Title: PSI 17/17 - Quantum Field Theory III - Lecture 10

Date: Feb 09, 2018 09:00 AM

URL: http://pirsa.org/18020099

Abstract:

Recap

D=2 CT

$z \to f(z) \quad \bar{z} \to \bar{f}(\bar{z})$

$[l_n, l_m] = (n-m)\, l_{n+m}$

global CT $f(z) = \frac{az+b}{cz+d}$ $\quad ad-bc=1$

$\frac{SL(2,\mathbb{C})}{\mathbb{Z}_2} \simeq SO(3,1)$

chiral (quasi-)primary field

primary $\tilde{\Phi}_h(f(z)) = (\partial f)^{-h}\, \Phi(z)$

$T^\mu{}_\mu = 0 \quad \partial_\mu T^{\mu\nu} = 0$

chiral

$T_{\mu\nu} \propto \begin{pmatrix} T(z) & 0 \\ 0 & \bar{T}(\bar{z}) \end{pmatrix}$

anti-chiral

goal: Virasoro Algebra

commutators of generators.

$\updownarrow$

operator product

$\downarrow$

OPE

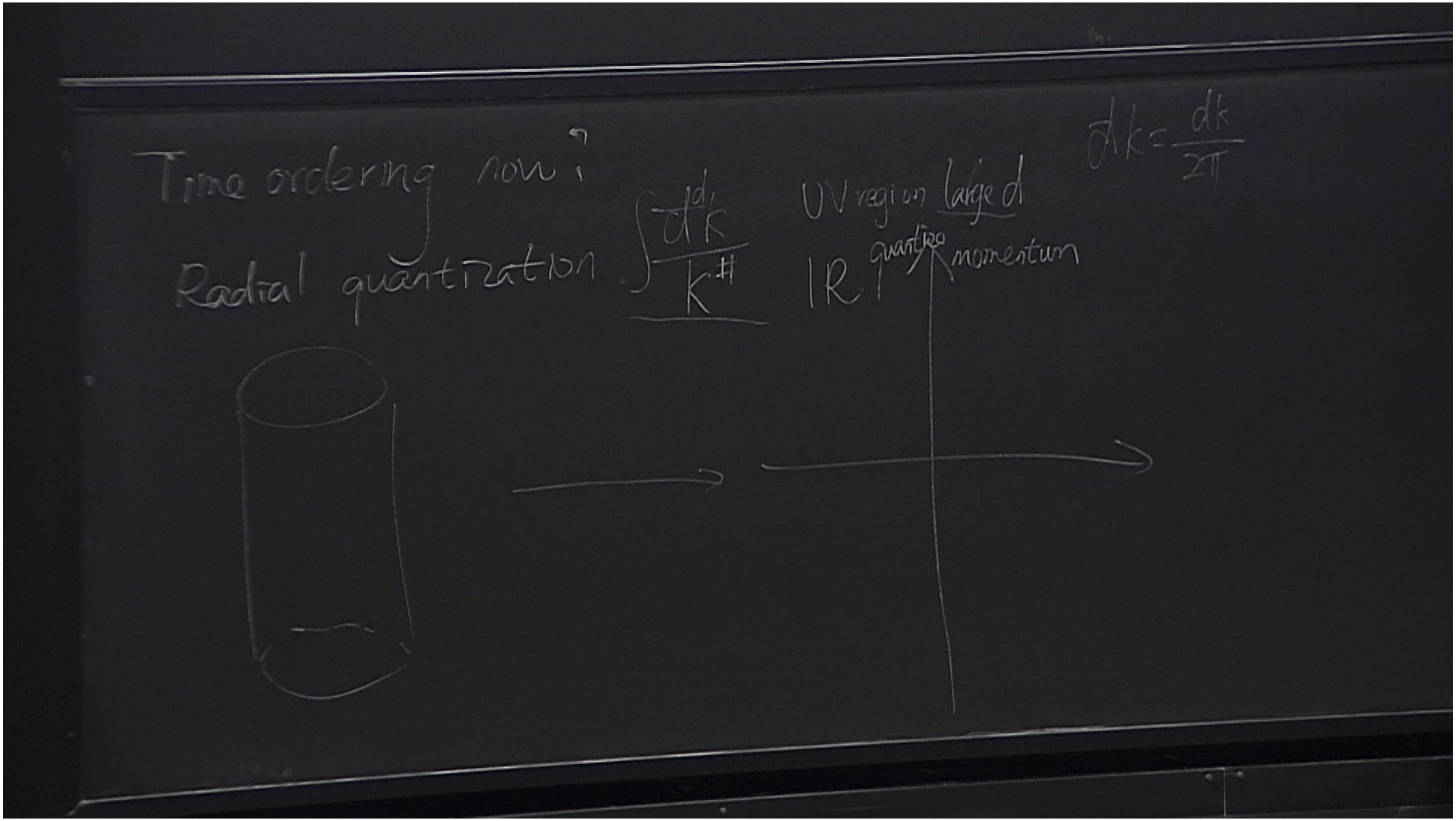
$$\delta\Phi = \sum_n \epsilon_n G_n \Phi$$

