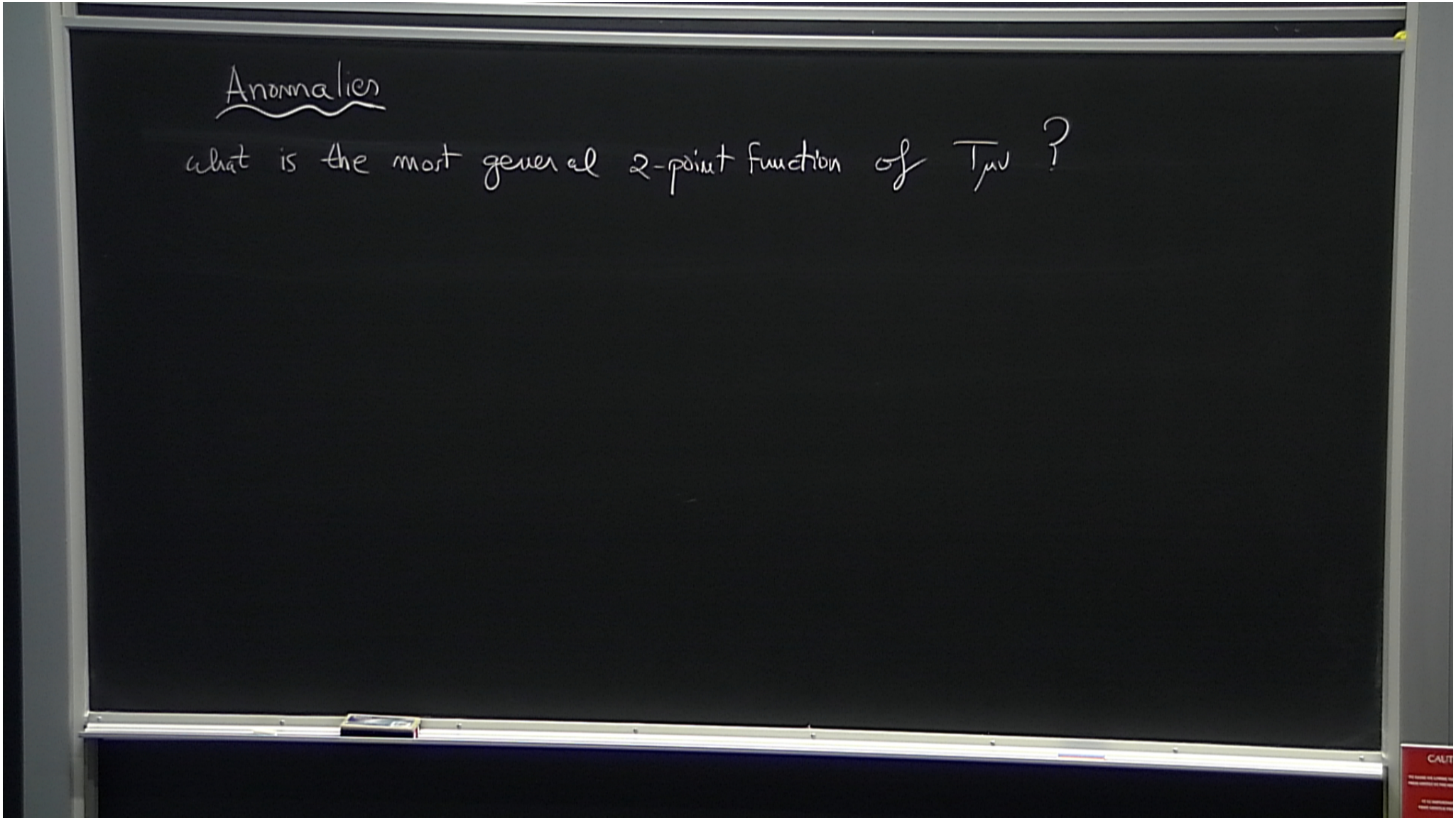
Title: PSI 17/18 - Quantum Field Theory III - Lecture 6

Date: Feb 02, 2018 03:45 PM

URL: http://pirsa.org/18020091

Abstract:


Anomalies
what is the most general 2-point function of $T_{\mu\nu}$?

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$$\langle T_{\mu\nu}(p) \quad T_{\rho\sigma}(-p) \rangle =$$

1. Compatible w/ Lorentz symmetry

Anomalies

what is the most general 2-point function of $T_{\mu\nu}$?

$$\langle T_{\mu\nu}(p) \; T_{\rho\sigma}(-p) \rangle = \eta_{\mu\nu}\eta_{\rho\sigma} F_1(p^2) + P_\mu P_\nu P_\rho P_\sigma F_2(p^2)$$

1. Compatible w/ Lorentz symmetry ($\eta_{\mu\nu}$, P_ρ)

Anomalies

what is the most general 2-point function of $T_{\mu\nu}$?

$\delta^2(0)$

$$\langle T_{\mu\nu}(p) \quad T_{\rho\sigma}(-p) \rangle = \delta^{(4)}(\quad) \Big(\eta_{\mu\nu}\eta_{\rho\sigma} F_1(p^2) + P_\mu P_\nu P_\rho P_\sigma F_2(p^2) + \dots \Big)$$

5 different structures.

