Title: Universal Formula for the Holographic Speed of Sound

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Abstract: <p>We consider planar hairy black holes in five dimensions with a real scalar field in the Breitenlohner-Freedman window and show that is possible to derive a universal formula for the holographic speed of sound for any mixed boundary conditions of the scalar field. As an example, we locally construct the most general class of planar black holes coupled to a single scalar field in the consistent truncation of type IIB supergravity that preserves the SO(3)xSO(3) R-symmetry group of the gauge theory. We obtain the speed of sound for different values of the vacuum expectation value of a single trace operator when a double trace deformation is induced in the dual gauge theory. In this particular family of solutions, we find that the speed of sound exceeds the conformal value. Finally, we generalize the formula of the speed of sound to arbitrary dimensional scalar-metric theories whose parameters lie within the Breitenlohner-Freedman window</p>

Universal formula for the holographic speed of sound

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1702.00017 [AA, D. Astefanesei, T. Andrade and R. Mann]

Outline

- A review of a no hair theorem
- How scalar fields in AdS avoid this theorem
- A review of scalar fields in AdS/CFT
- The BF window
- Hairy Black Holes and holography
- Speed of Sound
- An example
- Universal Formula for the Speed of Sound

Black Holes have no-hair: **A theory independent proof [Bekenstein]**

$$ds^2 = -f(r)dt^2 + g(r)dr^2 + r^2 d\Sigma$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right) - \frac{\partial V}{\partial\phi} = 0$$

$$\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right)\left(\frac{\partial V}{\partial\phi}\right)-\left(\frac{\partial V}{\partial\phi}\right)^2\sqrt{-g}=0$$

$$\int_{r_+}^{\infty}dr\left[\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right)\left(\frac{\partial V}{\partial\phi}\right)-\left(\frac{\partial V}{\partial\phi}\right)^2\sqrt{-g}\right] \quad = \quad 0$$

$$\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\left(\frac{\partial V}{\partial\phi}\right)\Big|_{r_+}^{\infty}-\int_{r_+}^{\infty}\sqrt{-g}dr\left[\left(\frac{\partial^2 V}{\partial\phi^2}\right)g^{\mu\nu}\partial_\nu\phi\partial_\mu\phi+\left(\frac{\partial V}{\partial\phi}\right)^2\right] \quad = \quad 0$$

$\phi=\phi(r)$ and $\frac{\partial^2 V}{\partial\phi^2}>0$ then $\phi(r)=\phi_0$

The requirement on the convexity of the potential can be thought as an statement on stability of the ground state. This indeed no longer relevant for asymptotically AdS spacetimes and scalar fields above the BF bound

$$m^2 \geq m^2_{BF} = -\frac{(D-1)^2}{4L^2}$$

For Dirichlet boundary conditions the perturbative stability of AdS is ensured

Scalar fields and AdS/CFT

$$ds^2 = \frac{L^2}{r^2}dr^2 + \frac{r^2}{L^2}\gamma_{\mu\nu}dx^\mu dx^\nu + O(1)$$

$$\varphi = \frac{L^{2\Delta_-}\alpha\left(x\right)}{r^{\Delta_-}} + \frac{L^{2\Delta_+}\beta\left(x\right)}{r^{\Delta_+}} + O(r^{-\Delta_- -1})$$

$$\Delta_\pm = \frac{(D-1)}{2}\left(1 \pm \sqrt{1 + \frac{4L^2m^2}{(D-1)^2}}\right)$$

