\partial t} + \nabla\cdot\left(|\Psi|^2\frac{\nabla S}{m}\right) = 0$$

Guidance equation $m\frac{d\mathbf{x}}{dt} = \nabla S$ applied to an *ensemble* (same ψ, different $\mathbf{x}_0$'s).

Distribution $P(\mathbf{x},t)$ obeys $$\frac{\partial P}{\partial t} + \nabla\cdot\left(P\frac{\nabla S}{m}\right) = 0$$

THEOREM (quantum equilibrium):

If $P(\mathbf{x},0) = |\Psi(\mathbf{x},0)|^2$, then $P(\mathbf{x},t) = |\Psi(\mathbf{x},t)|^2$ for all *t*

(Generalisation: replace $\mathbf{x}(t)$ by general configuration $q(t)$)

System with configuration $q(t)$ and wave function(al) $\psi(q,t)$

$$i\frac{\partial \psi}{\partial t} = \hat{H}\psi$$

$$\frac{dq}{dt} = \frac{j}{|\psi|^2}$$

These equations define a *pilot-wave dynamics* for any system whose Hamiltonian $\hat{H}$ is given by a differential operator (Struyve and Valentini 2009)

where $j = j\,[\psi] = j(q,t)$ is the Schrödinger current

[Requires an underlying preferred foliation with time function *t*.
Valid in any globally-hyperbolic spacetime (Valentini 2004)]

By construction $\rho(q,t)$ will obey

$$\frac{\partial \rho}{\partial t} + \partial_q \cdot (\rho v) = 0 \qquad \frac{dq}{dt} = v$$

and $\rho(q,t) = |\psi(q,t)|^2$ is preserved in time (Born rule).

In pilot-wave theory the Born rule is stable in two senses:

(1) $\rho(q,t) = |\psi(q,t)|^2$ is an equilibrium state:
if true at t=0, remains true at t>0

(2) deviations from equilibrium relax to equilibrium
(see colloquium yesterday)

Here we are concerned mainly with (1):

the *existence* of an equilibrium state described by the Born rule (or indeed by any rule)

Our central question: is there an equilibrium state in the deep quantum gravity regime? We suggest not

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(Warm-up argument 1)

There is already reason to doubt the stability of quantum equilibrium at the level of quantum field theory on a classical spacetime background

Stability is clear on a background globally-hyperbolic spacetime

But the spacetime generated by the formation and evaporation of a black hole is arguably not globally hyperbolic

Unclear how to demonstrate the existence of a stable state of quantum equilibrium in such conditions

Stable equilibrium on a globally-hyperbolic spacetime

$$d\tau^2 = (N^2 - N_i N^i)dt^2 - 2N_i dx^i dt - g_{ij} dx^i dx^j$$

massless and minimally-coupled real scalar field ϕ

Schrödinger equation   wave functional $\Psi[\phi, t]$

$$i\frac{\partial \Psi}{\partial t} = \int d^3x \, \frac{1}{2} N\sqrt{g}\left(-\frac{1}{g}\frac{\delta^2}{\delta\phi^2} + g^{ij}\partial_i\phi\partial_j\phi\right)\Psi$$

continuity equation

$$\frac{\partial |\Psi|^2}{\partial t} + \int d^3x \, \frac{\delta}{\delta\phi}\left(|\Psi|^2 \frac{N}{\sqrt{g}}\frac{\delta S}{\delta\phi}\right) = 0$$

de Broglie velocity field $\dfrac{\partial\phi}{\partial t} = \dfrac{N}{\sqrt{g}}\dfrac{\delta S}{\delta\phi}$

$$\frac{\partial P}{\partial t} + \int d^3x \, \frac{\delta}{\delta\phi}\left(P\frac{N}{\sqrt{g}}\frac{\delta S}{\delta\phi}\right) = 0$$

Stable state of quantum equilibrium (Born rule preserved) (AV 2004)

