Title: From superradiance to superfluidity: effective theories of point-like particles

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Abstract: <p>We are fortunate to live in an era of great discoveries in particle physics and cosmology, and most of the theoretical understanding that made this possibile is based on effective field theories. In this talk, I will show how these powerful techniques can be applied across the spectrum of theoretical physics, and allow us to draw unexpected connections among very different systems. To illustrate this, I will discuss two interesting but very different phenomena, and show how they can both be described using a point-like particle effective theory. The first phenomenon I will consider is superradiance---a dissipative phenomenon by which a spinning object can amplify the intensity of scattered radiation, or be unstable by creation of particles in a bound state. This phenomenon is particularly interesting when applied to spinning black holes, and in this context it has recently received significant attention as a possible probe of ultra-light particle beyond the Standard Model. The second phenomenon I will discuss is the existence of roton excitations in superfluid He4. By treating rotons as point-like particles--albeit of a very special kind--I will write down an effective theory that describes the motion of rotons in the medium at equilibrium as well as their couplings to sound waves and generic ï¬‚uid ï¬‚ows. This approach allows us to correct classic results by Landau and Khalatnikov on roton-phonon scattering, and to pose and answer the intriguing question: do rotons sink or float? </p>

From Superradiance to Superfluids

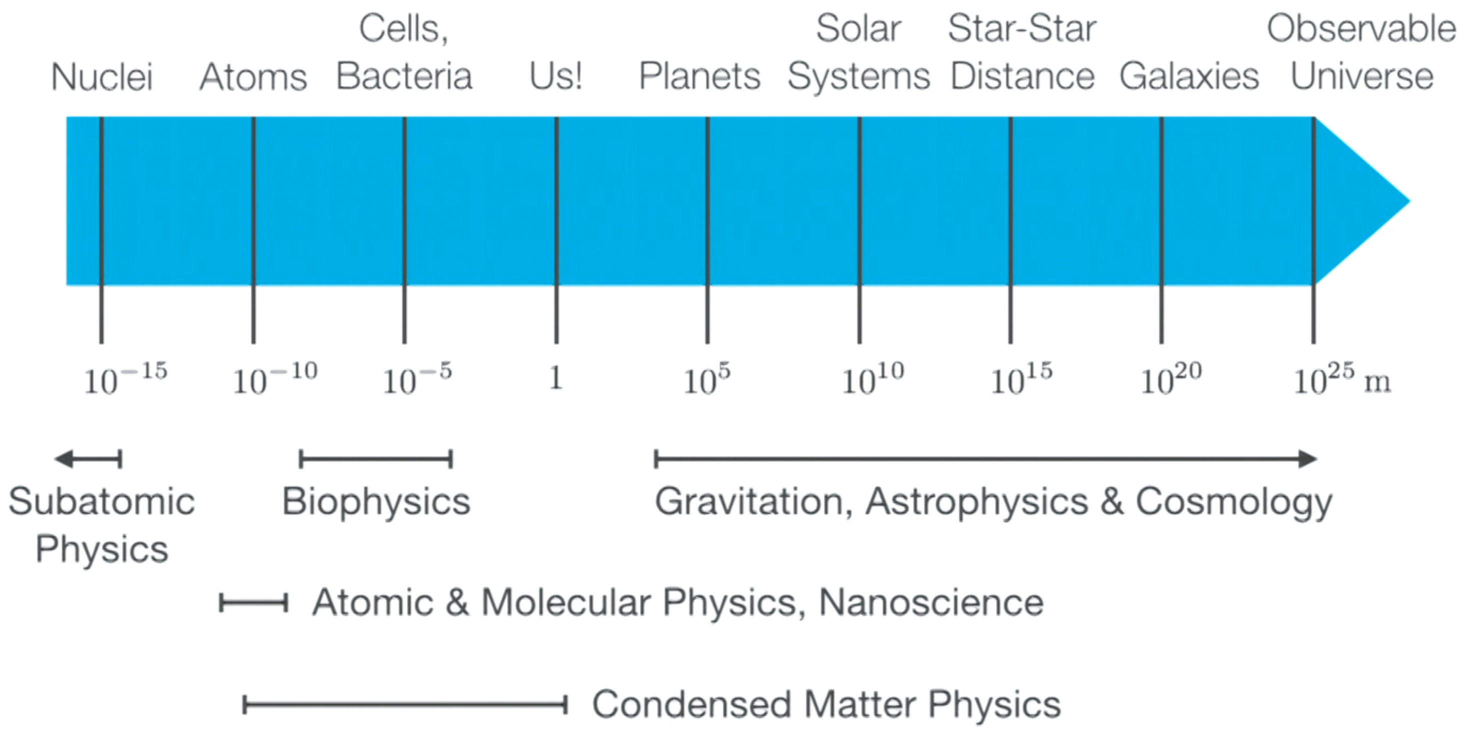
Effective theories of point-like particles.

Riccardo Penco
University of Pennsylvania

Perimeter Institute

2.22.18

Length Scales of Physics



What is an Effective Theory?

"Effective theory" is one of the more important notions within science—and outside it. The idea is to determine what you can actually measure and [...] find a theory appropriate to those measurable quantities. The theory that works might not be the ultimate truth, but it's as close an approximation to the truth as you need and is also the limit to what you can test at any given time.

— ***Lisa Randall***

Things I've worked on

Inflation	[C.Armendariz-Picon, L.Hui, **J.Neelakanta**, M.Trodden]
Dark Matter	[B.Famaey, J.Khoury, **A.Sharma**]
Astrophysical spinning objects	[**L.Delacretaz**, S.Endlich, A.Monin, F.Riva]
Black hole solutions	[**M.Carillo-Gonzalez**, **G.Franciolini**, L.Hui, L.Santoni, E.Trincherini, M.Trodden]
Field theories at finite density	[T.Brauner, **L.Delacretaz**, S.Endlich, A.Monin, A.Nicolis, F.Piazza, R.Rattazzi, R.Rosen]
Holographic superfluids and solids	[**A.Esposito**, **S.Garcia-Saenz**, A.Nicolis]
Gapped excitations in superfluids and magnets	[B.Horn, A.Nicolis, I.Rothstein]

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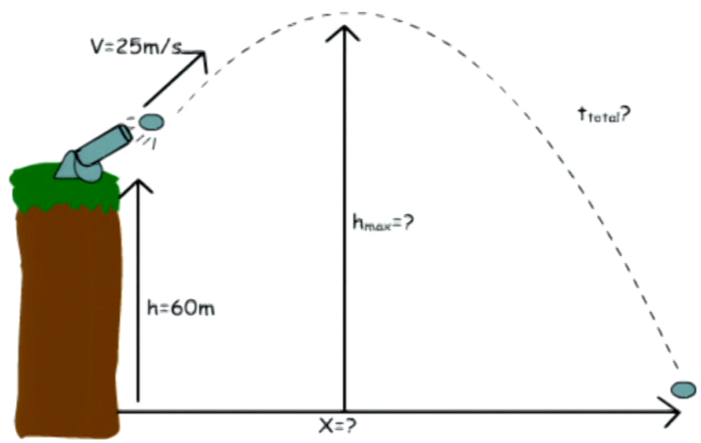
The simplest effective theory

