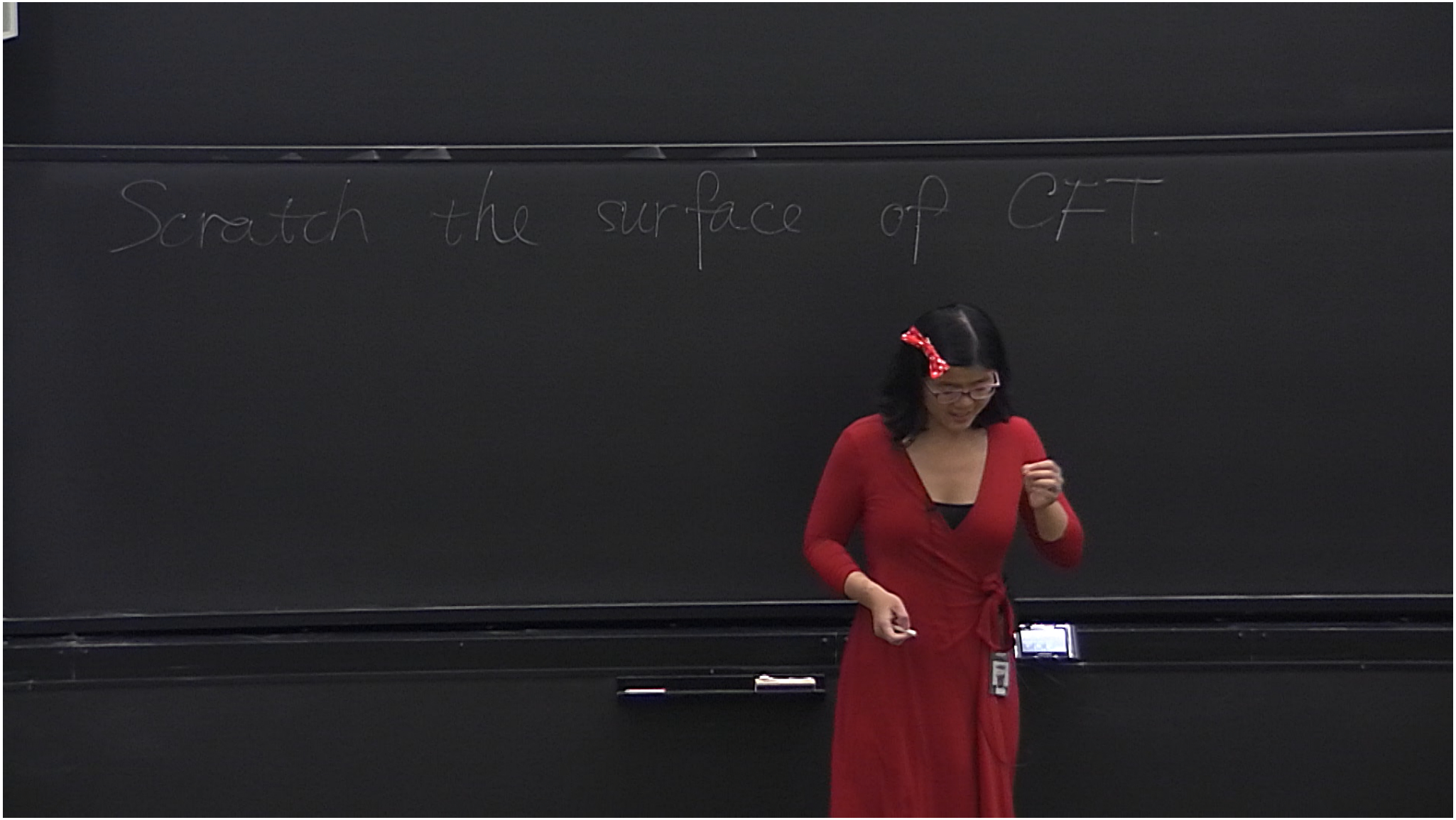
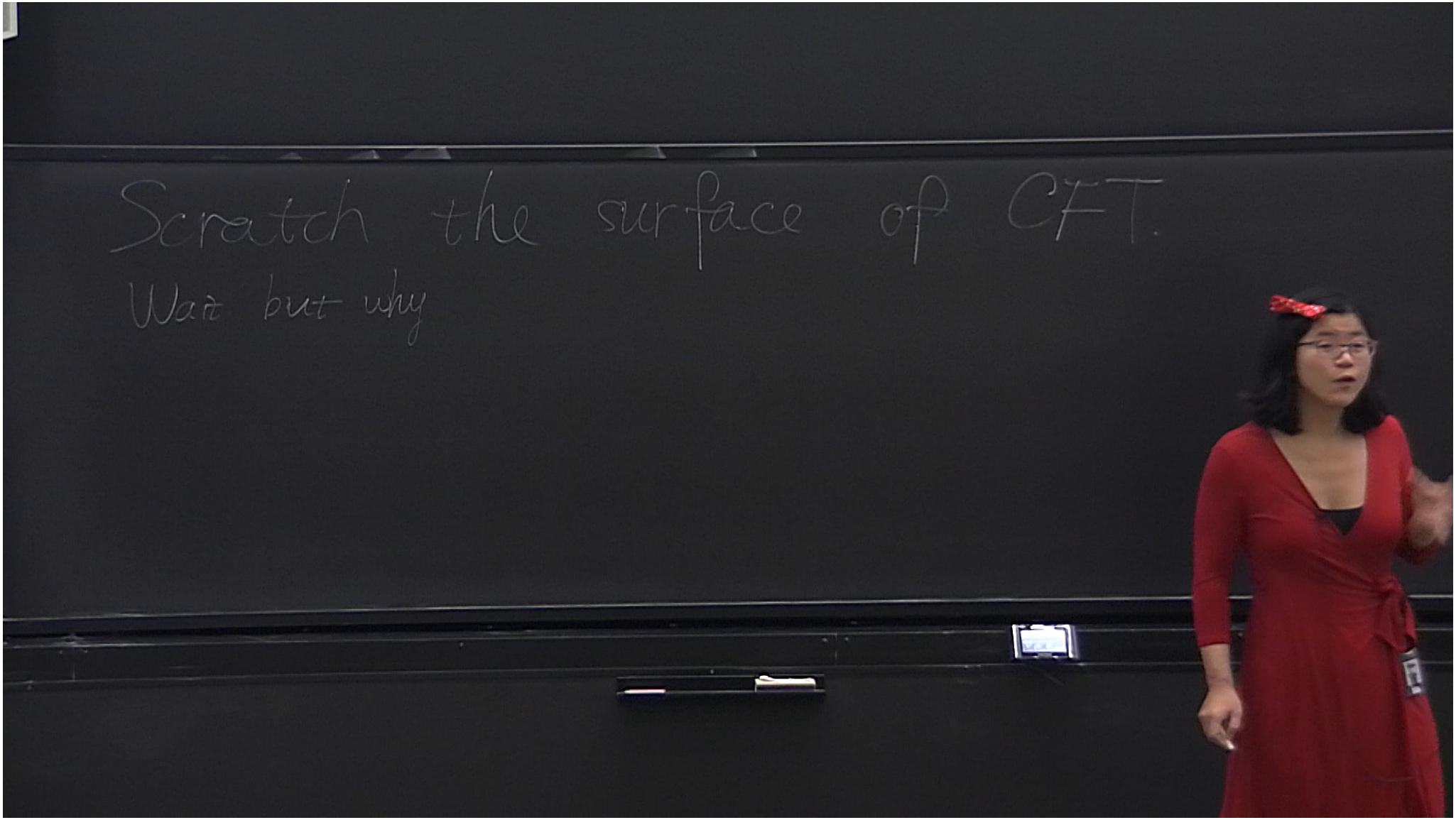
Title: PSI 17/18 - Quantum Field Theory III - Lecture 8

Date: Feb 08, 2018 09:00 AM

URL: http://pirsa.org/18020098

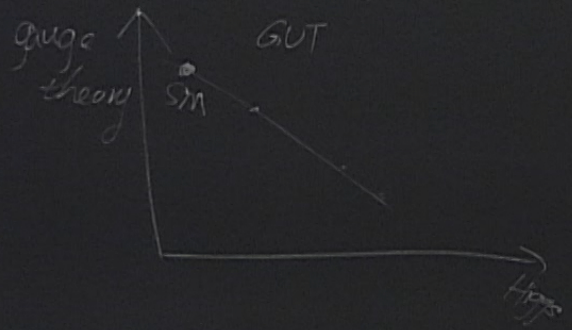
Abstract:


Scratch the surface of CFT.