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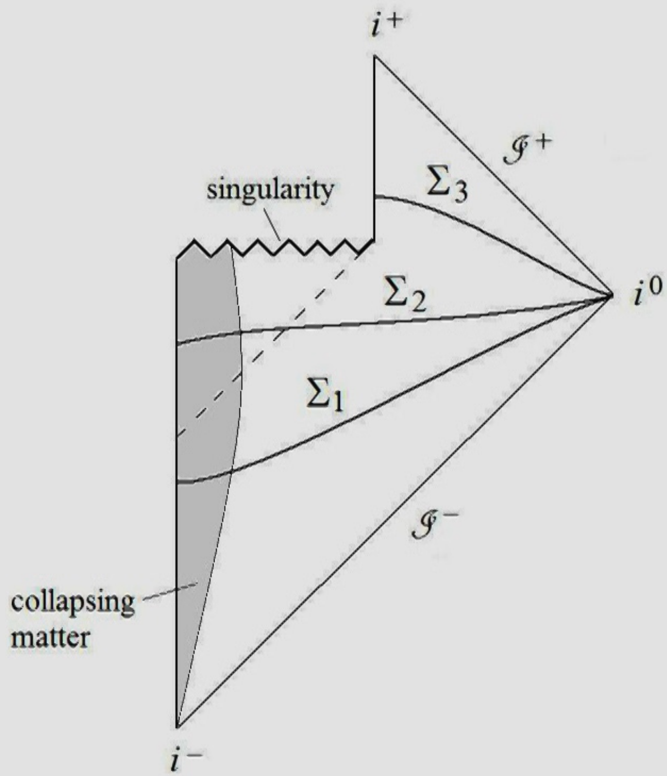
Formation and complete evaporation of a black hole



Resulting spacetime is arguably not globally hyperbolic (controversial)

How can we construct a stable equilibrium state?

Standard quantum field theory in curved spacetime, usually assumes globally hyperbolic:

-- canonical commutation relations on a Cauchy surface

-- expand field in terms of mode functions that are solutions of classical field equations with a well-posed initial value problem

Non-globally hyperbolic spacetime:

-- algebraic QFT, limited?
-- how discuss conservation of Born rule over time?

Perhaps these problems can be resolved.

Or: perhaps they are hints that the Born rule can run into difficulties in a gravitational context, even at the level of a classical spacetime background

Nonequilibrium Hawking radiation? (AV 2004, 2007)

(cf. recent work on "state-dependent operators" and black-hole horizons, with "violations of Born rule", e.g. Marolf and Polchinski 2016)

(Warm-up argument 2)

Regularisation of pilot-wave theory: instability at small scales?

$$m\frac{d\mathbf{x}}{dt} = \nabla S$$

Briefly:

-- pilot-wave dynamics diverges at nodes ($\psi = 0$), requires regularization at short distances

-- if the regularization is time-dependent, the Born rule becomes unstable (initial Born distribution can evolve to a final non-Born distribution, AV 2014)

Creation of quantum nonequilibrium at the Planck scale?

And now our main argument ...

(Warm-up argument 2)

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Quantum equilibrium and the Wheeler-DeWitt equation

$$\left(-G_{ijkl}\frac{\delta^2}{\delta g_{ij}\delta g_{kl}} - g^{1/2}R\right)\Psi = 0$$