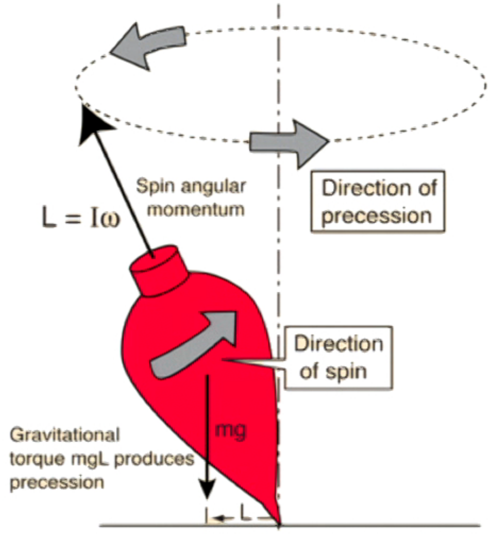
Point particles



We need at most 3 linear coordinates …

The simplest effective theory

Point particles

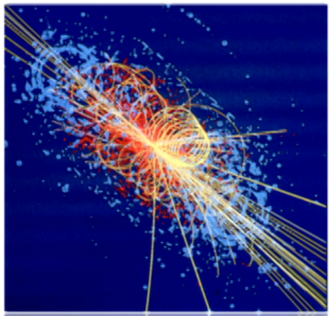


… and 3 Euler angles.

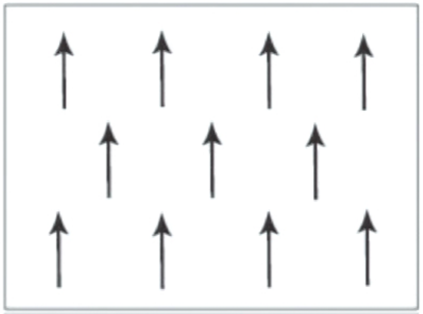
Spontaneous symmetry breaking

Occurs at all scales

SSB = ground state has fewer symmetries than equations



Electroweak force



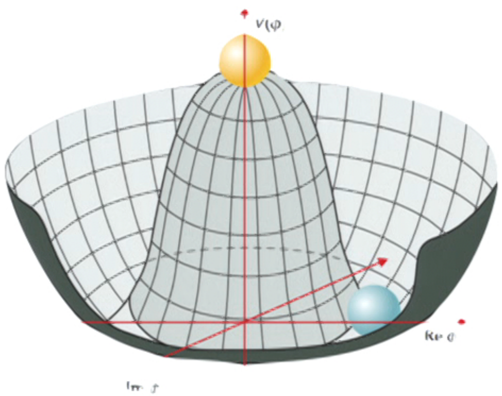
Ferromagnets

Sugar

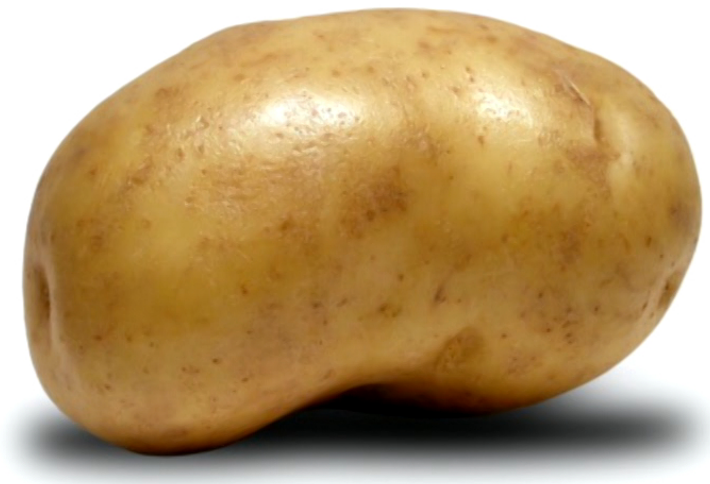
Goldstone Modes

- When **global continuous** symmetries are spontaneously broken, **Goldstone theorem** ensures existence of gapless modes

- Goldstone modes often dominate large distance / low energy behavior of the system



Coordinates are Goldstones



Coordinates + Euler angles = Goldstone modes of point-particle

Coordinates are Goldstones

Coordinates + Euler angles = Goldstone modes of point-particle

Point-like spinning objects

Relativistic effective theory

Effective action for relativistic spherical objects:

$$S = \int d\tau \left\{ -mc^2 + \frac{I}{2}\vec{\Omega}\cdot\vec{\Omega} + \frac{J}{4}(\vec{\Omega}\cdot\vec{\Omega})^2 + \cdots \right\}$$

$$\left(\Omega_i = \frac{1}{2}\epsilon_{ijk}\Lambda_\mu{}^j \frac{d}{d\tau}\Lambda^{\mu k} \right)$$

Expansion parameter: Ω/Ω_0

[Delacretaz Endlich Monin Penco Riva 14]

Higher-derivative terms

Encode finite-size effects

An object with a finite size will deform when spinning…

$$I\Omega^2 + J\Omega^4 \to (I + J\bar{\Omega}^2)\delta\Omega^2$$

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Dissipation

Large scales

$x^i, \theta^i, \phi, A_\mu, g_{\mu\nu}, \ldots$

Small scales

$\mathcal{O}, \mathcal{O}^i, \mathcal{O}^{ij}, \ldots$

We can encode dissipation by adding to the Lagrangian couplings that involve the $\mathcal{O}$'s:

$$S \supset \int d\tau \left\{ \mathcal{O}\phi + \mathcal{O}^i (\Lambda^{-1})_i{}^\mu \partial_\mu \phi + \cdots \right\}$$

[Goldberger Rothstein 06 / Porto 08 / …]

Absorption

of a scalar

- For simplicity, consider a spinning object with zero velocity.
- Interaction: $S_{\rm int} = \int dt\, \partial^i \phi\, R_i{}^j \mathcal{O}_j$
- Absorption process: $X_i + (\omega, \ell, m) \to X_f$
- Absorption probability:

$\langle \mathcal{O}_i \mathcal{O}_j \rangle \equiv \delta_{ij} \Delta$

$$P_{\rm abs} = \sum_{X_f} \frac{|\langle X_f; 0 | T e^{i S_{\rm int}} | X_i; \omega, \ell, m \rangle|^2}{\langle \omega, \ell, m | \omega, \ell, m \rangle} \simeq \frac{k^4 \delta_1^\ell}{6\pi\, v\, \omega} \Delta(\omega - m\Omega)$$

[Endlich Penco 16]

Spontaneous Emission

of a scalar

- Same object with zero velocity, same interaction.