Scratch the surface of CFT.
Wait but why

Scratch the surface of CFT.

Wait but why



MSSM

→ NMSSM

→ N^2MSSM

→ N^nMSSM

Scratch the surface of CFT.

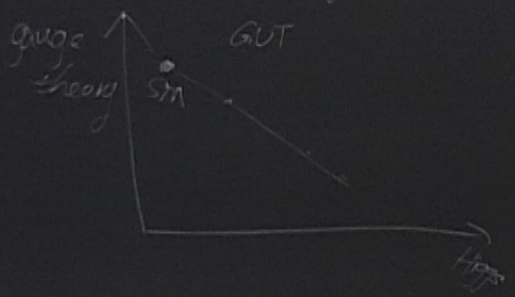
Wait but why

MSSM

→ NMSSM

→ N^2MSSM

→ N^nMSSM



d=2

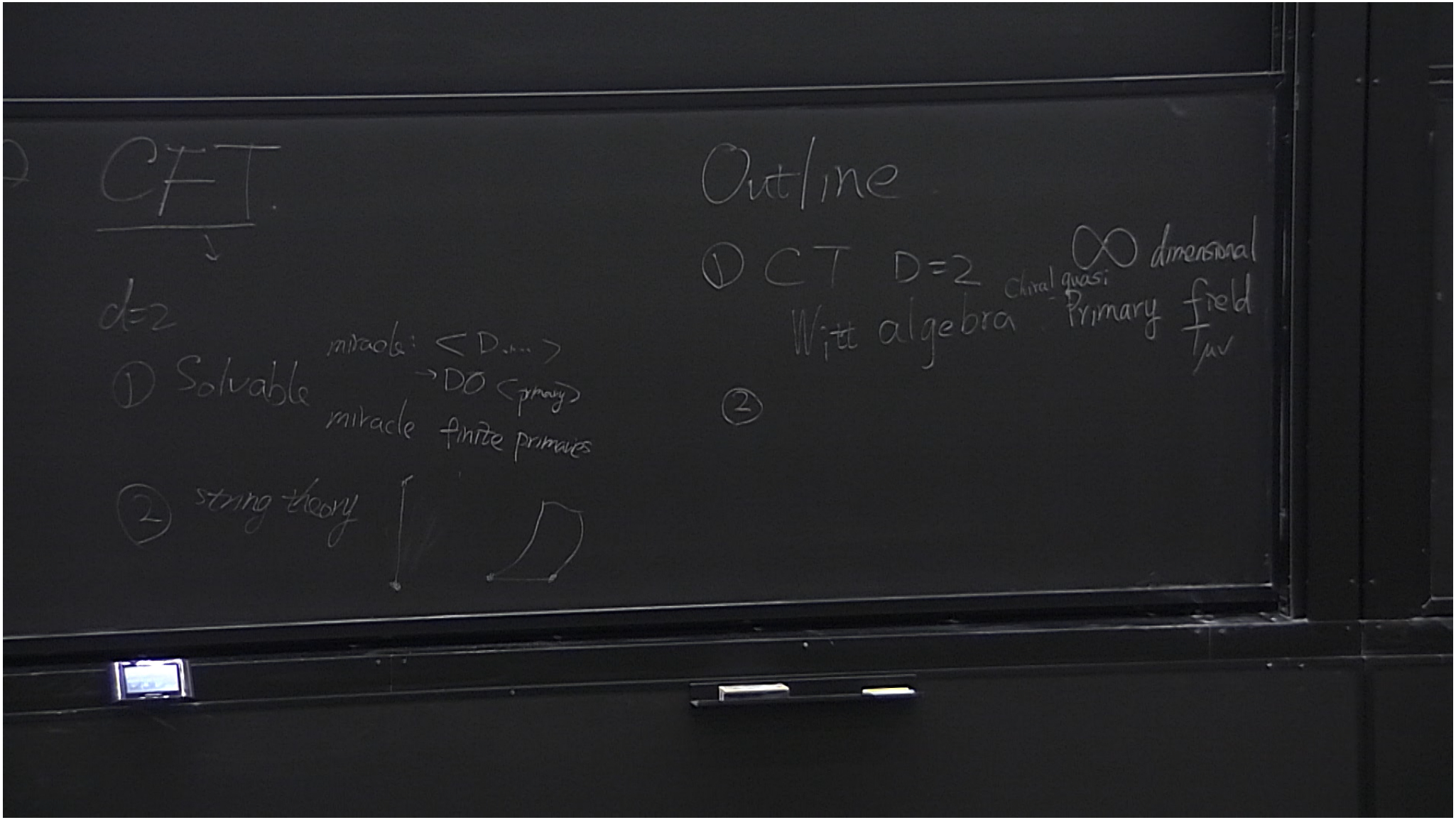
① Solvable

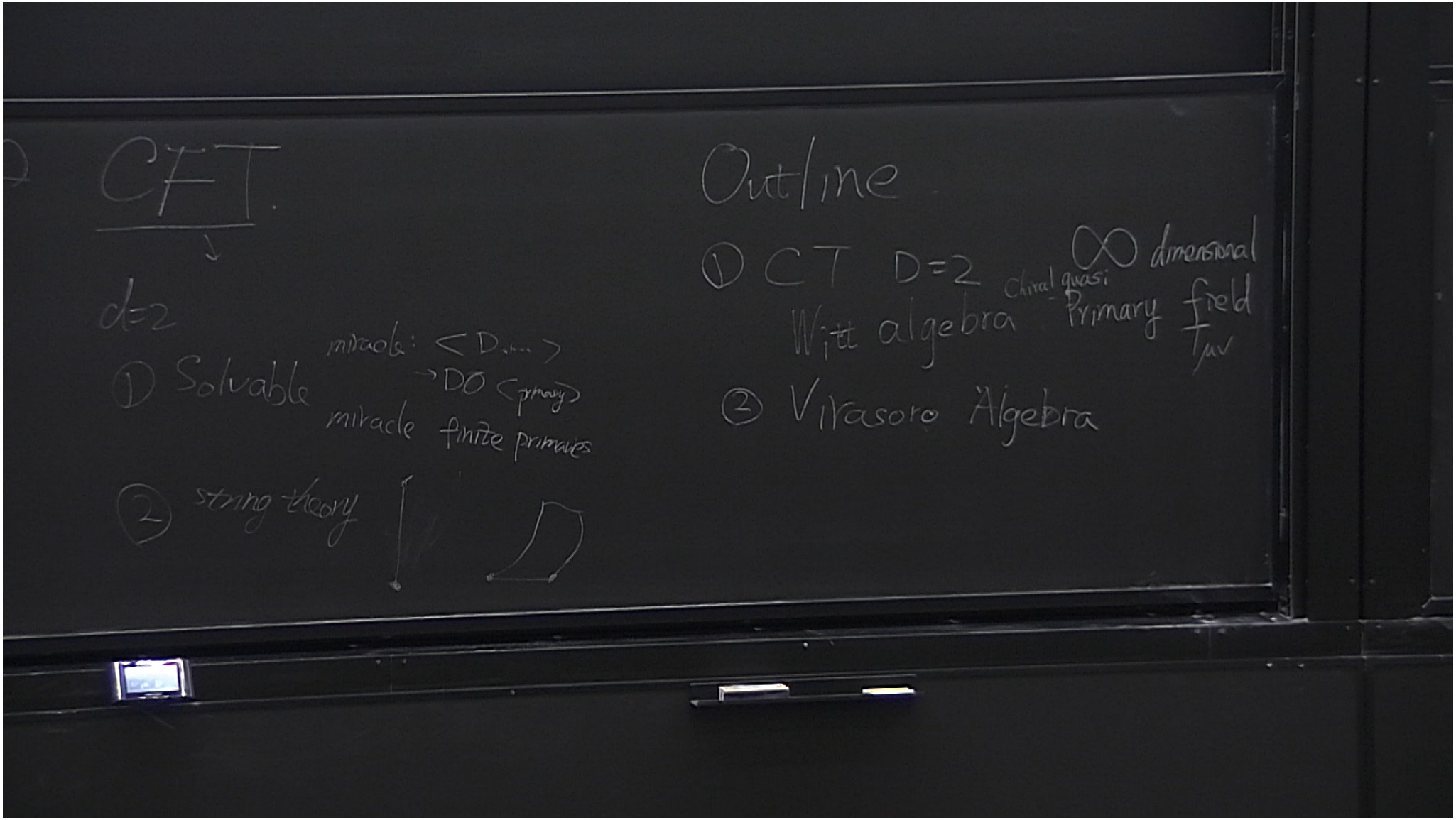
miracle: <D....>

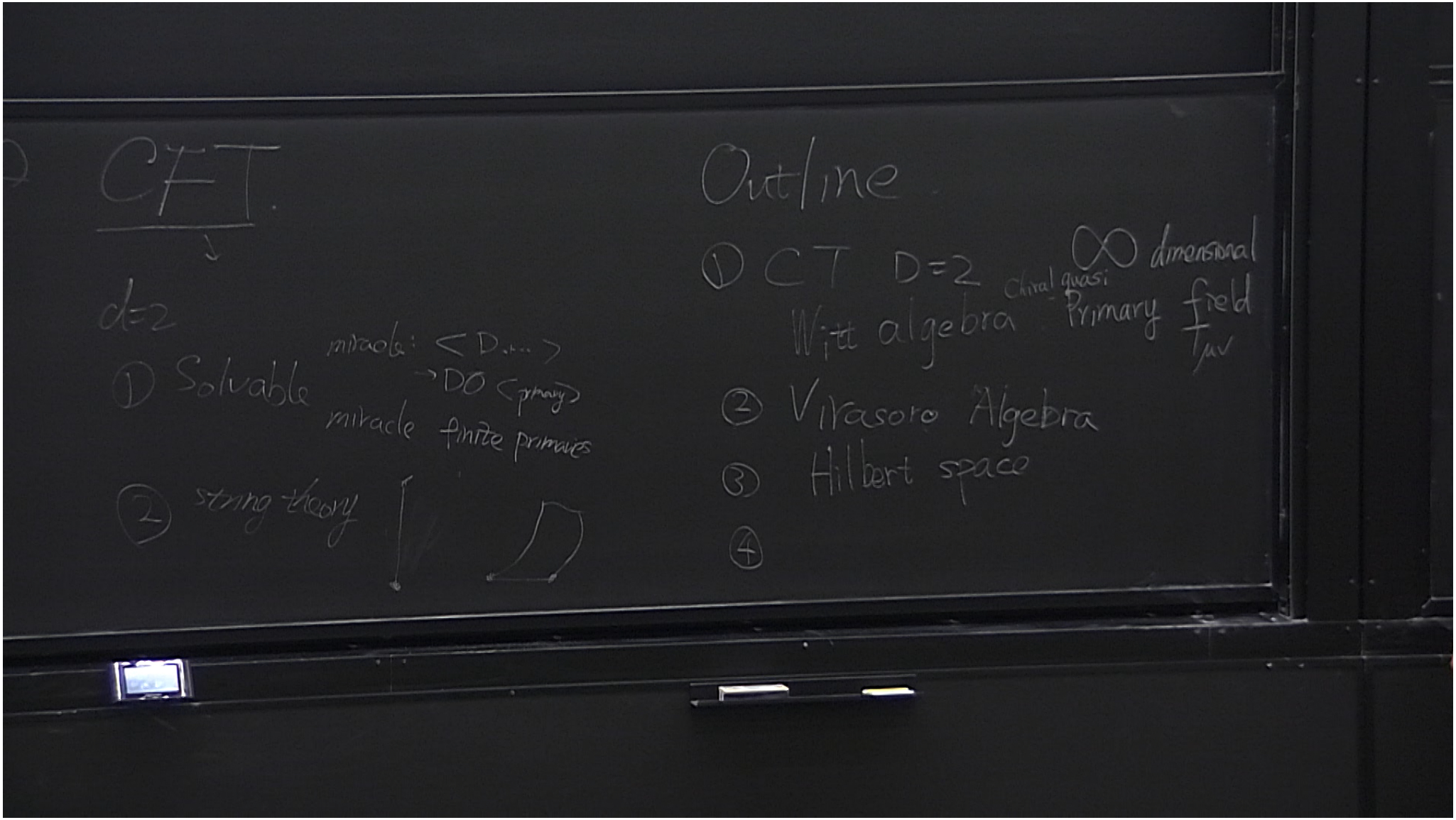
→ DO <group>

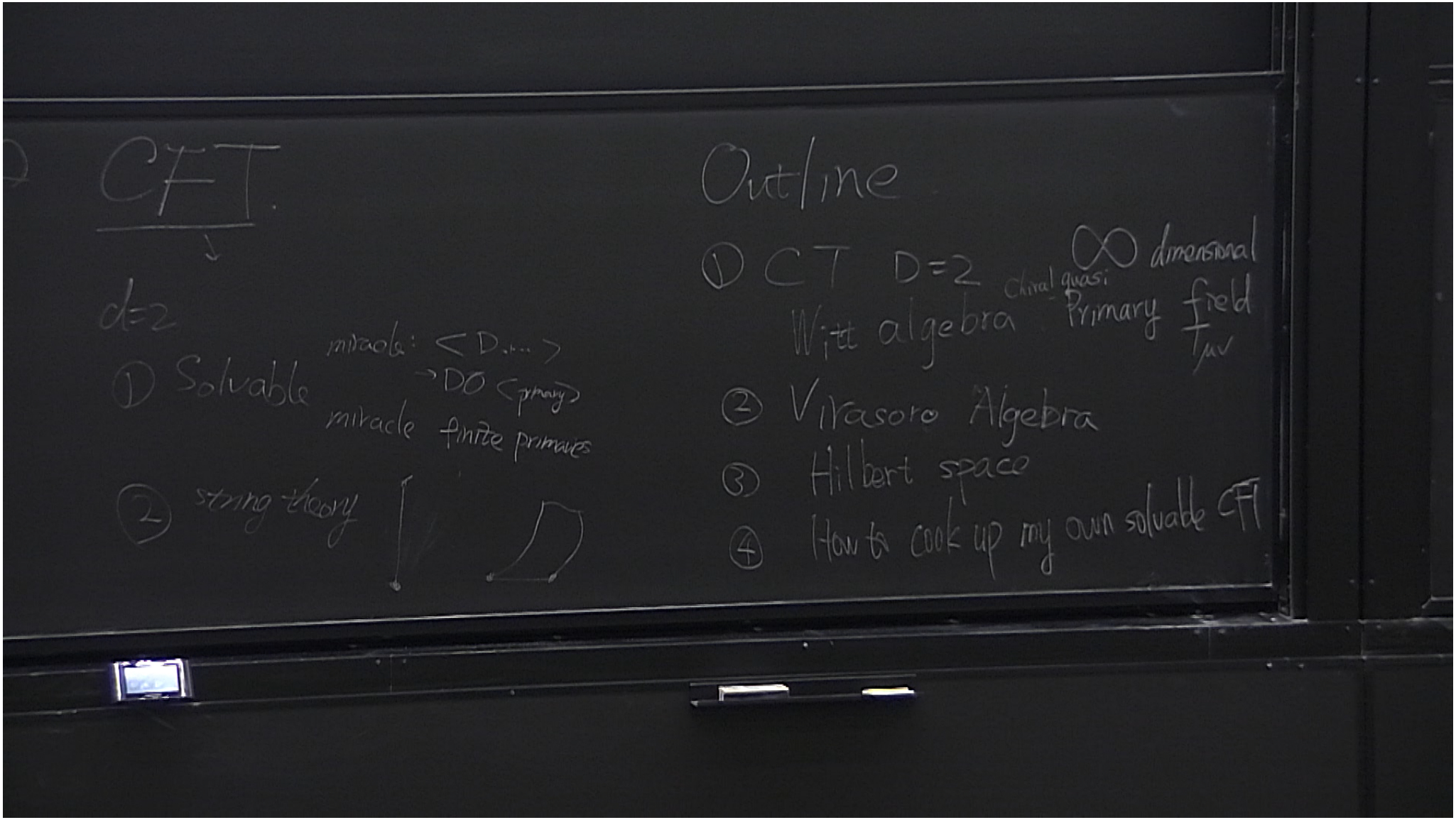