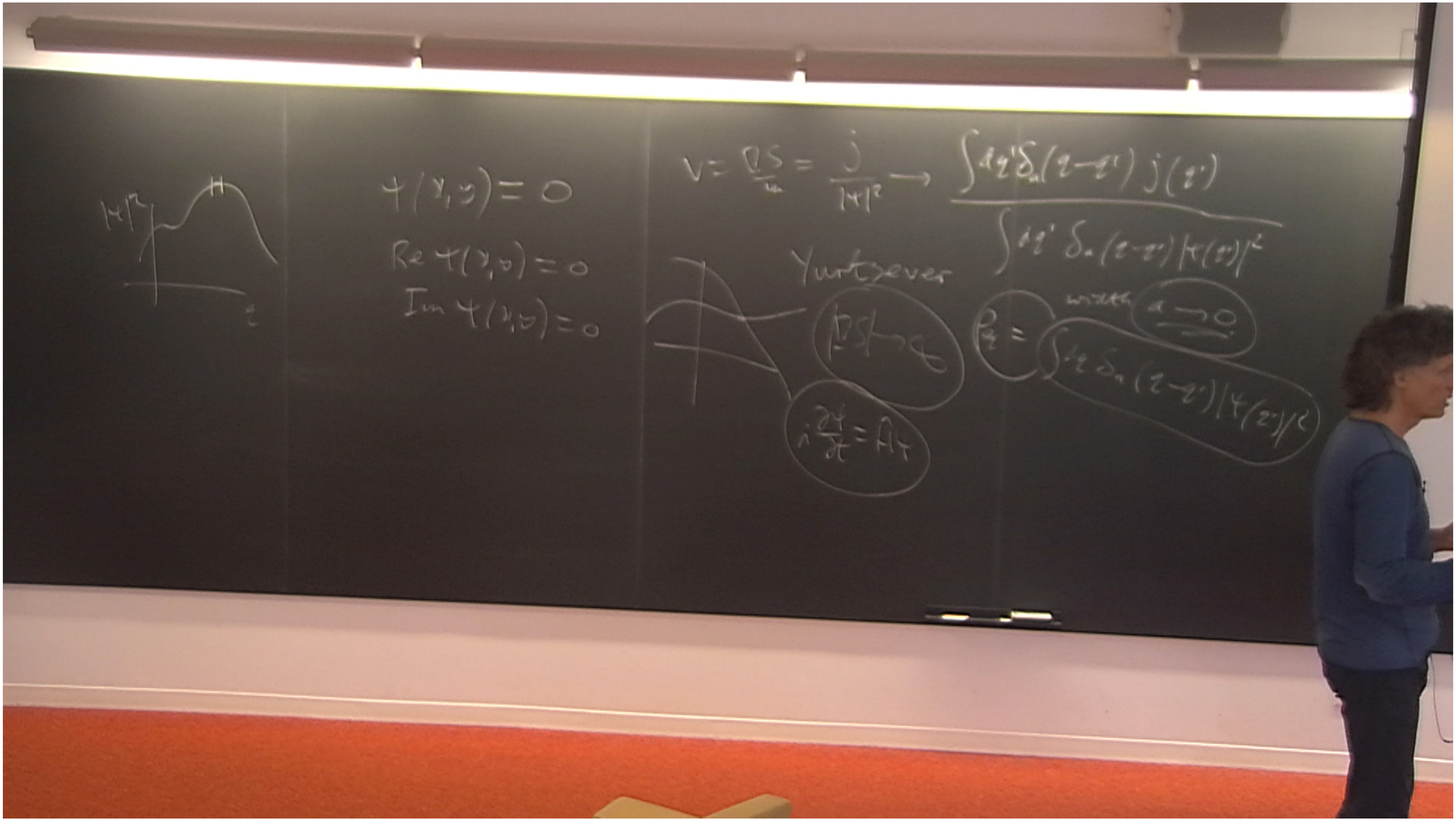
$$\Psi = \Psi[g_{ij}]$$

$$(\delta\Psi/\delta g_{ij})_{;\,j} = 0$$

-- a seemingly "timeless" theory

-- usually assume that conventional quantum mechanics with the Born rule still applies (somehow)

-- physical interpretation remains controversial (problem of time)


ψ(x,y) = 0
Re ψ(x,y) = 0
Im ψ(x,y) = 0
Yurtsever

The Problem of Time

How can we recover an approximate time-dependent Schrödinger equation and time-dependent probabilities (e.g. in the limit of a classical background) from an underlying theory with no time?

-- preferred time? (e.g. AV 1992, 1996; Smolin; others)

-- no time? (e.g. Rovelli, Barbour; De Witt in 1967)

-- ongoing controversy since the 1960s
(see e.g. recent book by E. Anderson, *The Problem of Time* (Springer 2017), 920 pages)

-- still unresolved, perhaps because something is wrong?
(physically, not just mathematically)

Key question of interest here:

In quantum gravity, can we establish the existence of a quantum equilibrium state?

Without recourse to a time-dependent Schrödinger equation and an associated conserved probability current?

Pilot-wave theory and the Wheeler-DeWitt equation

$$\left(-G_{ijkl}\frac{\delta^2}{\delta g_{ij}\delta g_{kl}} - g^{1/2}R\right)\Psi = 0$$

$$\Psi = \Psi[g_{ij}]$$

$$(\delta\Psi/\delta g_{ij})_{;\,j} = 0$$

$$\frac{\partial g_{ij}}{\partial t} = 2NG_{ijkl}\frac{\delta S}{\delta g_{kl}} + N_{i\,;\,j} + N_{j\,;\,i}$$

$$\Psi = |\Psi|\,e^{iS}$$

(general, not just WKB)
(motivated canonically, not from current)

Fundamental dynamics with time t

-- nonlocality; consistent with arbitrary lapse and shift?

-- dynamics claimed to be well-posed, breaking of local Lorentz invariance (Pinto-Neto and Santini 2002)

-- but problem remains: how to connect dynamics of single systems to theory of a quantum equilibrium ensemble

Common attempts to construct a time-dependent quantum probability density:

extract an effective time variable $\tilde{t}[g_{ij}]$ *from the 3-metric*

(e.g. scale factor *a* of expanding universe)

$\Psi[g_{ij}]$ effectively becomes of the schematic form $\Psi[\tilde{g}_{ij}, \tilde{t}]$

where $\tilde{g}_{ij}$ are the remaining metric variables.