1. Compatible w/ Lorentz symmetry ($\eta_{\mu\nu}$, P_ρ)

Anomalies

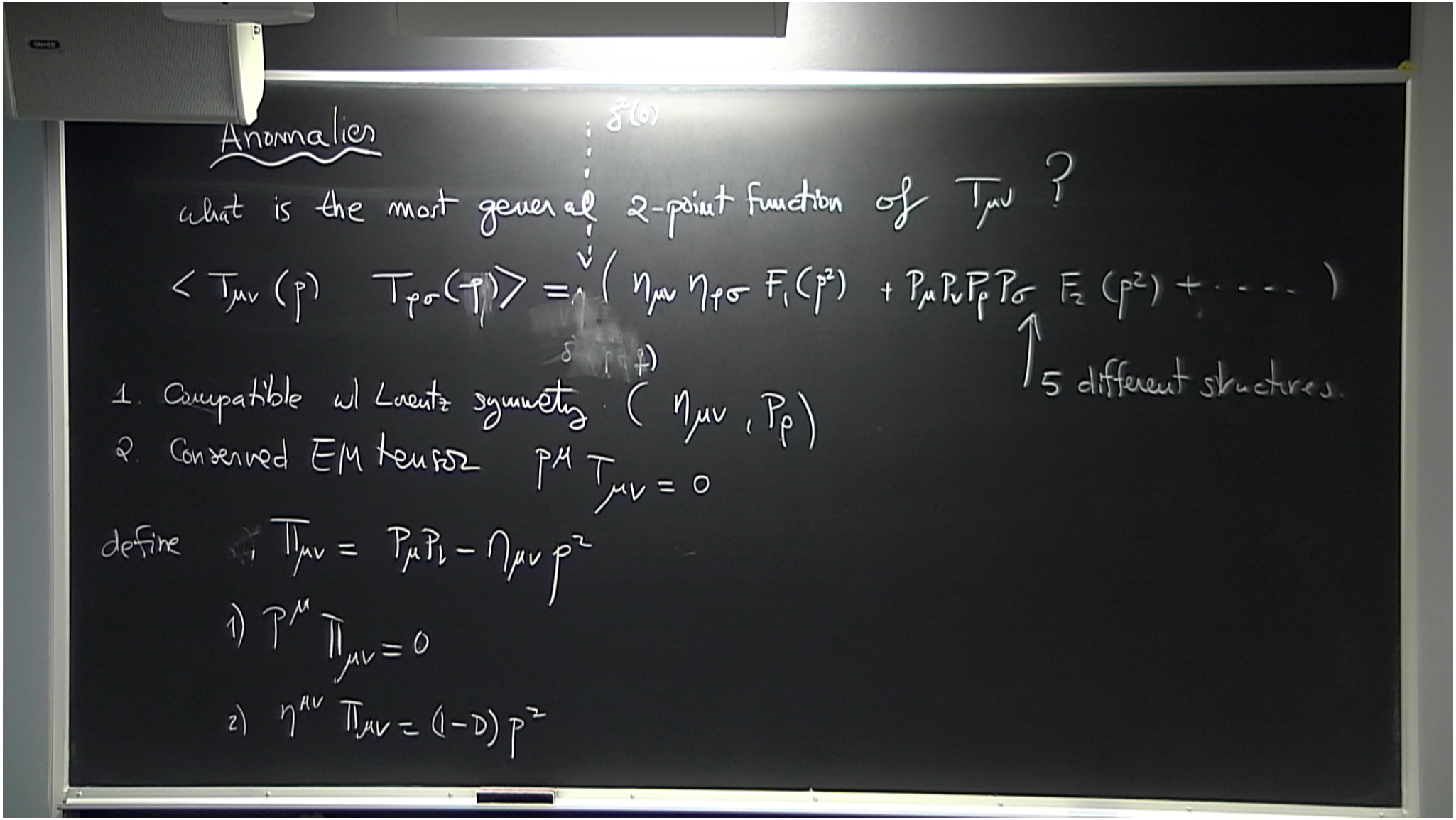
$\delta^3(0)$

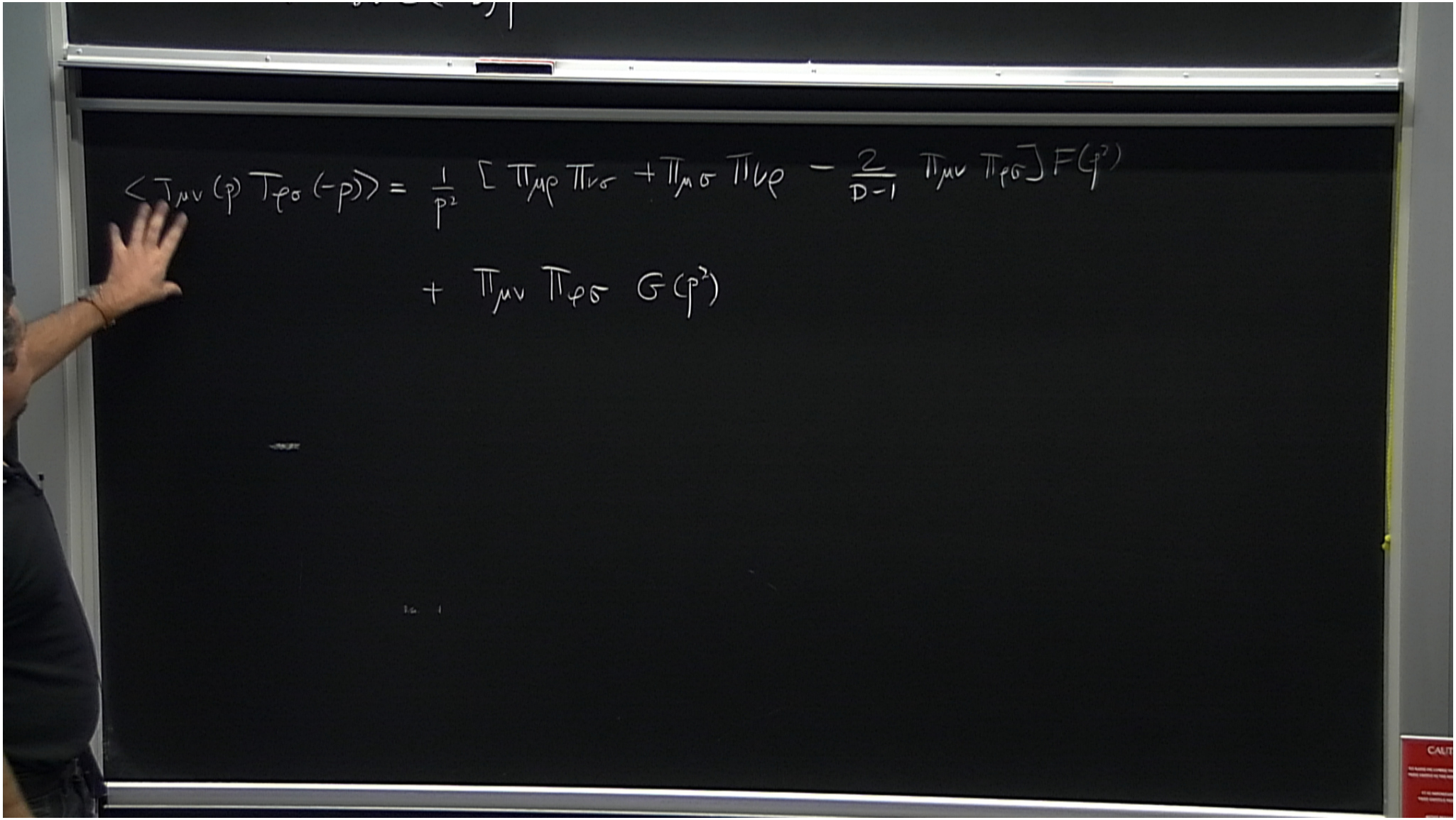
what is the most general 2-point function of $T_{\mu\nu}$?

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↑ 5 different structures.

1. Compatible w/ Lorentz symmetry ($\eta_{\mu\nu}$, P_ρ)
2. Conserved EM tensor $\partial^\mu T_{\mu\nu} = 0$


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5 different structures.
1. Compatible w/ Lorentz symmetry ($\eta_{\mu\nu}$, P_ρ)
2. Conserved EM tensor $p^\mu T_{\mu\nu} = 0$
define $\Pi_{\mu\nu} = P_\mu P_\nu - \eta_{\mu\nu} p^2$
1) $p^\mu \Pi_{\mu\nu} = 0$
2) $\eta^{\mu\nu} \Pi_{\mu\nu} = (1-D) p^2$


$\langle T_{\mu\nu}(p)\, T_{\rho\sigma}(-p)\rangle = \frac{1}{p^2}\left[\Pi_{\mu\rho}\Pi_{\nu\sigma} + \Pi_{\mu\sigma}\Pi_{\nu\rho} - \frac{2}{D-1}\Pi_{\mu\nu}\Pi_{\rho\sigma}\right] F(p^2)$
$+ \Pi_{\mu\nu}\Pi_{\rho\sigma}\, G(p^2)$

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M is a 2×2 matrix

$M^{\mu\nu}\ M^{\alpha\beta}$

$D=2$: In $D=2$ 1st term vanishes.

$$\Pi_{\mu\nu} = -\tilde{P}_\mu \tilde{P}_\nu$$

$$\tilde{P}_\mu = \epsilon_{\mu\rho} P^\rho$$

$$\epsilon_{\mu\alpha}\,\epsilon_{\nu\beta} = \eta_{\mu\nu}\eta_{\alpha\beta} - \eta_{\mu\beta}\eta_{\alpha\nu}$$

$$2\det M = (\mathrm{tr}\, M)^2 - \mathrm{tr}\, M^2$$

In $D=2$

$$\langle T_{\mu\nu}(p)\, T_{\rho\sigma}(-p)\rangle = G(p^2)\, \Pi_{\mu\nu}\, \Pi_{\rho\sigma}$$

Scale invariance constraint: $\left(T^{\mu}_{\mu} = 0 \right) \Leftrightarrow$ conformal invariance.

$$G(p^2) =$$

$T_{\mu\nu}(x)$

In $D=2$

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$T_{\mu\nu}(x) \qquad \Delta = D.$

$$T_{\mu\nu}(p) = \int d^D x\, e^{ipx}\, T_{\mu\nu}(x)$$

In $D=2$

$$\langle T_{\mu\nu}(p)\, T_{\rho\sigma}(-p)\rangle = \frac{c}{p^2}\,\Pi_{\mu\nu}\Pi_{\rho\sigma}$$