- Spontaneous emission process: $X_i \to X_f + (\omega, \ell, m)$

- Spontaneous emission probability:

$$P_{\rm em} = \sum_{X_f} \frac{|\langle X_f; \omega, \ell, m | T e^{iS_{\rm int}} | X_i; 0 \rangle|^2}{\langle \omega, \ell, m | \omega, \ell, m \rangle} \simeq \frac{k^4 \delta_1^\ell}{6\pi \, v \, \omega} \, \Delta(m\Omega - \omega)$$

[Endlich Penco 16]

Matching

Absorption cross section of black hole at rest

- From a classical perspective, a black hole is a system in its ground state:

$$\Delta(\omega) = \theta(\omega)\rho(\omega)$$

Spectral Density

- Absorption cross section in the EFT when $\Omega = 0$:

$$\sigma_{\text{abs}}(\omega, \ell = 1) = \frac{3\pi P_{\text{abs}}}{k^2} = \frac{k^2}{2v\omega}\rho(\omega)$$

[Endlich Penco 16]

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$$\sigma_{\rm abs}(\omega, \ell = 1) = \frac{3\pi P_{\rm abs}}{k^2} \simeq \frac{k^2}{2v}\gamma$$

[Endlich Penco 16]

- Absorption cross section in General Relativity:

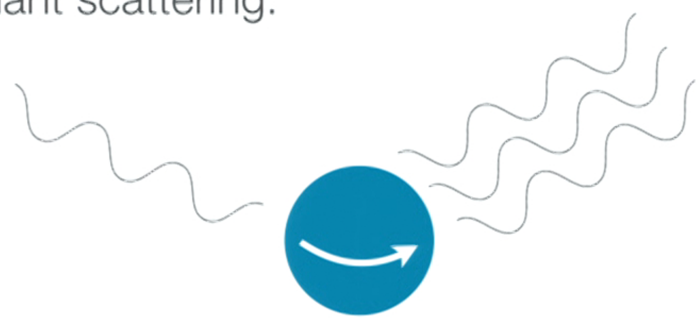
$$\sigma_{\rm abs}(\omega, \ell = 1) = \frac{\pi k^2 r_s^4}{3v} \longrightarrow \boxed{\gamma = \tfrac{2}{3}\pi r_s^4}$$

[Unruh 76]

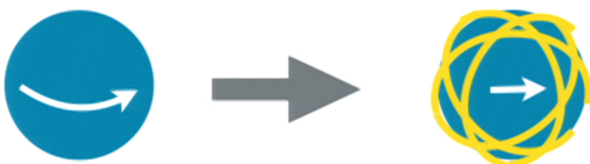
Superradiance

in a nutshell

- Superradiant scattering:



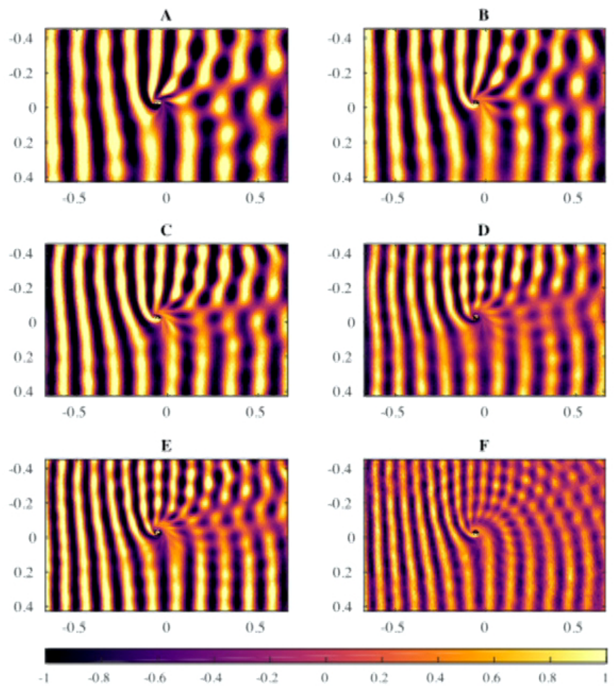
- Superradiant instability:

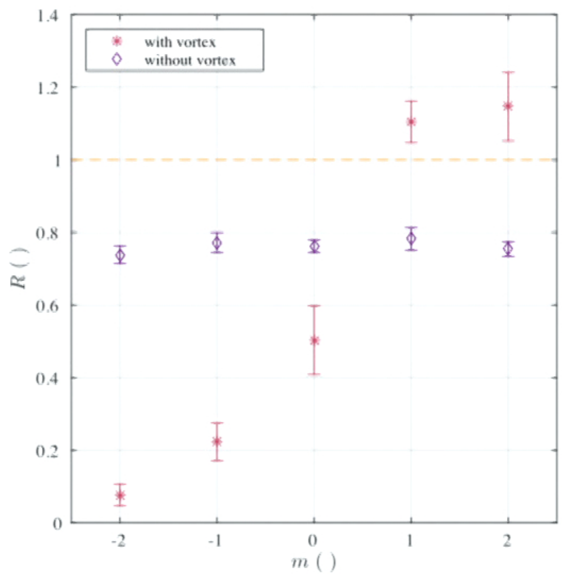


[Zel'dovich 71, 72 / Misner 72 / Press Teukolsy 72 / Starobinsky Churilov 73 / …]

Superradiant Scattering

First observed a year ago!





[Torres *et al.* Dec 16]

Superradiant Instability

Black holes as probes for axions

[Arvanitaki *et al.* 09, Arvanitaki Dubovski 10]
[ALSO SPIN 1: Baryakhtar Lasenby Teo 16, East Pretorius 17, East 17]

Superradiant Instability

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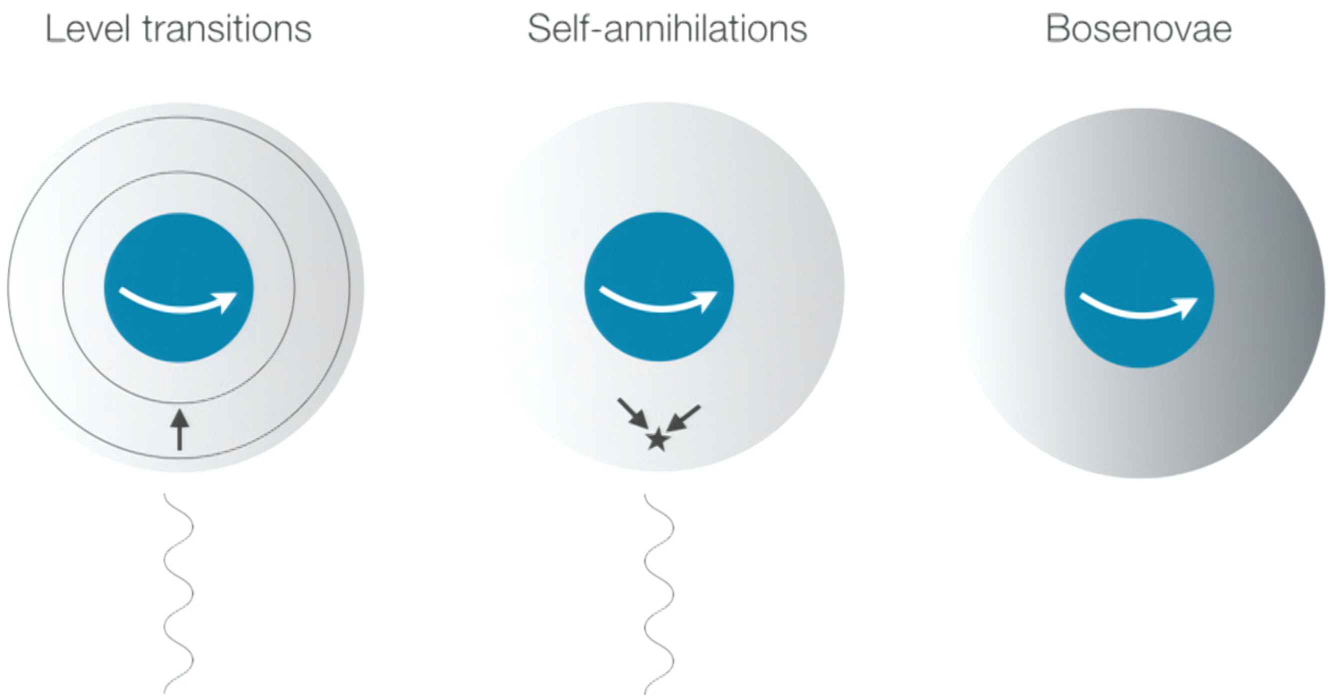
Superradiant Instability