Usually works only in some limited regime
(e.g. if universe recontracts then *a* 'runs backwards')

No generally well-behaved equilibrium current or time-dependent density for $\tilde{g}_{ij}$

The Schrödinger regime (approximate)

Approximate Schrödinger-like equation $i\partial\Psi/\partial\tilde{t} = \widehat{\tilde{H}}\Psi$ for $\Psi[\tilde{g}_{ij},\tilde{t}]$ with effective Hamiltonian $\widehat{\tilde{H}}$ and effective time parameter $\tilde{t}$.

Associated continuity equation $\dfrac{\partial|\Psi|^2}{\partial\tilde{t}} + \int d^3x\, \dfrac{\delta J_{ij}}{\delta\tilde{g}_{ij}} = 0$

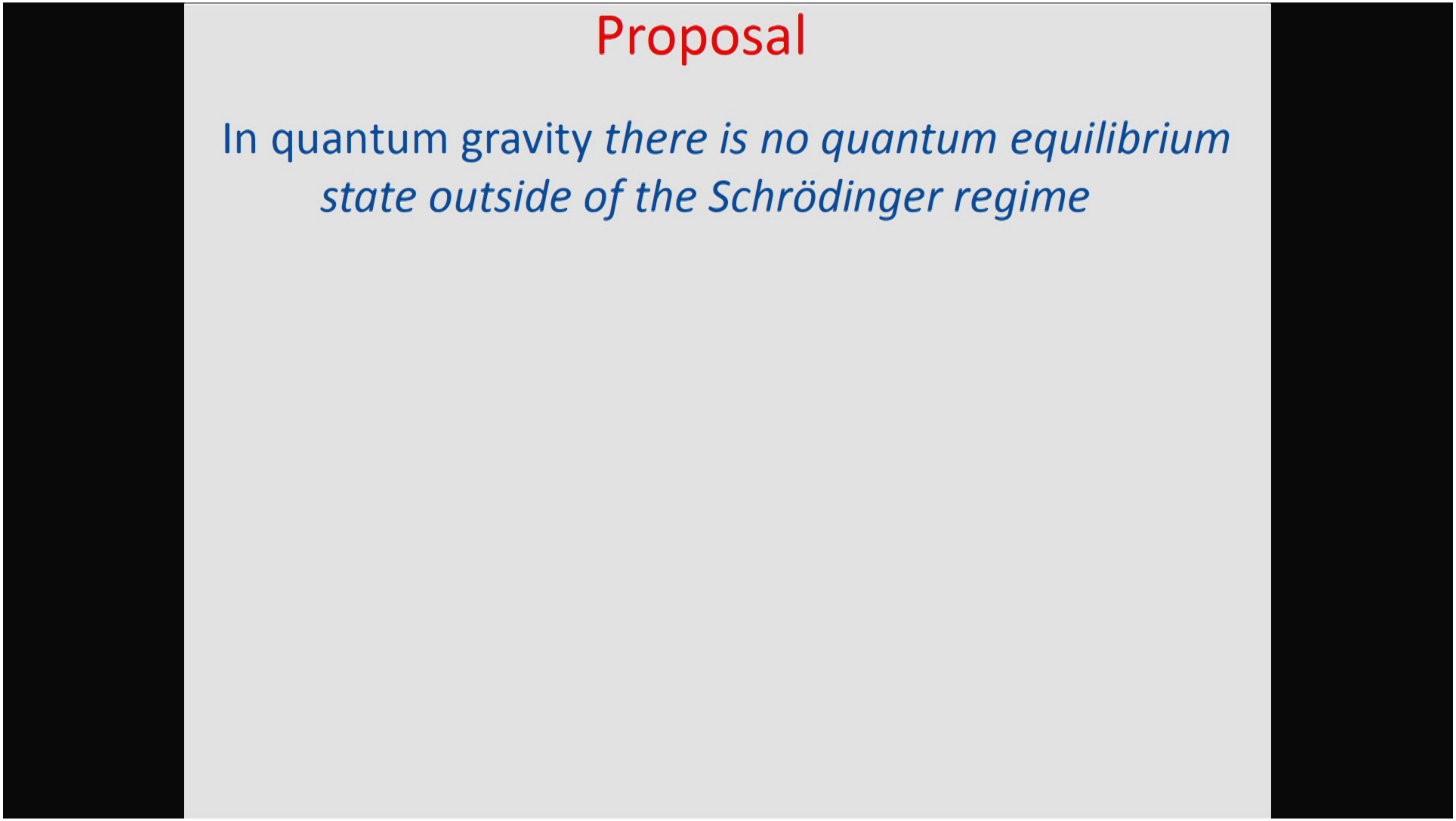
Can define a de Broglie velocity field

$$\partial\tilde{g}_{ij}/\partial\tilde{t} = J_{ij}/|\Psi|^2$$

Arbitrary distribution *P* of metrics $\tilde{g}_{ij}$ will then evolve via

$$\frac{\partial P}{\partial\tilde{t}} + \int d^3x\, \frac{\delta}{\delta\tilde{g}_{ij}}\left(P\frac{\partial\tilde{g}_{ij}}{\partial\tilde{t}}\right) = 0$$