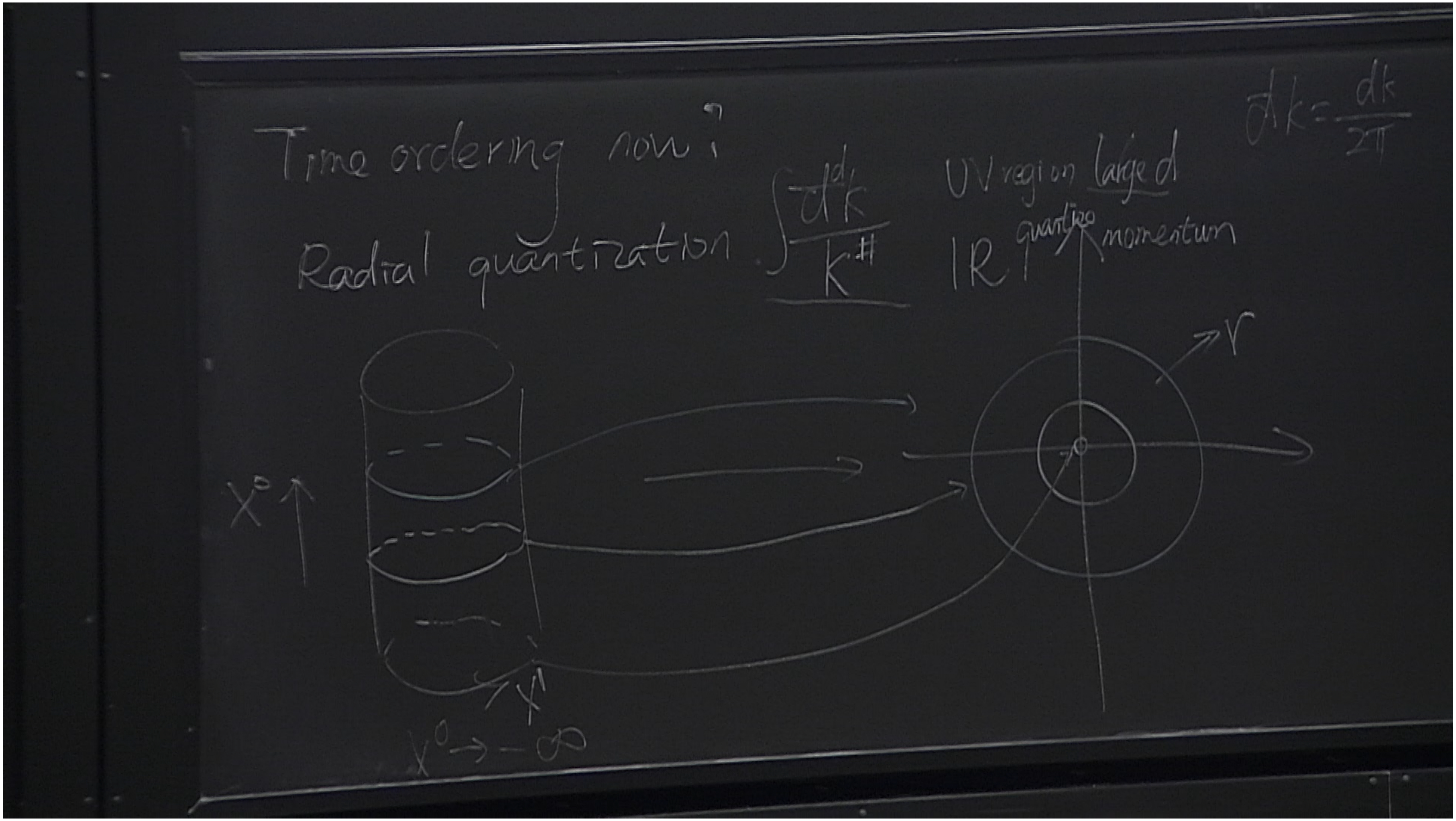
$$\parallel$$

$$\tilde{\Phi}(z) - \Phi(z)$$

Road map

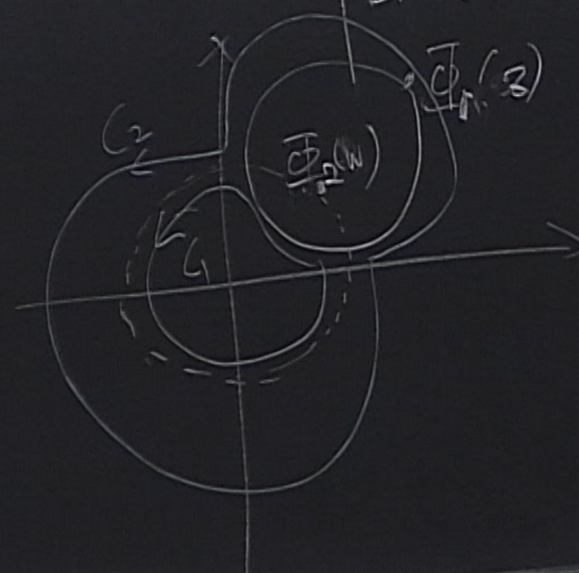
Virasoro Algebra


Time ordering now?
Radial quantization
$\int \frac{d^d k}{k^{\#}}$
UV region large d
IR
momentum
$đk = \frac{dk}{2\pi}$


Time ordering now?
Radial quantization
$\int \frac{d^d k}{k^{\#}}$
UV region large d
IR
momentum
$đk = \frac{dk}{2\pi}$
x^0
x^1
r
$x^0 \to -\infty$

