Title: Projective symmetry of partons in the Kitaev honeycomb model

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Abstract: <p>Low-energy states of quantum spin liquids are thought to involve partons living in a gauge-field background. We study the spectrum of Majorana fermions of the Kitaev honeycomb model on spherical clusters. The gauge field endows the partons with half-integer orbital angular momenta. As a consequence, the multiplicities do not reflect the point-group symmetries of the cluster, but rather its projective symmetries, operations combining physical and gauge transformations. The projective symmetry group of the ground state is the double cover of the point group.</p>

Projective symmetry of partons in Kitaev's honeycomb model

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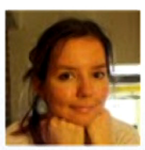

in collaboration with Olga Petrova and Oleg Tchernyshyov

Perimeter Institute, Waterloo, Feb 2018

Context

What we do

Study the spectrum of Majorana fermions of the Kitaev honeycomb model on spherical clusters.

Why

1. To examine the properties of partons in the Kitaev honeycomb spin model.
2. To find clean applications of projective symmetry to exactly solvable models of spin liquids.

What we found

- Parton excitations in the Kitaev honeycomb model on spherical clusters have half-integer orbital angular momenta due to a gauge background resembling the field of a magnetic monopole with a half-integer charge.
- For spherical clusters the projective symmetry group for the ground state is the double cover of the point group.

Quantum Spin Liquids

Conjectured states of matter that have no long-range magnetic order and thus cannot be distinguished by their physical symmetries.

Their low-energy physics is often described in terms of partons — matter particles with fractional quantum numbers — interacting with emergent gauge fields.

Solvable models in more than one spatial dimension are hard to find.

Motivation

Spin variables in terms of Abrikosov fermions or Schwinger bosons. Hamiltonian, quartic in parton fields, treated at the mean-field level.

$$\vec{S} = \frac{1}{2} a_\alpha^\dagger \vec{\sigma}_{\alpha\beta} a_\beta$$

$$\alpha, \beta = \uparrow, \downarrow$$

Spin Exchange

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$$\alpha, \beta = \uparrow, \downarrow$$

Spin Exchange

$$\vec{S}_i \cdot \vec{S}_j \approx -\underbrace{\frac{1}{2} \langle a^{\dagger}_{i\alpha} a_{j\alpha} \rangle}_{t_{ij}} a^{\dagger}_{j\beta} a_{i\beta} + \dots$$

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Spin Exchange

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$$H_{\rm MFT} = \sum_{i,j} t a^\dagger_{i\alpha} a_{j\alpha} + \Delta(a^\dagger_{i\uparrow} a^\dagger_{j\downarrow} + a^\dagger_{j\uparrow} a^\dagger_{i\downarrow}) + \ldots$$

Justified when $N \to \infty$

N: parton flavors

Projective symmetry group, **PSG**

Two spin liquids (**SL**) cannot be distinguished by their physical symmetries

Classify **SL** on the basis of Projective Symmetry

Wen 2002, PhysRevB.65.165113

PSG: A combination of physical and gauge symmetries.

association of projective symmetry with *ad hoc* fractionalization schemes

The **PSG** does not need to be tied to to a mean field theory

Landau levels on a sphere.

Massive particle as a rigid rotor—a particle pivoted on a massless rod of length r —with mutually orthogonal axes ξ , η, $\zeta = \mathbf{r}/r$

L_ξ L_η and L_ζ commute with L_x L_y and L_z, so we may use as basis vectors the simultaneous eigenstates of L^2 L_z and L_ζ

$$H = \frac{L_\xi^2}{2I_\xi} + \frac{L_\eta^2}{2I_\eta} + \frac{L_\zeta^2}{2I_\zeta}$$

$$I_\xi = I_\eta = mr^2$$

$$I_\zeta = 0 \to L_\zeta = 0 \to H = \frac{L^2}{2mr^2}$$

Particle in a uniform magnetic field

Charged particle moving on a sphere with a magnetic monopole **g** at the center.

$$\vec{B}(\vec{r}) = \frac{g\vec{r}}{r^3}$$

H is modified by replacing $L = r \times p \to \Lambda = r \times (p - A)$

$$H = \frac{(\vec{L} - \vec{r} \times \vec{A})^2}{2mr^2}$$

Although the magnetic field is spherically symmetric, <u>the vector potential is not.</u>

Landau levels on a Sphere

Performing a rotation will change **A** !!

Gauge away the change induced in **A**: follow it up with a gauge transformation.

The combined operation—a gauged rotation—leaves the vector potential, and H invariant.

Generator of gauged rotations $$\vec{J} = \vec{L} - \vec{r} \times \vec{A} - \frac{g\vec{r}}{r}$$

$$J = \Lambda - g\zeta$$

It satisfies the standard algebra of angular momentum

$$J_\zeta = L_\zeta - g = -g$$

This constraint restricts g to integer and half-integer values

Gauged angular momentum

gauged angular momentum $\quad j = |g|, |g|+1, |g|+2, \ldots,$

$\left(\text{ordinary orbital angular momentum} \quad l = 0, 1, 2, \ldots, \right)$

$$H = \frac{(\Lambda_\xi^2 + \Lambda_\eta^2)}{2mr^2} \qquad H = \frac{(|\vec{J}|^2 - g^2)}{2mr^2}$$

The **PSG** does not need to be tied to to a mean field theory.