Stable quantum equilibrium: $P = |\Psi|^2$ is conserved in time


Proposal
In quantum gravity *there is no quantum equilibrium state outside of the Schrödinger regime*

Proposal

In quantum gravity *there is no quantum equilibrium state outside of the Schrödinger regime*

In general: there is no Born rule in quantum gravity

-- probabilities P are in principle arbitrary, they need have no connection with Ψ (also true in deBB without gravity)

-- there is no stable equilibrium state for P (and P will not "relax to equilibrium" as there is no equilibrium state to relax to)

-- the equilibrium Born rule is applicable only in the approximate Schrödinger regime

Perhaps implement with pilot-wave theory (if consistent)

$$\left(-G_{ijkl}\frac{\delta^2}{\delta g_{ij}\delta g_{kl}} - g^{1/2}R\right)\Psi = 0$$

$$\frac{\partial g_{ij}}{\partial t} = 2NG_{ijkl}\frac{\delta S}{\delta g_{kl}} + N_{i\,;\,j} + N_{j\,;\,i}$$

dynamics of single systems,
arbitrary ensembles,
no Born rule

arbitrary ensemble with distribution *P* will evolve via

$$\frac{\partial P}{\partial t} + \int d^3x\ \frac{\delta}{\delta g_{ij}}\left(P\frac{\partial g_{ij}}{\partial t}\right) = 0$$ (by construction)

-- defines a dynamics of general ensembles

-- we suggest this has no stable equilibrium state

(in which case an arbitrary initial *P* will not “relax”)

If the above is true ...

Expect to find an intermediate regime with quantum-gravitational corrections to the Schrödinger equation, such that the Born rule is unstable

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$$\frac{\partial P}{\partial t} + \int d^3x\ \frac{\delta}{\delta g_{ij}}\left(P\frac{\partial g_{ij}}{\partial t}\right) = 0 \qquad \text{(by construction)}$$

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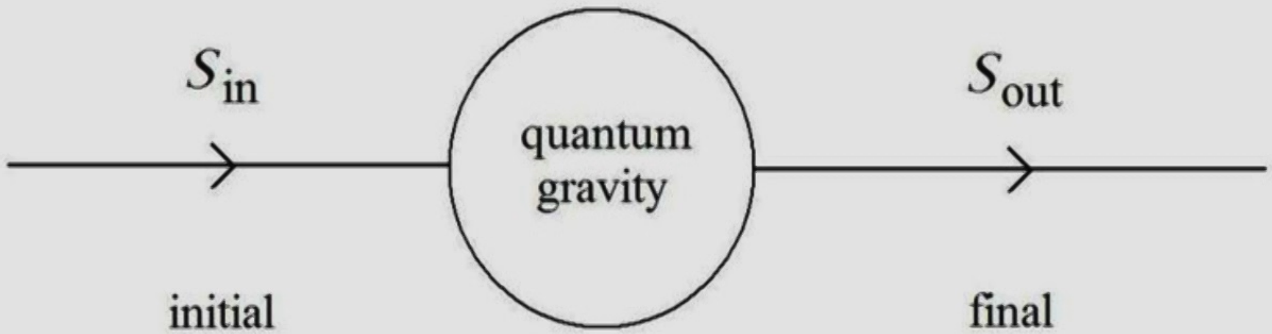
quantum nonequilibrium ?



initial
Schrödinger regime

$$i\frac{\partial\Psi_{\text{in}}}{\partial\tilde{t}_{\text{in}}} = \hat{\hat{H}}_{\text{in}}\Psi_{\text{in}}$$

final
Schrödinger regime

$$i\frac{\partial\Psi_{\text{out}}}{\partial\tilde{t}_{\text{out}}} = \hat{\hat{H}}_{\text{out}}\Psi_{\text{out}}$$

intermediate
non-Schrödinger regime

Gravitational corrections to the Schrödinger equation (Kiefer *et al*, 1991—2018)