Possible gravitational wave signals



[Arvanitaki Dubovski 10 / Yoshino Kodama 11,12 / Arvanitaki Baryakhtar Huang 14]

Superradiant Instability

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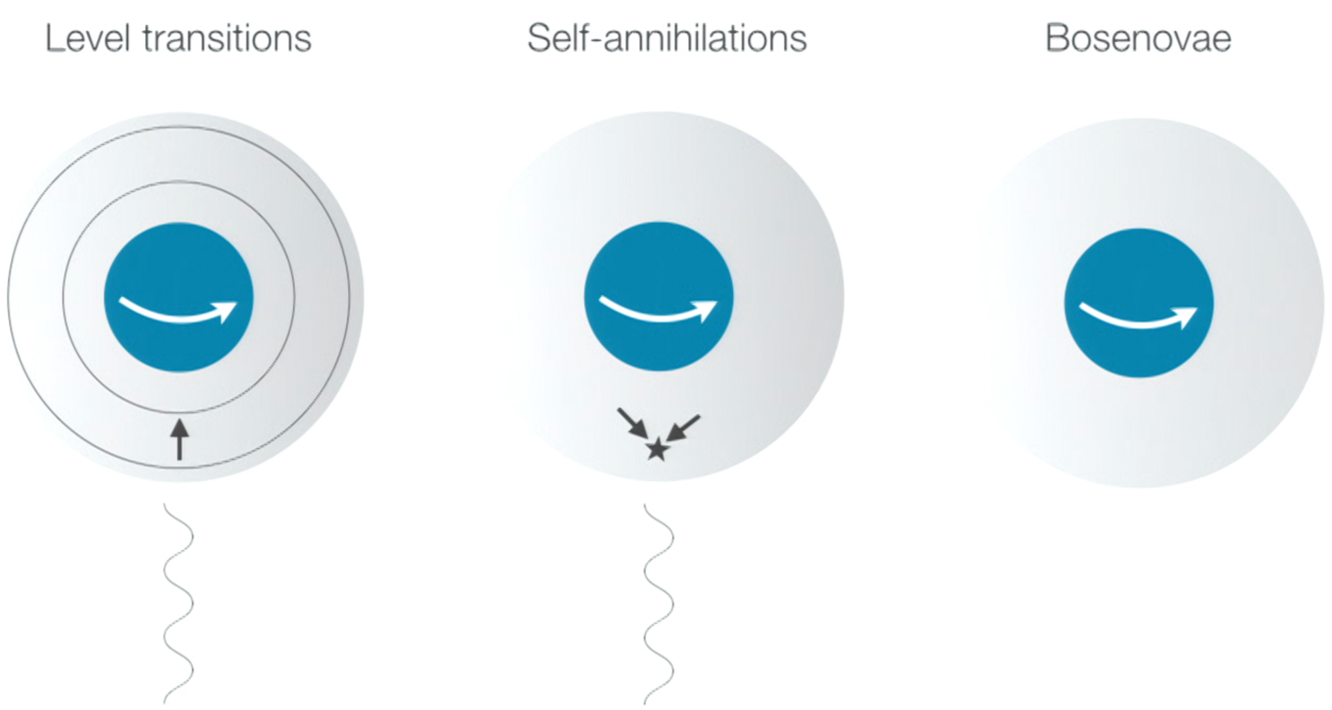
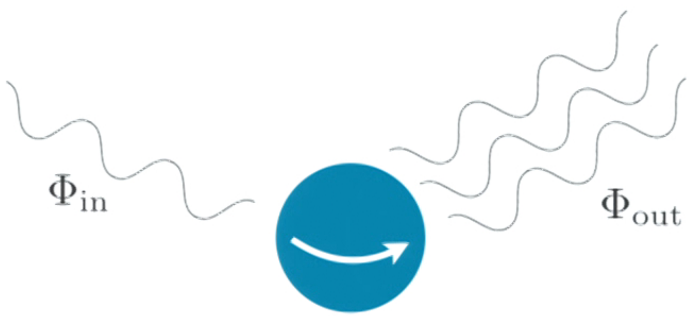


[Arvanitaki Dubovski 10 / Yoshino Kodama 11,12 / Arvanitaki Baryakhtar Huang 14]

Superradiant Scattering

of a scalar



$$\Phi_{\rm out} = \Phi_{\rm in}(1 - P_{\rm abs}) + \Phi_{\rm in} P_{\rm em}$$

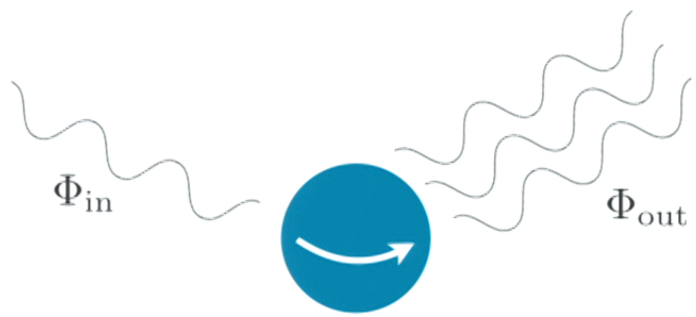
Absorption

Stimulated Emission

[Endlich Penco 16]

Superradiant Scattering

of a scalar

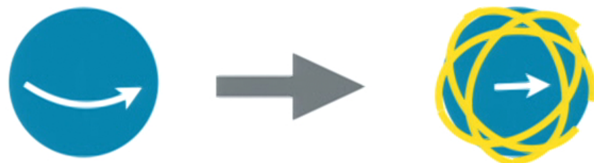


$$\frac{\Phi_{\rm out} - \Phi_{\rm in}}{\Phi_{\rm in}} = P_{\rm em} - P_{\rm abs} = -\frac{k^4}{6\pi\, v\, \omega}\, \rho(\omega - m\Omega)$$

[Endlich Penco 16]