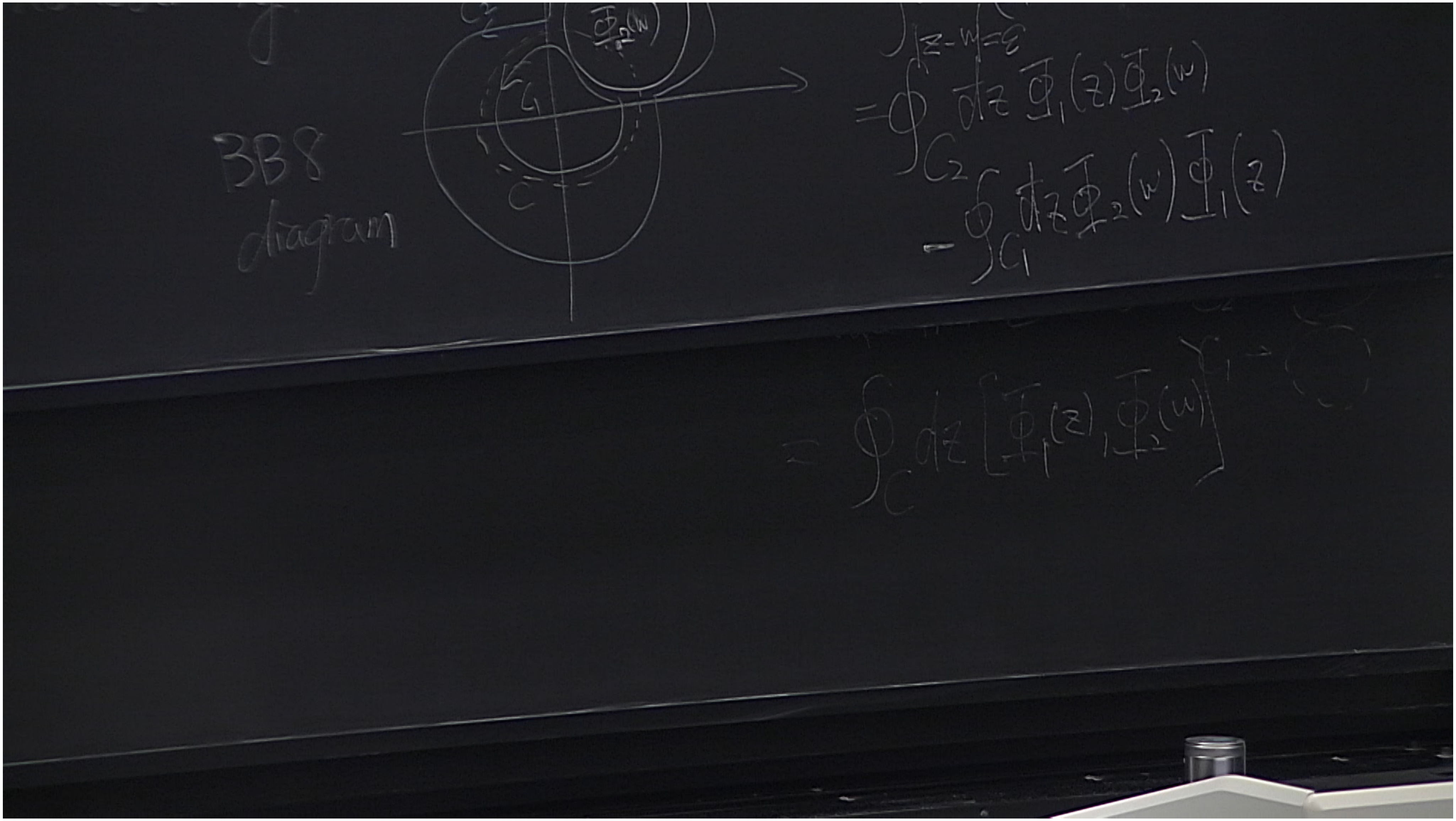
$$R\big(\Phi_1(z)\Phi_2(w)\big) \equiv \begin{cases} \Phi_1(z)\Phi_2(w) & \text{if } |z| \geq |w| \\ \Phi_2(w)\Phi_1(z) & \text{if } |z| < |w| \end{cases}$$

monodromy.



$$\oint dz\, R\big(\Phi_1(z)\Phi_2(w)\big)$$

$$= \oint_{C_2} dz\, \Phi_1(z)\Phi_2(w) - \oint_{C_1} dz\, \Phi_2(w)\Phi_1(z)$$


BB8
diagram
C_1
C
$\Phi_2(w)$
$= \oint_{C_2} dz\, \Phi_1(z)\Phi_2(w)$
$- \oint_{C_1} dz\, \Phi_2(w)\Phi_1(z)$
$= \oint_{C} dz\, [\Phi_1(z), \Phi_2(w)]$

goal: Virasoro Algebra

commutators of generators.