Matter field ϕ wave functional $\Psi[g_{ij}, \phi]$

Born-Oppenheimer expansion in powers of m_{P}^2

$$\Psi = e^{iS} \qquad S = m_{\mathrm{P}}^2 S_0 + m_{\mathrm{P}}^0 S_1 + m_{\mathrm{P}}^{-2} S_2 + \ldots$$

S_0 obeys a background classical Hamilton-Jacobi equation

Define "WKB time" $\dfrac{\delta}{\delta \tau} \equiv 2G_{ijkl} \dfrac{\delta S_0}{\delta g_{ij}} \dfrac{\delta}{\delta g_{kl}}$

$2G_{ijkl} \dfrac{\delta S_0}{\delta g_{ij}}$ is the WKB/deBB velocity $\dot{g}_{kl}$ generated by S_0

WKB/deBB trajectories for the background 3-metric define an effective time parameter ("WKB time")

Next order …

Next order:

-- define an effective matter wave functional χ for ϕ (evolving on the background)

-- obeys a corrected Schrödinger equation

$$i\frac{\delta\chi}{\delta\tau} = \hat{\mathcal{H}}_m\chi + \ldots$$

where $\hat{\mathcal{H}}_m$ is the matter Hamiltonian

-- find a small Hermitian correction

-- plus an even smaller non-Hermitian correction (violates unitarity, i.e. breaks conservation of Born rule)

Similar calculations for mini-superspace models

Effective wave function of matter field obeys a Schrödinger-like equation

$$i\frac{\partial\psi}{\partial t} = \left(\hat{H}_\phi + \hat{H}_\phi^A + i\hat{H}_\phi^B\right)\psi$$

$\hat{H}_\phi$ is the field (or "matter") Hamiltonian

$\hat{H}_\phi^A$ is small and Hermitian

$i\hat{H}_\phi^B$ is even smaller and non-Hermitian

Kiefer and Kraemer (2012), Bini *et al* (2013) study effect of $\hat{H}_\phi^A$ on the CMB spectrum (small effects at large scales)

Ignore $i\hat{H}_\phi^B$, which violates unitarity (i.e. breaks conservation of the Born rule)

Non-Hermitian corrections: fact or artifact?

Unclear and controversial:

Can eliminate by redefining the wave function, but artificial?
(unclear which is the true physical wave function, Bini *et al*)

Recent paper (last week!) by Kiefer and Wichmann makes another attempt to eliminate these terms

In the context of our proposal

Might such corrections be a signature of the breakdown of the stability of the Born rule?

Such a process can be described by pilot-wave dynamics

Pilot-wave theory with a non-Hermitian Hamiltonian

$\hat{H} = \hat{H}_1 + i\hat{H}_2$ $\quad\quad$ $\hat{H}_1$ and $\hat{H}_2$ are both Hermitian

$$i\frac{\partial\psi}{\partial t} = (\hat{H}_1 + i\hat{H}_2)\psi$$

$\Longrightarrow$

$$\frac{\partial |\psi|^2}{\partial t} + \partial_q \cdot j_1 = s$$

standard current j_1 associated with $\hat{H}_1$

$s = 2\,\mathrm{Re}\left(\psi^* \hat{H}_2 \psi\right)$ is an effective 'source' term

$\Longrightarrow$

$$\frac{d}{dt}\int dq\ |\psi|^2 = 2\left\langle \hat{H}_2 \right\rangle$$

(where $\left\langle \hat{H}_2 \right\rangle \equiv \int dq\ \left(\psi^* \hat{H}_2 \psi\right)$ is real)

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$$i\frac{\partial\psi}{\partial t} = (\hat{H}_1 + i\hat{H}_2)\psi$$

Define de Broglie velocity field $\quad v = \frac{j_1}{|\psi|^2}$

For an ensemble, density ρ satisfies $\quad \frac{\partial\rho}{\partial t} + \partial_q \cdot (\rho v) = 0$

Defines a pilot-wave theory with an unstable Born rule

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Define de Broglie velocity field $\quad v = \dfrac{j_1}{|\psi|^2}$

For an ensemble, density ρ satisfies $\quad \dfrac{\partial\rho}{\partial t} + \partial_q \cdot (\rho v) = 0$

whereas $|\psi|^2$ satisfies $\quad \dfrac{\partial\,|\psi|^2}{\partial t} + \partial_q \cdot (|\psi|^2\, v) = s$

The ratio $f \equiv \dfrac{\rho}{|\psi|^2}$ is not conserved along trajectories

($df/dt \neq 0$ $\quad d/dt = \partial/\partial t + v \cdot \partial_q$)