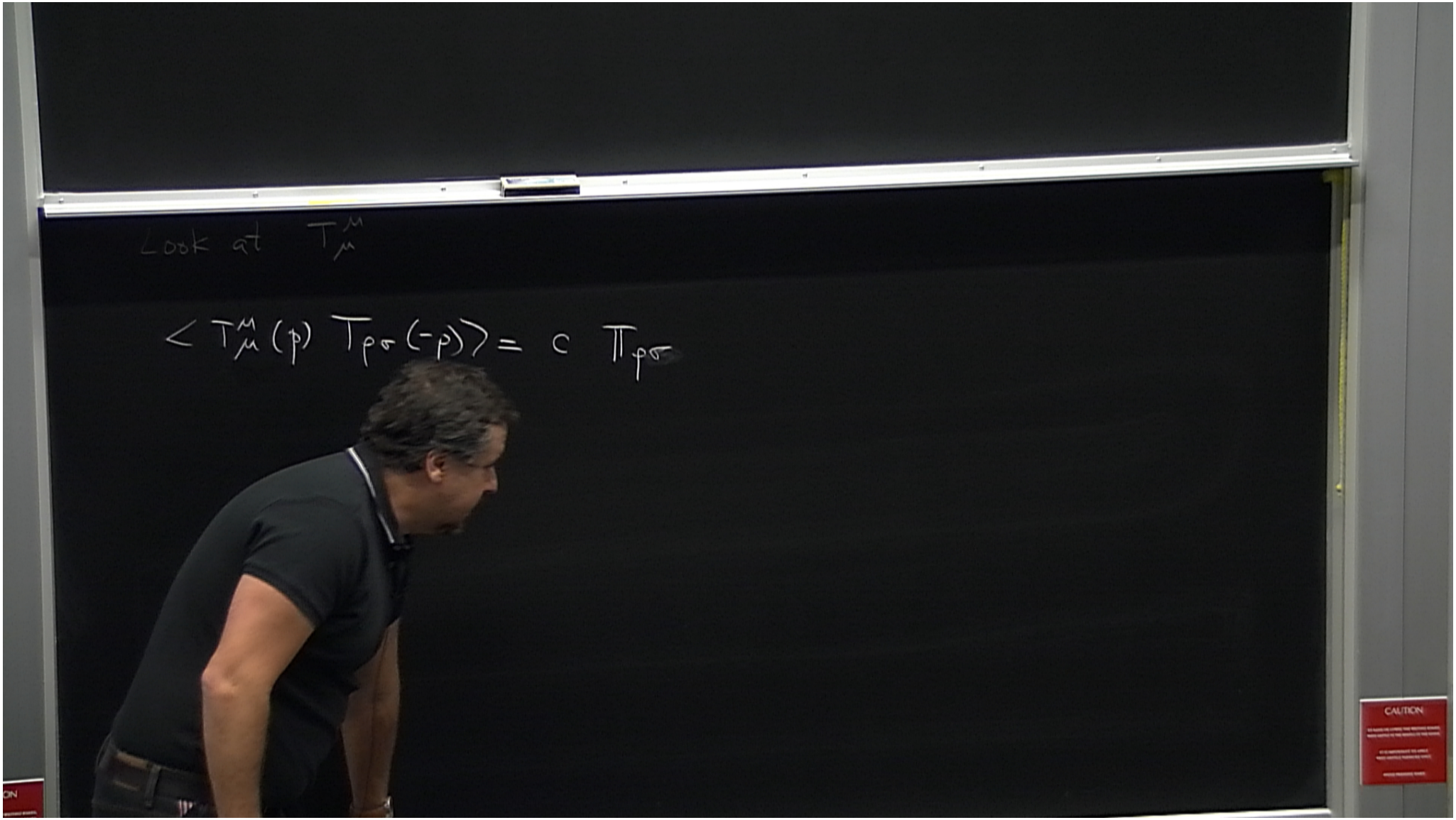
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Look at T^{μ}_{μ}
$\langle T^{\mu}_{\mu}(p)\, T_{\rho\sigma}(-p)\rangle = c\, \Pi_{\rho\sigma}$
CAUTION

Look at T^{μ}_{μ}

$\langle T^{\mu}_{\mu}(p)\, T_{\rho\sigma}(-p)\rangle = c\, \Pi_{\rho\sigma}$ conformal anomaly.

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$\langle T^{\mu}_{\mu}(x)\, T_{\rho\sigma}(0)\rangle = c\,(\partial_\mu\partial_\nu - \eta_{\mu\nu}\Box)\,\delta^2(x)$ local violation (support at $x=0$).

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Look at T^{μ}_{μ} Unitarity $\Rightarrow$ $\boxed{c>0}$