$$\delta \Phi = \sum_n a_n G_n \Phi$$

$$\parallel$$

$$\tilde{\Phi}(z) - \Phi(z)$$

$\updownarrow$ ✓

operator product

$\downarrow$

OPE

Road map

Virasoro Algebra

Generators:

$$\delta\Phi = [Q, \Phi]$$

$$\partial_\mu j^\mu = 0$$

$$Q = \int dx^1 j^0$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$j^\nu = \epsilon_\mu T^{\mu\nu}$$

$$\downarrow$$

$$Q = -\oint dz\, \epsilon(z)\, T(z)$$

$$\delta\Phi = -\oint_C dz \left[\epsilon(z)\, T(z), \Phi(w) \right]$$

$$= -\oint_{C'} dz\, \epsilon(z)\, T(z)\, \Phi(w)$$

$$\epsilon(z) = \sum_n a_n z^{n+1}$$

take ϵ_n

$$-\oint_{C'} dz\, a_n z^{n+1}\, T(z)\, \Phi(w)$$

Math Integral

$$\oint dz \frac{f(z)}{(z-z_0)^{n+1}} = f^{(n)}(z_0)$$

$$\delta\Phi = -\oint_C dz \left[\epsilon(z) T(z), \Phi(w)\right]$$

$$= -\oint_{C'} dz \, \epsilon(z) T(z) \Phi(w)$$

$$\epsilon(z) = \sum_n a_n z^{n+1}$$

take ϵ_n

$$-\oint_{C'} dz \, a_n z^{n+1} T(z) \Phi(w)$$

$$= -a_n L_n \Phi(w)$$

$$\delta\Phi = \sum_n a_n G_n \Phi$$

$$T(z) = \sum_m \frac{L_m}{z^{m+2}}$$

$$\delta \Phi = \sum_n a_n G_n \Phi$$

$$T(z) = \sum_m \frac{L_m}{z^{m+2}} \Longleftrightarrow L_m = \oint dz\, T(z)\, z^{m+1}$$

$$[L_n, L_m]$$

Commutators of . generators

$\updownarrow$

operator product

$\updownarrow$

OPE.

$= - a_n$

OPE

$$\langle O_1(z)\, O_2(w) \cdots \rangle = \sum_k \langle C_k(z-w)\, O_k(w) \cdots \rangle$$

primary $\Phi(z)$

$$= - a_n L_n \Phi(w)$$

$$O_k(w) \ldots \rangle$$

$$T(z)\Phi(w) \, ?$$

$$\delta \Phi = \delta \Phi$$

$$\tilde{\Phi}(f(z)) = (\partial f)^{-h} \Phi(z)$$

$$\delta \Phi = \tilde{\Phi}(z) - \Phi(z)$$

$a_n L_n \Phi(w)$

$T(z)\Phi(w)$?

$$\delta\Phi = \delta\Phi$$

$$\tilde{\Phi}(f(z)) = (\partial f)^{-h}\Phi(z) \underset{f = z+\epsilon}{=} \tilde{\Phi}(z) + \epsilon(z)\,\partial\tilde{\Phi}(z)$$

$$\delta\Phi = \tilde{\Phi}(z) - \Phi(z) \qquad f(z) = z + \epsilon(z),$$

$$T(z)\Phi(w)$$

$$L_n \Phi(w)$$

$T(z)\Phi(w)$?

$$\delta\Phi = \delta\Phi$$

$$\tilde{\Phi}(f(z)) = (\partial f)^{-h}\Phi(z) \xlongequal{f=z+\epsilon} \tilde{\Phi}(z) + \epsilon(z)\partial\tilde{\Phi}(z) = (1+\partial\epsilon)^{-h}\Phi(z)$$

$$\delta\Phi = \tilde{\Phi}(z) - \Phi(z) \qquad f(z) = z+\epsilon(z), \qquad \tilde{\Phi}(z) + \epsilon(z)\partial\Phi(z) = (1-h\partial\epsilon)\Phi(z)$$

$$= -\epsilon(z)\partial\Phi(z) - h\partial\epsilon(z)\Phi(z)$$

Time ordering now?