H-function (minus relative entropy) $H = \int dq \; \rho \ln(\rho / |\psi|^2)$

Easy to show that $$\frac{dH}{dt} = -\int dq \; \frac{\rho}{|\psi|^2} s$$

Close to equilibrium $\rho \approx |\psi|^2$ we have

$$\frac{dH}{dt} \approx -\int dq \; s = -2 \left\langle \hat{H}_2 \right\rangle$$

Define timescale τ_{noneq} as $$\tau_{\text{noneq}} \left| \frac{dH}{dt} \right| \sim 1$$

(time needed for *H* to change by ~ 1)

Find $$\tau_{\text{noneq}} \sim \frac{1}{2\left|\left\langle \hat{H}_2 \right\rangle\right|}$$ (as before)

Application to a scalar field mode during inflation

Estimate timescale $\tau_{\rm noneq} \sim \dfrac{1}{2\left|\left\langle \hat{H}_2 \right\rangle\right|}$ with $\left\langle \hat{H}_2 \right\rangle \approx \left\langle \hat{H}_2 \right\rangle_{\rm B-D}$

(take expectation value in Bunch-Davies vacuum $\psi^{(0)}_{\mathbf{k}r}$, without gravitational corrections)

The term $\hat{H}_2$ comes from Kiefer *et al*, gravitational corrections to the Schrödinger equation for a scalar field during inflation, corrected wave function $\psi^{(1)}_{\mathbf{k}r}$ obeys

$$i\frac{\partial \psi^{(1)}_{\mathbf{k}r}}{\partial t} = \hat{H}_{\mathbf{k}r}\psi^{(1)}_{\mathbf{k}r} - \frac{a^3}{2m_{\rm P}^2 H^2 \psi^{(0)}_{\mathbf{k}r}}\left[\frac{1}{a^6}\hat{H}^2_{\mathbf{k}r}\psi^{(0)}_{\mathbf{k}r} + i\frac{\partial}{\partial t}\left(\frac{1}{a^6}\hat{H}_{\mathbf{k}r}\right)\psi^{(0)}_{\mathbf{k}r}\right]\psi^{(1)}_{\mathbf{k}r}$$

where *H* is the Hubble parameter and

$$\hat{H}_{\mathbf{k}r} = -\frac{1}{2a^3}\frac{\partial^2}{\partial q^2_{\mathbf{k}r}} + \frac{1}{2}ak^2 q^2_{\mathbf{k}r}$$

(standard single-mode Hamiltonian)

$$\phi_{\mathbf{k}} = \frac{\sqrt{V}}{(2\pi)^{3/2}}\left(q_{\mathbf{k}1} + iq_{\mathbf{k}2}\right)$$

$$i\frac{\partial \psi^{(1)}_{\mathbf{k}r}}{\partial t} = \hat{H}_{\mathbf{k}r}\psi^{(1)}_{\mathbf{k}r} - \frac{a^3}{2m_{\mathrm{P}}^2 H^2 \psi^{(0)}_{\mathbf{k}r}}\left[\frac{1}{a^6}\hat{H}^2_{\mathbf{k}r}\psi^{(0)}_{\mathbf{k}r} + i\frac{\partial}{\partial t}\left(\frac{1}{a^6}\hat{H}_{\mathbf{k}r}\right)\psi^{(0)}_{\mathbf{k}r}\right]\psi^{(1)}_{\mathbf{k}r}$$

(derived for a closed universe, *a* has dimensions of length)

Corrections are multiplicative, one real and one imaginary

In our notation with $\hat{H} = \hat{H}_1 + i\hat{H}_2$ we have

$$\hat{H}_1 = \hat{H}_{\mathbf{k}r} - \frac{1}{2m_{\mathrm{P}}^2 H^2 \psi^{(0)}_{\mathbf{k}r}}\frac{1}{a^3}\hat{H}^2_{\mathbf{k}r}\psi^{(0)}_{\mathbf{k}r}$$

$$\hat{H}_2 = -\frac{a^3}{2m_{\mathrm{P}}^2 H^2 \psi^{(0)}_{\mathbf{k}r}}\frac{\partial}{\partial t}\left(\frac{1}{a^6}\hat{H}_{\mathbf{k}r}\right)\psi^{(0)}_{\mathbf{k}r}$$

where the Bunch-Davies wave function $\psi^{(0)}_{\mathbf{k}r}$ satisfies

$$i\frac{\partial \psi^{(0)}_{\mathbf{k}r}}{\partial t} = \hat{H}_{\mathbf{k}r}\psi^{(0)}_{\mathbf{k}r}$$

(with the Minkowski boundary condition for the limit $H \longrightarrow 0$)