miracle finite primaries

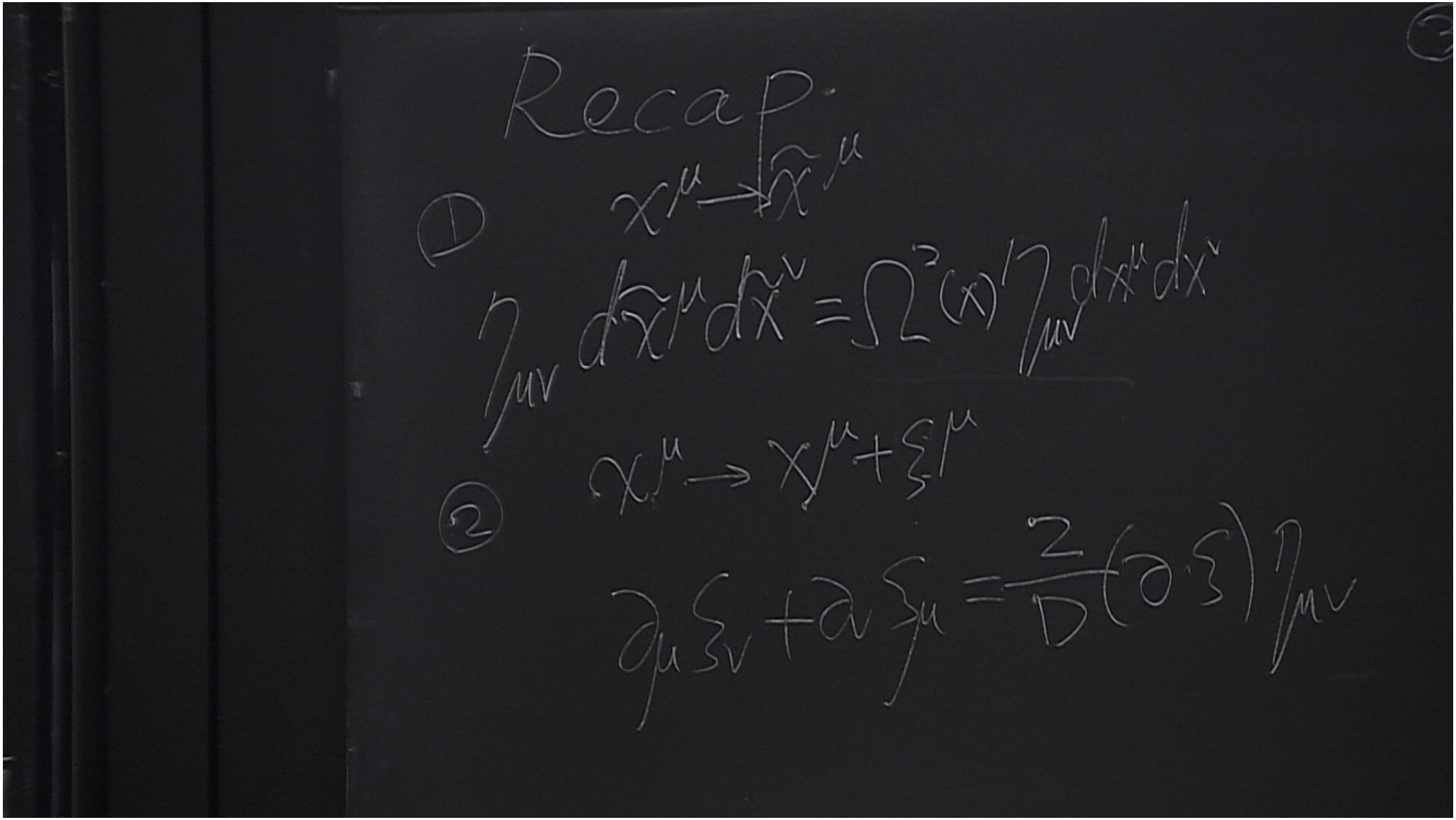
② string theory

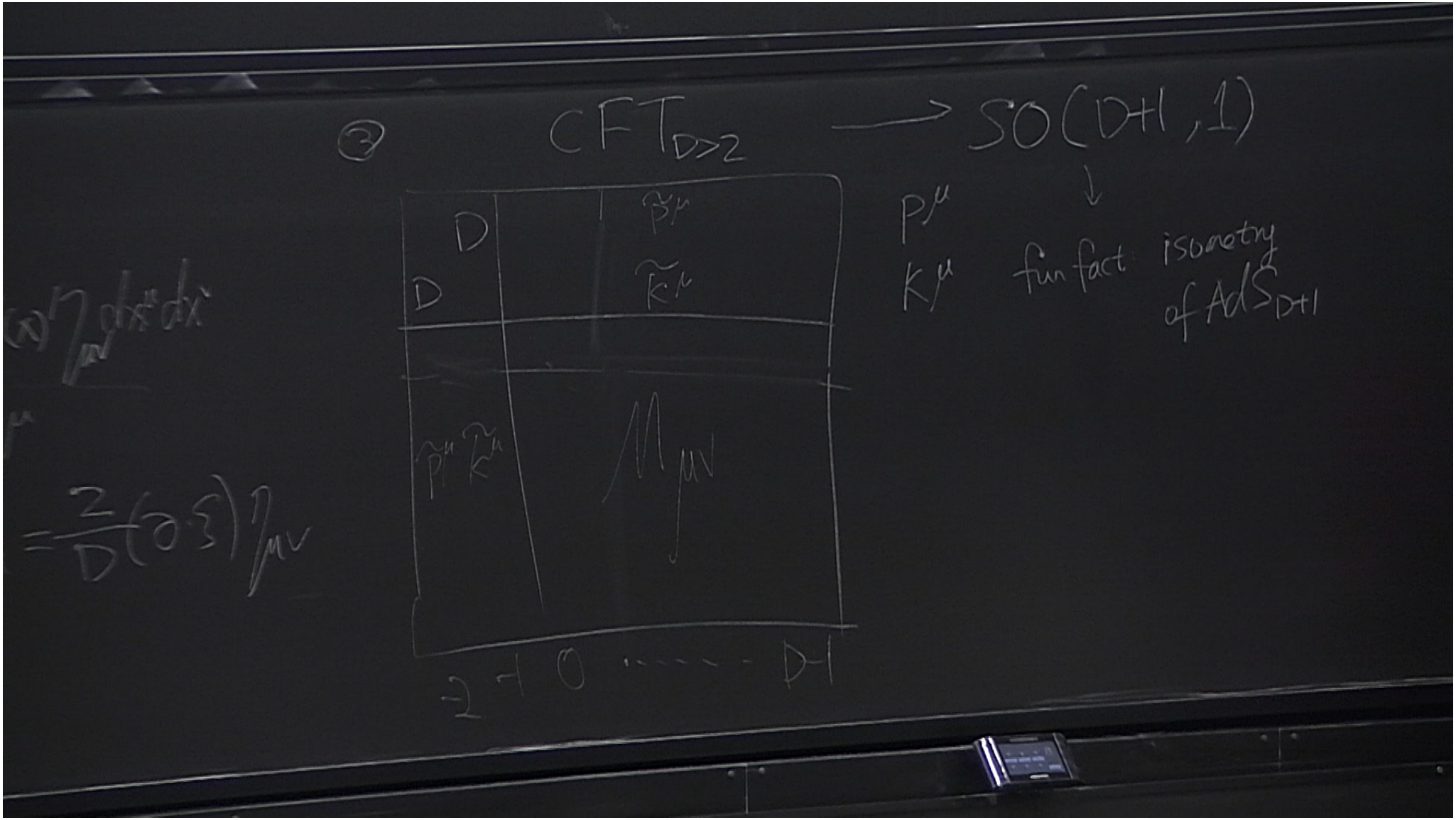

CFT
d=2
① Solvable
miracle: <D,...>
→ DO <primary>
miracle finite primaries
② string theory
Outline
① CT D=2 ∞ dimensional