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$\downarrow$

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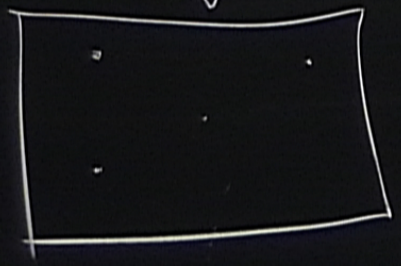
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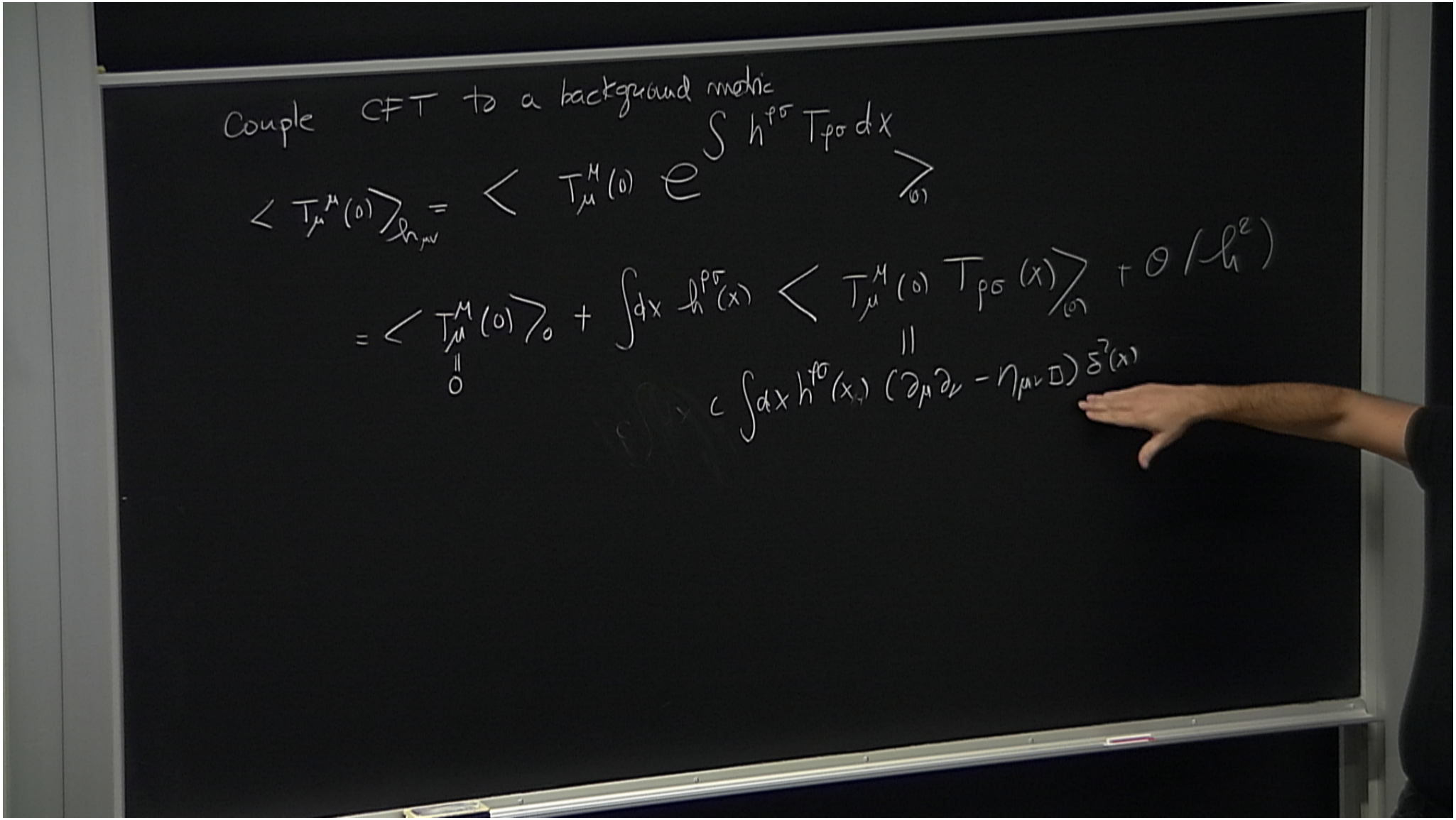
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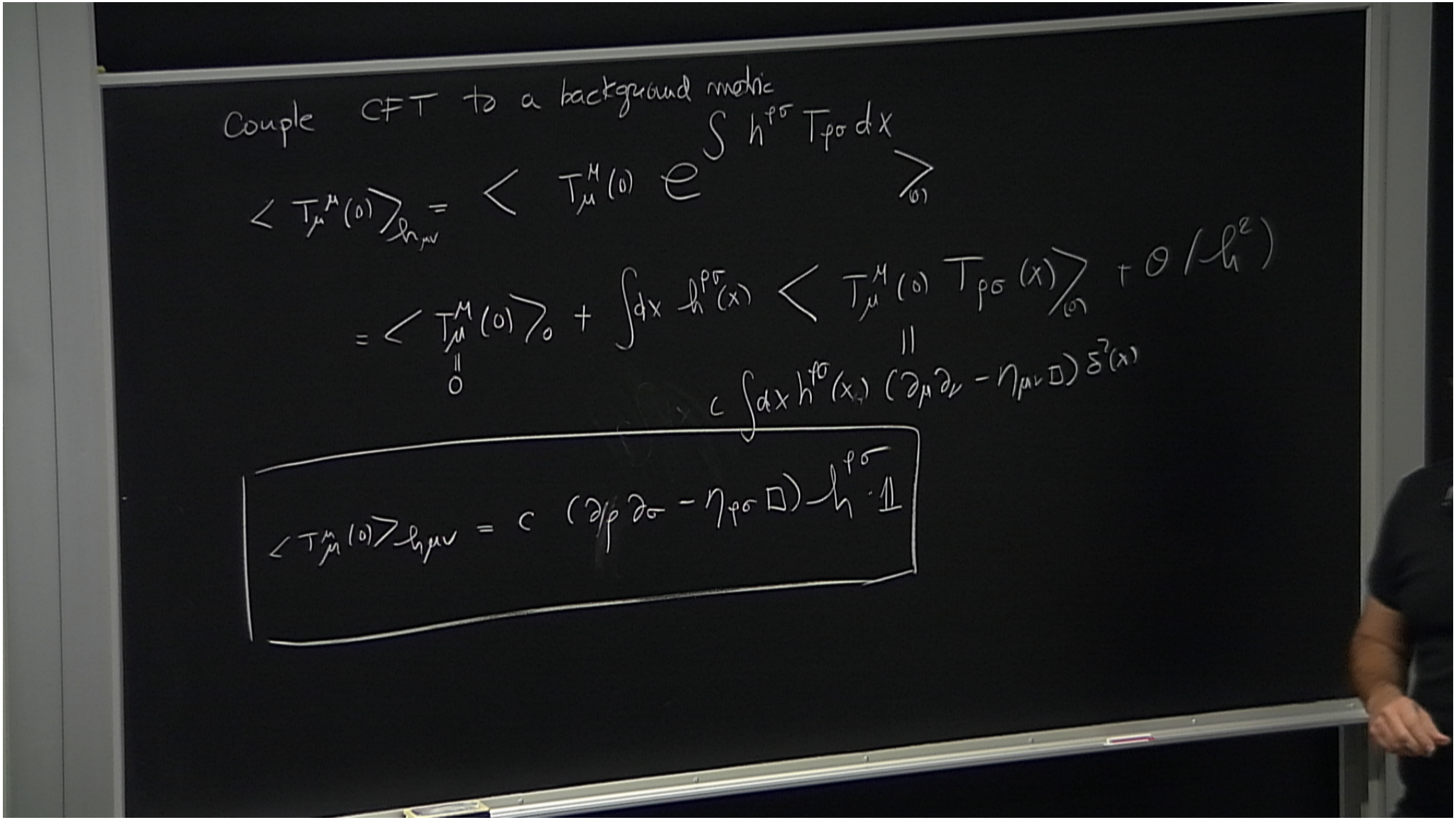