Gauged angular momentum

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(ordinary orbital angular momentum $l = 0, 1, 2, \ldots,$)

The resulting group of gauged rotations is a **PSG**

$$H = \frac{(\Lambda_\xi^2 + \Lambda_\eta^2)}{2mr^2}$$

$$H = \frac{(|\vec{J}|^2 - g^2)}{2mr^2}$$

The **PSG** does not need to be tied to to a mean field theory.

What we do about it?

We construct the **PSG** for a solvable model of a **SL** on a honeycomb lattice introduced by Kitaev.

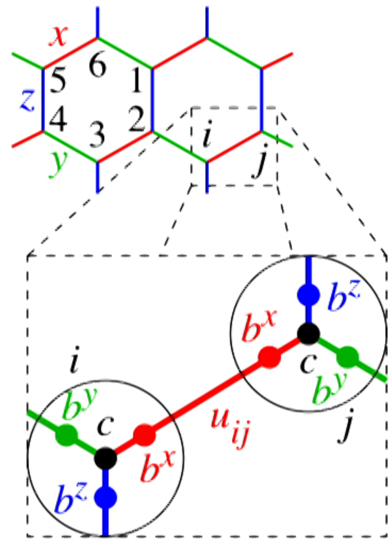
Partons in this model are **Majorana** fermions moving in a background of a $\mathbf{Z_2}$ gauge field.

To find that

The spectrum of Majorana fermions on symmetric clusters make sense in the light of gauged symmetry.

The Kitaev's model *Spins 1/2 on sites of a Honeycomb lattice*

$$H = -J_x \sum_{x\ \mathrm{links}} \sigma^x_m \sigma^x_n - J_y \sum_{y\ \mathrm{links}} \sigma^y_m \sigma^y_n - J_z \sum_{z\ \mathrm{links}} \sigma^z_m \sigma^z_n$$



Spins ➠ 4 Majorana fermions

$$\sigma_1^{\alpha_{12}} \sigma_2^{\alpha_{12}} = (i b_1^{\alpha_{12}} c_1)(i b_2^{\alpha_{12}} c_2) = -i u_{12}\, c_1 c_2$$

$$u_{mn} = -u_{nm} = i b^\alpha_m b^\alpha_n = \pm 1$$ Z_2 gauge variable

$$H = -\frac{1}{2} \sum_m \sum_n t_{mn} c_m c_n \qquad t_{mn} = -2iJ_{mn} u_{mn}$$

Hopping matrix
hermitian

Z_2 and U(1) fluxes

$$W = (-i)^n u_{12} u_{23} \ldots u_{n1}$$

Z_2 magnetic flux

Different gauge representations {u} of the same flux pattern {W} are related by a gauge transformation

$$u'_{mn} = \Lambda_m u_{mn} \Lambda_n,$$

$$c'_n = \Lambda_n c_n,$$

$$\Lambda_n = \pm 1$$

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$$\Lambda_n = \pm 1$$

$$W = e^{i\Phi}$$

U(1) gauge flux

Flux pattern in the GS

$$\Phi = \pm\pi/2$$ For any loop of perimeter L odd

$$\Phi = 0$$ L = 2 mod 4 (ex: hexagon)

$$\Phi = \pi$$ L = 0 mod 4 (ex:octagon)

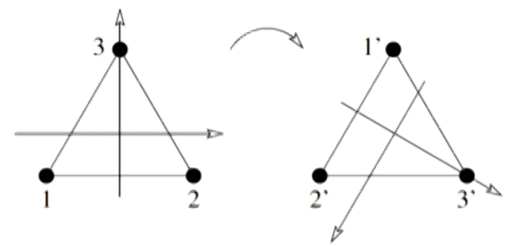
Petrova, Mellado, Tchernyshyov. PhysRevB.90.134404 (2014)

Complex fermions on a triangle

$$H = -\sum_{m=1}^{3} \sum_{n \neq m} t_{mn} \chi_m^\dagger \chi_n.$$

$$t = \begin{pmatrix} 0 & t_{12} & t_{13} \\ t_{21} & 0 & t_{23} \\ t_{31} & t_{32} & 0 \end{pmatrix} \qquad t = \begin{pmatrix} 0 & -1 & -1 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \end{pmatrix}.$$

Symmetry transformations



$$R = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \quad \mapsto t' = R\,t\,R^\dagger = \begin{pmatrix} 0 & t_{31} & t_{32} \\ t_{13} & 0 & t_{12} \\ t_{23} & t_{21} & 0 \end{pmatrix}$$

If H remains invariant after transformation R , R is a symmetry of H. All symmetries of H form a group.

$$R_+ = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},\ R_- = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},\ \sigma_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},\ \sigma_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix},\ \sigma_3 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

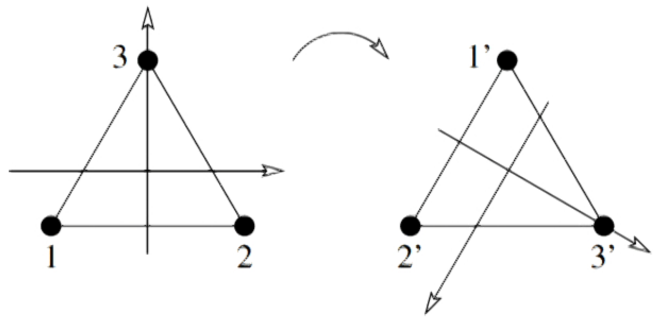
t invariant under +/-120 rotations and reflections exchanging two sites of the triangle. E, two rotations, three reflections: symmetry group of the equilateral triangle.