Superradiant Instability

due to scalar bound states



$$X_i + (n, \ell, m) \to X_f \quad \text{vs} \quad X_i \to X_f + (n, \ell, m)$$

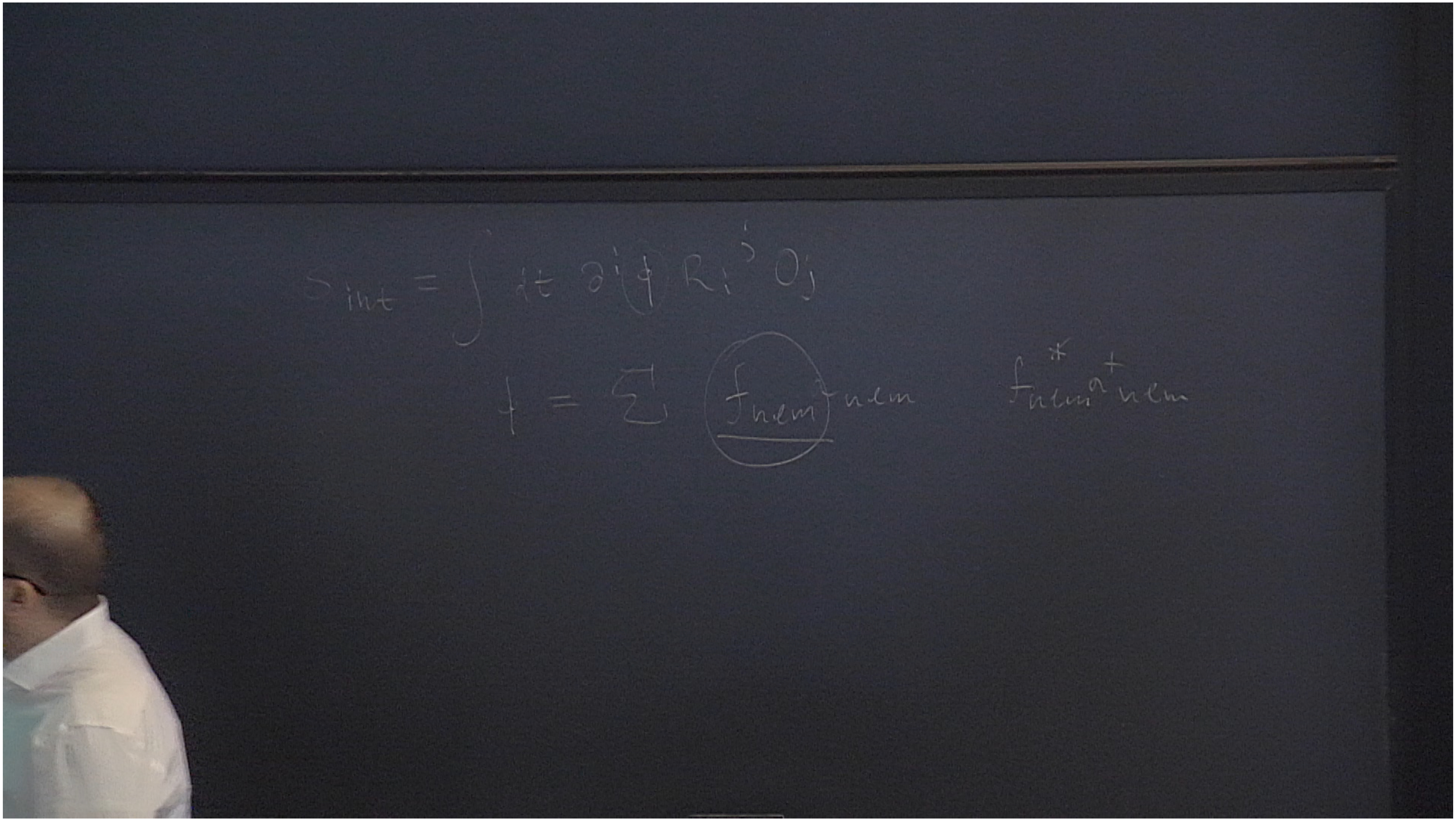
Absorption — Spontaneous Emission

$$\Gamma_{\text{em}} - \Gamma_{\text{abs}} \simeq \left(\frac{GM\mu^2}{2}\right)^5 \frac{\Omega\gamma}{2\pi\mu} \qquad (\ell = 1, m = 1)$$

For a Black Hole: $\gamma = \frac{2}{3}\pi r_s^4$

[Ternov Khalilov Chizov Gaina 78 / Detweiler 80]

[Endlich Penco 16]


S int = ∫ dt ∂i φ Ri j Oj
φ = Σ fnlm anlm
f*nlm a†nlm

A few comments

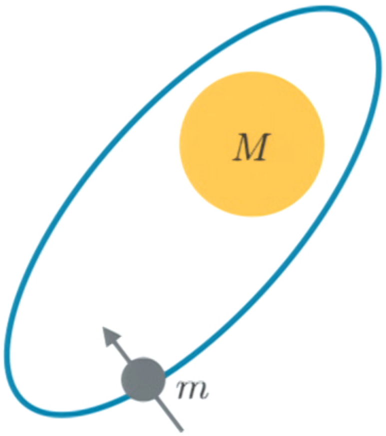
- This applies to any object in the long-wavelength limit
- Connects absorption, superradiant scattering / instability
- Matching needs to be done once, can be done for $\Omega = 0$
- Easily extended to higher multipoles / higher spins

e.g. **gravitational waves (spin 2)**: $\Delta L = \mathcal{O}_E^{ij}\tilde{C}_{0i0j} + \mathcal{O}_B^{ij}\tilde{C}_{0ijk}$

Tidal dynamics

In the non-relativistic limit, the effective Lagrangian is:

$$L = \frac{m\vec{v}^2}{2} - m\Phi + \frac{I\vec{\Omega}^2}{2} + \frac{1}{2}\partial_i\partial_j\Phi R^i{}_k R^j{}_l \mathcal{O}_E^{kl}$$



[Endlich Penco 15]

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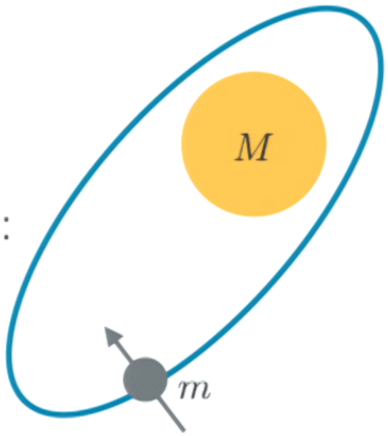
Assuming that $\vec{\Omega} \perp$ orbital plane, the eom's are:

$$m\dot{\vec{v}} = -\frac{GMm}{r^2}\hat{r} - \frac{9\gamma G^2 M^2(\dot{\theta} - \Omega)}{r^7}\hat{\theta}$$

$$I\dot{\Omega} = \frac{9\gamma G^2 M^2(\dot{\theta} - \Omega)}{r^6}$$

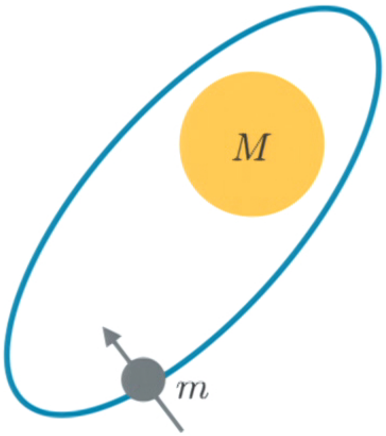


[Endlich Penco 15]

Tidal dynamics

Let's make the orbit slightly elliptical, tidally **un**locked:

$$r \simeq a(1 - e\cos\theta) \qquad\qquad \Omega \neq \dot{\theta}$$



These perturbations decay with **two** characteristic time scales:

$$T_e \propto \frac{ma^8}{\gamma G^2 M^2} \qquad\qquad T_\Omega \sim T_e \frac{I}{ma^2}$$

$$\frac{I}{ma^2} \sim \frac{\ell^2}{a^2} \ll 1$$

[Endlich Penco 15]

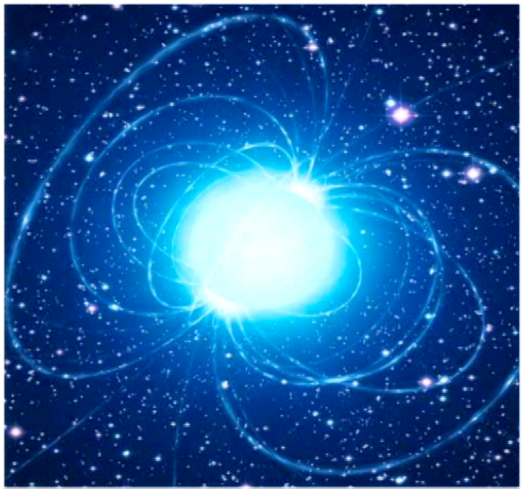
Our Moon agrees.