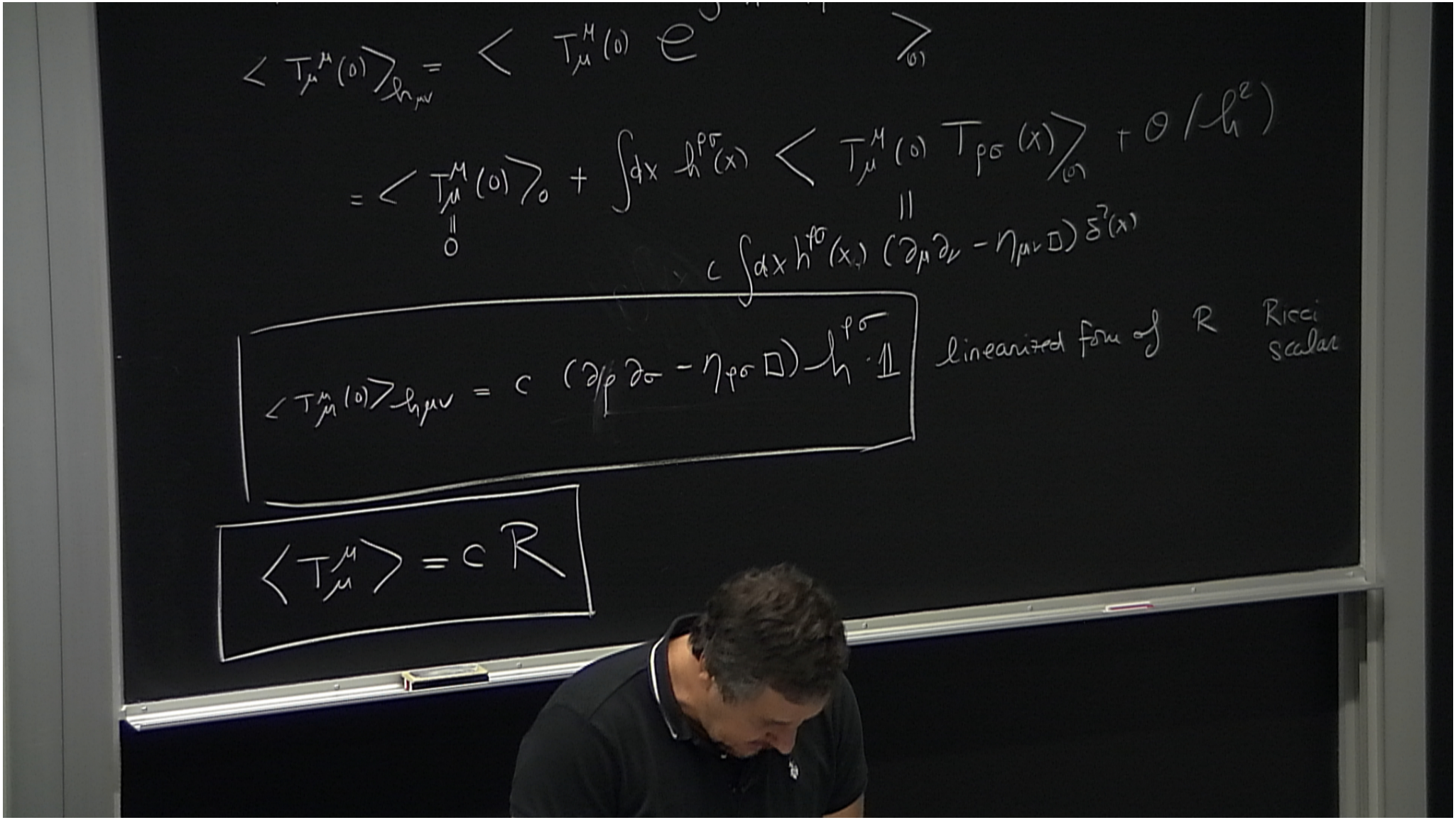
Couple CFT to a background metric

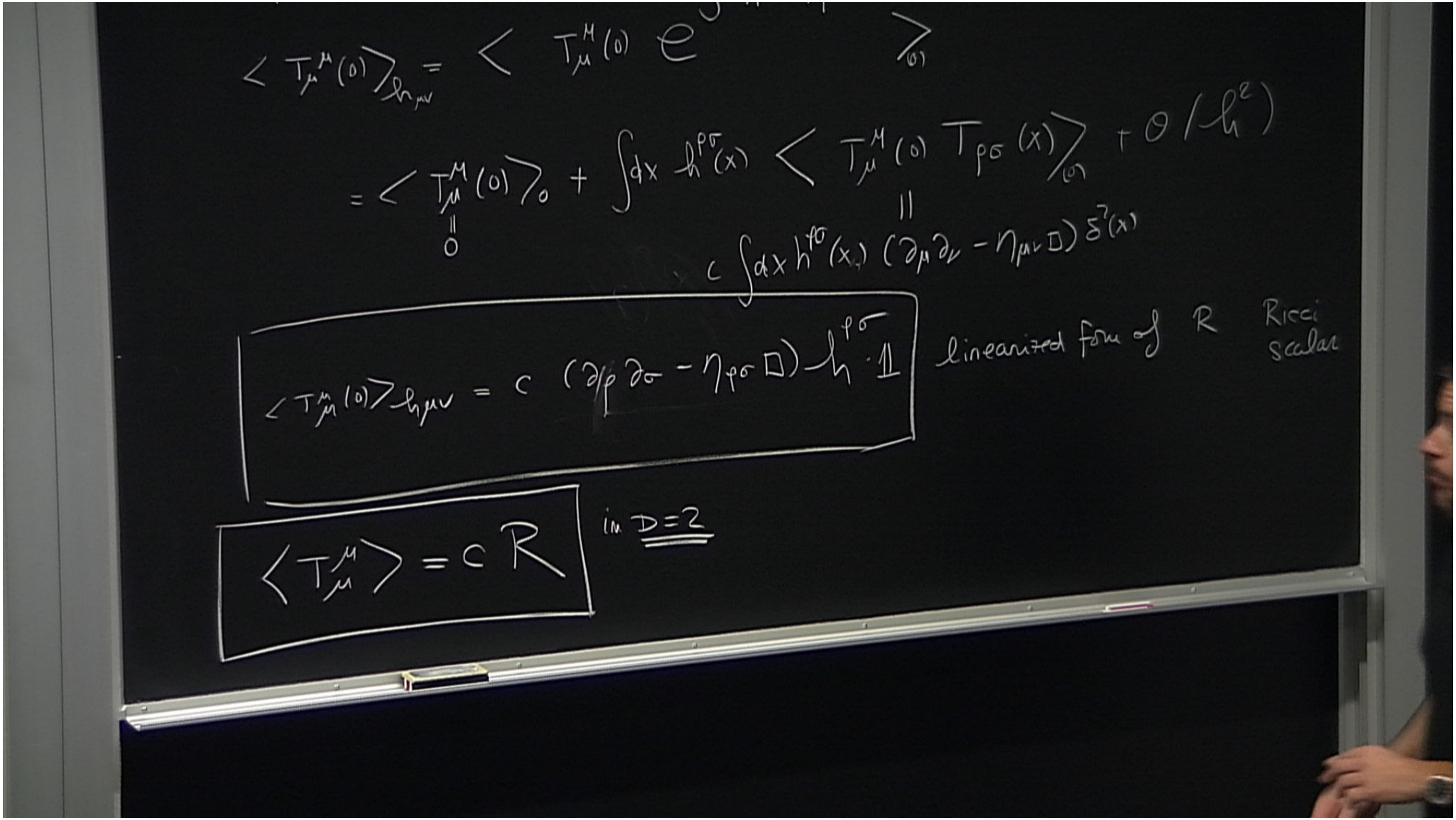
$$\langle T^{\mu}_{\mu}(0) \rangle_{h_{\mu\nu}} = \left\langle T^{\mu}_{\mu}(0)\, e^{\int h^{\rho\sigma} T_{\rho\sigma}\, dx} \right\rangle_{0}$$

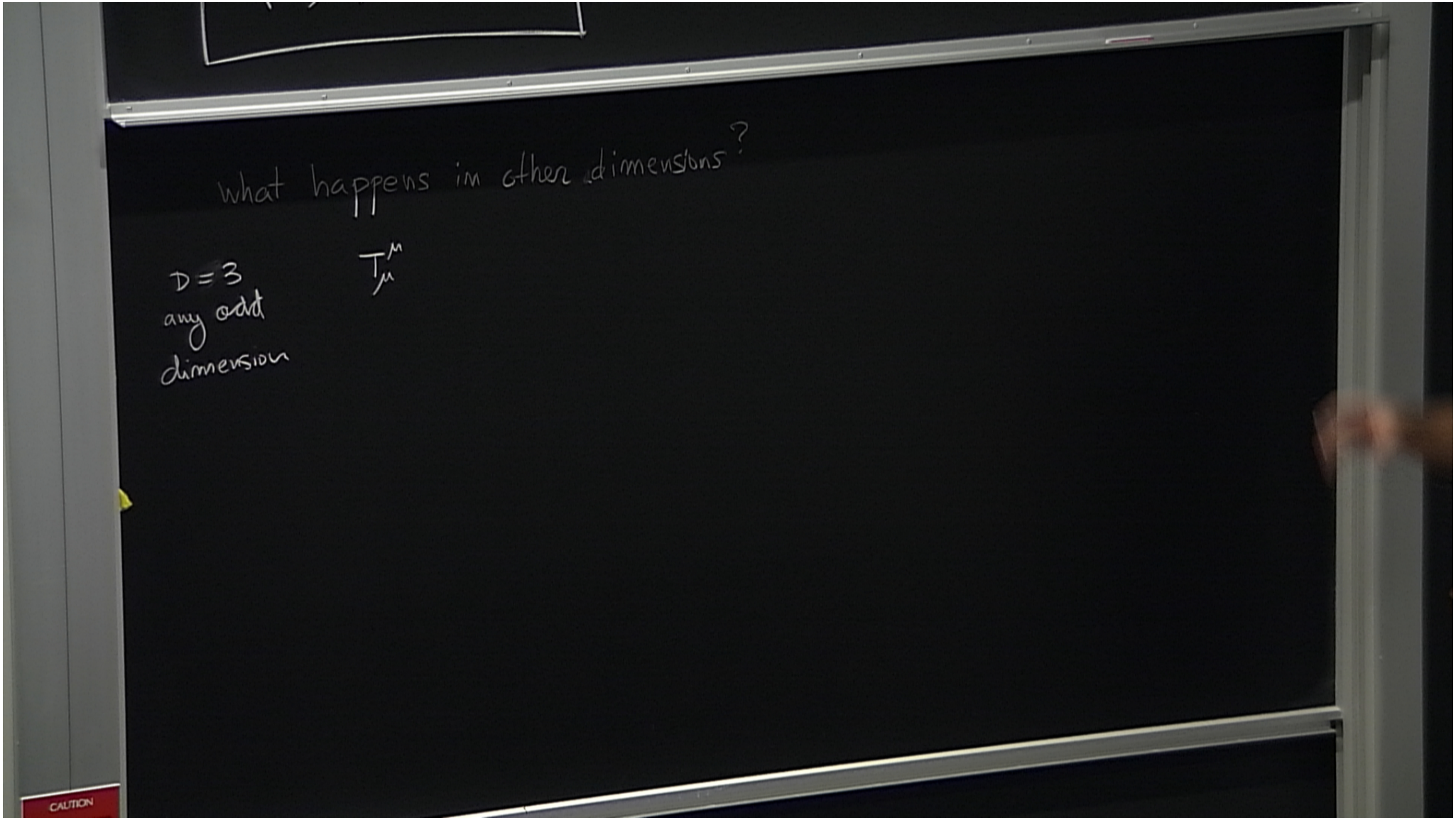
$$= \langle \underbrace{T^{\mu}_{\mu}(0)}_{\| \; 0} \rangle_0 + \int dx\, h^{\rho\sigma}(x) \left\langle T^{\mu}_{\mu}(0)\, T_{\rho\sigma}(x) \right\rangle_{0} + O(h^2)$$