The symmetry group of t

3- dimensional (reducible) representation of the triangle group.

irrep	E	$2R$	3σ
1	1	1	1
$1'$	1	1	-1
2	2	-1	0

3 conjugacy classes



3 irreducible representations: $1^2+1^2+2^2 = 6$: group order.

- The fermion operators transform in terms of each other under these symmetries.

- the reducible representation is a sum of irreps 1 and 2. We expect the energy levels to have degeneracies 1 and 2.

- Diagonalizing t with all off diagonal elements equal to 1 yields eigenvalues -2, 1 and 1

Gauge transformations

$c_1 \longrightarrow - c_1$

$$t = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$$

The energy levels remain the same but the symmetry of the Hamiltonian is now lower $\sigma_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ $E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$R_+ = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Symmetry of H restored if we follow the symmetry operations with gauge transformation $c_3 \longrightarrow -c_3$

$$\Lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

alters the signs of hopping on bonds 13 and 23.

$$\Lambda_3 R_+ t R_+^\dagger \Lambda_3^\dagger = t \qquad \Lambda_{12} R_+ t R_+^\dagger \Lambda_{12}^\dagger = t$$

And similarly with all the other transformations…

any combined symmetry+gauge operation has a twin obtained by acting with the global gauge transformation, which alters the signs of all fermions:

$c_m \longrightarrow - c_m$

$$G = \{e, \Lambda_{123}, \Lambda_3 R_+, \Lambda_{12} R_+, \Lambda_2 R_-, \Lambda_{13} R_-, \sigma_1, \Lambda_{123}\sigma_1, \Lambda_2\sigma_2, \Lambda_{13}\sigma_2, \Lambda_3\sigma_3, \Lambda_{12}\sigma_3\}$$

c Majorana fermions in a static magnetic background

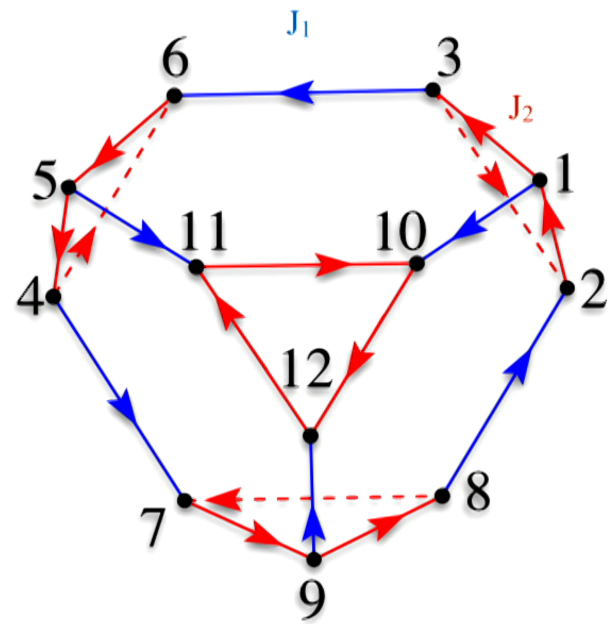
$$\gamma_k = \sum_n \psi_n^{(k)} c_n \qquad -\sum_n t_{mn}\psi_n = \epsilon\psi_m$$