Chiral quasi
With algebra Primary field
②

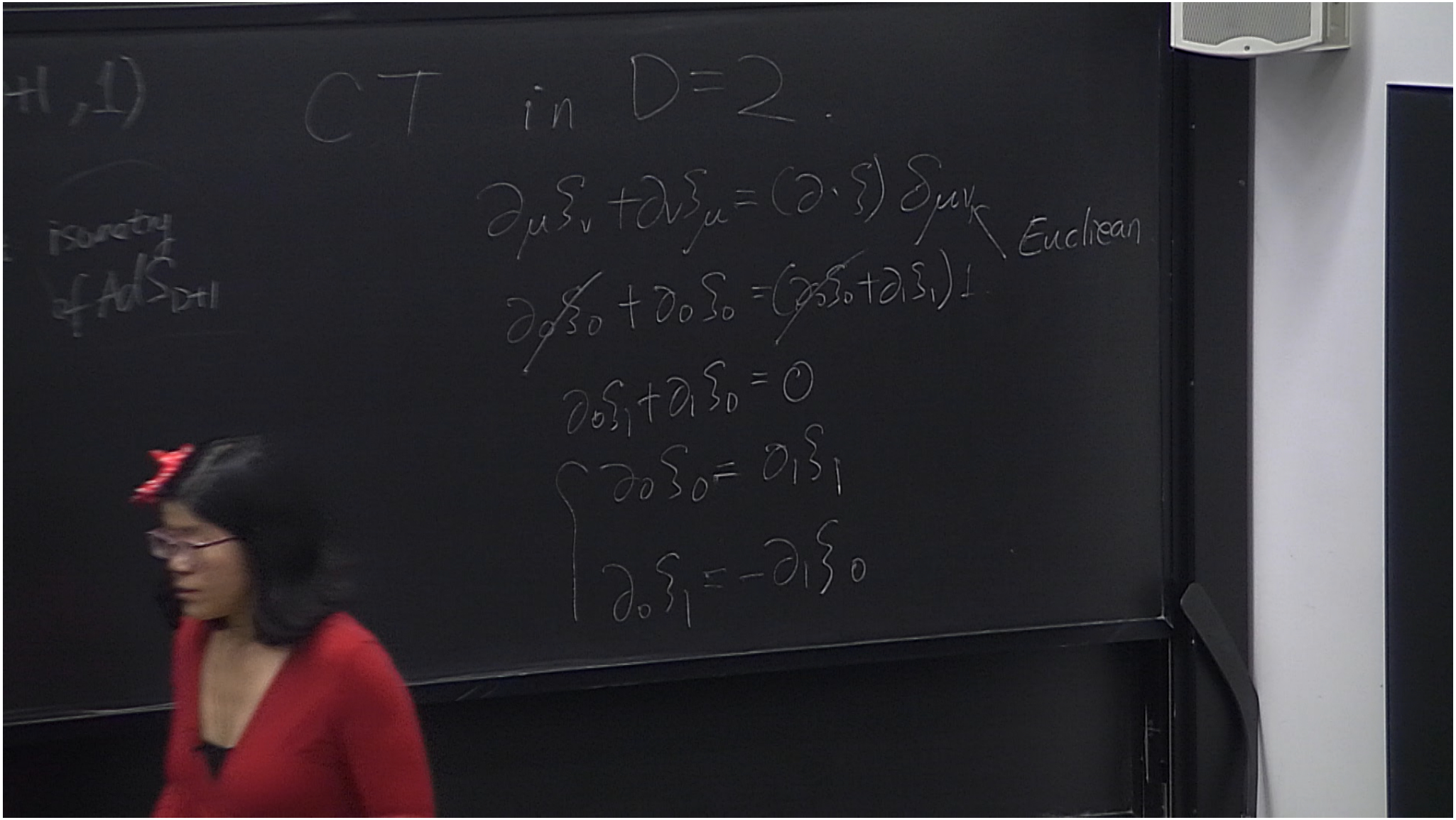

CFT
d=2
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Outline
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② Virasoro Algebra