Scalar fields and AdS/CFT

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$$\varphi = \frac{L^{2\Delta_-}\alpha(x)}{r^{\Delta_-}} + \frac{L^{2\Delta_+}\beta(x)}{r^{\Delta_+}} + O(r^{-\Delta_- -1})$$

$$I^E[\varphi] = \frac{1}{\kappa}\int_M \left[\frac{1}{2}(\partial\varphi)^2 + V(\varphi)\right]\sqrt{g}d^Dx + \frac{\Delta_-}{2L\kappa}\int_{\partial M}\varphi^2\sqrt{h}d^{D-1}x$$

$$\delta I^E = \frac{1}{\kappa}\int_{\partial M}\left[N^\mu\partial_\mu\varphi + \Delta_-\frac{\varphi}{L}\right]\delta\varphi\sqrt{h}d^{D-1}x$$

$$N^\mu = \delta^\mu_r\frac{r}{L} \qquad \delta\varphi = L^{2\Delta_-}\frac{\delta\alpha}{r^{\Delta_-}} + L^{2\Delta_+}\frac{\delta\beta}{r^{\Delta_+}} + O(r^{-\Delta_- -1})$$

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$$\delta I^E = (\Delta_- - \Delta_+)\frac{L^{D-2}}{\kappa}\int_{\partial M}\beta\delta\alpha\sqrt{\gamma}d^{D-1}x$$

$$e^{-I_E} = \int d[\Phi]G(\Phi)e^{-S[\Phi]-N^{\#}\mathcal{C}\int J(x)O(x)d^{D-1}x} = \left\langle e^{-N^{\#}\mathcal{C}\int J(x)\mathcal{O}(x)d^{D-1}x}\right\rangle$$

$$\frac{L^{D-2}}{\kappa} = N^{\#}\mathcal{C}$$

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$$\frac{1}{(\Delta_- - \Delta_+)}\left[-\frac{1}{\sqrt{\gamma}}\frac{\delta I_E}{\delta\alpha}\right]_{\alpha=0} = N^{\#}\mathcal{C}\,\langle\mathcal{O}\rangle = \frac{L^{D-2}}{\kappa}\beta$$

The BF window. [Breitenlohner-Freedman, Klebanov-Witten, Ishibashi-Wald]

$$m_{BF}^2 + L^{-2} > m^2 \geq m_{BF}^2$$

In the BF window both modes are normalisable.

AdS/CFT with Multitrace deformations

$$e^{-I_E} = \left\langle e^{-N^{\#}\mathcal{C}\int [J(x)\mathcal{O}(x)]\sqrt{\gamma}d^{D-1}x} \right\rangle$$

$$e^{-I_E} = \left\langle e^{-N^{\#}\mathcal{C}\int \left[J(x)\mathcal{O}(x)+W(\mathcal{O})-\left[W(\beta)+(\mathcal{O}-\beta)\dot{W}(\beta)\right]\right]\sqrt{\gamma}d^{D-1}x} \right\rangle$$

$$e^{-I_E} e^{-\frac{L^{D-2}}{\kappa}\int \left(W(\beta)-\beta\dot{W}(\beta)\right)\sqrt{\gamma}d^{D-1}x} = \left\langle e^{-N^{\#}\mathcal{C}\int [J(x)\mathcal{O}(x)+W(\mathcal{O})-\mathcal{O}\dot{W}(\beta)]\sqrt{\gamma}d^{D-1}x} \right\rangle$$

$$e^{-I_E^W} = \left\langle e^{-N^{\#}\mathcal{C}\int [J_W(x)\mathcal{O}(x)+W(\mathcal{O})]\sqrt{\gamma}d^{D-1}x} \right\rangle$$

AdS/CFT with Multitrace deformations

$$e^{-I_E^W} = \left\langle e^{-N^{\#}\mathcal{C}\int [J_W(x)\mathcal{O}(x)+W(\mathcal{O})]\sqrt{\gamma}d^{D-1}x} \right\rangle$$

$$\delta I_E^W \quad = \quad \frac{L^{D-2}}{\kappa}(\Delta_- - \Delta_+)\int \beta\delta\alpha\sqrt{\gamma}d^{D-1}x - \frac{L^{D-2}}{\kappa}\int \beta\ddot{W}(\beta)\,\delta\beta\sqrt{\gamma}$$