Eigenvalues of t_{mn} come in pairs $\pm\epsilon$

Positive eigenvalues are the excitation energies of the Majorana eigenmodes

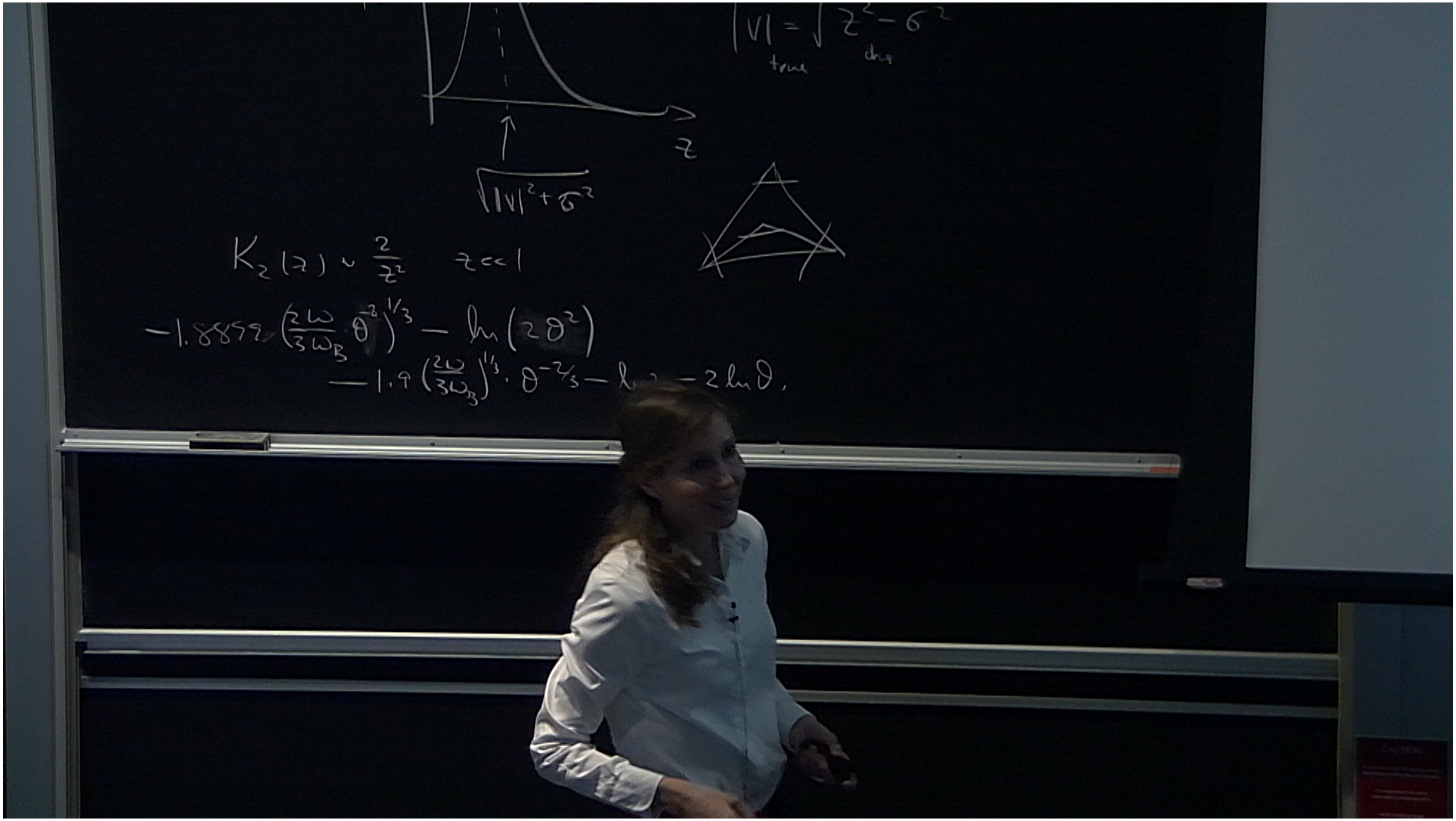
Archimedean Solids

Links in two flavors



- $J_1 > 0$ on edges inherited from Platonic solids
- And $J_2 > J_1 > 0$ on the edges resulting from truncation.

The presence of a gauge field endows edges with a sense of direction and thereby reduces the symmetry.



|V| = √(z² − σ²)
true
z
√(|V|² + σ²)
K₂(z) ~ 2/z²   z ≪ 1

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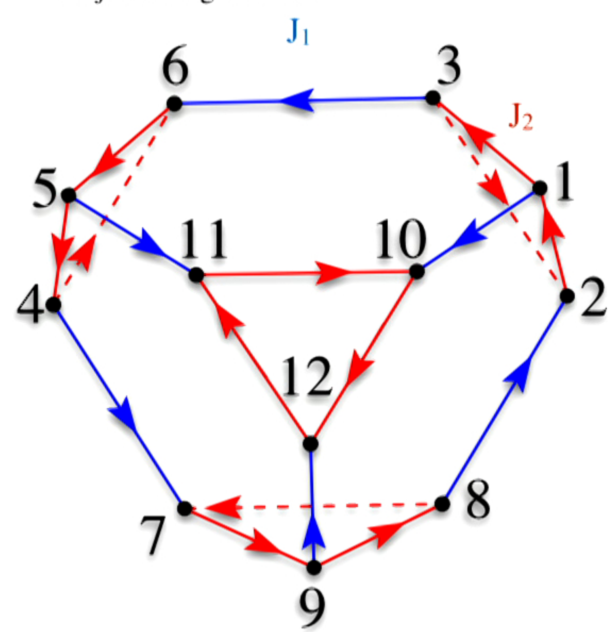