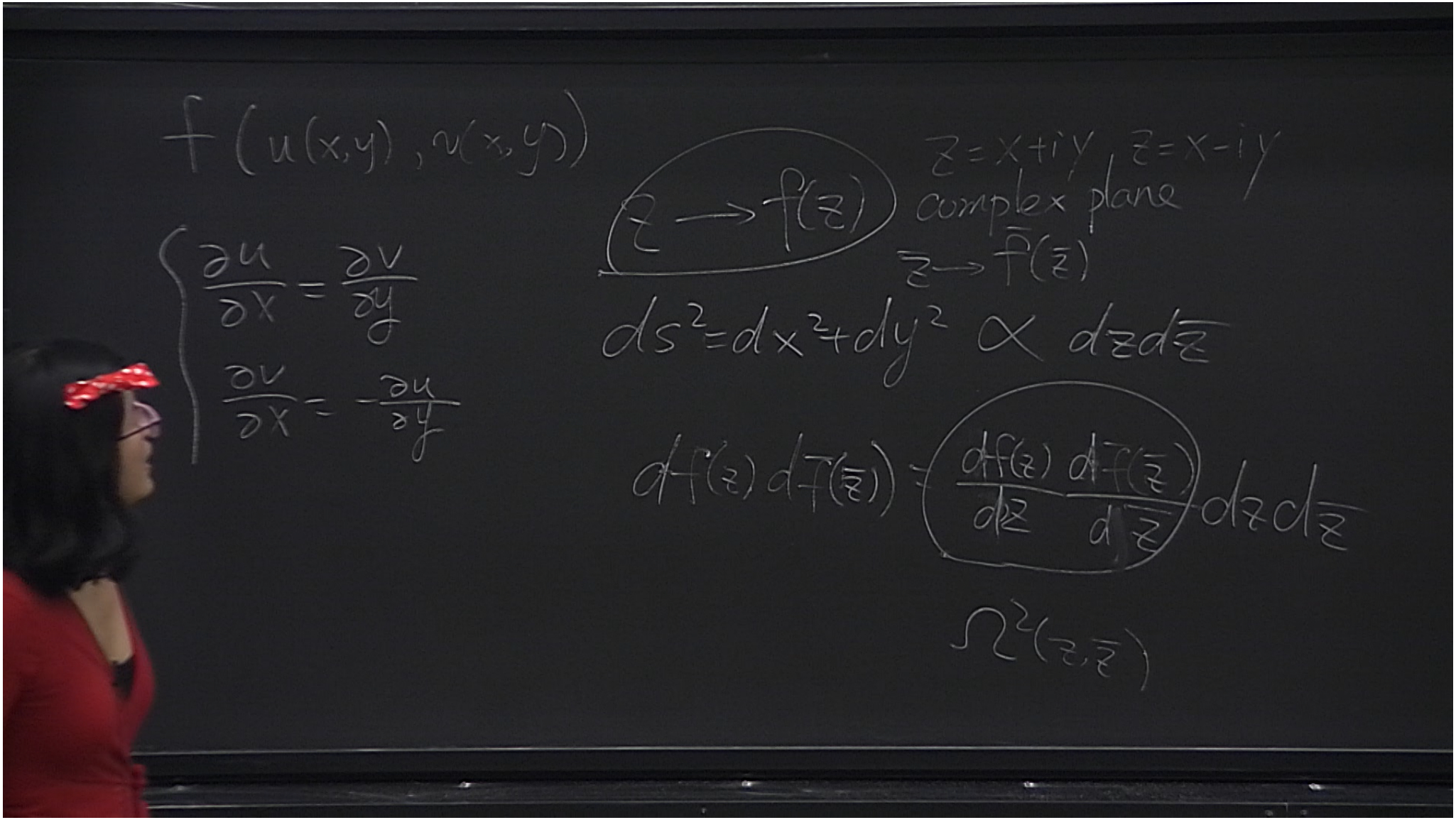

CFT
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② Virasoro Algebra
③ Hilbert space
④

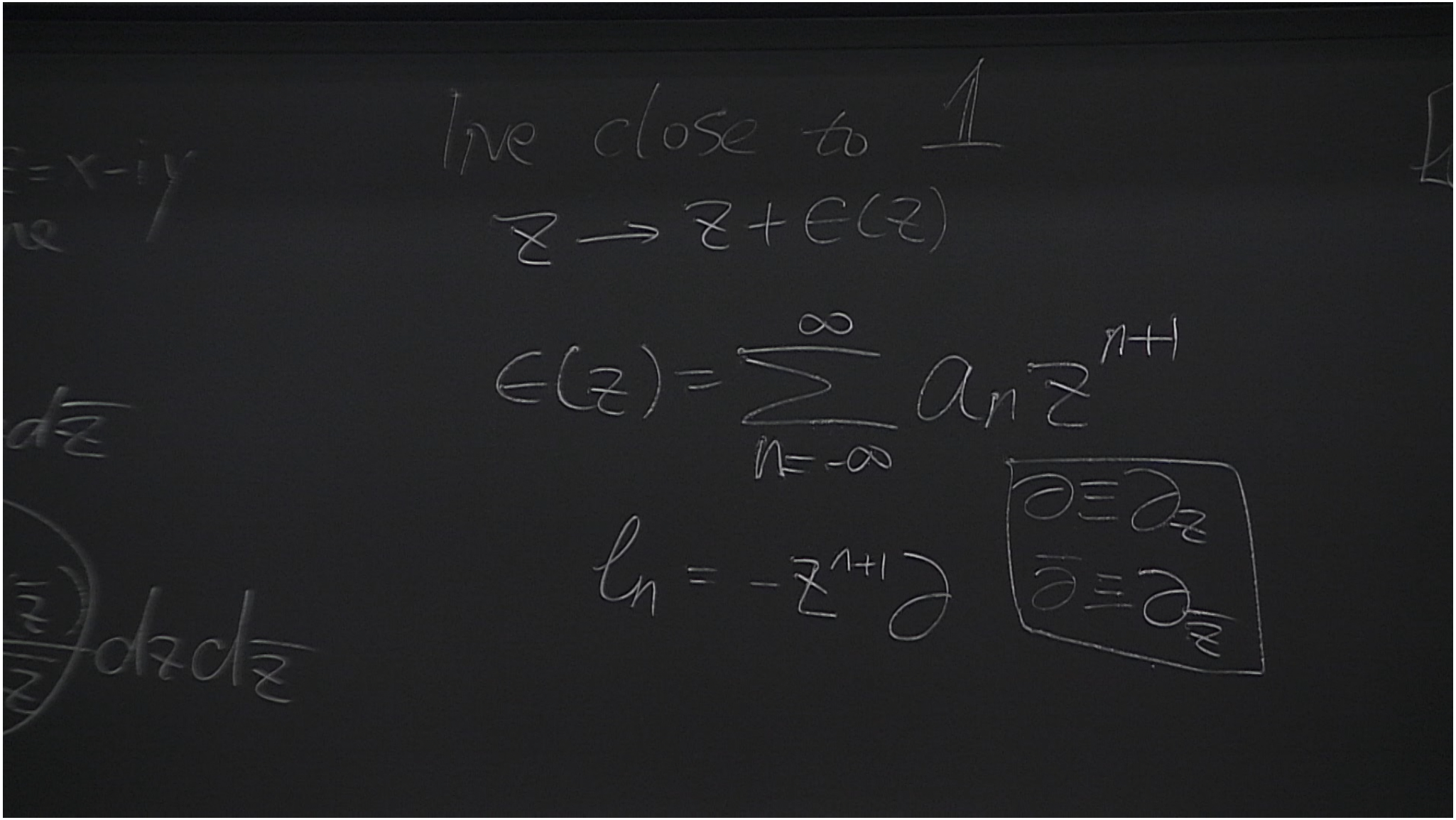

CFT
d=2
① Solvable
miracle: <D,...>
→ DO <primary>
miracle finite primaries
② string theory
Outline
① CT D=2 ∞ dimensional
With algebra Chiral quasi Primary field
② Virasoro Algebra
③ Hilbert space
④ How to cook up my own solvable CFT


Recap:
① $x^\mu \to \tilde{x}^\mu$
$\eta_{\mu\nu} d\tilde{x}^\mu d\tilde{x}^\nu = \Omega^2(x) \eta_{\mu\nu} dx^\mu dx^\nu$
② $x^\mu \to x^\mu + \xi^\mu$
$\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = \frac{2}{D}(\partial \cdot \xi)\eta_{\mu\nu}$