AdS/CFT with Multitrace deformations

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$$\delta I_E^W = \frac{L^{D-2}}{\kappa}\int \beta\delta\left[(\Delta_- - \Delta_+)\,\alpha - \dot{W}(\beta)\right]\sqrt{\gamma}d^{D-1}x$$

$$\langle\mathcal{O}\rangle_W = \beta \qquad J = \alpha\,(\Delta_- - \Delta_+) - \dot{W}(\beta)$$

Hairy black holes in AdS. [Sudarsky-Gonzales, Torii-Maeda-Narita]

Holography and Hairy Black Holes with multi-trace deformations [Buchel, Hertog-Horowitz, Gubser-Nellore]

Holographic speed of sound and Hairy Black Holes

$$\kappa I\left[g,\varphi\right]=\int_M d^5x\sqrt{-g}\left[\frac{R}{2}-\frac{1}{2}\left(\partial\varphi\right)^2-V\left(\varphi\right)\right]+I_{ct}+I_{ct}^{\varphi}+\kappa\Delta I$$

$$-\frac{2}{\sqrt{-\gamma}}\frac{\delta I}{\delta\gamma^{ab}}=\langle\mathcal{T}_{ab}\rangle=\left(\rho+p\right)u_a u_b+p\gamma_{ab}$$

$$c_s^2=\left(\frac{\partial p}{\partial\rho}\right)$$

The speed of sound as a function of Bulk quantities [0804.0434]

$$c_s^2 = \left(\frac{\partial p}{\partial \rho}\right)$$

$$c_s^2 = \frac{d\log T}{d\log s}$$

The speed of sound as a function of Bulk quantities [0804.0434]

$$c_s^2 = \frac{d \log T}{d \log s}$$

$$V(\phi) = V_0 e^{\gamma \phi}$$

$$c_s^2 = \frac{1}{3} - \frac{\gamma^2}{2}$$

When there is an asymptotic region the speed of sound must necessarily depend on the boundary quantities. The right formula is

$$c_s^2 = \left(\frac{\partial p}{\partial \rho} \right)_J$$

The hairy black hole problem in D=5

$$ds^2 = e^A(-f\,dt^2 + d\Sigma) + e^B \frac{dr^2}{f}$$

$$Z = \frac{d\varphi}{dA} \qquad Y = Zf\frac{d\varphi}{df}$$

$$\frac{dZ}{d\varphi} = \left(3\frac{dV}{d\varphi} + 4ZV\right)\frac{(2Z^2Y - 6Y - 3Z^2)}{12VZY}$$

$$\frac{dY}{d\varphi} = \left(3\frac{dV}{d\varphi} + 2ZV\right)\frac{(2Z^2Y - 6Y - 3Z^2)}{6VZ^2}$$

The hairy black hole problem in D=5

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$$-3Z\left(3\frac{dV}{d\varphi} + 4ZV\right)\frac{d^2Z}{d\varphi^2} + \left(-9\frac{dV}{d\varphi} + 12ZV\right)\left(\frac{dZ}{d\varphi}\right)^2 + \left[(Z^2+3)\,8ZV + (18Z^2+1)\frac{dV}{d\varphi} + 9Z\frac{d^2V}{d\varphi^2}\right]\frac{dZ}{d\varphi} = 0$$

The hairy black hole problem in D=5

$$A(r) = \int_{\varphi_h}^{\varphi(r)} \frac{d\varphi}{Z} + \frac{2}{3}\ln(\mathcal{A})$$

$$s = \frac{\mathcal{A}}{4G}$$

$$\delta\rho = T\delta s$$

Take a scalar field saturating the BF bound

$$\varphi = \frac{\alpha l^4}{r^2} \ln\left(\frac{r}{r_0}\right) + \frac{\beta l^4}{r^2} + O\left(\frac{\ln(r)^2}{r^3}\right)$$

$$e^A \varphi \sim \alpha l^2 \ln(\tfrac{r}{r_0})$$

$$2e^A\left(\varphi + \frac{d\varphi}{dA}\right) \sim \alpha l^2$$

$$l^2 \alpha(\varphi_h, \mathcal{A}) = \lim_{\varphi \to 0} 2(Z + \varphi) e^A = \alpha_h(\varphi_h) \mathcal{A}^{\frac{2}{3}}$$