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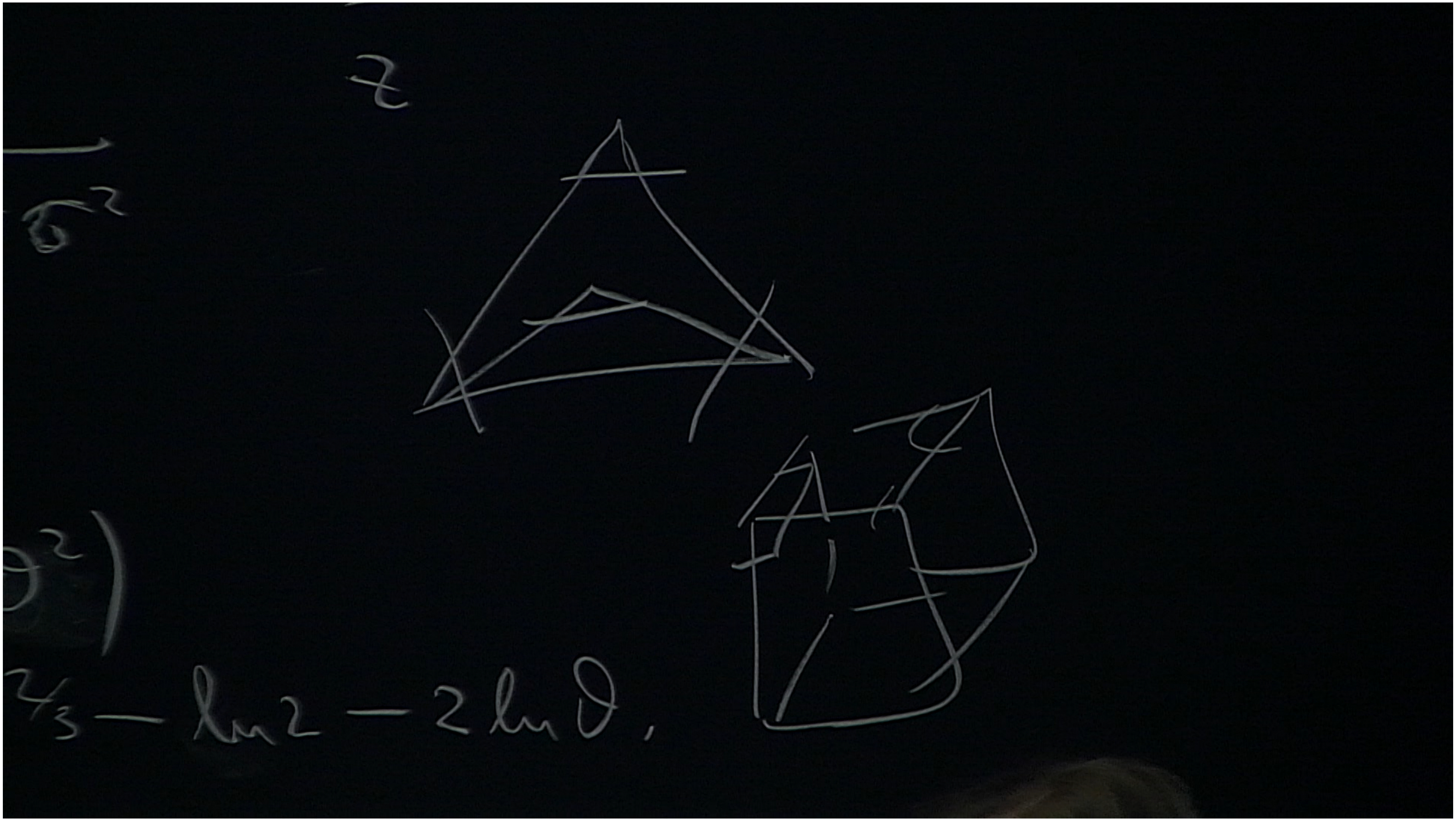
z
2/3 − ln 2 − 2 ln ϑ,

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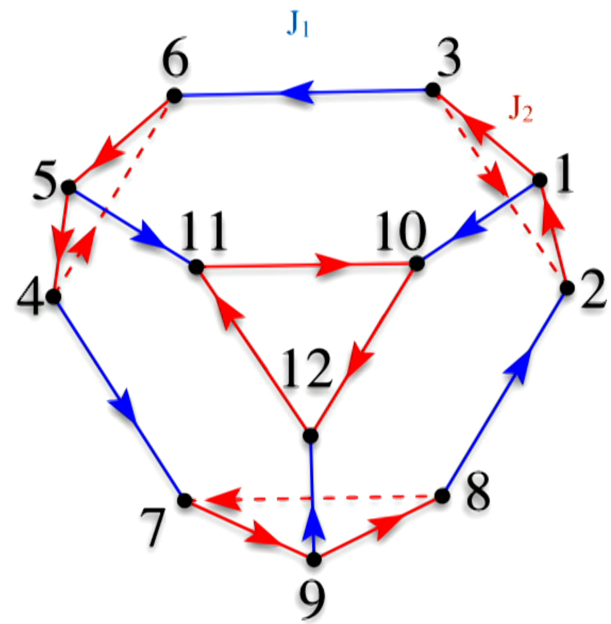

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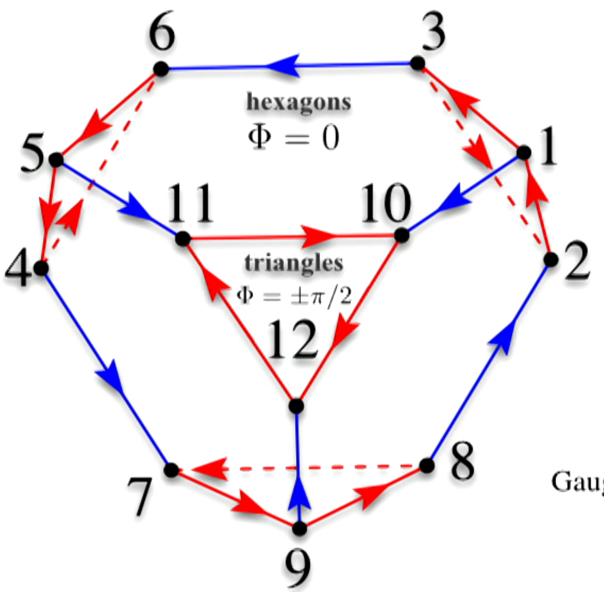
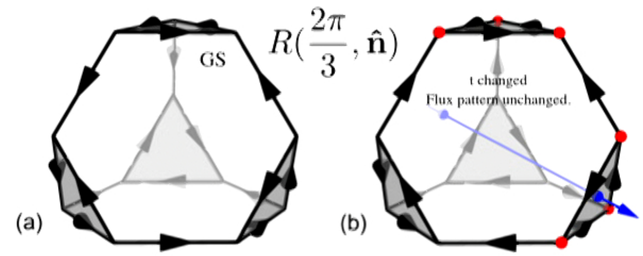
PSG of Majoranas on Archimedean Solids

2 **GS**





restore t by a Z_2 gauge transformation on vertices labeled with dots.