Superfluidity

Liquid Helium4, as temperature is lowered below 2.17K

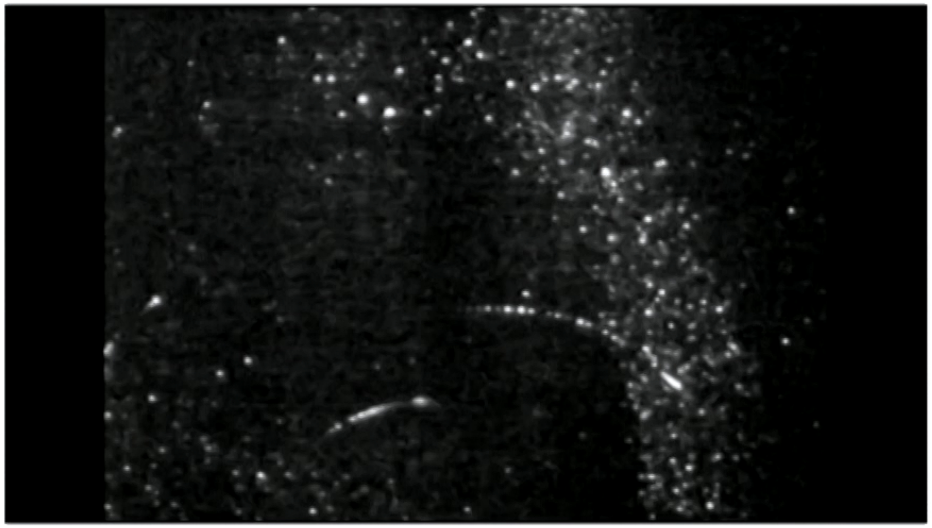
Superfluids in the Sky

From neutron stars…



… to galaxies.

Vortex Lines in Superfluids

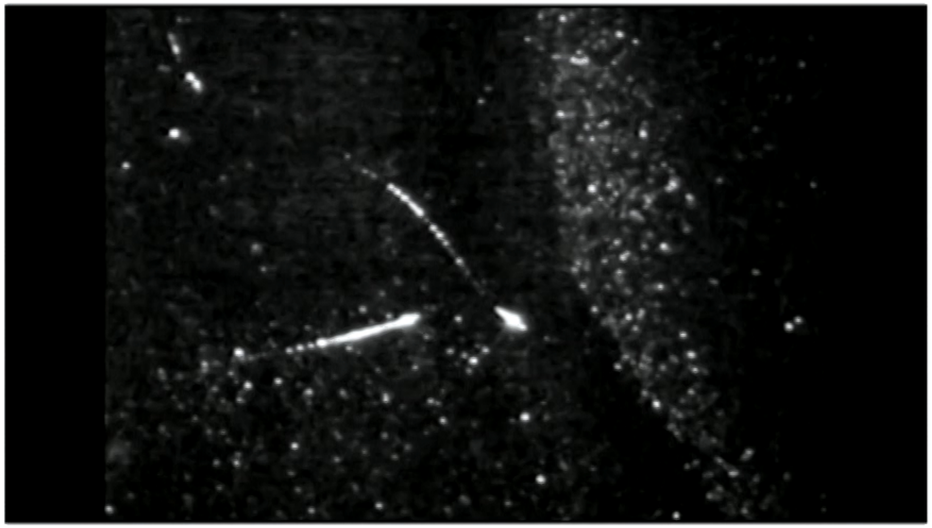


Despite appearances, vortex lines don't behave like usual strings.

[Horn Nicolis Penco 15]

[Video credit: E. Fonda, D. P. Meichle, N. T. Ouellette, S. Hormoz, K. R. Sreenivasan, D. P. Lathrop]

Vortex Lines in Superfluids



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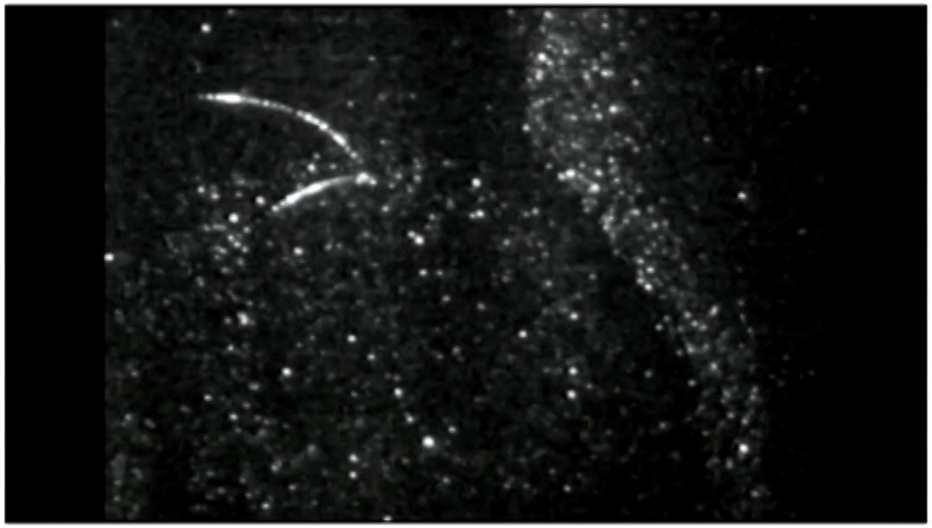

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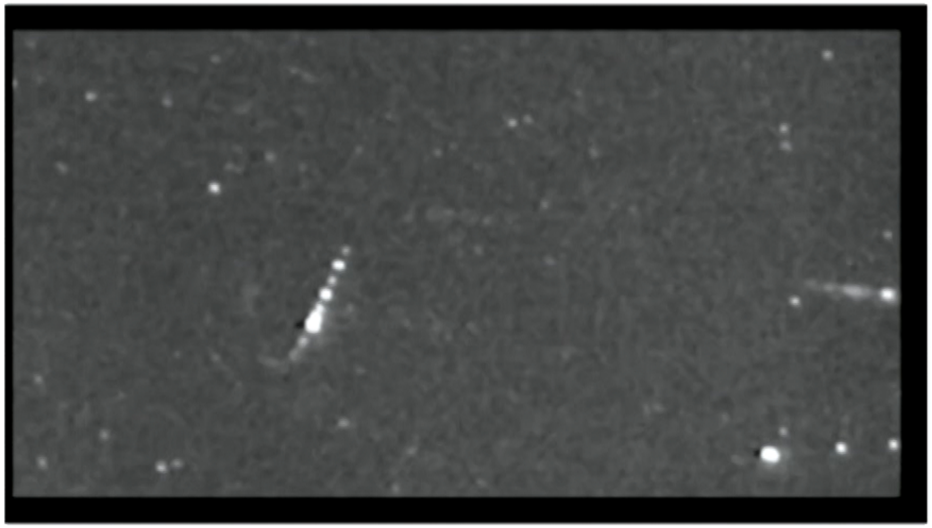