Radial quantization $\int \frac{d^d k}{k^{\#}}$

UV region large d

$đk = \frac{dk}{2\pi}$

$R(\Phi$

$T(z)\,\Phi(w) = \frac{\partial\Phi(w)}{z-w} + \frac{h\,\Phi(w)}{(z-w)^2} + \cdots$ monod

$-\epsilon(w)\,\partial\Phi(w)$

$= -\oint dz\, \frac{\epsilon(z)}{z-w}\,\partial\Phi(w)$

$-h\,\partial\epsilon(w)\,\Phi(w)$

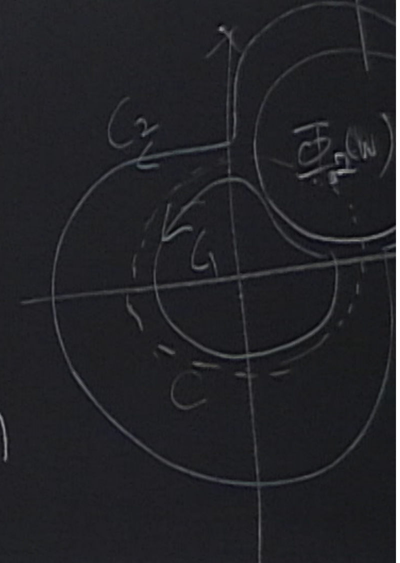
$= -\oint dz\, \frac{\epsilon(z)}{(z-w)^2}\,\Phi(w)$

$$T(z)\,\Phi(w) = \frac{\partial\Phi(w)}{z-w} + \frac{h\,\Phi(w)}{(z-w)^2} + \ldots\ldots$$ monodromy

def of primary field

BB8

$$T(z)\,T(w) = \frac{\partial T(w)}{(z-w)} + \frac{2\,T(w)}{(z-w)^2} + \frac{c/2}{(z-w)^4} + \ldots$$ diagram



$$= -a_n L_n \Phi(w)$$

$$[L_n, L_m] = \left[\oint dz\, z^{n+1} T(z), \oint dw\, w^{m+1} T(w)\right]$$

$$= \oint dw\, w^{m+1} \oint dz\, z^{n+1} \left(\frac{\partial T(w)}{z-w} + \frac{2T(w)}{(z-w)^2} + \frac{c/2}{(z-w)^4} \right)$$

$$= \oint dw\, w^{m+1} \left(\frac{c\,(n+1)n(n-1)\,w^{n-2}}{12} + w^{n+1}\partial T(w) + (n+1)\, w^{n}\, 2T(w) \right)$$

$$z\, z^{n+1} T(z), \oint dw\, w^{m+1} T(w)$$

$$z^{n+1}\left(\frac{\partial T(w)}{z-w} + \frac{2T(w)}{(z-w)^2} + \frac{c/2}{(z-w)^4}\right)$$

$$\frac{\ldots 1)n(n-1)\, w^{n-2}}{2} + w^{n+1}\partial T(w) + (n+1)\, w^n\, 2T(w)\Big)$$

$$(n-1)\, w^{m+n-1} + \oint dw\, w^{n+m+2}\,\partial T(w) + \oint 2(n+1)\, w^{n+m+1}\, T(w)$$

$$= \frac{c}{12} n(n+1)(n-1)\, \delta_{n+m,0}$$

$$- (n+m+2) \oint dw\, w^{n+m+1} T(w)$$

$$+ 2(n+1) \oint dw\, w^{n+m+1} T(w)$$

$$2T(w))$$

$$+ \oint 2(n+1) w^{n+m+1} T(w)$$

$$[L_n, L_m] = \frac{c}{12} n(n+1)(n-1)\, \delta_{n+m,0} + (n-m) L_{n+m}$$

$$= \frac{c}{12} n(n+1)(n-1)\, \delta_{n+m,0}$$

$$- (n+m+2) \oint dw\, w^{n+m+1} T(w)$$

$$+ 2(n+1) \oint dw\, w^{n+m+1} T(w)$$

$$[L_n, L_m] = \frac{c}{12} n(n+1)(n-1)\, \delta_{n+m,0} + (n-m) L_{n+m}$$

$$2T(w))$$

$$+ \oint 2(n+1) w^{n+m+1} T(w)$$