$\Lambda(\frac{2\pi}{3}, \hat{\mathbf{n}})$ Z_2 Gauge transformation

Gauge rotation $\mathcal{R}(\frac{2\pi}{3}, \hat{n}) = \Lambda(\frac{2\pi}{3}, \hat{n}) R(\frac{2\pi}{3}, \hat{n})$ leaves t invariant

and the complementary $-\Lambda(\frac{2\pi}{3}, \hat{n}) R(\frac{2\pi}{3}, \hat{n})$ too

Spectrum Majorana modes form 3 doublets

$\sqrt{2(7-\sqrt{33})}\ (2),\ 2\sqrt{2}\ (2),\ \sqrt{2(7+\sqrt{33})}\ (2)$

Every point-group symmetry generates two gauged symmetries

T has irreps 1 1\` 1\`\` 3

$$\mathcal{R}(2\pi, \hat{n}) = \mathcal{R}^3(\frac{2\pi}{3}, \hat{n}) = -1 \equiv \Lambda(\frac{2\pi}{3}, \hat{n}) R(\frac{2\pi}{3}, \hat{n})$$

Regular v/s gauged rotations

regular rotations

$$R^3(\frac{2\pi}{3}) = R(2\pi) = 1$$

gauged rotations

$$\mathcal{R}^3(\frac{2\pi}{3}) = -1$$

includes irreps with double dimensions

$$\mathbf{T} \subset \mathbf{SO(3)} \quad \Longrightarrow \quad \tilde{\mathbf{T}} \subset \mathbf{SU(2)}$$

$\mathbf{Z_2}$ symmetry broken by **u**

Use the *irreps* for which a rotation through 2π yields a factor of -1. The double cover of T has three such *irreps*: 2 , 2`, and 2``

PSG of the GS of Majorana fermions on the tetrahedron: $\tilde{T}$

generalization of T to half-integer spins

Blame the gauge field felt by Majorana fermions.

$$\Phi_{\text{net}} = \pm 2\pi \qquad \Phi = 4\pi g \qquad g = \pm\frac{1}{2}$$

g : charge of a magnetic monopole at the cluster's center.

1. L of a parton with unit electric charge incremented by the angular momentum of the electromagnetic field g.

2. g is half integer in the GS of Archimedean solids.

3. The net angular momentum is converted from integer to half-integer. Point group must be enlarged.

Similar scenarios apply to other spherical clusters

PSG of the ground state

$\tilde{\mathbf{G}} \subset \mathbf{SU(2)}$

PSG construction for the ground-state flux sector

1. Construct two **gauged** rotations $\mathcal{R}(\frac{2\pi}{3}, \hat{\mathbf{n}}_1)$ and $\mathcal{R}(\frac{2\pi}{3}, \hat{\mathbf{n}}_2)$

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2. Use multiplication table of **SU(2)** (of its subgroup $\widetilde{\mathrm{T}}$) to generate new elements and label them accordingly:

$$\mathcal{R}(\frac{2\pi}{3}, \hat{\mathbf{n}}_2)\mathcal{R}^{-1}(\frac{2\pi}{3}, \hat{\mathbf{n}}_1) = \mathcal{R}(\pi, \hat{\mathbf{n}}_3)$$

Perimeter Institute, Waterloo, Feb 2018

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3. Check that each new element is a gauge transformation: a composition of

$$R(\phi, \mathbf{n}) \in T \quad \text{and} \quad \Lambda(\phi, \mathbf{n})$$

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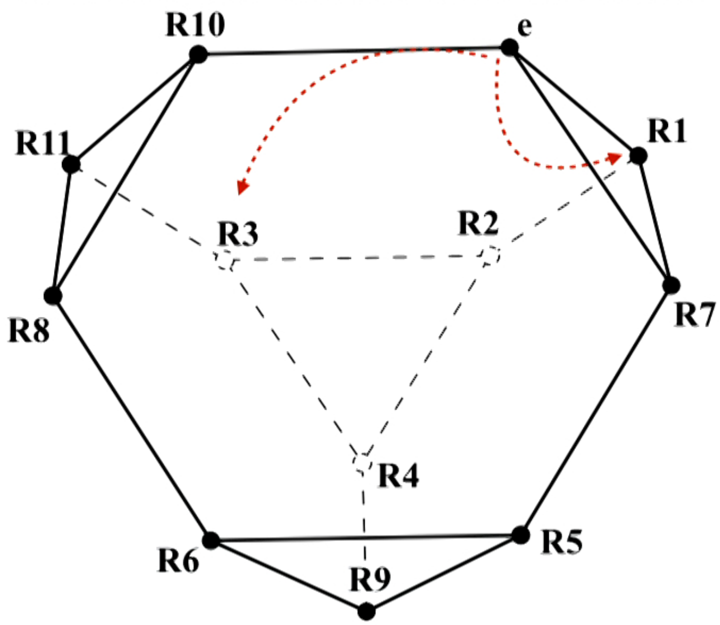
$$R(\phi, \mathbf{n}) \in T \quad \text{and} \quad \Lambda(\phi, \mathbf{n})$$

4. Check that the multiplication tables of the new group and $\widetilde{\mathrm{T}}$ are the same.

Multiplets of fermions



Number of vertices = Order of **G⊂SO(3)**

States of a fermion living on the vertices transform under the regular representation of the group **G**

$$R_2|R_1\rangle = |R_2R_1\rangle.$$

Regular representation of **G = T**

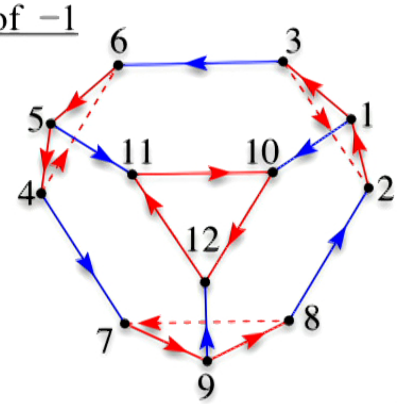
$$12 = 1\times 1 + 1\times 1' + 1\times 1'' + 3\times 3$$