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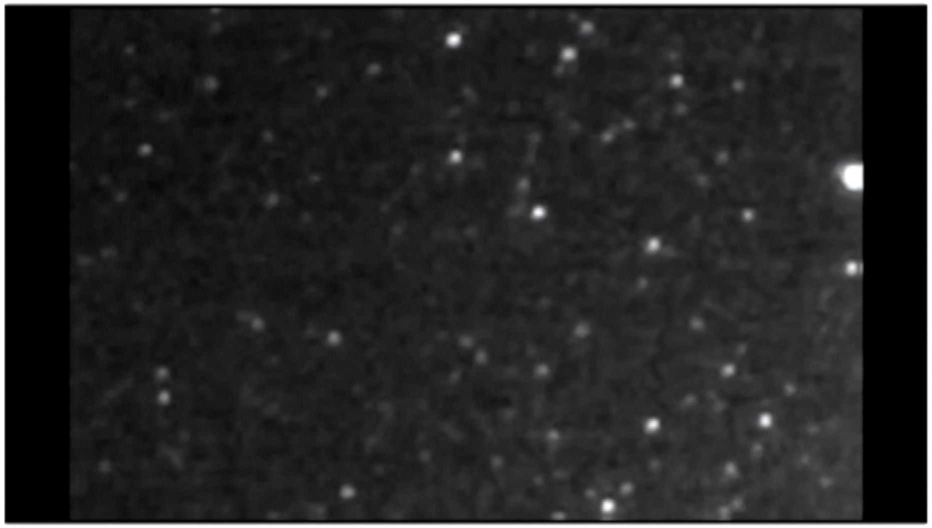
[Video credit: E. Fonda, D. P. Meichle, N. T. Ouellette, S. Hormoz, K. R. Sreenivasan, D. P. Lathrop]

$$S_{int} = \int dt\, \partial^i \phi\, R_i^{\ j}\, O_j \qquad \omega \sim k^2 \log k$$

$$\langle O^{ij}_E \, O^{kl}_E \rangle \qquad \phi = \sum f_{nlm}\, a_{nlm} \qquad f^*_{nlm}\, a^\dagger_{nlm}$$

$$\delta \quad \delta \quad \theta(\omega)\, \underbrace{\rho(\omega)}_{\sim \gamma \omega}$$

Vortex Lines in Superfluids



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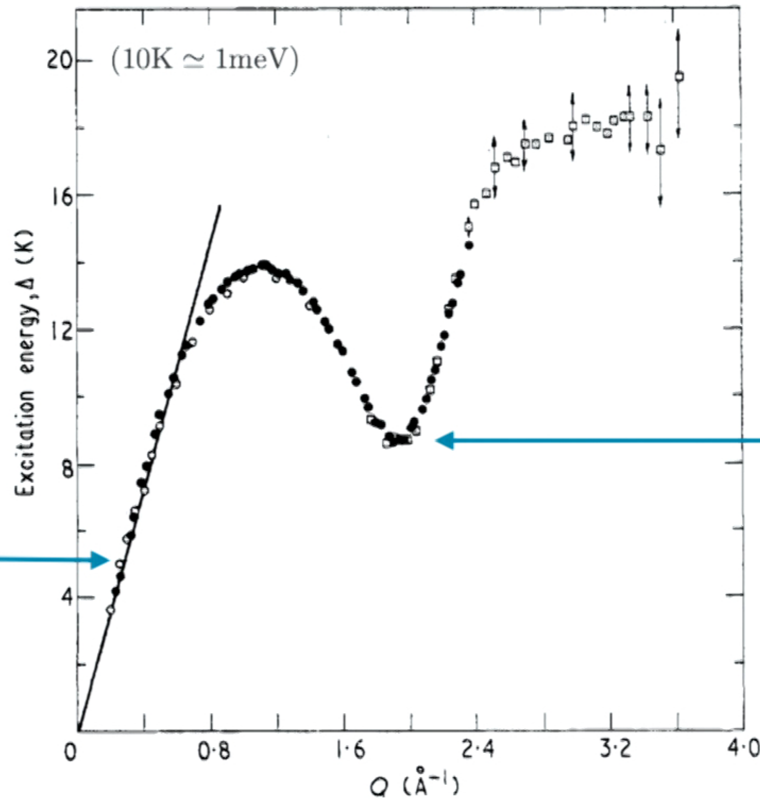
Collective Excitations in He4



Phonons

$E \simeq c_s p$

Rotons

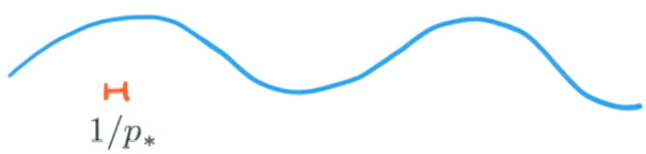
$$E \simeq \Delta + \frac{(p - p_*)^2}{2m_*}$$

[Cowley Woods 71]

Phonons vs Rotons

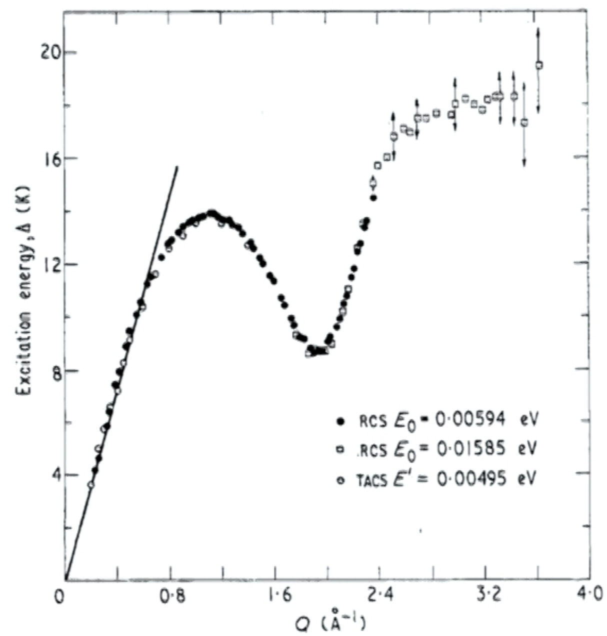
Phonons are Goldstone modes: we can write an effective theory valid for soft momenta $p \ll 1/a$

Rotons are not, and have a typical momentum $p_* \sim 1/a$



IDEA: treat rotons as point-like particles

[Nicolis Penco 17]



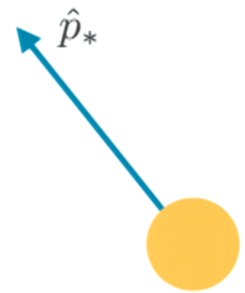
What is the difference between a ***black hole*** *seen from far away and a* ***roton****?*

Goldstone Modes of Rotons

Position: 3 coordinates

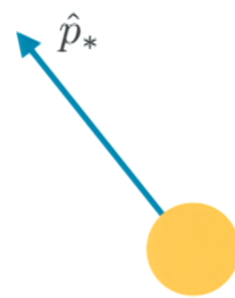
Direction of momentum: 2 angles

Goldstone modes = $\vec{x},\ \hat{p}$



[Nicolis Penco 17]

Goldstone Modes of Rotons



Position: 3 coordinates

Direction of momentum: 2 angles

Goldstone modes = $\vec{x}, \; \hat{p}$

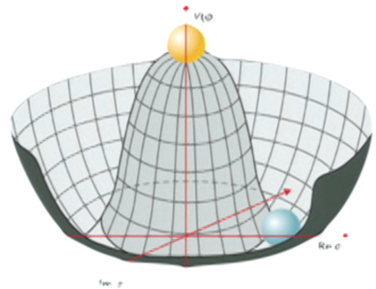
Heuristic derivation of effective Lagrangian:

$$H = \Delta + \frac{(p - p_*)^2}{2m_*} \longrightarrow L = -\Delta + p_* \dot{\vec{x}} \cdot \hat{p} + \frac{m_*}{2} (\dot{\vec{x}} \cdot \hat{p})^2$$

[Nicolis Penco 17]

What about Phonons?