③
$CFT_{D>2}$
$SO(D+1,1)$
P^μ
K^μ
fun fact: isometry of AdS_{D+1}
D
D
$\tilde{P}^\mu$
$\tilde{K}^\mu$
$\Pi_{\mu\nu}$
2 1 0 ..... D-1


CT in D=2.
$\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = (\partial \cdot \xi)\,\delta_{\mu\nu}$
Euclidean
$\partial_0 \xi_0 + \partial_0 \xi_0 = (\partial_0 \xi_0 + \partial_1 \xi_1)$!
$\partial_0 \xi_1 + \partial_1 \xi_0 = 0$
$\partial_0 \xi_0 = \partial_1 \xi_1$
$\partial_0 \xi_1 = -\partial_1 \xi_0$


$f(u(x,y), v(x,y))$
$\begin{cases} \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \\ \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} \end{cases}$
$z \to f(z)$
$z = x+iy$, $\bar{z} = x-iy$
complex plane
$\bar{z} \to \bar{f}(\bar{z})$
$ds^2 = dx^2 + dy^2 \propto dz d\bar{z}$
$df(z)\, d\bar{f}(\bar{z}) = \frac{df(z)}{dz} \frac{d\bar{f}(\bar{z})}{d\bar{z}} dz d\bar{z}$
$\Omega^2(z, \bar{z})$


I've close to 1
$z \to z + \epsilon(z)$
$\epsilon(z) = \sum_{n=-\infty}^{\infty} a_n z^{n+1}$
$l_n = -z^{n+1}\partial$
$\partial \equiv \partial_z$
$\bar{\partial} \equiv \partial_{\bar{z}}$
dz
$d\bar{z}$
$dz\,d\bar{z}$

② string theory

③ Hilbert space

④ How to cook up my own soluable CFT

live close to 1

$z \to z + \epsilon(z)$

$$\epsilon(z) = \sum_{n=-\infty}^{\infty} a_n z^{n+1}$$

$$l_n = -z^{n+1}\partial$$

$\partial \equiv \partial_z$

$\bar{\partial} \equiv \partial_{\bar{z}}$

$$[l_n, l_m] = -z^{n+1}\partial(-z^{m+1}\partial)$$

$$-(-z^{m+1}\partial)(-z^{n+1}\partial)$$

$$= z^{n+1}(m+1)z^m\partial - z^{m+1}(n+1)z^n\partial$$

$$-(m-n)z^{m+n+1}\partial$$

$$[l_n, l_m] = -(n-m)l_{n+m}$$

$$[\bar{l}_n, \bar{l}_m] = -(n-m)\bar{l}_{n+m}$$

$$[l_n, \bar{l}_m] = 0$$

$$[l_n, l_m] = -z^{n+1}\partial(-z^{m+1}\partial)$$
$$-(-z^{m+1}\partial)(-z^{n+1}\partial)$$
$$= z^{n+1}(m+1)z^m\partial - z^{m+1}(n+1)z^n\partial$$
$$= (m-n)z^{m+n+1}\partial$$

Witt Algebra

$$[l_n, l_m] = -(n-m)\,l_{n+m}$$
$$[\bar{l}_n, \bar{l}_m] = -(n-m)\,\bar{l}_{n+m}$$
$$[l_n, \bar{l}_m] = 0$$

local conformal algebra

global conformal algebra.

$l_n = -z^{n+1}\partial$ $n \in$ Integers $(-\infty, \infty)$

(a) $z=0$ $n+1 \geq 0$ $n \geq -1$

(a) $z=\infty$ $w = \frac{1}{z}$ $\partial_z = \frac{\partial w}{\partial z}\partial_w$

$= -\frac{1}{z^2}\partial_w$

$= -w^2\partial_w$

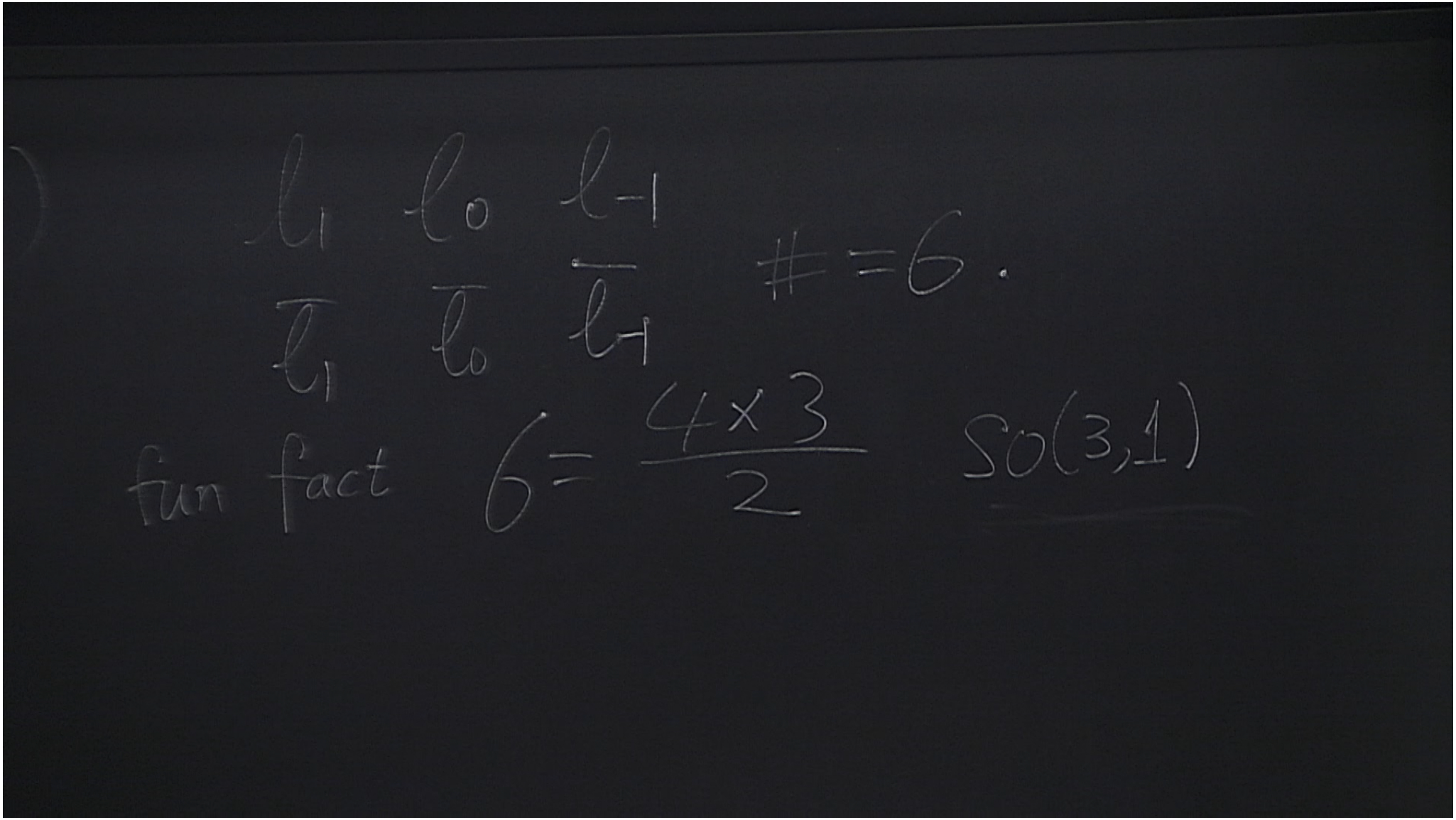
$$l_n = -\left(\frac{1}{w}\right)^{n+1} \cdot (-w^2 \partial_w)$$

$$= w^{-n-1+2} \partial_w$$

$$= w^{-n+1} \partial_w$$

$$-n+1 \geq 0$$

$$n \leq 1$$


l_1 l_0 l_{-1}
$\bar{l}_1$ l_0 $\bar{l}_{-1}$ #=6.
fun fact $6 = \frac{4 \times 3}{2}$ $SO(3,1)$

l_{-1}: $\epsilon(z) = a_{-1} z^{-1+1} = a_{-1}$

$z \to z + a_{-1}$ translation.