Complex fermions transform under the regular representation of the point group **G**

Majorana multiplets

Use the *irreps* for which a rotation through 2π yields a factor of -1

$$\mathbf{12} = 2 \times \mathbf{2} + 2 \times \mathbf{2}' + 2 \times \mathbf{2}''$$ regular representation



Spectrum for Majorana

$$\sqrt{2(7-\sqrt{33})}\ (2),\ 2\sqrt{2}\ (2),\ \sqrt{2(7+\sqrt{33})}\ (2)$$

Summary of **PSG**

Solid	Multiplicities	Φ	g	PSG
Truncated tetrahedron	2, 2, 2	2π	1/2	$\tilde{T}$
Truncated octahedron	4, 4, 4	6π	3/2	$\tilde{O}$
Truncated cube	4, 2, 4, 2	2π	1/2	$\tilde{O}$
Truncated icosahedron	6, 2, 4, 6, 2, 6, 4	6π	3/2	$\tilde{I}$

In agreement with exact diagonalization

Majorana multiplets

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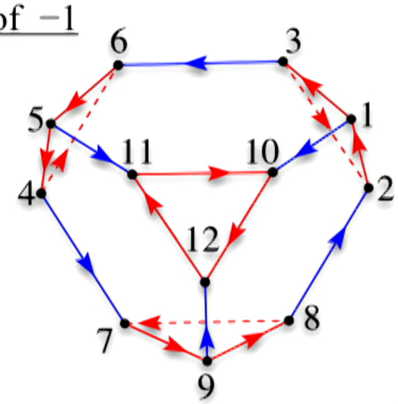

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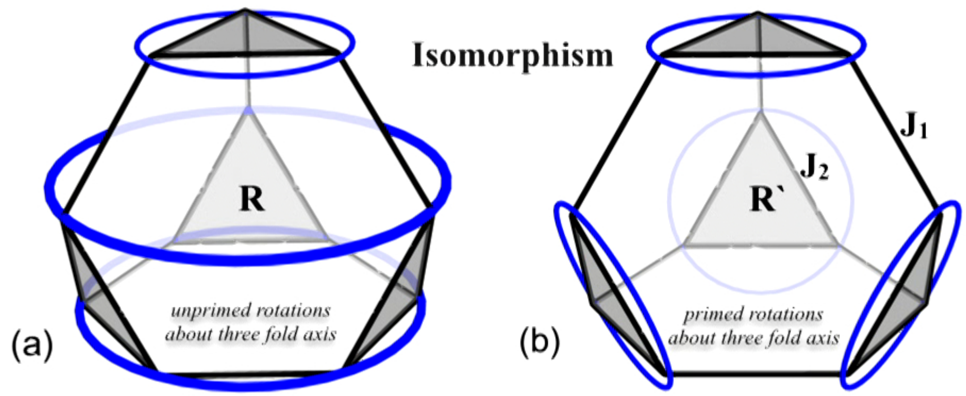
Truncated cube	4, 2, 4, 2	2π	1/2	$\tilde{O}$
Truncated icosahedron	6, 2, 4, 6, 2, 6, 4	6π	3/2	$\tilde{I}$

In agreement with exact diagonalization

Parton spectrum

R` (follows right multiplication) analogs of rotations about axes attached to a rigid body



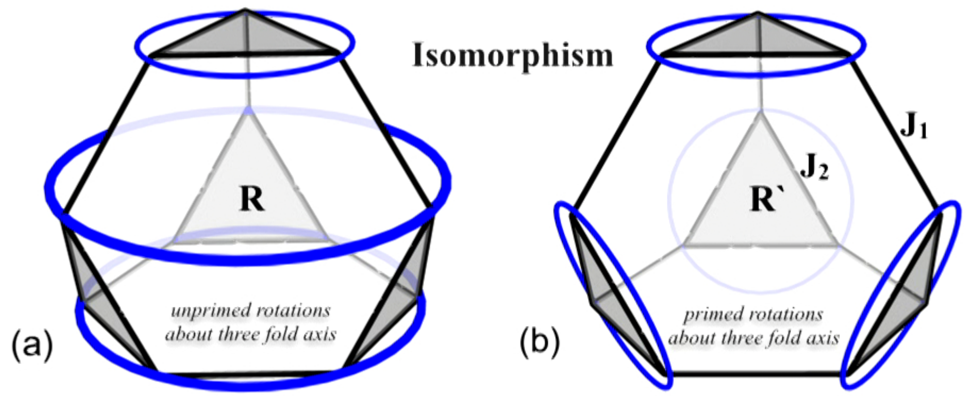
unprimed **(a)** and primed **(b)** rotations commute $R_3 R_2' |R_1\rangle = |R_3 R_1 R_2\rangle = R_2' R_3 |R_1\rangle$

t commutes with unprimed rotations

Primed rotations form $\tilde{T}$

Parton spectrum

R` (follows right multiplication) analogs of rotations about axes attached to a rigid body