- Superfluid = particle number is
    1. at finite density
    2. spontaneously broken



→ Goldstone = phonon

[Greiter Wilczek Witten 89, Son Wingate 05]
[Nicolis Penco Piazza Rosen 13, Nicolis Penco Rosen 14]

Coupling Rotons to Phonons

$$S = \int dt \left[-\Delta + p_* \dot{\vec{x}} \cdot \hat{p} + \frac{m_*}{2} (\dot{\vec{x}} \cdot \hat{p})^2 + \cdots \right]$$

$$S = \int dt \left\{ -\Delta(X) + p_*(X)(\dot{\vec{x}} - \vec{u}) \cdot \hat{p} + \frac{m_*(X)}{2} [(\dot{\vec{x}} - \vec{u}) \cdot \hat{p}]^2 + \cdots \right\}$$

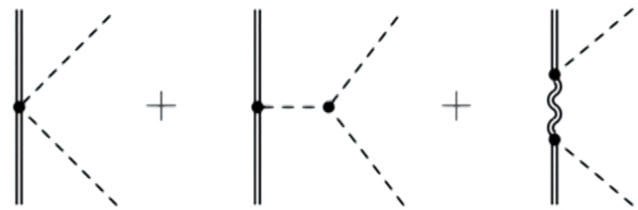
$$X = \mu/m + \dot{\pi} - \tfrac{1}{2}(\nabla\pi)^2$$

$$\vec{u} = -\vec{\nabla}\pi$$

[Nicolis Penco 17]

Phonon-Roton Scattering

- At finite temperatures, thermally excited phonons+rotons form a **second fluid** [Tisza 40, Landau 41]
- Phonon-roton scattering contributes to its viscosity [Landau Khalatnikov 49]
- Scattering amplitude:

$i\mathcal{M} =$ 

[Nicolis Penco 17]

- Scattering cross section: $\dfrac{d\sigma}{d\Omega} = \dfrac{|\mathcal{M}|^2}{16\pi^2 c_s^4}$

Phonon-Roton Scattering

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- $$i\mathcal{M}\big|_{v_0=0} = - \frac{i c_s p_* k^2}{\rho}\Bigg\{(\hat{k}_i \cdot \hat{k}_f)(\hat{k}_i + \hat{k}_f)\cdot \hat{p}_0 + \frac{p_*}{m_* c_s}(\hat{k}_i \cdot \hat{p}_0)^2(\hat{k}_f \cdot \hat{p}_0)^2 + \frac{\rho^2}{c_s p_*}\left[\frac{d^2\Delta}{d\rho^2} + \frac{1}{m_*}\left(\frac{dp_*}{d\rho}\right)^2\right] + B \cdot \frac{\rho}{p_* c_s}\frac{d\Delta}{d\rho} \boxed{- \frac{\rho}{m_* c_s}\frac{dp_*}{d\rho}\left[(\hat{k}_i \cdot \hat{p}_0)^2 + (\hat{k}_f \cdot \hat{p}_0)^2\right]}\Bigg\},$$

  where

  $$B \equiv 1 - 2\frac{\rho}{c_s}\frac{dc_s}{d\rho} + (\hat{k}_i \cdot \hat{p}_0)(\hat{k}_f \cdot \hat{p}_0)\left[2 + \frac{\rho}{m_* c_s^2}\frac{d\Delta}{d\rho} + \frac{p_*}{m_* c_s}(\hat{k}_i + \hat{k}_f)\cdot \hat{p}_0\right] - \frac{\rho}{m_* c_s}\frac{dp_*}{d\rho}(\hat{k}_i + \hat{k}_f)\cdot \hat{p}_0 \, .$$

[Nicolis Penco 17]
[Castin Sinatra Kurkjian 17]

*Do rotons **float** or **sink**?*

Rotons in a gravitational field

- Roton action w/out phonons:

$$S = \int dt \left\{ -\Delta(\mu/m) + p_*(\mu/m)\dot{\vec{x}} \cdot \hat{p} + \frac{m_*(\mu/m)}{2}(\dot{\vec{x}} \cdot \hat{p})^2 + \cdots \right\}$$

- Coupling to gravity shifts the energy/particle: $\mu/m \to \mu/m - \Phi$

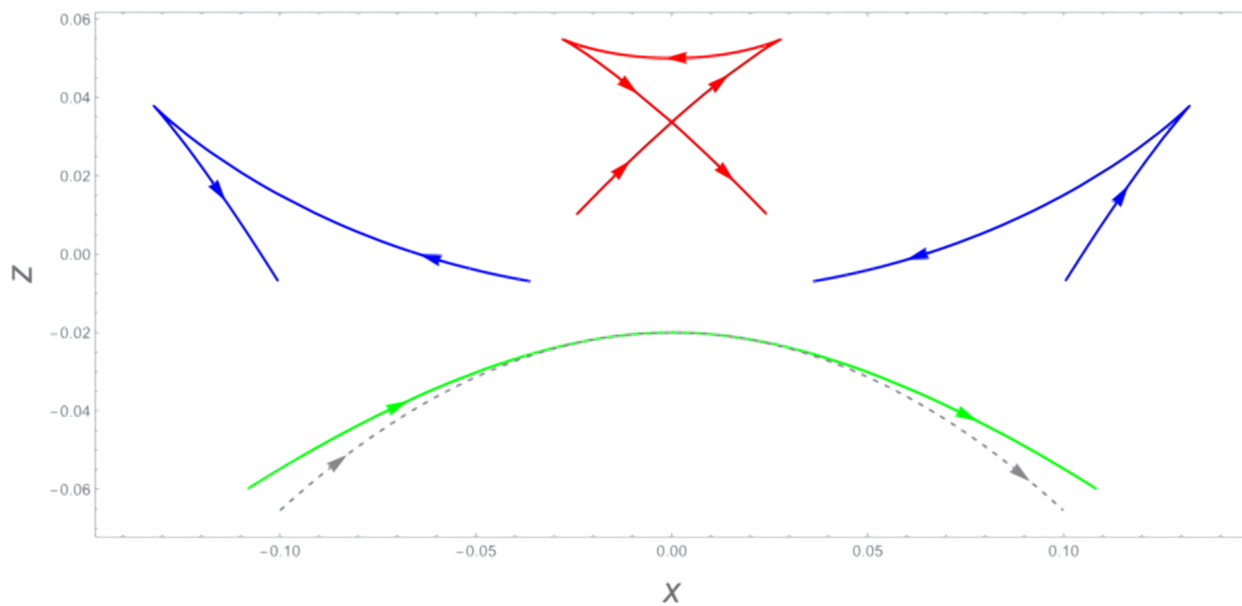
- Gravitational mass of rotons: $m_{\rm g} = -\Delta' = -\rho \dfrac{d\Delta}{dP} > 0$

- Non-trivial relation between momentum and velocity gives rise to interesting trajectories

[Nicolis Penco 17]

Rotons sink

(eventually)



(btw, phonons float instead)

[Nicolis Penco 17]

Final Remarks

- Effective field theories are a very flexible and powerful tool
- Allow us to connect seemingly different phenomena
- “Point-like” black holes: Maximal spin? AdS/dS?
- Rotons: relativistic extension, neutron stars?


Thank you,
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