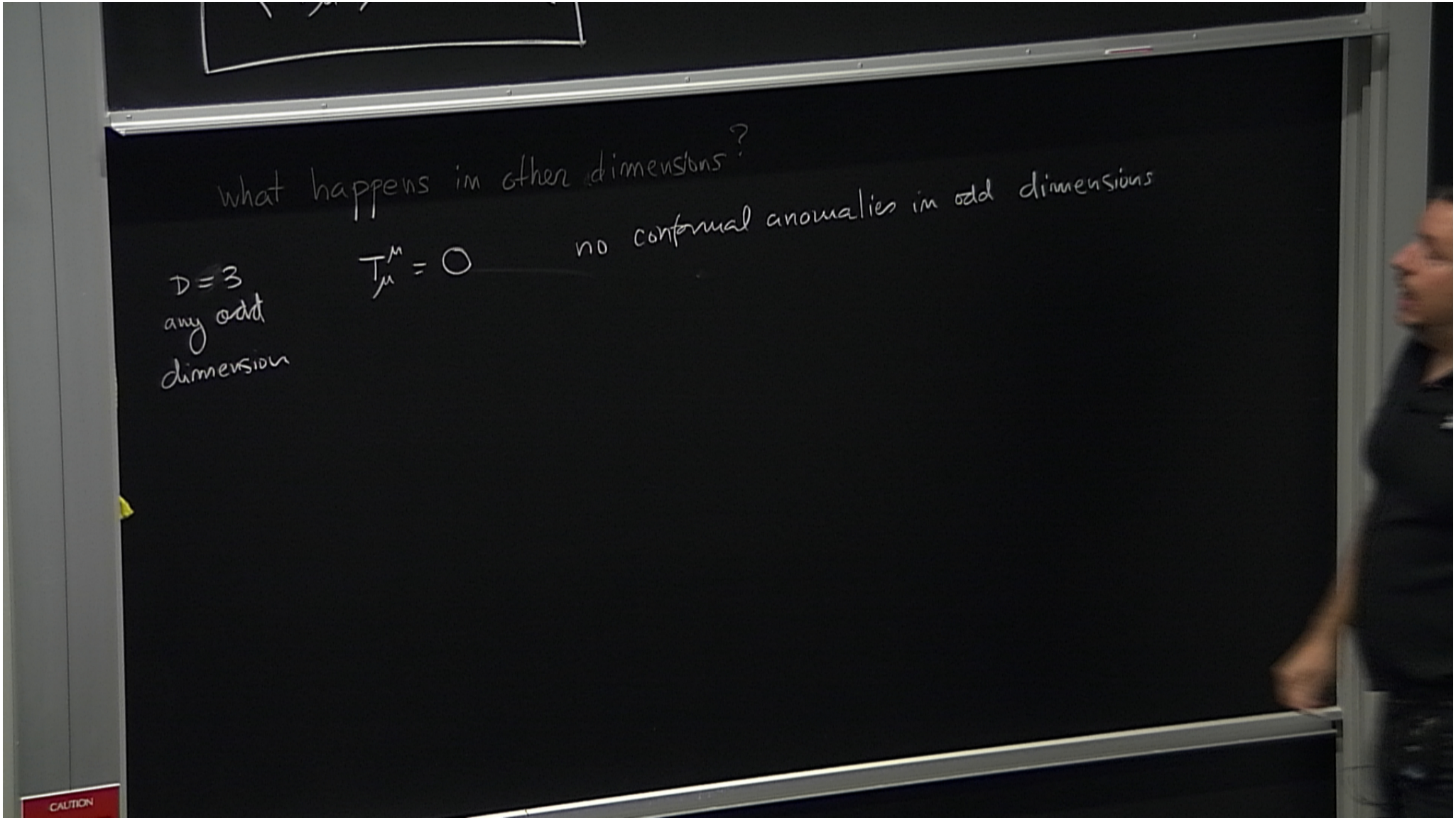

Couple CFT to a background metric
$\langle T^{\mu}_{\mu}(0) \rangle_{h_{\mu\nu}} = \langle T^{\mu}_{\mu}(0)\, e^{\int h^{\rho\sigma} T_{\rho\sigma}\, dx} \rangle_{(0)}$
$= \langle T^{\mu}_{\mu}(0) \rangle_0 + \int dx\, h^{\rho\sigma}(x) \langle T^{\mu}_{\mu}(0)\, T_{\rho\sigma}(x) \rangle_{(0)} + O(h^2)$
$T^{\mu}_{\mu} = 0$
$= c \int dx\, h^{\rho\sigma}(x) (\partial_\mu \partial_\nu - \eta_{\mu\nu} \Box) \delta^2(x)$


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$\langle T^\mu_\mu(0) \rangle_{h_{\mu\nu}} = c\, (\partial_\rho \partial_\sigma - \eta_{\rho\sigma} \Box) h^{\rho\sigma} \cdot \mathbb{1}$


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$\langle T^\mu_\mu(0) \rangle_{h_{\mu\nu}} = c\, (\partial_\rho \partial_\sigma - \eta_{\rho\sigma} \Box) h^{\rho\sigma}$
linearized form of R Ricci scalar
$\langle T^\mu_\mu \rangle = c\, R$


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linearized form of R Ricci scalar
$\langle T^\mu_\mu \rangle = c\, R$
in $D=2$


What happens in other dimensions?
D = 3
any odd
dimension
T^{μ}_{μ}
CAUTION


What happens in other dimensions?
D = 3
any odd
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$T^{\mu}_{\mu} = 0$
no conformal anomalies in odd dimensions
CAUTION