The same argument applies to all the integration constants

$$e^A f = \frac{r^2}{l^2} - \frac{\mu l^6}{r^2} + O(r^{-3}) \ , \qquad e^A = \frac{r^2}{l^2} + O(r^{-3}) \ , \qquad \frac{e^B}{f} = \frac{l^2}{r^2} + O(\frac{\ln(r)^2}{r^6})$$

$$\mu\left(\varphi_h, \mathcal{A}\right) = l^{-4} \mu_h\left(\varphi_h\right) \mathcal{A}^{\frac{4}{3}}$$

$$\beta\left(\varphi_h, \mathcal{A}\right) = -\frac{1}{2} \alpha \ln\left|\frac{\alpha}{\alpha_0}\right| + \alpha z_h\left(\varphi_h\right)$$

Hence the system is fully characterized by the function F(z) that we call the black hole line. In analogy with a soliton introduced by Horowitz and Hertog

$$\frac{\mu}{\alpha^2} = F(z)$$

The formula for the speed of sound with boundary condition $z = \omega(\alpha)$

$$p = \frac{l^3}{\kappa}\left[\frac{\mu}{2} + \frac{1}{8}\left(\alpha^2 - 4\alpha\beta + 8W(\beta)\right)\right]$$

$$\rho = \frac{l^3}{\kappa}\left[\frac{3\mu}{2} - \frac{1}{8}\left(\alpha^2 - 4\alpha\beta + 8W(\beta)\right)\right]$$

$$\left(\frac{\partial p}{\partial \alpha}\right)_J = \frac{l^3}{\kappa}\left[\alpha F + \frac{\alpha^2 \dot{F}\omega'}{2} + \frac{1}{2}\alpha^2\omega'\right], \qquad \left(\frac{\partial \rho}{\partial \alpha}\right)_J = \frac{l^3}{\kappa}\left[3\alpha F + \frac{3}{2}\alpha^2 \dot{F}\omega' - \frac{1}{2}\alpha^2\omega'\right]$$

$$c_s^2 = \left(\frac{\partial p}{\partial \rho}\right)_J = \frac{1}{3} + \frac{4}{3}\frac{\alpha^2\omega'}{6\alpha F + 3\alpha^2\dot{F}\omega' - \alpha^2\omega'}$$

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$$e^A f = \frac{r^2}{l^2} - \frac{\mu l^6}{r^2} + O(r^{-3}) \, , \qquad e^A = \frac{r^2}{l^2} + O(r^{-3}) \, , \qquad \frac{e^B}{f} = \frac{l^2}{r^2} + O(\frac{\ln(r)^2}{r^6})$$

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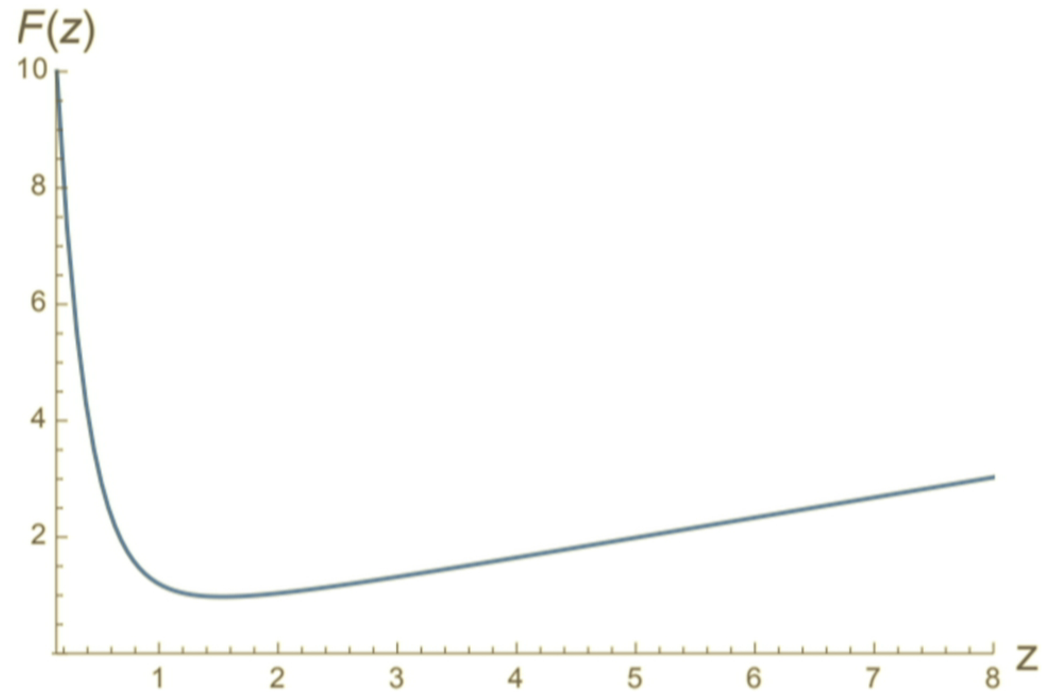
An Example from type IIB supergravity