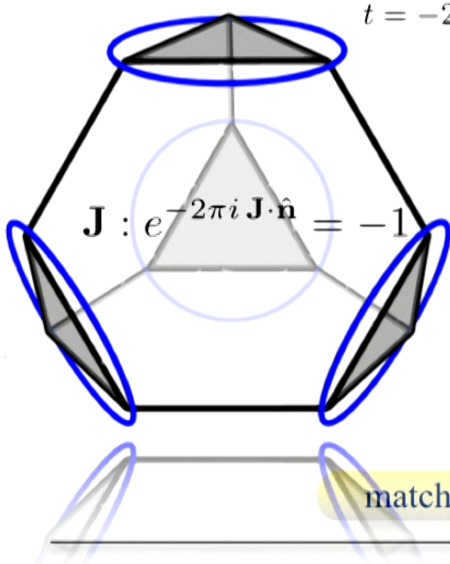
unprimed **(a)** and primed **(b)** rotations commute $R_3 R'_2 |R_1\rangle = |R_3 R_1 R_2\rangle = R'_2 R_3 |R_1\rangle$

t commutes with unprimed rotations

$$t = -2i[J_1 \mathcal{R}'(\pi, \hat{\mathbf{n}}_1) - J_2 \mathcal{R}'(\tfrac{2\pi}{3}, \hat{\mathbf{n}}_2) + J_2 \mathcal{R}'(-\tfrac{2\pi}{3}, \hat{\mathbf{n}}_2)]$$

t superposition of primed rotations Primed rotations form $\tilde{T}$

Use *irreps* of R′ to block diagonalize t



$t = -2i[J_1\mathcal{R}'(\pi, \hat{\mathbf{n}}_1) - J_2\mathcal{R}'(\frac{2\pi}{3}, \hat{\mathbf{n}}_2) + J_2\mathcal{R}'(-\frac{2\pi}{3}, \hat{\mathbf{n}}_2)]$

Replace block of *irrep* λ by SU(2) rotation matrices

$$\mathcal{R}'(\phi, \hat{\mathbf{n}}) \to \mathcal{D}^{(\lambda)}(-\phi, \hat{\mathbf{n}}) = e^{i(\sigma\cdot\hat{n})\phi/2}$$

1. *irrep* **2** of $\tilde{T}$ (spin-1/2) block 2x2

$$t^{(\mathbf{2})} = -2J_1\sigma_z + 2J_2(\sigma_x + \sigma_y + \sigma_z)$$

$$\epsilon = 2\sqrt{J_1^2 - 2J_1J_2 + 3J_2^2}$$

matches the energy of one of the Majorana doublets

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2. *irrep* $\mathbf{2}' + \mathbf{2}''$ (spin-3/2) block 4x4

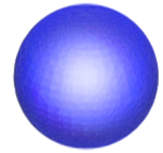
$$P(\epsilon) = \epsilon^4 - (3J_1^2 + 2J_1J_2 + 2J_2^2)\epsilon^2 + 16(J_1 + J_2)^2J_2^2$$

matches the energy of the other two Majorana doublets

Diagonalization procedure also works correctly for the ground states of the other spherical clusters

Connection to Haldane continuum model

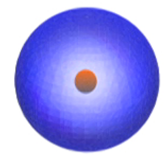
1) **t real and > 0** $\Phi = 0$



- **GS**: $\Psi_n = 1$ (lattice analog) s state.

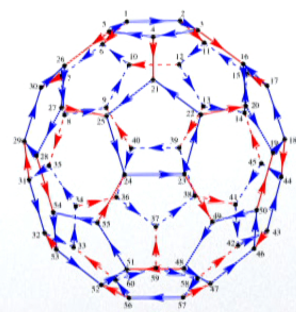
- **Excited states**: l = 1, 2, 3,... **Multiplicities**: 2l+1

2) $\Phi = 4\pi g$



Energy eigenstates: j = |g|, |g|+ 1, |g|+ 2,... **Multiplicities**: 2j+1

The positive eigenvalues of the Majorana hopping matrix mirror the negative ones. The parton multiplet with the highest energy will have angular momentum j = |g|, followed by multiplets with j = |g| + 1, |g| + 2, . . . until the continuum approximation breaks down.



Buckyball $g = \dfrac{3}{2}$

Highest energy partons form a quartet j = 3/2 and a sextet j = 5/2

Conclusions

- Parton excitations in Kitaev's honeycomb model on spherical clusters have half-integer orbital angular momenta due to a gauge background resembling the field of a magnetic monopole with a half-integer charge.

- Parton multiplets have even dimensions, incompatible with point-group symmetry. Their structure can be understood in the framework of projective symmetries.

- For spherical clusters, the PSG for the ground state is the double cover of the point group. First application of projective symmetries in a solvable model of a spin liquid.

Mellado, Petrova, Tchernyshyov, Phys. Rev. B 91, 041103(R)(2015)

Open question

A disclination is a line defect in which rotational symmetry is violated. It can be obtained by removing a pi/3 sector of a honeycomb lattice and gluing the resulting edges together. Because of the presence of odd-length cycles, the honeycomb lattice can no longer be globally partitioned into A and B sublattices.

In graphene, the boundary conditions across the cut have drastic consequences for the low energy wavefunctions: they switch the sub lattice and the valley.

Consider the case of the buckyball:

This is an example of a honeycomb net with 12 disclinations realized by 12 pentagons. When the coupling joining pentagons is made smaller than the exchange around pentagons, Majorana modes localize on them. These finding calls for further analysis.

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