$\langle T^{\mu}_{\mu} \rangle = \boxed{c} R$ in $D=2$

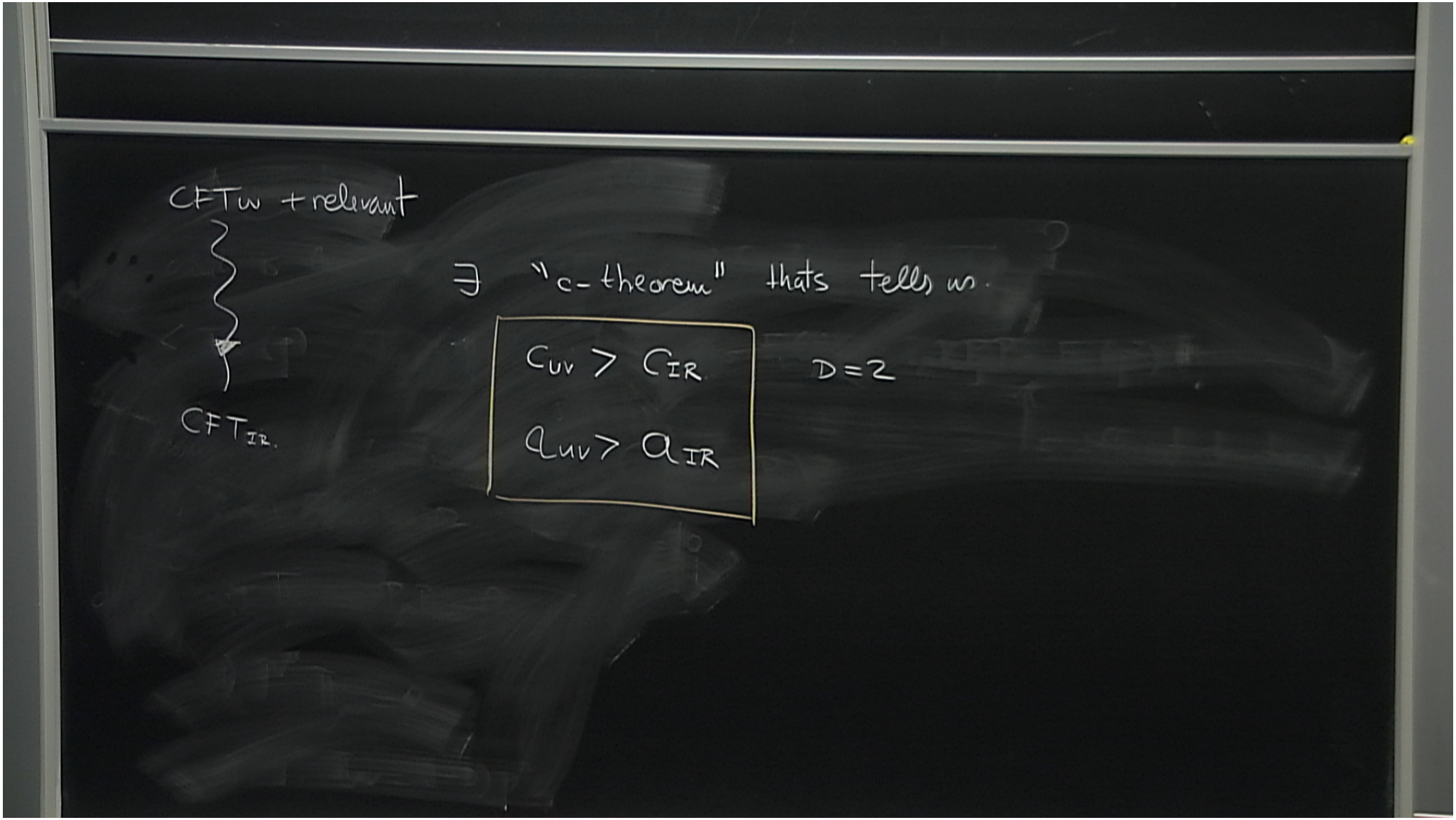
What happens in other dimensions:

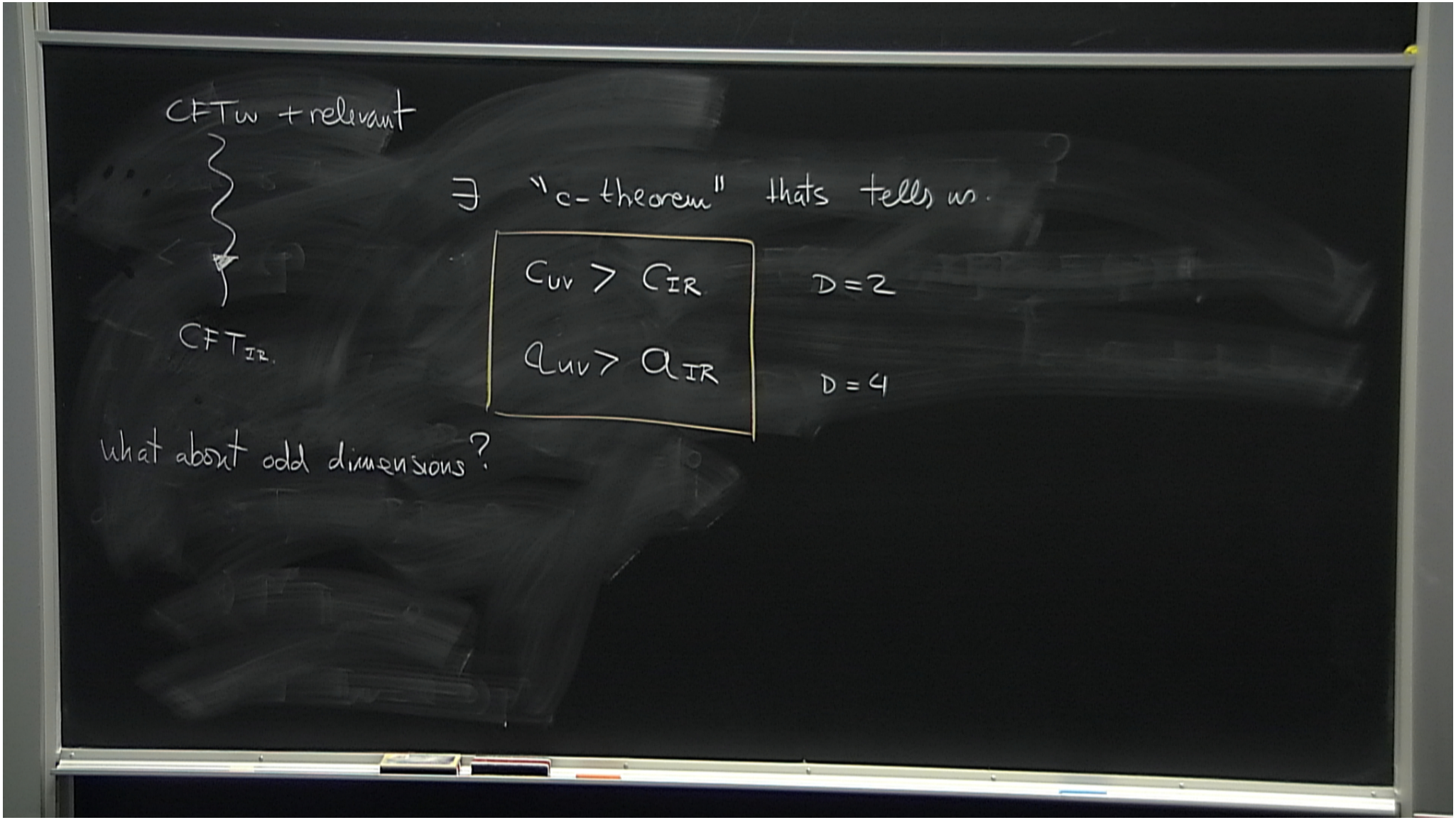
$D=3$ any odd dimension: $T^{\mu}_{\mu} = 0$ — no conformal anomalies in odd dimensions