$$V(\varphi) = -\frac{3}{2l^2}\left[3 + \cosh\left(\frac{2\sqrt{6}}{3}\varphi\right)\right]$$

$$\mathcal{O} = \frac{1}{N}\mathrm{Tr}\left(\phi_1^2 + \phi_2^2 + \phi_3^2 - \phi_4^2 - \phi_5^2 - \phi_6^2\right)$$

An Example from type IIB supergravity



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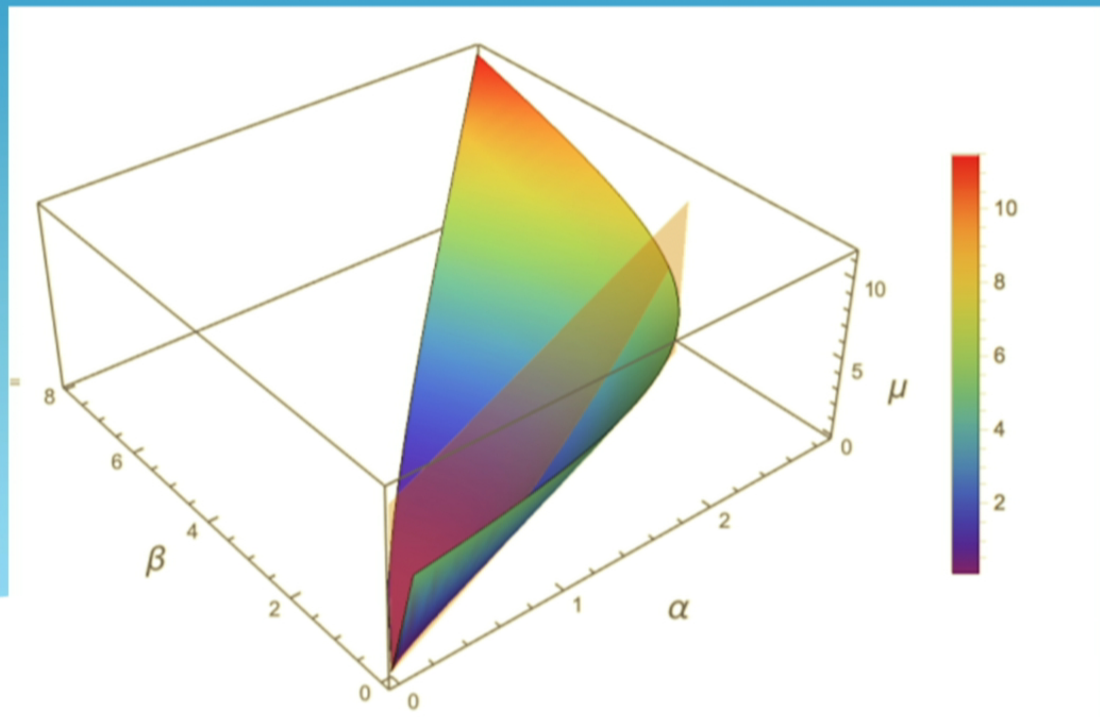
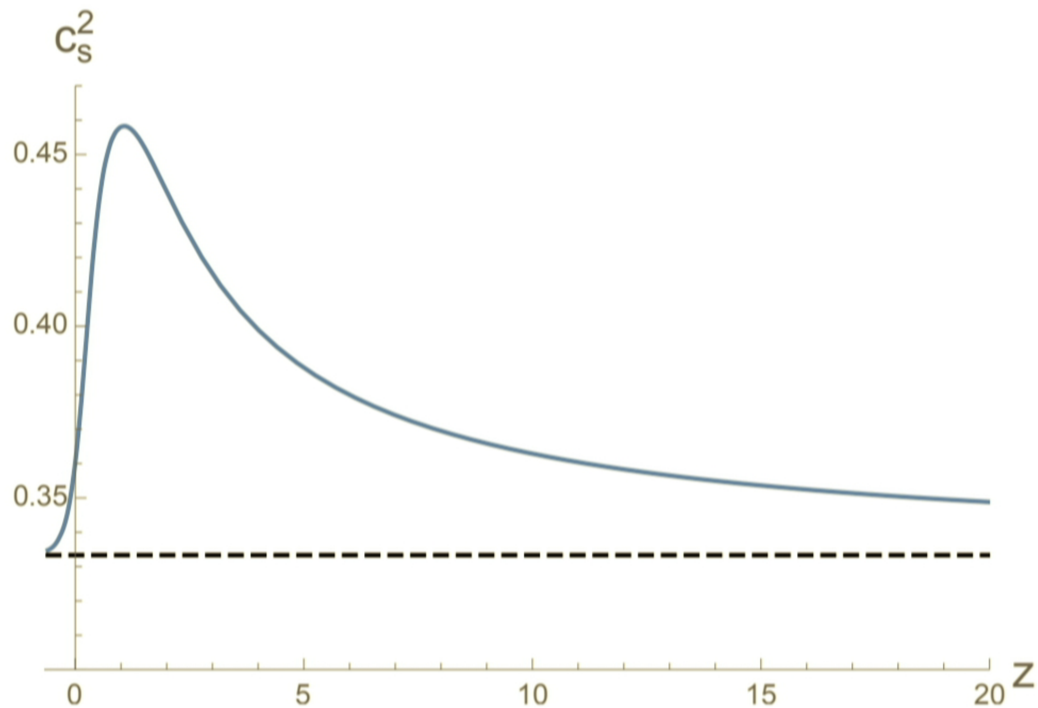


The speed of sound is always larger than the conformal value



The energy is bounded from below

$$\rho = \frac{\alpha^2 l^3}{\kappa}\left(\frac{3}{2}F(z) - \frac{1}{8}\right).$$

The Universal Formula

$$\alpha = \alpha_h(\varphi_h)\,\mathcal{A}^{\frac{\Delta_-}{D-2}}\ , \qquad \mu(\varphi_h,\mathcal{A}) = \mu_h(\varphi_h)\,\mathcal{A}^{\frac{D-1}{D-2}}\ , \qquad \beta = z(\varphi_h)\,\alpha^{\frac{\Delta_+}{\Delta_-}}$$

$D-1$

$$c_s^2 = \frac{1}{D-2} + 2\frac{D-1}{D-2}\frac{\omega'\alpha(D-1-2\Delta_-)\Delta_-^2}{(D-2)(D-1)\left((D-1)F+\omega'\alpha\Delta_-\dot{F}\right) - 2\Delta_-^2(D-1-2\Delta_-)\omega'\alpha}$$

Conclusions

- Every theory in the BF window has infinite number of hairy black hole solutions
- Every hairy black hole solution has different thermodynamics.
- The speed of sound can be larger than the conformal value for theories with an holographic dual
- The characterization of the theory in terms of a single function allows to codify the numerical integration of the system in a simple form