l_0: $\epsilon(z) = a_0 z^{0+1} = a_0 z = r e^{i\phi} z$

$z \to z + a_0 z = (1 + a_0) z$ 1 dilatation $l_0 + \bar{l}_0 \to r\partial_r$

1 rotation $l_0 - \bar{l}_0 \to \partial_\phi$

$(l_n, l_m) = 0$

$\epsilon(z) = a_1 z^{1+1} = a_1 z^2$

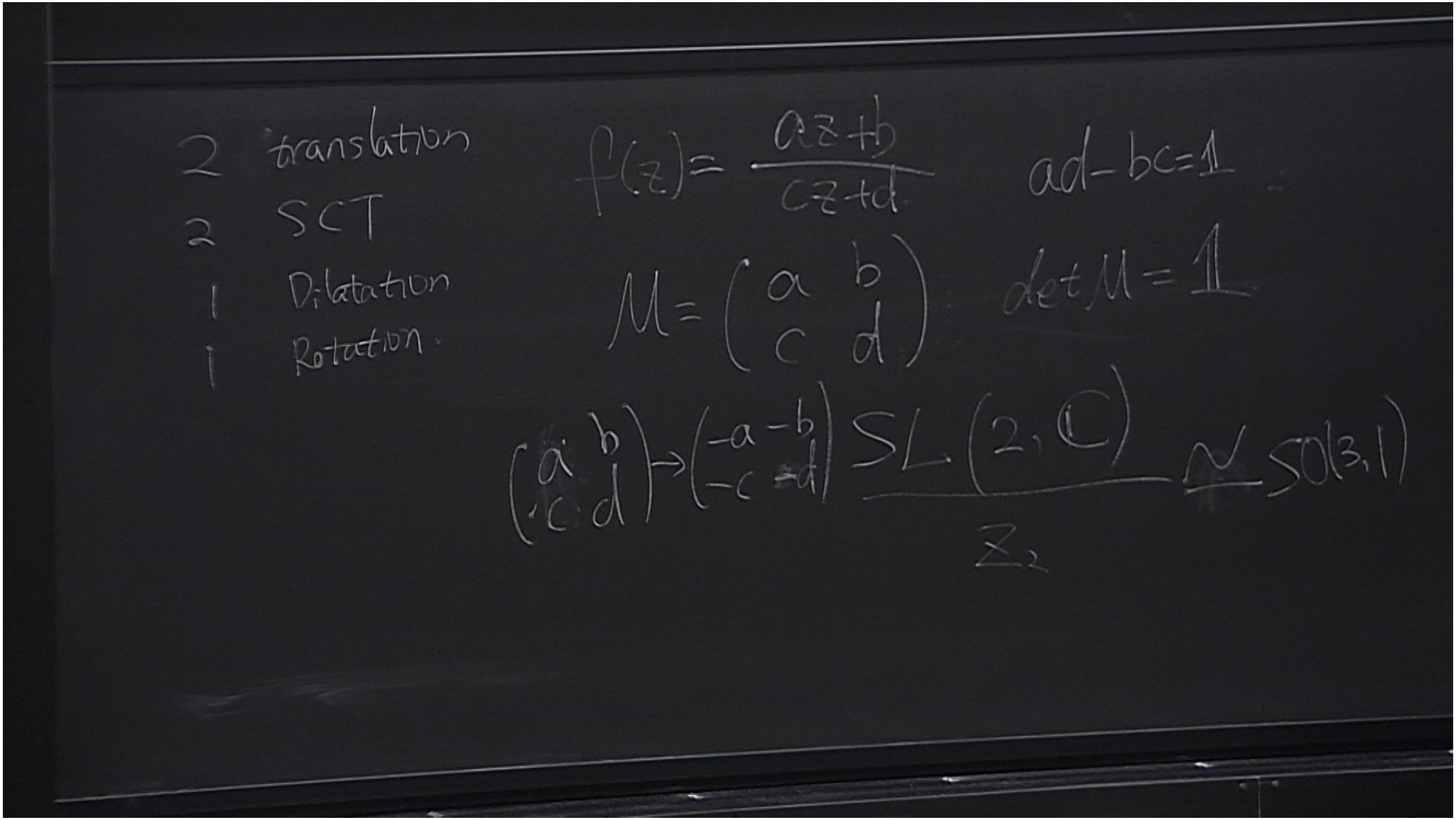
$z \to z + a_1 z^2 = z(1 + a_1 z) + \cdots$

$\tilde{z} = \frac{z}{1 - a_1 z}$ $\quad K_\mu = I P_\mu I$

$z = \frac{1}{w}$ $\quad \tilde{z} = \frac{1}{\tilde{w}}$

$\frac{1}{\tilde{w}} = \frac{\frac{1}{w}}{1 - \frac{a_1}{w}} = \frac{1}{w - a_1}$

$\tilde{w} = w - a_1$


2 translation
2 SCT
1 Dilatation
1 Rotation.
$f(z) = \frac{az+b}{cz+d}$
$ad-bc=1$
$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$
$\det M = 1$
$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \to \begin{pmatrix} -a & -b \\ -c & -d \end{pmatrix}$
$\frac{SL(2,\mathbb{C})}{\mathbb{Z}_2} \cong SO(3,1)$

$z \to z + a_0 z = (1+a_0) z$ dilatation $\ell_0 + \bar{\ell}_0 \to r\partial_r$

rotation $\ell_0 - \bar{\ell}_0 \to \partial_\phi$

$\frac{1}{\tilde{w}} = \frac{\frac{1}{w}}{1 - a_1 \frac{1}{w}}$

$\tilde{w} = w - a_1$

Primary fields (Powerful)

primary : VIP

$$\tilde{\Phi}_{h,\bar{h}}(\lambda z, \bar{\lambda}\bar{z}) = \lambda^{-h}\bar{\lambda}^{-\bar{h}}\Phi(z,\bar{z})$$

$h + \bar{h} = \Delta$

$h - \bar{h} = s$

$$\tilde{\Phi}_{h,\bar{h}}(f(z), \bar{f}(\bar{z})) = \left(\frac{df(z)}{dz}\right)^{-h}\left(\frac{d\bar{f}(\bar{z})}{d\bar{z}}\right)^{-\bar{h}}\Phi(z,\bar{z})$$

any $f(z)$ → primary field

only $f(z) = \frac{az+b}{cz+d}$ → quasi-primary field.

chiral field $\Phi(z,\bar{z}) \equiv \Phi(z)$

anti-chiral $\Phi(z,\bar{z}) \equiv \bar{\Phi}(\bar{z})$

$\left(\frac{dz}{dz}\right)^{-h} \left(\frac{d\bar{f}(\bar{z})}{d\bar{z}}\right)^{-\bar{h}} \Phi(z,\bar{z})$

field ≡ local operators

ϕ, $\partial\phi$, $e^{i\alpha\phi}$