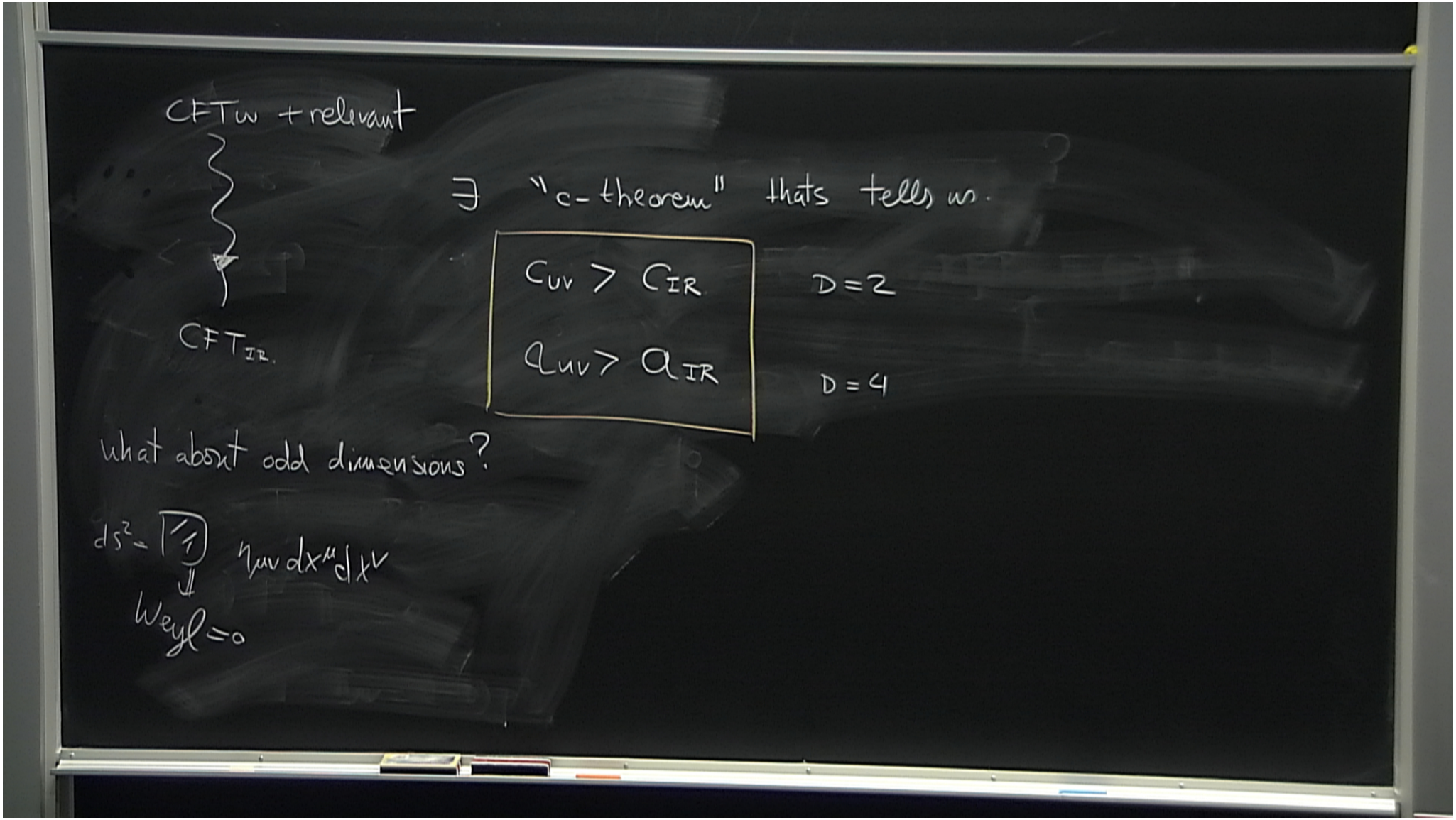
$D=4$: $T^{\mu}_{\mu} = \boxed{a}\,\text{Euler} + c\,\text{Weyl}^2$

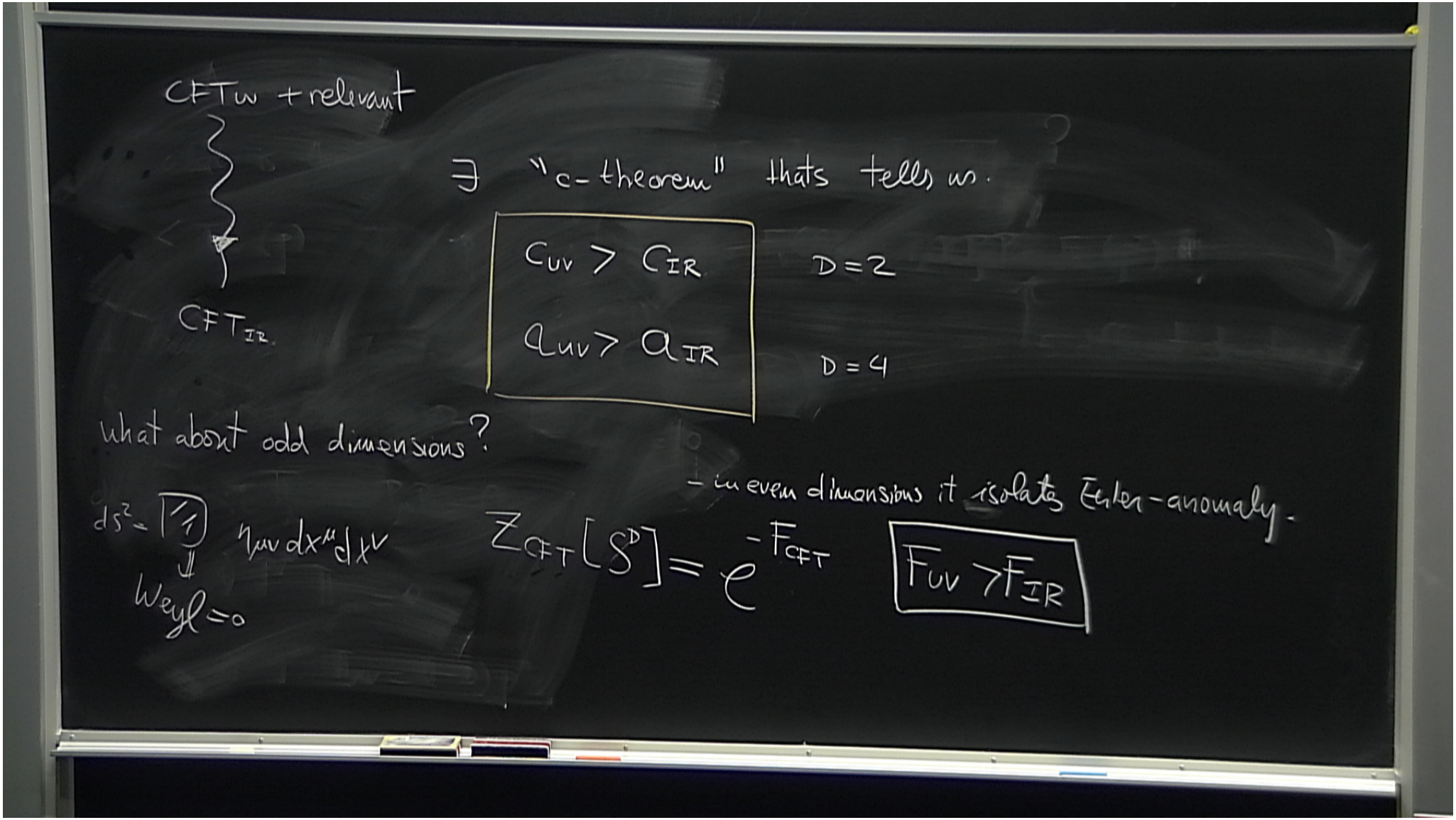
$\uparrow$

$R_{\mu\lambda\rho\sigma}^2 + R_{\mu\nu}^2 + R^2$


CFT_UV + relevant
CFT_IR.
∃ "c-theorem" thats tells us.
C_UV > C_IR
a_UV > a_IR
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What about odd dimensions?


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What about odd dimensions?
ds² = ?
η_μν dx^μ dx^ν
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∃ "c-theorem" thats tells us.
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what about odd dimensions?
- in even dimensions it isolates Euler-anomaly.
ds² = η_μν dx^μ dx^ν
Weyl=0
Z_CFT[S^D] = e^{-F_CFT}
F_UV > F_IR