Estimate $\tau_{\text{noneq}} \sim \dfrac{1}{2\left|\left\langle \hat{H}_2 \right\rangle\right|}$

Use $\dot{a} = Ha$ to write

$$\hat{H}_2 = \frac{1}{2m_{\mathrm{P}}^2 H} \frac{1}{a^3} \frac{1}{\psi_{\mathbf{k}r}^{(0)}} \hat{H}'_{\mathbf{k}r} \psi_{\mathbf{k}r}^{(0)}$$

With

$$\hat{H}'_{\mathbf{k}r} = -9\frac{1}{2a^3}\frac{\partial^2}{\partial q_{\mathbf{k}r}^2} + 5\frac{1}{2}ak^2 q_{\mathbf{k}r}^2$$

Calculate

$$\left\langle \hat{H}_2 \right\rangle_{\mathrm{B-D}} = \frac{1}{2m_{\mathrm{P}}^2 H}\frac{1}{a^3}\left\langle \hat{H}'_{\mathbf{k}r} \right\rangle_{\mathrm{B-D}}$$

for Bunch-Davies wave function

$$\psi_{\mathbf{k}r}^{(0)} = \left|\psi_{\mathbf{k}r}^{(0)}\right| e^{is_{\mathbf{k}r}^{(0)}}$$

$$\psi_{\mathbf{k}r}^{(0)} = \left|\psi_{\mathbf{k}r}^{(0)}\right| e^{is_{\mathbf{k}r}^{(0)}}$$

$$\left|\psi_{\mathbf{k}r}^{(0)}\right| = \frac{1}{(2\pi\Delta_k^2)^{1/4}} e^{-q_{\mathbf{k}r}^2/4\Delta_k^2}$$

$$s_{\mathbf{k}r}^{(0)} = -\frac{ak^2 q_{\mathbf{k}r}^2}{2H(1+k^2/H^2a^2)} + \frac{1}{2}\frac{k}{Ha} - \frac{1}{2}\tan^{-1}\left(\frac{k}{Ha}\right)$$

$$\Delta_k^2 = \frac{H^2}{2k^3}\left(1 + \frac{k^2}{H^2a^2}\right)$$

(AV, PRD 2010)

(replacing k by $a_0 k$)

Order of magnitude for the timescale

$$\tau_{\text{noneq}} \sim 2m_{\text{P}}^2 a^3 \frac{x}{7+\frac{5}{2}x^2}$$

$$x = \frac{1}{2\pi}\frac{\lambda_{\text{phys}}}{H^{-1}}$$

Take $H \sim 10^{-3} m_{\text{P}}$ and x in some small range around $x \sim 1$

(so λ_{phys} is not too close to l_{P} and not too far outside H^{-1})

For $x \sim 1$ we have $\tau_{\text{noneq}} \sim m_{\text{P}}^2 a^3$ so for $a \sim H^{-1}$ we have

$$\tau_{\text{noneq}} \sim m_{\text{P}}^2 \frac{1}{H^3} \sim 10^9 t_{\text{P}}$$

Compare to time $H^{-1} \sim 10^3 t_{\text{P}}$ mode spends in regime $x \sim 1$

Only a fraction 10^{-6} of τ_{noneq}

Crude estimate suggests: effects will be small (as expected)

Trans-Planckian modes and inflation

-- some argue that trans-Planckian field modes are likely to contribute significantly to the inflationary spectrum (Brandenberger and Martin 2001)

-- if so, can use inflation to probe physics at the Planck scale and beyond (review, Brandenberger and Martin 2013)

-- our proposal suggests that field modes will exit the "Planck radius" in a state of quantum nonequilibrium (owing to quantum-gravitational effects)

-- future work: need detailed predictions (too small to be observed?)

Laboratory systems?

-- *expect that quantum nonequilibrium is being continuously generated at the Planck scale*

-- possible significant cumulative effect over long times?

-- will be counter-acted by the natural relaxation process that occurs when a system is in a superposition of energy states

-- **speculative prediction**: *if a laboratory system is sufficiently isolated and kept in an energy eigenstate for a very long time, it will eventually evolve significantly away from quantum equilibrium and violate the Born rule (details to be developed)*

NB: *small* perturbations do not cause relaxation (Kandhadai and Valentini 2016), so isolating sufficiently is unlikely to be a problem but waiting for long enough may well be.

Discussion

General idea is independent of Kiefer *et al*'s results
(in case latter are spurious)

We should investigate the borderline between the Schrödinger and non-Schrödinger regimes

Perhaps we will find that gravitation makes quantum equilibrium unstable

Assuming the Born rule in quantum gravity may have been a long-standing mistake